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GENERALIZED REGIONAL AGRICULTURE  
MODEL (GRAM): BASIC VERSION

Murat Albegov

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INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS  
A-2361 Laxenburg, Austria



## PREFACE

Interest in regional development problems is continuing to increase throughout the world. It is at the regional level that the consequences of inadequate decisions about economic growth are most clearly displayed. Adequate organization of growth requires a comprehensive consideration of the essential elements constituting the socioeconomic regional system. These elements should be integrated for the purposes of analysis, planning, and management.

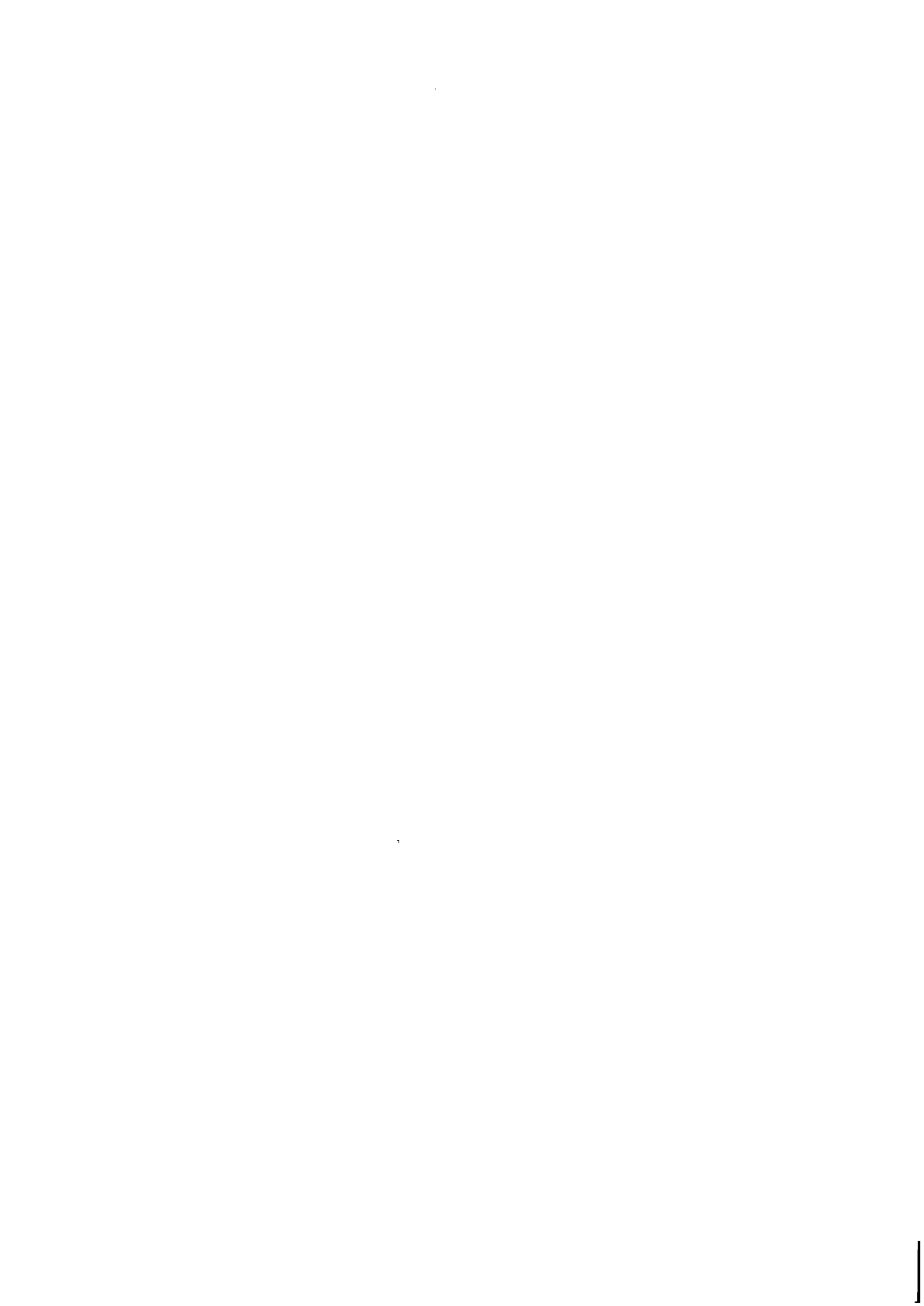
The work of the Regional Development Task at the International Institute for Applied Systems Analysis (IIASA) focuses on problems of medium- and long-term regional development. As an overall objective, the Task aims to collect, generalize, and disseminate improved methods for planning the development of regions and to improve understanding of the strategic choices that must be made by policymakers at the regional or national level. The Task, in collaboration with other Areas and Programs at IIASA as well as external institutions, is preparing a system of regional models and an approach to multisectorial analysis that will fulfill the above objective. The models will represent specific sectors of the regional economy (agriculture, industry, water supply, and labor resources and others) and they will be capable of joint and integrated use.

Several regional case studies have been planned. They will serve to test, refine, and demonstrate the set of models and the analytical approach. At present, case studies of the Silistra region in Bulgaria and the Notec region in Poland are in progress. For these studies, an agriculture model has been developed and, as a result of several trial simulations, it has been possible to formulate a more general model that could be used in many other case studies. The general model is

not intended to be a specialized agriculture model, but rather it is to be used as a means of contributing to multisectorial analysis of regional development. This paper presents the Generalized Regional Agriculture Model (GRAM).

## ABSTRACT

The Generalized Regional Agriculture Model (GRAM) presented in this paper is to form part of a system of regional models. It is not intended as a specialized agriculture model but rather as a means of reflecting the agricultural sector in the model system. This model, being general, may be used in various socioeconomic systems and yet it is sufficiently detailed to be capable of providing practical results. The main purpose of the model is to solve, by means of linear programming, large-scale problems of regional agricultural specialization. It has been designed to include all significant feedbacks and results from the other models in the system. Information will be transferred both directly and indirectly from these other models to GRAM and vice versa. GRAM includes a comprehensive description of factors such as land use, production structure, animal-feed rations, technology choices, and availability of resources. These factors affect decisions about agricultural specialization. The model includes both monetary and nonmonetary objective functions.



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INTRODUCTION

Case studies of the Silistra region in Bulgaria and the Notec region in Poland are being carried out by the International Institute for Applied Systems Analysis (IIASA) with the participation of Bulgarian and Polish Institutes. These studies examine development problems in the above regions and for this purpose a scheme for constructing a system of regional models has been developed (see Figure 1). An essential part of the system is a regional agriculture model (GRAM), which has been implemented in the above case studies. The Silistra and Notec regions have some distinctive features, thus, a specific version of GRAM was in fact used. However, the results of several runs provided some hints for the formulation of a more general model.

IIASA, as an international institute, cannot limit its activities to one or two case studies only. This paper, therefore, presents a basic version of GRAM that is general enough to be used in various socioeconomic systems and yet sufficiently detailed to be capable of solving practical problems of future regional agricultural specialization within the framework of the model system. It is expected that results of GRAM can be used for policy formulation at the regional or national level.

