ABOUT IIASA

IIASA, a nongovernmental, multidisciplinary, international research institution, was founded in October 1972 by the academies of science and equivalent scientific organizations of 12 nations from both East and West. Its goal is to bring together scientists from around the world to work on problems of common interest, particularly those resulting from scientific and technological development.

The Institute now has 17 National Member Organizations:

- The Academy of Sciences of the Union of Soviet Socialist Republics
- The Canadian Committee for IIASA
- The Committee for IIASA of the Czechoslovak Socialist Republic
- The French Association for the Development of Systems Analysis
- The Academy of Sciences of the German Democratic Republic
- The Japan Committee for IIASA
- The Max Planck Society for the Advancement of Sciences, Federal Republic of Germany
- The National Committee for Applied Systems Analysis and Management, People's Republic of Bulgaria
- The National Academy of Sciences, United States of America
- The National Research Council, Italy
- The Polish Academy of Sciences
- The Royal Society of London, United Kingdom
- The Austrian Academy of Sciences
- The Hungarian Committee for Applied Systems Analysis
- The Swedish Committee for Systems Analysis
- The Finnish Committee for IIASA
- The Foundation IIASA–Netherlands

MIGRATION PATTERNS AND POPULATION REDISTRIBUTION

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FOREWORD

Interest in human settlement systems and policies has been a critical part of urban-related work at IIASA since its inception. During the past several years this interest has given rise to a concentrated research activity focusing on migration dynamics and settlement patterns. Four subtasks have formed the core of this research effort:

- The study of spatial population *dynamics*
- The definition and elaboration of a new research area called *demometrics* and its application to migration analysis and spatial population forecasting
- The analysis and design of migration and settlement *policy*
- A *comparative study* of national migration and settlement patterns and policies

This paper gives an overview of recent migration and redistribution research at IIASA. Fundamental concepts of migration measurement are set out, and several multiregional demographic models dealing with the redistributional dynamics of national populations are outlined.

Reports, summarizing previous work on migration and settlement at IIASA, are listed at the end of this report.

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MIGRATION PATTERNS AND POPULATION REDISTRIBUTION*

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Received March 1979

This paper is a broad overview of recent research on the multiregional analysis of migration patterns and redistributional dynamics. Fundamental concepts regarding problems of migration measurement are set out, and several multiregional demographic models dealing with the spatial dynamics of national populations are outlined.

1. Introduction

The unexpected postwar baby boom in the United States had a salutary influence on demographic research in that it stimulated studies of improved methods for *measuring* fertility and for understanding the *dynamics* by which it, together with mortality, determines the age composition of a population. Because attention was principally directed at national population growth, measurement of internal migration and the *spatial* dynamics by which it affects national patterns of redistribution were neglected. This neglect led Kirk (1960) to conclude, in his 1960 Presidential Address to the Population Association of America, that the study of migration was the 'stepchild' of demography. Sixteen years later, Goldstein (1976, pp. 19–21) echoed a similar theme in *his* Presidential address to the same body:

> '... improvement in the quantity and quality of our information on population movement has not kept pace with the increasing significance of movement itself as a component of demographic change ... Redistribution has suffered far too long from neglect within the pro-

*This paper is a revised version of one prepared for presentation at the Quetelet Chair Seminar held on April 27–28, 1978, at the Catholic University of Louvain, Belgium.

**As will be evident to the reader, I have been greatly influenced by the scholarly contributions of two outstanding mathematical demographers: Ansley Coale and Nathan Keyfitz, and have been generously assisted in my own research by four former graduate students and subsequent colleagues at IIASA: Luis Castro, Jacques Ledent, Richard Raquillet, and Frans Willekens. As the many references to our joint papers indicate, I have borrowed liberally from this collaborative work.

fession . . . It behooves us to rectify this situation in this last quarter of the twentieth century, when redistribution in all its facets will undoubtedly constitute a major and increasingly important component of demographic change . . .'

Improved methods for measuring migration and understanding its important role in spatial population dynamics have been receiving increasing attention in recent years. The search for improved methods for measuring migration has, for example, stimulated research on the construction of multiregional life tables and demographic accounts [Rogers (1973a, b), Schoen (1975), Rogers and Ledent (1976), Rees (1977), Rees and Wilson (1977), and Ledent (1978)], and the need for a better understanding of spatial population dynamics has fostered mathematical analyses of the fundamental processes of spatial population growth and redistribution [Rogers (1966, 1968 and 1975a), Stone (1968), Drewe (1971), LeBras (1971), Feeney (1970 and 1973), Willekens (1977), and Liaw (1978)].

This paper reviews some of the work carried out during the past decade that has been directed at more rigorous methods for measuring migration and for establishing the fundamental redistributional dynamics through which it influences the evolution of spatial human populations. The second section of the paper deals with the measurement, the third with dynamics.

2. Measurement

The migration literature has until very recently adopted a curiously ambivalent position with regard to migration measurement. Definitions of migration rates and probabilities, construction of life tables that include migration flows, and differences between counts of migrations and of migrants, all are relatively recent topics of interest and concern. This paucity of work in migration measurement problems is in distinct contrast to the corresponding demographic literature in mortality and fertility – a literature that is richly endowed with detailed discussions of measurement problems.

It is natural to look to the state of mortality and fertility measurement for guidance in developing measures of migration. Like mortality, migration may be described as a process of interstate transfer; however, death can occur but once, whereas migration is potentially a repetitive event. This suggests the adoption of a fertility perspective and a focus on counts rather than durations. However, the dependence of migration on spatial boundaries introduces difficulties of measurement that do not occur in fertility analysis.

2.1. Migration rates and schedules

The most prominent regularity exhibited by empirical schedules of age-specific migration rates is the selectivity of migration with respect to age.

Young adults in their early twenties generally show the highest migration rates and mid-teenagers the lowest. The migration rates of children mirror the rates of their parents; thus the migration rates of infants exceed those of adolescents. Finally, migration streams directed toward regions with warmer climates and cities with relatively high levels of social services and cultural amenities often exhibit a 'retirement peak' at ages in the mid-sixties.

Fig. 1 illustrates a typical age-sex-specific migration schedule with a retirement peak. Several important points along the age profile may be identified: the *low point*, x_1, the *high peak*, x_p, and the *retirement peak*, x_r. Associated with the first two points is the labor force shift, X, which is defined to be the difference in years between the ages of the low point and the high peak, i.e., $X = x_p - x_1$. Associated with this shift is the *jump*, B, the increase in the migration rate of individuals aged x_p over those aged x_1.

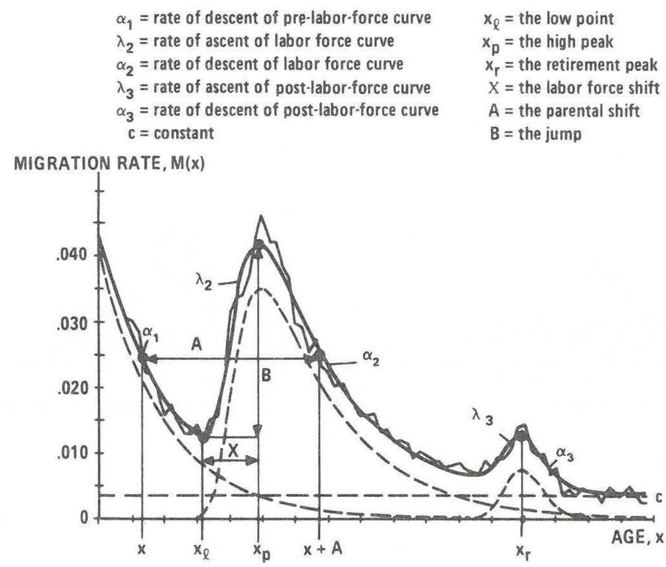

Fig. 1. The model migration schedule. [*Source:* Rogers, Raquillet and Castro (1978).]

The close correspondence between the migration rates of children and those of their parents suggests another important shift in observed migration schedules. If, for each point x on the pre-labor force part of the migration curve, we obtain by interpolation the point, $x + A_x$ say, with the identical rate of migration on the labor force curve, then the average of the values of A_x will be defined to be the observed *parental shift*, A.

The decomposition of the migration schedule described in fig. 1 suggests the following simple sum of four curves [Rogers, Raquillet and Castro

(1978)]:

$$M(x)=a_1\exp\{-\alpha_1 x\} + a_2\exp\{-\alpha_2(x-\mu_2)-e^{-\lambda_2(x-\mu_2)}\} + a_3\exp\{-\alpha_3(x-\mu_3)-e^{-\lambda_3(x-\mu_3)}\} + c, \qquad x=0,1,2,\ldots \tag{1}$$

The 'full' model schedule in eq. (1) has 11 parameters: a_1, α_1, a_2, μ_2, λ_2, a_3, α_3, μ_3, λ_3, and c. Migration schedules *without* a retirement peak may be represented by a 'reduced' model with 7 parameters, because in such instances the third component of eq. (1) is omitted. The profile of the full model schedule is defined by 7 of the 11 parameters: α_1, α_2, μ_2, λ_2, α_3, μ_3, and λ_3.

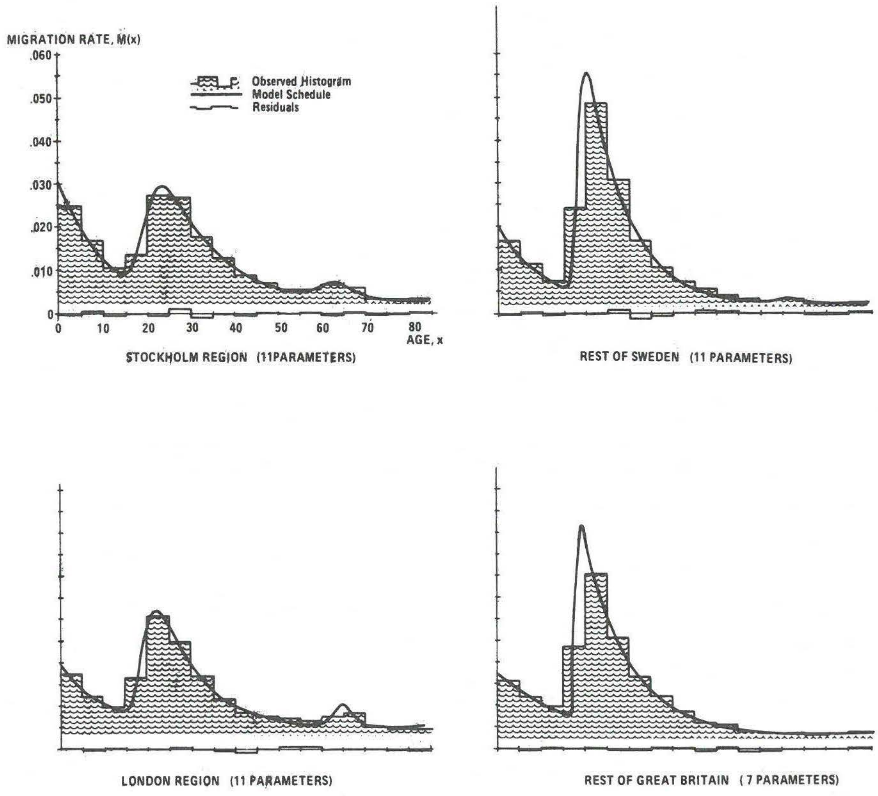

Fig. 2. Observed and model migration schedules: Sweden and Great Britain. [*Source*: Rogers (1978).]

