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Towards Detection of Early Warning Signals on Financial Crises

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Contents

Abstract

The financial crises of 2001-2002 and 2008-2009 had a significant impact on the world economy. In this paper, we investigate whether early warning signals can be seen in financial time series preceding the crises. In our analysis, we use data on the Dow Jones Industrial Average and Federal Reserve Interest Rate. We construct a random process describing the occurrence of positive and negative signals in a time series preceding the financial crisis of 2001-2002. We use the constructed random process and a time series for the period 2001-2008 to assess the probability of a crisis to occur in 2008-2009. We show that the probability exhibits a steady growth and conclude that the proposed method demonstrates an ability to register early warning signals on the global financial crisis of 2008-2009.

Keywords: collapse analysis, early warning signals, financial crisis, data processing, binary model, encoding rules, stochastic processes

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Towards Detection of Early Warning Signals on Financial Crises

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1 Introduction

Financial crises have strong impacts on the economy, including labor markets, household incomes, and the profitability of companies. Substantial literature is devoted to understanding how financial crises develop. Papers [1]-[3], [8], [10]-[13] review background information on the latest Global Financial Crisis of 2008-2009 and the Dot-com boom of 2001-2002, and explore how these crises developed and how collapse of the Dow Jones market spread globally. In [6] the US housing market is simulated in order to measure a systematic impact of the ratchet effect, which caused the Global Financial Crisis. In [5], behavior of six leading US economic indicators is analyzed in the context of forecasting the onset and the end of recessions; the authors use a pattern recognition algorithm developed for predicting infrequent events. In [9] it is argued that catastrophic bubbles such as the Global Financial Crisis are caused by the formation of increasing and decreasing trends, and that smaller crises can provide statistical laws for bubble formation and financial collapse.

In this paper we view financial crises as extreme events. Compared to fluctuations in values of the indicators of a system's performance, extreme events are usually understood as qualitative shifts in the system's behavior. In this context, signals on the upcoming extreme events can be characterized in terms of tendencies rather than predictions on particular quantities. Roughly, one can group the tendencies in two categories – tendencies to a crisis (an extreme event of a negative character) and tendencies to avoiding a crises. Under that paradigm, early warning signals can be treated in a binary way – as either "minus" signals registering a tendency to a crisis, or "plus" signals registering a tendency to avoiding a crisis (see [7]).

Based on this binary approach, we develop a three-stage research pattern for identifying tendencies to crises in application to two recent financial crises – the Dot-com crisis of 2001-2002, and the latest global financial crisis of 2008-2009.

A first stage is *recognition*. Assessing an eight-year-long financial time series (on the Dow Jones Industrial Average and Federal Reserve Interest Rate) preceding the crisis of 2001-2002, we identify some "minus" and "plus" signals. We understand the "minus" signals as short (four-month-long) patterns in the time series, which occur, primarily, close to the time of the crisis, and the "plus" signals as those occurring, primarily, in earlier periods. We propose a *binary encoding rule* that transforms short data patterns into "minus" and "plus" signals.

A second stage is a *statistical analysis*. We use the binary encoding rule to transform a long (1954-2001) time series preceding the crisis of 2001-2002 into a sequence of "minus" and "plus" signals, and analyze the frequencies of a "minus" and a "plus" to follow each short *binary window* in the sequence (in our analysis each binary window is formed by three subsequent overlapping signals covering six months). We treat the frequencies as transition probabilities, which define a *binary random process* operating in the space of the binary windows. In our analysis the binary random process serves as a model describing the mechanism for the "plus" and "minus" signals to occur in the operation of the financial system under consideration. Two important features of the model are the following. Firstly, as ensured by the recognition analysis, the model recognizes early warning signals on the crisis of 2001-2002. Secondly, as ensured by the statistical analysis, the model captures the dynamics of signals occurring in a long historical time series.

A third stage is *testing the forecasting ability* of the model. We use the model to assess, retrospectively, the probability of a financial crisis to occur in October 2008 (the latest global financial crisis was registered in the period from October 2008 to February 2009). We show that the probability grows steadily starting from October 2007 and reaches value 1 in August 2008. Thus, our binary stochastic model based on analysis of data preceding the crisis of 2001-2002, demonstrates an ability to register early warning signals on the global financial crisis of 2008-2009.

In section 2 we provide a brief overview on the Dot-com crisis of 2001-2002 and the global financial crisis of 2008-2009.

In sections 3, 4 and 5 we present, respectively, the recognition stage, statistical analysis stage and testing stage of our research effort.

2 Recent financial crises

Here we provide a brief overview on the Dot-com crisis of 2001-2002 and the global financial crisis of 2008-2009.

2.1 The Dot-com crisis, 2001-2002

According to [3] and [17], in 2001 the stock markets, which followed the rapid upswing in technology stocks in the late 1990's, achieved a high point and a bubble was formed. Valuations of stocks did not correspond to real values. The bubble collapsed. Since the bubble was essentially powered by the rise of Internet sites and the technology industry in general, many of these companies went bankrupt, which gave the crisis the name the "Dot-com crisis". The Dow Jones Industrial Average (DJIA) fell down from 11497 points in December 1999 to 7581 points in September 2002, a drop of 34% in nominal terms.

The Dot-com crisis resulted from a rapid appreciation of technology stocks, which led to a steady growth in DJIA. Growth in DJIA was essentially motivated by a significant level of confidence in new and emerging technology businesses related to technology stocks, which created a high level of confidence and strength in the economy in the US, and its spread world wide.

To put some pressure on the economy expanding too quickly (in December 1999, DJIA reached an extreme height of around 11,500 points) and to constrain money supply, the Federal Reserve started to increase the Federal Reserve Interest Rate (FRIR) around

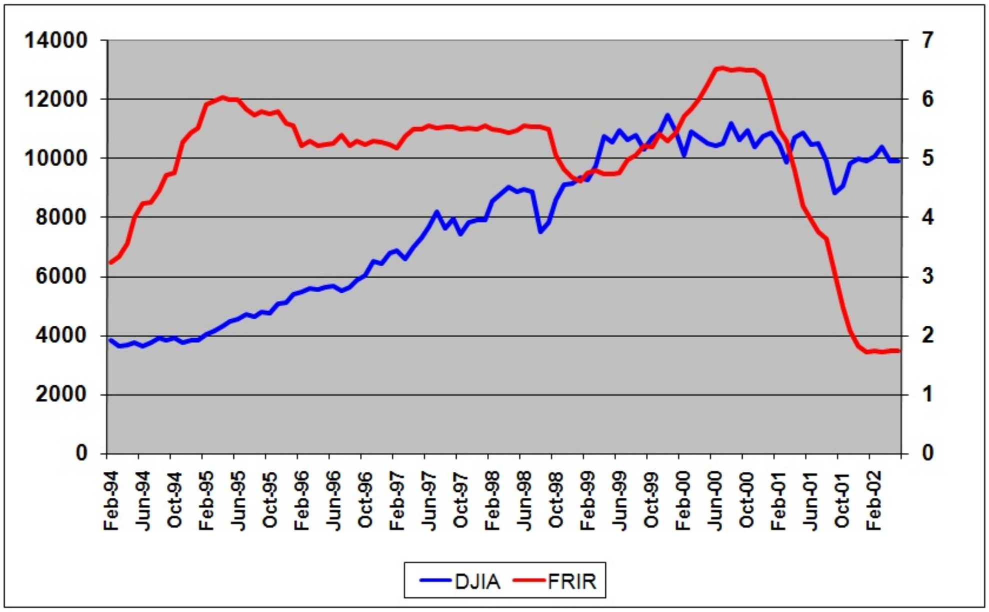

Figure 1: The Dow Jones Industrial Average (DJIA) and Federal Reserve Interest Rate (FRIR) in the period from February 1994 to May 2002 (monthly data, 100 months).

January 1999 (see the red curve in Figure 1). The upward pressure on interest rates reached a peak in June 2000, which was maintained until, approximately, December 2000. DJIA started to oscillate in a range between, approximately, 10,000 and 11,000 points. A collapse in DJIA became visible in June 2001 (see the blue curve in Figure 1); in October 2001, DJIA fell down to 9,000 points. The Federal Reserve started to decrease interest rates as the US economy seemed to be stalled. The Federal Reserve reduced interest rates from 6.5% in November 2000 to 1.75% in February 2002 to stimulate the economy. This had the effect of mitigating a fall in the DJIA and the broader world economy going into a recession.

