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Sustainable Growth: Modelling, Issues and Policies

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Abstract

This document is a literature review of sustainable growth. Because there are many definitions of sustainable growth, we use the following one that is very common in economics. Economic growth is *sustainable*, if it meets the needs of the present generations without compromising the ability of future generations to meet their own needs. This concerns both the availability of resources for future generations and the environmental impacts of current decisions on future activities.

Section I, we consider issues and policy measures related to the resource problem. We introduce dynamic models in which exhaustible resources are used in production and apply them to several cases: an open economy, a backstop technology and the relationship of climate and economic growth. We also examine a transition of dirty to clean technology and the consequences of this to public finance and intergenerational equity.

In Section II, we consider macroeconomic performance with natural resources: origins and effects of resource abundance, patterns

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of development for world prices, resource depletion, peak production, "Dutch disease" and external debt. If economic growth affects environment, then the abatement of environmental damages must be included into the discussion of sustainable economic growth.

In Section III, we present resource extraction and the environmental impacts of economic activities in the context of multiple decision makers. This introduces strategic interactions of agents, e.g. firms, households and nations. We consider collusion as well as Cournot, Bertrand games and discuss on diverse micro and macro policies that consider incentive compatibility.

In the Appendix, we introduce a finite horizon procedure called Nonlinear Model Predictive Control (NMPC) by which the models presented in this survey can be numerically solved.

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Introduction

There is a large body of literature on sustainable development and economic growth with an emphasis on economies with abundant natural resources. This paper extensively reviews the modeling of sustainable growth with exhaustible resources, where exhaustible resource is used as an input for production activities. Within this context, a basic model of a dynamic decision problem with two dynamic constraints – changes in capital accumulation, and constraints imposed by the finiteness of the resource stock – is presented. This modeling approach is based on literature by Dasgupta and Heal (1974), Stiglitz (1974) and Solow (1973). As Dasgupta and Heal (1974) recognize, an economy's growth potential is constrained by resources, which have a limited long-run supply, and technological feasabilities.[1]

The basic model of a closed economy can be augmented to an open economy model by adding a constraint of changes in foreign debt accumulation. Blanchard (1983) sketches the open economy model for the country with large external debt based on work by Bardhan (1966) and Bruno (1967) and in more recent research by Obstfeld (1980, 1982), Svensson and Razin (1983), Sachs (1981, 1982). Blanchard and Fischer (1989) present a closed economic model with dynamic budget constraint; they also extend their model to an open economy by taking current account deficits, which can be financed with external debt, into account. They also show a relationship between the current account, savings, and investment. This paper discusses two ways of intertemporal budget constraints' formulations presented by Sachs (1982).

Mansoorian (1991) examines the long-run problem of resource discovery and models heavy borrowing of resource-abundant economies in an optimizing framework. In his paper, a Dutch disease model[2] with three sectors: non-traded goods, manufacturing and resources[3] using the overlapping generation framework of Blanchard (1985), is presented. Eastwood and Venables (1982) analysis of a macroeconomic response of a country to the discovery of a large natural resource in the United Kingdom is also discussed. Hamilton

[1] A more extensive survey of the basic approaches is given in Greiner et al. (2008)

[2] This paper reviews definitions of the Dutch disease and standard models presented by Corden (1984) and Neary and Van Wijinbergen (1986).

[3] Non-traded and manufacturing goods are consumable and the resource sector's output is exported.

(2001) studies the nature of the development path in resource-rich countries and presents theory and measurement for genuine saving in countries concerned with sustainable development.

If we assume that a resource is available in unlimited quantities, but at different grades and costs, then the model can be augmented by introducing a backstop technology,[4] as suggested by Heal (1993). Moreover, variations of the model can stress the climate change problem by including the additional dynamic constraints of carbon dioxide emissions and global temperature. Recent growth models such as Acemoglu et al. (2012) stress the distinction between "clean" and "dirty"[5] inputs for the production and use "endogenous and directed technical change." Acemoglu et al. (2012, p. 131) point out that "when inputs are sufficiently substitutable, sustainable growth can be achieved with temporary taxes/subsidies that redirect innovation toward clean inputs." In addition, the extension of the model can include public capital[6] and sovereign debt dynamics.[7]

In regard to contributors to wealth, comprehensive measurement of welfare that captures reproducible capital, human capital, natural capital, health capital and technological change along the line Arrow et al. (2012) will be discussed. This paper also studies the literature related to problem of intergenerational equity and compares different criteria to deal with this problem. Commonly used discounted utility and welfare criteria will be compared to alternative approaches, including the Rawlsian criterion, the Chichilnisky's criterion of satisfaction of basic needs, as well as the Ramsey (1928), Von Weizsäcker (1967) and Heal (1985) criteria.

We review the surveys on origins and causation of natural resource abundance. In this context, widely used the literature, such as Sachs and Warner (1995, 1999, 2001), on the negative relationship between the natural resource abundance and economic growth is examined with its critiques and proposed

[4]According to Heal (1976), through backstop technology, such as resource extraction from marine sources, the resource can be "inexhaustible but available at various grades and at various costs."

[5]Production with dirty input uses non-renewable resource (Acemoglu et al., 2012).

[6]In Semmler et al. (2007, p. 4), public capital is represented by "public infrastructure to support market production as well as facilities for health and education services."

[7]Blanchard and Fischer (1989) present dynamic budget constraint, and Greiner et al. (2007) use the evolution of public debt accumulation with breakdown of return to government bond, public consumption, transfers, public investment, and tax revenues.

measurement and econometric problems. Out review of studies on scarcity of mineral resource start with Hotelling (1931) and Barnett and Morse (1963). The basic Hotelling rule for perfectly competitive market indicates that a difference between market price and marginal cost should increase at interest rate. Modifications of the Hotelling basic model take into account extraction costs and consider a monopolistic owner of the resources. Barnett and Morse (1963) presented the first comprehensive theoretical and empirical analysis of impact of resource scarcity on growth. Their empirical test results show a decreasing scarcity for mineral resources.

Empirically one finds for non-renewable resources that prices tend to rise as their scarcity increases. Early studies such as Hotelling's (1931) suggest an increasing trend in prices. Greiner et al. (2012b) suggest a numerical solution, using dynamic programming with infinite decision horizon, that indicates monotonically rising prices and monotonically declining extraction rates if the initially known [8] stock is large. However, in the case of small initial stock, the extraction rate will rise $--$ if there is a further discovery of resources $--$ but will later decline. Then, the optimal extraction rate may have an inverted-U shape and the price can show a U-shaped path. Similar results have been shown by Pindyck (1978). Solutions of the optimal exploration and production of an exhaustible resource, by Pindyck (1978), in both competitive and monopolistic markets, also indicate a U-shaped price evolution if the initial stock of the reserve is small.

This paper not only examines literature on mineral resource exports and economic growth but also on macroeconomic performance and economic problems of the resource-rich country including resource depletion, volatility of resource price, Dutch disease, and the threads that may arise from extensive external debt. External debt problems are discussed by comparing countries that borrowed heavily after the resource discovery that led to a resource boom. During the period of high commodity prices in the 1970s, countries that discovered large deposits of resources used their resources as collateral for debt. However, when the prices declined in 1980s, these countries experienced a debt crisis because they had extensive erxternal liabilities and were unable to continue borrowing for economic activitities from abroad.

[8]Greiner et al. (2012b) refer to discovered reserves as “known” resource.

Overall, since resource scarcity gives rise to extraction cost, and spot and future prices of resoruces, exhaustible resource prices tend to rise as scarcity increases. Thus early studies such as Hotelling (1931) and Solow (1974) suggest an increasing trend for resource prices. However, some of the later studies, e.g., Barnett and Morse (1963), did not show a rising trend in mineral prices over time. More recenty, a number of studies, e.g., Slade (1982), Pindyck (1978), Livernois and Uhler (1987), Swierzbinski and Mendelshohn (1989), Greiner et al. (2012b), and Nyambuu and Semmler (2013), suggest a U-shaped mineral price trend.

The paper presents decision making and policy options as well and studies environmental or carbon tax in a more complicated setting, where there is a strategic dependence between the decisions of agents. In this context, decision making is mostly studied in some game theoretical set ups. Yet, in strategic decision making, complicated issues arise concerning incentive compatible decision and policy options on both micro and macro levels. Examples of strategic dependence are worked out first with static games on the firm level with respect to oligopolistic competition. Furthermore, models with growth and diverse set ups for strategic decision making and further policy options are discussed. In this context, topics such as dirty investment, Pigouvian taxation, lobbying, and agents' interdependence in the economy are considered.

For the numerical solutions to the dynamics, the method of nonlinear model predictive control (NMPC) is used that operates with a finite time horizon procedure. NMPC is a discrete-time model and is used as an approximation of infinite-horizon optimal control problem (Grüne and Pannek, 2011). In the case of a very long time horizon, NMPC approximates the infinite time horizon model well, and even with a small number of periods, important issues in a model can still be investigated.[9] NMPC only computes single (approximate) optimal trajectories rather than computing the optimal value function for all possible initial states. As summarized in Grüne et al. (2013), this method can be used in dynamic decision problems in economics.

The remainder is divided into three main parts including the basic modeling and model variants, macroeconomic performance, and numerical solu-

[9]See Greiner et al. (2012a, p. 5).

tion. Part I focuses on theoretical aspects of modeling growth with natural resources. It begins with the definitions of sustainable development followed by problem of intergenerational equity, and measurement of wealth. The basic model and its extensions, including backstop technology, open economy, climate change constraints and other variations are presented. In Part II, a study of the relationship between growth and natural resource abundance is followed by the literature on resource depletion and price trends of the resources. The following sections discuss the problem of Dutch disease and the accumulation of external debt. In Part III we introduce multiple agent's decision making with strategic dependence between different agents, framed in terms of a game theoretical set up. Here then diverse micro and macro policies are considered that are incentive compatible. The appendix sketches a numerical solution technique called NMPC.

1 Basic models and issues

1.1 Notion of sustainable development

There exist different definitions of sustainable development. A commonly accepted point is that economic activity is unsustainable if the relevant biological or social system is degraded. In general, long-run development and consumption is affected by resource and environmental constraints. Pezzey (1989, p. 14) shows the difference between sustainable growth and sustainable development where "non-declining output or consumption over time" represents sustainable economic growth and "non-declining utility over time" can be used for sustainable economic development. Pearce et al. (1990, p. 4) note that "natural capital stock should not decrease over time."[10]

Similarly, the Brundtland report of The World Commission on Environment and Development (WCED, 1987, p. 52) states that "If needs are to be met on a sustainable basis the Earth's natural resource base must be conserved and enhanced." Repetto (1986, p. 15) also highlights conservation of natural capital: "Sustainable development, as a goal rejects policies and practices that support current living standards by depleting the productive base, including natural resources, and that leaves future generations with poorer prospects and greater risks than our own."

Policy makers are concerned with equity between generations or so-called intergenerational equity in the achievement of sustainable development. Pezzey (1992) presents different definitions of the sustainable development concepts. According to the sustainability criterion presented by Tietenberg (1984, p. 30), "future generations should be left no worse off than current generations." Pearce (1987, p. 13) points out that for sustainability we must ensure that "the conditions necessary for equal access to the resource base be met for each generation." Solow (1991, p. 181) examines definitions on sustainability proposed by UNESCO, U.N. Environment Programs and the World Conservation Union, and suggests "an obligation to conduct ourselves so that we leave to the future the option or the capacity to be as well off as we are" as an indicator of sustainability.

[10] See Pearce et al. (1990, p. 1).

Brundtland report emphasizes the problem of a society that might "compromise its ability to meet the essential needs of its people in the future – by overexploiting resources" (WCED, 1987, p. 42). Use of non-renewable resources reduces the stock of the resources available for future generations. As the Commission suggests, for the achievement of sustainable development, "the rate of depletion of non renewable resources should foreclose as few future options as possible" (WCED, 1987, p. 43).

1.2 Intergenerational equity

A fair treatment of different generations can be formulated through a number of approaches proposed in the literature. Intergenerational equity can be accounted for simply by assigning the same weight to the present and future generations, in other words by taking a zero utility discount rate. Alternatively, discounted utility and welfare criteria can be used for the selection of the development paths. These criteria include the Rawlsian criterion and the Chichilnisky's criterion of satisfaction of basic needs, Ramsey (1928), Von Weizsäcker (1967) and Heal (1985).

Weizsäcker (1967) proposed an overtaking criterion that compares two consumption paths C_t^1 and C_t^s and corresponding utilities. Greiner and Semmler (2008, p. 150) express this as follows:

$$\int_0^T U\left(C_t^1\right) dt \geq \int_0^T U\left(C_t^2\right) dt.$$

However, the zero discount rate and overtaking criterion, however, have analytical and technical drawbacks compared to the discounted utility criterion. Greiner and Semmler (2008, p. 150) point out that "for a zero discount rate the set of attainable values of the integral may be open, and the way of ranking consumption paths according to the overtaking criterion is incomplete." Similarly, Chichilnisky (1995, p. 236) argues that the "overtaking criterion fails to rank any two paths which switch between overtaking and being overtaken by others" which makes the criterion "seriously incomplete or indecisive ranking of alternative paths."

Rawls (1972) presented the Rawlsian criterion that is concerned with the welfare of the less advantaged generation, and as Greiner and Semmler

(2008, p. 150) note it requires that the "welfare level to be the same for all generations." Greiner and Semmler (2008, p. 150) show the Rawlsian criterion as follows:

$$max_{feasiblepaths} min_{generationst} \left(Welfare_t\right).$$

Greiner and Semmler (2008) draw upon Solow's (1974) statement that the non-renewable resource stock will last longer if the discounted utility criterion is used instead of Rawlsian criterion, and note that "since the utilitarian rule demands higher savings, earlier generations will have a lower standard of living than the constant max-min would generate." Solow (1974, p. 41) points out that "the max-min criterion seems to be reasonable criterion for intertemporal planning decisions except for two important difficulties: (a) it requires an initial capital stock big enough to support a decent standard of living, else it perpetuates poverty, but it can not tell us why the initial capital stock should ever have been accumulated; and (b) it seems to give foolishly conservative injunctions when there is stationary population and unlimited technical progress."

Another welfare criterion was proposed by Chichilnisky (1995, p. 231); she defined two axioms for sustainable development: "the axioms require that neither the present nor the future should play a dictatorial role."[11] Axiom 1 states that "the present generation should not dictate the outcome in disregard for the future" and the Axiom 2 states that "the welfare criterion should not be dictated by the long-run future, and thus requires sensitivity to the present." Preferences are sustainable if the welfare criterion satisfies both axioms (p. 237).

As presented in Greiner and Semmler (2008, p. 151), Chichilnisky's criterion, where "positive weight is placed on the present and on the very long-run properties of a growth path," can be expressed in following way:

$$\alpha \int\limits_0^\infty U\left(C_t, S_t\right) \triangle(t) dt + (1-\alpha) \lim_{t\to\infty} U\left(C_t, S_t\right).$$

where $\alpha \in (0,1)$ and $\int_0^\infty \triangle(t)dt = 1$. More weight is placed on the future with the Chichilnisky criterion in comparison to the discounted utility criterion (Greiner and Semmler 2008, p. 151).

