Jules et Jim and the vortex of life

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Abstract

Love stories are dynamic processes that begin, develop, and often stay for a relatively long time in a stationary or fluctuating regime, before possibly fading. Although they are, undoubtedly, the most important dynamic processes in our life, they have as yet not been cast in a formal theoretical framework. In particular, why it is so difficult to predict the evolution of love affairs continues to be largely unexplained. Here we conjecture that a love story can be unpredictable—that is, chaotic, in the technical sense of the word—on the sole basis of the interplay of the characters involved. To do this, we analyzed the celebrated triangular love story described in *Jules et Jim*, a novel by Henri-Pierre Roché, using a mathematical model. The results fully support our conjecture and also highlight the genius of François Truffaut, whose film—one of the masterpieces on love and friendship—is based on the novel.

Introduction

The romantic relationship involving Helen Grund, her husband Franz Hessel, and his friend Henri-Pierre Roché, is analyzed using a mathematical model. The novel *Jules et Jim*, published by Roché in 1953 (1), describes this triangular relationship and is interesting for two reasons. First, because, being autobiographical, it is a reliable source of information. Second, because it conveys the central idea of Roché's philosophy, namely, that one should not try to possess or constrain the people one loves, but leave them free to engage in other relationships. This anti-bourgeois ideology of "free love"—popular during the social movement of 1968—is exploited in the formulation of the model.

Reading the novel, where Helen Grund is Kathe and Hessel and Roché are Jules and Jim, respectively, one has the impression that the evolution of their love story is turbulent and difficult to predict. Actually, the uncertainty over their future creates, in them and in the reader, a remarkable tension that ceases only when Kathe and Jim commit suicide (2)

> *Jules would never have again the fear that had been with him since the day he met Kate, first that she would deceive him—and then, quite simply, that she would die, for she had now done that too* (p. 236)

Technically, the triangle seems to evolve in accordance with the principles of deterministic chaos (3) because of the psycho-physical peculiarities of the three characters. Confirming this impression with a mathematical model would formally prove the conjecture that a love story can be chaotic, not only by reflecting the complexity of the surrounding social environment, but also because the characters involved generate antagonistic forces of attraction and repulsion that make the future unpredictable.

The first allusion to this conjecture was made by S.H. Strogatz (4), who mentioned the “many-body problem” of celestial mechanics when presenting his naïve model of Romeo and Juliet. A more technical hint can be found in a paper by J.C. Sprott (5), where a stylized extension of Strogatz’s model to the case of a hypothetical triangle is briefly discussed. Here we do much more because we validate the conjecture with a model based on a well documented love story. For this, the main psycho-physical characteristics of Kathe, Jules, and Jim are first identified and then encapsulated in the model, which is analyzed using numerical mathematical techniques and shown to be chaotic.

Mathematical models of love stories

Since the publication of Strogatz’s pioneering paper (4), love stories have been modeled with increasing success in terms of differential or difference equations. Many attempts describe the story from the state of indifference—in which we are when we first meet—to the establishment of a permanent (stationary or fluctuating) regime, while others (6, 7) focus on the phase of marital dissolution. To minimize the number of equations, it is convenient to assume (as first suggested by G. Levinger (8)) that the interest of one person in another can be captured by a single variable called *feeling*. Low and high positive feelings correspond to friendship and love, while negative feelings indicate antagonism and hate. Feelings vary over time because of the interplay of consumption and regeneration mechanisms, assumed, for the sake of simplicity, to be time-invariant processes. This means that adaptivity, learning, and aging are ruled out of the model.

The basic consumption mechanism is oblivion: it explains why a person gradually forgets the partner after being abandoned. As for regeneration processes, we will distinguish, as is done in all studies surveyed in (9) and in (10–14), the reaction to love and the reaction to appeal—the mix of beauty, talent, wealth, and other characteristics that are independent of the feelings.

The model of Jules, Jim, and Kathe

To show that a romantic relationship can evolve unpredictably, even if it develops in a steady social environment, the story described by Roché is reduced to that of a pure triangle. The model should therefore be characterized by six variables, namely, the feelings of each person for the others. However, Jules and Jim have a deep and permanent friendship

> *In twenty years Jim and he had never quarrelled. Such disagreements as they did have they noted indulgently* (p. 237)

It is thus reasonable to consider only the feelings of Kathe for Jules and Jim and those of the two friends for her.

The three follow almost to perfection the principles of free love

> *In her mind, each lover was a separate world, and what happened in one world was no concern of the others* (p. 108)

It is thus spontaneous to split Kathe into two independent women, one in love with Jules and one with Jim, and to describe the triangle by means of two independent models of pairwise relationships: the Kathe-Jules model and the Kathe-Jim model. These two models are presented in the supporting online material and their ensemble is henceforth called the *free love model*.

Adopting the jargon used in the literature (15), we can say that the main peculiarities of the three characters are as follows:

- Jules is secure (his reaction to Kathe's love increases with her feeling) and platonic (his reaction to Kathe's appeal is reduced the more in love he is with her)

  Really, Jules is happy, in his own way, and just wants things to go on. He's seeing you often, in idyllic circumstances, and he's living on hope (p. 24)

- Jim is insecure, as all "Don Juan" are to avoid deep involvements (his reaction to Kathe's love decreases when she becomes too much in love with him)

  'Oh, when,' she said to him one day,—'when are you going to stop giving me bits of yourself and give me everything?' (p. 207)

- Kathe is insecure with Jules (because she reacts negatively to his platonic nature)

  But then, he wasn't the husband she needed, and she wasn't the woman to bear that (p. 89)

  She is secure with Jim and synergic with both (she finds them more appealing the more she is involved with them).

