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COST ALLOCATION IN WATER RESOURCES DEVELOPMENT - A CASE STUDY OF SWEDEN

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September 1979
WP-79-77

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PREFACE

Water resource systems have been an important part of resources and environment related research at IIASA since its inception. As demands for water increase relative to supply, the intensity and efficiency of water resources management must be developed further. This in turn requires an increase in the degree of detail and sophistication of the analysis, including economic, social and environmental evaluation of water resources development alternatives aided by application of mathematical modelling techniques, to generate inputs for planning, design, and operational decisions.

During the year of 1978 it was decided that parallel to the continuation of demand studies, an attempt would be made to integrate the results of our studies on water demands with water supply considerations. This new task was named "Regional Water Management" (Task 1, Resources and Environment Area).

This paper is oriented towards the application of systems analysis techniques to water management problems in Western Skåne, Sweden. These problems concern the allocation of scarce water and related land resources among several mutually conflicting users, e.g., municipal, industrial, agricultural and recreational water use.

The paper is part of a collaborative study on water resources problems in Western Skåne, Sweden, between the Swedish Environmental Protection Board, the University of Lund and IIASA. The paper concerns a joint municipal water supply project. The viability of the project depends on how many municipalities will participate in it. Specifically addressed in this paper are methodological problems involved in allocating costs of the joint project to provide incentives for the participants.

ACKNOWLEDGEMENTS

We particularly wish to thank Lennart de Maré, on leave at IIASA from the University of Lund, who provided us with much useful data and background information on the situation in Sweden and commented extensively on the manuscript. We are also especially grateful to Janusz Kindler, who as leader of the Regional Water Management Task at IIASA provided the inspiration for undertaking this study, and gave us much encouragement and advice along the way.

ABSTRACT

Methods for allocating the joint costs of a water supply facility among the different users are systematically compared using basic principles from game theory and fair division. It is shown that some of the more widely used methods, including the separable cost remaining benefit method, fail to satisfy some of these basic principles and that other lesser known methods appear to be more satisfactory. Application is made to a cost sharing problem among a group of Swedish municipalities developing a joint municipal water supply.

key words:

cost allocation/game theory/water resources management

1. INTRODUCTION

The water resources field has an extensive literature on the problem of how the costs of constructing and operating a joint facility, such as a multipurpose reservoir or common municipal water supply, should be shared among the different users.* There is also a significant literature in game theory that deals implicitly with this kind of problem, but only recently have these ideas been applied to the evaluation of water resource projects (Loehman and Whinston 1974, Suzuki and Nakayama 1976, Bogardi and Szidarovsky 1976, Okada 1977).

This paper examines some of the available methods from both sources in the context of a concrete example: a cost sharing problem among a group of municipalities in Sweden developing a joint municipal water supply. This example not only illustrates differences in the behavior of methods in practice, but suggests certain basic principles that methods should satisfy - based on considerations of equity, common sense, and the need to provide sufficient incentives for the participants to cooperate. This

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* See, for example, (Loughlin 1977) for a discussion of the literature.

comparative analysis reveals that some of the more widely used cost allocation methods are suspect, while at least one lesser known method from the game theory literature appears to be considerably more satisfactory.

This analysis of cost allocation procedures came about as a part of a broader study of regional resource management problems focusing on southern Sweden. The root of the problem there appears to be a lack of established institutional procedures to resolve conflicts over a resource which is in increasingly short supply. A particular but significant aspect of the problem is the lack of a really acceptable procedure for sharing joint costs. This problem is becoming of even greater importance now due to the formation of consortia of municipalities (e.g. the Sydvatten Company) which are engaged in large scale, long-term water supply projects having sizable fixed investment costs.

We begin with a general discussion of cost allocation methods and the formulation of some basic properties that these methods should satisfy. The methods are then applied to the Swedish cost data (beginning in Section 9) and the results used to compare the relative merits of the different approaches. From this comparison the conclusion emerges that one method seems particularly well suited to cost allocation problems of the type encountered in Sweden.

2. JOINT COST FUNCTIONS

The nature of the cost-sharing problem can be conveniently illustrated by a small hypothetical example.

Consider three neighboring municipalities A,B, and C, who can either supply themselves with municipal water by building separate facilities or by building a joint facility. We suppose that the total cost of constructing a joint facility is cheaper due, in part, to economies of scale. We also assume that the water use is inelastic.

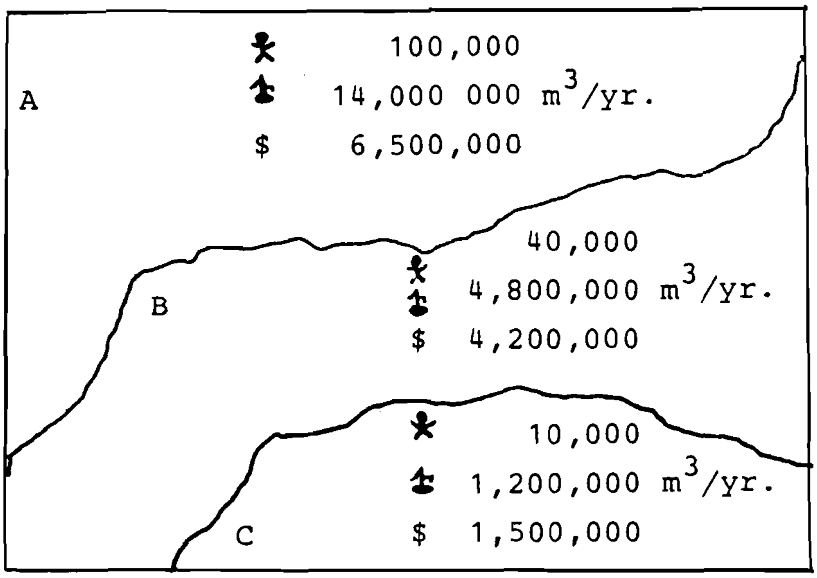

Figure 1

Region	Use per capita (m^3/yr.)	Unit cost($/$m^3$yr.)
A	140	.46
B	120	.88
C	120	1.25

Table 1. Use Rates and Unit Costs for 3 Hypothetical Municipalities

Figure 1 shows the populations of the municipalities, the targeted amounts of water to be supplied to each, and the cost of building separate facilities. Note that water use per capita varies between the municipalities, as does the unit cost of supplying water (Table 1). For present purposes it is assumed that water use is independent of price (a "requirement"); more realistic treatment would have to include demand as a function of price.

Table 2 shows the costs of supplying the same amount by joint facilities. All possibilities are considered: e.g. A,B build a joint facility but C goes alone; B and C cooperate but A goes alone; A and C coperate but B goes alone.

A	6.5	{A,B}	10.3		
B	4.2	{A,C}	8.0 (= 6.5 + 1.5)	{A,B,C}	10.6
C	1.5	{B,C}	5.3		

Table 2. Annual Costs of Supply (Millions of dollars).

The cost figures say that building a facility to serve all three communities will save a total of 1.6 million dollars as compared with building separate facilities. However, A and B can also realize savings of $400,000 without including C, likewise B and C can save $400,000 without including A. A and C, being geographically separated by B, cannot do any better by building a joint facility than they can by building separate facilities. Overall, the most cost-effective way of supplying water would be to build a joint facility serving all three communities.

More generally, let $\{1,2,\ldots,n\} = N$ represent a group of prospective participants in a cooperative venture to provide a service that is insensitive to price. The cost of serving a subgroup S, denoted by c(S), is found by considering the least-cost alternative of providing the same service, either jointly or singly, to the members of S alone. The *joint cost function* c(S) so defined has the property that $c(S) + c(T) \geq c(S \cup T)$ for any two nonoverlapping groups S and T, because the ways of serving S together with T include the possibiliy of serving S alone and T alone. Thus, in the above example, municipalities A and C might find it quite costly to build a single joint facility because of their geographical separation; hence their least-cost alternative is to have two facilities: one for A and one for C.

If the cost of serving any group of users is simply the sum of the costs of serving them singly, then the cost allocation problem is trivial. The more interesting (and typical) case is that the cost of serving several users by some joint facility is *less* than the sum of serving them singly, that is $c(N) < \sum_N c(i)$. To illustrate the different types of situations that can arise, suppose that the cost of serving a group S depends only on the number of members of S, and that there are cost savings from larger groups due to economies of scale. One possibility is to have constantly declining marginal costs as shown in Figure 2. Another and more typical situation is to have first declining and then rising marginal costs as in Figure 3.

It will turn out that justifying a "fair" allocation of costs for the latter case is considerably more difficult than for the former (see Section 5).