2.2 The global financial crisis, 2008-2009

In this subsection we use [1], [2], [11]-[13] and [16] as sources.

The global financial crisis of 2008-2009 is considered by many economists as the worst financial crisis since the Great Depression of the 1930s. It resulted in the collapse of many large banks in the US, for example, Bear Strearns, Merrill Lynch, the Lehman Brothers. The US government undertook bailout measures for some banks and let other banks buy banks damaged by the collapse. Internationally, this scene was repeated around the world particularly in Europe, but less so in South East Asia and Australia. As a consequence, this led to the downturns in stock markets around the world. In many countries, the housing markets also suffered, resulting in numerous people being evicted from their homes, increased bank foreclosures and prolonged vacancies. This was particularly so in the US where the housing bubble reached extreme proportions created by the banks and other lenders whose lending policies were out of control. The trust of self-monitoring of banks and other financial institutions failed in a significant way. This was particularly the case in the US where there was a large drop in the value of house prices, which in turn contributed to the failure and insolvency of key businesses and finally the decline in consumer wealth and confidence. This led to a significant decline in economic activity and to a severe global economic recession in 2008.

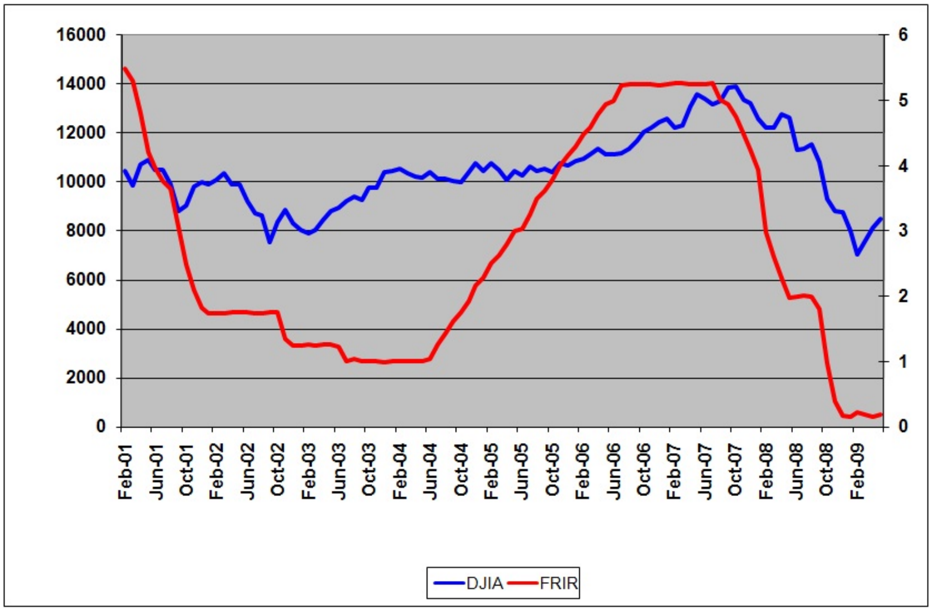

Figure 2: The Dow Jones Industrial Average (DJIA) and Federal Reserve Interest Rate (FRIR) in the period from February 2001 to May 2009 (monthly data, 100 months).

These failures were estimated to have been in the order of trillions of US dollars in the US alone. The global financial crisis was triggered by a cash shortfall in the US banking system in 2008 and the collapse of the US housing bubble which peaked in 2007. This damaged financial institutions globally and damaged investor confidence had an impact on global stock markets such as the Dow Jones, where shares suffered large losses during 2008 and early 2009. During this period economies worldwide slowed down as credit tightened and international trade declined. Governments and central banks responded with unprecedented fiscal stimulus packages, monetary policy expansion and institutional bailouts. Although there have been aftershocks, the financial crisis itself ended sometime between late-2008 and mid-2009.

The crash began on October 6, lasting a week. DJIA fell 18%, its worst weekly decline. On October 24, 2008 many of the world's stock exchanges experienced their worst declines in history, with drops of around 10% in most indices. By February 2009 the DJIA dropped 49,3% to 7063 points from its peak of 13930 points reached in October 2007, in just 16 months (see Figure 2).

Figure 2 shows that DJIA (the blue curve) reaches the highest level of 13930 points in October 2007 whilst FRIR (the red curve) climbs from 1% to 5.25%. The Federal Reserve increased the interest rates to stop the economy from overheating that could be attributed to the banking system not being in control of the housing market and toxic debt world wide. In October 2007 there was a common understanding that some major banks had liquidity problems. As this became known to the populace, the stock market collapsed. From October 2007 to February 2009 DJIA dropped 6867 points. The Federal Reserve attempted to support the economy by decreasing interest rates to almost zero from October 2007 to November 2008. The interest rates was staying at a nearly zero percent for three years showing the depth of the collapse.

3 Recognition: Dot-com crisis

In the recognition stage of our analysis, our goal is to identify pre-cursors of the Dot-com crisis of 2001-2002. We use time series for DJIA and FRIR (see Appendix 2 and Figure 1), representing the monthly data for the period from February 1994 to May 2002 – 100 months that precede and cover the Dot-com crisis.

3.1 Binary encoding rule

Here, we describe a *binary encoding rule* that transforms each short pattern in the above mentioned 100-month-long time series, y_1, y_2, ..., y_{100}, that precedes and covers the Dot-com crisis, into either a "minus" signal registering a tendency to a crisis, or a "plus" signal registering no tendency to a crisis. Here, $y_i = (y_i^1, y_i^2)$, y_i^1 is the value for DJIA and y_i^2 is the value for FRIR in month i ($i = 1, \ldots, 100$).

We discretize the data. For this purpose, we introduce a grid in the two-dimensional y space, with steps

$$h_1 = \frac{y^1_{max} - y^1_{min}}{N}$$

and

$$h_2 = \frac{y^2_{max} - y^2_{min}}{N}$$

on the y^1 axes and y^2 axes, respectively; here y^1_{max} and y^1_{min} are the maximum and minimum values for DJIA over months $1, \ldots, 100$; y^2_{max} and y^2_{min} are the maximum and minimum values for FRIR over months $1, \ldots, 100$; and N is a natural number, which we call the *grid dimension.* For each node on the grid, $\bar{y} = (\bar{y}^1, \bar{y}^2) = (y^1_{min} + Y^1 h_1, y^2_{min} + Y^2 h_2)$, where Y^1, Y^2 are integers located between 0 and N, we define its *grid coordinates* as a pair $Y = (Y^1, Y^2)$ and do not distinguish between $\bar{y}$ and Y.

We approximate each vector, y, in the time series y_1, y_2, ..., y_{100} by a node on the grid, $Y = (Y^1, Y^2)$, that is closest to y in the Euclidean metric. Using this approximation, we transform the time series y_1, y_2, ..., y_{100} into a sequence of nodes, Y_1, Y_2, ..., Y_{100}, which we call the *grid series.*

Take a natural m much smaller than 100; we call it the *pattern length.* We define *pre-patterns* as $(m+1)$-long subsequences in the grid series, $Y_i, Y_{i+1}, ..., Y_{i+m}$, where $i = 1, \ldots, 100 - m$. Any sequence of the form

$$z = (z_1, z_2, \ldots, z_{m+1}) = (Y_i - l_i, Y_{i+1} - l_i, ..., Y_{i+m-1} - l_i, Y_{i+m} - l_i)$$

where $Y_i, Y_{i+1}, ..., Y_{i+m}$ is a pre-pattern, $l_i = (l_i^1, l_i^2)$, $l_i^1 = \min\{Y_i^1, ..., Y_{i+m}^1\}$, $l_i^2 = \min\{Y_i^2, ..., Y_{i+m}^2\}$ ($i = 1, \ldots, 100 - m$), will be said to be a *pattern.* In contrast to the pre-patterns, the patterns are insensitive to the locations of the $(m+1)$-long subsequences on the grid series, representing their shapes only.