The shape, or *profile*, of an age-specific schedule of migration rates is a feature that may be usefully studied independently of its intensity, or *level*. This is because there is considerable empirical evidence that although the latter tends to vary significantly from place to place, the former is remarkably similar in various localities. Some evidence on this point appears in the schedules set out in figs. 2 and 3; their parameters appear in table 1. For ease of comparison the areas under each curve were fixed at unity, i.e., the gross migraproduction rate (GMR) was scaled to unity.

The schedules illustrated in figs. 2 and 3 describe migration out of and into the capital region of each of four nations: Sweden, Great Britain, Bulgaria, and Japan. Observed data by five-year age groups (i.e., histograms) were disaggregated into one-year age groups by graduation-interpolation with the model schedule.

Four of the eleven parameters defining the model schedule refer only to migration *level*: a_1, a_2, a_3, and c. Their values in table 1 are for a GMR of unity; to obtain corresponding values for other levels of migration, we simply

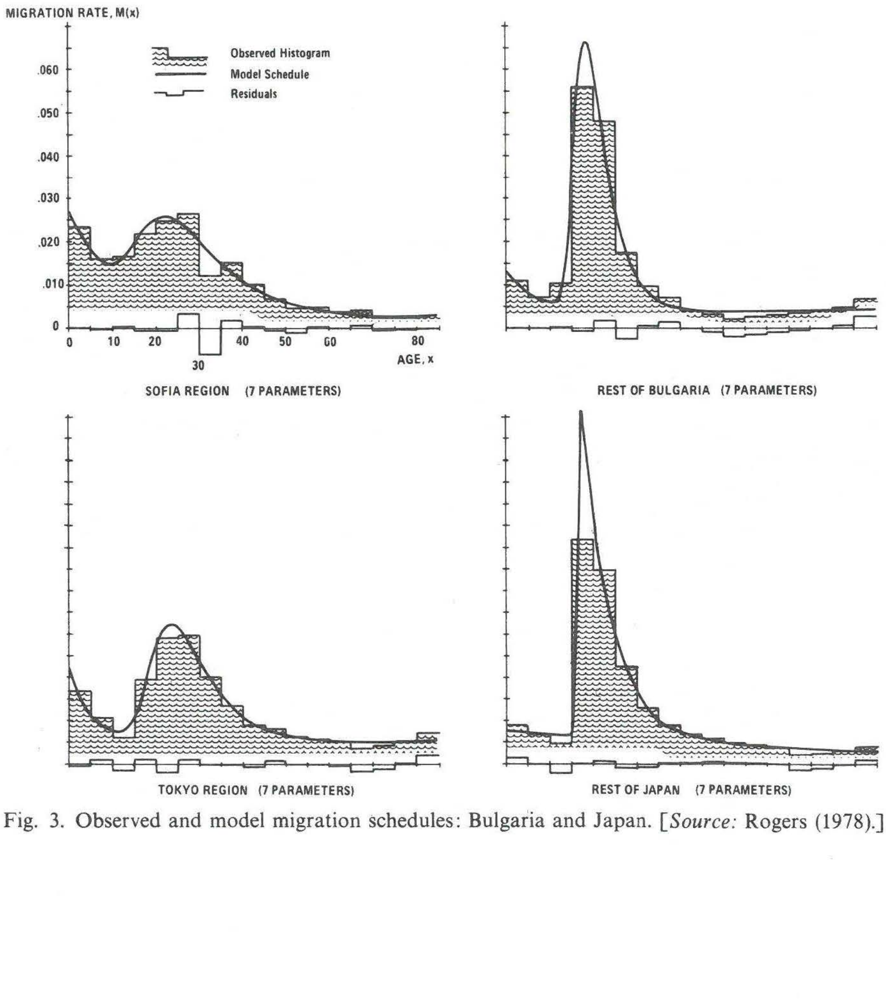

Fig. 3. Observed and model migration schedules: Bulgaria and Japan. [*Source:* Rogers (1978).]

Table 1

Parameters and variables defining the model migration schedule: Sweden, Great Britain, Bulgaria, and Japan.[a]

Parameters and variables	Sweden, 1974		Great Britain, 1970		Bulgaria, 1975		Japan, 1969	
	Stockholm	R.S.	London	R.G.B.	Sofia	R.B.	Tokyo	R.J.
Population (000)	1,487	6,670	17,316	36,871	1,070	7,657	29,496	75,169
GMR	1.45	0.28	1.04	0.44	0.29	0.10	2.60	0.71
a_1	0.0285	0.0189	0.0153	0.0143	0.0257	0.0099	0.0188	0.0079
α_1	0.1032	0.1033	0.1008	0.0687	0.0918	0.1503	0.1986	0.0110
a_2	0.0452	0.0762	0.0446	0.0519	0.0504	0.1549	0.0688	0.0909
α_2	0.0912	0.1151	0.1045	0.1042	0.0901	0.2279	0.1320	0.1528
μ_2	20.16	18.22	19.03	18.26	20.18	17.35	21.69	16.61
λ_2	0.3441	0.8913	0.4585	3.1953	0.1434	0.3735	0.2016	3.3391
a_3	0.0000	0.0000	0.0001					
α_3	0.6851	1.1593	1.2231					
μ_3	79.00	74.81	72.93					
λ_3	0.1148	0.2023	0.2209					
c	0.0029	0.0022	0.0051	0.0035	0.0026	0.0040	0.0051	−0.0002
$\bar{n}$	29.21	27.19	32.90	29.44	27.48	27.46	32.34	28.57
σ_2	3.77	7.74	4.39	30.67	1.59	1.64	1.53	21.85
σ_3	0.17	0.17	0.18					
x_1	15.32	15.97	15.01	17.59	12.00	12.00	12.18	15.90
x_p	23.71	20.48	22.12	19.31	22.33	18.66	23.74	18.00
x_r	63.20	60.15	65.2					
X	8.39	4.51	7.11	1.73	10.32	6.66	11.56	2.10
A	26.72	29.95	29.48	29.36	26.26	27.46	34.49	33.01
B	0.0206	0.0500	0.0235	0.0444	0.0113	0.0610	0.0253	0.0752

[a]*Source*: Rogers (1978).

multiply the four numbers shown in the table by the desired level of *GMR*. For example, the observed *GMR* for migration out of the Stockholm region in 1974 was 1.45. Multiplying $a_1 = 0.0285$ by 1.45 gives 0.0413, the appropriate value of a_1 with which to generate the migration schedule having a *GMR* of 1.45.

The remaining seven model schedule parameters in table 1 refer to migration *profile*: α_1, α_2, μ_2, λ_2, α_3, μ_3, and λ_3. Their values remain constant for all levels of the *GMR*. Taken together, they define the age profile of migration from one region to another (e.g., from the Stockholm region to the rest of Sweden). Schedules without a retirement peak yield only the four profile parameters: α_1, α_2, μ_2, and λ_2.

Set out below the model schedule parameters in table 1 are several 'derived' variables – variables derived either from the original parameters or from the migration curve generated by them. In addition to the mean age of migration, $\bar{n}$, they are:

(i) the measures of labor force and retirement curve asymmetry: $\sigma_2 = \lambda_2/\alpha_2$, and $\sigma_3 = \lambda_3/\alpha_3$, respectively,
(ii) the ages associated with the low point, x_1, the high peak, x_p, and the retirement peak, x_r,
(iii) two shifts: the labor force shift, X, and the parental shift, A, and
(iv) the labor force jump, B.[1]

Two major classes of migration profiles are illustrated in figs. 2 and 3: migration from the capital region to the rest of the nation, i.e., *capital outflow*, and migration from the rest of the nation to the capital region, i.e., *capital inflow*. A cursory visual examination reveals that the two sets of flows exhibit strikingly different age profiles. The parameters and variables in table 1 articulate more precisely these differences.

The most apparent difference between the age profiles of the capital outflow and inflow migration schedules is the dominance of young labor force migrants in the latter, i.e., proportionately more migrants aged 15 to 24 appear in capital inflow schedules. As a result, the rate of ascent of the labor force curve, λ_2, is always much more steeper in the inflow schedules than in the outflow schedules, i.e., $\lambda_2(i) > \lambda_2(0)$. We shall call this characteristic *labor dominance.*

A second profile attribute is the degree of asymmetry in the labor force curve of the migration schedule, i.e., the ratio of the rate of ascent λ_2, to the rate of descent α_2, designated by σ_2 in table 1. In all of the four countries examined, the labor force curve of the capital inflow profile is more asymmetric than that of the corresponding outflow profile, i.e., $\sigma_2(i) > \sigma_2(0)$. We shall refer to this characteristic as *labor asymmetry.*

[1]A retirement jump could also be defined and studied in an analogous manner.

Examining the observed rates of descent of the labor and pre-labor force curves, α_2 and α_1, respectively, we find that they are close to being equal in the outflow schedules of London and Sofia (i.e., $\alpha_2 = \alpha_1$), and quite different in the case of Tokyo (i.e., $\alpha_2 < \alpha_1$). In all four capital inflow profiles, however, $\alpha_2(i) > \alpha_1(i)$. Profiles with significantly different values for α_2 and α_1, will be said to be *irregular*.

A number of derived variables such as x_1, x_p, X, A, and B, tend to move together. For example, labor dominant profiles (e.g., capital inflow schedules) exhibit lower values for x_p and X; on the other hand, profiles that are regular (e.g., capital outflow schedules) show higher values for x_p and X, and lower values for x_1, A, and B.