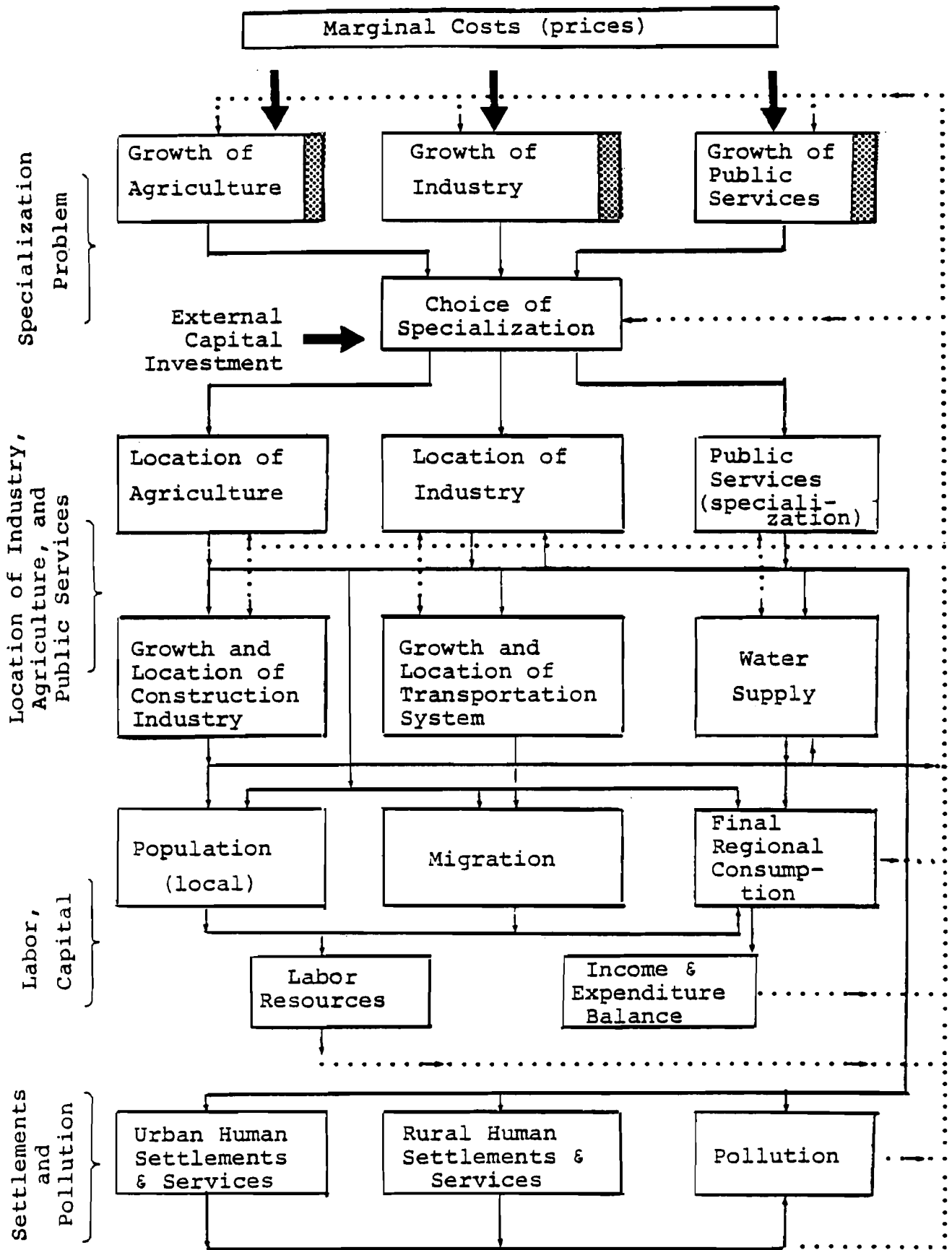


Figure 1

- external data
- iterative precision feedback data
- information flow
- fixed direction of specialization

The two case studies for which the model was originally developed have already shown that many differences exist in the regional agricultures of planned economies. However, in this respect, we only discuss differences in types of property ownership, in economies of scale, in attitudes towards technological innovation, and in the organizational structure of farms. Additional problems would be encountered if the features specific to agriculture in market economies were considered.

The main characteristics of regional agricultural development that are included in the model are:

1. Regional agricultural specialization;
2. Crop and livestock production in disaggregated form;
3. Land-use problems, with reference to irrigation, drainage, the use of pastures, and so on;
4. Alternative animal-feed compositions (protein, rough and green forage, and so on) for balanced animal-feed rations;
5. Crop-rotation conditions;
6. Possibilities for second crop production; and
7. Availability of regional supplies of labor, capital investment, fertilizers, water, and so on.

The inclusion of the above features results from not only theoretical considerations but also practical experience in dealing with development problems in the Silistra and Notec regions.

To analyze these and other problems, one must be confident that the related model can be solved. This implies that, due to the size of the regional problems discussed, the linear programming (LP) approach is the only one that is practical. However, even using the LP approach it is still necessary to simplify the computational effort and the matrix should thus be generated within the computer program. This part of the research, which is very important for practical implementation of the model, will be described in a separate report.

## LINKAGE OF GRAM WITH THE OTHER MODELS IN THE SYSTEM

A scheme of regional development models is described in detail by Albegov (1978) and is outlined in Figure 1. It consists of many models, which together form a hierarchical system. At the top are models for determining regional specialization (Level 1), these are followed by models for determining the location of sectorial activities (Level 2), by labor, capital, income, and expenditure balance models (Level 3), and finally by models of settlements, services, and pollution (Level 4).

Viewed in another way, the system is composed of four main "blocks" models - agriculture, industry, water supply, and labor resources. The interdependence of these regional models is outlined in Figure 2. The main information flows are represented in this figure and it is evident that three types of interrelationship exist.

1. The flow of "direct" information indicates that the results of one model should be transferred unchanged to another model.
2. The flow of "indirect" information indicates that the information from one model only indirectly influences another model. Therefore, an intermediate stage may exist in the coordination process, at which point information from one model is modified before being included in the other model. For example, the level of labor use or the level of wages in the regional industry and agriculture models indirectly influence the birthrate and, depending on the type of demography model, these data may be modified before being included in this model.
3. In addition to the flows of direct and indirect information, there are several directions of information coordination (in this context "coordination" refers to intervention in the calculation process by the introduction of another model or by alteration of the model being used). Coordination takes place between the industry and agriculture models in respect of the