We categorize the patterns into two groups, a "$-$" *group* and a "$+$" *group* in such a way that the "$-$" patterns concentrate closer to month 100, the time of the crisis, and the "$+$" patterns concentrate closer to month 1, far away from the time of the crisis. We interpret these locations of the "$-$" and "$+$" patterns in time so that the "$-$" patterns "feel" crisis, whereas the "$+$" patterns do not.

In our study we set $m = 3$. Therefore, the patterns, z, have length 4,

$$z = (z_1, z_2, z_3, z_4), \quad z_j = (z_j^1, z_j^2) = (DJ_j, FR_j) \quad (j = 1, 2, 3, 4). \tag{1}$$

We define a pattern z (1) to be a "$-$" *pattern* if it satisfies one of the following conditions:

(A$-$) $DJ_j \geq DJ_{j+1}$ and $FR_j < FR_{j+1}$ for some $j = 1, 2, 3$;

(B$-$) $DJ_j \geq DJ_{j+1}$ and $FR_j > FR_{j+1}$ for some $j = 1, 2, 3$;

(C$-$) the number of $j = 1, 2, 3, 4$ such that $DJ_j \geq 1$ is not less than 2 and $FR_j = 0$ for all $j = 1, 2, 3, 4$.

We define a pattern z (1) to be a "+" *pattern* if it satisfies one of the following conditions:

(A+) $DJ_j < DJ_{j+1}$ and $FR_j < FR_{j+1}$ for some $j = 1, 2, 3$;

(B+) $DJ_j < DJ_{j+1}$ and $FR_j > FR_{j+1}$ for some $j = 1, 2, 3$;

(C+) the number of $j = 1, 2, 3, 4$ such that $DJ_j \geq 1$ is less than 2 and $FR_j = 0$ for all $j = 1, 2, 3, 4$.

It is easily seen that for each pattern z (1) at least one of the conditions (A$-$), (B$-$), (C$-$), (A+), (B+), (C+) is satisfied. If two conditions, one from the "$-$" group: (A$-$), (B$-$), (C$-$) and one from the "+" group: (A+), (B+), (C+), are satisfied for some z, we define it to be a "+" pattern. Thus, each pattern belongs to either the "$-$" group or to the "+" group.

Let us give a rough economic interpretation of the above definitions.

Suppose DJIA goes down and FRIR grows. Growth in FRIR implies that the economy gets less cheap money, and investment becomes more expensive. The decrease in DJIA shows that the economy fails to cope with that financial pressure, and gives us a negative signal. This can explain (A$-$).

Suppose both DJIA and FRIR go down. In this situation, the economy does not react positively to the fact that the interest rates go down. In other words, the economy is pessimistic about its future prospects. A further drop in the interest rates is needed in order to stimulate the economy. This can be qualified as a negative signal. In this manner, we explain (B$-$).

Suppose DJIA varies and FRIR stays constant. In this situation the economy is not regulated financially. We treat it as a negative signal and come to (C$-$).

Suppose DJIA grows, showing that the economy is strong, and let FRIR grows as well. Growth in FRIR is an indication of an effort to slow down economic growth and implies that the economy is very strong. A strong economy provides no signal on an upcoming crisis. This can justifies (A+).

Suppose DJIA grows and FRIR goes down. The decrease in FRIR shows an effort to stimulate the economy. The effort is efficient, since DJIA grows, showing that market is expected to be stronger. Again, no signal on a crisis is seen. In this manner, we can justify (B+).

Finally, suppose both DJIA and FRIR stay constant. This is an equilibrium situation. We treat it positively and we come to (C+).

To come to the above definitions of the "$-$" and "+" groups, we have used a special *pattern classification algorithm* described in Appendix 1 and implemented as a software (see Appendix 4). The definitions of the "$-$" and "+" groups provide us with a *binary encoding rule*, using which we transform every pattern z (1) into either a "$-$" signal supposed to register a tendency to a crisis, or a "+" signal supposed to register no tendency to a crisis. Namely, for every "$-$" pattern z we say that "$-$" is the *code* of z, and for every "+" pattern z we say that "+" is the *code* of z.

3.2 Verification of the binary encoding rule

We use the binary encoding rule to transform the grid series $Y_1, \ldots, Y_{100}$ into a sequence of "$-$" and "+" signals, $s_1, \ldots, s_{96}$, where s_i is either a "$-$" or a "+" $(i = 1, \ldots, 96)$. Namely, for $i = 1, \ldots, 96$ we set s_i to be the code of the pattern z^i that starts at month i,

$$z^i = (z_1^i, z_2^i, z_3^i, z_4^i) = (Y_i - l_i, Y_{i+1} - l_i, Y_{i+2} - l_i, Y_{i+3} - l_i), \quad (2)$$

$$l_i = (l_i^1, l_i^2), \quad l_i^1 = \min\{Y_i^1, Y_{i+1}^1, Y_{i+2}^1, Y_{i+3}^1\}, \quad l_i^2 = \min\{Y_i^2, Y_{i+1}^2, Y_{i+2}^2, Y_{i+3}^2\}.$$

To verify that the proposed encoding rule allows us to "feel" the approach of the Dot-com crisis, we consider the evolution of F_i, the fraction of the "$-$" signs in the moving 10-long window of signals, $s_i, \ldots, s_{i+9}$ $(i = 1, \ldots, 87)$, for $N = 4$. Figure 3 shows that F_i stays below 0.6 before November 1998 and above 0.6 after November 1998. In the period from February 1998 to May 1999 F_i exhibits a rapid growth, and it never drops below 0.7 after December 1998. Thus, the "$-$" signals concentrate, primarily, in the period adjoining the time of the Dot-com crisis. We interpret this as an evidence for the fact that in the original time series we have satisfactorily recognized the "$-$" signals registering a tendency to a crisis.

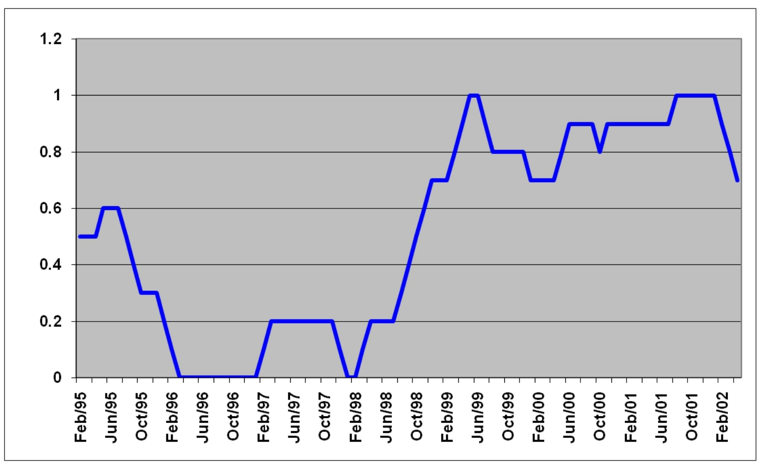

Figure 3: Behavior of F_i, the fraction of the "$-$" signals in the moving 10-long window of signals, $s_i, \ldots, s_{i+9}$ $(i = 1, \ldots, 87)$; $m = 3$, $N = 4$.

Remark 1 It is well-known that robustness to variations in parameter values is an important property of methods of assessment of uncertain systems; it serves as an indication of the reliability of the assessment results. Our experiments show that for a relatively wide range of values for the pattern length, $m > 3$, and grid dimension, $N > 4$, we satisfactorily recognize "$-$" signals in the original time series if we use (A$-$) and (B$-$) and generalize (C$-$) as

(D$-$) the number of $j = 1, \ldots, m+1$ such that $DJ_j \geq 1$ is not less than $m - 1$ and $FR_j = 0$ for all $j = 1, \ldots, m+1$.

Accordingly, we satisfactorily recognize "+" signals using (A+) and (B+) and generalizing (C+) as

(D+) the number of $j = 1, \ldots, m+1$ such that $DJ_j \geq 1$ is less than $m - 1$ and $FR_j = 0$ for for all $j = 1, \ldots, m+1$.

In other words, the proposed binary encoding rule is robust to variations in the parameter values of our recognition method.