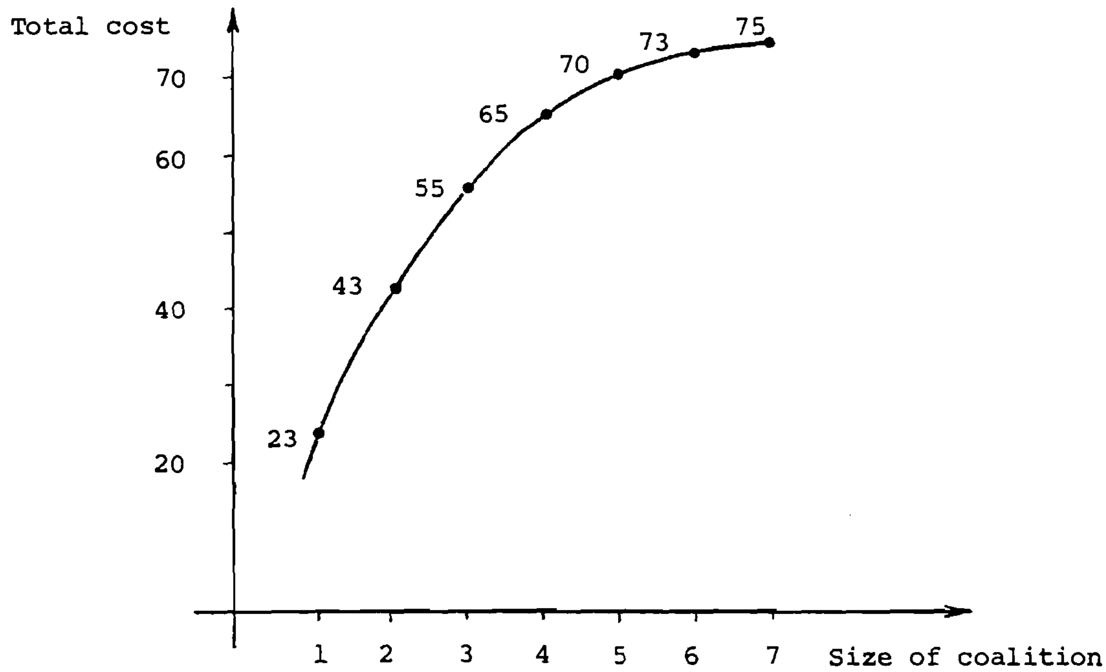

Figure 2

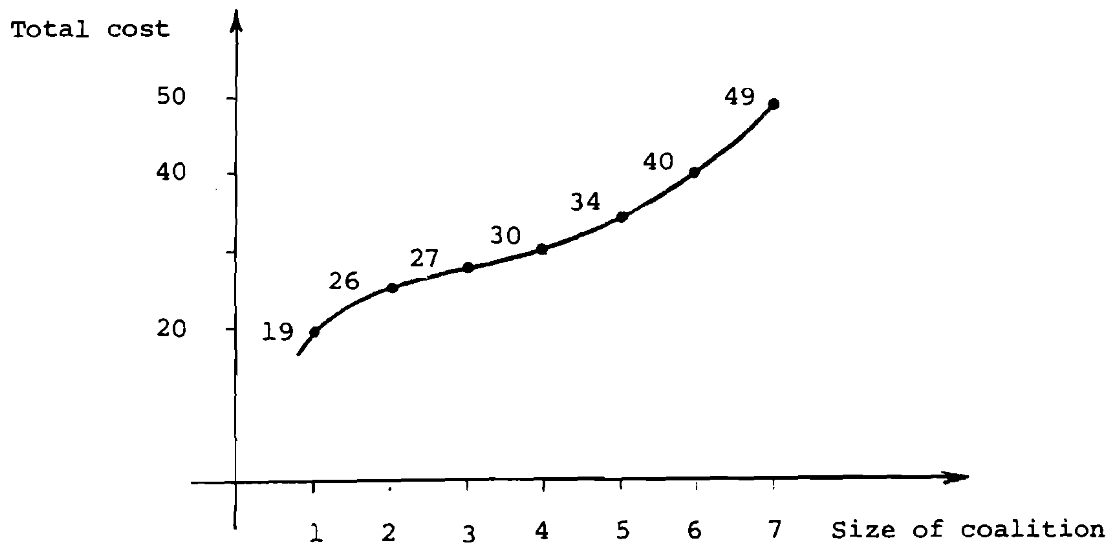

Figure 3

3. PROPORTIONAL ALLOCATION METHODS AND THE "RATIONALITY" PRINCIPLE

One of the commonest cost allocation approaches is to simply dispense with the evaluation of alternative costs for different groups of users and allocate the cost of the whole project in proportion to some given criteria, such as use of facilities or populations. This approach is better suited to single purpose than to multipurpose projects since, in the latter case, the natural units in which the different uses or purposes are expressed may not be comparable.

The greatest advantage of the proportionality approach is realized when the proportionality criterion selected seems "fair", is easy to compute, and the accuracy of the numbers is not open to serious dispute. Compared with the prospect of evaluating alternative costs of all possible combinations of users - numbers whose accuracy may indeed be disputed - this approach seems quite practical.

Set against this is the serious difficulty that costs allocated according to such a criterion may not provide adequate incentives for some potential users to participate. Nor should it be assumed -because the project analyst finds calculating alternative costs cumbersome - that this is beyond the capacity (or self-interest) of the users involved.

In the municipality example above, an allocation of costs according to *populations* would result in the shares:

A	B	C
$\frac{100}{150}(10.6) = 7.067$	$\frac{40}{150}(10.6) = 2{,}827$	$\frac{10}{150}(10.6) = .707$

But municipality A would find it hard to accept such an allocation, since it could provide the same amount of water on its own for only 6.5 million dollars.

An allocation according to use gives even worse results for A:

A	B	C
$\frac{14}{20}(10.6) = 7.420$	$\frac{4.8}{20}(10.6) = 2.540$	$\frac{1.2}{20}(10.6) = .636$

The difficulty with "proportional" allocation methods is that they ignore a fundamental datum of the problem - the *alternative costs* embodied in the joint cost function c(S). In particular, a minimum requirement of a "fair" allocation is that no user should pay more in the joint venture than he would have to pay on his own. This principle, known as *individual rationality* in the game theory literature (von Neumann and Morgenstern 1943, Nash 1950) is also well established in the project evaluation literature (U.S.Dept. of Agriculture 1964, U.S. Dept.of the Interior 1959). This is a "fairness" idea since it means that no participant is penalized for his participation. But if cooperation is voluntary, then as a practical matter it is also *necessary* since it constitutes a minimum inducement to join. To be individually rational the costs allocated to A,B, and C in the above example must satisfy $x_A \leqq 6.5$, $x_B \leqq 4.2$, $x_C \leqq 1.5$, where $x_A + x_B + x_C = 10.6$ millions of dollars.

Similar arguments can be applied to *groups* of participants as well as to individuals. Consider, for example, neighboring municipalities A and B: they can build a joint facility to service just the two of them for \$10.3 million, hence it would certainly be unfair to allocate them more than \$10.3 million in total costs. Not only would it be unfair, if cooperation is voluntary then with an allocation of $x_A + x_B > 10.3$, there is a risk of A and B backing out, since they can do better on their own. The condition that no group pay more than its alternative cost is known as the *group rationality* principle. Since a group may consist of a single participant, group rationality implies individual rationality. For the example above the cost allocations obeying group rationality form a subset of the set of all cost allocations x_A, x_B, x_C, where $x_A, x_B, x_C \geqq 0$ and $x_A + x_B + x_C = 10.6$ (see Figure 4).

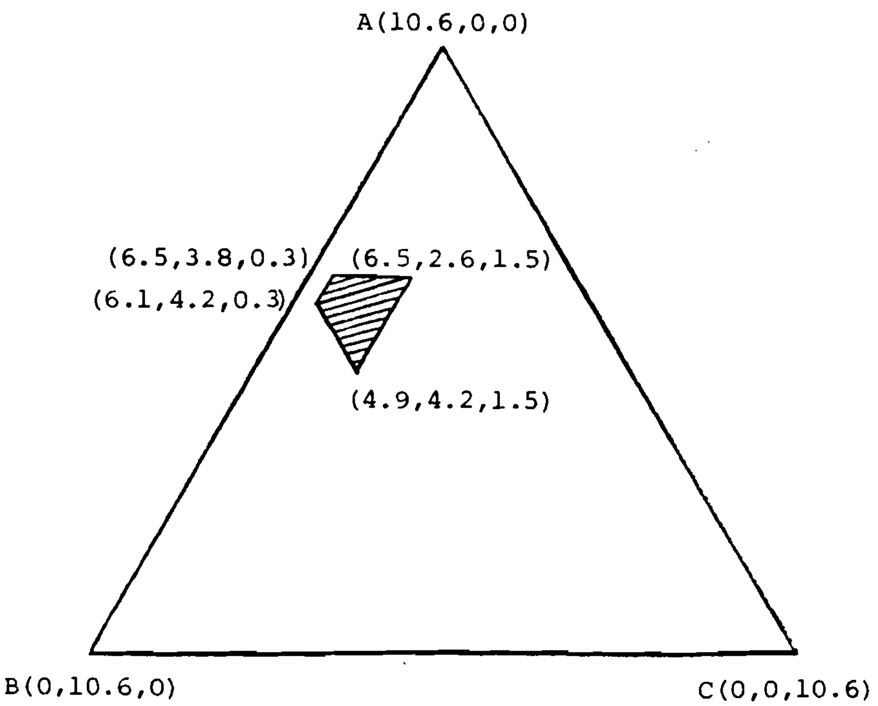

Figure 4. The Core

In the general case, where there are n independent users $\{1,2,\ldots,n\} = N$ and alternative costs are given by the function $c(S)$, the group rationality condition for a cost allocation $\underset{\sim}{x} = (x_1, x_2, \ldots, x_n)$ is that

$$(1) \qquad \sum_S x_i \leqq c(S) \qquad \text{for every subset } S \text{ of } N,$$

and

$$\sum_N x_i = c(N) \quad .$$

4. MARGINAL COSTS

A second way of approaching the cost allocation problem is to ask: what does an individual user contribute *at the margin?* Specifically, if A,B, C are to share a joint facility, one may legitimately ask how much it costs to serve A at the margin, that is, how much could be saved by leaving out A. In the example \$10.6 - \$5.3 = \$5.3 million could be saved by building a joint facility that serves all but A. Therefore, if A is included it seems only fair that A should be assessed at *least* \$5.3 million. This idea is well grounded in the project evaluation literature as a basic test of reasonableness (U.S. Dept. of Agriculture, 1964, and U.S. Dept. of Interior, 1959) and will be called here the *individual marginal cost principle.* The marginal costs for A,B,C are shown in Table 3.