INTERDEPENDENCE BETWEEN INDUSTRY, AGRICULTURE, WATER SUPPLY, AND LABOR FORCE MODELS

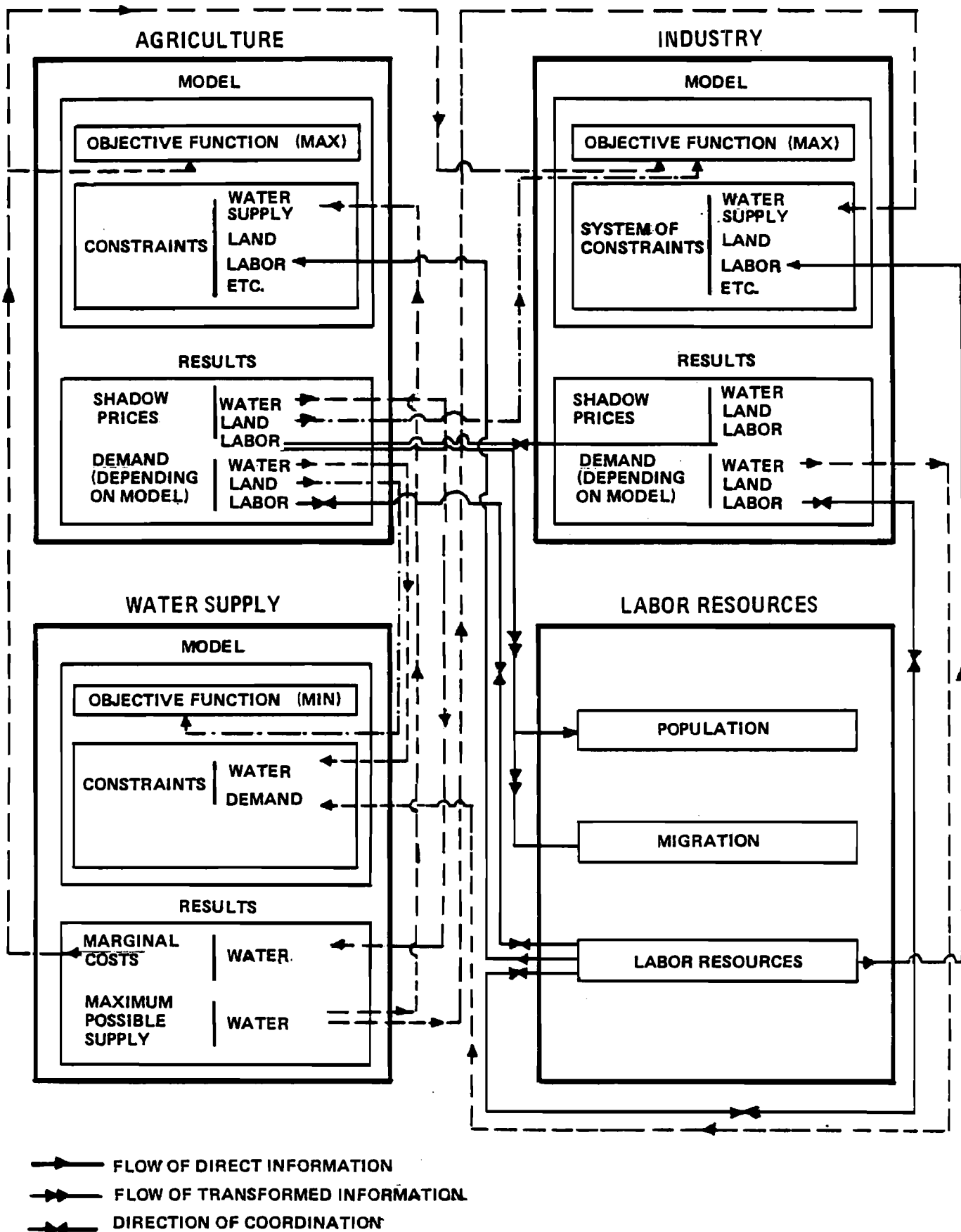


Figure 2

labor force. It also occurs between regional labor demand (agriculture and industry models) and supply (labor resources model), for example, when implementation of the migration model does not balance these. There is also coordination between regional water demand (agriculture model) and supply (corresponding water supply model), for example, when prescribed limits for water supply are too low to obtain an optimal solution.

Many other directions of exchange involving qualitative (prices) and quantitative (constraints) types of information also exist. These require an interactive calculation process that must be limited in time. It seems important, therefore, to omit interrelations of lesser importance, for example, the influence of regional industrial growth and the location of industry on the system of marginal costs of land, from the model description. The same situation is applicable to the interrelationship between the models for labor resources and water supply.

Special "reaction functions" must be prepared in order to coordinate the main blocks (industry and agriculture). This should be followed by implementation of the Bellman approach to problem solution (see Albegov, 1978, pages 2-5). In general, the reaction function, which demonstrates the effect on the regional economy of the use of limited resources by different sectors, is a function with several variables. If the number of variables is too great, however, it is not certain that solution of the model would be possible. Therefore, as an approximate yet reasonable starting point, we define this effect as a function with two variables: labor and capital investment. This means that in the process of developing the agriculture model the possibility of investigating not only the optimal solution but also near-optimal solutions for the sector must be considered. In this way, the extent to which the reaction function is dependent on the amount of labor and capital investment can be calculated.

## THE PURPOSE AND SCOPE OF THE MODEL

The principal purpose of the model is to achieve results that can be used for the formation of policy regarding future regional agricultural specialization. This specialization depends on issues such as land use, production structure, animal-feed rations, technology choices, labor use, availability of resources, which are examined in this section.

GRAM should be strictly limited to solving agricultural problems but also must be able to include all significant feedbacks and results from other subsystems, such as water, industry, and labor. A regional development problem should be separated according to its sectorial components, so that each component can be solved by the corresponding model within the framework of the set of regional models. Such an approach would allow each subproblem to be described in as much detail as is necessary and would allow each subproblem to be described in as much detail as is necessary and would avoid the use of "hybrid" models. (We use "hybrid" in the sense that these models include elements of several sectors, for example, water and industry or agriculture.) During the interaction between the agriculture and the other regional models, it should be possible to change the coefficients of the constraints or objective functions in accordance with the results of these other models.

Although we have emphasized the importance of including detail in GRAM, it is essentially intended as a general model and as such must describe a variety of agricultural and technological conditions, for example, all aspects of land use, all possibilities for land improvement, and alternative animal-feed compositions. It is also necessary to account for all types of property ownership - state, cooperative, and private - in the model. Let us now consider the main issues of agricultural specialization in a relatively broad perspective.

### Land Use

To obtain a comprehensive description of regional land use the following points need to be examined.

1. The possibility of implementing all types of land-improvement techniques, such as irrigation, drainage, terracing, chemical application;
2. The variations in the quality of the land; and
3. The possibilities of cultivating a second crop in some areas and also the conditions for crop rotation.

The effectiveness of implementing land-improvement technologies depends on the quality of the land: in general, the better the quality, the greater the crop yields. Thus, the economic efficiency of the capital investment in such projects is greater. This efficiency is also influenced by the situation of the land; for example, the closer the area requiring irrigation to a river, the more economically effective would be the irrigation scheme. GRAM accounts for the land-improvement factor by including several different types of technology in the model description.

Crop production conditions cannot be considered uniform for all subregions because of the differences in soil quality and consequently in the results from land improvement. These differences can be described adequately by accounting for a large number of subregions. In GRAM the regions are divided according to the soil quality and the model is capable of handling 40-50 subregions. In general, such a division must meet the modeling requirements of not only the agricultural sector but also other sectors, such as industry, water supply, and the system of settlements. It is impossible to achieve a division of the region's land area that is "ideal" for all sectors. Thus, the boundaries of the subregions should be defined by some factor of importance for the leading sector of the regional economy.

In some regions it is possible to harvest a second crop and this should be represented in the model description. An important problem connected with land use is the question of how to define the ratio of perennial to annual crop production. It is possible to find the exact proportions for a particular year by using a model that describes an average annual harvest.



Perennial production may change from year to year but the way in which these changes occur (that is, the dynamics of production) can be assumed to be constant for any given 5-year period. These conditions are formulated in GRAM in terms of appropriately defined land areas that may be utilized for the purposes mentioned above.

### Production Structure

To obtain practical results, a detailed model is required in which no less than 20-30 main agricultural products, including livestock and annual and perennial crops, are described. In the Soviet Union (Albegov, 1975) it has been shown that at the national level no fewer than 15 crop products should be described in the model (spring wheat, winter wheat, rye, oats, barley, maize, beans, potatoes, forage and sugar beets, annual and perennial grass, different types of animal-feed products). At the regional level the crops specified must generally be no fewer in number and type than those specified at the national level. It is obvious that some constraints on crop production should be introduced into the model. These constraints are numerous, because they are dependent on factors such as crop rotation, soil quality, and type of farm.

In most cases, the agricultural processes directly involve the dynamics. Thus, GRAM should be oriented towards analysis of the dynamics of agricultural development. However, in order to consider problems of a general nature, such as regional agricultural specialization, it is not necessary to specify details of the dynamics, for example, the year-to-year changes in the area of land used for cultivation of a particular crop and in the livestock production structure. Detailed time-span analysis is, however, more important when a significant variation in the volume of production is observed, for example, when the volume of production of some important crop or livestock product increases or decreases dramatically over time. The introduction of time-variability into agriculture models has been shown to increase significantly the dimensions of the problem. Hence, a choice between two approaches to the problem solution has to be decided upon. Either the description of intraregional problems can be

simplified significantly, or the complexity of the dynamic problems can be reproduced as accurately as possible by restricting the description to one specific aspect, for example, consecutive 5-year intervals of dynamic analysis.

The dynamics of regional livestock production is reflected directly in herd structure, which in turn influences the structure and volume of livestock products. Thus, not only should these products be included in GRAM in an aggregated form, but also the livestock specialization should be represented: cattle breeding for meat, milk, or both; sheep breeding for meat or wool; poultry breeding for meat or eggs; and so on. The model (if compared with Gouevsky and Maidment, 1977), therefore, has to describe the structure of future regional livestock production, taking into account all available alternatives. Regional agricultural activities, such as gardening and vegetable growing, might also be incorporated; for example, the production of apricots could be included for the Silistra region. The above points are included in GRAM by the use of indices representing appropriate technology and specialization in the variables concerning livestock production. The herd structure, however, is not involved directly; it has to be determined exogenously.