4 Statistical analysis: time series for 1954-2001

In the second stage of our analysis we use the binary encoding rule and a time series for a long period preceding the Dot-com crisis (1954-2001) to construct a random process describing the occurrence of "−" and "+" signals in the operation of the financial system. Following subsection 3.1, we set $m = 3$ and $N = 4$.

4.1 Historical transition probabilities

We use the constructed binary encoding rule (see subsection 3.1) to transform the DJIA and FRIR time series for the period 1954-2001 into a *historical binary sequence* of "−" and "+" signals, following each other with one-month steps, and carry out a statistical analysis of the historical binary sequence.

Namely, we consider every three subsequent signals in the historical binary sequence (covering, together, six months) as a *binary window*. For every binary window, $\bar{\sigma} = (\bar{\sigma}_1, \bar{\sigma}_2, \bar{\sigma}_3)$, we collect all subsequent binary windows, and for every subsequent binary window, $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ (which, due to its overlap with $\bar{\sigma}$, is necessarily such that $\sigma_1 = \bar{\sigma}_2$ and $\sigma_2 = \bar{\sigma}_3$), we compute the frequency of its occurrence after $\bar{\sigma}$. We treat that frequency as a historical probability, $p(\sigma|\bar{\sigma})$, for σ to follow $\bar{\sigma}$, or a historical *transition probability* for $\bar{\sigma}$ to be transformed to σ in a one-step transition. We organize the transition probabilities as a *matrix of transition probabilities*, Z, whose columns correspond to the binary windows, $\bar{\sigma}$, and rows correspond to the subsequent binary windows, σ. The matrix has the form

$$Z = \begin{array}{cccccccc|c}
+++ & ++- & +-+ & +-- & -++ & -+- & --+ & --- & \\
453/479 & 0 & 0 & 0 & 26/29 & 0 & 0 & 0 & +++ \\
26/479 & 0 & 0 & 0 & 3/29 & 0 & 0 & 0 & ++- \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +-+ \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & +-- \\
0 & 0 & 1/2 & 0 & 0 & 0 & 29/33 & 0 & -++ \\
0 & 0 & 1/2 & 0 & 0 & 0 & 4/33 & 0 & -+- \\
0 & 0 & 0 & 9/33 & 0 & 0 & 0 & 24/45 & --+ \\
0 & 0 & 0 & 24/33 & 0 & 0 & 0 & 21/45 & ---
\end{array}.$$

4.2 Binary random process and the probability of crisis

The matrix of transition probabilities, Z, defines a random process operating in the space of the binary windows; we call it the *binary random process* (see [4]).

Based on an understanding that a crisis occurs after a long series of "−" signals (see subsection 3.2), we assume that a crisis occurs in month k if a 12-month-long sequence of "−" signals, with no more than one exceptional "+" signal, occurs in months $k-12, \ldots, k-1$. In terms of the binary windows, we understand a crisis as the realization of a sequence of binary windows, $\bar{\sigma}^1, \ldots, \bar{\sigma}^{10}$, such that one of the four conditions are satisfied: (a) $\bar{\sigma}^1 = \ldots = \bar{\sigma}^{10} = (-,-,-)$; (b) $\bar{\sigma}^1 = \ldots = \bar{\sigma}^9 = (-,-,-)$ and $\bar{\sigma}^{10} = (-,-,+)$; (c) $\bar{\sigma}^1 = \ldots = \bar{\sigma}^8 = (-,-,-)$, $\bar{\sigma}^9 = (-,-,+)$, and $\bar{\sigma}^{10} = (-,+,-)$; (d) there is a single $s \leq 8$ such that $\bar{\sigma}^s = (-,-,+)$, $\bar{\sigma}^{s+1} = (-,+,-)$, $\bar{\sigma}^{s+2} = (+,-,-)$, and $\bar{\sigma}^j = (-,-,-)$ for all $j \neq s$, $j \neq s+1$ and $j \neq s+2$. Let C denote the set of all such sequences of binary windows, $\bar{\sigma}^1, \ldots, \bar{\sigma}^{10}$. Formally, we understand a *crisis* as the random event C in the set of all trajectories of the binary random process.

Given that a binary window $\hat{\sigma}^i$ occurs in month i ($i \leq k-12$), the binary random process evaluates the probability of a sequence $\bar{\sigma}^1, \ldots, \bar{\sigma}^{10}$ to occur in months $k-12, \ldots, k-1$ as follows:

$$P_k(\bar{\sigma}^1, \ldots, \bar{\sigma}^{10}|\hat{\sigma}^i) = \sum_{\sigma^{i+1}, \ldots, \sigma^{k-11} \in S} \gamma(\sigma^{k-11}) p(\sigma^{k-11}|\sigma^{k-12}) \ldots p(\sigma^{i+2}|\sigma^{i+1}) p(\sigma^{i+1}|\hat{\sigma}^i)$$

where S is the set of all binary windows and

$$\gamma(\sigma^{k-11}) = p(\bar{\sigma}^{10}|\bar{\sigma}^9) p(\bar{\sigma}^9|\bar{\sigma}^8) \ldots p(\bar{\sigma}^1|\sigma^{k-11}).$$

Thus, for the probability of a crisis to occur in month k we get

$$P_k(C|\hat{\sigma}^i) = \sum_{(\bar{\sigma}^1, \ldots, \bar{\sigma}^{10}) \in C} P_k(\bar{\sigma}^1, \ldots, \bar{\sigma}^{10}|\hat{\sigma}^i).$$

In case $i > k-12$, when the 12-month-long sequence of signals in months $k-12, \ldots, k-1$ overlaps the binary window $\hat{\sigma}^i$, the random process evaluates the probability of a crisis to occur in month k as follows:

$$P_k(C|\hat{\sigma}^i) = \sum_{(\bar{\sigma}^{i-k+12}, \ldots, \bar{\sigma}^{10}) \in C_i} p(\bar{\sigma}^{10}|\bar{\sigma}^9) p(\bar{\sigma}^9|\bar{\sigma}^8) \ldots p(\bar{\sigma}^{i-k+12}|\hat{\sigma}^i)$$

where C_i is the set of $(k-i-1)$-long sequences of binary windows, $\bar{\sigma}^{i-k+12}, \ldots, \bar{\sigma}^{10}$, which satisfy (a), (b), (c) or (d).

To compute $P_k(C|\hat{\sigma}^i)$, we developed a special software (see Appendix 4).

5 Testing: global financial crisis

In this part of our analysis our goal is to test the forecasting ability of the binary random process as a qualitative model describing the operation of the financial system. Like in section 4, we set $m = 3$ and $N = 4$.

5.1 Original time series and grid series

We consider the time series for DJIA and FRIR in a period that precedes the global financial crisis and overlaps with it – 100 months from February 2001 to May 2009 (see Figure 2 and Appendix 2). We introduce a 100-node time grid with a one-month step, in which February 2001 is numbered 1. We discretize the data using a two-dimensional grid of dimension N in the DJIA, FRIR plane and form a *grid series*, $Y_1, \ldots, Y_{100}$, as explained in subsection 3.1. (See Appendix 3 for more details on data processing.)

5.2 Imitated on-line assessment of the probability of a crisis

We use an imitation of an on-line experiment to generate the values for the probability of a crisis to occur in October 2008 (month 93) – the actual time, at which the global financial crisis occurred (see section 2).