A	B	C
5.3	2.6	.3

Table 3: Marginal Costs of 3 Municipalities (\$10.6)

There is an obvious extension of this idea to groups. For example, the marginal cost of including the group A,B is 10.6 - 1.5 = 9.1. The *marginal cost principle* says that *every collection of users should be charged at least as much as the additional cost of serving them.* This is only fair, since otherwise if the allocation were such that $x_A + x_B < 9.1$, then C would be *subsidizing* A and B's participation.

The idea of finding prices in which no users subsidize others in the use of a given public service or facility is well-known in the theory of public relation (Faulhaber 1975).

Stated in general terms the *marginality principle* is that a cost allocation $\tilde{x}$ should satisfy

(2) $$\sum_S x_i \geq c(N) - c(N-S) \qquad \text{for all subsets } S \text{ of } N,$$

where

$$\sum_N x_i = c(N).$$

5. THE CORE

An inspection of the notions of group rationality (1) and marginality (2) reveals that in fact they are *equivalent*, given the assumption that $\Sigma x_i = c(N)$ (i.e. that all costs are allocated). The argument for group rationality rests on strategic considerations, i.e. providing sufficient incentive for users to cooperate, whereas marginality can be viewed as a general fairness concept that can be applied even if cooperation is mandated. The set of all allocations $\tilde{x}, \sum_N x_i = c(N)$, satisfying (1), (equivalently (2)) is called the core.*

The core provides a *guideline* for cost allocation, but no more than that, since it narrows consideration to a class of allocations (as in Figure 4) without, typically, identifying a "unique" answer. Worse, however, is the possibility that there may be *no* core allocations; that is, no allocations that are *either* group rational *or* satisfy marginality. That this can happen in perfectly reasonable situations is seen from the example of Figure 3. Here there are increasing returns to scale but marginal costs are first declining, then rising. The minimum cost of serving all users is 49, but no matter how this is allocated

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* The cost function c(S), or more precisely - c(S), can be interpreted as the *characteristic function* of a cooperative n-person game played by the prospective users.

among the seven, *some* group of five must be assessed at least 35 even though their alternative cost is lower (34).

In such an example a quick test for the core is to draw a line from the origin to the point corresponding to the total number of users; this line segment must lie below the cost curve for the core to exist (Shapley and Shubik 1973). In more complicated examples the conditions (1) or (2) can be checked for feasibility using linear programming. In the three-municipality example, the core exists but is not large and if total costs had happened to be 12.0 instead of 10.6 million it would be empty.

No guarantee can be given that core allocations need exist for a particular problem; however, in general the greater the economies of scale, the more likely it is to exist. Even if the core does exist, it does not typically provide a unique answer.

6. ALLOCATION METHODS EXTENDING THE CORE

One approach to resolving these difficulties has been to look for some natural way of strengthening (or, as the case may be, relaxing) the inequalities (1) defining the core. This is the most common approach in the game theory literature. Three such approaches will be discussed; the least core(and nucleolus), the "proportional" least core, and the"weak"least core.

The Least Core and Nucleolus

If the core of the cost function c is empty this means that the best alternatives of some subgroups are very good - in a certain sense "too" good - relative to the best alternative of the whole group. Hence one could imagine imposing a tax on all proper subgroups as a way of encouraging the whole group to stick together. The "least core" is found by asking for the smallest uniform tax ε such that if all coalitions other than the whole coalition are taxed by an amount ε, then a "core" allocation exists. Thus one finds the least ε for which there exists an allocation $\underset{\sim}{x}$ satisfying:

$$(3) \qquad \sum_S x_i \leqq c(S) + \varepsilon \qquad \text{all } S \subset N,$$

$$\sum_N x_i = c(N) \ ,$$

and the *least core* is the set of all allocations $\underset{\sim}{x}$ satisfying (3) for this least ε (Shapley and Shubik 1973).

Suppose on the other hand that the function c already has a core; then one way of narrowing down the choice of an allocation is to imagine *subsidizing* all coalitions other than the whole coalition by a uniform amount ε. This amounts to solving (3) for smallest ε and allowing ε to go negative.

Computing the least core involves solving a linear program. For the three-municipality example, this program (in millions of dollars) is:

$$\begin{aligned} &\min \varepsilon \\ &\text{subject to} \quad x_A \leqq 6.5 + \varepsilon, \; x_B \leqq 4.2 + \varepsilon, \; x_C \leqq 1.5 + \varepsilon \\ &\qquad x_A + x_B \leqq 10.3 + \varepsilon \\ &\qquad x_A + x_C \leqq 8.0 + \varepsilon \\ &\qquad x_B + x_C \leqq 5.3 + \varepsilon \\ &\qquad x_A + x_B + x_C = 10.6 \end{aligned}$$

The answer is $\varepsilon = -.533$, and the unique corresponding cost allocation is

$$x_A = 5.967 \ , \ x_B = 3.667 \ , \ x_C = .967$$

Normally (that is, except for "degenerate" cases) the linear program (3) produces a *unique* cost allocation. If it has multiple solutions the following tie-breaking device may be used: for any allocation $\underset{\sim}{x} = (x_1,\ldots,x_n)$ and coalition S define the *excess* of $\underset{\sim}{x}$ on S to be $\varepsilon(S,\underset{\sim}{x}) = \sum_S x_i - c(S)$. Let $\varepsilon_1(\underset{\sim}{x})$ be the largest excess of any coalition relative to $\underset{\sim}{x}$, $\varepsilon_2(\underset{\sim}{x})$ the next largest excess and so forth. The *nucleolus* is a cost allocation $\bar{\underset{\sim}{x}}$ for which

(4) $\varepsilon_1(\bar{x}) \leqq \varepsilon_1(x)$ for all x (i.e. $\bar{x}$ is a least core allocation)

(5) $\varepsilon_2(\bar{x}) \leqq \varepsilon_2(x)$ for all x satisfying (4),

(6) $\varepsilon_3(\bar{x}) \leqq \varepsilon_3(x)$ for all x satisfying (4) and (5), etc.

It may be proved that there is only one such allocation $\bar{x}$ for a given cost function c (Schmeidler, 1969).

The Proportional Least Core

This method is a variation of the least core that seems equally plausible, but has not to our knowledge been discussed in the literature on cost allocation. Instead of imposing a *uniform* tax on project alternatives (or giving a uniform subsidy) make the tax or subsidy *proportional* to the cost of the alternative. One finds the least tax rate r (per unit of costs) such that

(7) $$\sum_S x_i \leqq (1+r)c(S) \quad \text{for all } S \subset N$$

$$\sum_N x_i = c(N) ,$$

has a solution x (r negative corresponds to a subsidy). To assure the existence of a minimum r assume that $c(S) > 0$ for all S. The set of all allocations x optimizing (7) is the *proportional least core.*

For the three-municipality example the linear program is

min r

subject to $x_A \leqq 6.5(1+r)$, $x_B \leqq 4.2(1+r)$, $x_C \leqq 1.5(1+r)$,

$$x_A + x_B \leqq 10.3\ (1+r)$$

$$x_A + x_C \leqq 8.0\ (1+r)$$

$$x_B + x_C \leqq 5.3\ (1+r)$$

$$x_A + x_B + x_C = 10.6$$

The solution is $r = -.102$ (a subsidy) and the unique cost allocation is

$$x_A = 5.839 \quad , \quad x_B = 3.414 \quad , \quad x_C = 1.347 \; .$$

The Weak Least Core

In this approach one imagines imposing a uniform tax on any *individual user* who undertakes an alternative other than the whole group. Thus one finds the least ε for which a solution $\underset{\sim}{x}$ exists to the system:

(8) $$\sum_S x_i \leqq c(S) + \varepsilon|S| \qquad \text{for all } S \subset N \; ,$$

$$\sum_N x_i = c(N)$$

The set of all corresponding allocations $\underset{\sim}{x}$ is the *weak least core*. (For arbitrary ε the solutions to (8) constitute the *weak* ε *core*; see (Shapley and Shubik 1973)). This method, while superficially similar to the least core, turns out to have a number of desirable properties not shared by the least core or the proportional least core (see Section 10).

For the above example the weak least core is calculated from the solution to the linear program

$$\min \varepsilon$$

$$\text{subject to} \quad x_A \leqq 6.5 + \varepsilon, \; x_B \leqq 4.2 + \varepsilon \; , \; x_C \leqq 1.5 + \varepsilon \; ,$$

$$\begin{aligned} x_A + x_B \quad\quad\; &\leqq 10.3 + 2\,\varepsilon \\ x_A \quad\quad + x_C &\leqq 8.0 + 2\,\varepsilon \\ x_B + x_C &\leqq 5.3 + 2\,\varepsilon \\ x_A + x_B + x_C &= 10.6 \end{aligned}$$

The solution is $\varepsilon = -.4$, and the unique cost allocation in the weak least core is

$$x_A = 6.1 \quad , \quad x_B = 3.4 \quad , \quad x_C = 1.1$$

If desired, ties in either this method or the preceding could be resolved by a device analogous to that used for finding the nucleolus.