Since the tendency to organize agriculture on the basis of agro-industrial complexes is becoming more widespread, the agricultural processing industry needs to be briefly discussed. The directions of regional agricultural development and the volume of production should be defined and all the main problems of regional agriculture should be described in as much detail as possible. After this has been done, the problem of where the processing plant should be located can be solved. The location depends to a large extent on the transport facilities available, since rapid transportation of the products to consumers within and outside the region is essential. The separation of the procedure into two stages, as proposed above, could introduce errors. However, these errors are not as significant as they would be if a detailed description of the processing industry were included

in the model. For this latter case, the description of crop and livestock production would have to be simplified because the model's size is restricted.

#### Animal-Feed Rations

To achieve regional livestock growth, it is essential that the livestock are provided with adequate and well-balanced animal-feed rations. Thus, the following main issues should be examined.

1. Is the region able to supply its livestock with a complete set of animal-feedstuffs (a balance of feed units, such as green, rough, and succulent, should be included in the model)?
2. What possibilities exist to export excess feed-stuffs produced?
3. What influence do internal and external animal-feed supplies have on regional livestock specialization and on the scale of future development of feed production?

Some models (Gouevsky and Maidment, 1977) treat animal-feed ration alternatives as fixed. This has both advantages and disadvantages. Although it may simplify the model description, it can lead to errors in cases where the real situation is complex. Therefore, the approach chosen for GRAM is the "free formation" of animal-feed rations. This allows a choice to be made about optimal animal-feed production according to the regional specialization in crop cultivation and available external supplies. In most cases such analysis is considered to be very important. It has been shown that in the USSR (Albegov, 1975) an economy of a few million tons of crops could be achieved by using certain balances of animal-feed rations.

The availability of intraregional and imported agricultural resources has to be included in the model. In GRAM this is reflected by the crop balances and upper bounds for import quotas, respectively. Imported cereals for poultry raising can be taken as an example of the latter type of resource. In this case

several approaches to estimating the cost of imported grain supplies are used in the model and should be reflected in the objective function.

Because a significant part of crop production is required for feeding livestock, it is important to obtain the optimal balance between crop and livestock production and to examine the problem of organizing the animal-feed processing industry separately. The solution of these problems requires that the following questions be answered. What is the best way of organizing animal-feed processing operations? Where should the feed processing plants be located? What facilities will these plants require? Each of these questions should be dealt with by separate models. However, in GRAM neither the organization of animal-feed production nor the agricultural processing industry are accounted for because the main aim is to determine merely the optimal structure of animal-feed production.

#### Choice of Technology

In the process of developing a regional agriculture model it is essential to examine the various types of agricultural technology that are available. They should be evaluated in relation to the particular conditions of the subregion, for example, the availability of capital investment, the cost of water and fertilizers, and the balance of labor. To determine the optimal choice of technology, some preliminary calculations have to be carried out without using the model. The results should be combined with a variety of possible technology options and then be included in the basic version of GRAM.

This approach is adopted in our case study of the Notec region, where private and state farms are located in the same area. The choice of technology used depends to a large extent on the size of the farm, which in turn depends on the type of property ownership. As a preliminary calculation, it was thus necessary to forecast the future size of each type of farm (by determining the optimal farm size). The method presented in Kulikowski (1978) can be used. For such a forecast, it is

necessary to have some idea of possible technologies that depend on machinery, fertilizers, water, use of manual labor, and so on. One obvious assumption is that the farmworker's chief aim is to maximize his income, but this is more applicable to state and collective farmworkers than to private farmers. The choice of technology does not have to be made simultaneously with the other calculations, since specialized programs could be used for this purpose. The solution to the problem of the optimal farm size could also be solved separately.

The farmer's response to modern technology is an important factor governing the success of the model's implementation. The private farmer must be convinced that the new technology will significantly improve his output over the long term before he will replace his old machinery and methods. If he is to receive a stable additional income that corresponds to this extra output, it may be necessary that, for example, the price of water (for irrigation purposes) be less than the marginal costs of the water supply. In this case it would be necessary to investigate the water pricing system. GRAM is constructed to indicate the influence of water costs on the structure and volume of regional agricultural output.

#### Use and Supply of Labor

Since the tendency for migration from the rural to the urban areas is a worldwide phenomenon, restrictions on the use of labor merit some discussion. At this point, however, it is unnecessary to consider the coordination of labor between the main economic sectors (industry, agriculture, services).

If regional limits to the labor supply are accounted for in GRAM, it should be possible to determine the regional agricultural structure and output when employees change their field of work. Additional limits to the labor supply may exist at the subregional level, and exchanges of labor between collective and state farms, for example, to provide support staff when required, should be considered. This may be done by introducing a constraint representing labor-supply restrictions on collective and state farms for the region.

### Capital Investment

The total capital investment required for regional agriculture has to be assessed. The capital investment needed by the farms of the region should, therefore, be estimated on an individual basis. Collective and private farms may be assessed in the same way, but another approach should be used for state farms. The differences in farm organization are reflected in these two approaches. On state farms all income goes to the state, which pays the farmworkers a wage. They are thus not so dependent on the results of annual production, as are the farmworkers on collective farms. The state also supplies the farms with all requirements, such as seed, fertilizers, and the capital investment necessary to achieve the desired level of growth in output and of expansion of activities. In the case of collective farms, it is the members who decide what proportion of the farm income should go towards capital investment. However, they are able to obtain some external funds for the expansion of activities, usually in the form of subsidies, from the local or central authorities. In the case of private farms, the owner is responsible for providing all capital investment necessary to increase his output or to expand his activities. In GRAM the capital investment constraints are considered only for collective and private farms.

The availability of capital investment is one of the main factors that determines the rate of regional agricultural growth. In this respect, constraints exist at the subregional and also the regional level. It is possible to ascertain the degree of dependence of the regional agricultural structure, output, and income on state finance by varying the level of state investment in agriculture.

### Water Resources

The interdependence of agriculture and water supply is obvious. The scale on which an irrigation scheme is introduced significantly affects the marginal costs of the water supply. Therefore, an optimal solution to the water supply problem in the agricultural region must be found. Our approach is to

separate water demand (described in the agriculture model) from water supply (described in a water supply model). Information about the price of water and the limits to the water supply is obtained from the water supply model and included in the agriculture model.

However, the water pricing system could be complicated as a result of the irregularity of the agricultural water demand, which during the spring and summer varies considerably but is much higher than in the autumn and winter. Therefore, to obtain an estimate of water demand, it is necessary to include only one value for the cost of water in the agriculture model. However, for a more precise calculation, several values for the cost of water should be introduced.

#### The Supply of Technological and Technical Resources

The general approach used in GRAM to solve the problem of the supply of technological and technical resources is to calculate the additional benefit to the enterprise resulting from the provision of an additional supply of technological and technical resources, such as machinery and fertilizers. These resources should be delivered to the farms of the region in accordance with our calculations. If supply restrictions on certain times exist, the corresponding constraints should be introduced into the model.

#### Objective Function

The type of objective function used is primarily dependent on the policy defining the agricultural development of the particular region. Two types of objective function are included in the model - the monetary and the nonmonetary type.

For the former, a cost-benefit comparison is made. For the latter, some policy-oriented objectives must be fulfilled, such as regional self-sufficiency in agricultural products, or maximization of the prescribed livestock production. The nonmonetary objective function is not often used and so let us consider the

monetary type in more detail. The system of prices used can change according to the product or the variations in the structure of the model. This factor should be considered in the analysis of regional agricultural benefit; for example, the prices of the products to be sold on the international market should be estimated higher than those sold for domestic consumption. Similarly, the prices of goods purchased on the international agricultural market should also be estimated higher than the prices of domestic products, because the amount of foreign exchange held in the country is limited.