In each current month, i, where $6 \le i \le 93$ ($i = 6$ corresponds to July 2001), we use the binary encoding rule to construct a current binary window – three subsequent signals, $\hat{\sigma}^i = (\hat{\sigma}^i_1, \hat{\sigma}^i_2, \hat{\sigma}^i_3)$, where $\hat{\sigma}^i_1$ is the code of the pattern

$$z^{i-5} = (z^{i-5}_1, z^{i-5}_2, z^{i-5}_3, z^{i-5}_4) = (Y_{i-5} - l_{i-5}, Y_{i-4} - l_{i-5}, Y_{i-3} - l_{i-5}, Y_{i-2} - l_{i-5}),$$

$$l_{i-5} = (l^1_{i-5}, l^2_{i-5}), \quad l^1_{i-5} = \min\{Y^1_{i-5}, Y^1_{i-4}, Y^1_{i-3}, Y^1_{i-2}\}, \quad l^2_{i-5} = \min\{Y^2_{i-5}, Y^2_{i-4}, Y^2_{i-3}, Y^2_{i-2}\},$$

which starts at month $i - 5$; $\hat{\sigma}^i_2$ is the code of the pattern

$$z^{i-4} = (z^{i-4}_1, z^{i-4}_2, z^{i-4}_3, z^{i-4}_4) = (Y_{i-4} - l_{i-4}, Y_{i-3} - l_{i-4}, Y_{i-2} - l_{i-4}, Y_{i-1} - l_{i-4}),$$

$$l_{i-4} = (l^1_{i-4}, l^2_{i-4}), \quad l^1_{i-4} = \min\{Y^1_{i-4}, Y^1_{i-3}, Y^1_{i-2}, Y^1_{i-1}\}, \quad l^2_{i-4} = \min\{Y^2_{i-4}, Y^2_{i-3}, Y^2_{i-2}, Y^2_{i-1}\},$$

which starts at month $i - 4$; and $\hat{\sigma}^i_3$ is the code of the pattern

$$z^{i-3} = (z^{i-3}_1, z^{i-3}_2, z^{i-3}_3, z^{i-3}_4) = (Y_{i-3} - l_{i-3}, Y_{i-2} - l_{i-3}, Y_{i-1} - l_{i-3}, Y_i - l_{i-3}),$$

$$l_{i-3} = (l^1_{i-3}, l^2_{i-3}), \quad l^1_{i-3} = \min\{Y^1_{i-3}, Y^1_{i-2}, Y^1_{i-1}, Y^1_i\}, \quad l^2_{i-3} = \min\{Y^2_{i-3}, Y^2_{i-2}, Y^2_{i-1}, Y^2_i\},$$

which starts at month $i - 3$.

In month i we estimate the probability of a crisis to occur in month 93 on the absolute time grid, as $P(C) = P_{93}(C|\hat{\sigma}^i)$ (see subsection 4.2).

In our imitated on-line assessment experiment we compute $P(C)$ as described above, sequentially, for months $i = 6, \ldots, 93$. Our calculations show that until July 2003 (month 30), $P(C)$ does not change, remaining extremely small; in the period between July 2003 and October 2007 $P(C)$ oscillates, still remaining extremely small; in the period between October 2007 and March 2008 $P(C)$ grows slowly but steadily; in the period between March 2008 and August 2008 $P(C)$ grows fast; and it reaches size 1 in August 2008, three months before the time the crisis had actually occurred. Figure 4 shows the behavior of $P(C)$ in the period from October 2007 to October 2008.

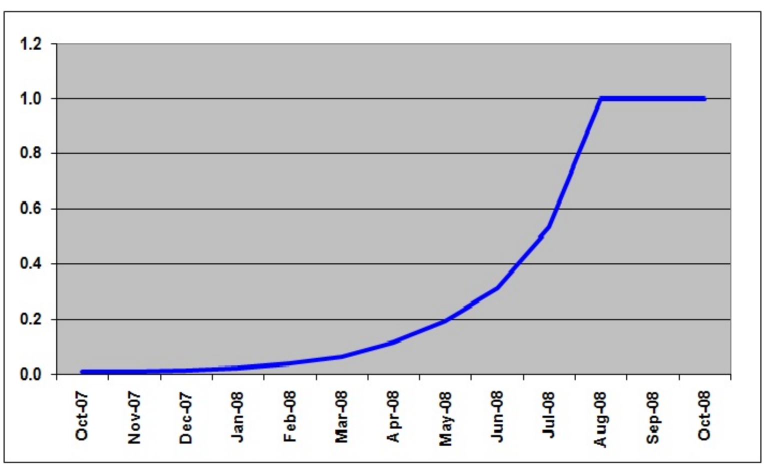

Figure 4: Behavior of $P(C)$, the probability of a crisis to occur in October, 2008.

6 Conclusion

The testing result presented in section 5 shows that the proposed method for a qualitative assessment of early signals on crises has, potentially, an ability to guess the actual tendencies. The fact that the model identified using data on an earlier crisis is sensitive to signals on a next crisis allows us to conjecture that either the two crises have a same nature, or the model is robust to the mechanisms that drove their development.

The presented work is preliminary. Next research phases will include analysis of the sensitivity of the method to variations in parameter values (a preliminary step in this direction is outlined in Remark 1); extensions to assessment of other historical financial crises; and extensions to analysis of pre-cursors of other types of extreme events.

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Appendix 1: a pattern classification algorithm

Here, for the case of an arbitrary pattern length, m (much smaller than 100), and an arbitrary grid dimension, N, we describe a pattern classification algorithm we used to find the binary encoding rule presented in subsection 3.1.

Let us set $M = 100 - m$ if m is even and $M = 99 - m$ otherwise; then M is even. We split the discrete time grid $1, \ldots, M$ into two equal parts: $1, \ldots, M/2$ and $M/2+1, \ldots, M$.

We perform the algorithm in steps. By n^- and n^+ we denote the numbers of patterns encoded by a "$-$" and by a "+", respectively.

Step 0. We encode every pattern z^i (2) where $i = 1, \ldots, M/2$ with a "+".

As a result we get $M/2$ "+" patterns concentrating at the "early" half of the time grid (far distant from the time of the crisis), $n^- = 0$, and $n^+ = M/2$. In the following steps we extend the encoding rule to new patterns and make sure that (i) the "$-$" patterns concentrate, primarily, in the "late" half of the time grid; (ii) in the end, the numbers of the "$-$" and "+" patterns, n^-, and n^+, approximately equalize, and (iii) in each step, encoding is *not contradictory* in the sense that all identical patterns that are encoded are encoded identically.

Step 1. We encode z^M with a "$-$". If $z^M \neq z^j$ for all $j \leq M/2$, we get $n^- = 1$ and $n^+ = M/2$ and go to Step 2. If $z^M = z^j$ for some $j \leq M/2$, we re-encode z^j with a "$-$", implying that $n^- = 2$, $n^+ = M/2 - 1$. In result we have the single "$-$" pattern, z^M, lying at the end of the "late" half of the time grid, and "+" patterns z^i covering the "early" half of the time grid ($i = 1, \ldots, M/2$) either with no gaps, or with the single "gap" – the exclusive pattern z^j encoded by a "$-$". Here we go to step 2.

Step s ($s < M/2$). Initially, all patterns located in the "early" part of the time grid, i.e., z^i with $i = 1, \ldots, M/2$, and all patterns following the time node $M - s$, i.e., z^i with $i = M - s + 1, \ldots, M$, are encoded by "$-$" or "+" signs, and all patterns z^i with $i = M/2 + 1, \ldots, M - s$ are not encoded. Moreover, encoding is not contradictory in the sense mentioned above.

We encode z^{M-s} with a "$-$".

If $z^{M-s} \neq z^j$ for all $j \leq M/2$ and all $j > M - s$, or if all z^j with $j \leq M/2$ and $j > M - s$ identical to z^{M-s} are "$-$" patterns, we go to the Switching rule (see below). In result, a new "$-$" pattern, z^{M-s}, appears, the original value for n^- grows for 1, n^+ does not change, and encoding is, again, not contradictory.

If $z^{M-s} = z^j$ for some $j \leq M/2$ or some $j > M - s$, and z^j is encoded with a "+", we re-encode z^j with a "$-$". In result, two new "$-$" patterns, z^{M-s} and z^j, appear, the original value for n^- grows for 2, the original value for n^+ decreases for 1, and encoding is, again, not contradictory. Here we go to the Switching rule.

Switching rule.

If $n^- < n^+$, we go to Step $s + 1$.

If $n^- \geq n^+$, we go to Sub-step $s.1$.

Sub-step $s.1$. We encode z^{M-s-1} with a "+".

If there are no "$-$" patterns z^j with $j \leq M/2$ or $j \geq M - s$, which are identical to z^{M-s-1}, we finalize Sub-step $s.1$; n^- remains unchanged and n^+ grows for 1.