Finally, the schedules for Japan and Sofia show upturns in the migration rates of post-labor force age groups that do not conform to the retirement curve of the model schedule in eq. (1). This may be an indication that a different model schedule is required, e.g., a reverse negative exponential for the retirement ages. However, the relatively uncertain quality of the data for these particular age groups make such a speculation premature.

In conclusion, the empirical migration data of four industrialized nations suggest the following hypothesis: *The migration profile of a typical capital inflow schedule is, in general, more labor dominant, more labor asymmetric, and more irregular than the migration profile of the corresponding capital outflow schedule, and it is much less likely to exhibit a retirement peak.*

The level of migration, like that of *mortality*, can be measured in terms of an expected duration time, for example, the fraction of a lifetime that is expected to be lived at a particular location. However, like *fertility*, migration is a potentially repetitive event, and its level therefore can be expressed in terms of an expected number of migrations per person.

The most common demographic measure of level is the notion of *expectancy*. Demographers often refer to life expectancies, for example, when speaking about mortality, and to reproduction expectancies when discussing fertility. Migration expectancies have been used in migration studies [Wilber (1963), and Long (1973)]. However, their definitions have been *non-spatial*; migration was viewed as an *event* occurring in a national population rather than as a *flow* arising between regional populations.

The study of *spatial* population dynamics can be considerably enriched by explicitly identifying the *locations* of events and flows. This permits one to define spatial expectancies such as the expectation of life at birth or the net reproduction rate of individuals born in region i [respectively, ${}_ie(0)$ and ${}_iNRR$, say], and the expected allocation of this lifetime or rate among the various constituent regions of a multiregional population system [${}_ie_j(0)$ and ${}_iNRR_j$, respectively, $j = 1, 2, \ldots, m$]. For example, it has been estimated [Rogers (1975a)] that the expectation of life at birth of a California-born woman exposed to the 1958 U.S. schedules of mortality and migration would be 73.86 years, out of which 24.90 years would be lived outside of California. The net reproduction rate of such a woman, on 1958 fertility rates, would be 1.69, with 0.50 of that total being born outside of California.

A spatial migration expectancy based on duration times, e.g., the expected number of years lived in region j by individuals born in region i, may be complemented by an alternative definition of spatial migration expectancy – one reflecting a view of migration as a recurrent event. Just as a net reproduction rate can be apportioned among the constituent regions of a multiregional system, so too can a *net migraproduction rate*, NMR say, be similarly disaggregated by place of birth and place of residence.

The net migraproduction rate ${}_iNMR_j$ describes the average lifetime number of migrations made out of region j by an individual born in region i. The summation of ${}_iNMR_j$ over all regions of destination $(j \neq i)$ gives ${}_iNMR$, the net migraproduction rate of individuals born in region i, i.e., the average number of migrations an i-born person is expected to make during a lifetime.

The *gross migraproduction rate* measures the intensity of migration between two regions at a particular point in time. The measure, therefore, has a basically cross-sectional character, in contrast to the NMR which measures the intensity of migration over a lifetime. Consequently, the gross migraproduction rate often may prove to be a more useful measure than the net rate in that it is a 'purer' indicator of migration, in the same sense as the gross reproduction rate. However, the gross rate measures the intensity of migration at a given moment and not over a lifetime. Hence, in instances where return migration is an important factor, the gross rate and the net rate may give differing indications of geographical mobility.

Table 2 shows that the allocation of the gross migraproduction rate from the Northeast region to the South region in the United States was larger in 1968 than the allocation to the same destination of the West region's gross rate (${}_1\varepsilon_3 = 0.5525 > {}_4\varepsilon_3 = 0.4853$). Yet the opposite was true of the corresponding allocations of the net rate (${}_1\gamma_3 = 0.0965 < {}_4\gamma_3 = 0.1008$). The cause of this reversal was the significantly higher return migration to the West region (${}_3\varepsilon_4 = 0.3302 > {}_3\varepsilon_1 = 0.2606$). Thus, because of the influence of return migration, the *lifetime level* of geographical mobility to the South region of a baby girl born in the Northeast region was lower, on 1968 rates of migration and mortality, than the corresponding mobility to the same destination of a baby girl born in the West region. The 1968 *intensity* of geographical mobility to the South region, however, was higher from the Northeast region than from the West region.

2.2. *Migration probabilities*

Vital statistics and censuses of the kind regularly collected in most developed nations provide the necessary data for the computation of rates.

Table 2

Gross and net migraproduction rates and allocations by region of residence and region of birth: United States female population, 1968.[a]

	Region of residence				
Region of birth	1	2	3	4	Total
(A) Gross migraproduction rates and allocations: ${}_iGMR_j$ *and* ${}_i\varepsilon_j$					
(1) Northeast	—	0.1258	0.3253	0.1377	0.5889
	(—)	(0.2137)	(0.5534)	(0.2339)	(1.00)
(2) North Central	0.0978	—	0.3296	0.2526	0.6801
	(0.1438)	(—)	(0.4847)	(0.3715)	(1.00)
(3) South	0.1462	0.2296	—	0.1853	0.5611
	(0.2605)	(0.4092)	(—)	(0.3303)	(1.00)
(4) West	0.1005	0.2374	0.3186	—	0.6564
	(0.1531)	(0.3616)	(0.4853)	(—)	(1.00)
(B) Net migraproduction rates and allocations: ${}_iNMR_j$ *and* ${}_i\gamma_j$					
(1) Northeast	0.4178	0.0364	0.0520	0.0326	0.5387
	(0.7756)	(0.0675)	(0.0965)	(0.0604)	(1.00)
(2) North Central	0.0233	0.4665	0.0547	0.0510	0.5956
	(0.0392)	(0.7833)	(0.0919)	(0.0857)	(1.00)
(3) South	0.0320	0.0578	0.4116	0.0447	0.5460
	(0.0586)	(0.1058)	(0.7538)	(0.0818)	(1.00)
(4) West	0.0242	0.0575	0.0613	0.4649	0.6078
	(0.0398)	(0.0946)	(0.1009)	(0.7648)	(1.00)

[a]*Source*: Rogers (1975b, pp. 9 and 11).

They may be used to answer questions, such as: what is the current rate at which 40-year-old males are dying from heart disease? or at which 30-year-old women are bearing their second child? But many of the more interesting questions regarding mortality and fertility patterns are phrased in terms of probabilities, for example: what is the current probability that a man aged 40 will outlive his 38-year-old wife, or that she will bear her third child before she is 45?

Demographers normally estimate probabilities from observed rates by developing a life table. Such tables describe the evolution of a hypothetical cohort of babies born at a given moment and exposed to an unchanging age-specific schedule of vital rates. For this cohort of babies, they exhibit a number of probabilities for changes of state, such as dying, and develop the corresponding expectations of years of life spent in different states at various ages.

The simplest life tables recognize only one class of decrement, e.g., death, and their construction is normally initiated by estimating a set of age-specific probabilities of leaving the population, e.g., dying, within each interval of age, $q(x)$ say, from observed data on age-specific exit rates, $M(x)$ say. The conventional calculation that is made for an age interval five years wide is [Rogers (1975a, p. 12)]

$$q(x)=5M(x)/[1+\tfrac{5}{2}M(x)],$$

or alternatively,

$$p(x)=1-q(x)=[1+\tfrac{5}{2}M(x)]^{-1}[1-\tfrac{5}{2}M(x)], \tag{2}$$

where $p(x)$ is the age-specific probability of remaining in the population, e.g., of surviving, between exact ages x to $x+5$.

Simple life tables, generalized to recognize several modes of exit from the population are known as *multiple-decrement life tables* [Keyfitz (1968, p. 333)]. They have been applied, for example, in studies of mortality by cause of death, of first marriage and death, of labor force participation and death, and of school attendance and death.

A further generalization of the life table concept arises with the recognition of entries as well as exits. Such *increment-decrement life tables* [Schoen (1975)] allow for multiple movements between several states, for example, transitions between marital statuses and death (married, divorced, widowed, dead), or between labor force statuses and death (employed, unemployed, retired, dead).

Multiple-radix increment-decrement life tables that recognize several regional populations each with a region-specific schedule of mortality and several destination-specific schedules of internal migration are called *multiregional life tables* [Rogers (1973a, b)]. They represent the most general class of life tables and were originally developed for the study of interregional migration between interacting multiple regional populations. Their construction is initiated by estimating a matrix of age-specific probabilities of surviving and migrating $\boldsymbol{P}(x)$ from data on age-specific death and migration rates, $\boldsymbol{M}(x)$. Rogers and Ledent (1976) show that the equation for this estimation may be expressed as the matrix analog of eq. (2),[2]

$$\boldsymbol{P}(x)=[\boldsymbol{I}+\tfrac{5}{2}\boldsymbol{M}(x)]^{-1}[\boldsymbol{I}-\tfrac{5}{2}\boldsymbol{M}(x)]. \tag{3}$$

One of the most useful statistics provided by a life table is the average expectation of life beyond age x, $e(x)$ say, calculated by applying the probabilities of survival $p(x)$ to a hypothetical cohort of babies and then observing their average length of life beyond each age.

[2]This formula is applicable only when migration is viewed as a move, i.e., an event. If the data treat migration as a transition, i.e., a transfer during a specified unit time interval, then eq. (3) yields only an approximation. See Ledent (1978).

Expectations of life in a multiregional life table reflect the influences of mortality and migration. Thus in addition to carrying out their traditional function as indicators of mortality levels, they also serve as indicators of levels of internal migration. For example, consider the regional expectations of life at birth that are set out in table 3 for the U.S. female population in 1968. A baby girl born in the West, and exposed to the multiregional schedule of mortality and migration that prevailed in 1968, could expect to live an average of 75.57 years, out of which total an average of 11.32 years would be lived in the South. Taking the latter as a fraction of the former, we have in $\theta = 0.1497$ a useful indicator of the (lifetime) *migration level* from the West to the South that is implied by the 1968 multiregional schedule. (Compare these migration levels with those set out earlier in table 2.)

Table 3

Expectations of life at birth and migration levels by region of residence and region of birth: United States female population, 1968.[a]

	Region of residence				
Region of birth	1	2	3	4	Total
(A) Expectations of life at birth: ${}_ie_j(0)$					
(1) Northeast	54.13	5.08	10.11	5.25	74.56
(2) North Central	3.76	52.14	10.48	8.05	74.44
(3) South	5.06	7.88	54.53	6.93	74.40
(4) West	3.90	7.94	11.32	52.41	75.57
(B) Migration levels: ${}_i\theta_j$					
(1) Northeast	0.7260	0.0681	0.1356	0.0704	1.00
(2) North Central	0.0506	0.7005	0.1408	0.1081	1.00
(3) South	0.0680	0.1060	0.7328	0.0931	1.00
(4) West	0.0516	0.1051	0.1497	0.6936	1.00

[a]*Source*: Rogers (1975b), p. 4).