Next we mention two other well-known cost allocation procedures.

7. THE SHAPLEY VALUE

The Shapley value for n players is given by the formula

$$(9) \qquad x_i = \sum_{s=1}^{n} \frac{(s-1)!(n-s)!}{n!} \sum_{\substack{S:i \in S \\ |S|=s}} [c(S) - c(S-i)]$$

This is one of the earliest allocation methods to be based on a consistent set of postulates about how an allocation should behave (Shapley 1953). The idea is to think of all players as "signing up" in some order. If a group S has already signed up and i is the last to arrive, then his *marginal cost contribution to S* is c(S) - c(S-i). The Shapley value is i's average marginal contribution if all orders for "signing up" are assumed to be equally likely.

The Shapley value for municipalities A,B,C is calculated as follows. There are six possible orders for signing up, opposite each is the marginal contribution of each municipality for that order:

	A	B	C
ABC	6.5	3.8	0.3
ACB	6.5	2.6	1.5
BAC	6.1	4.2	0.3
BCA	5.3	4.2	1.1.
CAB	6.5	2.6	1.5
CBA	5.3	3.8	1.5
Total	36.2	21.2	6.2
Shapley Value =(1/6)×Totals	6.033	3.533	1.033

8. THE SEPARABLE COST REMAINING BENEFIT METHOD

The last method to be considered here is one that is commonly used in practice for evaluating multi-purpose water development projects. This method differs somewhat from the others, however, in that it requires a specific knowledge of benefits as well as costs.

The SCRB method (Eckstein 1958, James and Lee 1971) begins with the assumption that the potential benefit of the contemplated service is known for each user, and that these benefits b(1), b(2),...,b(n) can be estimated *independently* for each of the users. The so-called "justifiable cost" of the service to user i is the minimum of b(i) and i's alternative cost, c(i).

What is known in this method as the "separable cost" is actually just the marginal cost, $c'(i) = c(N) - c(N-i)$, and the "nonseparable cost" is what remains from c(N) after separable costs are subtracted.* The "remaining benefit" is the justifiable cost minus the marginal cost: $r(i) = \min\{b(i),c(i)\} - c'(i)$. (If $r(i)<0$, the marginal cost of including i exceeds the justifiable cost and user i should not be included in the project). Assuming that $r(i) \geq 0$ for all i the SCRB allocates costs according to the formula:

$$x_i = c'(i) + [r(i)/\sum_N r(i)][c(N) - \sum_N c'(j)] \quad . \tag{10}$$

In order to apply this method to our example let us assume that each municipality's benefit from water supplied exceeds its "stand-alone" cost. From the marginal costs in Table 2 (Section 4) the remaining benefits for A,B,C are calculated to be:

* The term "non-separable costs" could be misleading since the sum of "seperable costs" may actually exceed the total project costs c(N), in which case "nonseparable costs" are negative and x_i is *less than* its marginal cost, c'(i). The possibility is shown by the example in Figure 3 (Section 2).

$$r(A) = 6.5 - 5.3 = 1.2$$
$$r(B) = 4.2 - 2.6 = 1.6$$
$$r(C) = 1.5 - .3 = 1.2$$
$$\text{Total} = 4.0$$

"Common costs" are 10.6 -(5.3 + 2.6 + .3) = 2.4, so the SCRB allocation is

$$x_A = 5.3 + \frac{12}{40}(2.4) = 6.020$$

$$x_B = 2.6 + \frac{16}{40}(2.4) = 3.560$$

$$x_C = 3.0 + \frac{12}{40}(2.4) = 1.020$$

Variations of the SCRB method include proposals for allocating the "nonseparable costs" in proportion to some criterion such as use, priority of use, population, etc. (James and Lee 1971). However, these variations have essentially the same advantages and disadvantages as this version.

Benefits constitute a feature of the SCRB method that does not occur in the description of the other methods. From the standpoint of project justification, one wants to know whether the benefit from including another user or purpose in a project exceeds the marginal cost of including it. However, to estimate whether benefits *exceed* costs is quite a different matter than to justify a particular *level* of benefits for inclusion in an allocation formula, since in practice, benefits are much more difficult to estimate reliably than are costs. Thus, in the application of the SCRB method to Sweden we simply assumed that benefits exceed stand-alone costs in order to avoid the conundrums involved in estimating benefit levels explicitly. This feature of the SCRB constitutes one of its drawbacks in applications. The whole question of whether, in principle, it is *benefits* or *costs* that should be allocated is an interesting one that unfortunately is beyond the scope of this study; suffice is to say that there is no *a priori* reason why the answers should be the same. The practical problem remains that benefits are harder to estimate, and in the end what gets allocated *concretely* is costs.

9. APPLICATION TO SWEDEN

The area of study consists of eighteen municipalities in the Skåne region of southern Sweden (Figure 5). At present most of the municipal water supply is drawn from three sources: local groundwater, and two separate pipeline systems which distribute water from two lakes Vombsjön and Ringsjön.

As early as the 1940's, some municipalities in the area realised the possibility of shortages in local water sources and turned their attention to off-site sources. An association called the Sydvatten company was formed by several of them to plan for long-term water supply and management of the region. In the late 1960's, this group (consisting presently of 12 of the municipalities) began to design a major project to obtain water from a lake outside the region (Lake Bolmen) via an 80 km. tunnel. Recently, this project has been undergoing a period of reconsideration and redesign as the actual increase in water demand over the past decade has turned out to be short of the original forecasts.

The viability of the project depends on how many municipalities will participate in the project, and this in turn is dependent on how much they will be obliged to pay by participating in such a development vis-à-vis the availability and costs of developing their own on-site sources. In fact, the costs of the project in Southwest Skåne has greatly escalated since the initiation of the project. This, together with more optimistic estimates of local resources and lower rates of demand growth, are some of the major factors that have led to a questioning of the project. The cost allocation problem is

Figure 5. Study Area in Skåne, Sweden

one important aspect of a deeper planning problem in the region which can be ultimately attributed to a lack of established rules, procedures, and institutions that can effectively cope with changing planning environments and conflicts that emerge during the planning process.

To illustrate how the basic concepts and methods developed in the preceding sections can be applied, the decade 1970-1980 was chosen as a recent planning period for which data and forecasts were available. The Sydvatten tunnel project mentioned earlier is not expected to have an impact until the 1980's, hence the alternatives considered for meeting incremental demands in the period are of a more conventional variety: extending the capacity of the pipeline system and increasing use of local groundwater sources where feasible. The year 1970 was taken as the "base" year and a water supply system was designed to satisfy the municipal "requirements" for 1980 as they were forecast in 1970. The different cost allocation methods described above were then applied to examine the relative positions of the different municipalities. The results help qualify how the different methods work, and why some of them may be preferable to others in practice.

Identifying Independent Actors

The first problem in defining the cost function is to identify the independent actors in the system. To try to develop the costs for each of $2^{18} - 1 = 262{,}143$ possible groupings of the 18 municipalities is impractical and unrealistic. In reality there are natural groupings of municipalities based on past associations, geographical proximity, pre-existing water transmission systems, and hydrological and geographical conditions that determine the natural routes for water transmission networks.

A careful study of these conditions led the grouping of the 18 municipalities into six independent units as shown in Figure 6. Group H for example consists of those municipalities already connected by the Ringsjön water supply system by 1970 together with the municipality of Svalöv, which would be located in the middle of the main transmission route. These groupings are treated as single units in the subsequent analysis of alternative costs, and will be referred to (somewhat loosely) as "municipalities", or sometimes "actors" or "players". The 1970 populations and forecast incremental demands for these six municipalities are shown in Table 4.

Municipality		Population (10^3)	Incremental Demands (Mm^3/yr)
A		85.0	6.72
H		176.3	8.23
K		26.8	3.75
L		69.0	3.53
M		287.3	14.64
T		59.5	5.39
	Totals	703.7	42.26

Table 4. Populations and Incremental Demands for Six Municipalities

Ambiguities in Defining the Cost Function: Direct Costs

In practice, ambiguities arise in defining the cost function due to the problem of distinguishing *direct costs*, that is, costs that would be incurred by a given municipality no matter what alternative action it pursued. For example, water distribution systems within municipalities are project components that are required whether the water is supplied jointly or separately. Therefore, the cost associated with it may be regarded as a direct cost. In principle, these costs could be excluded from the cost function on the grounds that they can be independently allocated. In practice, the borderline between direct and indirect costs is not always clear. In some municipalities, for instance, the water delivered by the regional supply network must first be pumped up to a reservoir before it is distributed further within the municipality, and facilities required for pumping depend on the pressure at the end of the transmission network. So in fact, the costs of these distribution facilities are *not* independent of how the water is supplied. The definition of the cost function naturally depends on what part of these costs are treated as direct costs.

Since some arbitrariness in defining the cost function always exists in practice, it is desirable that the cost allocation method not be sensitive to the inclusion or exclusion of "direct costs". One of the difficulties with the proportional least core and SCRB as defined above is that they are sensitive in some cases to the inclusion of direct costs (see Note 2 in the Appendix).