The reader can simulate the free love model (16), by assigning values to the parameters identifying the consumption and regeneration mechanisms that are different from the reference values we have proposed. For parameter values not too far from ours, the result is invariably as follows. The two couples start from the state of indifference (since Kathe, Jules, and Jim are all present the first time they meet) and develop, as shown in Fig.1, positive feelings characterized by recurrent ups and downs that tend to become periodic as time goes on (17).

The results in Fig.1 are quite interesting because they fit with some of the facts explicitly mentioned by Roché:

- During the first years Kathe is more attracted to Jules (she marries him).

- Jim's ups and downs are more relevant than those of Jules
  'Jim was easy for her to take, but hard to keep. Jim's love drops to zero when Kate's does, and shoots up to a hundred with hers. I never reached their zero or their hundred' (p. 231)

- The drops in interest of Kathe for Jules anticipate those of Jules for Kathe

  The danger was that Kate would leave. She had done it once already… and it had looked as if she didn't mean to return… She was full of stress again, Jules could feel that she was working up for something (p. 89)

- The drops in interest of Jim for Kathe anticipate those of Kathe for Jim

  He himself was incapable of living for months at a time in close contact with Kate, it always brought him into a state of exhaustion and involuntary recoil which was the cause of their disasters (p. 189)

The free love model has, however, two weaknesses. The first is that it gives rise (Fig.1, bottom panel), to a subdivision of the entire period of interest into 10 shorter periods, each characterized by one of the two lovers being preferred by Kathe. Assuming that Kathe changes partner each time she changes preference, the free love model predicts nine changes of partner, while in the novel there are seven. A second weakness of the free love model is that it cannot support our conjecture, as the feelings within each couple tend to become periodic and are therefore predictable after some time.

A remedy for the first weakness would be to modify the parameters of the model, with a view to obtaining exactly seven changes of partner. But this would fail to resolve the second weakness because the new free love model would still be composed of two independent models, each tending toward a periodic behavior (17). It is therefore more promising to introduce a weak interaction between the two models of pairwise relationships, by deviating a little from the principles of free love. This is sensible, as Roché himself describes specific behaviors on the part of the two friends that violate the principles of free love. Jules is pleased when Kathe is with Jim because he believes it makes her happier. This characteristic, which is peculiar to Jules, is consistent with his platonic nature and is well described by Roché

> *'…I'm terrified of losing her, I can't bear to let her go out of my life. Jim—love her, marry her, and let me go on seeing her. What I mean is, if you love her, stop thinking that I'm always in your way'* (p.27)

Although jealousy is at odds with the principles of free love, Jim is jealous

> *She bestowed her graciousness on each in turn… and Jim was jealous* (p. 97)

These two characteristics are incorporated into the model by means of a small parameter that slightly amplifies [attenuates] the reaction of Jules [Jim] when Kathe is more involved with Jim [Jules].

We let also Kathe slightly deviate from the principles of free love by assuming that she systematically forgets her current partner less quickly than she forgets the other. This asymmetry is introduced into the model by means of a second parameter. In conclusion, the model of the triangle differs from the free love model because of the presence of two parameters that interpret weak interferences between the two couples (18).

As Roché does not give any indication that could help us quantify these interferences, we studied the behavior of the model for all small values of the two parameters. We looked for particular combinations for which the model is chaotic and predicts, at the same time, seven partner changes in the period of interest. The result is summarized in Fig.2, where the green-to-red color scale represents the so-called Lyapunov exponent (19) computed from the state of indifference (20). For parameter combinations in the red region, the model is therefore chaotic, while the shaded area, obtained through extensive simulations, shows where there are seven partner changes. Figure 2 proves that it is indeed possible to satisfy all the desired requirements. For example, for the parameter values corresponding to the white dot, the love story predicted by the model is as in Fig.3 and the times at which Kathe changes preference compare very favorably with the partner changes indicated by Roché (the correlation is 0.97!). Thus, in conclusion, the model explains all the significant features of the story and proves the conjecture.

The film by François Truffaut

François Truffaut, one of the prominent directors of the "Nouvelle Vague", made *Jules et Jim* in 1961, after discussing the idea with Roché. Jeanne Moreau and Oskar Werner, already well known, played Kathe and Jules, while Henri Serre, selected by Truffaut because of a certain resemblance to Roché, played Jim. Truffaut omits many minor characters of the novel, but successfully reproduces the feelings between the two friends and Helen Grund. Indeed, after watching the film, she writes a letter to Truffaut in which she says:

> *But what disposition in you, what affinity could have enlightened you to the point of recreating—in spite of the odd inevitable deviation and compromise—the essential quality of our intimate emotions?* (21)

Truffaut actually adds, here and there, explicit elements pointing to the fact that love can be chaotic because of attracting and repelling forces. The first of these elements anticipates the beginning of the film. While the screen is still dark, Jeanne Moreau sends this lapidary message:

> *You told me: "I love you." I told you: "Wait." I almost said: "Yes." You said: "Go."* (22)

Symmetrically, the film finishes by again stressing the mechanisms of attraction and repulsion

> *The ashes were placed in an urn. Jules might have mixed them. Catherine wanted hers to be cast to the wind* (22)

Other stylistic elements which mirror the chaotic dynamics are the use of the handy camera and of scenes in quick cuts, and the voice-over technique, that allows Truffaut to lump together long periods of calm and focus on the moments when Kathe is ready to change partner.