[11] See Chichilnisky (1995, p. 231).

Economists are well aware of the difficulty in finding a welfare criterion that not only permits intergenerational equity, but also operates well technically. Although discounted utility emphasizes the present over the future, it is still one of the widely used approaches (Greiner and Semmler 2008). Chichilnisky (1995, p. 235) pointed out that "no criterion has achieved the analytical clarity of the discounted sum of utilities."

1.3 Measurement of wealth

There are different approaches to the measurement of comprehensive wealth. In his analysis of wealth, Dasgupta (2005) considers not only manufactured and human capital but also natural capital. While The World Bank (2011) estimates the shadow values of natural, human and reproducible capitals, a study by Arrow, et al. (2012) takes into account two more contributors of wealth, namely health improvements and technological change. In contrast to studies based on income, Arrow et al. (2012) examine the sustainability of economic growth by focusing on the measurement of wealth. In their sustainability analysis of intergenerational well-being, sustainability is reached if a comprehensive wealth measure is maintained. The sustainability criterion is met if per capita wealth is increasing.

In the paper by Arrow et al. (2012), natural capital includes not only non-renewable energy and mineral resources but also renewable forest and land resources. They compare resource trading countries and stress that while non-renewable resource exporting countries have enjoyed capital gains on stocks of their resources, importers have experienced capital losses. In open economies, since higher prices of exhaustible resources bring profit to exporters, the wealth of an exporter is higher than would be found by using current prices. Resource owners receive capital gains as a rental value of a non-renewable resource increases over time. However, real wealth will be reduced because consumers will have to pay higher real prices (see Arrow et al., 2012).

Arrow et al. (2012) apply the model to five countries including the United States, China, Brazil, India and Venezuela. Using data from 1995 to 2000 they examine whether economic development was sustainable. The estimation results indicate the importance of technological change, natural capital, and health capital in achieving sustainability. They point out that the role of these capitals in sustainable development differs across countries. For example, their findings show that in the case of the United Stated and Brazil, growth in comprehensive wealth is primarily due to investment in reproducible, human and knowledge capital. In all countries except China, the negative impact of natural capital depletion is outweighed by the changes in human capital. For China and India, a reduction in natural capital can be offset by investments in reproducible capital. In Venezuela, a significant decrease in natural capital leads to the substantial changes in capital stocks. Sizeable capital gains on oil stocks contributed to a positive investment. The estimated increase in scarcity rents for Venezuela's oil accounts for large share of the growth in comprehensive wealth (for details see Arrow et al., 2012).

1.4 The basic model

Early studies by Forrester (1971), Meadows et al. (1972), and Dasgupta and Heal (1974) emphasize the possibility of resource constraints on economic growth. Dasgupta and Heal (1974, p. 4) point out that the resource can be essential "if output of final consumption goods is nil in the absence of the resource." In analyzing the depletion of non-renewable resources, Dasgupta and Heal (1974), Stiglitz (1974) and Solow (1973) use a growth model where the mineral resource is one of production inputs.

In the basic theoretical model presented by Dasgupta and Heal (1974), a welfare function for households, where the inter-temporal utility as function of consumption is maximized with two constraints including evolution of the capital stock and non-renewable resource. As in Dasgupta and Heal (1974), Greiner and Semmler (2008, p. 142) assume that the utility function is strictly concave, which means it should have the properties of $U'(C_t) < 0$, $U"(C_t) > 0$.

The model has two state variables – the capital stock, K_t, and the remainder of the non-renewable resource, S_t – and two decision variables – the consumption, C_t, and the flow of the exhaustible resource, R_t. Dasgupta and Heal (1974) assume the production function, $F(K_t, R_t)$, to be increasing, strictly concave, and twice continuously differentiable. Following Dasgupta and Heal (1974), Greiner and Semmler (2008, pp. 141-143) present the optimization problem with its solutions followed. Dasgupta and Heal (1974), Stiglitz (1974), and in Solow (1973) presented the basic growth model with two inputs of production, $F(K, R)$, e.g., capital stock and non-renewable resources. Dynamic maximization problem with objective function of utility as function of consumption for households, $U(C)$, has constraints of the capital stock accumulation, K, and the remainder of the non-renewable resource, S, which is diminished by the flow of the exhaustible resource, R. With a discount rate of r and given initial stock of the remainder of the resource S_0, Greiner and Semmler's (2008, pp. 141-3) model is the following: maximize[12]

$$\int_0^\infty U(C_t)e^{-rt}dt \tag{1}$$

subject to

$$\dot{K}_t = F(K_t, R_t) - C_t, \quad \dot{S}_t = -R_t, \quad S_t = S_0 - \int_0^t R_\tau d\tau.$$

The current value Hamiltonian with two constraints is

$$H = U(C_t) + \alpha\left(F(K_t, R_t) - C_t\right) + \beta(-R_t),$$

where α and β are co-state variables or shadow prices of capital accumulation and resource constraints respectively. The necessary optimality conditions are obtained by

$$U'(C) = \alpha, \tag{2}$$

$$\alpha F_R = \beta,$$

$$\dot{\alpha} = r\alpha - \alpha F_K, \tag{3}$$

$$\dot{\beta} = r\beta,$$

with $F_R = \frac{\partial F(K,R)}{\partial R}$ and $F_K = \frac{\partial F(K,R)}{\partial K}$.

Dasgupta and Heal (1974, p. 11) and Greiner and Semmler (2008, pp. 165-6) show the following path of the consumption:

$$\frac{\dot{C}}{C} = \frac{F_K - r}{\varepsilon(C)}, \quad \text{where} \quad \varepsilon(C) = -\frac{CU''(C)}{U'(C)}.$$

[12]Greiner and Semmler's (2008) dynamic optimization problem is based on Dasgupta and Heal (1974). Here, extraction cost is not considered.

Greiner and Semmler (2008) point out that higher discount rate is associated with further fall of the rate of consumption over time. Dasgupta and Heal (1974, p. 11) state that the following condition implies "the equality of the rates of return on the two assets (the exhaustible resource and reproducible capital)."

$$F_K = \frac{\partial F_R}{\partial t} \frac{1}{F_R} \tag{4}$$

with a production function of homogenous of degree one, a ratio of two inputs can be expressed as $x_t = K_t/R_t$ with $f(x_t) = F(K_t/R_t, 1)$. Following Greiner and Semmler (2008, p. 166), the capital-resource ratio along the optimal path can be obtained by substituting $F_R = f(x_t) - x_t f'(x_t)$ and $F_K = f'(x_t)$ in Eq. (4):

$$\frac{\dot{x}_t}{x_t} = \sigma \frac{f(x_t)}{x_t},$$

where an elasticity of substitution between two production inputs K and R are defined by

$$\sigma = -\frac{f'(x_t)\left(f(x_t) - x_t f'(x_t)\right)}{x_t f(x_t) f"(x_t)} \in [0, \infty).$$

Dasgupta and Heal (1974) stress the importance of the elasticity of substitution in the properties of an optimal path. They use a CES production function with constant elasticity of substitution:

$$F(K, R) = [\beta K^{(\sigma-1)/\sigma} + (1-\beta) R^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)}, \quad \text{where} \quad 0 < \beta < 1.$$

With the Cobb-Douglas production function, Solow (1973) notes that if the share of capital exceeds the share of natural resources, sustained per capita consumption can be a feasible objective. Dasgupta and Heal (1974, pp. 14-9) analyze different forms of production functions, in particular, the case when $R = 0$, with different values of the elasticity of substitution and the results for each cases as shown below. They indicate whether the resource is essential to production and finitely or infinitely valuable at the margin. They show that an exhaustible resource is essential to production only when $\sigma \leq 1$ (pp. 14-15). If $\sigma \leq 1$, then it holds true that

$$F(K, 0) = 0, \quad \rho = \underset{t \to \infty}{lt} f'(x) = \underset{t \to \infty}{lt} \frac{f(x)}{x} = 0,$$

$$\underset{t\to\infty}{lt}\,(f(x) - xf'(x)) = \underset{t\to\infty}{lt}\, f(x) = \infty.$$

In this case, the non-renewable resource is essential to production and infinitely valuable at the margin. The price of the resource relative to fixed capital is rising and tends to be infinite. Additionally, the asymptotic value of marginal productivity of capital equals zero. On the other hand, if $\sigma < 1$, then it holds true that

$$F(K, 0) = 0, \quad \rho = \underset{t\to\infty}{lt}\, f'(x) = \underset{t\to\infty}{lt}\, \frac{f(x)}{x} = 0,$$

$$\underset{t\to\infty}{lt}\,(f(x) - xf'(x)) = \underset{t\to\infty}{lt}\, f(x) = (1-\beta)^{\sigma/(\sigma-1)}.$$

The resource is essential but finitely valuable at the margin when the rate of utilization of the resource is zero. Dasgupta and Heal (1974, p. 16) point out that this is the most pessimistic case because as total output is bounded, if $\sigma < 1$, then $C_t \to 0$ as $t \to \infty$. Thus, a positive and non-decreasing consumption is not sustainable.

If $\infty > \sigma > 1$, then it holds true that

$$F(K, 0) = \beta^{\sigma/(\sigma-1)}K, \quad \rho = \underset{t\to\infty}{lt}\, f'(x) = \underset{t\to\infty}{lt}\, \frac{f(x)}{x} = \beta^{\sigma/(\sigma-1)} > 0,$$

$$\underset{t\to\infty}{lt}\,(f(x) - xf'(x)) = \underset{t\to\infty}{lt}\, f(x) = \infty.$$

In this case, the non-renewable resource is not essential to production and infinitely valuable at the margin when the rate of utilization of the resource is zero.

Stiglitz (1974) points out that resource limitations to growth can be offset by economic forces, e.g., technical change, the substitution of capital for natural resources, and changes in the returns to scale profile. In his study of optimal growth paths, optimal extraction and optimal saving rates in the presence of non-renewable resources, Stiglitz examines the feasibility of sustainable levels of per capita consumption. He points out that sustained growth in consumption per capita can be feasible in his simple model of economic growth, with exhaustible resources.

1.5 Extension of the model

1.5.1 Backstop technology

In the basic model, it is assumed that the mineral resources are exhaustible. This model can be extended by introducing a backstop technology that was used by Nordhaus (1973) and Heal (1976). Heal (1976, p. 371) points out that the resource can be "inexhaustible but available at various grades and at various costs" and "extracted from marine sources or crustal rocks." Particularly, at higher prices, the resource can be available in unlimited supply.

Heal (1976, p. 371) states that "cost is assumed to increase with cumulative extraction up to a point, but then to remain constant as a "backstop" supply is reached." As examples, he suggests metals such as aluminum and manganese. These metals' ores can be extracted from a current deposit, which is exhaustible and cheap. But these metals can also be extracted from the sea or rock formations, with indefinite supplies at much higher prices (Nordhaus 1974; Heal 1976). Similarly, Nordhaus (1973) discusses the extraction of exhaustible oil using extraction from shale as a backstop.

Following Heal (1976, pp. 373-7), Greiner and Semmler (2008, pp. 146-7) sketch a formulation of the model with backstop technology where the total amount of the resource at time t which is z_t expressed by

$$z_t = \int_0^t R_\tau d\tau.$$

They state that the backstop technology takes over when the conventional low-cost deposits are exhausted at date T. When these resources are exhausted it switches to an extraction with higher costs. The extraction cost increases with cumulative extraction up to the level z_T; then the backstop technology is available at a constant cost per unit, b. The cost of extracting a unit of the resource, $g(z_t)$, has a following property:

$$\partial g/(\partial z_t) = g^{'}(z_t) > 0 \; for \; 0 \leq z_t \leq z_T \quad \text{and} \quad g(z_T) = b > 0 \; for \; z_t \geq z_T.$$

The basic model with backstop technology is solved by applying two maximization problems successively.[13] First, before the current deposits are

[13] For details, see Greiner and Semmler (2008, pp. 146-7).

exhausted, total extraction costs are given by $g(z_t)R_t$ and the problem is

$$\max \int_0^T U(C_t)e^{-rt}dt \tag{5}$$

subject to

$$\dot{K}_t = F(K_t, R_t) - C_t - g(z_t)R_t, \quad \dot{S}_t = -R_t, \quad S_t = S_0 - \int_0^t R_\tau d\tau.$$

Second, after the backstop technology has taken over, total extraction costs are given by bR_t and the problem is

$$\max \int_T^\infty U(C_t)e^{-rt}dt \tag{6}$$

subject to

$$\dot{K}_t = F(K_t, R_t) - C_t - bR_t. \tag{7}$$

The Hamiltonian for the first problem is given by[14]

$$H = U(C_t) + \alpha\left(F(K_t, R_t) - C_t - g(z_t)R_t\right) + \beta(-R_t),$$

where α and β are co-state variables or shadow prices of capital accumulation and resource constraints respectively. The necessary optimality conditions are obtained by

$$U'(C_t) = \alpha, \quad \alpha F_R - \alpha g(z_t) = \beta, \tag{8}$$

$$\dot{\alpha} = r\alpha - \alpha F_K, \tag{9}$$

$$\dot{\beta} = r\beta + \alpha g^{'}(z_t)R_t, \tag{10}$$

with $F_R = \frac{\partial F(K,R)}{\partial R}$ and $F_K = \frac{\partial F(K,R)}{\partial K}$.

The consumption rate along an optimal path is the following:

$$\frac{\dot{C}}{C} = \frac{F_K - r}{\varepsilon(C)}, \quad \text{where} \quad \varepsilon(C) = -\frac{CU''(C)}{U'(C)},$$

$$F_K = \frac{\partial F_R}{\partial t}\frac{1}{F_R} + \frac{F_K g(z_t)}{F_R}. \tag{11}$$

[14]cf. Greiner and Semmler (2008, pp. 167-168).

Following Greiner and Semmler (2008, p. 168), the capital-resource ratio along the optimal path can be obtained as shown:

$$\frac{\dot{x}_t}{x_t} = \sigma\frac{f(x_t)}{x_t} + \frac{f^{'}(x_t)}{x_t f^{"}(x_t)}\frac{g(z_t)}{x_t}.$$

Correspondingly, the Hamiltonian and the necessary optimality conditions for the second problem [eqs. (6) and (7)] are given by

$$H = U(C_t) + q_1 \left(F(K_t, R_t) - C_t - bR\right)$$

$$U'(C_t) = q_1, \quad q_1 F_R = q_1 b, \tag{12}$$

$$\dot{q_1} = r q_1 - q_1 F_K \tag{13}$$

Heal (1976) concludes that initially, the stocks of the resource with lower-cost are exhausted according to equations (8) and (10). He states that "the initial conditions are chosen that the difference between prices and extraction costs ... declines according to (10) and just reaches zero as the lower-cost stocks are exhausted".[15] After this point, the economy behaves according to equations (12) and (13), thus, the "extraction costs of the resource always equal its price." Heal (1976) stresses that "the relationship between the price of a resource and its marginal extraction cost along an optimal path ... depends on the nature of the extraction technology"[16].