Life tables are normally calculated using observed data on age-specific vital rates. However, in countries without reliable vital registration systems, recourse is often made to inferential estimation methods that rely on *model* schedules of mortality or fertility. These methods may be extended to multiregional demographic analysis by the introduction of the notion of a *model multiregional life table* [Rogers (1975a, pp. 146–154)].

Model multiregional life tables approximate the regional mortality and migration schedules of a multiregional population, by drawing on the regularities exhibited by the mortality and migration schedules of comparable populations. A collection of such tables may be entered with empirically determined survivorship proprtions (disaggregated by region of birth and region of residence) to obtain the particular combination of regional expectations of life at birth (disaggregated by region of birth and region of residence) that best matches the mortality and migration levels implied by these observed proportions [Rogers 1975a, pp. 172–189)].

Age-specific probabilities of migrating, $p_{ij}(x)$, in empirical multiregional life tables mirror the fundamental regularities exhibited by observed migration rates. The migration risks experienced by different age and sex groups of a given population are strongly interrelated, and higher (or lower) than average migration risks among one segment of a particular population normally imply higher (or lower) than average migration risks for other segments of the same population. This association stems in part from the fact that if socioeconomic conditions at a location are good or poor for one group in the population, they are also likely to be good or poor for other groups in the same population. Since migration is widely held to be a response to spatial variations in socioeconomic conditions, these high intercorrelations between age-specific migration risks are not surprising.

A relatively close accounting of the regularities shown by empirically estimated migration probabilities may be obtained with the zero-intercept linear regression model

$$p_{ij}(x) = \beta(x)_i\theta_j. \tag{4}$$

Estimates of the regression coefficients $\beta(x)$ may be used in the following way. First, starting with a complete set of multiregional migration levels ${}_i\theta_j$ one calculates the matrix of migration probabilities $\boldsymbol{P}(x)$ for every age, using eq. (4). With $\boldsymbol{P}(x)$ established, one then may compute the usual life table statistics, such as the various region-specific expectations of life at each age. The collective results of all these computations constitute a *model multiregional life table.*

3. Dynamics

Until about a decade ago, the contribution of internal migration to population growth was assessed in non-spatial terms. The evolution of regional populations affected by migration was examined by adding the contribution of *net* migration to that of natural increase. The dynamics of redistribution, therefore, were expressed over time but not over space; the evolution of a system of interacting regional populations was studied one region at a time.

Beginning in the mid-1960's, efforts to express the dynamics of spatial change in matrix form began to appear in the demographic literature and had considerable success in describing processes of geographical redistribution in multiregional population systems. Such studies, typically, adopted a process of change in which a population disaggregated into several classes and set out as a vector, is premultiplied by a matrix that advances the population forward over time, and geographically across space.

The spatial distribution of a multiregional population across its constituent regions and the age compositions of its regional populations are determined by the interactions of fertility, mortality, and interregional migration. People are born, age with the passage of time, reproduce, migrate, and ultimately die. In connecting these events and flows to determine the growth rate of each population, one also obtains the number of people in each region and their age composition.

Spatial processes of population growth and *redistribution* may be studied with the aid of multiregional generalizations of the discrete Leslie model [Rogers (1966)] or of the continuous Lotka renewal equation [LeBras (1971)]. These formal representations of multiregional population growth and change permit one, for example, to focus on the mathematical analysis and design of particular *intervention* policies for redirecting the spatial population system's growth path toward a target multiregional distribution [Rogers (1968 and 1971), Willekens (1976), Willekens and Rogers (1977)]. Such models also permit one to examine more rigorously the dynamics of *urbanization* [Rogers (1978)].

3.1. Population redistribution

Multiregional generalizations of the classical models of mathematical demography project the numerical consequences, to an initial (single-sex) multiregional population, of a particular set of assumptions regarding future fertility, mortality, and internal migration. The mechanics of such projections typically revolve around three basic steps. The first ascertains the starting age-region distributions and the age-specific regional schedules of fertility, mortality, and migration to which the multiregional population has been subject during a past period; the second adopts a set of assumptions regarding the future behavior of such schedules; and the third derives the consequences of applying these schedules to the initial population.

The *discrete* model of multiregional demographic growth expresses the population projection process by means of a matrix operation in which a multiregional population, set out as a vector, is multiplied by a growth matrix that survives that population forward over time. The projection calculates the region and age-specific survivors of a multiregional population of a given sex and adds to this total the new births that survive to the end of the unit time interval. This process may be described by the matrix model

$$\{K(t+1)\}=G\{K(t)\}, \tag{5}$$

where the vector $\{K(t)\}$ sets out the multiregional population disaggregated by age and region, and the matrix G is composed of zeroes and elements that represent the various age–region-specific components of population change.

As in the single-region model, survival of individuals from one moment in time to another, say 5 years later, is calculated by diminishing each regional population to take into account the decrement due to mortality. In the multiregional model, however, we also need to include the decrement due to outmigration and the increment contributed by inmigration. An analogous problem is presented by surviving children born during the 5 year interval. Some of these migrate with their parents; others are born after their parents have migrated but before the unit time interval has elapsed.

It is well known that a population undisturbed by migration will, if subjected to an unchanging regime of mortality and fertility, ultimately achieve a stable constant age distribution that increases at a constant stable growth ratio, λ say. In Rogers (1966) it is shown that this same property obtains region-by-region in the case of a multiregional population system that is closed to external migration and subjected to an unchanging multiregional schedule of mortality, fertility, and internal migration. Knowledge of the asymptotic properties of such a population projection helps us understand the meaning of observed age-specific birth, death, and migration rates. In particular, the quantity $r=0.2\ln\lambda$ gives the intrinsic rate of growth that is implied by the indefinite continuation of observed schedules of mortality, fertility, and migration.

A related but equally useful demographic measure is the *stable equivalent*, Y [Keyfitz (1969)], of each region and its proportional allocation across age groups in that region, $C_i(x)$, which is the region's *stable age composition*. The former may be obtained by projecting the observed multiregional population forward until it becomes stable and dividing the resulting age–region-specific totals by the stable growth ratio λ raised to the nth power, where n is the number of iterations that were needed to achieve stability. Summing across all age groups in a region gives the regional stable equivalent Y_i; dividing the number in each age group in region i by Y_i gives $C_i(x)$, region i's age composition at stability. Finally, dividing each region's stable equivalent by the sum total of all regional stable equivalents gives SHA_i, region i's *stable regional share* of the total multiregional population at stability.

The growth, spatial distribution, and regional age compositions of a 'closed' multiregional population are completely determined by the recent history of fertility, mortality, and internal migration it has been subject to. Its current crude regional birth, death, migration, and growth rates are all governed by the interaction of the prevailing regime of growth with the current regional age compositions and regional shares of the total population. The dynamics of such growth and change are clearly illustrated, for example, by the four-region population system exhibited in tables 4 and 5, and fig. 4, which describe the evolution of the U.S. total population resident in the four Census Regions that collectively exhaust the national territory: (1) the Northeast Region, (2) the North Central Region, (3) the South Region, and (4) the West Region.

Table 4

Projected annual regional rates of growth [$r_i(t)$]: United States total population.[a]

	Region *i*				
Time *t*	(1) Northeast	(2) North Central	(3) South	(4) West	Total
(A) Base year: 1958					
1958	0.008484	0.011421	0.016831	0.027227	0.014777
1968	0.009335	0.013217	0.017296	0.026612	0.015896
1978	0.012085	0.015817	0.018111	0.026624	0.017776
1988	0.014067	0.017446	0.019041	0.026256	0.019060
1998	0.016221	0.019284	0.020158	0.026261	0.020483
2008	0.018264	0.020653	0.021190	0.025739	0.021574
Stability			0.021810		
(B) Base year: 1968					
1968	0.003808	0.006633	0.011606	0.014698	0.008890
1978	0.005500	0.008549	0.011317	0.014101	0.009734
1988	0.004323	0.006853	0.008900	0.011126	0.007756
1998	0.004663	0.007056	0.008621	0.010408	0.007703
2008	0.005085	0.006953	0.008088	0.009466	0.007435
2018	0.004555	0.006175	0.007204	0.008380	0.006630
Stability			0.005769		

[a]*Source*: Rogers and Castro (1976, p. 59).

The prevailing growth regime is held constant and two sets of spatial population projections are obtained. These offer interesting insights into the growth rates, regional shares, and regional age compositions that evolve from a projection of current trends into the future, taking 1958 and 1968 as alternative base years from which to initiate the projections.

Table 4 shows that between the two base years (1958 and 1968) the regional growth rates of the South and West Regions were higher than the national average, whereas those of the Northeast and North Central Regions were lower. By virtue of the assumption of a linear model and a constant

Table 5

Observed and projected regional shares $[SHA_i(t)]$: United States total population.[a]

Time t	Region i (1) North-east	(2) North Central	(3) South	(4) West	Total
(A) Base year: 1958					
1958	0.2503	0.2955	0.3061	0.1481	1.0000
1968	0.2347	0.2861	0.3122	0.1670	1.0000
1978	0.2202	0.2792	0.3157	0.1850	1.0000
1988	0.2084	0.2740	0.3164	0.2012	1.0000
1998	0.1986	0.2699	0.3161	0.2154	1.0000
2008	0.1907	0.2668	0.3150	0.2275	1.0000
Stability	0.1443	0.2525	0.3061	0.2971	1.0000
(B) Base year: 1968					
1968	0.2413	0.2784	0.3090	0.1713	1.0000
1978	0.2306	0.2728	0.3198	0.1768	1.0000
1988	0.2216	0.2699	0.3243	0.1841	1.0000
1998	0.2143	0.2676	0.3280	0.1901	1.0000
2008	0.2082	0.2660	0.3307	0.1950	1.0000
2018	0.2035	0.2647	0.3328	0.1989	1.0000
Stability	0.1764	0.2617	0.3425	0.2194	1.0000

[a] *Source*: Rogers and Castro (1976, p. 60).

regime of growth, all four regional growth rates ultimately converge to the same intrinsic rate of increase: 0.021810 in the case of the 1958 growth regime, and 0.005699 in the case of the 1968 growth regime. However, what is interesting is that the trajectories converging toward these two intrinsic rates are quite different. Only in the case of the West Region is a decline in the long-run growth rate projected under either of the two observed growth regimes. Also of interest is the substantial difference between the two intrinsic growth rates themselves, which clearly documents the dramatic drop in fertility levels that occurred during the decade in question.