Calculating the Cost Function

To avoid inconsistencies in defining the cost function, it will be assumed in this study that the pressure condition at each demand point is given irrespective of how the water is transmitted to that point. Then the cost of distributing the water within each municipality does not depend on the arrangement by which the water is supplied, and this cost element can be eliminated from the cost function as a "direct cost".

The water delivered to municipalities is assumed to have the same quality level. The water taken from different sources is treated accordingly at the source and the costs of treatment are included in the cost function.

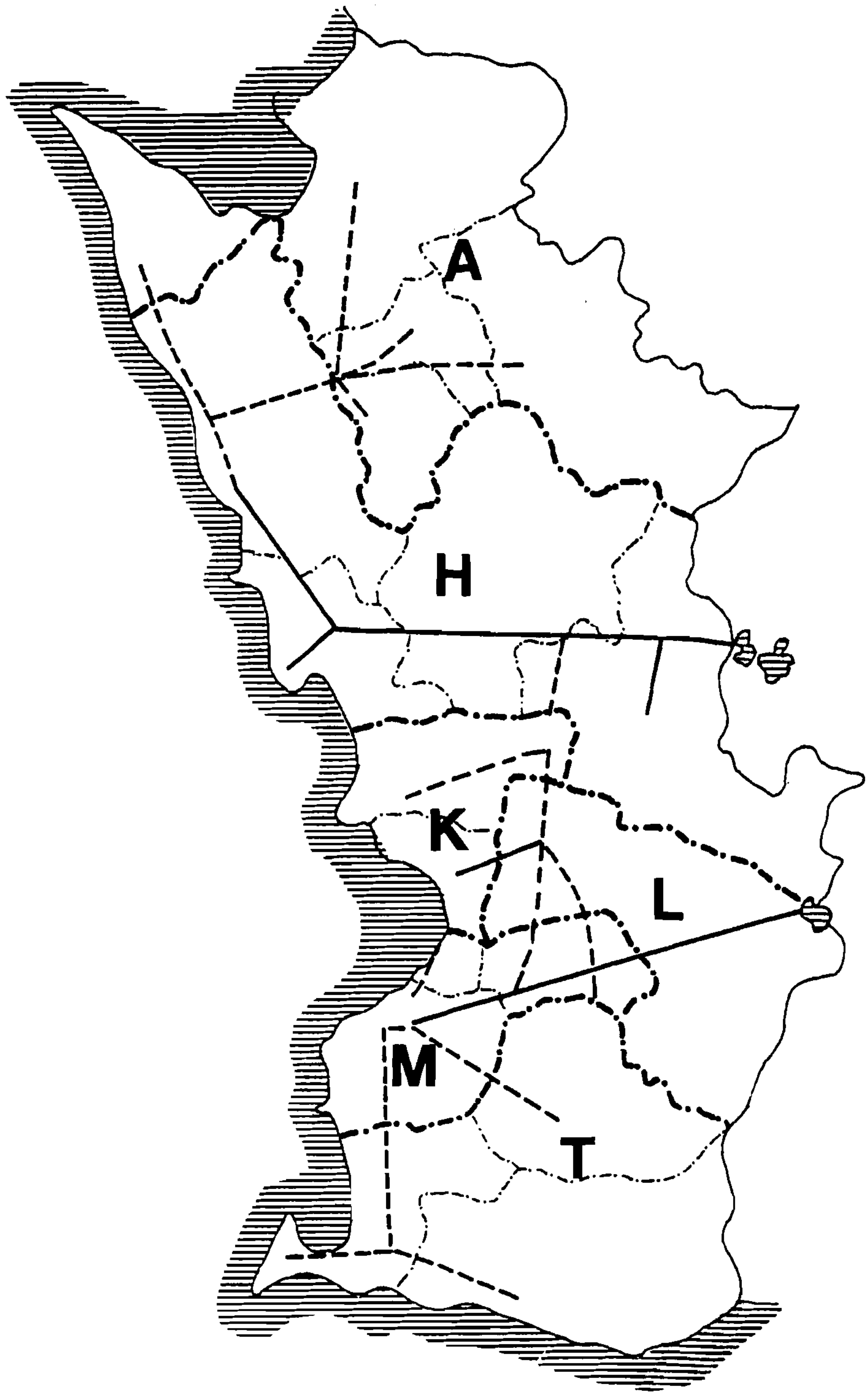
Included in the water supply system are Vombsjön and Ringsjön, one major groundwater aquifer (Alnarp) and other minor on-site sources (see Figure 6). The possible routes of the water transmission network (based on a preliminary analysis) are shown in Figure 7. Also shown are the distances between each pair of points, the elevation of each junction or demand point (in parenthesis) and the incremental demand for each municipality (whose name is circled) up to 1980.

A mathematical programming model may be used to determine the least-cost combination of the alternative supply sources for each coalition S. In the present case we assume that inherent economies of scale are such that all the incremental demands of a party will be met by the regional water supply system once they are connected to it.* Moreover, the assumption of the fixed

* This, of course, does not mean all the parties should be connected to the regional system. In general, there exists an optimal "degree of aggregation" which depends on types of facilities involved in a project. In this connection, it is useful to distinguish between what may be called point facilities (eg. a reservoir) and distributed facilities (eg. water distribution network). The former usually exhibit more significant economies of scale, while for the latter type of facilities even scale diseconomies may be observed. Our water supply system consists of both types of facilities.

Figure 6. Grouping of 18 municipalities

Group	Municipalities in the Group
A	Ängelholm, Höganäs, Klippan, Åstorp, Bjuv
H	Helsingborg, Landskrona, Svalöv, Eslöv
K	Kävlinge, Lomma
L	Lund
M	Malmö, Burlöv, Staffanstorp
T	Trelleborg, Vellinge, Svedala



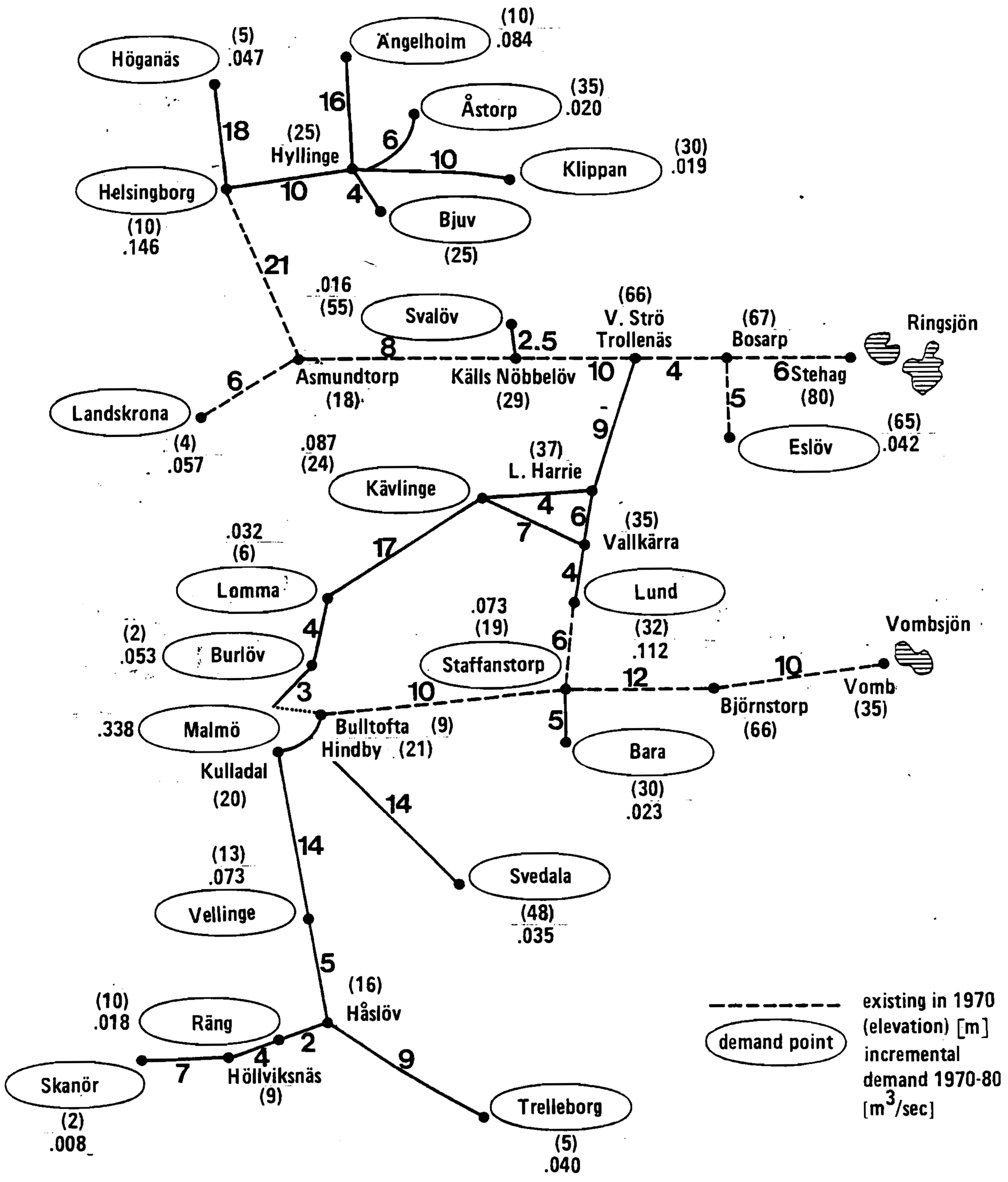


Figure 7. Water transmission network

pressure condition at each demand point allows us to treat each arc of the transmission network independently of the other parts. Thus, the cost-effective design of the network is carried out arc by arc. The water transmission cost for each arc of the least-cost design is derived in the Appendix (Note 1) and the results are given in Table 5.