1.5.2 Technical progress

Weitzman (1997, p. 1) defines sustainability as "the annualized equivalent of the present discounted value of consumption that the economy is capable of achieving" and emphasizes the connection between sustainability, Green Net National Product[17] (NNP), and technological progress.[18] Due to importance of technological progress in long-term sustainability, the Green NNP requires a significant upward correction in the presence of the technological progress. In the following formulation Weitzman (1997) shows how the Solow residual

[15] This equation refers to the equation in this paper that shows the evolution of co-state variable of resource constraint which is denoted by q_t.

[16] See Heal (1976, p. 377).

[17] Green NNP is an adjusted national income, where depreciation of capital and depletion of natural resources at current market prices are deducted from GNP (Weitzman,1997).

[18] In Weitzman (1997) technological progress is measured as Solow residual.

connects the sustainability and the Green NNP. Weitzman (1997, p. 5) presents an optimal control problem that maximizes present discounted value of consumption, C, where r denotes real interest rate, K is capital stock that includes natural resource, S represents the production possibilities, P stands for the price of

$$\int_0^\infty C_t e^{-rt} dt \tag{14}$$

subject to

$$(C_t,\ \dot{K}_t) \in S(K_t; t), \quad K(0) = K_0.$$

The sustainability, Γ_t, and the Green NNP function in Weitzman (1997, p. 7), Y_t^*, are defined as follows:

$$\Gamma_t = r \int_t^\infty C^*(s) e^{-r(s-t)} ds, \quad Y_t^* = C_t^* + P_t \dot{K}_t^* = G(K_t^*, P_t; t).$$

Growth rates of aggregate output, g, and the residual, λ, are:

$$g \equiv \frac{\dot{Y}^*}{Y^*}, \quad \lambda = \frac{1}{Y^*} \frac{\partial Y^*}{\partial t}.$$

Weitzman (1997, p. 8) derives the technological progress premium:

$$\Theta = \frac{\lambda}{r - g}.$$

The connection of the current Green NNP and future sustainability is [19]

$$\Psi_0 = Y_0^*(1 + \Theta).$$

Weitzman (1997, p. 12) argues that "NNP, whether conventionally measured or green-inclusive, likely understates an economy's sustainability."

1.5.3 Open economy

Closed economy optimization models can be modified to represent an open economy problem. An open economy model with debt and current account is seen in recent research by Obstfeld (1980, 1982), Svensson and Razin (1983),

[19] See Weitzman (1997, p. 8)

Sachs (1981, 1982), Blanchard (1983), Cohen and Sachs (1986), and Barro et al. (1995).

As Semmler and Sieveking (2000) note, with the assumption of free access to capital markets, the country's debt can be increased by an amount equivalent to the current account deficits. Sachs (1982) stresses the importance of the current account in macroeconomic adjustments and shows how to determine it. He presents two formulations for the intertemporal budget constraint. Current account (CA) equals financial claims on the rest of the world, which is denoted by B and can be expressed in the following way:[20]

$$B = CA = Q + r^* B - C - G,$$

where Q stands for the gross domestic product, $Q + r^* B$ is the gross national product, C is household consumption, G is government fiscal expenditure, $r*$ is short-term financial assets' fixed real yield. A positive B implies that the country is a creditor, and a negative B suggests a debtor.

The following condition, as in Sachs (1982, p. 148) is added to avoid Ponzi games:

$$\lim_{t \to \infty} e^{-r^* t} B = 0.$$

Sachs (1982, p. 149) points out that the initial net indebtedness is balanced by the discounted value of trade surplus, thus this relationship is shown as the second formulation of the intertemporal budget constraint:

$$\int_0^\infty e^{-r^* t} \left(Q - G - C \right) dt = -B(0).$$

Sachs (1982) states that the simple model does not include investment of physical capital, which plays important role in cross-country current account behavior.

Sachs, Cooper, and Fischer (1981) examine how an increase in investment leads to a current account deficit and raises the real exchange rate. His empirical evidence shows the importance that investment demand had in the behavior of current accounts and exchange rates in the 1970s.

In an open economy model, Blanchard and Fischer (1989) show that temporary imbalances of saving and investment, which are the current account

[20] See Sachs (1982, p. 148).

deficits, can be financed by external debt. With a constant world interest rate, θ, for foreign borrowing or lending, the change in the foreign debt of an open economy in per capita term is shown to be:[21]

$$\frac{db_t}{dt} = c_t + i_t\left[1 + T\left(\frac{i_t}{k_t}\right)\right] + \theta b_t - f(k_t),$$

where b_t stands for per capita debt, c_t and i_t are per capita consumption and investment spending, and $f(k_t)$ is the production function. In addition, there is a "cost of installing investment goods," in the transformation of goods into capital $T(\cdot)$ amount per unit of investment is used (see Blanchard and Fischer, 1989, p. 59).

From the above budget constraint, foreign debt dynamics are determined by the difference between spending and output. Blanchard and Fischer (1989, 59-60) state that "the change in foreign debt is the current account deficit ... the current account deficit is equal to the excess of absorption over production."

Blanchard and Fischer (1989, p. 60) show that using national income accounting, the current account deficits can be equal to the difference between interest payment and net exports of goods:

$$db/dt = \theta b - nx,$$

where nx stands the trade surplus. They show that the current account deficit is equal to the difference between investment, $i(1+T(\cdot))$, and savings, s.[22]

$$db/dt = i\left(1 + T(\cdot)\right) - s.$$

Similarly, a simple model presented by Blanchard (1983, p. 188) on external debt and current account deficits in Brazil is shown below:

$$\max_{C,I} \int_0^\infty e^{-\theta t} U(C_t) dt$$

subject to

$$\dot{B}_t = \theta B_t + C_t + I_t\left[1 + \psi\left(I_t\right)\right] - F\left(K_t, \overline{L}\right), \quad \dot{K}_t = I_t,$$

[21] See Blanchard and Fischer (1989, p. 58).

[22] See Blanchard and Fischer (1989, 60).

where $\psi_t' > 0$, $F_k > 0$ and $F_{kk} > 0$.

Blanchard (1983) notes that spending is the sum of consumption,C, and investment spending, I, but the investment spending consists of investment and installation costs, $I_t\psi(I_t)$, where $\psi(\cdot)$ is an increasing function of I. The country has current account deficits when the sum of spending,

$$C_t + I_t\left[1 + \psi(I_t)\right],$$

and interest on debt, θB_t, exceeds output, $F\left(K_t, \overline{L}\right)$. Here, initial values for K_0 and B_0 are given.

In the extension of the above simple model, Blanchard (1983, 190) relaxes the assumptions so that the population grows at rate n, the rate of discount may change over time, and time varying Harrod neutral technological progress, β_t, and a positive rate of capital depreciation, δ, time varying discount rate, θ, and constant world interest rate, r, are introduced. In this case, the extended model is

$$\max_{C,I} \int_0^\infty L_t U\left(\frac{C_t}{L_t}\right) e^{-\int_0^t \theta_s ds} dt$$

subject to

$$\dot{B}_t = C_t + I_t\left[1 + \psi\left(\frac{I_t}{K_t}\right)\right] + rB_t - F\left(K_t, L_t e^{-\int_0^t \beta_s ds}\right),$$

$$\dot{K}_t = I_t - \delta K_t.$$

Semmler and Sieveking (2000, p. 1124) present an optimal control problem of a resource-rich open economy with debt dynamics. The utility, which is a function of consumption and renewable resource, is maximized with constraints determined by the resource stock and foreign debt.

$$\max_{c,q} \int_0^\infty e^{-\delta t} U(R, c) dt$$

subject to

$$\dot{R} = g(r) - qR \; 0 \leq q \leq Q,$$

$$\dot{B} = h(B) - pf(qR) + c, \; 0 \leq c \leq C,$$

where R_t denotes the resource stock, B_t is accumulated debt, $q_t R_t$ stands for the extraction rate of the resource, q_t is the extraction rate, f is the exportable goods' production function, g is the reproduction function, and $h(B)$ is the interest payment on debt.

Open economy formulations often use the Dutch disease models. In the 1980's, the phenomenon "Dutch disease" was in the center of interest. Domestic sectors particularly manufacturing in certain countries, had declined due to adverse effects of a resource boom. Corden (1984, p. 359) defines the Dutch disease as "the adverse effects on Dutch manufacturing of the natural gas discoveries of the nineteen sixties, essentially through the subsequent appreciation of the Dutch real exchange rate."

Kremer (1986) examines the impacts of Dutch gas discovery on its small and open economy with free international movement of capital. In the Netherlands, gas exploitation required limited inputs. Capital inputs do not seem to have put any pressure on other sectors. In the case of the Dutch disease in the Netherlands, he states that the real exchange rate overshooting models might be inapplicable and a model without inputs to the booming sector should be considered.

Neary and Van Wijinbergen (1986) consider the effects of the discovery of resources in a series of theoretical models and conclude that a resource boom can lead to a real appreciation, i.e., an increase in the price of non-traded goods relative to traded goods. In addition, all their models show deindustrialization, i.e., a decline in output and employment in the manufacturing sector.

Mansoorian (1991) examines a problem of resource discovery and models heavy borrowing in an optimizing framework. He shows that resource discovery may cause a sharp deterioration in net foreign asset positions in the long-run. Under the overlapping generation framework of Blanchard (I985), Mansoorian (1991) presents a Dutch Disease model with three sectors, non-traded, manufacturing and resource. While non-traded and manufacturing goods are consumable, the resource sector's output is exported.

The dynamics of the model are the current account, the rate of change of aggregate expenditure, and the evolution of the difference between the price of a title to a unit of factor (capital, specific factor, resource) and the partial derivative of the GDP function. As Mansoorian (1991) states, this procedure is based on Blanchard (1985, pp. 228-9) and Matsuyama (1987, pp. 304-5). The standard equation for current account is defined in Mansoorian (1991, p. 1502) as:

$$\dot{B}_t = rB_t + g(p_t, L, K, R, T) - Z_t,$$

where Z_t is aggregate expenditure, B_t is aggregate bond holdings of the economy at time t. In addition, $g[p(t), L, K, R, T]$ is the value of the gross domestic product with the price of the non-traded good, p, labor, L, used in non-traded and manufacturing sectors, capital, K, used in the extraction of resources, R, and T is a specific factor.

Mansoorian (1991, p. 1499) states that in standard Dutch disease models "if the resource sector is an integral part of the economy then a resource discovery need not cause both real appreciation and deindustrialization." He further notes that as shown by Corden and Neary (1982) a resource discovery may lead to real appreciation and pro-industrialization because the extraction of new resources promotes production in manufacturing sector. In contrast, Neary and Purvis (1982) emphasize a possibility of real depreciation and de-industrialization because the extraction of new resources encourage production of non-tradables. In his long-run modeling, Mansoorian (1991) shows that real depreciation and pro-industrialization, that cannot be predicted by traditional Dutch disease as stated by Neary and Van Wijnbergen (1986), is actually possible. This may happen because of the fall in aggregate expenditure and in demand in non-tradables.

Eastwood and Venables (1982) analyze a macroeconomic response of a country to the discovery of a large natural resource, e.g., the discovery of oil in The North Sea. They assume zero extraction costs, that oil is perfectly tradeable, and that foreign currency denominated exogenous oil price increases at the foreign interest rate. The model before the oil shock consists of money market equilibrium, m, assumptions of perfect capital mobility and perfect foresight in the foreign exchange market, r, demand function for domestic output, d, demand determined output, $y = d$, and Phillips curve, $\dot{p}$ (p. 287).

$$m = \phi y - \lambda r + \alpha p + (1 - \alpha)e,$$

$$r = r^* + \dot{e},$$

$$d = \delta(e - p) + \gamma y - \sigma(r - \dot{p}),$$
$$y = d,$$
$$\dot{p} = \beta(y - \bar{y}),$$
$$\phi, \lambda, \delta, \gamma, \sigma, \beta > 0;\ 0 \leq \alpha \leq 1.$$

They consider resource discovery as a "foreign exchange increment to national wealth" (289) and show how demand is affected by the discovery. A demand function, d, for domestic output before the resource discovery depends on the relative prices of domestic, p, and imported goods with an exchange rate, e, income, y, and the real interest rate with domestic interest rate of, r.

As a result of natural discovery, the demand function changes to

$$d = \delta(e - p) + \gamma y - \sigma(r - \dot{p}) + \eta(f + e - p),$$

where $(f + e - p)$ expression is the value of oil revenue in domestic currency with elasticity of aggregate demand with respect to oil revenue denoted by η (p. 290).

Analysis of the dynamic adjustment of the economy to a resource discovery by Eastwood and Venables (1982) show that additional demand from oil revenue results in exchange rate appreciation. They impose a slow price adjustment, with a "time lag between the resource discovery and the spending of the resource revenue" and show that a "deflationary interval will follow the initial exchange rate appreciation" (p. 297).

Hamilton (2001) examines whether a resource-abundant country consumes wealth along the development path and whether the development of the economy can be sustained. In his model, the supply of resources is considered as one of the inputs of domestic production. He develops a theoretical approach to assess the sustainability path and provides empirical evidence on that sustainability. Sustainability is measured by genuine savings, G, which is the difference between the net national product and consumption as

$$G = GNP - C - \delta K - n_s R - n_l(h - g) - \sigma e + E,$$

where R stands for the depletion of subsoil resources, n_s is the unit resource rental rate, $(h - g)$ is net depletion of living resources and its rental rate is n_l,

e is net pollution accumulation and its social marginal cost is σ; E denotes the current educational expenditures.

According to Hamilton's (2001) model of depletion and discovery of sub-soil resources, negative genuine saving indicates that an optimal development path is not sustainable. A resource extracting economy can be sustainable if resource rents are invested in other productive assets. Hamilton derives the net income and genuine saving and presents an optimal growth problem that the present value of welfare, U, is maximized (pp. 47-48):

$$\max \int_t^\infty U(C_s)e^{-\rho s}\,ds.$$

The problem has several constraints in Hamilton (2001). When the genuine savings are negative wealth is falling, in turn welfare will be declined as shown (pp. 47-48):

$$U + U_G G = \rho W, \quad U_G G = \rho \int_t^\infty U_s e^{-\rho(s-t)}\,ds - U = \dot{W}.$$

Furthermore, in Hamilton (2001), domestic supply of resource, R, depletion and accumulation for resource deposits, S, and cumulative discoveries, D, are defined by

$$R = \sum_{i=1}^{N-1} R_i, \quad \dot{S}_i = -\dot{R}_i, \ \ (i = 1, ..., N-2, \quad \dot{S}_{N-1} = -R_{N-1} + D,$$

$$\dot{S_N} = -R_N, \quad \dot{Q} = D.$$

Foreign trade with foreign assets, A, repatriation of assets, M, fixed foreign interest rate, r, and international resource price path , can be expressed by the following equation:

$$\dot{A} = rA + pR_N - M.$$

Domestic production function with fixed labor and population has two inputs namely produced capital, K, and the supply of the resources, R, is $F = F(K, R)$. The supply and disposition in the economy is given by

$$F + M = C + \dot{K} + \sum_{i=1}^{N} f_i(R_i) + \nu.$$

According to the efficiency condition for foreign trade, the domestic resource price, which is the marginal product of the resource, should be equal to the international price:

$$F_R = p.$$

Optimal time paths for C, R, D and M should be chosen as indicated in Hamilton (2001).