But the most explicit reference to chaos is *Le tourbillon de la vie* (i.e., the vortex of life), the soundtrack sung by Jeanne Moreau accompanied on the guitar by Albert, a friend of Kathe. This song is a beautiful hymn to chaos, characterized by continuous phases of attraction (folding) and separation (stretching)

> *We met with a kiss*
> *A hit, then a miss*
> *It wasn't all bliss*
> *And we parted*
> *We went our own ways*
> *In life's whirlpool of days*
> *I saw her again one night*
> *Again she was an enchanted sight* (22)

And if that weren't enough, Truffaut reinforces the message visually, by introducing in the next scene an effective representation of the stretching mechanism (divergence of nearby trajectories): Kathe, Jules, Jim, and Albert are bicycling in the countryside, when suddenly Albert leaves the group by taking a side road.

All this was done by Truffaut in 1961, two years before the publication of the first relevant paper on chaos (23). Thus, Truffaut has intuitively anticipated, as only an artist can do, an important scientific discovery and its founding principles, and has used them with grace and skill.

Concluding remarks

We have shown how a love story can be analyzed using a mathematical model. The approach is standard in psychoanalysis. First, the main psycho-physical characteristics of the individuals involved are identified, in this case from a careful reading of Roché's novel. Then, these characteristics are used to develop a mathematical model (the formal analog of the verbal models used in psychology), which is finally studied to detect the key features of the romantic relationship (e.g., the fact that the unpredictability of the love story is due to the small deviations of Jules, Jim, and Kathe from the principles of free love).

The main result is the validation—through a detailed analysis of a paradigmatic love story—of the conjecture that romantic relationships can be unpredictable on the sole basis of the characters involved. A second important result is to show how a mathematical model can be used to highlight the genius of an artist—in this case François Truffaut—who featured Roché's novel in his most important film.

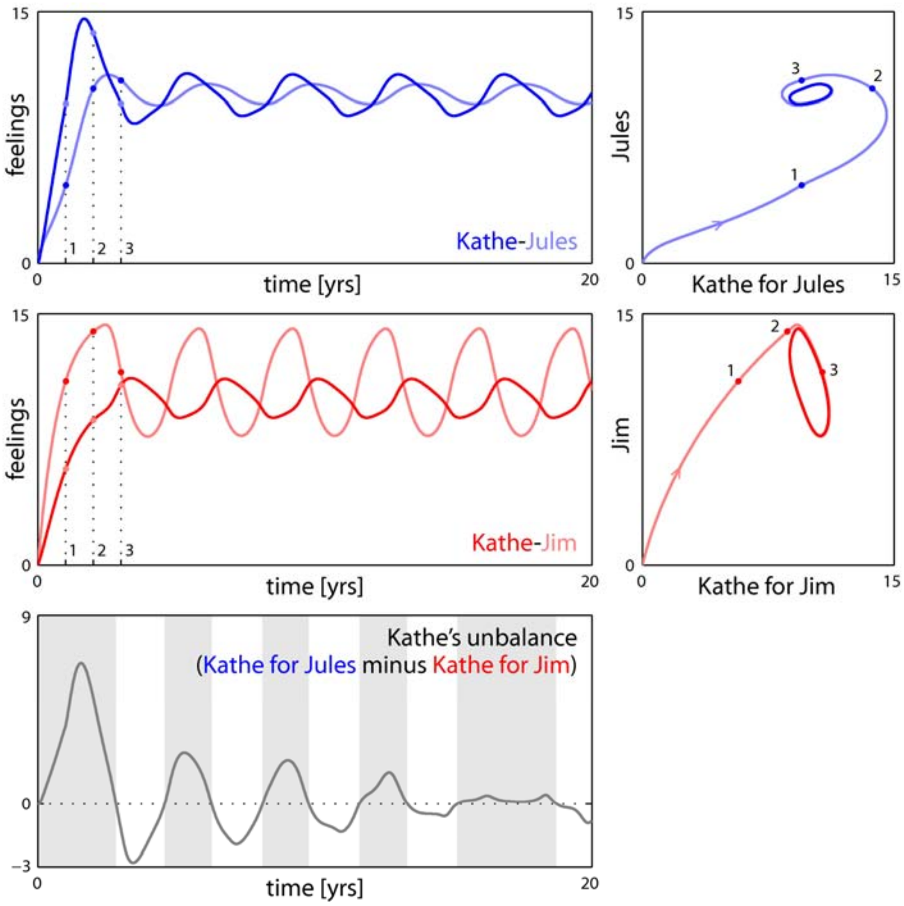

Figure 1: The love story predicted by the free love model.
The bottom panel identifies the preference of Kathe for the two lovers.