Both in 1958 and in 1968 approximately 31 percent of the U.S. population resided in the South. This regional share remains relatively unchanged in the projection under the 1958 growth regime but increases to over 34 percent under the 1968 growth regime. Thus the ultimate spatial allocation of the national population changed in favor of the South during the decade between 1958 and 1968. According to table 5, a large part of this change occurred at the expense of the West's regional share, which declined from roughly 30 percent to about 22 percent. Despite this decline, the West's projected share of the national population nonetheless shows a substantial increase over the base year allocation. This increase and that of the South

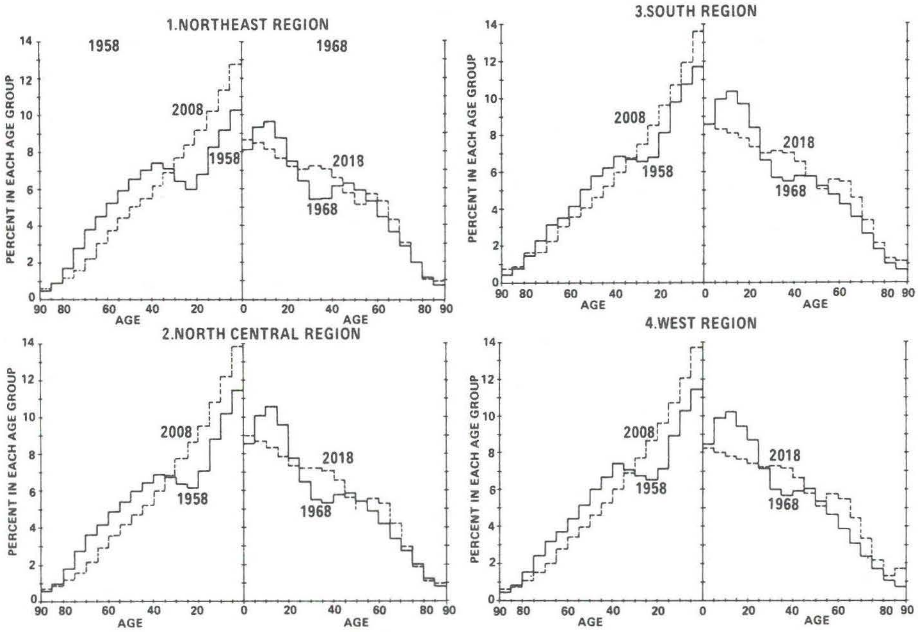

Fig. 4. Observed and projected regional age compositions: United States total population. [*Source*: Rogers and Castro (1976, p. 13).]

match the decrease in the regional shares of the Northeast and North Central Regions. Thus, under either projection, the 'North's' share of the U.S. population is headed for a decline while that of the 'South West' is due to increase.

Fig. 4 vividly illustrates the impact that a high growth rate has on age composition. The four regional graphs depict both the age compositions observed at the time of the base year and those projected 50 years forward on the assumption of an unchanging regime of growth. Since the regional growth regimes in 1958 produced a relatively high time series of growth rates after a period of 50 years, the age compositions of the left-hand side of fig. 4 show a relatively steep slope. Because the 1968 growth regimes, on the other hand, produced relatively low regional growth rates after 50 years, the regional age compositions on the right-hand side show a relatively shallow slope.

The growth dynamics of empirical populations are often obscured by the influences that particular initial conditions have on future population size and composition. Moreover, the vast quantities of data and parameters that go into a description of such empirical dynamics make it somewhat difficult to maintain a focus on the broad general outlines of the underlying demographic process, and instead often encourage a consideration of its more peculiar details. Finally, studies of empirical growth dynamics are constrained in scope to population dynamics that have been experienced and recorded; they cannot be extended readily to studies of population dynamics that have been experienced but not recorded or that have not been experienced at all. In consequence, demographers frequently have resorted to examinations of the dynamics exhibited by hypothetical *model* populations that have been exposed to hypothetical *model* schedules of growth and change.

The study of population dynamics by means of model schedules and model stable populations has been pioneered by Ansley Coale. In a series of articles and books published during the past decade, he and his collaborators have established a paradigm that has become the standard approach of most mathematical demographers. This paradigm is developed in an early study in which Coale and Demeny (1966) present two sets of model (single-region) stable populations that evolve after a long and continued exposure to particular combinations of unchanging schedules of growth. Each population is identified by two non-redundant indices of variation relating to fertility and mortality, respectively, and evolves out of a particular combination of a model life table and an intrinsic rate of growth or gross reproduction rate. The former are referred to as the 'growth rate' stable populations; the latter are called the '*GRR*' stable populations and rely on a model fertility schedule with a given mean age of childbearing $\bar{m}$, which is assumed to be 29 years. Symbolically, the two sets of model stable populations may be expressed as

(1) Growth rate stable populations: $f[e(0), r]$,
(2) *GRR* stable populations: $g[e(0), GRR]$,

where $e(0)$ is the expectation of life at birth, r is the intrinsic annual rate of growth, and *GRR* is the gross reproduction rate.

The paradigm introduced by Coale and Demeny may be extended to multiregional populations. In such an extension, a particular model multiregional life table is linked with an intrinsic rate of growth or set of gross reproduction rates. In the former case one must also specify a set of additional indices that relate to spatial distribution, for example, the spatial distribution of births or of people [Rogers (1975a) and Rogers and Willekens (1976)]. Symbolically, the two sets of model multiregional stable populations may be expressed as

(1) Growth rate multiregional stable populations: $f(\boldsymbol{EXP}, r, \boldsymbol{SRR}, \boldsymbol{\theta})$ or $h(\boldsymbol{EXP}, r, \boldsymbol{SHA}, \boldsymbol{\theta})$,
(2) *GRR* multiregional stable populations: $g(\boldsymbol{EXP}, \boldsymbol{GRR}, \boldsymbol{\theta})$,

where $\boldsymbol{EXP}$ is a diagonal matrix of regional expectations of life at birth, ${}_ie(0)$;

$\boldsymbol{SRR}$ is a matrix of stable radix ratios SRR_{ji}; $\boldsymbol{SHA}$ is a diagonal matrix of stable regional shares SHA_i; $\boldsymbol{\theta}$ is a matrix of migration levels ${}_j\theta_i$; and $\boldsymbol{GRR}$ is a diagonal matrix of regional gross reproduction rates GRR_i. (Alternatively, we could instead have adopted gross migraproduction rates GMR_{ji} in place of the migration levels ${}_j\theta_i$. In this event the matrix $\boldsymbol{\theta}$ would be replaced by the matrix $\boldsymbol{GMR}$.)

Tables 6 and 7 set out several specimen model multiregional stable populations that were generated by means of specific combinations of model schedules of fertility, mortality, and migration. The model fertility schedules were obtained by applying Coale and Demeny's (1966) basic age profile, for a mean age of childbearing of 29 years, to different values of GRR; model mortality schedules were taken from their 'WEST' family; and the model migration schedules were calculated using the 'AVERAGE' regression equations set out in Appendix Table D.2 of Rogers and Castro (1976). Each of the populations in the two tables may be expressed symbolically by any one of the three forms described earlier.

Model multiregional stable populations readily reveal the long-run consequences of particular changes in fertility, mortality, and migration levels. For example, consider several of the more interesting aspects of population dynamics that are manifested in the stable populations presented in tables 6 and 7. First, identical schedules of regional fertility and mortality produce identical stable regional age compositions. The stable regional shares of such populations, however, will vary inversely with the ratio of their respective migration levels. Second, higher values of the intrinsic growth rate lead to stable (regional) populations that taper more rapidly with age and, in consequence, include a higher proportion of the population below every age. Third, fertility affects not only the rate of growth of a stable population, but also its regional distribution. Fourth, mortality and migration schedules affect the form of the stable regional age compositions and the stable regional shares in an obvious way, and any idiosyncracies in the age patterns of such schedules will be reflected in the age patterns of the corresponding regional populations.

Somewhat surprising is the relative insensitivity of regional age compositions and birth rates to changes in migration levels. For example, consider the case of unequal migration levels with $GRR_1 = 1$, $GRR_2 = 3$, and that with $GRR_1 = 3$, $GRR_2 = 1$. In the first case the region with the larger (by a factor of 2) outmigration has the higher fertility level; in the second case the situation is reversed. Yet in both instances the population of the region with the higher fertility level has an average age of approximately 23 years and a birth rate of approximately 41 per 1000. This insensitivity to migration behavior does not extend to aggregate systemwide measures, however. For the same example, the intrinsic growth rate and systemwide birth rate are considerably lower in the first case than in the second; the higher fertility

Table 6

Model growth rate multiregional (two-region) female stable populations with equal mortality levels: ${}_1e(0)={}_2e(0)=70$ years, intrinsic rate of growth $r=0.00,\ldots,0.03$.[a]

		$r=0.00$			$r=0.01$			$r=0.02$			$r=0.03$		
		Region			Region			Region			Region		
Growth rate set[b]		(1)+(2)	(1)	(2)	(1)+(2)	(1)	(2)	(1)+(2)	(1)	(2)	(1)+(2)	(1)	(2)
(A)	*SHA*	1.0000	0.5000	0.5000	1.0000	0.5000	0.5000	1.0000	0.5000	0.5000	1.0000	0.5000	0.5000
	b	0.0143	0.0143	0.0143	0.0203	0.0203	0.0203	0.0276	0.0276	0.0276	0.0358	0.0358	0.0358
${}_1\theta_2={}_2\theta_1=0.3$	Δ	0.0143	0.0143	0.0143	0.0103	0.0103	0.0103	0.0076	0.0076	0.0076	0.0058	0.0058	0.0058
$SRR_{12}=SRR_{21}=1$	a	37.92	37.92	37.92	32.82	32.82	32.82	28.16	28.16	28.16	24.11	24.11	24.11
(B)	*SHA*	1.0000	0.5999	0.4001	1.0000	0.5919	0.4081	1.0000	0.5839	0.4162	1.0000	0.5762	0.4238
	b	0.0143	0.0119	0.0179	0.0203	0.0172	0.0249	0.0276	0.0236	0.0331	0.0358	0.0311	0.0422
${}_1\theta_2=0.2, {}_2\theta_1=0.4$	Δ	0.0143	0.0119	0.0179	0.0103	0.0072	0.0149	0.0076	0.0036	0.0131	0.0058	0.0011	0.0122
$SRR_{12}=SRR_{21}=1$	a	37.92	39.24	35.94	32.82	34.20	30.82	28.16	29.52	26.26	24.11	25.38	22.37

[a]*Source*: Rogers and Castro (1976, p. 49).