1.5.4 Climate change effects

In the further extension of the open economy growth model with exhaustible resources and backstop technology, as described earlier, the climate change effects can be incorporated with additional constraints on greenhouse gases (GHGs) and temperature.

Social welfare should be maximized with an effort towards the limitation of environmental degradation. In Greiner et al. (2012a, p. 6) an increase in GHGs is expressed by M:

$$\dot{M} = \beta_1 u - \mu(M - \kappa M_o),\ M(0) = M_0 \geq M_o,$$
$$\mu \in (0, 1),\ \beta_1 \in (0, 1).$$

where M_0 is the pre-industrial level of the GHGs, u is the amount of fossil fuels used, β_1 is the part of the GHGs not taken up by oceans, and μ denotes the inverse of the atmospheric lifetime of GHGs.

Bondarev et al. (2013) study a dynamic endogenous growth model that considers both environmental and economic variables with technological progress. They emphasize the importance of the efficiency of the technology in an increase of the welfare. Their results show that less environmental damage, which is less GHGs emissions and lower temperature increase, can be generated from the endogenous technical change compared to exogenous technical change.

In the model with environmental damage, Bondarev et al. (2013, p. 4) use m for the GHG concentration in the atmosphere. It is increased by economic activity, which has a weakening effect through controlled abatements and exogenous improvement in cleaning technology. This constraint of GHG concentration on the growth is shown as:

$$\dot{m}_t = -vm_t + (1 - a_t)e_t Y_t,$$

where Y_t denotes total output, v is the rate of recovery of the atmosphere due to natural absorption, a_t is the abatement rate, and e_t is the reduction of intensity of emissions from economic activities.

Besides using the GHG concentrations, Bondarev et al. (2013, p. 4) use temperature as one of the constraints:

$$\dot{\tau}_t = -\lambda \tau_t + d(m_t),$$

where τ_t is the temperature increase from pre-industrial levels. This evolution of the temperature is shown as a function of GHG concentration.

1.5.5 DICE model

Nordhaus (2008) presents a Dynamic Integrated Model of Climate and the Economy (DICE) in the framework of growth theory. His model includes natural capital and adds CO_2 emissions, climate change impacts, climatic damages, and climate-change policies. In his DICE-2007 model, there are equations of damage, Ω_t, abatement-cost function, Λ_t, total carbon emission, E_t (a sum of emissions from industry and land), mass of carbon in earth's "reservoir," M_t (including atmospheric, upper ocean, and lower ocean), total radiative forcing, F_t, global mean surface temperature, $T_{AT,t}$, temperature of lower ocean, $T_{LO,t}$, and abatement cost as participation cost markup, π_t (which describes climate change effects and policy).[23]

$$\Omega_t = \frac{1}{1 + \pi_1 T_{AT,t} + \pi_2 T_{AT,t}^2}, \quad \Lambda_t = \pi_t \theta_{1,t} \mu_t^{\theta_2}, \quad E_t = E_{Ind,t} + E_{land,t},$$

$$E_{Ind,t} = \sigma_t (1 - \mu_t) A_t K_t^{\gamma} L_t^{1-\gamma},$$

$$M_{AT,t} = E_t + \phi_{11} M_{AT,t-1} + \phi_{21} M_{UP,t-1},$$

$$M_{UP,t} = \phi_{12} M_{AT,t-1} + \phi_{22} M_{UP,t-1} + \phi_{32} M_{LO,t-1},$$

$$M_{LO,t} = \phi_{23} M_{UP,t-1} + \phi_{33} M_{LO,t-1},$$

$$F_t = \eta \left\{ log_2 \left(\frac{M_{AT,t}}{M_{AT,1750}} \right) \right\} + F_{EX,t},$$

$$T_{AT,t} = T_{AT,t-1} + \xi_1 \left\{ F_t - \xi_2 T_{AT,t-1} - \xi_3 \left(T_{AT,t-1} - T_{LO,t-1} \right) \right\},$$

[23] See the model equations in the appendix of Nordhaus (2008, 205).

$$T_{LO,t} = T_{LO,t-1} + \xi_4 \left(T_{AT,t-1} - T_{LO,t-1} \right), \quad \pi_t = \varphi_t^{1-\theta_2},$$

where μ_t is an emissions-control rate, φ_t denotes a participation rate, σ_t is a ratio of uncontrolled industrial emissions to output, γ is an elasticity of output with respect to capita, ξ is a temperature-forcing parameter, ϕ stands for parameter of the carbon cycle, ψ is a parameter of damage function, θ is a parameter of the abatement-cost function, and ξ is a parameter of climate equations.

1.5.6 Clean technology

In Bondarev et al. (2013, p. 4) impose the following constraint of state of technology with R&D investments, g_t, on growth, which can be expected to decline in the absence of investments:

$$\dot{x_t} = \beta g_t - \delta_2 x_t.$$

Studies of growth models such as Acemoglu et al. (2012), stress the distinction between "clean" and "dirty" technologies and uses "endogenous and directed technical change." They use "dirty,"[24] Y_d, and "clean," Y_c, inputs for the production of final goods and point out that "when inputs are sufficiently substitutable, sustainable growth can be achieved with temporary taxes/subsidies that redirect innovation toward clean inputs" (p. 131). The aggregate production function with elasticity of substitution, ε,[25] is given by (p. 135)

$$Y_t = \left(Y_{ct}^{(\varepsilon-1)/\varepsilon} + Y_{dt}^{(\varepsilon-1)/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}.$$

One of the constraints on the growth model in Acemoglu et al. (2012, p. 137) is the evolution of the quality of the environment, S_t, as shown by:

$$S_{t+1} = -\xi Y_{dt} + (1+\delta) S_t,$$

where ξ denotes the rate of "environmental degradation" and δ stands for the rate of "environmental regeneration." Their findings suggest that when two inputs are substitutes with high enough elasticity "immediate switch of R&D resources to clean technology, followed by a gradual switch of all production to clean inputs" is needed (p. 159).

[24]In Acemoglu et al. (2012), production with dirty input uses non-renewable resource.
[25]Acemoglu et al. (2012) assume that $\varepsilon > 1$ which implies substitutes of two sectors.

1.5.7 Public capital

The model can be extended by adding another constraint: evolution of public capital. Semmler et al. (2007) study a relationship between the composition of public expenditure and economic growth. Their model has private physical capital, human capital and public capital. Public capital includes public infrastructure, and health and education related facilities.

They use the following Cobb-Douglas production function for the market goods with k as private capital, h is human capital, g is public capital and only a part of it goes to private production of the market $(v_1 g)$[26]

$$Ak^{\alpha}(u_1 h)^{\beta}(\nu_1 g)^{\gamma}.$$

In Semmler et al. (2007, p. 7), evolution of public capital is shown as:

$$\dot{g} = i_p^f + \alpha_1 e_p - (\delta_g + n)g,$$

where i_p^f denotes foreign aid, α_1 is a part of public resources used for new public capital, e_p is resources absorbed by the public sector, and δ_g is a depreciation of public capital, and n is the growth rate of population.

1.5.8 Sovereign debt

Blanchard and Fischer (1989) point out that the government can be financed not only by tax receipts, but also through borrowing from private sector. As an additional constraint, sovereign debt dynamics can be imposed on the growth.

The dynamic budget constraint is presented in Blanchard and Fischer (1989, p. 54) as follows:

$$\frac{db_t}{dt} + nb_t = g_t - \tau_t + r_t b_t,$$

where db_t/dt is an increase in per capita government debt b_t, nb_t is the debt amount with population growth n, and the right hand side of the equation shows the "excess of government outlays" with government purchases, g_t, and interest payment, $r_t b_t$, over tax receipts, τ_t.

[26]See Semmler et al. (2007, p. 4).

Greiner et al. (2007, p. 3) use the public debt accumulation with real public debt B_t ,

$$\dot{B}(t) = B_t r_t - S_t = rB_t + G_t - T_t,$$

where r_t stands for real interest rate and S_t represents real government surplus, T_t denotes tax revenue, and G_t is public spending excluding interest payments.

Greiner et al. (2005, p. 165) show the breakdown of the per capita budget constraint of the government in the following way:

$$\dot{B} = r_2 B + C_p + T_p + I_p - T - nB,$$

where r_2 denotes the return to government bond or public debt of B, C_p is public consumption, T_p are transfers, I_p is public investment, T stands for tax revenue that equals $T = \tau(w + r_1 K + r_2 B)$ with income tax rate, τ, wage rate, w, the return to physical capital,r_1, and labor supply grows at the rate of n.

2 Macroeconomic performance

Resource booms, in general, can contribute to the economic development of the resource abundant countries by raising a welfare of the country. Cross-country comparison shows that the mining industry has contributed to a high growth of certain resource-rich economies, for example in Latin America since the 1990s, with increased foreign investment in mining and security of mining investments. Wright and Czelusta (2007) discuss the economic growth of Latin America particularly in Chile and Brazil (Exploration in South America, 2001). Although some of the mineral resource exporting countries have succeeded, a number of studies indicate that some resource-rich countries have worse growth performance compared to those without, mainly due to side effects. In contrast to resource-rich countries, a number of resource-poor countries, such as Singapore and South Korea, have promoted exports of manufactured goods and achieved rapid economic growth. Certain resource-rich countries have experienced adjustment problems including Dutch disease and accumulation of external debt.

2.1 Natural resource and growth

Economists have been debating the relation of resource scarcity and economic growth for many years. Barnett and Morse (1963) presented the first comprehensive theoretical and empirical analysis of the impact of resource scarcity on growth. Their comprehensive study presented three historical models: The Utopian model with no resource scarcity, Malthusian scarcity with fixed resource supply, and Ricardian scarcity with declining quality, where the scarcity has a negative impact on economic growth. Following John Stuart Mill's argument that Ricardian model was more significant than the other two models, Barnett and Morse (1963) consider and extend the Ricardian model in their study. They state that in a Ricardian model "resources are readily available, but only under conditions of declining economic quality" and summarize the Ricardian economic growth model in the following way:[27]

1. There is "an endogenous, scale-induced decline in the economic quality of freely available resource.

[27]The model is discussed in detail in Barnett and Morse (1963, pp. 107-115).

2. The parameters are strictly invariant.

3. The social production function has constant returns to scale.

The important characteristics of the resource-abundant country are the large export of natural resources. Based on deteriorating terms of trade in 1950's, economists were concerned about the growth of economies rich in natural resources (Prebisch, 1950 and 1964; Singer, 1950). Prebisch (1950) in his study of Latin America along with Singer (1950) argue that developing countries underperformed compared to industrialized countries mainly due to specialization in primary commodities, slower technological progress in this sector and deteriorating commodity terms of trade. A number of recent studies show a negative relationship between natural resource abundance and economic performance. These studies include Auty (1990, 1993, 2001), Sachs and Warner (1995, 1999, 2001), and Smith (2004).

Sachs and Warner (1995) show a negative relationship between natural resource abundance and economic growth during the period 1970-1990. Furthermore, in extension of their research, Sachs and Warner (2001) show that resource-abundant countries tended to have higher price levels. In addition, because of uncompetitive export sector resource-rich economies failed to pursue export-led growth. They state that "almost without exception, the resource-abundant countries have stagnated in economic growth since the early 1970s, inspiring the term 'curse of natural resources'. Empirical studies have shown that this curse is a reasonably solid fact." (Sachs and Warner, 2001, p. 837).

In most of above mentioned studies, growth rate is taken as per capita income, and resource abundance is measured as the ratio of primary product exports to GDP or the share of primary products in total exports. There also exist studies that propose measures of resource endowments other than the export ratios. Consistent with the Heckscher-Ohlin framework, Leamer (1984) suggests to use net exports of resources per worker as a proxy. Maloney (2002), Lederman and Maloney (2003) use Leamer's measure of resource abundance in their analysis.

Maloney (2002, p. 1) argues that slow growth of resource-rich Latin America was due to barriers to technological adoption and innovation. Lederman and Maloney (2003) emphasize the role of trade in growth of resource-rich economies. They show how a natural resource-abundant economy can have a higher growth. Using data from 1975 to 1999, it is shown that the estimated coefficient on the Leamer index is strongly significant and positive in panel estimation, but insignificant in cross section regressions. By examining data on energy and mineral reserves, Stijins (2005) points out that natural resource abundance was not a main contributor to the economic growth in the period of 1970-1989. He argues that natural resource effect on growth seem to take both positive and negative channels.

Manzano and Rigobon (2001) argue that in studies such as by Sachs and Warner (1995, 2001), where a negative relationship between resource abundance and growth is presented, there are two econometric problems: "First, the result might depend on factors that are correlated with primary exports but that have been excluded from the regression. Second, total GDP includes the production in the resource sector that has been declining in the last 30 years."[28] Their estimations use panel data and alternative measures such as the "GDP net of resource exports" to solve these problems. According to their findings, negative relationship is present only in cross-section data. Manzano and Rigobon (2001) argue that low growth of the resource-rich economy might be due to debt overhang.

2.2 Resource depletion

A number of early studies, including Hotelling (1931) and Barnett and Morse (1963), were conducted on long-run availability of mineral resource. The basic Hotelling rule indicates that net price should increase at the rate of interest. This theoretical framework is used by economists to model the supply and long-run price of the non-renewable resources (Livernois, 2009).

Barnett and Morse (1963) emphasize the distinction between economic scarcity and physical availability of resources. They assess historical trends of scarcity of natural resources for the United States from 1870 to 1957 and present the empirical evidence of decreasing scarcity[29] for most of the resources. Barnett and Morse (1963, p. 199) state that "the trend in the unit

[28] This is stated in Manzano and Rigobon (2001, p. 2).

[29] In the empirical test by Barnett and Morse (1963), the "trend of real cost of extractive output" is used for measuring the scarcity of natural resources.

cost of extractive goods as a whole has been down – not up." Their data on labor-capital cost per unit of output for minerals decreased over time from average of 211 between 1870-1900 to 47 in 1957.

Barnett and Morse (1963) point out that the technological progress can offset an increasing extraction cost that occurred due to an increase in demand and depletion. An extension of this study by Barnett (1979) that considers additional countries and updated data until 1970 yields similar results of decreasing trend in labor per unit extracted. Similarly, other studies also suggest that technological progress is crucial in overcoming resource scarcity and can ensure efficient management of natural resources (Krautkraemer, 1998; Tilton, 2003).