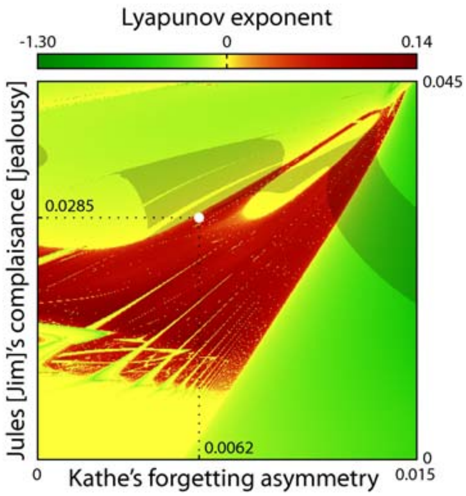

Figure 2: Behavior of the full model w.r.t. the interference parameters.

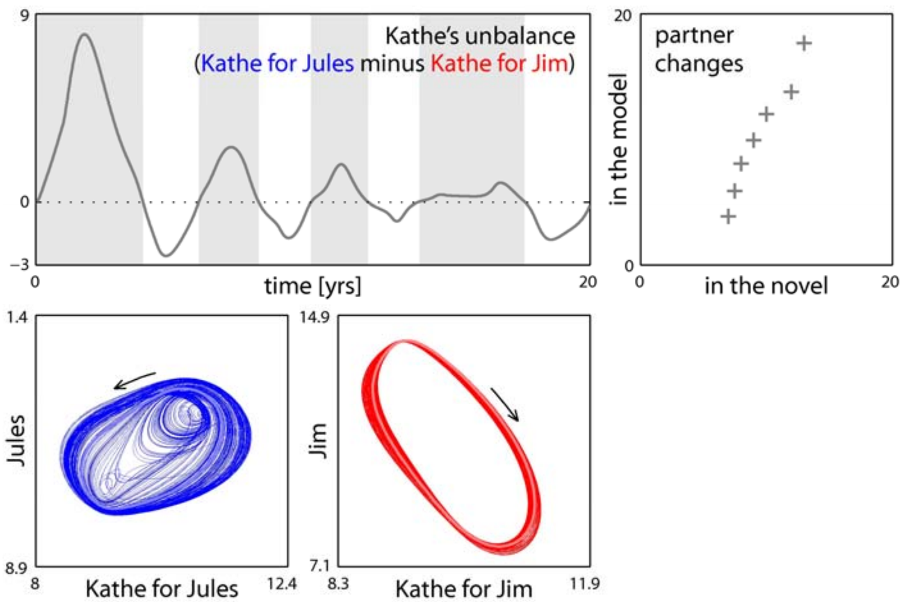

Figure 3: The love story predicted by the full model.
The interference parameters are set to the values corresponding to the white dot of Fig.2. (Top panels) Kathe's preference and comparison with the novel. (Bottom panels) Projections of the chaotic attractor (the limit cycles of Fig.1 are superimposed for comparison with the free love model).

References and Notes

1. H.-P. Roché, *Jules et Jim* (Éditions Gallimard, Paris, 1953, in French; Marion Boyars, 1993, 2nd ed., English translation by P. Evans).

2. Quotes refer to the English translation of *Jules et Jim* (1).

3. Deterministic chaos is the most complex behavior of dynamical systems. It is the result of a *stretching* mechanism—divergence of nearby trajectories—and of a *folding* mechanism that keeps trajectories bounded. Chaotic dynamics are aperiodic and unpredictable (24).

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15. See (9) and references therein.

16. An Adobe Flash simulator is available at http://home.dei.polimi.it/dercole/julesetjim/en/sim/. The two models of pairwise relationships and the full model can be simulated in three different tabs. The free love model can be obtained from the full model by setting to zero the interferences between the two couples.

17. This property is generic in two-dimensional dynamical systems (24), where periodic regimes are the most complex asymptotic behavior. Formally, the asymptotic behavior of the free model is quasi-periodic, because the frequencies of the periodic regimes of the two models of pairwise relationships are generically incommensurable.

18. Recall that coupled oscillators have quasi-periodic regimes when coupling is weak, while chaos is expected for stronger coupling (24).

19. The (largest nontrivial) Lyapunov exponent—a measure of the mean divergence of nearby trajectories—is positive, zero, and negative in chaotic, quasi-periodic, and periodic regimes, respectively (24).

20. The figure is in full agreement with the theory of dynamical systems (24) and shows regions where the asymptotic regime is quasi-periodic and chaotic, as well as the very narrow regions where the two oscillators behave periodically by locking their frequencies.

21. Taken from the Introduction by F. Truffaut to the English translation of *Jules et Jim* (1).

22. Taken from the English subtitles of the film (Fox Lorber World Class Cinema Collection). Opening original screenplay: *Tu m'as dit: “Je t'aime.” Je t'ai dit: “Attends.” J'allais dire: “Prends-moi.” Tu m'as dit: “Va-t'en.*” Soundtrack original: *On s'est connus, on s'est reconnus—On s'est perdus de vue, on s'est r'perdus d'vue—On s'est retrouvés, on s'est réchauffés—Puis on s'est séparés—Chacun pour soi est reparti—Dans l'tourbillon de la vie—Je l'ai revue un soir, hàie, hàie, hàie—Ça fait déjà un fameux bail*.