[b]Parameters under stability: SHA = regional share, b = birth rate, Δ = absence rate, a = average age, and SSR = stable radix ratio.

Table 7

Model GRR multiregional (two-region) female stable populations with equal mortality levels: ${}_1e(0) = {}_2e(0) = 70$ years, gross reproduction rates $GRR_1 = 1, 2, 3$ and $GRR_2 = 1$.[a]

GRR set[b]		$GRR_1=1$, $GRR_2=1$ Region			$GRR_1=2$, $GRR_2=1$ Region			$GRR_1=3$, $GRR_2=1$ Region		
		(1)+(2)	(1)	(2)	(1)+(2)	(1)	(2)	(1)+(2)	(1)	(2)
(A)	SHA	1.0000	0.5000	0.5000	1.0000	0.6168	0.3832	1.0000	0.6801	0.3199
	b	0.0131	0.0131	0.0131	0.0232	0.0282	0.0152	0.0331	0.0409	0.0165
${}_1\theta_2 = {}_2\theta_1 = 0.3$	Δ	0.0153	0.0153	0.0153	0.0091	0.0140	0.0010	0.0063	0.0141	−0.0103
	r	−0.0022	—	—	0.0142	—	—	0.0268	—	—
	a	39.08	39.08	39.08	30.80	28.84	33.96	25.34	23.06	30.17
	SRR_{21}	1.000	—	—	0.335	—	—	0.189	—	—
(B)	SHA	1.0000	0.6667	0.3333	1.0000	0.7556	0.2444	1.0000	0.7976	0.2024
	b	0.0131	0.0131	0.0131	0.0254	0.0286	0.0156	0.0363	0.0413	0.0167
${}_1\theta_2 = 0.2$,	Δ	0.0153	0.0153	0.0153	0.0082	0.0114	−0.0016	0.0057	0.0107	−0.0139
${}_2\theta_1 = 0.4$	r	−0.0022	—	—	0.0172	—	—	0.0306	—	—
	a	39.08	39.08	39.08	29.42	28.25	33.04	23.88	22.56	29.09
	SRR_{21}	0.500	—	—	0.176	—	—	0.103	—	—
		$GRR_1=1, GRR_2=1$			$GRR_1=1, GRR_2=2$			$GRR_1=1, GRR_2=3$		
(C)	SHA	1.0000	0.6667	0.3333	1.0000	0.5391	0.4609	1.0000	0.4550	0.5450
	b	0.0131	0.0131	0.0131	0.0208	0.0148	0.0277	0.0293	0.0161	0.0404
${}_1\theta_2 = 0.2$,	Δ	0.0153	0.0153	0.0153	0.0101	0.0042	0.0171	0.0071	−0.0061	0.0182
${}_2\theta_1 = 0.4$	r	−0.0022	—	—	0.0106	—	—	0.0222	—	—
	a	39.08	39.08	39.08	32.52	35.08	29.52	27.22	31.52	23.63
	SRR_{21}	0.500	—	—	1.603		—	3.010	—	—

[a]*Source*: Rogers and Castro (1976, p. 50).
[b]See footnote b of table 6.

region, however, assumes a stable regional share of only 54 percent in the first case but of 80 percent in the second.

Finally, it is important to underscore the powerful influence that past patterns of fertility, mortality, and migration play in the determination of present regional age compositions and shares, inasmuch as the latter arise out of a history of regional births, deaths, and internal migration. For example, a region experiencing high levels of fertility will have a relatively younger population, but if this region also is the origin of high levels of outmigration, a large proportion of its young adults will move to other regions, producing a higher growth rate in the destination regions while lowering the average age of its own population. This suggests that inferences made, say about fertility, on the basis of a model that ignores internal migration may be seriously in error. For example, table 7(A) illustrates the significant impact on the ultimate stable age composition and regional share of Region (2) that is occasioned by a doubling and tripling of fertility levels in Region (1) while everything else is held constant. The mean age of the population in Region (2) declines by 5.1 and 8.9 years, respectively, while its regional share decreases by 24 percent in the first instance and by 36 percent in the second.

3.2. Intervention

Public concern over population matters generally arises when the demographic acts of individuals affect societal welfare to produce a sharp divergence between the aggregation of individual net benefits and social well-being. In such situations, population processes properly become the focus of public debate and the object of public policy.

Because a policy to increase mortality is not only politically infeasible but also morally offensive, reductions in the size of regional populations must be brought about by reductions in their birth rates or by some control of internal migration.

The effects of birth or migration control in a multiregional population system governed by the growth dynamics defined in eq. (5) may be introduced by an intervention vector, $\{\boldsymbol{f}\}$ say, which is added to the population in each time period [Rogers (1968, p. 53)],

$$\{\boldsymbol{K}(t+1)\} = \boldsymbol{G}\{\boldsymbol{K}(t)\} + \{\boldsymbol{f}\}. \tag{6}$$

Starting with an initial population distribution at a given moment in time $t=0$, we may trace out the cumulative impact of a particular intervention vector, acting under an unchanging growth regime, by repeatedly applying eq. (6) to derive [Rogers (1971, p. 99)]

$$\{\boldsymbol{K}(t)\} = \boldsymbol{G}^t\{\boldsymbol{K}(0)\} + (\boldsymbol{I}-\boldsymbol{G})^{-1}(\boldsymbol{I}-\boldsymbol{G}^t)\{\boldsymbol{f}\}.$$

Assuming now that a vector of target populations at the planning horizon year T, has been defined, the intervention vector that will bring this about is readily calculated as

$$\{f\}=(I-G^T)^{-1}[(I-G)[\{K(T)\}-G^T\{K(0)\}]]. \qquad (7)$$

By way of example, consider the following hypothetical illustration [Rogers (1971, pp. 101–102)]. Imagine a national population, undisturbed by international migration, that is disaggregated into two sectors, urban and rural, and is described by a two-region components-of-change population growth model of the following form:

$$\{K(1965)\}=G\{K(1950)\}=\begin{bmatrix}\frac{5}{6} & \frac{1}{4}\\ \frac{1}{4} & \frac{5}{6}\end{bmatrix}\begin{bmatrix}288\\576\end{bmatrix}=\begin{bmatrix}384\\522\end{bmatrix}.$$

According to this model, one-fourth of the urban population of 288 individuals moves to rural areas during the 15-year unit time interval, and one-fourth of the rural population of 576 individuals moves in the opposite direction. Consider the growth of this population over two consecutive time intervals,

$$\{K(1980)\}=G^2\{K(1950)\}=\begin{bmatrix}\frac{109}{144} & \frac{5}{12}\\ \frac{5}{12} & \frac{109}{144}\end{bmatrix}\begin{bmatrix}288\\576\end{bmatrix}=\begin{bmatrix}458\\556\end{bmatrix},$$

and assume that it is desirable to achieve a redistribution, such that by 1980 the national population is equally divided among the two regions. What intervention vector $\{f\}$ will achieve such a goal?

First, we compute $(I-G^2)^{-1}$ and recall that our target distribution is

$$\{K(1980)\}=\begin{bmatrix}507\\507\end{bmatrix},$$

next we find

$$(I-G)[\{K(1980)\}-G^2\{K(1950)\}],$$

and then use eq. (7) to obtain

$$\{f\}=(I-G^2)^{-1}\{(I-G)[\{K(1980)\}-G^2\{K(1950)\}]\}=\begin{bmatrix}\frac{588}{19}\\ -\frac{588}{19}\end{bmatrix}.$$

As a check, observe that

$$\{K(1965)\}=G\{K(1950)\}+\{f\}=\begin{bmatrix}\frac{5}{6} & \frac{1}{4}\\ \frac{1}{4} & \frac{5}{6}\end{bmatrix}\begin{bmatrix}288\\576\end{bmatrix}+\begin{bmatrix}\frac{588}{19}\\ -\frac{588}{19}\end{bmatrix}=\begin{bmatrix}\frac{7884}{19}\\ \frac{9900}{19}\end{bmatrix}$$

and

$$\{\boldsymbol{K}(1980)\} = \boldsymbol{G}\{\boldsymbol{K}(1965)\} + \{\boldsymbol{f}\} = \begin{bmatrix} \frac{5}{6} & \frac{1}{4} \\ \frac{1}{4} & \frac{5}{6} \end{bmatrix} \begin{bmatrix} \frac{7884}{19} \\ \frac{9900}{19} \end{bmatrix} + \begin{bmatrix} \frac{588}{19} \\ -\frac{599}{19} \end{bmatrix} = \begin{bmatrix} 507 \\ 507 \end{bmatrix}.$$

Drewe (1971) has used the above intervention model to demonstrate that a rather major redirection of internal migrants would be necessary to achieve national plans for regional population targets in the year 2000 for the three northern provinces of the Netherlands (Groningen, Friesland, and Drenthe). In a more recent paper, he updates his analysis in the light of more current data and a revised plan [Drewe (1977)].

Willekens (1976) has developed the intervention perspective much further in his dissertation. He shows that the model in eq. (6) may be usefully extended along three important directions:

(1) the introduction of economic control variables and the specification of their impacts on the population distribution,
(2) the expansion of the initial period control problem to a truly dynamic control problem, and
(3) the admission of other constraints on both the state and the control variables, and the formulation of policy objectives in terms of variables other than population targets.