If we find a "$-$" pattern z^j with $j \leq M/2$ or $j \geq M - s$, which is identical to z^{M-s-1}, we encode z^j with a "+" and finalize Sub-step $s.1$; n^- decreases for 1 and n^+ grows for

2, and encoding is non-contradictory.

If $n^- < n^+$, we go to Step $s + 2$.

Otherwise we go to Sub-step $s.2$.

Sub-step $s.k$**.** Initially, all patterns located in the "early" part of the time grid, i.e., z^i with $i = 1, \dots, M/2$, and all patterns following the time node $M - s - k$, i.e., z^i with $i = M - s - k, \dots, M$, are encoded by "$-$" or "+" signs, and all patterns z^i with $i = M/2 + 1, \dots, M - s - k - 1$ are not encoded. Encoding is not contradictory.

We encode $z^{M-s-k-1}$ with a "+".

If there are no "$-$" patterns z^j identical to $z^{M-s-k-1}$, we finalize Sub-step $s.k$; n^- remains unchanged and n^+ grows for 1.

If we find a "$-$" pattern z^j identical to $z^{M-s-k-1}$, we encode z^j with a "+" and finalize Sub-step $s.k$; n^- decreases for 1 and n^+ grows for 2, and encoding is non-contradictory.

If $n^- < n^+$, we go to Step $s + k + 1$.

Otherwise we go to Sub-step $s.k + 1$.

The described pattern classification algorithm provides us with a binary encoding rule such that the "$-$" patterns concentrate, primarily, in the "late" half of the time grid; and the numbers of the "$-$" and "+" patterns, n^- and n^+, are approximately equal (one can prove that $|n^- - n^+| \leq 2$). The encoding rule is non-contradictory in the sense that all identical patterns are encoded identically. We call it the *basic* encoding rule. Next, we notice that the basic encoding rule differs from the one described in subsection 3.1 non-essentially in the sense that the number of the patterns encoded differently by those two binary encoding rules is small enough. In our analysis, we use the latter binary encoding rule, since it is more interpretable in economic terms.

Appendix 2: data

Table 1: DJIA and FRIR data from February 1994 until May 2002 (sources [14] and [15]).

	MONTH	DJIA	FRIR
1	Feb-94	3832.02	3.25
2	Mar-94	3635.96	3.34
3	Apr-94	3681.69	3.56
4	May-94	3758.37	4.01
5	Jun-94	3624.96	4.25
6	Jul-94	3764.5	4.26
7	Aug-94	3913.42	4.47
8	Sep-94	3843.19	4.73
9	Oct-94	3908.12	4.76
10	Nov-94	3739.23	5.29
11	Dec-94	3834.44	5.45
12	Jan-95	3843.86	5.53
13	Feb-95	4011.05	5.92
14	Mar-95	4157.69	5.98
15	Apr-95	4321.27	6.05
16	May-95	4465.14	6.01
17	Jun-95	4556.1	6.00
18	Jul-95	4708.47	5.85
19	Aug-95	4610.56	5.74
20	Sep-95	4789.08	5.80
21	Oct-95	4755.48	5.76
22	Nov-95	5074.49	5.80
23	Dec-95	5117.12	5.60
24	Jan-96	5395.3	5.56
25	Feb-96	5485.62	5.22
26	Mar-96	5587.14	5.31
27	Apr-96	5569.08	5.22
28	May-96	5643.18	5.24
29	Jun-96	5654.63	5.27
30	Jul-96	5528.91	5.40
31	Aug-96	5616.21	5.22
32	Sep-96	5882.17	5.30
33	Oct-96	6029.38	5.24
34	Nov-96	6521.7	5.31
35	Dec-96	6448.27	5.29
36	Jan-97	6813.09	5.25
37	Feb-97	6877.74	5.19
38	Mar-97	6583.48	5.39
39	Apr-97	7008.99	5.51

Continued on next page

Table 1 – continued from previous page

	MONTH	DJIA	FRIR
40	May-97	7331.04	5.50
41	Jun-97	7672.79	5.56
42	Jul-97	8222.61	5.52
43	Aug-97	7622.42	5.54
44	Sep-97	7945.26	5.54
45	Oct-97	7442.08	5.50
46	Nov-97	7823.13	5.52
47	Dec-97	7908.25	5.50
48	Jan-98	7906.5	5.56
49	Feb-98	8545.72	5.51
50	Mar-98	8799.81	5.49
51	Apr-98	9063.37	5.45
52	May-98	8899.95	5.49
53	Jun-98	8952.02	5.56
54	Jul-98	8883.29	5.54
55	Aug-98	7539.07	5.55
56	Sep-98	7842.62	5.51
57	Oct-98	8592.1	5.07
58	Nov-98	9116.55	4.83
59	Dec-98	9181.43	4.68
60	Jan-99	9358.83	4.63
61	Feb-99	9306.58	4.76
62	Mar-99	9786.16	4.81
63	Apr-99	10789.04	4.74
64	May-99	10559.74	4.74
65	Jun-99	10970.8	4.76
66	Jul-99	10655.15	4.99
67	Aug-99	10829.28	5.07
68	Sep-99	10336.95	5.22
69	Oct-99	10729.86	5.20
70	Nov-99	10877.81	5.42
71	Dec-99	11497.12	5.30
72	Jan-00	10940.53	5.45
73	Feb-00	10128.31	5.73
74	Mar-00	10921.92	5.85
75	Apr-00	10733.91	6.02
76	May-00	10522.33	6.27
77	Jun-00	10447.89	6.53
78	Jul-00	10521.98	6.54
79	Aug-00	11215.1	6.50
80	Sep-00	10650.92	6.52
81	Oct-00	10971.14	6.51
82	Nov-00	10414.49	6.51
83	Dec-00	10787.99	6.40

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Table 1 – continued from previous page

	MONTH	DJIA	FRIR
84	Jan-01	10887.36	5.98
85	Feb-01	10495.3	5.49
86	Mar-01	9878.8	5.31
87	Apr-01	10735	4.8
88	May-01	10911.9	4.21
89	Jun-01	10502.4	3.97
90	Jul-01	10522.8	3.77
91	Aug-01	9949.8	3.65
92	Sep-01	8847.6	3.07
93	Oct-01	9075.1	2.49
94	Nov-01	9851.6	2.09
95	Dec-01	10021.6	1.82
96	Jan-02	9920	1.73
97	Feb-02	10106.1	1.74
98	Mar-02	10403.9	1.73
99	Apr-02	9946.2	1.75
100	May-02	9925.3	1.75

Table 2: DJIA and FRIR data from February 2001 until May 2009 (sources [14] and [15]).

	MONTH	DJIA	FRIR
1	Feb-01	10495.3	5.49
2	Mar-01	9878.8	5.31
3	Apr-01	10735	4.8
4	May-01	10911.9	4.21
5	Jun-01	10502.4	3.97
6	Jul-01	10522.8	3.77
7	Aug-01	9949.8	3.65
8	Sep-01	8847.6	3.07
9	Oct-01	9075.1	2.49
10	Nov-01	9851.6	2.09
11	Dec-01	10021.6	1.82
12	Jan-02	9920	1.73
13	Feb-02	10106.1	1.74
14	Mar-02	10403.9	1.73
15	Apr-02	9946.2	1.75
16	May-02	9925.3	1.75
17	Jun-02	9243.3	1.75
18	Jul-02	8736.6	1.73
19	Aug-02	8663.5	1.74
20	Sep-02	7580.97	1.75

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Table 2 – continued from previous page