Krautkraemer (1998, 6) points out that "technological progress has ameliorated the scarcity of natural resource commodities, but resource amenities have become more scarce, and it is unlikely that technology alone can remedy that." He points out the distinction between natural resource commodities and resource amenities. While natural resource commodities are used to produce material goods and services, resource amenities include other goods and services, such as the basic life support systems of the earth, the climate, the sinks, and the ecosystems, provided by the natural environment (Krautkraemer, 1998, 9).

As production of exhaustible resources rises at a high speed, it will ultimately reach its peak point and will start falling until the resources are depleted if necessary measures are not taken. According to the Peak-Oil theory by Hubbert (1956), oil production in the United States has already reached its peak in 1960s and started declining ever since. There are more recent studies conducted on the peak of the oil including the Organization for the Study of Peak Oil (ASPO), Campbell and Laherrere (1998), Schindler and Zittel (2008). Schindler and Zittel (2008) show that oil production of major oil producing economies has reached its maximum level and in 2006 world oil production reached its peak.[30]

Production of mineral resources in particular fossil fuel such as oil, natural gas and coal has been rising over time. Thus reserves-to-production (R/P)[31]

[30]See Schindler and Zittel (2008, p. 12).

[31]As defined in the BP Statistical Review (2012, p. 6), reserves-to-production shows the length of time that those remaining reserves would last if production were to continue

ratios of fossil fuel show a decreasing trend. According to the BP Statistical Review (2012), world oil reserves at the end of 2011 reached 1652.6 billion barrels that were sufficient to meet 54.2 years of global production. Oil production as well as oil consumption has been growing in most regions since early 1960s. While major producers of oil are Middle East, Europe and Eurasia and North America, the majority of consumption in recent years account for Asia Pacific followed by North America and Europe and Eurasia (BP Statistical Review, 2012).

Due to the recovery of the global economy after the recent Global Financial Crisis in 2008-2010, demand in mineral products has been increasing substantially in the past several years. As a result, prices of mineral resources on the world commodity market have increased which have led to the growth of mining industry production and its exports. According to the BP Statistical Review (2012), coal is the fastest growing source of fossil fuel and its share of global energy consumption in 2011 reached 30.3 per cent which is the highest share since 1969.

According to the BP Statistical Review (2012), world proved coal reserves almost reached 861 billion tons in 2011, which were sufficient to provide 112 years of global production of coal. Coal has the largest R/P ratio compared to other fossil fuels. However, the R/P ratio has been declining over time and it fell from 2000 R/P of 210 years due to a rising production. In Asia pacific region, particularly in China, coal production and consumption have risen drastically accounting for almost half of the world's coal.

2.3 Prices of natural resources

There is a wide range of literature on statistical trends in the long run prices of resources. While some empirical studies demonstrate a rising trend of the prices of resources, other studies present a declining trend, or even a U-shaped pattern of the price. Empirical findings of Barnett and Morse (1963) and Barnett (1979) show a relative decline in mineral prices. Other studies, for example by Sullivan et al. (2000), claim a downward trend of resource prices over the twentieth century.

Economists use extraction cost, price and user cost to measure resource

at that rate.

scarcity. The advantage of the price is that they reflect the demand for the resources (Brown and Field 1978; Fisher 1979). For non-renewable resources, prices tend to rise as the scarcity increases. Early studies by Hotelling (1931) and Solow (1974) suggest an increasing trend of prices. However, some of the studies such as by Barnett and Morse (1963) did not show a rising trend of the mineral prices over time. Later on, by using data between 1900 and 1973, Smith (1979) did not find any statistically stable trend in mineral price index. However, a number of studies, such as by Slade (1982), Pindyck (1978), Livernois and Uhler (1987), Swierzbinski and Mendelshohn (1989), suggest a U-shaped mineral price trend.

As mentioned earlier, Hotelling (1931) rule indicates that price of oil increases at the rate of interest rate that is used in discounting the profit. However, Hotelling model is very basic and simple. As noted in Livernois (2009, p. 23), Hotelling states that his basic model fails to take into account an extraction cost of resources. If an extended Hotelling model includes extraction costs, then the net price of the resource would rise at the interest rate. In addition, the basic Hotelling model assumes a perfectly competitive market. Thus, further modification of the model considers imperfect competition, in particular a monopolistic market structure where a monopolist resource owner can control the price of the resource at least to a certain extent. Extensions of the Hotelling model that consider factors such as a backstop technology, durability of the mineral, and recycling are shown in Krautkraemer (1998) and Gaudet (2007) (Livernois, 2009, 24).

Livernois (2009, 23) points out that the most important empirical significance of the Hotelling rule is that "market price must rise over time in real terms, provided that costs are time-invariant." His empirical evidence suggests that scarcity rent might be the least significant factor that influences the price evolution. In addition to the Hotelling rule, other determinants such as technological progress and market structure might have a more significant role in price changes. Livernois (2009, p. 37) points out that "if the Hotelling Rule is only one among many supply-side factors that influences price, all kinds of price paths are possible."

In the 1970s, a scarcity of natural resource attracted attention because of events like the Arab embargo and oil price increases from oil exporting countries (Neary and Van Wijnbergen, 1986). In addition, report of the Club of Rome "Limits to Growth", which stressed the scarcity of minerals and arable land as constraint to growth, was published (Meadows et al., 1972).

Frankel (2011, pp. 6-7) compares different studies on price trends of the resources and points out that upward or downward price trends seems possibly connected with "the date of the end of the sample." He summarizes that "studies written after the commodity price increases of the 1970s found an upward trend, but those written after the 1980s found a downward trend."[32] Frankel (2011, 6-7) lists studies such as Cuddington (1992), Cuddington et al. (2007), Cuddington and Urzua (1989), Grilli and Yang (1988), Pindyck (1999), Hadass and Williamson (2003), Reinhart and Wickham (1994), Kellard and Wohar (2006), Balagtas and Holt (2009) and Harvey et al. (2010).

Prices of non-renewable resources are highly volatile compared to renewable resources such as agricultural products. Greiner et al. (2012b) using dynamic programming and Nyambuu and Semmler (2014) using NMPC solution methods show that in a model of optimal control with a monopolistic resource owner the price will be monotonically rising if the initially known stock is large and does not need to be discovered.

In the above model, the total stock of non-renewable resources consists of known and unknown resources. As the known resource could not increase, the extraction rate would decline. However, the optimal extraction rate has an inverted-U shape when the initial stock is small. Since initially known stock of resources is small, the extraction rate will rise due to a further discovery of resources, and then will decline. In this case, the price of exhaustible resources first declines and then rapidly increases indicating a U-shaped path because the resource will be depleted eventually (Greiner et al., 2012b; Nyambuu and Semmler, 2014).

Pindyck's (1978) findings in show a U-shaped price evolution in the case of the small initial stock of the reserve. Pindyck (1978, p. 841) notes that "at first production will increase as reserves are developed, and later production will decline as both exploratory activity and the discovery rate fall." The U-shaped price movement is also found in the optimal exploration and

[32] See Frankel (2011, pp. 6-7).

extraction model by Liu and Sutinen (1982) where "net benefits of extraction increase with an increase in the known resource stock and exploration costs increase with an increase in cumulative exploration."[33]

Slade's (1982) empirical study of long-run price movements examines data for the period of 1870-1978 for major metals and fuels of twelve commodities except gold. Relative price[34] is used as a proxy of resource scarcity. Her model with exogenous technical change and endogenous change in the grade of ores mined suggests a U-shaped pattern for relative prices. She incorporates cost-increasing degradation effects and cost-decreasing technological progress in the Hotelling rule. Slade (1982) points out that mineral prices can decline initially but eventually prices increase because the effects of cost increase and price increasing effect of the Hotelling rule dominate the effects of cost-decreasing technological progress (Livernois, 2009, p. 24). Slade (1982, p. 136) concludes that these results are consistent with findings of Smith (1979) where linear-trend coefficient is unstable over time for all aggregate prices including mineral sector. Empirical data indicate an upward trend of prices of exhaustible resources in the long run. Energy Information Administration Outlook releases the projection of fossil fuel prices up to 2040: this shows an upward price trend of all types of non-renewable energy. Specifically, crude oil spot prices are predicted to grow faster in the long run.

2.4 Dutch disease

Mineral resource discoveries can lead to commodity export booms that can have short-run monetary effects such as balance of payment surpluses and accumulation of international reserves in the Central Bank. Edwards (1986) presents a model of a resource-rich developing country and shows that commodity export booms result in short-run increase in money creation, inflation and a real appreciation.

Corden and Neary (1982) examine a resource boom in natural gas and oil and assess a decline in the traditional manufacturing sector. Their is a simple model of a small open economy; it shows a fall in manufacturing output and

[33]See Liu and Sutinen (1982, 159-160).

[34]Relative price is defined as the ratio of an extractive-industry price index to an overall price index (Slade, 1982, 123).

employment, deteriorating balance of trade, and a fall in the real return to factors concerned with manufacturing. In addition, they demonstrate a real appreciation[35] caused by the boom. In their analysis, they emphasize the importance of the distinction between resource movement effects and the spending effects of the boom in the study of the Dutch disease. These effects are explained as follows:

"The boom in the energy sector raises the marginal products of the mobile factors employed there and so draws resources out of other sectors, giving rise to various adjustments in the rest of the economy, one mechanism of adjustment being the real exchange rate. This is the resource movement effect. If the energy sector uses relatively few resources that can be drawn from elsewhere in the economy this effect is negligible and the major impact of the boom comes instead through spending effect. The higher real income resulting from the boom leads to extra spending on services which raises their price (i.e. causes a real appreciation) and thus leads to further adjustments" (see Corden and Neary, 1982, p. 827)

In the analysis of British and Australian resource booms, Forsyth (1986) shows structural changes, Dutch disease effects in particular. While Australia had a substantial resource movement effect, the resource boom in Britain was characterized by a large spending effect. Primary production has been increasing mainly due to the Common Agricultural Policy. The manufacturing sector has declined dramatically since 1979. The service sector showed significant growth. Thus, British economy seems consistent with Dutch disease effects (Forsyth, 1986). Frankel (2011, p. 12) lists the following adverse effects of the export commodity price boom that reflects Dutch disease:

- A large real appreciation in the currency.
- An increase in government spending.
- An increase in the price of non-traded goods, relative to traded goods.
- A shift of labor and land out of non-export-commodity traded goods.
- An increase in external debt.

[35] Real appreciation is calculated as an increase in the relative price of non-traded relative to traded goods (Corden and Neary, 1982).

2.5 External debt

Resource boom results in rapid increase in public spending and domestic money supply. In order to finance spending and consumption, governments of the mineral-exporting countries often borrow extensively from international capital markets. For example, when Mexico discovered huge reserves of oil in 1977, the government expenditures were increased significantly. According to Buffie and Krause (1989), Mexico's total debt increased from USD 27.9 billion in 1977 to USD 87.6 billion in 1982 mainly due to lack of sufficient savings. Harberger et al. (1985) note that the purpose of the large external debt in some of the Latin American countries, such as Mexico, was to fund consumption.

In comparison to some of the oil-exporting countries, Indonesia's external debt did not rise much in the late 1970s. In this way, Indonesia could overcome the debt crisis of early 1980s (Warr, 1986). Frankel (2011, p. 17) suggests that debt crises of Mexico in 1982 and Indonesia, Russia and Ecuador in 1998 with debt-service problem, could be avoided "if their debts had been indexed to the oil price."

As stated by Manzano and Rigobon (2001), in the 1970s, resource-rich countries were encouraged to borrow against their mineral reserves because of high commodity prices. However, in 1980's when the prices dropped these countries experienced debt crisis because they were unable to continue borrowing and had to repay their debts. Manzano and Rigobon (2001) claim that poor performance of the resource-abundant economies can be associated with debt overhang.

3 Decision making and strategic dependence

If economic growth is supposed to be sustainable, the depletion of natural resources and the environmental effects of economic activities need to be taken care of. Policies have to be developed that conserve resources and preserve the environment such as natural resources, soil, air, water, climate, and biodiversity. Yet, what we have not sufficiently considered so far is decision making and policy options, for example environmental or carbon tax, in a more complicated setting where one can observe that there is strategic dependence between the decisions of agents. In this context, decision making is mostly studied in some game theoretical set ups. Here then, in strategic decision making, complicated issues arise concerning incentive compatible decision and policy options on the micro as well macro level. We first study prototype examples of strategic dependence which have been worked out first on the firm level with respect to Cournot and Bertrand games. So we start with static games. Then we introduce again models with growth and diverse set ups for strategic decision making and discuss further policy options.

3.1 Strategic dependence

3.1.1 Static case

Dixit (1986) constructs the basic principles of a quantity-setting oligopoly, depicting the reactions and conjectures of oligopolists in a static simultaneous-move game. This model has two particular merits. First, by specifying the conjectures appropriately, it integrates many different and familiar models of oligopolistic behavior into the same formal framework. These models include Cournot, Bertrand, and the case with consistent or rational conjectures. Second, the conjectural variations, treated as parameters, allow us to capture the idea of varying degrees of competition. The basic principles of this model are the following.

Let there be a set I of strategically interlinked agents in the economy. Let y_i be the output of agent $i \in I$ and

$$y_{-i} \doteq \{y_j | j \neq i\} = \{y_j | I \setminus i\}$$

be the vector of the outputs of all the other agents $j \neq i$. Agent $i \in I$ faces the revenue function which is homogeneous of degree one:

$$r^i(y_i, y_{-i}) \text{ with } r^i_j \doteq \frac{\partial r^i}{\partial y_j} \text{ and } r^i_j\big|_{y_\kappa = y\, \forall \kappa \in [0,1]} = 1 \text{ for } j \in I. \tag{15}$$

When all agents $j \in I$ increase their outputs y_j in the same proportion, then also their revenues r^j increase in the same proportion.

Dixit's (1986) main idea is to introduce the following assumption:

Assumption 1 *Agent i believes that another agent $j \neq i$ follows its choice of output y_i according to the conjectural variation relation*

$$\frac{y_i}{y_j}\frac{dy_j}{dy_i} = v(y_i, y_{-i}, \beta) \text{ for } j \neq i, \tag{16}$$

where the function $v(y_i, y_{-i}, \beta)$ is homogeneous of degree zero with respect to outputs (y_i, y_{-i}) and β is the expectations parameter.

Note that the cross elasticity (16) is symmetric for all agents i. The homogeneity of the function v means that when all agents $j \in I$ increase their outputs y_j in the same proportion, then the anticipated elasticity of the output y_j of any other agent $j \neq i$ with respect to the output of agent i remains constant.