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Supporting Online Material for

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This PDF file includes:

Materials and Methods

Figs. S1 and S2

Tab. S1

Other Supporting Online Material for this manuscript includes the following:

(available at http://home.dei.polimi.it/dercole/julesetjim/en/sim)

Adobe Flash simulator

Materials and Methods

In this appendix we present the mathematical model of the triangle, using a didactic style that should make our study accessible also to non-technically oriented readers. In particular, we present the model in discrete time, as a formal rule that allows the feelings of Kathe, Jules, and Jim to be recursively updated from one day to the next.

The variables

George Levinger (8) has been the first to use graphs to represent the time evolution of the feelings of one person for another. Of course, love stories can be very different one from another. For example, in the story depicted in Fig. S1 (top-left panel), she develops from the very beginning a positive feeling for him, while he is initially antagonistic. In contrast, in the other story (bottom-left panel), both she and he are always positively involved, but suffer from remarkable ups and downs.

A love story can also be represented by drawing in the plane of the feelings (*state* plane) a curve, called *trajectory*, showing the contemporary evolution of the feelings (see the right panels of Fig. S1). Of course, the trajectory starts from the point of the plane corresponding to the feelings that she and he have one for the other at the beginning of the story. Thus, the starting point is the origin of the plane if the two individuals are initially indifferent to each other.

Triangular love stories are more difficult to be represented through graphs, because each individual is characterized by two distinct feelings, one for each of the two others. In the following, we indicate with x_1 and x_2 the feelings of Kathe for Jules and Jim, respectively, and with y_1 and y_2 the feelings of Jules and Jim for Kathe. As their friendship is deep and permanent, the feelings between Jules and Jim are not considered because basically invariant.

The model of the triangle is constructed by first considering the couples Kathe-Jules and Kathe-Jim separately. The ensemble of the two models of pairwise relationships is the “free love” model, while the complete model of the triangle also includes the weak interferences between the two couples.

Models of pairwise relationships

Consider a couple and denote by x and y the feelings that she and he have one for the other. In general, the feelings evolve over time, so they are more properly denoted by $x(t)$ and $y(t)$, where t is an integer number

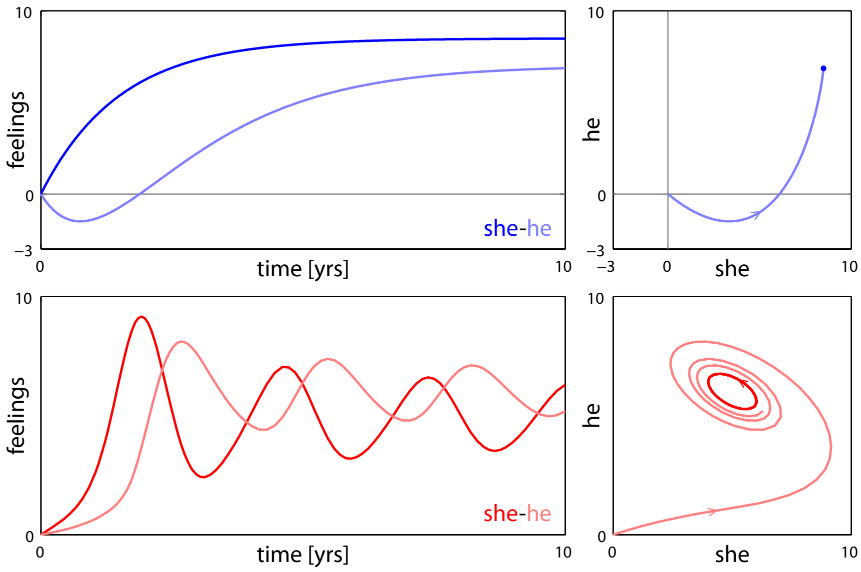

Fig. S1: Graphical representation of two hypothetical love stories. (Left) Feelings' time series. (Right) Trajectories in the plane of the feelings.

that allows us to order all days sequentially. A model is simply a balance of the feelings between any day t and the following day $(t + 1)$. In words, her feeling tomorrow is equal to that of today minus the loss of interest between today and tomorrow due to oblivion, plus the recharge of interest, again between today and tomorrow, due to her reaction to his love and appeal.

The loss of interest due to oblivion can be described with a function $F(x)$ increasing with x, to express the idea, usually confirmed in physics and chemistry, that the rate at which a given property is lost is positively correlated with the abundance of the property. Typically, the loss is assumed to be proportional to x, so that the function $F(x)$ is the product of a proportionality coefficient f and x. The parameter f, called *forgetting coefficient*, represents the portion of interest lost in one day through oblivion.

As for the recharge of the feeling, we must separately consider the reaction to the partner love $R_L(y)$, where R stands for reaction and L for love, from the reaction $R_A(a_y)$ to the appeal of the partner, here indicated with a_y and assumed to be invariant.