In general, there are two possible approaches to describing the problem of expenditure: static and dynamic. When the static approach is employed and the most important technical and economic data are constant over time (capacity and operational costs), all expenditure can be expressed as the following form of production costs (this approach is widely adopted in planned economies):

$$Z_i^* = E_i^* + r^* C_i^* , \quad (1)$$

where

$Z_i^*$  = production cost per unit of commodity  $i$ ,

$E_i^*$  = operational cost per unit of commodity  $i$ ,

$r$  = rate of efficiency (as a percentage of capital investment), which is the same for all development alternatives and which corresponds to a marginal (in this case, minimal) rate of interest (in planned economies it also includes the rate of return on capital investment),

$C_i^*$  = capital investment per unit of commodity  $i$ .

In the above case, the depreciation is determined as:

$$E_{ix}^* = \frac{r}{(1+r)^{t'} - 1} , \quad (2)$$



where

$E_{ir}^*$  = annual depreciation cost as a percentage of capital investment per unit commodity  $i$ ,

$t'$  = estimated life of capital stock in number of years.

When the plant capacity, capital investment, and output vary according to the length of time that the plant has been in operation, the average operational costs are determined as follows:

$$Z_i = \frac{\sum_{t=1}^T (C_{ti} + E_{ti}) \beta_t}{\sum_{t=1}^T A_{ti} \beta_t} , \quad (3)$$

where

$Z_i$  = average operational cost of commodity  $i$  over over the period  $T$ ,

$t$  = specific year in the period under analysis,

$C_{ti}$  = capital investment in the plant that produces commodity  $i$  in year  $t$ ,

$E_{ti}$  = annual operational cost of the plant that produces commodity  $i$  in year  $t$ ,

$A_{ti}$  = output of commodities  $i$  in year  $t$ ,

$T$  = length of time that the plant has been in operation,

$\beta_t$  = coefficient of discount, which is determined

$$\text{as } \beta_t = \frac{1}{(1+r)^t} .$$

When the dynamic approach is used, a comparison between different commodities can be made on the basis of their production costs over the period under analysis (aggregated production costs). Such a comparison makes it possible to determine the commodity that it is most profitable to produce.

$$Z_i^a = \sum_{t=1}^T (C_{ti} + E_{ti}) \beta_t, \quad (4)$$

where

$Z_i^a$  = sum of discounted production costs for the commodity  $i$  for the period under analysis.

The operational costs vary according to the type of farm. In general, they are expressed as:

$$E^* = \sum_j b_j^* P_j^* + E_r^*,$$

where

$E^*$  = operational costs per unit of commodity  $i$   
(index  $i$  is omitted from now on),

$j$  = index of production factor,

$b_j^*$  = use of factor  $j$  per unit of commodity  $i$ ,

$P_j^*$  = price per unit of factor  $j$ ,

$E_r^*$  = depreciation cost per unit of commodity  $i$   
(determined in accordance with equation (2)).

The production factors can be fertilizers, energy, water, seeds, wages, and so on. It should be stressed that for state and collective farms the cost of land can be zero and that in collective and private farms the cost of labor is not directly included in the production costs. In the latter case, the

minimum level of wages can be included in the cost of production to ensure that this factor is taken into account and that a part of the benefit obtained from the output of the production process can be directed towards a further growth in wages. In GRAM this minimum income level condition is also included as a constraint for private and collective farms.

It should also be emphasized that when coordinating different types of models (for example, the agriculture and water supply models) improved data on the price of water can be included in equation (5). Therefore, the procedure for calculating E can be used separately in order to obtain a more precise value of the operational costs.

#### DESCRIPTION OF THE MODEL

This section contains definitions of the indices, coefficients, decision variables, constraints, and objective functions used in the model.

#### Indices

- $i$  - type of crop,
- $i \in I^1 = \{1, 2, 3, \dots, I_1\}$  - grain crops,
- $i \in I^2 = \{I_1+1, I_1+2, \dots, I_2\}$  - industrial crops,
- $i \in I^3 = \{I_2+1, I_2+2, \dots, I_3\}$  - starchy root crops,
- $i \in I^4 = \{I_3+1, I_3+2, \dots, I_4\}$  - vegetables,
- $i \in I^5 = \{I_4+1, I_4+2, \dots, I_5\}$  - garden crops, grapes, etc.,
- $i \in I^6 = \{I_5+1, I_5+2, \dots, I_6\}$  - forage from meadows and pastures;
- $w$  - crop rotation group  $I^w$ ;

- j - livestock (including poultry);
- k - livestock-breeding specialization,
  - k = 1 - meat production,
  - k = 2 - milk production,
  - k = 3 - mixed, meat and milk,
  - k = 4 - egg production,
  - k = 5 - wool production;
- m - type of livestock product (such as meat milk, eggs, skins);
- r - subregion which can either correspond to an administrative division or a division of the land according to soil quality;
- n - animal-feed components,
  - n = 1 - feed units,
  - n = 2 - protein,
  - n = 3 - rough feed,
  - n = 4 - green feed;
- l - type of market on which a particular commodity is sold (purchased),
  - l = 1 - internal state market,
  - l = 2 - internal private market,
  - l = 3 - external (world) market;
- p - type of property ownership,
  - p = 1 - state farm,

$p = 2$  - collective farm,

$p = 3$  - private farm;

$s$  - type of technology used for crop production,

$s = 1$  - technology for unimproved land,

$s = 2$  - technology for land on which there are limited possibilities for irrigation,

$s = 3$  - technology for land on which there are unlimited possibilities for irrigation,

$s = 4$  - technology for land that requires terracing,

$s = 5$  - technology for land that requires drainage,

$s = 6$  - technology for land that requires the application of chemicals;

$s'$  - type of livestock- or poultry-breeding technology;

$\alpha$  - type of land, differentiated according to soil quality;

$f$  - type of fertilizer used;

$\beta_i$  - index of the best second crop, if any, following the  $i$ -th crop.

#### Coefficients

$a_{fiprs}$  - demand for fertilizer  $f$  to produce one unit of crop  $i$  by property  $p$  on land  $\alpha$  in subregion  $r$ , when technology  $s$  is used;

$\hat{a}_{fjk}$  - manure produced from one unit of livestock  $j$  of specialization  $k$ , expressed in units of fertilizer  $f$ ;

- $b_{jkprs}'$  - demand for labor to produce one unit of livestock  $j$  of specialization  $k$  by property  $p$  in subregion  $r$ , when technology  $s'$  is used;
- $b_{iprs\alpha}$  - demand for labor to produce one unit of crop  $i$  by property  $p$  on land  $\alpha$  in subregion  $r$ , when technology  $s$  is used;
- $c_{jkprs}'$  - demand for capital to produce one unit of livestock  $j$  of specialization  $k$  by property  $p$  in subregion  $r$ , when technology  $s'$  is used;
- $c_{iprs\alpha}$  - demand for capital investment to produce one unit of crop  $i$  by property  $p$  on land  $\alpha$  in subregion  $r$ , when technology  $s$  is used;
- $\bar{c}_{iprs\alpha}$  - additional capital investment required to produce one unit of crop  $i$  by property  $p$  on land  $\alpha$  in subregion  $r$ , when technology  $s$  is used for land improvement;
- $d_{jkprs}'$  - annual demand for water to produce one unit of livestock  $j$  of specialization  $k$  by property  $p$  in subregion  $r$ , when technology  $s'$  is used;
- $\hat{d}_{jkprs}'$  - demand for water at peak periods to produce one unit of livestock  $j$  of specialization  $k$  by property  $p$  in subregion  $r$ , when technology  $s'$  is used;
- $d_{iprs\alpha}$  - annual demand for water to produce one unit of crop  $i$  by property  $p$  on land  $\alpha$  in subregion  $r$ , when technology  $s$  is used;
- $\hat{d}_{iprs\alpha}$  - demand for water at peak periods to produce one unit of crop  $i$  by property  $p$  on land  $\alpha$  in subregion  $r$ , when technology  $s$  is used;