Using the conjectural variation relation (16) when differentiating the revenue function (15), one obtain the perceived marginal revenue for agent i:

$$\begin{aligned} m^i(y_i, y_{-i}, \beta) &= r^i_i(y_i, y_{-i}) + \sum_{j \neq i} r^i_j(y_i, y_{-i}) \frac{dy_j}{dy_i} = r^i_i + \sum_j r^i_j v(y_i, y_{-i}, \beta) \frac{y_j}{y_i} \\ &= r^i_i(y_i, y_{-i}) \left[1 + \frac{v(y_i, y_{-i}, \beta)}{r^i_i(y_i, y_{-i}) y_i} \sum_{j \neq i} r^i_j(y_i, y_{-i}) y_j \right]. \end{aligned} \tag{17}$$

Because the function (15) is homogeneous of degree one, then its partial derivative r^i_j is homogeneous of degree zero. Noting this, (16) and (17), one obtains the following result which will be useful later on:

Proposition 1 *The perceived marginal revenue function $m^i(y_i, y_{-i})$ is homogeneous of degree zero.*

The cross elasticity (16) characterizes conjectural variation. There is perfect competition for $v = 0$ and full collusion for $v = 1$. In cases $v \in (0,1)$, the parameter v characterizes an agent's subjective probability of the possibility that the others $j \neq i$ will increase their output y_j in response to the an increase in the agent's own output y_i. The agents mimic Bertrand behavior (19) for negative values $v < 0$: when any of them plans to increase its output, it expects the others to decrease their output in order to prevent their output prices from falling due to higher supply.

A special case of the conjectural variation is $v(y_j, y_{-j}, \beta) \equiv \beta$, where β is a public policy instrument. If, for instance, the agents are oligopolists, then the probability of their collaboration is decreased by anti-trust policy β.

3.1.2 Example: oligopolistic competition

The conjectural variation model can be applied to oligopolistic competition as follows. Let us specify the revenue of agent i as follows:

$$r^i(y_i, y_{-i}) \doteq p^i(y_i, y_{-i})y_i - c^i(y_i),$$

where

$$p^i(y_i, y_{-i}) \text{ with } p^i_j \doteq \partial r^i/\partial y_j \text{ for } j \in I$$

is the inverse demand function and $c(y_i)$ the cost function for agent i. The model integrates the following cases into a unified framework:

Cournot: Agent j conjectures that the other agents $j \neq i$ hold their output y_j constant as it changes its output y_i:

$$v(y_i, y_{-i}, \beta) \equiv 0, \quad m^i = p^i + p^i_i y_i. \tag{18}$$

Competitive: Agent i conjectures that its own price p^i will remain unchanged as it changes its output y_i. Differentiating

$$p^i(y_i, y_{-i}) = \text{constant}$$

totally and noting (15) yield

$$0 = p^i_i + \sum_{j \neq i} p^i_j \frac{dy_j}{dy_i} = p^i_i + v \sum_{j \neq i} p^i_j \frac{y_j}{y_i} = p^i_i \left[1 + \frac{v(y_i, y_{-i}, \beta)}{p^i_i y_i} \sum_{j \neq i} p^i_j y_j \right]$$

and the expectations parameter β adjusts so that

$$v(y_i, y_{-i}, \beta) = -\frac{p_i^i y_i}{\sum_{j \neq i} p_j^i y_j}, \quad m^i = p^i, \tag{19}$$

holds true in the equilibrium.

Bertrand: Agent i conjectures that the other agents $j \neq i$ hold their prices R_j constant as it changes its output y_i. Differentiating

$$p^j(y_i, y_{-i}) = p^j(y_j, y_{-j}) = \text{constant}$$

totally and noting (15) yield

$$0 = p_i^j + \sum_{\kappa \neq i} p_\kappa^j \frac{dy_\kappa}{dy_i} = p_i^j + v \sum_{\kappa \neq i} p_\kappa^j \frac{y_\kappa}{y_i} = p_i^j \left[1 + \frac{v(y_i, y_{-i}, \beta)}{p_i^i y_i} \sum_{\kappa \neq i} p_\kappa^j y_\kappa \right]$$

and the expectations parameter β adjusts so that

$$v(y_i, y_{-i}, \beta) = -\frac{p_i^i y_i}{\sum_{\kappa \neq i} p_\kappa^j y_\kappa},$$

holds true in the equilibrium.

Collusion: Agent j conjectures that the other agents $j \neq i$ change their output y_j in proportion to its output y_i:

$$v(y_i, y_{-i}, \beta) \equiv 1, \quad m^j = p^i + \sum_{j=1}^{I} p_j^i y_j.$$

The comparison of this result to the Cournot case (18) reveals that the agents behave in this case as if they were a single monopoly producing all goods $i \in I$.

3.2 Dirty investment

The simplest way of introducing economic growth is to assume that output is produced from reproducible resources according to constant returns to scale. The common way of doing this is the following (cf. Barro and Sala-i-Martin 1995, section 1.3.1). Define labor as human capital that accumulates with education. Aggregating the two reproducible resources, human and physical capital, into capital in general creates the AK production function where output is produced in fixed proportion to capital.

Assume that agent i makes its output y_i in fixed proportion to capital k_i:

$$y_i = k_i. \tag{20}$$

Agent $i \in I$ accumulates capital k_i by saving that is the difference of its revenue r^i and consumption c_i:

$$\dot{k}_i \doteq \frac{dk_i}{dt} = r^i(y_i, y_{-i}) - c_i = r^i(k_i, k_{-i}) - c_i, \tag{21}$$

where t is time. Define the vector of the capital stocks of all the other agents $j \neq i$ relative to the capital stock of agent i as follows:

$$\frac{k_{-i}}{k_i} = \left\{ \frac{k_j}{k_i} \middle| j \neq i \right\}. \tag{22}$$

Assume $v(y_i, y_{-i}, \beta) \equiv \beta$, for simplicity. In that case, the function (17) becomes

$$m^i(y_i, y_{-i}, \beta) = r^i_i(y_i, y_{-i}) \left[1 + \frac{\beta}{r^i_i(y_i, y_{-i}) y_i} \sum_{j \neq i} r^i_j(y_i, y_{-i}) y_j \right],$$

$$\frac{\partial m^i}{\partial \beta} = \frac{1}{y_i} \sum_{j \neq i} r^i_j(y_i, y_{-i}) y_j > 0. \tag{23}$$

The result $\partial m^i / \partial \beta > 0$ means that an increase in the intensity of competition (i.e. the decrease of β) lowers the marginal revenue m^i for all agents $i \in I$.

We assume that pollution is a by-product of capital accumulation. With the awareness of environmental quality, individual preferences depend on consumption c_i and environmental quality ϕ. To obtain an explicit solution, let us specify that function in the exponential form as follows:

$$U(c_i, \phi) = \phi^\nu c_i^{1-\theta} / (1 - \theta), \quad \theta \in (0, 1) \cup (1, \infty), \quad \nu > 0, \tag{24}$$

where the parameter ν is the weight of environmental quality ϕ and θ the inter-temporal elasticity of substitution in consumption. Furthermore, assume that environmental quality ϕ is determined by

$$\phi = e^{\epsilon t} / \Phi(k_i, k_{-i}), \quad 0 < \epsilon < 1, \quad \Phi_j \doteq \frac{\partial \Phi}{\partial k_j} > 0, \quad \Phi^i_j \big|_{k_\kappa = k \, \forall \kappa \in [0,1]} = 1,$$

$$\Phi \text{ linearly homogeneous}, \tag{25}$$

where ϵ is a constant. This relation is equivalent to

$$\frac{\dot{\phi}}{\phi} = \frac{\log \phi}{dt} = \epsilon - \frac{1}{\Phi}\int_{i\in I}\frac{\partial \Phi}{\partial k_j}\dot{k}_j di = \epsilon - \frac{1}{\Phi}\int_{i\in I}\Phi_j \dot{k}_j di, \tag{26}$$

where environmental quality ϕ improves with time, but deteriorates with any investment $\dot{k}_j$ in "dirty" capital for $j \in I$. If there is no investment, $\dot{k}_j = 0$ for all j, then the nature improves environmental quality at the rate ϵ.

Let ρ be the constant rate of time preference. Given (24), the intertemporal utility of agent i becomes

$$\int_0^\infty U(c_i,\phi)e^{-\rho t}dt = \int_0^\infty \frac{\phi^\nu c_i^{1-\theta}}{1-\theta}e^{-\rho t}dt = \int_0^\infty \frac{c_i^{1-\theta}}{1-\theta}\frac{e^{-\delta t}}{\Phi(k_i,k_{-i})^\nu}dt, \tag{27}$$

where $\delta \doteq \rho - \nu\epsilon$ is a constant. Agent i maximizes its intertemporal utility (27) by its consumption c_i subject to production (20) and capital accumulation (21), given the capital stocks of the others', k_{-j}. The Hamiltonian of this maximization is

$$H_i = c_i^{1-\theta}\Phi(k_i,k_{-i})^{-\nu}/(1-\theta) + \mu_i[r^i(k_i,k_{-i}) - c_i], \tag{28}$$

where the shadow price μ_i for capital k_i evolves according to [cf. (17) and (20)]

$$\begin{aligned}&\dot{\mu}_i = \frac{d\mu_i}{dt} = \delta\mu_i - \frac{\partial H_i}{\partial k_i} = [\delta - m^i(k_i,k_{-i},\beta)]\mu_i + \frac{\nu c_i^{1-\theta}}{1-\theta}\frac{\Phi_i(k_i,k_{-i})}{\Phi(k_i,k_{-i})^{\nu+1}},\\ &\lim_{t\to\infty}\mu_i k_i e^{-\delta t} = 0.\end{aligned} \tag{29}$$

The first-order condition for this maximization is

$$\frac{\partial H_i}{\partial c_i} = \frac{c_i^{-\theta}}{\Phi(k_i,k_{-i})^\nu} - \mu_i = 0.$$

Given (29), this is equivalent to the Euler equation

$$\begin{aligned}\frac{\dot{c}_i}{c_i} &= -\frac{1}{\theta}\left[\frac{\dot{\mu}_i}{\mu_i} + \nu\frac{\Phi_i(k_i,k_{-i})}{\Phi(k_i,k_{-i})}\right] = \frac{1}{\theta}\left[m^i(k_i,k_{-i},\beta) - \delta - \frac{1}{\mu_i}\frac{\nu c_i^{1-\theta}}{1-\theta}\frac{\Phi_i(k_i,k_{-i})}{\Phi(k_i,k_{-i})^{\nu+1}}\right]\\ &= \frac{1}{\theta}\left[m^i(k_i,k_{-i},\beta) - \delta - \frac{\nu c_i}{1-\theta}\frac{\Phi_i(k_i,k_{-i})}{\Phi(k_i,k_{-i})}\right].\end{aligned} \tag{30}$$

Because the revenue function r^i and the environmental constraint Φ are homogeneous of degree one and the perceives marginal revenue function m^i

homogenous of degree zero with respect to (k_i, k_{-i}), then the partial derivative Φ_i is homogeneous of degree zero with respect to (k_i, k_{-i}). In the stationary state, consumption c_i and capital k_i grow at the same rate. From (21), (22) and (30) it then follows that

$$\begin{aligned}
&\frac{1}{\theta}\left[m^i\left(1, \frac{k_{-i}}{k_i}, \beta\right) - \delta - \frac{\nu}{1-\theta}\frac{\Phi_i(1, k_{-i}/k_i)}{\Phi(1, k_{-i}/k_i)}\frac{c_i}{k_i}\right] \\
&= \frac{1}{\theta}\left[m^i(k_i, k_{-i}, \beta) - \delta - \frac{\nu c_i}{1-\theta}\frac{\Phi_i(k_i, k_{-i})}{\Phi(k_i, k_{-i})}\right] = \frac{\dot{c}_i}{c_i} = \frac{\dot{k}_i}{k_i} = r^i\left(1, \frac{k_{-i}}{k_i}\right) - \frac{c_i}{k_i}.
\end{aligned}$$

Solving for the propensity to consume $c_i/y_i = c_i/k_i$ and noting (23), one obtains

$$\begin{aligned}
&\frac{c_i}{k_i} = \left\{r^i\left(1, \frac{k_{-i}}{k_i}\right) + \frac{1}{\theta}\left[\delta - m^i\left(1, \frac{k_{-i}}{k_i}, \beta\right)\right]\right\}\left[1 - \frac{\nu/\theta}{1-\theta}\frac{\Phi_i(1, k_{-i}/k_i)}{\Phi(1, k_{-i}/k_i)}\right]^{-1} \\
&\text{with } \frac{\partial}{\partial\beta}\left(\frac{c_i}{k_i}\right) = -\frac{1}{\theta}\frac{\partial m^i}{\partial\beta}\left(1 - \frac{\nu/\theta}{1-\theta}\frac{\Phi_i}{\Phi}\right)^{-1} \quad \text{for } i \in I.
\end{aligned}$$

This shows that there exists a stationary state in which all industries $i \in I$ grow at the same rate g: $g = \dot{k}_j/k_j$ for $j \in I$. In other words:

Proposition 2 *For given initial capital stocks, $k_i(0)$ for $i \in I$, there exists a stationary state where capital k_i, output y_i and revenue r^i grow at the same rate for all agents $i \in I$.*

In this stationary state, given (25) and (26), the function Φ grow at the same rate g as all capital stocks k_i, and environmental quality evolves according to

$$\frac{\dot{\phi}}{\phi} = \epsilon - \frac{1}{\Phi}\int_{i\in I}\Phi_j \dot{k}_j di = \epsilon - \frac{g}{\Phi}\int_{i\in I}\Phi_j k_j di = \epsilon - g.$$

Proposition 3 *If the natural abatement rate ϵ is higher (lower) than the growth rate g, then environment improves $\dot{\phi} > 0$ (deteriorates $\dot{\phi} < 0$).*

The analysis in this section shows that the AK model approach creates a well-functioning model to analyze sustainable development. Its usefulness depends on the justification of the assumption that output is produced from reproducible resources according to constant returns to scale.

3.3 Pigouvian taxation

The model of this subsection is a modification of the model in Palokangas (2008). Let there be a fixed number n of similar agents. Agent $i \in I \doteq \{1, ..., n\}$ possesses a fixed amount L of labor, of which the amount l_i is used in production and the rest z_i in R&D:

$$L = l_i + z_i. \tag{31}$$

Emissions m_i are in fixed proportion to inputs used in production, l_i, for each agent j. By a proper choice of units, the emissions of agent i, m_i, and total emissions m can be written as follows:

$$m_i = l_i, \quad m \doteq \sum_{i=1}^{n} m_i = \sum_{i=1}^{n} l_i. \tag{32}$$

Each agent $i \in \{1, ..., n\}$ produces a different good and competitive firms produce a consumption good from all these goods according to

$$\sum_{i=1}^{n} c_i = y = \prod_{i=1}^{n} y_i^{1/n}, \tag{33}$$

where c_i is the consumption of agent j, y_i the output of agent j, and y total consumption in the union. With some complication, the same results can be generalized for any neoclassical production function with constant returns to scale. Let p_i be the price for good j. With Cobb-Douglas technology (33), the consumption price p is obtained by minimizing the unit cost

$$\frac{1}{y}\sum_{i=1}^{n} p_i y_i = \sum_{i=1}^{n} p_i \frac{y_i}{y}$$

of the consumption good by the input-output ratios $(y_1/y, ..., y_n/y)$:

$$p = \min_{y_1/y, ..., y_n/y} \left\{ \sum_{i=1}^{n} p_i \frac{y_i}{y} \middle| \prod_{i=1}^{n} \left(\frac{y_i}{y}\right)^{1/n} = 1 \right\} = \prod_{i=1}^{n} p_j^{1/n}.$$

Because in the model there is no money that would pin down the nominal price level at any time, the consumption price p can be normalized at unity:

$$1 = p = \prod_{i=1}^{n} p_i^{1/n}. \tag{34}$$

Let t be time. It is assumed that total emissions m degrade, but the nature improves environmental quality ϕ:

$$\dot{\phi} \doteq \frac{d\phi}{dt} = h\phi - m. \tag{35}$$

3.3.1 The agents

Assume that all agents $i \in \{1, ..., n\}$ share the same preferences, deriving utility from their consumption c_i and environmental quality ϕ. The utility of agent i at time T is then given by

$$E \int_T^\infty \frac{1}{1-\sigma} c_i^{1-\sigma} \phi^\nu e^{-\rho(t-T)} dt, \tag{36}$$

where E is the expectation operator, $\rho > 0$ the constant rate of time preference, σ the constant rate of risk aversion and ν the wight of environmental quality. The constant $1 - 1 - \sigma \in (0, 1)$ is the constant rate of risk aversion.