	MONTH	DJIA	FRIR
21	Oct-02	8393.84	1.75
22	Nov-02	8896.09	1.34
23	Dec-02	8345.33	1.24
24	Jan-03	8076.36	1.24
25	Feb-03	7907.87	1.26
26	Mar-03	8062.86	1.25
27	Apr-03	8480.09	1.26
28	May-03	8850.26	1.26
29	Jun-03	8994.73	1.22
30	Jul-03	9229.39	1.01
31	Aug-03	9415.82	1.03
32	Sep-03	9277.21	1.01
33	Oct-03	9797.79	1.01
34	Nov-03	9782.46	1
35	Dec-03	10439.48	0.98
36	Jan-04	10488.07	1
37	Feb-04	10583.92	1.01
38	Mar-04	10396.15	1
39	Apr-04	10249.5	1
40	May-04	10188.45	1
41	Jun-04	10440.27	1.03
42	Jul-04	10139.71	1.26
43	Aug-04	10163.16	1.43
44	Sep-04	10063.31	1.61
45	Oct-04	10027.47	1.76
46	Nov-04	10435.33	1.93
47	Dec-04	10787.45	2.16
48	Jan-05	10491.06	2.28
49	Feb-05	10779.08	2.5
50	Mar-05	10522.97	2.63
51	Apr-05	10092.53	2.79
52	May-05	10471.61	3
53	Jun-05	10271.06	3.04
54	Jul-05	10640.91	3.26
55	Aug-05	10482.64	3.5
56	Sep-05	10568.22	3.62
57	Oct-05	10440.07	3.78
58	Nov-05	10805.63	4
59	Dec-05	10717.5	4.16
60	Jan-06	10868.3	4.29
61	Feb-06	10994.05	4.49
62	Mar-06	11138.53	4.59
63	Apr-06	11367.14	4.79
64	May-06	11164.31	4.94

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Table 2 – continued from previous page

	MONTH	DJIA	FRIR
65	Jun-06	11166.31	4.99
66	Jul-06	11183.44	5.24
67	Aug-06	11382.83	5.25
68	Sep-06	11679.07	5.25
69	Oct-06	12075.93	5.25
70	Nov-06	12230.26	5.25
71	Dec-06	12463.15	5.24
72	Jan-07	12621.53	5.25
73	Feb-07	12262.3	5.26
74	Mar-07	12354.35	5.26
75	Apr-07	13120.94	5.25
76	May-07	13626.91	5.25
77	Jun-07	13408.62	5.25
78	Jul-07	13200.36	5.26
79	Aug-07	13344	5.02
80	Sep-07	13895.63	4.94
81	Oct-07	13930.01	4.76
82	Nov-07	13382.94	4.49
83	Dec-07	13251.17	4.24
84	Jan-08	12614.35	3.94
85	Feb-08	12262.81	2.98
86	Mar-08	12266.15	2.61
87	Apr-08	12813.37	2.28
88	May-08	12638.32	1.98
89	Jun-08	11336.42	2
90	Jul-08	11370.78	2.01
91	Aug-08	11543.96	2
92	Sep-08	10859.58	1.81
93	Oct-08	9325.01	0.97
94	Nov-08	8829.04	0.39
95	Dec-08	8771.69	0.16
96	Jan-09	7996.72	0.15
97	Feb-09	7062.93	0.22
98	Mar-09	7596.65	0.18
99	Apr-09	8161.11	0.15
100	May-09	8500.57	0.18

Appendix 3: patterns, binary sequence and frequency of negative signals

Table 3: Processed data for the period from February 1994 to April 2002 – the patterns (DJIA and FRIR), binary sequence (BS) and the fractions of the "−" signals in 10-month-long moving windows (F); $m = 3$, $N = 4$; 1 stands for a "+" and 0 for a "−".

	MONTH	DJIA	FRIR	BS	F
1	May-94	0000	0011	0	
2	Jun-94	0000	0111	0	
3	Jul-94	0000	0000	1	
4	Aug-94	0000	0000	1	
5	Sep-94	0000	0000	1	
6	Oct-94	0000	0001	0	
7	Nov-94	0000	0011	0	
8	Dec-94	0000	0111	0	
9	Jan-95	0000	0000	1	
10	Feb-95	0000	0000	1	0.5
11	Mar-95	0000	0001	0	0.5
12	Apr-95	0000	0011	0	0.5
13	May-95	0000	0111	0	0.6
14	Jun-95	0000	0000	1	0.6
15	Jul-95	0001	1110	1	0.6
16	Aug-95	0011	1100	1	0.5
17	Sep-95	0111	1000	1	0.4
18	Oct-95	0000	0000	1	0.3
19	Nov-95	0000	0000	1	0.3
20	Dec-95	0000	0000	1	0.3
21	Jan-96	0000	0000	1	0.2
22	Feb-96	0000	0000	1	0.1
23	Mar-96	0000	0000	1	0
24	Apr-96	0000	0000	1	0
25	May-96	0000	0000	1	0
26	Jun-96	0000	0000	1	0
27	Jul-96	0000	0000	1	0
28	Aug-96	0000	0000	1	0
29	Sep-96	0000	0000	1	0
30	Oct-96	0000	0000	1	0
31	Nov-96	0000	0000	1	0
32	Dec-96	0000	0000	1	0
33	Jan-97	0001	0000	1	0
34	Feb-97	0011	0000	0	0.1
35	Mar-97	0111	0000	0	0.2

Continued on next page

Table 3 – continued from previous page

	MONTH	DJIA	FRIR	BS	F
36	Apr-97	0000	0000	1	0.2
37	May-97	0000	0000	1	0.2
38	Jun-97	0000	0000	1	0.2
39	Jul-97	0000	0000	1	0.2
40	Aug-97	0000	0000	1	0.2
41	Sep-97	0000	0000	1	0.2
42	Oct-97	0000	0000	1	0.2
43	Nov-97	0000	0000	1	0.2
44	Dec-97	0000	0000	1	0.1
45	Jan-98	0000	0000	1	0
46	Feb-98	0001	0000	1	0
47	Mar-98	0011	0000	0	0.1
48	Apr-98	0111	0000	0	0.2
49	May-98	0000	0000	1	0.2
50	Jun-98	0000	0000	1	0.2
51	Jul-98	0000	0000	1	0.2
52	Aug-98	1110	0000	0	0.3
53	Sep-98	1100	0000	0	0.4
54	Oct-98	1001	0000	0	0.5
55	Nov-98	0011	0000	0	0.6
56	Dec-98	0111	1110	0	0.7
57	Jan-99	0000	1100	0	0.7
58	Feb-99	0000	1001	0	0.7
59	Mar-99	0000	0011	0	0.8
60	Apr-99	0001	0111	0	0.9
61	May-99	0011	0000	0	1
62	Jun-99	0111	0000	0	1
63	Jul-99	0000	0000	1	0.9
64	Aug-99	0000	0000	1	0.8
65	Sep-99	1110	0000	0	0.8
66	Oct-99	1101	0000	0	0.8
67	Nov-99	1011	0000	0	0.8
68	Dec-99	0111	0000	0	0.8
69	Jan-00	0000	0000	1	0.7
70	Feb-00	1110	0000	0	0.7
71	Mar-00	1101	0000	0	0.7
72	Apr-00	1011	0001	0	0.7
73	May-00	0111	0011	0	0.8
74	Jun-00	1110	0111	0	0.9
75	Jul-00	1101	0000	0	0.9
76	Aug-00	1011	0000	0	0.9
77	Sep-00	0111	0000	0	0.9
78	Oct-00	0000	0000	1	0.8
79	Nov-00	1110	0000	0	0.9

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Table 3 – continued from previous page

	MONTH	DJIA	FRIR	BS	F
80	Dec-00	1101	0000	0	0.9
81	Jan-01	1011	0000	0	0.9
82	Feb-01	0110	1110	0	0.9
83	Mar-01	1100	1100	0	0.9
84	Apr-01	1001	1000	0	0.9
85	May-01	0011	1110	0	0.9
86	Jun-01	0110	1100	0	0.9
87	Jul-01	1101	1000	0	0.9
88	Aug-01	1010	0000	0	1
89	Sep-01	0100	1110	0	1
90	Oct-01	1000	1100	0	1
91	Nov-01	0000	2110	0	1
92	Dec-01	0000	1100	0	1
93	Jan-02	0000	1000	0	1
94	Feb-02	0000	0000	1	0.9
95	Mar-02	0000	0000	1	0.8
96	Apr-02	0000	0000	1	0.7

Table 4: Processed data for the period from February 2001 to October 2008 – the patterns (DJIA and FRIR), binary sequence (BS) and the fractions of the "−" signals in 10-month-long moving windows (F); $m = 3$, $N = 4$; 1 stands for a "+" and 0 for a "−".