- $e_{iprs\alpha}$  - demand for machinery to produce one unit of crop  $i$  by property  $p$  on land  $\alpha$  in subregion  $r$ , when technology  $s$  is used;
- $f_{nj\kappa}^{\min}, f_{nj\kappa}^{\max}$  - minimum and maximum demand for animal-feed component  $n$  per unit of livestock  $j$  of specialization  $\kappa$ ;
- $g_{in}$  - content of animal-feed component  $n$  in one unit of crop  $i$ ;
- $g_{mn}$  - content of animal-feed component  $n$  in one unit of livestock product  $m$ ;
- $h_{mj\kappa p s'}$  - output of livestock product  $m$  in one unit of livestock  $j$  of specialization  $\kappa$  from property  $p$ , when technology  $s'$  is used;
- $B$  - maximum amount of labor available in the whole region;
- $B_{pr}$  - maximum amount of labor available on property  $p$  in subregion  $r$ ;
- $C$  - total (external and internal) capital investment available for regional agriculture;
- $C_{pr}$  - total (external and internal) capital investment available for agriculture for property  $p$  in subregion  $r$ ;
- $D$  - maximum annual water supply available in the whole region;
- $\hat{D}$  - maximum water supply available at peak periods in the whole region;
- $D_{pr}$  - maximum annual water supply available for property  $p$  in subregion  $r$ ;

- $\hat{D}_{pr}$  - maximum water supply available at peak periods for property p in subregion r;
- E - maximum amount of agricultural machinery available for the whole region;
- $F_i^{\min}, F_i^{\max}$  - minimum and maximum levels of consumption of crop i in the whole region;
- $F_m^{\min}, F_m^{\max}$  - minimum and maximum levels of consumption of livestock product m in the whole region;
- $F_{ipr}^{\min}, F_{ipr}^{\max}$  - minimum and maximum production of crop i on property p in subregion r;
- $G_f$  - maximum volume of fertilizer f available in the whole region;
- $G_{fpr}$  - maximum volume of fertilizer f available for property p in subregion r;
- $H_{il}$  - maximum volume of external purchases of crop i on market l for livestock in the whole region;
- $I_{il}$  - maximum volume of external purchases of crop i on market l for human consumption in the whole region;
- $I_{ml}$  - maximum volume of external purchases of livestock product m on market l for human consumption in the whole region;
- $\bar{I}_{il}$  - sale limitation of crop i on market l;
- $\bar{I}_{ml}$  - sale limitation of livestock product m on market l;



- $L_{ipr}^{\min}, L_{ipr}^{\max}$  - minimum and maximum area of land (state, collective, or private) that, in accordance with crop rotation, could be used for cultivating crop  $i$  of rotation group  $w$  on property  $p$  in subregion  $r$ ;
- $L_{pra}^{\min}, L_{pra}^{\max}$  - minimum and maximum area of land  $a$  available on property  $p$  in subregion  $r$ ;
- $L_{prsa}^{\min}, L_{prsa}^{\max}$  - minimum and maximum area of land  $a$  on property  $p$  in subregion  $r$  that can be improved using technology  $s$ ;
- $L_{pr}$  - maximum area of arable land on property  $p$  in subregion  $r$ ;
- $L_{pr}^m$  - area of meadows and pastures on property  $p$  in subregion  $r$ ;
- $M_{jpr}^{\min}, M_{jpr}^{\max}$  - minimum and maximum possible production of livestock  $j$  on property  $p$  in subregion  $r$ ;
- $N_i$  - number of nutrition units per unit of crop  $i$ ;
- $N_{mjk}$  - number of nutrition units per unit of livestock product  $m$  obtained from livestock  $j$  of specialization  $k$ ;
- $P_i^l$  - price per unit of garden crop  $i$  purchased on market  $l$ ;
- $P_m^l$  - price per unit of domestically produced livestock product  $m$  purchased on market  $l$ ;
- $P_{il}^{\text{imp}}$  - price per unit of crop  $i$  purchased for animal-feed on market  $l$ ;

- $\bar{p}_{il}^{imp}$  - price per unit of crop  $i$  purchased for human consumption on market  $l$ ;
- $p_{ml}^{imp}$  - price per unit of livestock product  $m$  purchased for human consumption on market  $l$ ;
- $S_{iprsa}$  - production cost per unit of crop  $i$  produced on property  $p$  on land  $\alpha$  in subregion  $r$ , when technology  $s$  is used;
- $S_{jkprs}'$  - maintenance cost per unit of livestock  $j$  of specialization  $k$  on property  $p$  in subregion  $r$ , when technology  $s'$  is used (expenditure on animal-feed is not included);
- $u_{iprsa}$  - average yield of first crop  $i$  on property  $p$  per unit of land  $\alpha$  in subregion  $r$ , when technology  $s$  is used;
- $u_{iprsa}^1$  - average yield of second crop  $i$  on property  $p$  per unit of land  $\alpha$  in subregion  $r$ , when technology  $s$  is used;
- $\bar{w}_p$  - minimum wage level per capita on property  $p$ .

#### Decision Variables

- $P_{iprl}$  - volume of purchase for animal-feed of crop  $i$  on market  $l$  by property  $p$  in subregion  $r$ ;
- $Q_{iprl}$  - volume of purchase for human consumption of crop  $i$  on market  $l$  by property  $p$  in subregion  $r$ ;
- $Q_{mprl}$  - volume of purchase for human consumption of livestock product  $m$  on market  $l$  by property  $p$  in subregion  $r$ ;
- $R_{iprl}$  - volume of sale of crop  $i$  on market  $l$  by property  $p$  in subregion  $r$ ;

- $R_{mprl}$  - volume of sale of product  $m$  on market  $l$  by property  $p$  in subregion  $r$ ;
- $W_{ipr}$  - human consumption of crop  $i$  on property  $p$  in subregion  $r$ ;
- $W_{mpr}$  - human consumption of livestock product  $m$  on property  $p$  in subregion  $r$ ;
- $X_{iprsa}$  - volume of first production of crop  $i$  on property  $p$  on land  $\alpha$  in subregion  $r$ , when technology  $s$  is used;
- $X_{jkprs}$  - number of livestock  $j$  of specialization  $k$  on property  $p$  in subregion  $r$ , when technology  $s$  is used;
- $Y_{iprsa}$  - volume of the second production of crop  $i$  on property  $p$  on land  $\alpha$  in subregion  $r$ , when technology  $s$  is used;
- $Z_{ipr}$  - consumption by livestock of crop  $i$  on property  $p$  in subregion  $r$ ;
- $Z_{mpr}$  - consumption by livestock of livestock product  $m$  on property  $p$  in subregion  $r$ ;

### Constraints

#### Land

The area of arable land belonging to the farms in subregion  $r$  that can be used for crop cultivation is constrained in the following way:

$$\sum_{i,s,\alpha} \frac{X_{iprsa}}{u_{iprsa}} \leq L_{pr} \quad , \quad \text{for all } p,r \quad . \quad (6)$$

However, for social or political reasons, the above inequality sign could be changed to a sign of equality.

The areas of quality land are limited:

$$L_{pra}^{\min} \leq \sum_{i,s} \frac{X_{iprsa}}{u_{iprsa}} \leq L_{pra}^{\max} , \quad \text{for all } p,r,\alpha . \quad (7)$$

The area of land occupied by crops from groups  $I^1, I^2, \dots, I^5$  on property  $p$  must be used in accordance with the limitations imposed by crop rotation:

$$L_{ipr}^{\min} \leq \sum_{\substack{s,\alpha, \\ i \in I^w}} \frac{X_{iprs}}{u_{iprs}} \leq L_{ipr}^{\max} , \quad (8)$$

for all  $p,r,w = 1,2,3,4,5$  .