A fundamental feature of population policy is the nondemographic character of its goals and instruments. Control of migration flows is rarely justified solely on the grounds of achieving target population totals. Nor is the control exercised directly on population flows. Rather, the goals and interventions are expressed in terms of economic variables such as regional incomes, employment, housing construction, and government expenditures. Therefore, let $\{\boldsymbol{u}\}$ be a vector of socioeconomic control variables and, for the sake of simplicity, assume the linear relationship $\{\boldsymbol{f}\} = \boldsymbol{A}\{u\}$, where $\boldsymbol{A}$ is a time invariant coefficient matrix. An element a_{ij} denotes the impact of the jth control variable on the ith element of $\{\boldsymbol{f}\}$. Substituting this relationship into eq. (6) gives

$$\{\boldsymbol{K}(t+1)\} = \boldsymbol{G}\{\boldsymbol{K}(t)\} + \boldsymbol{A}\{u\}. \tag{8}$$

Eq. (8) links the population distribution at a given time to the population distribution at a preceding point in time, and to socioeconomic policy variables. The model is closely related to the static policy model developed by Tinbergen (1963). A solution exists if the rank of $\boldsymbol{A}$ is equal to the number of targets. The solution is unique of $\boldsymbol{A}$ is non-singular, i.e., if, in the jargon of Tinbergen, the number of instruments is equal to the number of targets. In that case,

$$\{u\} = A^{-1}[\{K(t+1)\} - G\{K(t)\}].$$

The policy models in eqs. (6) and (8) are not truly dynamic. Although the control vector varies over time, its trajectory is fixed once the instruments of the initial time period have been chosen. Relaxing this restriction leads to the multiperiod control model

$$\{K(t+1)\} = G\{K(t)\} + A\{u(t)\}, \tag{9}$$

and its solution,

$$\{K(t)\} = G^t\{K(0)\} + \sum_{i=0}^{t-1} G^{(t-1-i)} A\{u(i)\}.$$

Two multiperiod policy problems now may be studied:

(1) the *horizon-oriented policy problem*, in which one seeks a sequence of control vectors $\{u(i)\}$ that guide the evolution of the initial population distribution $\{K(0)\}$ toward a target vector at time T, assuming fixed coefficient matrices, and
(2) the *trajectory-oriented policy problem*, in which the principal question addressed is whether there exists a sequence of control vectors $\{u(i)\}$ such that any sequence of target vectors can be realized, given a specific initial condition and unchanging coefficient matrices.

In mathematical systems theory, the first policy problem is known as state controllability. The second problem is called dynamic trajectory controllability. Both are formally analyzed in Willekens (1976).

The policy models considered thus far assume that the policy-maker's objectives can be expressed completely in terms of population targets, and that the achievement of these targets is constrained only by the equation that describes the system's dynamic behavior. No direct constraints were placed on population totals, and the control variables were constrained only through the introduction of linear dependencies.

In practical policy applications, the values taken on by population and control vectors are likely to be restricted by political and socioeconomic considerations. This suggests the desirability of adding instrumental variables to population variables to define an explicit objective function.

It also may be desirable to constrain each element of the control vector inside of a lower and upper bound,

$$\underline{u}_i(t) \leqq u_i(t) \leqq \bar{u}_i(t),$$

and to assume a budget constraint for each period,

$$\{c(t)\}'\{u(t)\} \leqq C(t),$$

and for the entire span of control,

$$\sum_{t=0}^{T-1} \{c(t)\}'\{u(t)\} \leqq C.$$

The cost vector $\{c(t)\}'$ contains the unit costs incurred by the use of each instrument.

The above constraints refer to the control vector. It also may be desirable to incorporate constraints on the population distribution vector itself. For example, the policy-maker may wish to define lower and upper bounds for the size of the population in each region in order to avoid social costs arising out of excessive density or of excessive depopulation. If the constraints set and the objective function are both linear, the policy model may be expressed as a dynamic linear programming problem [Propoi and Willekens (1978)]. If the objective function is quadratic, the computational task is considerably more complex [Evtushenko and MacKinnon (1976)].

The most general formulation of a dynamic population policy problem may be conveniently expressed as an *optimal control problem* with (i) a state equation describing the dynamics of the system, (ii) a set of constraints on the state and control variables, (iii) a set of boundary conditions, and (iv) an objective function. Such a formulation combines several fundamental themes in two related but largely independent bodies of literature: the mostly mathematical literature in systems engineering that deals with the control of complex systems describable by sets of differential or difference equations, and the more substantive literature in the formal theory of economic growth and policy. The logical structures of the two paradigms are similar, and their formalisms can be fruitfully transferred to the field of population policy [Willekens and Rogers (1977)].

3.3. *Urbanization*

Urbanization is a structural transformation all nations go through in their transition from an agrarian to an industrial society. Such transitions can be depicted by attenuated S-shaped curves which tend to show a swift rise around 20 percent, a flattening out at a point somewhere between 40 and 60 percent, and a halt or even a decline in the proportion urban at levels above 75 percent.

Accelerated rates of population growth and urbanization are direct consequences of higher rates of natural increase and net urban inmigration. Explanations of the temporal and spatial variations exhibited by these two fundamental components of population change frequently have adopted descriptive generalizations called 'transitions' or 'revolutions'. Specifically, the *vital revolution* is commonly held to be the process whereby societies with high birth and death rates move to low birth and death rates. The *mobility revolution* is the transformation experienced by societies with low migration rates as they advance to a condition of high migration rates. These two revolutions occur simultaneously, and they jointly constitute the *demographic transition.*

Urbanization results from a particular spatial interaction of the vital and the mobility revolutions. It is characterized by distinct and urban–rural differentials in fertility–mortality levels and patterns of decline, and by a massive, largely voluntary, net transfer of population from rural to urban areas through internal migration. An especially notable example of a structural transformation involving high fertility, massive rural to urban migration, and rapid urbanization is offered by the development history of Mexico. Indeed, studies of agriculture's role in economic development strategy, and the process of structural change that it induces in developing countries, often point to Mexico as a polar prototype to countries such as Japan [Johnston (1970, pp. 86–87)]:

> 'Most developing countries face a basic issue of agricultural development strategy that can be crudely defined as a choice between the 'Japanese model' and the 'Mexican model' . . . the increase in farm output and productivity in Japan resulted from the widespread adoption of improved techniques by the great majority of the nation's farmers whereas in Mexico a major part of the impressive increase in agricultural output in the postwar period has been the result of extremely large increases in production by a very small number of large-scale, highly commercial farm operators,'

The urban-demographic consequences of the Japanese and Mexican success stories differed significantly; it is, therefore, important to also keep them in mind when evaluating each of the two experiences. The aggregate annual population growth rate of Meiji, Japan was less than 1 percent; that of Mexico today is over three times as high. Urbanization proceeded at a relatively moderate pace in Japan during its structural transformation; in Mexico its pace has been startlingly high, with Mexico City alone projected to have a population in excess of 30 million by the end of this century [United Nations (1976)].

Analyses of the causes and consequences of urbanization and development can usefully be carried out within the framework of formal models of demographic and economic (demoeconomic) development. A notable example is the now classic analysis of Coale (1969), which identified some of the ways in which alternative demographic trends might affect the economic development of less developed countries. Coale focused on national rather than regional populations, considered only a single future course for mortality, and examined the demoeconomic consequences of two alternative future courses for fertility:

(A) maintenance at its current level, and
(B) a rapid decline to half its current level over a period of twenty-five years.

After generating the two alternative projections or 'scenarios', Coale (1969, p. 63) went on to

> '. . . inquire what effects these contrasting trends in fertility would have on three important population characteristics: first, the burden of dependency, defined as the total number of persons in the population divided by the number in the labor force ages (fifteen to sixty-four), second, the rate of growth of the labor force, or, more precisely, the annual percent rate of increase of the population fifteen to sixty-four, and third, the density of the population, or, more precisely, the number of persons at labor force age relative to land area and other resources. Then we shall consider how these three characteristcs of dependency, rate of growth, and density, influence the increase in per capita income.'

In order to assess some of the important demographic consequences of rapid urbanization, we have elsewhere disaggregated Coale's scenario-building approach by dividing his national population into urban and rural sectors and by introducing the impacts of rural–urban migration on their regional age compositions and population totals [Rodgers (1978)]. Since our focus here is on Mexico as a case study, we shall replace Coale's hypothetical national population of a million people with the 1970 population of Mexico [Colosio, Castro, and Rogers (1978)].

Table 8 summarizes our particular assumptions regarding future patterns of urban–rural fertility, mortality, and migration, and it also sets out Coale's parametric assumptions for purposes of comparison. Scenario (A), like that of Coale, assumes a continuation of current levels of fertility; Scenario (B), again like that of Coale, assumes a sudden reduction in fertility levels. The future courses of mortality and internal migration are assumed to follow identical paths in both scenarios; thus fertility is the sole population change variable considered to be responsive to governmental policy. The study of migration as a policy variable is currently being carried out, within the framework of a modified Kelley–Williamson–Cheetham demoecononomic model [Colosio (1979), Kelley, Williamson and Cheetham (1972)].

Both scenarios start with the observed 1970 population as the initial population. But the projection exercise includes a historical projection (for the 1940 to 1970 period) that 'tracks' the observed trajectories remarkably

Table 8

Initial values and assumptions in the two projection models.

	Coale model	IIASA–Mexico model	
		Urban	Rural
Initial values (1970)			
Population (000s)	1,000	28,329	20,048
Death rate[a]	14/1000	9.3/1000	13.0/1000
Birth rate[a]	44/1000	43.9/1000	44.5/1000
Outmigration rate[a]	——	3.0/1000	23.0/1000
Future paths			
Mortality	Decline over 30 years to level with an expectation of life at birth of 70 years; then unchanged	Decline as in Coale's model, but over 25 years; then unchanged	Decline as in Coale's model, but over 35 years; then unchanged
Fertility	(A) Unchanged (B) Reduction of 50% over 25 years; then unchanged	(A) Unchanged (B) Reduction as in Coale's model, but over 25 years; then unchanged	(A) Unchanged (B) Reduction as in Coale's model, but over 30 years; then unchanged
Migration		Unchanged	Increase of 120% over 25 years; then a reduction to 80% of that peak over 40 years; then unchanged

[a]Rates for Mexico are for 1970 and were obtained by rough estimations using historical data.

well, with the projected urban population, for example, always falling within 7 percent of the recorded values.