Transmission Cost			
Pipelines		$C_1 = (150+477\ D^{1.60})L$	Skr.
Pumps		$C_2 = 39.2\ QH \times 10^3$	Skr.
Electricity		$C_3 = 9.54\ QH \times 10^3$	Skr/yr.
Treatment Cost			
Vombsjön	Capital Cost	$C_K^V = 2.31\ \tilde{Q}^{.74} \times 10^6$	Skr.
	O /M cost	$C_M^V = .162\ \tilde{Q}^{.91} \times 10^6$	Skr/yr.
Ringsjön	Capital Cost	$C_K^R = 3.68\ \tilde{Q}^{.64} \times 10^6$	Skr.
	O /M cost	$C_M^R = .410\ \tilde{Q}^{.64} \times 10^6$	Skr/yr.
Groundwater	Capital cost	$C_K^G = 2.38\ \tilde{Q}^{.58} \times 10^6$	Skr.
	O /M cost	$C_M^G = .263\ \tilde{Q}^{.82} \times 10^6$	Skr/yr.

$[D] = m$, $[Q] = m^3/sec.$, $[\tilde{Q}] = Mm^3/yr.$,

$[H] = m$.

Table 5. Cost functions for the water supply systems.

The cost of treating water at Vombsjön includes the cost of infiltration, pumping and chlorination. The treatment at Ringsjön consists of screening, sedimentation, coagulation and filtration. Unfortunately, the particular cost data on these unit processes were not available for this study, so capital costs and total operation/maintenance costs were estimated based on available data. The costs of treating on-site groundwater including the costs of pumping, filtration and chlorination were estimated similarly.

With these assumptions the alternative costs of each coalition can be evaluated. The results are summarized in Table 6 below. (Commas signify that the least cost option of that coalition is to break up into the subcoalitions indicated.)

A	21.95	AHK	40.74	AHKL	48.95
H	17.08	AHL	43.22	AHKM	60.25
K	10.91	AH,M	55.50	AHK,T	62.72
L	15.88	AH,T	56.67	AHL,M	64.03
M	20.81	A,KL	48.74	AHL,T	65.20
T	21.98	A,KM	53.40	AH,MT	74.10
		A,KT	54.85	A,K,LM	63.96
AH	34.69	A,LM	53.05	A,K,L,T	70.72
A,K	32.86	A,L,T	59.81	A,LMT	73.41
A,L	37.83	A,MT	61.36	HKL,M	48.07
A,M	42.76	HKL	27.26	HKL,T	49.24
A,T	43.93	HKM	42.55	HKMT	59.35
HK	22.96	HK,T	44.94	HLMT	64.41
HL	25.00	HL,M	45.81	KLMT	56.61
H,M	37.89	HL,T	46.98	A,K,MT	72.27
H,T	39.06	H,MT	56.49	AHKLM	69.76
K,L	26.79	K,LM	42.01	AHKMT	77.42
KM	31.45	K,L,T	48.77	AHLMT	83.00
K,T	32.89	K,MT	50.32	AHKL,T	70.93
LM	31.10	LMT	51.46	AKLMT	73.97
L,T	37.86			HKLMT	66.46
MT	39.41				
				AHKLMT	83.82

Table 6. Joint Cost Characteristic function c(S) (Millions of Swedish Crowns)

Discussion

The cost function reveals the relative strength of the different actors, which depends on such factors as the cost and availability of local resources and access to others' resources. For example, L has a high unit cost of going alone but at the same time its location is advantageous, close to both regional sources. Municipality K is also favourably located. L and K have a strong incentive to participate in developing a regional water supply system, although L seems to be in a weaker position since it cannot do as well as K can by going alone. Municipalities A and T also have rather high unit costs, but their locations are unfavourable. As they have higher requirements for water than K or L, however, they may expect to take advantage of economies of scale by being connected to a regional supply system. The effect is bilateral, of course; other parties can also benefit from the parties A and T joining the system.

Municipalities H and M have low unit costs, as they own the Ringsjön and Vombsjön supply systems respectively. By including other municipalities significant economies of scale can be realized and their own costs will be reduced. Between the two of them, however, exist differences in availablity of water and excess capacity of existing treatment facilities at Ringsjön and Vombsjön. This aspect is also reflected in the values of coalitions that include H or M. Compare, for instance, the coalitions HKL and KLM. The incremental cost of including municipalities K and L is much higher for the coalition KLM than for the coalition HKL since the treatment plant at Vombsjön has to be expanded if K and L are connected to this system:

$$C(HKL) - C(H) = 10.18$$
$$C(KLM) - C(M) = 21.20$$

This implies that, in terms of savings, municipality H has more to offer K and L than M does. This will ultimately affect the costs allocated to each party by a strategic allocation method.

10. COMPARISON OF METHODS

Using the cost function developed above we now examine how the different methods described in the preceding sections compare in their cost allocations. The results are shown in Table 7 for two proportional schemes (by use and by population) and for the SCRB method, Shapley value, Nucleolus, Proportional Least Core,

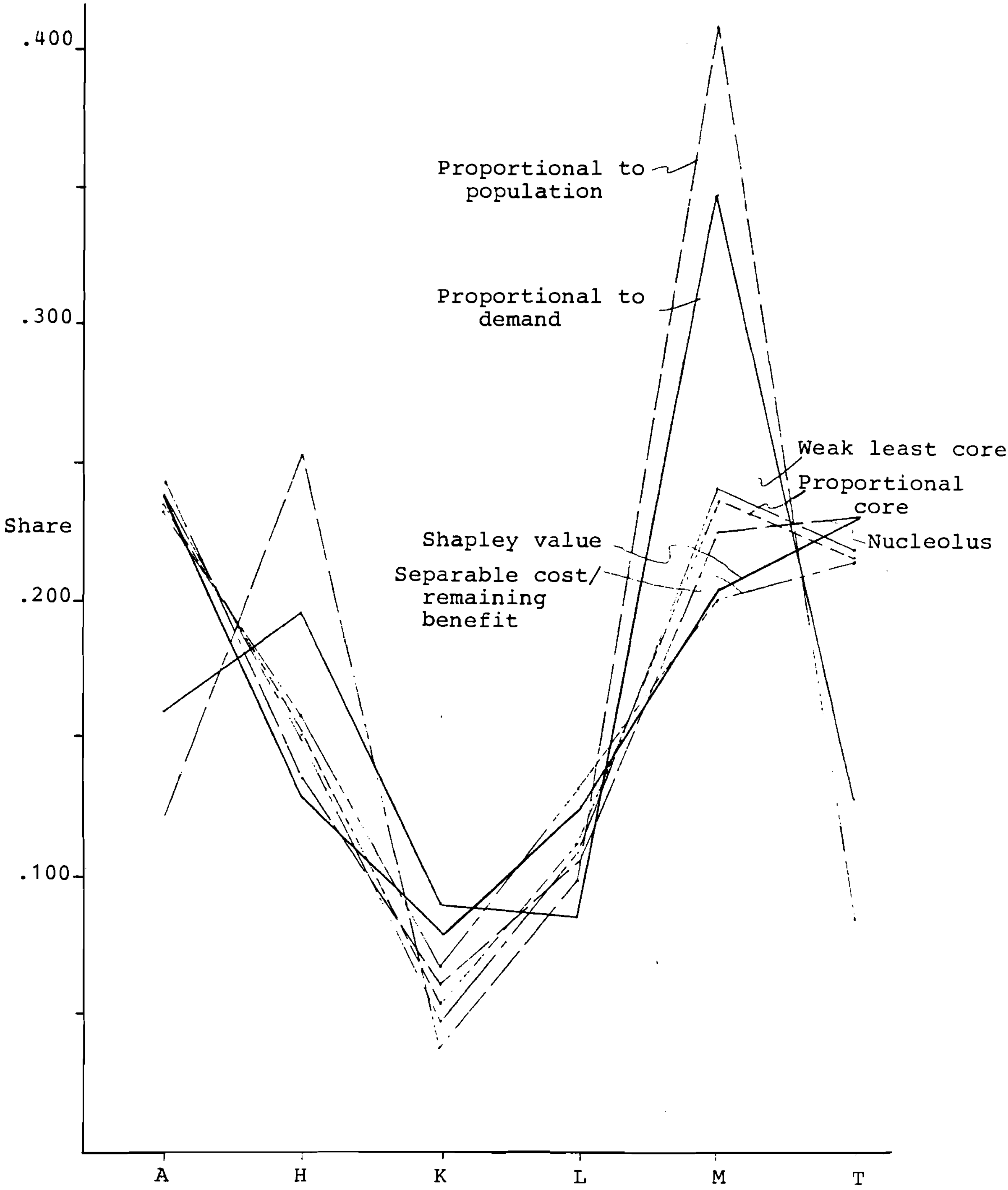


Figure 8. Municipalities' share of total costs

and Weak Least Core. The cost *shares* allocated by the seven methods are graphed in Figure 8 to facilitate comparisons.