	MONTH	DJIA	FRIR	BS	F
1	May-01	0000	1100	0	
2	Jun-01	0000	1000	0	
3	Jul-01	0000	0000	1	
4	Aug-01	0000	0000	1	
5	Sep-01	1110	1110	0	
6	Oct-01	1100	1100	0	
7	Nov-01	1001	2110	1	
8	Dec-01	0011	1100	1	
9	Jan-02	0111	1000	1	
10	Feb-02	0000	0000	1	0.4
11	Mar-02	0000	0000	1	0.3
12	Apr-02	0000	0000	1	0.2
13	May-02	0000	0000	1	0.2
14	Jun-02	1110	0000	0	0.3
15	Jul-02	1100	0000	0	0.3
16	Aug-02	1000	0000	1	0.2
17	Sep-02	1110	0000	0	0.3

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Table 4 – continued from previous page

	MONTH	DJIA	FRIR	BS	F
18	Oct-02	1101	0000	0	0.4
19	Nov-02	1011	0000	0	0.5
20	Dec-02	0111	0000	0	0.6
21	Jan-03	0000	0000	1	0.6
22	Feb-03	1110	0000	0	0.7
23	Mar-03	1101	0000	0	0.8
24	Apr-03	1011	0000	0	0.8
25	May-03	0111	0000	0	0.8
26	Jun-03	0000	0000	1	0.8
27	Jul-03	0000	0000	1	0.7
28	Aug-03	0000	0000	1	0.6
29	Sep-03	0000	0000	1	0.5
30	Oct-03	0001	0000	1	0.4
31	Nov-03	0011	0000	0	0.5
32	Dec-03	0111	0000	0	0.5
33	Jan-04	0000	0000	1	0.4
34	Feb-04	0000	0000	1	0.3
35	Mar-04	0000	0000	1	0.2
36	Apr-04	0000	0000	1	0.2
37	May-04	0000	0000	1	0.2
38	Jun-04	0000	0000	1	0.2
39	Jul-04	0000	0000	1	0.2
40	Aug-04	0000	0000	1	0.2
41	Sep-04	0000	0000	1	0.1
42	Oct-04	0000	0000	1	0
43	Nov-04	0000	0000	1	0
44	Dec-04	0000	0001	0	0.1
45	Jan-05	0000	0011	0	0.2
46	Feb-05	0000	0111	0	0.3
47	Mar-05	0000	0000	1	0.3
48	Apr-05	0000	0000	1	0.3
49	May-05	0000	0000	1	0.3
50	Jun-05	0000	0000	1	0.3
51	Jul-05	0000	0000	1	0.3
52	Aug-05	0000	0001	0	0.4
53	Sep-05	0000	0011	0	0.5
54	Oct-05	0000	0111	0	0.5
55	Nov-05	0000	0000	1	0.4
56	Dec-05	0000	0000	1	0.3
57	Jan-06	0000	0000	1	0.3
58	Feb-06	0000	0000	1	0.3
59	Mar-06	0000	0000	1	0.3
60	Apr-06	0001	0000	1	0.3
61	May-06	0010	0001	0	0.4

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Table 4 – continued from previous page

	MONTH	DJIA	FRIR	BS	F
62	Jun-06	0100	0011	0	0.4
63	Jul-06	1000	0111	0	0.4
64	Aug-06	0001	0000	1	0.3
65	Sep-06	0011	0000	0	0.4
66	Oct-06	0111	0000	0	0.5
67	Nov-06	0000	0000	1	0.5
68	Dec-06	0000	0000	1	0.5
69	Jan-07	0000	0000	1	0.5
70	Feb-07	0000	0000	1	0.5
71	Mar-07	0000	0000	1	0.4
72	Apr-07	0001	0000	1	0.3
73	May-07	0011	0000	0	0.3
74	Jun-07	0111	0000	0	0.4
75	Jul-07	0000	0000	1	0.3
76	Aug-07	0000	0000	1	0.2
77	Sep-07	0000	0000	1	0.2
78	Oct-07	0000	1110	0	0.3
79	Nov-07	0000	1100	0	0.4
80	Dec-07	0000	1000	0	0.5
81	Jan-08	1110	0000	0	0.6
82	Feb-08	1100	1110	0	0.7
83	Mar-08	1000	1100	0	0.7
84	Apr-08	0000	1000	0	0.7
85	May-08	0000	1110	0	0.8
86	Jun-08	1110	1100	0	0.9
87	Jul-08	1101	1000	0	1
88	Aug-08	1011	0000	0	1
89	Sep-08	0110	0000	0	1
90	Oct-08	2210	0000	0	1

Appendix 4: software

To carry out numerical analysis presented in this work, two software packages were developed.

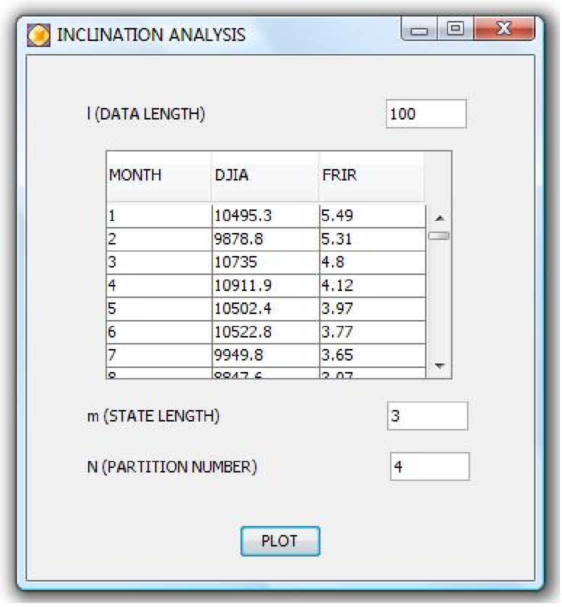

Figure 5: Software package 1: construction of a binary sequence for a given time series.

Package 1 (see Figure 5) allows the user to construct a binary sequence for a given time series. The user chooses the length of the time grid in months (in our case, 100); puts in monthly data for DJIA and FRIR; and chooses parameter values: the pattern length (m) and dimension of the grid (N). The package finds the encoding binary sequence and plots the graph of the fraction of the "$-$" signals in the moving 10-long window of signals (see Figure 3).

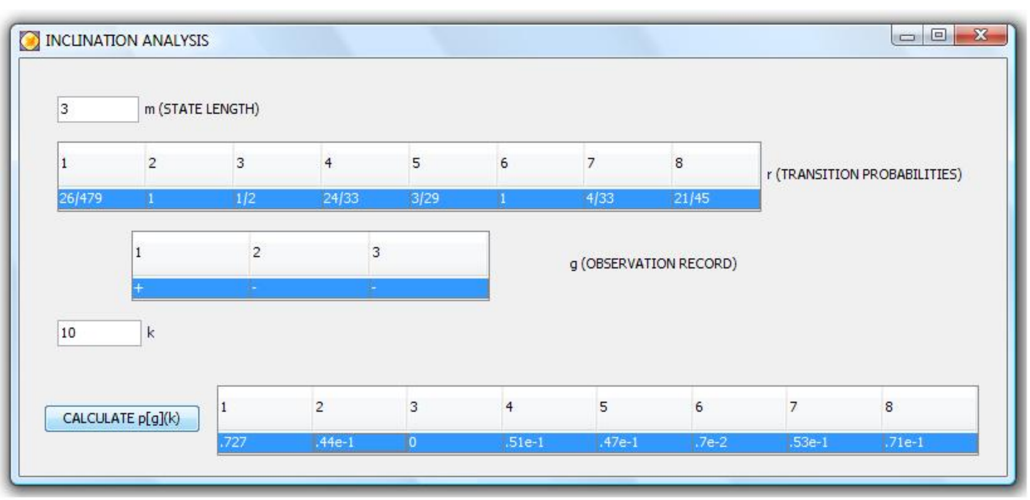

Figure 6: Software package 2: computation of the probability of a crisis.

Package 2 (see Figure 6) finds the probability of a crisis to occur in month k, $P_k(C|\hat{\sigma}^i)$

(see subsection 4.2). The user chooses k, the pattern length (m), the matrix of transition probabilities (see subsection 4.1) and the binary window, $\hat{\sigma}^i$.