To model the reaction to love, we distinguish between *secure* individuals—who positively react to any increase in the love of the partner—and *insecure* ones—who avoid high involvements by negatively reacting when the love of the partner is too high. Secure individuals are therefore characterized by functions $R_L(y)$ increasing with the love y of the partner. Among these functions, we have linear functions, which however correspond to rather extreme individuals who have an unbounded capacity of recharge. In contrast, insecure

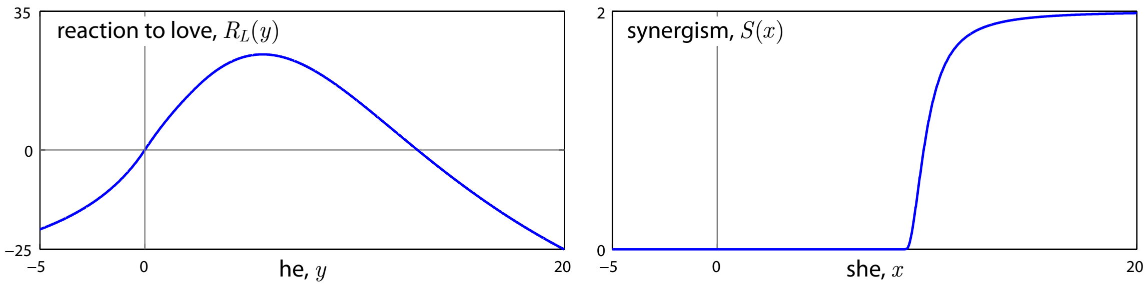

Fig. S2: (Left) Reaction to love typical of an insecure individual. (Right) Typical synergism function.

individuals are characterized by functions $R_L(y)$ which are decreasing at high values of y (see, e.g., Fig. S2, left).

Another important characteristic of an individual is the propensity to react to the appeal of the partner in a biased way, depending on the state of involvement. For example, parents often see their own children more beautiful than they really are. But the same phenomenon, that we call "synergism", can also occur in love relationships. In this case, the reaction to the partner appeal can be written in the form $(1 + S(x))\, R_A(a_y)$, where the function S, called *synergism*, has a sigmoid shape for positive x and is zero for negative x (Fig. S2, right). The opposite behavior is also possible, like in *platonic* individuals described by a reaction to appeal of the form $(1 - P(x))\, R_A(a_y)$, where the function P (shaped as S) measures the loss of sexual interest with respect to the state of involvement x. Individuals who are neither synergic nor platonic are those who are not biased by their own feelings.

The couple Kathe-Jules

The main characteristic of Jules is to be strongly platonic. In other words, he reduces his reaction to Kathe's appeal when he is more in love with her. Assuming that Jules has linear forgetting and reaction functions, the equation regulating his feeling for Kathe is therefore given by

$$y_1(t+1) = y_1(t) - f_1\, y_1(t) + r_1\, x_1(t) + (1 - P(y_1(t)))\, r_{A1}\, a.$$

Kathe is definitely annoyed by the platonic nature of Jules. For this reason, her reaction R_L to Jules' love is of the insecure type (Fig. S2, left). Moreover, Kathe is very synergic, so that her reaction to Jules' appeal is amplified by the factor $(1 + S)$, where S is Kathe's synergism (Fig. S2, right).

In conclusion, assuming that Kathe's forgetting function and her reaction to appeal are linear, the model of the couple Kathe-Jules is composed of the following two equations:

$$x_1(t+1) = x_1(t) - f\, x_1(t) + R_L(y_1(t)) + (1 + S(x_1(t)))\, r_A\, a_1,$$
$$y_1(t+1) = y_1(t) - f_1\, y_1(t) + r_1\, x_1(t) + (1 - P(y_1(t)))\, r_{A1}\, a.$$

The model can be used repeatedly, for $t = 1, 2, 3$ and so on, to compute the time evolution of the feelings of Kathe and Jules. For this, we must first assign suitable values to all parameters appearing in the model. For example, we fix the appeal of Jules to 4 and that of Kathe to 20, because she is, by far, more fascinating than him. All the details about the functions R_L, S, and P and the parameter values can be found in Tab. S1.

Now, assuming that the day they meet for the first time, say $t = 0$, Kathe and Jules are completely indifferent one to each other, we can fix x_1 and y_1 equal to zero for $t = 0$ and use the two equations to compute the values of the two feelings during the next day, thus obtaining $x_1(1) = r_A\, a_1$ and $y_1(1) = r_{A1}\, a$. It is interesting to note that only appeal matters at the beginning of a love story, when feelings are still latent.

To go on to the next day, it is sufficient to increase the time of one unit and use the same equations written for $t = 1$, rather than for $t = 0$, to compute the feelings at day $t = 2$. Note that also the forgetting functions and the reactions to love are now involved. Once the values of x_1 and y_1 at day 2 have been obtained, one can repeat the same operations to compute the feelings of Kathe and Jules on day 3, and continue like this for months or even years. The results provided by the model can easily be portrayed to show the evolution of the love story in a time interval of interest. In this way we obtain the graphs in the top line of Fig. 1, where the points indicated with 1, 2, and 3 represent the feelings of Kathe and Jules at the end of the first, second, and third year of their relationship.

Kathe and Jules are always positively involved, but their love story does not reach a plateau. Indeed, as time goes on, their feelings tend to oscillate with a period of about 4 years, more precisely 3 years and 10 months. At the beginning of their relationship, Kathe and Jules are increasingly involved, until Kathe has the first drop in interest, generating in Jules the fear of being abandoned. According to the model, these drops in interest are recursive, and this is in agreement with both the novel and the film.