The efficiency of input l_i in production in agent j is a^{γ_i}, where a>1 is a constant and γ_i is the serial number of technology. In the advent of technological change in agent j, this efficiency increases from a^{γ_i} to a^{γ_i+1}. Total output in agent j is given by

$$y_i = a^{\gamma_i} l_i. \tag{37}$$

In production, firms employ labor l_i up to the point where the wage w_i is equal to the output price p_i times the marginal product of labor, $\partial y_i / \partial l_i$:

$$w_i = p_i \frac{\partial y_i}{\partial l_i} = p_i a^{\gamma_i}. \tag{38}$$

It is assumed that in a small period of time dt, the probability that R&D leads to development of a new technology is given by $\lambda z_i dt$, while the probability that R&D remains without success is given by $1 - \lambda z_i dt$, where λ is the productivity of R& D. This defines a Poisson process q_i with

$$dq_i = \begin{cases} 1 & \text{with probability } \lambda z_i dt, \\ 0 & \text{with probability } 1 - \lambda z_i dt, \end{cases} \tag{39}$$

where dq_i is the increment of the process q_i.

It is assumed that the central planner imposes a uniform tax τ on the product $w_i m_i$ of wages w_i and emissions m_i, and pays a uniform subsidy b to labor income $w_i L$ throughout all agents j in the union. Thus, given (32), (37) and (38), consumption in agent j is determined by

$$c_i = p_i y_i - \tau w_i m_i + b w_i L = p_i a^{\gamma_i}[(1-\tau)l_i + bL], \tag{40}$$

where $p_i y_i$ is income from production, $\tau w_i m_i$ emission taxes and $b w_i L$ subsidies for labor. Noting (32), the central planner's budget constraint is then

$$b \sum_{i=1}^{n} w_i L = \tau \sum_{i=1}^{n} w_i m_i = \tau \sum_{i=1}^{n} w_i l_i, \tag{41}$$

where $b \sum_{i=1}^{n} w_i L$ is subsidies and $\tau \sum_{i=1}^{n} w_i m_i$ emission taxes in the union. Planner $i \in \{1, ..., n\}$ forms expectations on the prospective responses of the other agents $\ell \neq j$ to its action. It anticipates the others $\ell \neq j$ to increase their emissions m_ℓ by the constant $\beta \in (-\infty, 1)$ units after it itself has increased its emissions m_i by one unit. This assumption and (32) imply $dl_\ell/dl_i = dm_\ell/dm_i = \beta$ for all $\ell \neq j$ and

$$m = M(l_i, n, \beta) \text{ with } \frac{\partial M}{\partial l_i} = 1 + \sum_{\ell \neq j} \frac{dl_\ell}{dl_i} = 1 + (n-1)\beta. \tag{42}$$

Here, $M(l_i, n, \beta)$ is planner j's *perceived supply function* of total emissions in the union. Because in the model all agents $i = 1, ..., n$ are in symmetric position, they have the same perceived supply function (42). An agent takes β as a constant, but at the level of the whole union the parameter β adjusts to keep the perceived supply of emissions, $M(l_i, n, \beta)$, equal to actual emissions m for all planners $i \in \{1, ..., n\}$.

3.3.2 General equilibrium

Noting (42) and (35), agent i's perceived pollution evolves according to

$$\dot{\phi} = h\phi - M(l_i, n, \beta), \quad \frac{\partial \dot{\phi}}{\partial l_i} = -\frac{\partial M}{\partial l_i} = -1 - (n-1)\beta. \tag{43}$$

Agent i maximizes its utility (36) by consumption c_i and labor input (l_i, z_i) subject to the resource constraint (31), Poisson technological change (39), the budget constraint (40), expectations (42) and pollution (43), on the assumption that the price p_i, the tax τ, the subsidy b and the parameter β are kept constant. Using stochastic dynamic programming for this problem, Palokangas (2008) obtains the following equilibrium condition:

$$(a^{1-\sigma}-1)\lambda+\frac{\nu}{P}[1+(n-1)\beta]=\frac{(1-\tau)1-\sigma}{(1-\tau)l_i+bL}\frac{1}{\vartheta_i}=$$
$$\frac{(1-\tau)1-\sigma}{(1-\tau)l_i+bL}\left[\rho+(1-a^{1-\sigma})\lambda(L-l_i)+\nu\frac{\dot{\phi}}{\phi}\right]_. \tag{44}$$

Because there is symmetry throughout agents $i=1,...,n$ in the model, noting (31), (32), (34) and (38), it is true that

$$l_i=l,\;\; z_i=z=L-l,\;\; c_i=c,\;\; M=m=nl,\;\; p_i=p=1,\;\; w_i=a^{\gamma_i}. \tag{45}$$

Given (33), (37) and (45), the consumption good is produced according to

$$c_i=c=\frac{y}{n}=\frac{1}{n}\prod_{i=1}^{n}y_i^{1/n}=\frac{1}{n}a^{\gamma}\prod_{i=1}^{n}l_i^{1/n}=\frac{l}{n}a^{\gamma},\quad \gamma\doteq\frac{1}{n}\sum_{i=1}^{n}\gamma_i, \tag{46}$$

where γ the serial number of the consumption-good technology. Because the improvement of productivity a^{γ_i} in the production of good j, (37), follows the Poisson process (39), and because labor inputs l_i are constant over time in the stationary state, then, given (33), the improvement of productivity in the production y of the consumption good follows the Poisson process q with

$$dq=\begin{cases}1 & \text{with probability } \lambda(L-l)dt,\\ 0 & \text{with probability } 1-\lambda(L-l)dt.\end{cases} \tag{47}$$

Thus, the expected growth rate of consumption y in the stationary state is

$$g\doteq E\big[\log\, a^{\gamma+1}-\log\, a^{\gamma}\big]=(\log a^{)}\lambda z=(\log a^{)}\lambda(L-l), \tag{48}$$

where E is the expectation operator. Noting (45), the budget constraint (41) changes into

$$b=\tau\sum_{i=1}^{n}w_il_i\bigg/\left(L\sum_{i=1}^{n}w_i\right)=\frac{\tau l}{L}. \tag{49}$$

Finally, given (45), the evolution of environmental quality (43) becomes

$$\dot{\phi} = h\phi - nl \quad \text{with} \quad \partial\dot{\phi}/\partial l = -n. \tag{50}$$

I consider the stationary state in which labor inputs (l_i, z_i) are kept constant over time. In that case ϕ is constant as well, and given (50), the following conditions hold true:

$$\dot{\phi} = 0, \quad h\phi = nl. \tag{51}$$

Inserting (45), (49) and (51) into (44) and solving for l, one obtains the equilibrium level of emissions in the stationary state:

$$l = \frac{1/n + (1 - 1/n)\beta}{(1-\tau)1-\sigma-1}\frac{\nu h/\lambda}{a^{1-\sigma}-1} + \frac{(1-\tau)1-\sigma}{1-(1-\tau)1-\sigma}\left(\frac{\rho/\lambda}{a^{1-\sigma}-1} - L\right). \tag{52}$$

3.3.3 Pareto optimum

Noting (32) and (46), the welfare in the union takes the form

$$U(c, m, T) = \int_T^\infty \frac{1}{1-\sigma} a^{(1-\sigma)\gamma} n^{\sigma-1} l^{1-\sigma} \phi^\nu e^{-\rho(t-T)} dt, \tag{53}$$

The central planner maximizes this by labor in production, l, subject to technological change (47) and the dynamics of pollution (50). Using stochastic dynamic programming for this problem, Palokangas (2008) obtains the Pareto-optimal level of emissions, l^*:

$$l^* \doteq \frac{(a^{1-\sigma}-1)^{-1}}{\sigma\lambda}\Big\{(\sigma-1)\big[\rho + (1-a^{1-\sigma})L\big] - h\nu\Big\}, \quad \frac{\partial l^*}{\partial(h\nu)} < 0. \tag{54}$$

Noting (48) and (54), the Pareto-optimal growth is $g^* = (\log A)(L - l^*)$, for which $\partial g^*/\partial(h\nu) > 0$. This result can be summarized as follows:

Proposition 4 *The Pareto-optimal growth rate g^* is the higher, the more environmental quality is liked (i.e. the bigger ν), or the higher the absorption rate of pollution, h.*

R&D spending for clean production promote economic growth. A high absorption rate eases the constraint for the central planner and boosts growth.

The central planner sets the tax parameter τ to establish the Pareto optimum $l = l^*$. Given $\rho + (1 - a^{1-\sigma})\lambda L < 0$, (52) and (54), one obtains

$$\tau^* \doteq \frac{1-\beta}{1-\sigma}\Big(1-\frac{1}{n}\Big)\frac{\sigma h\nu/[\rho+(1-a^{1-\sigma})\lambda L]}{1-h\nu/[\rho+(1-a^{1-\sigma})\lambda L]} > 0, \quad \frac{\partial\tau^*}{\partial(h\nu)} < 0. \tag{55}$$

Thus, the following result is obtained:

Proposition 5 *The optimal emission tax τ^* [cf. (55)] increases with the number n of agents. When the union is a single jurisdiction, $n = 1$, the tax is zero. The tax is the lower, the greater is the other agent's anticipated response β. The less environmental quality is liked (i.e. the smaller ν) or the smaller the absorption rate of pollution, h, the higher the emission tax.*

The more agents (i.e. the bigger n), the higher proportion of the emissions of an agent falls upon the other agents and the less an agent is willing to reduce emissions. Thus, a higher tax is needed to make a agent to reduce its emissions. Noting (42) and the symmetry $l_i/m \approx 1/n$, the elasticity of total emissions m with respect to emissions in a single agent l_i is given by

$$\frac{l_i}{m}\frac{\partial M}{\partial l_i} = [1+(n-1)\beta]\frac{l_i}{m} \approx \frac{1}{n} + \Big(1-\frac{1}{n}\Big)\beta.$$

With estimating this elasticity, one can estimate β. The more a agent expects the others to follow its policy (i.e. the bigger β), the less space it has for raising its emissions and the smaller tax (55) is adequate for reducing its emissions. Preferring pollution (i.e. a high ν) or high absorption rate h strengthens the welfare effect of the tax. In that case, a smaller tax τ^* is needed for maintaining the Pareto optimum, $\partial\tau^*/\partial(h\nu) < 0$.

3.4 Negotiation games

Because there is no worldwide benevolent central planner, the problem is to find out how the agents could cooperate to mimic the Pareto-optimum. A key to such cooperation is *mutual confidence*: once the agents reach an agreement on the coordination of their behavior, each of them must trust that the other will not cheat and renege. Reputation could be a basis for mutual confidence. Because the agents gain reputation by their behavior, a successful cooperation today in alleviating global warming creates confidence that helps to run such cooperation in future. Thus, international climate policy might be constructed as a learning process which, on certain conditions, can lead to full confidence and a Pareto optimum in future.

Haurie et al. (2006) have constructed the necessary optimality conditions for the following negotiation game:

1. The accumulation of *greenhouse gases (GHGs)* in the atmosphere in exogenously constrained.

2. An agreement must be reached on the relative development paths of the different countries and their use of GHGs to foster their development.

3. GHGs are by-products of economic production process.

4. There must be a non-cooperative equilibrium of the strategic game between the countries.

Theories for environmental policy commonly assume benevolent policy makers. In a coalition of countries, however, international agencies tend to be self-interested, thus being prone to lobbying from the member countries. This interplay brings an additional flavor to the maintenance of sustainable development. Lobbying can be modelled either by the *all-pay auction model*, in which the lobbyist making the greater effort wins with certainty, or by the *menu-auction model*, in which the lobbyists announce their bids contingent on the politician's actions. In the all-pay auction model, lobbying expenditures are incurred by all the lobbyists before the planner takes an action A good example of this is Johal and Ulph (2002) in which local interest groups lobby to influence the probability of getting their favorite type of government elected. In the menu-auction model, it is not possible for a lobbyist to spend money and effort on lobbying without getting what he lobbied for. Because the menu-auction model characterizes better the case in which the central planner's decision variables (regulatory constraints, subsidies) are continuous – so that the interest groups obtain marginal improvements in their position by lobbying – it is chosen as a starting point in this document.

Palokangas (2009, 2014) have extended Haurie's (2006) framework so that the constraint on global emissions is not given, but endogenously determined by negotiation between the countries and the international agency before environmental policy takes place. In that case, the entire negotiation game starts from the determination of emission caps rather than from its subgame starting from the imposition of emission caps.

3.4.1 The economy

There is large number ("continuum") of economic agents that are placed evenly in the limit $[0,1]$. They have different endowments of labor and land, different productions functions in manufacturing and different technology in R&D. Their emissions have different impacts on global pollution. All agents produce the same consumption good from land, labor and energy. That good is chosen as the numeraire in the model, for convenience.

Each agent $i \in [0,1]$ supplies land A_i and labor L_i inelastically, and devotes l_i units of labor to production and the remainder

$$z_i = L_i - l_i \tag{56}$$

to R&D. There exists an emitting input called energy the extraction costs of which are ignored, for simplicity. It is assumed that emissions are proportional to the use of energy, m_i, in each agent i. Pollution m is a linearly homogeneous function M of the emissions of all agents $i \in [0,1]$:

$$m = M\big(m_i | \, i \in [0,1]\big), \quad M \text{ homogeneous of degree one.} \tag{57}$$

In global warming problems, it is the stock of GHGs that causes damages and not the flow. In this document, however, the flow is used instead to simplify the dynamics. In the model, pollution affects the economy in two ways. First, pollution decreases utility globally. Second, local pollution harms local production. Except realism, there is also a technical reason to introduce the "local" effect: it enables the existence of the laissez-faire equilibrium in the case there is no international agent running emission policy.