The area of land that can be improved by irrigation, terracing, and the like is limited:

$$L_{prsa}^{\min} \leq \sum_i \frac{X_{iprsa}}{u_{iprsa}} \leq L_{prsa}^{\max} , \quad \text{for all } \alpha,p,r,s . \quad (9)$$

The area of pastures and meadows is limited:

$$\sum_{s,\alpha, i \in I^6} \frac{X_{iprsa}}{u_{iprsa}} \leq L_{pr}^m , \quad \text{for all } p,r . \quad (10)$$

### Crop and Livestock Balances

The crop balance for each property in subregion  $r$  is represented as:

$$\sum_{s,\alpha} (X_{iprsa} + Y_{iprsa}) + \sum_l (Q_{iprl} + P_{iprl}) \quad (11)$$

$$- W_{ipr} - Z_{ipr} - \sum_l R_{iprl} = 0 ,$$

for all  $i,p,r$ .

The balances of first and second crop production for each property in subregion r are represented as:

$$\sum_{s,\alpha} \frac{X_{iprsa}}{u_{iprsa}} - \sum_{\substack{s,\alpha, \\ i \in B_i}} \frac{Y_{iprsa}}{u_{iprsa}} \geq 0, \quad \text{for all } p,r. \quad (12)$$

The balance of livestock products for each property in subregion r is represented as:

$$\sum_{j,k,s'} h_{mjkps'} X_{jkprs'} + \sum_1 Q_{mprl} - W_{mpr} - Z_{mpr} - X_{mpr} - \sum_1 R_{mprl} = 0, \quad (13)$$

for all m,p,r .

The demand for each animal-feed component n of livestock j of specialization k on property p in subregion r must be satisfied either by resources belonging to the property itself or by feed purchased within, or outside, the region under analysis. The components of animal-feed rations should be balanced to satisfy the physiological requirements of the livestock.

For collective and private farms:

$$\sum_{j,k,s'} f_{njk}^{\min} X_{jkprs'} \leq \sum_i g_{in} Z_{ipr} + \sum_{i,1} g_{in} P_{ipr1} + \sum_m g_{mn} Z_{mpr} \leq \sum_{j,k,s'} f_{njk}^{\max} X_{jkprs}, \quad (14)$$

for all n,p = 2,3 .

For state farms, which are able to balance animal-feed supply and demand at a regional level:

$$\begin{aligned} \sum_{j,k,r,s'} f_{njk} X_{jkprs'} &\leq \sum_{i,r} g_{in} Z_{ipr} + \sum_{i,r,l} g_{in} P_{iprl} \\ &+ \sum_{m,r} g_{mn} Z_{mpr} \quad (15) \\ &\leq \sum_{j,k,r,s'} f_{njk}^{\max} X_{jkprs'} \end{aligned}$$

for all  $n, p = 1 \dots$

It is assumed that the minimum requirements for the production of crops and livestock products should be supplied from within the region.

For crops:

$$F_i^{\max} \geq \sum_{p,r} W_{ipr} \geq F_i^{\min}, \quad \text{for } i \in I^1, I^3, I^4, I^5 \quad (16)$$

For livestock products:

$$F_m^{\max} \geq \sum_{p,r} W_{mpr} \geq F_m^{\min}, \quad \text{for all } m \quad (17)$$

#### Production Limits

The volume of production of crop and livestock products may be limited at the subregional or regional level:

$$F_{ipr}^{\min} \geq \sum_{s,\alpha} (X_{iprs\alpha} + Y_{iprs\alpha}) \geq F_{ipr}^{\max}, \quad (18)$$

for all  $i, p, r$ ,

and

$$M_{jpr}^{\min} \leq \sum_{k,s'} X_{jkprs'} \leq M_{jpr}^{\max}, \quad \text{for all } j, p, r \quad (19)$$

#### Resource Constraints

Labor resources can be limited at the subregional level:

$$\sum_{i,s,\alpha} b_{iprsa} (X_{iprsa} + Y_{iprsa}) + \sum_{j,k,s'} b_{jkprs'} X_{jkprs'} \leq B_{pr} \quad (20)$$

for all  $p,r$  ,

and at the regional level:

$$\sum_{\substack{i,p,r, \\ s,\alpha}} b_{iprsa} (X_{iprsa} + Y_{iprsa}) + \sum_{\substack{j,k,p, \\ r,s'}} b_{jkprs'} X_{jkprs'} \leq B \quad (21)$$

In the system of regional models, the price of water should be specified (at each interaction) in the cost of agricultural products. It may also be necessary for quantitative restrictions on the annual or peak-period water supply at the regional and subregional levels to be represented in the cost.

The annual water supply available for each subregion is represented as:

$$\sum_{i,s,\alpha} d_{iprsa} (X_{iprsa} + Y_{iprsa}) + \sum_{j,k,s'} d_{jkprs'} X_{jkprs'} \leq D_{pr} \quad (22)$$

for all  $p,r$  .

The peak-period water supply available for each subregion is represented as:

$$\sum_{i,s,\alpha} \hat{d}_{iprsa} (X_{iprsa} + Y_{iprsa}) + \sum_{j,k,s'} \hat{d}_{jkprs'} X_{jkprs'} \leq \hat{D}_{pr} \quad (23)$$

for all  $p,r$  .

The annual water supply available for the whole region is represented as:

$$\sum_{\substack{i,p,r, \\ s,\alpha}} d_{iprsa} (X_{iprsa} + Y_{iprsa}) + \sum_{\substack{j,k,p, \\ r,s'}} d_{jkprs'} X_{jkprs'} \leq D . \quad (24)$$

The peak-period water supply available for the whole region is represented as:

$$\sum_{\substack{i,p,r, \\ s,\alpha}} \hat{d}_{iprsa} (X_{iprsa} + Y_{iprsa}) + \sum_{\substack{j,k,p, \\ r,s'}} \hat{d}_{jkprs'} X_{jkprs'} \leq \hat{D} . \quad (25)$$

The supply of machinery required for crop production in the whole region may be limited:

$$\sum_{\substack{i,p,r, \\ s,\alpha}} e_{iprsa} (X_{iprsa} + Y_{iprsa}) \leq E . \quad (26)$$

The demand for machinery should be defined taking into account the machinery and draft horses already in use.

The regional and subregional supply of fertilizers may be limited.

For the whole region:

$$\sum_{\substack{i,p,r, \\ s,\alpha}} a_{fiprsa} (X_{iprsa} + Y_{iprsa}) - \sum_{\substack{j,k,p, \\ r,s'}} \hat{a}_{fjks'} X_{jkprs'} \leq G_f , \quad (27)$$

for all f .



For the subregions:

$$\sum_{i,s,\alpha} a_{fiprsa} (X_{iprsa} + Y_{iprsa}) - \sum_{j,k,s} \hat{a}_{fjks} X_{jkprs} \leq G_{fpr} \quad (28)$$

for all  $f,p = 2,3$  .

#### Purchase and Sale Limits

The possibilities for purchasing certain crops and live-stock products for the whole region on a particular market  $l$  could be limited.

The volume of crops purchased for animal-feed is:

$$\sum_{p,r} P_{iprl} \leq H_{il} \quad , \quad \text{for all } l, i \in I^1, I^3, I^4, I^5, I^6 \quad (29)$$

The volume of crops purchased for human consumption is:

$$\sum_{p,r} Q_{iprl} \leq I_{il} \quad , \quad \text{for all } l, i \in I^1, I^3, I^4, I^5, I^6 \quad (30)$$

The volume of livestock products purchased for human consumption is:

$$\sum_{p,r} Q_{mprl} \leq I_{ml} \quad , \quad \text{for all } m,l \quad (31)$$

The possibilities for selling certain kinds of crops and livestock products for the whole region on market  $l$  may be limited.

The volume of crops sold is:

$$\sum_{p,r} R_{iprl} \leq \bar{I}_{il} \quad , \quad \text{for all } l, i \in I^1, \dots, I^5 \quad (32)$$

The volume of livestock products sold is:

$$\sum_{p,r} R_{mprl} \leq \bar{I}_{ml} , \quad \text{for all } m,l . \quad (33)$$

If some products (for example, those derived from meadows and pastures) cannot be sold or purchased but can only be consumed at the place of production, upper limits in the above constraints should be set at zero.