The couple Kathe-Jim

The main characteristic of Jim is to be insecure, as all "Don Juan" are to avoid deep involvements. Thus, this is Jim's equation

$$y_2(t+1) = y_2(t) - f_2\, y_2(t) + R_{L2}(x_2(t)) + r_{A2}\, a,$$

if one assumes that his forgetting function and his reaction to appeal are linear. In contrast, his reaction R_{L2} to Kathe's love is that of an insecure individual and is therefore nonlinear (Fig. S2, left).

Kathe is secure in her relationship with Jim, because he is not platonic, but she is definitely synergic. This is therefore Kathe's equation

$$x_2(t+1) = x_2(t) - f\, x_2(t) + r_L\, y_1(t) + (1 + S(x_2(t)))\, r_A\, a_2,$$

where S is her synergism function.

In conclusion, the Kathe-Jim model is composed of the following two equations:

$$\begin{aligned} x_2(t+1) &= x_2(t) - f\, x_2(t) + r_L\, y_1(t) + (1 + S(x_2(t)))\, r_A\, a_2, \\ y_2(t+1) &= y_2(t) - f_2\, y_2(t) + R_{L2}(x_2(t)) + r_{A2}\, a. \end{aligned}$$

Once all parameters have been fixed at reasonable values (see Tab. S1), the model can be repeatedly used to compute the time evolution of the feelings of Kathe and Jim. The result is that of the second line of Fig. 1. In this case too, the involvements of Kathe and Jim increase during the first phase of their relationship and then tend toward a swinging regime with a period of 3 years and 4 months. This time, the first to invert the positive trend is Jim, who being insecure refuses too deep involvements.

The triangle

If the principles of free love were rigorously followed, the feelings of Kathe, Jules and Jim would evolve as previously determined by the models of the two pairwise relationships. As discussed in the paper, the free love model has two major weaknesses—the incorrect number of partner changes and the absence of chaos—but already captures many characteristics of the love story.

In particular, it predicts that during the first years Kathe is more attracted to Jules (see Fig. 1, bottom panel). This is consistent with the real story and with Roché's novel, where Kathe and Jules get married a few months after their first encounter. Actually, the model predicts that at the very beginning of the story, Kathe is more attracted to Jim (this is not visible at the scale of the figure). In fact, Jules is not as appealing as Jim, namely $a_1 < a_2$, and this implies that during the very first days, the feeling of Kathe for Jules is lower than that for Jim. But then, according to the model, after a couple of weeks Kathe's preference is in favor of Jules. Surprisingly, this detail is also mentioned in the novel, where talking about a missed appointment between Kathe and Jim, Roché writes:

> *If Kate and Jim had met at the café, things might have turned out very differently* (p. 80)

To overcome the two weaknesses, the free love model is slightly modified by introducing some realistic extra-characteristics in the behaviors of the three individuals: Kathe does not live in fully separated worlds; Jules is pleased by the relation between Kathe and Jim; and Jim is jealous. These small changes imply that the free love principles are not fully followed anymore.

To implement the first change, we assume that Kathe forgetting capabilities depend upon her state of involvement. More precisely, we assume that at any given time she forgets her current partner less quickly than the other. This is realized by multiplying, in the two equations for Kathe (see below), the forgetting coefficient f by an exponential term which is greater than 1 in one equation and smaller than 1 in the other. In order to deviate only slightly from the principles of free love, ϵ must be a small positive parameter.

Jules does not suffer when Kathe is with Jim. In fact, he is pleased because he knows that she is more happy. This peculiar characteristic is consistent with the platonic nature of Jules and is well described by Roché. In order to take Jules' complaisance into account, his reaction to Kathe's love is multiplied by an exponential factor greater than 1 when she is with Jim, namely when x_2 is greater than x_1 (see Jules' eq.).

Although jealousy is at odds with free love, Jim is jealous. In order to take Jim's jealousy into account, his reaction to Kathe's love is multiplied by an exponential factor smaller than 1 when she is with Jules, namely when x_1 is greater than x_2 (see Jim's eq.). For simplicity, Jules' complaisance and Jim's jealousy are quantified by the same positive parameter δ, that must also be small if we like to avoid large deviations from the free love principles.

In conclusion, the model of the triangle is composed of the following four difference equations:

$$x_1(t+1) = x_1(t) - f\exp(\epsilon(x_2(t)-x_1(t)))x_1(t) + R_L(y_1(t)) + (1+S(x_1(t)))r_A a_1, \quad \text{(Kathe for Jules)}$$

$$x_2(t+1) = x_2(t) - f\exp(\epsilon(x_1(t)-x_2(t)))x_2(t) + r_L y_1(t) + (1+S(x_2(t)))r_A a_2, \quad \text{(Kathe for Jim)}$$

$$y_1(t+1) = y_1(t) - f_1 y_1(t) + r_1 x_1(t)\exp(\delta(x_2(t)-x_1(t))) + (1-P(y_1(t)))r_{A1} a, \quad \text{(Jules)}$$

$$y_2(t+1) = y_2(t) - f_2 y_2(t) + R_{L2}(x_2(t))\exp(-\delta(x_1(t)-x_2(t))) + r_{A2} a, \quad \text{(Jim)}$$

and differs from the free love model due to the presence of the two small parameters ϵ and δ, which interpret the small interactions among the two couples.

The simulator

This section is a brief userguide to the online simulator (16). The main page is organized in three “tabs”: the first two are dedicated to the models of the two couples and the third, set as the default tab, to the model of the triangle.