Method	A	H	K	L	M	T
Proportional to Population	10.13	21.00	3.19	8.22	34.22	7.07
Proportional to Demand	13.33	16.32	7.43	7.00	29.04	10.69
S.C.R.B.	19.54	13.28	5.62	10.90	16.66	17.82
Shapley Value	20.01	10.71	6.61	10.37	16.94	19.18
Nucleolus	20.35	12.06	5.00	8.61	18.60	19.21
Proportional Least Core	19.81	12.57	4.35	9.25	19.85	17.99
Weak Least Core	20.03	12.52	3.94	9.07	20.11	18.15

Table 7. Cost Allocations of 83.82 Million Crowns by Seven Methods.

Notice that the proportional allocations differ markedly from the others. In fact, a comparison of the proportional allocations with the individual actors' alternative costs (Table 8) reveals that these methods assign some participants in the joint project higher costs than they would have had to pay on their own. Allocation by population penalizes M for participating, while allocation by demand penalizes both H and M.

	A	H	K	L	M	T
Individual Alternative Costs	21.95	17.08	10.91	15.88	20.81	21.98
Marginal Costs	17.36	9.85	.82	6.40	12.90	14.06

Table 8. Individual and Marginal Costs for Swedish Example.

This failure to satisfy the requirement of individual rationality derives from the fact that proportional methods do not take into account crucial differences among the participants in their access to sources of supply. The proportional procedures work against H and M, which are populous and in favour for example of the outlying regions A and T. However, in spite of their smaller populations, the inclusion of A and T is relatively costly because they are both remote from the major sources of supply (Vombsjön and Ringsjön). This fact is reflected in their high marginal costs (Table 8). Whereas they should be charged at least the marginal costs of including them, both proportional methods fail to do so. These tendencies constitute a fatal drawback of the proportional methods.

More seemingly reasonable, but almost as ill-behaved in fact, is the SCRB method. SCRB is individually rational, that is, does not assess an individual participant more than his alternative cost provided we assume that alternative costs are less than corresponding benefits (otherwise it may not even satisfy this condition). But in general the SCRB method does not satisfy group rationality. That is, the SCRB allocation may not be in the core even when core allocations exist (as they do in this case).

The three adjacent municipal groups H,K,L can provide municipal water for themselves at a cost of 27.26 million Swedish crowns, but the SCRB method would assess them 29.81 million crowns if they shared in a regional facility. With such an assessment it would not be in this group's interest to participate. Put another way, if H,K,L are assessed these amounts then they are, in effect, being forced to *subsidize* the others' water supply costs. Since there exist other assessments in which no group subsidizes another, such an allocation must be branded as inequitable.

That the SCRB suffers from this defect is not surprising, since it only considers the marginal costs of including *individual* participants, not the marginal cost of groups. In this case, the marginal cost of including both M *and* T is much higher than the sum of their individual marginal costs (since if one is served, the *additional* expense of serving the other is low) but the SCRB method fails to pick up this fact. In sum, while fewer cost elements need be estimated in order to calculate the SCRB allocation, this short-cut has a price: it may result in a less equitable allocation overall.

By contrast, the Shapley value requires knowing the alternative costs for all subsets. Unfortunately, however, it also fails the group rationality (alternatively, the marginality test) in this case on the same coalition H,K.L. The Shapley value would assess this coalition 440,000 crowns more than their alternative cost.

Since the core idea seems essential from both the standpoint of equity and of providing sufficient incentives to cooperate, the remaining three methods - the nucleolus, weak least core, and proportional least core - seem potentially to be the most desirable. They always produce a core allocation if one exists, and it may be checked that all three allocations in the Swedish case satisfy the core conditions. Moreover, there seems to be little difference between the results they give. One might be tempted to say that therefore it does not matter which is used.

But we are analyzing the behavior of *methods*, not solutions to one problem only. The question is, how do the methods compare over different problems - do they always give similar results? By considering variations in the problem data some crucial differences between these methods come to light.

Typically in a construction project costs cannot be known in advance with certainty. Rather, the project must be undertaken based on some "best estimate" of total costs and the costs of alternatives. Assuming that the project represented by the whole group is undertaken, the true level of total costs c(N) will be revealed once the project is completed, but the estimated costs of alternatives will remain as before (assuming that no better information on the alternatives has been gained).

Therefore, in practice an agreement on how to allocate costs must involve an agreement on how to allocate *different* levels of costs c(S) for all $S \subsetneq N$. It is essential that a method should behave reasonably for different levels of costs; in particular, if costs turn out to be higher than expected then no participant's allocation should go down, and *vice versa*. Such a method is said to be *monotone* (Megiddo, 1974). Monotonicity seems to be essential for any "fair" allocation procedure in which the total amount to be allocated is variable or uncertain. This property has also played a key role in discrete allocation or "apportionment" problems (Huntington 1928, Balinski and Young 1974, 1975, 1979a, 1979b).

Unfortunately, not all of the methods considered above are monotone. In the Swedish example, suppose that the costs of a regional facility are discovered to involve unforeseen additional costs of 4 million crowns so that now total costs are 87.82 instead of 83.82, all alternative costs remaining the same as before.

Comparing the new allocations with the old (Table 9), we notice that the nucleolus assesses both K and T *less* even though the total cost of the project has increased. Similarly, under SCRB, K's assessment would be 150,000 crowns less in a more costly project. This nonmonotone behavior casts grave doubt on the nucleolus and SCRB as reasonable cost allocation procedures.

	A	H	K	L	M	T	Total
Proportional to population	10.61	22.00	3.35	8.62	35.86	7.40	87.82
	10.13	21.00	3.19	8.22	34.22	7.07	83.82
Proportional to demand	13.96	17.10	7.97	7.33	30.42	11.20	87.82
	13.33	16.32	7.43	7.00	29.04	10.69	83.82
SCRB	21.42	14.19	5.46	10.97	17.31	18.47	87.82
	19.54	13.28	5.62	10.90	16.66	17.82	83.82
Shapley Value	20.67	11.38	7.29	11.03	17.60	19.84	87.82
	20.01	10.71	6.61	10.37	16.94	19.18	83.82
Nucleolus	20.76	13.25	4.51	9.80	20.52	18.97	87.82
	20.35	12.06	5.00	8.61	18.60	19.21	83.82
Proportional Least Core	20.75	13.17	4.56	9.69	20.80	18.85	87.82
	19.81	12.57	4.35	9.25	19.85	17.99	83.82
Weak Least Core	20.70	13.19	4.61	9.74	20.77	18.82	87.82
	20.03	12.52	3.94	9.07	20.11	18.15	83.82

Table 9. Allocations Under Increased Costs (Millions of Crowns).

On the other hand, it is easy to verify that the proportional allocation methods and the Shapley value are always monotone. So also is the weak least core, since the way the weak least core operates under a change in total costs is to allocate the change equally among the players (Note 3). The proportional least core, on the other hand, operates by allocating the *change* in costs in proportion to the players' previous assessments (Note 4). This feature of the proportional least core seems rather reasonable - provided the cost changes are relatively small, and that every player was assessed costs before the change.

But there are also perfectly natural examples where certain players can reasonably expect to *be paid* to cooperate (rather than pay) and for these examples the proportional least core does not seem so reasonable. In fact, it is not monotone (See Note 5).

12. CONCLUSION

Given the practical need to allocate costs when there are different users of a joint water resource facility, the problem is how to choose rationally among the many different available methods.

In itself, the use of a definite computational procedure gives some semblance of rationality. However, the justification of a method does not lie in the computational procedure it employs, but rather in its behavior in practice. Hence the need to formulate basic cost allocation principles that can be used to systematically compare the merits of different methods.

Two principles seem very broadly applicable. One is the "core", which says that an allocation should provide sufficient incentive for every group of users to cooperate (provided such an allocation is possible); it can also be stated as the requirement that no group should have to "subsidize" another in the use of the facilities (provided such an allocation is possible). The second principle, "monotonicity", says that no reasonable method would assess some user *less* if total costs were to go *up*; nor by the same token would it assess *more* if the costs were to go *down*.

Six different approaches have been selected for comparison from among the various methods discussed in the project evaluation and game theory literature, and their performance evaluated in relation to an actual municipal cost allocation problem in Sweden. The findings were as follows:

The *proportional* approach is seen at its greater advantage when the allocation criterion chosen (eg. allocation according to offical census populations) seems equitable, and gives numbers whose accuracy is not seriously disputable. It is also relatively easy to compute. The difficulty with this approach is that it ignores the alternative costs of the prospective partners, and hence may not provide sufficient incentives for them to cooperate. This possibility actually arose in the Swedish example.

Of the methods using costs to determine allocations one of the most widely used in the separable cost remaining benefit (SCRB) method. But this method only considers the marginal costs of individual participants, not of groups. Hence, as seen in the Swedish example, one may have allocations that are not group rational, or in other words which subsidize some groups at the expense of others. In addition, this method is not monotone.

A more sophisticated and widely used method from game theory is the Shapley value. This method is individually rational and monotone but unfortunately - as shown by the Swedish example - may not be group rational.