Fig. 5 indicates that the urbanization trajectory projected for Mexico accords well with the historical experience of nations that have already become highly urbanized. Mexico's 1970 urban population (here defined as the population living in places with more than 2,500 inhabitants) of 28 million constituted roughly 55 percent of the national total. By the turn of this century, about three-fourths of Mexico's population is projected to be

Table 9

Relative sizes of Mexico's urban, rural, and total populations: Two alternative projections.[a]

	Scenario (A)			Scenario (B)		
Year	Urban	Rural	Total	Urban	Rural	Total
Historical projection						
1940	100.0	100.0	100.0	100.0	100.0	100.0
1945	119.6	109.7	113.2	119.6	109.7	113.2
1950	148.9	122.2	131.6	148.9	122.2	131.6
	185.3	135.2	152.8	185.3	135.2	152.8
1960	233.0	147.8	177.7	233.1	147.8	177.7
	295.0	161.3	208.2	295.0	161.3	208.2
1970	378.9	173.8	245.8	378.9	173.8	245.8
Scenario projection						
	485.4	184.6	290.2	485.4	184.6	290.2
1980	611.5	196.7	342.3	611.5	196.7	342.3
	762.8	209.8	403.9	744.7	206.4	395.3
1990	945.4	224.1	477.2	882.8	213.7	448.5
	1165.4	239.9	564.6	1016.8	218.1	498.4
2000	1429.4	257.3	668.6	1166.6	219.1	551.6
	1743.4	278.0	792.3	1321.5	217.2	604.8
2010	2110.7	306.6	939.7	1473.9	218.9	659.3
	2541.1	344.9	1115.6	1623.7	223.6	714.9
2020	3046.2	394.8	1325.2	1775.3	231.5	773.2
	3638.8	459.3	1575.0	1933.9	242.9	836.3
2030	4334.0	542.2	1872.9	2101.4	258.5	905.2
	5147.4	649.1	2227.6	2278.1	279.4	980.8
2040	6097.6	786.9	2650.6	2465.9	306.3	1064.2
	7204.3	965.3	3154.7	2667.1	340.6	1157.0
2050	8488.0	1197.0	3755.5	2883.0	384.0	1260.9
	10004.1	1479.2	4470.8	3124.1	432.6	1377.1
2060	11797.0	1822.0	5322.4	3391.9	487.0	1506.4
	13916.3	2238.6	6336.6	3689.8	548.2	1650.7
2070	16420.1	2745.2	7544.0	4012.9	616.2	1808.2
	19378.3	3360.6	8981.5	4369.8	692.4	1982.9
2080	22874.8	4107.2	10693.2	4755.7	776.6	2173.0
	27008.7	5012.4	12731.3	5184.1	870.6	2384.3
2090	31986.0	6109.2	15158.4	5643.0	973.5	2612.1

[a]*Source*: Colosio, Castro and Rogers (1978, p. 12).

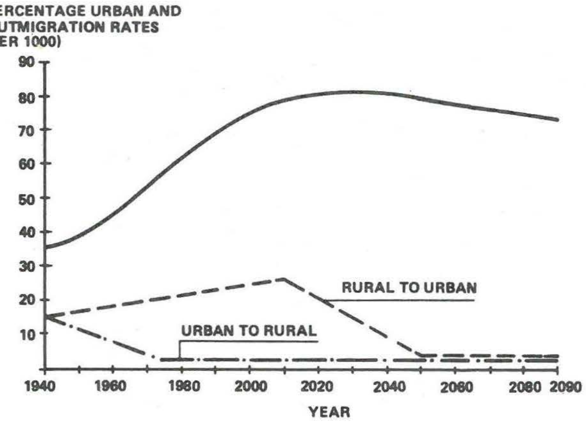

Fig. 5. Percentage urban, and rural–urban and urban–rural outmigration rates. [*Source*: Colosio, Castro and Rogers (1978, p. 11).]

urban in each of the two scenarios. According to table 9, at this time the urban population will have increased to 14 times its 1940 level, if fertility is maintained at 1970 levels, and to just over 11 times, if fertility is sharply reduced in the manner defined by Scenario (B). The corresponding multiples of the 1970 urban population are approximately four and three, respectively.

Fig. 6 shows how the three population characteristics studied by Coale (1969), vary in their significance in the short, medium, and long runs in our two scenarios of Mexico's future population growth and urbanization. The first principal impact of the decline in fertility is a 25 percent decrease in the dependence burden over two generations, followed in the subsequent two generations by an increase that brings the ratio to approximately 85 percent of its current level. The medium-run impact of fertility reduction begins to appear about 15 to 20 years after the onset of the fertility decline, producing an annual rate of labor force growth that decreases for about 60 years and then rises, over the next 40 years, to a level that remains relatively fixed thereafter. Finally, the long-run effects of reduced fertility start to become significant after 60 years; at this point the size of the high fertility population is roughly twice that of the one with reduced fertility, and this ratio assumes ever increasing dimensions thereafter.

The introduction of migration as a component of change and the concomitant spatial disaggregation of a national population into urban and rural sectors brings into sharp focus urban–rural differentials in dependency burdens and in the patterns of their decline following fertility reduction. This is also true of the differentials in the initial growth rates of the labor force population and the paths by which they converge to their long-run levels.

The dependency ratio in urban areas in Mexico was over 20 points lower

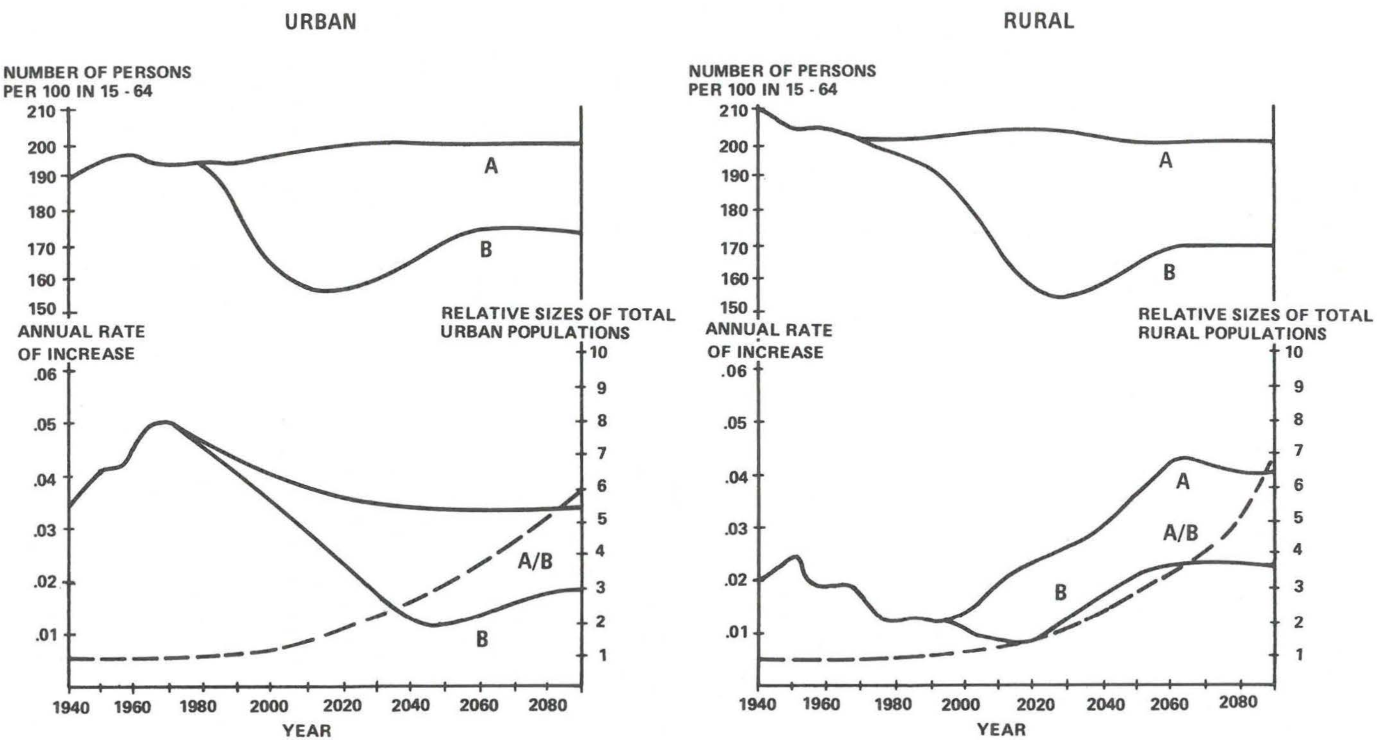

Fig. 6. Dependency burden annual rate of increase of population aged 15–64 years and relative sizes of total populations: Alternative urban–rural projections. [*Source*: Colosio, Castro and Rogers (1978, p. 14).]

than its rural counterpart in 1940, but a convergence of the two ratios reduced the difference to 7 points by 1970. This difference ultimately drops to practically zero in both scenarios, with the ratio stabilizing at just over 200 in the constant fertility projection and leveling off at about 30 points under that total in the reduced fertility scenario.

The annual rates of growth of the labor force population in urban and rural areas in 1940 were 0.035 and 0.020, respectively. By 1970 the difference between these two rates more than doubled, with the urban rate peaking at 0.050 percent per annum. In Scenario (A) this rate declines to a stable level of 0.034; it drops even further in the reduced fertility projection, stabilizing at a level of 0.018.

The rural rate, declining at first, begins to 'turn-around' by the end of the century in Scenario (A) and after some twenty years later in Scenario (B). In the constant fertility projection it levels off at an annual rate of increase of 0.040 percent; in the reduced fertility scenario the stabilization comes earlier and stands at the lower rate of 0.023, just exceeding its 1940 level.

The economic consequences of the projected patterns of dependency, growth, and density in the two urbanization scenarios are similar to those described by Coale (1969), but they now include a spatial dimension. First, the pressure for allocating a much higher proportion of the national product toward consumption is likely to be greater in the high fertility population because of its greater dependency burden. The capacity to raise net investment levels in such populations, therefore, will be seriously impaired. But if urban households save a larger fraction of their income than do rural households, rapid urbanization could have a positive influence on the national savings rate.

The short-run depressing influence of a higher burden of dependency on savings and investment in the higher fertility population is exacerbated in the middle-run by a higher growth rate of the labor force. The population with the higher rate of labor force growth will find it more difficult to increase the per worker productivity of its economy. This difficulty will be especially severe in the nation's urban areas, where high levels of rural–urban migration reduce the per capita endowment of capital and social infrastructure in cities and contribute to high rates of unemployment and underemployment.

Growing urban unemployment and underemployment in today's less developed countries have sharply underscored the urgent need for an efficient and equitable allocation of human resources between the urban and rural sectors of national economies. The determinants of rural–urban migration and the consequences of such migration for economic development warrant careful study. An important contribution to such study can come from improved demoeconomic models of dualistic development.

It has been said that models are always based on assumptions known to be false, and that this is what differentiates them from the phenomena they purport to describe. Demographic models are no exception to this dictum, and all population projections, for example, are generated on the basis of assumptions that are almost certain to be violated. One cannot foresee the future, and important insights into the dynamics of human populations can indeed be revealed by relatively simple linear models based on rather restrictive assumptions. As has been demonstrated in this paper, such models can be used to structure data collection efforts; they often generate hypotheses for empirical confirmation; they can suggest potential policy problems and issues; and they provide indices for comparative studies [Keyfitz (1971)].

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