For each couple, one can see the computed time series of her and his feelings in the 20 years period, as well as the trajectory in the state space, that is drawn for a much longer time to facilitate the emergence of the attractor. Upon convergence, the equilibrium point or the cycle are highlighted and, in the case of a cycle, the period is shown in the regime indicator. The comparison with the results obtained with our reference parameter values is straightforward, because the reference solution always appears in gray in the background.

The button “change the parameters” activates a window with tabs through which it is possible to modify, quite consistently, all the parameters of the model of the couple, and check all the mathematical details of the model. In particular, the equations are shown in the first side tab, while the other tabs concern the nonlinear reactions to love and appeal and show both their analytical expression and their graph. With the button “Reference values” it is always possible to reset the parameter values of the tab to their reference values, while the button “Simulate” starts the simulation of the model.

On the tab concerning the triangle, the time series of the unbalance of Kathe between Jules and Jim, $x_1 - x_2$, is shown, together with the projections of the trajectory in the planes of the feelings of the two couples. When the changes of partner, identified with the changes of sign of Kathe’s unbalance over the 20 years, are 7, as in Roché’s novel, the times of the changes τ_i are compared with the times t_i indicated by Roché. The yellow crosses in the plane (t, τ) can be compared with the gray ones obtained with the reference values of the parameters.

As with the couples, the simulation also goes on for more than 20 years, to show the projections of the attractor. At the same time, the Lyapunov exponent (19) is computed and the regime indicator reveals if the story is chaotic or not.

Through the window for changing the parameters, it is also possible to modify the parameters ϵ and δ that quantify the small deviations from the principles of free love. In particular, by setting them to zero, one can simulate the triangle in the case of free love.

Table S1

Nonlinear functions (specified for nonnegative feelings)

Character	Symbol	Expression	Description
Kathe	$R_L(y_1)$	$r_I \dfrac{y_1/y_L}{1+y_1/y_L} \begin{cases} \dfrac{1-((y_1-\tau_I)/y_I)^2}{1+((y_1-\tau_I)/y_I)^2} & \text{if } y_1 \geq \tau_I \\ 1 & \text{if } y_1 < \tau_I \end{cases}$	Kathe's reaction to Jules' love
	$S(x)$	$\begin{cases} s\,\dfrac{((x-\tau_S)/x_S)^2}{1+((x-\tau_S)/x_S)^2} & \text{if } x \geq \tau_S \\ 0 & \text{if } x < \tau_S \end{cases}$	Kathe's synergism
Jules	$P(y_1)$	$\begin{cases} p\,\dfrac{((y_1-\tau_P)/y_P)^2}{1+((y_1-\tau_P)/y_P)^2} & \text{if } y_1 \geq \tau_P \\ 0 & \text{if } y_1 < \tau_P \end{cases}$	Jules' platonicity
Jim	$R_{L2}(x_2)$	$r_{I2} \dfrac{x_2/x_L}{1+x_2/x_L} \begin{cases} \dfrac{1-((x_2-\tau_{I2})/x_I)^2}{1+((x_2-\tau_{I2})/x_I)^2} & \text{if } x_2 \geq \tau_{I2} \\ 1 & \text{if } x_2 < \tau_{I2} \end{cases}$	Jim's reaction to Kathe's love

Parameters

Character	Context	Symbol	Value	Description
Kathe	forgetting	f	$2/365$	Kathe's forgetting coefficient
		ϵ	0.0062	Kathe's forgetting asymmetry
	reaction to love	r_L	$1/365$	Kathe's reaction coefficient to Jim's love
	$R_L(y_1)$	r_I	$80/365$	Kathe's-to-Jules maximum insecureness
		y_L	10	Sensitivity of Kathe's reaction to Jules' love
		τ_I	2.5	Kathe's-to-Jules insecureness threshold
		y_I	10.5	Sensitivity of Kathe's-to-Jules insecureness
	reaction to appeal	r_A	$1/365$	Kathe's reaction coefficient to appeal
	$S(x)$	s	2	Kathe's maximum synergism
		τ_S	9	Kathe's synergism threshold
		x_S	1	Sensitivity of Kathe's synergism
	appeal	a	20	Kathe's appeal
Jules	forgetting	f_1	$1/365$	Jules' forgetting coefficient
	reaction to love	r_{L1}	$1/365$	Jules' reaction coefficient to love
	reaction to appeal	r_{A1}	$0.5/365$	Jules' reaction coefficient to appeal
	$P(y_1)$	p	1	Jules' maximum platonicity
		τ_P	0	Jules' platonicity threshold
		y_P	1	Sensitivity of Jules' platonicity
	appeal	a_1	4	Jules' appeal
Jules/Jim	reaction to love	δ	0.0285	Jules/Jim's complaisance/jealousy coefficient
Jim	forgetting	f_2	$2/365$	Jim's forgetting coefficient
	$R_{L2}(x_2)$	r_{I2}	$20/365$	Jim's maximum insecureness
		x_L	10	Sensitivity of Jim's reaction to love
		τ_{I2}	9	Jim's insecureness threshold
		x_I	1	Sensitivity of Jim's insecureness
	reaction to appeal	r_{A2}	$1/365$	Jim's reaction coefficient to appeal
	appeal	a_2	5	Jim's appeal