The three remaining methods - least core (and nucleolus), proportional least core and weak least core - satisfy the core principle by definition. However, neither the least core (nor the nucleolus) nor the proportional least core need be monotone, though the latter only breaks down for a somewhat "special" class of examples.

Thus only the weak least core satisfies both basic principles over the most general class of examples. The proportional least core also behaves satisfactorily over a fairly wide range of situations and might even be preferred from an equity standpoint in some cases.

The conclusion is that a systematic investigation of methods in the light of what they do in practice shows significant differences between them. While the decision about what principles should apply in a given situation must be left to the decision makers involved, such a choice has definite implications for what method should be used.

APPENDIX

Note 1. The following procedure is used to determine the sizes of pipes and pumps for water transmission to cope with the water requirements expected by the end of the period (the year 1980 in our case) which the system is designed to serve.

The cost of water transmission includes the following components:

Cost of pipelines :	$C_1 = c_1 L = (\gamma + \alpha D^{\beta}) L$	Skr.
Cost of pumps :	$C_2 = c_2 f P$	Skr,
Cost of electricity:	$C_3 = c_3 P$	Skr/yr.

where

c_1	= unit cost of piping	Skr/m
L	= length of pipe	m
c_2	= unit cost of pump	Skr/kw
f	= safety factor	
c_3	= unit cost of electricity	Skr/kw/yr.
P	$= \frac{9.81}{E}$ QH = effective capacity of pump	kw
Q	= flow of water in pipe	m^3/sec.
H	$= H_o + I\,L$ = required pumping head	m
H_o	= difference in attitude between origin and destination of pipe	m
I	= hydraulic gradient	
E	= pumping efficiency	
D	= pipe diameter	m
α, β, γ	= coefficients	

The total annual cost of transmission is given by

$$C = (C_1 + C_2) \text{ CRF} + C_3 ,$$

where $CRF = \frac{(1+i)^n\ i}{(1+i)^n - 1}$ = capital recovery factor

i = interest rate

n = amortization period in years

The total cost C is a function of the pipe diameter D, the flow Q, the pumping head H, and the length of pipe L. These factors are related by the Hazen-Williams formula:

$$H = \frac{10.7}{Cw^{1.85}} \quad D^{-4.87} \ Q^{1.85} \ L \quad ,$$

where

Cw = Hazen-Williams coefficient.

The economical pipe diameter D^* is obtained as a function of the flow Q by lettering $\frac{\partial C}{\partial D} = 0$:

$$D^* = \left\{ \frac{4.87\ ab}{\alpha\beta\ CRF} \right\}^{1/(\beta + 4.87)} Q^{2.85/(\beta + 4.87)}$$

where

$$a = (c_2 f\ CRF + c_3)\ \frac{9.81}{E}$$

$$b = \frac{10.7}{Cw^{1.85}}$$

Similarly, the economic hydraulic gradient I* is obtained as

$$I^* = \frac{\frac{\alpha\beta\ CRF}{4.87\,a}^{\beta/4.87}}{} \quad Q^{(1.85\beta - 4.87)/(\beta+4.87)}$$

The parameters are determined from Swedish data as follows:

$\alpha = 477$ Skr , $\beta = 1.60$, $\gamma = 150$ Skr

$E = .63$, $Cw = 100$, $f = 1.33$

$CRF = .0871$ based on $i = .06$, $n = 20$ years

$C_2 = 1893$ skr/kw, $C_3 = 613$ skr/kw-year.

The results are given below

$$D^* = .928\, Q^{.43} \quad m$$

$$I^* = 2.99\, Q^{-.28} \times 10^3$$

$$C_1 = (150 + 477\, D^{1.60})L \quad \text{Skr.}$$

$$C_2 = 39.2\, QH \times 10^3 \quad \text{Skr.}$$

$$C_3 = 9.54\, QH \times 10^3 \quad \text{Skr/yr.}$$

Note 2. Since the choice of what to call direct costs may be arbitrary, the inclusion or exclusion of such costs should give equivalent results. Specifically, if $(d_1, d_2, \ldots, d_n)$ are direct costs, then subtracting them from the cost function c results in the *strategically equivalent* cost function c' defined by $c'(S) = c(S) - \sum_S d_i$ for all subsets S of N. The allocation method is *strategically invariant* if the allocations for c' are equivalent to the allocations for c; that is, $(x_1, x_2, \ldots, x_n)$ is an allocation for c if and only if $(x_1 - d_1, x_2 - d_2, \ldots, x_n - d_n)$ is an allocation for c'.

It is easy to check that the Shapley value, least core, nucleolus, and weak least core, as defined in the text, are strategically invariant. So also is SCRB provided the benefit side is ignored (i.e. provided benefits are always assumed to exceed individual costs). The proportional least core and the proportional methods are not strategically invariant. However, a simple device to remedy this problem is to first *normalize* the cost function c by defining $\bar{c}(S) = \sum_S c(i) - c(S)$, then apply the methods to the cost function $\bar{c}$ to obtain an allocation $(\bar{x}_1, \ldots, \bar{x}_n)$. The actual cost allocation $(x_1, \ldots, x_n)$ is the difference between the individual costs and the $\bar{x}_i : x_i = c(i) - \bar{x}_i$.

Note 3. By definition the weak least core is the set of $\underset{\sim}{x}$ optimizing the linear program

(11) $$\min \varepsilon$$

subject to

$$\sum_N x_i = c(N)$$

$$\sum_S x_i \leqq c(S) + \varepsilon|S| \quad \text{for all } S \subsetneqq N \quad .$$

This is equivalent to

$$\max c(N) - n\varepsilon = \sum_N (x_i - \varepsilon)$$

subject to

$$\sum_S (x_i - \varepsilon) \leqq c(S) \quad \text{for all } S \subsetneqq N \quad ,$$

which is equivalent to solving

(12) $$\max \sum_N y_i$$

subject to

$$\sum_S y_i \leqq c(S) \quad \text{for all } S \subsetneqq N \quad ,$$

and letting $\varepsilon = [c(N) - c^*(N)]/n$, $\underset{\sim}{x} = \underset{\sim}{y} + \varepsilon\underset{\sim}{e}$ where $\underset{\sim}{e} = (1,1,\ldots,1)$ and $c^*(N) = \sum_N y_i$ is the optimal value of (12). In particular, if c^+ satisfies $c^+(N) > c(N)$ and $c^+(S) = c(S)$ for all $S \subsetneqq N$, then $\underset{\sim}{x}^+$ is the weak least core of c^+ if and only if $\underset{\sim}{x} = \underset{\sim}{x}^+ - [[c^+(N) - c(N)]/n]\underset{\sim}{e}$ is in the weak least core of c. Thus the weak least core is monotone.

Note 4. Assume that $c(S) > 0$ for all S.

By definition the proportional least core is the set of $\underset{\sim}{x}$ optimizing

$$\min r$$

$$\text{subject to } \Sigma_N x_i = c(N)$$

$$\Sigma_S x_i \leqq (1+r)\, c(S) \quad \text{for all } S \subsetneqq N,$$

This is equivalent to

$$\max c(N)/_{(1+r)} = \sum_N x_i/_{(1+r)}$$

subject to

$$\sum_S x_i/_{(1+r)} \leqq c(S) \quad \text{for all } S \subsetneq N \quad ,$$

which is equivalent to solving (12) and letting $r = [c(N)/c^*(N)] - 1$ and $\underset{\sim}{x} = (1+r)\underset{\sim}{y}$, where $c^*(N) = \sum_N y_i$ is the optimal value of (12). Notice that $c^*(N) > 0$ by virtue of the hypothesis that $c(S) > 0$ for all S. With c^+ defined as in Note 3, it follows that $\underset{\sim}{x}^+$ is in the proportional least core of c^+ if and only if $\underset{\sim}{x} = [c(N)/c^+(N)]\underset{\sim}{x}^+$ is in the proportional least core of c.

Note 5. To see how the proportional least core can fail to be monotone, consider a hypothetical example similar to that of Section 1 but where municipality B owns a lake with sufficient capacity to serve all municipalities at low cost compared to the alternatives. The cost function might be as follows:

A	6.5	{A,B}	4.5		
B	1.0	{A,C}	8.0	{A,B,C}	6.0
C	3.0	{B,C}	1.5		

Thus the costs of supply are reduced *provided* B *is included*. Including B has a marginal value of 2.0 (that is, B's marginal cost is -2.0) hence B might well *be paid*, rather than *pay*, to participate, i.e. B may be able to sell its water to the others at some negotiated price. The cost allocations by the five "strategic" methods are given in Table 9 (second row). Notice that B gets a credit in every case.* Now if total costs increase to 7.0, then A and C's marginal costs increase and B's marginal value decreases so one would expect A and C to be assessed more and B to be paid less. But the proportional least core actually *increases* the amount paid to B. Note the interesting circumstance that all cost allocations of 7.0 are the same except for the Shapley value.

* By contrast, the proportional allocation procedures would assess B positive costs.

	A	B	C	C(N)
SCRB	5.50	-1.00	2.50	7
	5.12	-1.08	1.96	6
Shapley value	5.41	-.59	2.16	7
	5.08	-.92	1.83	6
Nucleolus	5.50	-1.00	2.50	7
	5.17	-1.33	2.17	6
Proportional Least Core	5.50	-1.00	2.50	7
	4.71	-.86	2.14	6
Weak Least Core	5.50	-1.00	2.50	7
	5.17	-1.33	2.17	6

Table 9

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