### Financial Constraints

For collective and private farms, the following financial limitation exists:

$$\begin{aligned} \sum_{i,s,\alpha} c_{iprsa} (X_{iprsa} + Y_{iprsa}) + \sum_{i,s,\alpha} \bar{c}_{iprsa} X_{iprsa} \\ + \sum_{j,k,p} c_{jkprs'} X_{jkprs'} \\ \leq C_{pr} , \end{aligned} \quad (34)$$

for all  $r,p$  .

There may be limited capital investment available for agriculture in the whole region:

$$\begin{aligned} \sum_{\substack{i,p,r, \\ s,\alpha}} c_{iprsa} (X_{iprsa} + Y_{iprsa}) + \sum_{\substack{i,p,r, \\ s,\alpha}} \bar{c}_{iprsa} X_{iprsa} \\ + \sum_{\substack{j,k,p \\ r,s'}} c_{jkprs'} X_{jkprs'} \\ \leq C . \end{aligned} \quad (35)$$

The output produced on private and collective farms should be sufficient to provide the farmers with the minimum level of income necessary to satisfy their needs:

$$\begin{aligned}
 & \sum_{i,l} P_i^1 R_{ipr1} + \sum_{i,l} P_m^1 R_{mpr1} - \sum_{i,s,\alpha} S_{iprsa} X_{iprsa} \\
 & + \sum_{j,k,s'} S_{jkprs'} X_{jkprs'} - \sum_{i,l} P_{il}^{imp} P_{ipr1} \\
 & - \sum_{i,l} \bar{P}_{il}^{imp} Q_{ipr1} - \sum_{m,l} P_{ml}^{imp} Q_{mpr1} \\
 & \geq \bar{W}_p^B B_{pr} ,
 \end{aligned} \tag{36}$$

for all  $r, p = 2, 3$  .

#### OBJECTIVE FUNCTIONS

As stated earlier, two types of objective function - monetary and nonmonetary - can be implemented.

1. The former type of objective function is used to maximize regional profit in agriculture:

$$\begin{aligned}
 & \sum_{i,p,r} [P_i^1 R_{ipr1} + P_i^2 R_{ipr2} + P_i^3 R_{ipr3}] \\
 & - \sum_{i,p,r, s,\alpha} S_{iprsa} X_{iprsa} \\
 & + \sum_{m,p,r} [P_m^1 R_{mpr1} + P_m^2 R_{mpr2} + P_m^3 R_{mpr3}] \\
 & - \sum_{j,k,p, r,s'} S_{jkprs'} X_{jkprs'} \\
 & - \sum_{i,p,r,l} P_{il}^{imp} P_{ipr1} - \sum_{i,p,r,l} \bar{P}_{il}^{imp} \\
 & - \sum_{m,p,r,l} P_{ml}^{imp} Q_{mpr1} + \max .
 \end{aligned} \tag{37}$$

2. Variations of equation (37) are possible but the effectiveness of their implementation is debatable. One of these is a monetary variant expressed in terms of the internal pricing system:

$$\begin{aligned}
 & \sum_{\substack{i,p,r, \\ s,\alpha}} P_i^1 X_{iprsa} + \sum_{\substack{m,j,k, \\ p,r,s'}} P_m^1 h_{mjkps'} X_{jkprs'} \\
 & - \sum_{i,p,r,l} P_{il}^{imp} P_{iprl} \\
 & - \sum_{i,p,r,l} P_{il}^{imp} Q_{iprl} \\
 & - \sum_{m,p,r,l} P_{ml}^{imp} Q_{mprl} \rightarrow \max .
 \end{aligned} \tag{38}$$

3. One type of nonmonetary objective function that maximizes agricultural production is expressed in terms of nutrition units:

$$\begin{aligned}
 & \sum_{\substack{i,p,r, \\ s,\alpha}} N_{i\alpha} [X_{iprsa} - \sum_l (P_{iprl} + Q_{iprl})] \\
 & + \sum_{\substack{m,j,k,p, \\ r,s'}} N_{mjk} (h_{mjkps'} X_{jkprs'} - \sum_l Q_{mprl}) \\
 & \rightarrow \max .
 \end{aligned} \tag{39}$$

4. To account for the problem of increasing the volume of livestock products, a monetary objective function may be used:

$$\begin{aligned}
 & \sum_{m,p,r,l} P_m^1 R_{mprl} - \sum_{m,p,r,l} P_{ml}^{imp} Q_{mprl} \\
 & - \sum_{i,p,r,l} P_{il}^{imp} P_{iprl} \rightarrow \max .
 \end{aligned} \tag{40}$$

5. The volume of livestock products can also be expressed in nutrition units:

$$\sum_{\substack{m,j,k,p \\ r,s'}} N_{mjk} (h_{mjkps} X_{jkprs} - \sum_1 Q_{mprl}) \rightarrow \max \quad . \quad (41)$$

6. The maximization of exports may be considered to be the most important problem of regional agriculture.

$$\begin{aligned} & \sum_{i,p,r} (P_i^3 R_{ipr3} - \bar{P}_{i3}^{imp} Q_{ipr3} - P_{i3}^{imp} P_{ipr3}) \\ & + \sum_{m,p,r} (P_m^3 R_{mpr3} - P_{m3}^{imp} Q_{mpr3}) \quad . \end{aligned} \quad (42)$$

The examples given above provide a comprehensive sample of objective functions.

#### DIRECTIONS FOR FURTHER MODEL DEVELOPMENT

##### Preparation of the Matrix of Constraints

The size of the matrix of constraints for GRAM is determined mainly by the number of animal-feed balance equations, crops, and subregions included in the model description. The animal-feed balance equations have a decisive impact on the number of rows (constraints) required and the crops and subregions affect the number of columns (variables) needed. During implementation of the model the detailed description of regional agriculture often causes large-scale linear programming problems, in some cases involving several thousands of variables and constraints. Therefore, the formation and eventual modification of such a matrix are rather difficult to perform. Thus, the use of an efficient computer software (DATAMAT system) is essential.

##### Dynamics

The characteristics of regional agricultural growth restrict the use of yearly dynamics in the agriculture model. It seems

acceptable to describe the dynamics of regional agricultural growth in consecutive 5-year intervals, within which yearly variations can be interpolated. Alternatively, a more accurate year-to-year analysis may be required. In this case it would be necessary to determine which part of the model should be omitted in order to reduce the dimensions, so that the yearly dynamics may be included. It seems that the detailed dynamic character of regional agricultural growth could best be studied taking the regional level as the basis for analysis. Variables representing agricultural growth at lower levels would, therefore, have to be aggregated at a regional level. If it is necessary to describe the dynamics in the model, a thorough analysis of the model modifications needed to meet this requirement should be made.

#### Irregularity of Water Supply

In many agricultural models the price of water is assumed to be constant and independent of the time of year. Such an assumption is, however, misleading. It is possible, using the electric energy supply as an analogue, that during the peak period the price of water might be estimated to be four or five times the price of water in the off-peak period. Thus, equations (17) to (20) might be added to a few others that divide the volume of water consumption by periods, so that expenditure on water may be calculated accurately according to the time of year. The variations in water price during the different seasons could be represented in the model not only for average but also for drought and high-precipitation conditions.

#### Additional Improvements

There should be a possibility for closely analyzing the production of some important products, for example, meat, wool, and milk in an agriculture model. In GRAM, such products are represented indirectly in the form of specified production targets of livestock units. If it is necessary to include them in a direct way explicit production targets for these products can easily be added. The restrictions on regional agricultural

growth resulting from environmental factors are also not explicitly included in the model description but are represented indirectly by some production constraints. For some regions, however, it would be essential to include such factors directly and the way in which they are introduced into the model would depend on the characteristics specific to the region.

Another important task is to consider some random effects on agriculture, for example, the weather, and to represent them in a linear model. The results would then form a basis for more rational proposals about future levels of agricultural output under randomly varying conditions.

It is usually inadequate to assume only one objective function in agriculture because of the different interests and objectives of the various administrative units and levels, property types, and so on. Therefore, an extension of the model to cover some multicriterial aspects should also be considered.

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