To enable that the agents can increase their efficiency and consequently grow at different rates in a stationary-state equilibrium, we eliminate

- the terms-of-trade effect by the assumption that all agents produced the same internationally-traded good, and
- international capital movements by the assumption that all agents share the same constant rate of time preference, ρ.

3.4.2 Single agents

Production

When agent i develops a new technology, it increases its productivity by constant proportion $a_i > 1$. The level of productivity in agent i is then equal to ${a_i}^{\gamma_i}$, where γ_i is its serial number of technology. The innovation of new technology in agent i increases γ_i by one.

Agent i produces its output y_i from land A_i, labor l_i and energy m_i. It is assumed that local emissions, which are proportional to energy input m_i, harm local production.[36] It is furthermore assumed that labor l_i and energy m_i form a composite input $\phi^i(l_i, m_i)$ through CES technology, but otherwise there is Cobb-Douglas technology:[37]

$$
\begin{aligned}
& y_i = {a_i}^{\gamma_i} f^i(l_i, m_i) m_i^{-\zeta}, \;\; f^i(l_i, m_i) \doteq A_i^{1-\alpha_i} \phi^i(l_i, m_i)^{\alpha_i}, \;\; 0 < \alpha_i < 1, \;\; \zeta > 0, \\
& f_l^i > 0, \;\; f_m^i > 0, \;\; \phi_l^i > 0, \;\; \phi_m^i > 0, \;\; \phi_{ll}^i < 0, \;\; \phi_{mm}^i < 0, \;\; \phi_{lm}^i > 0, \qquad (58)
\end{aligned}
$$

where the subscripts l and m denote the partial derivative of the function with respect to l_i and m_i, respectively, ${a_i}^{\gamma_i}$ is total factor productivity, α_i a parameter and ζ is the constant elasticity of output with respect to emissions m_i. The higher ζ, the more local emissions m_i harm local production.

When the markets are perfect in agent i, one can interpret $1 - \alpha_i$ as the expenditure share of land and α_i that of labor and energy taken together. Noting (58), the expenditure shares of energy and labor in production are

$$
\begin{aligned}
& \frac{m_i f_m^i(l_i, m_i)}{f^i(l_i, m_i)} = \alpha_i \frac{m_i \phi_m^i(l_i, m_i)}{\phi^i(l_i, m_i)} = \alpha_i \frac{\phi_m^i(l_i/m_i, 1)}{\phi^i(l_i/m_i, 1)} \doteq \varpi^i\left(\frac{l_i}{m_i}\right) \in (0, \alpha_i), \\
& \frac{l_i f_l^i(l_i, m_i)}{f^i(l_i, m_i)} = \alpha_i \frac{l_i \phi_l^i(l_i, m_i)}{\phi^i(l_i, m_i)} = \alpha_i \left[1 - \frac{m_i \phi_m^i(l_i, m_i)}{\phi^i(l_i, m_i)}\right] = \alpha_i - \varpi^i\left(\frac{l_i}{m_i}\right) \\
& \qquad \in (0, \alpha_i). \qquad (59)
\end{aligned}
$$

[36] Without this assumption, agent i would use an indefinitely large amount of energy in the case of laissez-faire (cf. section 3.4.3).

[37] The use of a general production function $y_i = a_i^{\gamma_i} F(A_i, l_i, m_i)$ would excessively complicate the analysis.

Because the composite input $\phi^i(l_i, m_i)$ is a CES function, one obtains

$$(\varpi^i)'\left(\frac{l_i}{m_i}\right) = \frac{d\varpi^i}{d(l_i/m_i)} \begin{cases} > 0 & \text{for } 0 < \sigma_i < 1, \\ < 0 & \text{for } \sigma_i > 1, \end{cases} \tag{60}$$

where σ_i is the constant elasticity of substitution between inputs l_i and m_i.

Research and development (R&D)

An increase in productivity in agent i, ${a_i}^{\gamma_i}$ [cf. the production function (58)], depends on labor devoted to R&D, z_i, in that agent: the probability that input z_i leads to development of a new technology with a jump from γ_i to $\gamma_i + 1$ in a small period of time $d\theta$ is given by $\lambda_i z_i d\theta$, while the probability that input z_i remains without success is given by $1 - \lambda_i z_i d\theta$, where $\lambda_i > 0$ is a constant. Noting (56), this defines a Poisson process q_i with

$$dq_i = \begin{cases} 1 & \text{with probability } \lambda_i z_i d\theta, \\ 0 & \text{with probability } 1 - \lambda_i z_i d\theta, \end{cases} \qquad z_i = L_i - l_i, \tag{61}$$

where dq_i is the increment of the process q_i.

Preferences

All agents have the same preferences: the expected utility of agent $i \in [0, 1]$ starting at time T is given by

$$E \int_T^\infty c_i m^{-\delta} e^{-\rho(\theta - T)} d\theta, \quad \delta > 0, \quad \rho > 0, \tag{62}$$

where E is the expectation operator, θ time, c_i consumption in agent i, ρ the constant rate of time preference and δ the constant elasticity of temporary utility with respect to economy-wide emissions m. The lower ρ, the more patient the agents are. Total pollution m decreases welfare in all agents $i \in [0, 1]$, but a single agent is so small that it ignores this dependence. The higher δ, the more pollution m is disliked.

3.4.3 Laissez-faire

Because all agents $i \in [0, 1]$ produce the same consumption good, then, without GHG emissions management, each agent i consumes what it produces,

$c_i = y_i$. Noting (58) and $c_i = y_i$, the expected utility of the agent starting at time T, (62), becomes

$$\Upsilon^i = E\int_T^\infty y_i m^{-\delta} e^{-\rho(\theta-T)} d\theta = E\int_T^\infty a_i^{\gamma_i} f^i(l_i, m_i) m_i^{-\zeta} m^{-\delta} e^{-\rho(\theta-T)} d\theta. \quad (63)$$

Assume for a while that energy input m_i is held constant. Agent i then maximizes its expected utility (63) by its labor devoted to production, l_i, subject to its technological change (61), given pollution m. Palokangas (2014) shows by stochastic dynamic programming that the solution of this maximization is the following:

Proposition 6 *The expected utility of agent i is*

$$\Upsilon^i = m^{-\delta}\Pi^i(\gamma_i, m_i, T), \ \textit{for which} \ \frac{\partial \Pi^i}{\partial m_i} = \frac{\Pi^i}{m_i}\left[\varpi^i\left(\frac{l_i}{m_i}\right) - \zeta\right]. \quad (64)$$

Agent i chooses its labor input l_i so that

$$\frac{(a_i - 1)\lambda_i l_i}{\rho + (1 - a_i)\lambda_i(L_i - l_i)} = \alpha_i - \varpi^i\left(\frac{l_i}{m_i}\right). \quad (65)$$

In the presence of laissez-faire, agent i can optimally determine its energy input m_i as well: it maximizes the value of its program, Υ^i, by m_i. Given (64), this leads to the first-order condition

$$\frac{\partial \Upsilon^i}{\partial m_i} = m^{-\delta}\frac{\partial \Pi^i}{\partial m_i} = m^{-\delta}\frac{\Pi^i}{m_i}\left[\varpi^i\left(\frac{l_i}{m_i}\right) - \zeta\right] = 0 \quad \text{and} \quad \varpi^i\left(\frac{l_i}{m_i}\right) = \zeta. \quad (66)$$

The second-order condition of the maximization is given by

$$\frac{\partial^2 \Upsilon^i}{\partial m_i^2} = -\underbrace{m^{-\delta}\frac{\Pi^i}{m_i}}_{+}(\varpi^i)'\underbrace{\frac{l_i}{m_i^2}}_{+} < 0 \quad \text{and} \quad (\varpi^i)' > 0.$$

Given this and (60), labor and energy are gross complements, $0 < \sigma_i < 1$, and $(\varpi^i)' > 0$ holds true everywhere. From this, (65) and (66) it follows that

$$\frac{(a_i - 1)\lambda_i l_i^L}{\rho + (1 - a_i)\lambda_i(L_i - l_i^L)} = \alpha_i - \zeta, \quad \varpi^i\left(\frac{l_i^L}{m_i^L}\right) = \zeta \ \text{with} \ (\varpi^i)' > 0, \quad (67)$$

where the superscript L denotes the laissez-faire equilibrium.

Given (31), (58), (59) and (67), it then holds true that

$$\rho+(\underbrace{1-a_i}_{-})\lambda_i(\underbrace{L_i-l_i^L}_{+})\underbrace{\xi^i}_{\in(0,1)}>\rho+(1-a_i)\lambda_i(L_i-l_i^L)>0,$$

$$\frac{(a_i-1)\lambda_i l_i^L}{\rho+(1-a_i)\lambda_i(L_i-l_i^L)}<\alpha_i-\beta<\alpha_i<1,\quad \rho+(1-a_i)\lambda_i L_i>0. \tag{68}$$

Noting (31), (67) and (68) yield

$$\frac{d}{dl_i^L}\log\left[\frac{(a_i-1)\lambda_i l_i^L}{\rho+(1-a_i)\lambda_i(L_i-l_i^L)}\right]=\frac{1}{l_i^L}\left[1-\underbrace{\frac{(a_i-1)\lambda_i l_i^L}{\rho+(1-a_i)\lambda_i(L_i-l_i^L)}}_{\in(0,1)}\right]>0 \text{ and}$$
$$\frac{d}{dl_i^L}\left[\frac{(a_i-1)\lambda_i l_i^L}{\rho+(1-a_i)\lambda_i(L_i-l_i^L)}\right]>0.$$

Noting this and differentiating the left-hand equation in (67), one obtains

$$\underbrace{\frac{d}{dl_i^L}\left[\frac{(a_i-1)\lambda_i l_i^L}{\rho+(1-a_i)\lambda_i(L_i-l_i^L)}\right]}_{+}dl_i^L+d\beta=0$$

and $dl_i^L/d\beta<0$. Given (31), this implies $dz_i^L/d\beta=-dl_i^L/d\beta>0$. Finally, differentiating the right-hand equation in (67), and noting (67), one obtains

$$\frac{dm_i^L}{d\beta}=\frac{m_i^L}{l_i^L}\left[\underbrace{\frac{dl_i^L}{d\beta}}_{-}-\underbrace{\frac{m_i^L}{(\xi^i)'}}_{+}\right]<0.$$

Thus, the following result is proven:

Proposition 7 *The more emissions harm locally (i.e. the higher ζ), the less there are emissions m_i^L, $dm_i^L/d\zeta<0$, and the more there is R&D (i.e. the higher z_i^L), $dz_i^L/d\zeta>0$.*

Because technological change generated by R&D decreases the need for polluting energy, there are incentives to perform R&D.

3.4.4 The central planner

Grandfathering means that emission caps have a base that is determined by the history, but updated over time. In models with discrete time, that base would be calculated by a moving average of past emissions. In the quality-ladders model of this document where time is continuous, the base is specified as follows. The central planner sets the pollutant caps in fixed proportion ε to the energy input of that agent under previous technology, $\widehat{m}_i$:

$$m_i \leq \varepsilon \widehat{m}_i \text{ for } i \in [0,1] \text{ and } \varepsilon > 0. \tag{69}$$

If the current number of technology is γ_i, then the allocation base $\widehat{m}_i$ is calculated by energy input under previous technology $\gamma_i - 1$ (cf. subsection 3.4.2). If the central planner tightens emission policy by decreasing ε below one, then the constraint (69) becomes binding for all agents $i \in [0,1]$. Because the function M in (57) is linearly homogeneous, one then obtains:

$$m_i = \varepsilon \widehat{m}_i \text{ for } i \in [0,1], \quad m = \varepsilon \widehat{m}, \quad \widehat{m} \doteq M\big(\widehat{m}_i \big| \, i \in [0,1]\big). \tag{70}$$

Thus, there is only one policy parameter ε in the grandfathering scheme.

The central planner can be benevolent or self-interested, but in both cases it chooses its grandfathering policy parameter ε from the class of *technology-invariant* controls: this is independent of technology changes in all agents.

3.4.5 The Pareto optimum

Because all agents $i \in [0,1]$ produce the same consumption good, total consumption is equal to total production, $\int_0^1 c_i dj = \int_0^1 y_i dj$. To construct the Pareto optimum, let us introduce a benevolent central planner that maximizes the welfare of the representative agent of the economy, $\mathcal{W}$. Given (62), (63), (64) and $\int_0^1 c_i dj = \int_0^1 y_i dj$, that welfare is

$$\begin{aligned}
\mathcal{W} &\doteq \int_0^1 \left[E \int_T^\infty c_i m^{-\delta} e^{-\rho(\theta - T)} d\theta \right] dj = E \int_T^\infty \left(\int_0^1 c_i dj \right) m^{-\delta} e^{-\rho(\theta - T)} d\theta \\
&= E \int_T^\infty \left(\int_0^1 y_i dj \right) m^{-\delta} e^{-\rho(\theta - T)} d\theta = E \int_T^\infty \left(\int_0^1 y_i m^{-\delta} e^{-\rho(\theta - T)} d\theta \right) dj \\
&= \int_0^1 \Upsilon^i dj = m^{-\delta} \int_0^1 \Pi^i(\gamma_i, m_i, T) dj \qquad (71)
\end{aligned}$$

which should be maximized by the policy parameter ε. Given (64) and (70), this leads to the first-order conditions

$$0 = \frac{d\mathcal{W}}{d\varepsilon} = m^{-\delta} \int_0^1 \frac{\partial \Pi^i}{\partial m_i} \underbrace{\frac{\partial m_i}{\partial \varepsilon}}_{= \widehat{m}_i} dj - \delta m^{-\delta - 1} \underbrace{\frac{\partial m}{\partial \varepsilon}}_{= \widehat{m}} \int_0^1 \Pi^i dj$$

$$
\begin{aligned}
&= m^{-\delta}\left[\int_0^1 \frac{\partial \Pi^i}{\partial m_i}\widehat{m}_i dj - \delta\frac{\widehat{m}}{m}\int_0^1 \Pi^i dj\right] \\
&= m^{-\delta}\left\{\int_0^1 \Pi^i\left[\varpi^i\left(\frac{l_i}{m_i}\right) - \zeta\right]\frac{\widehat{m}_i}{m_i}dj - \delta\frac{\widehat{m}}{m}\int_0^1 \Pi^i dj\right\} \\
&= m^{-\delta}\int_0^1 \Pi^i\left\{\left[\varpi^i\left(\frac{l_i}{m_i}\right) - \zeta\right]\frac{\widehat{m}_i}{m_i} - \delta\frac{\widehat{m}}{m}\right\}dj. \qquad (72)
\end{aligned}
$$

In the stationary state, all inputs (l_i, m_i) for all agents $i \in [0,1]$ must be constant. Once the economy attains the stationary state, the emissions under the previous and current technology become equal: $\widehat{m} = m$ and $\widehat{m}_i = m_i$ for $i \in [0,1]$. Plugging these conditions and into (72) yields

$$
0 = m^{-\delta}\int_0^1 \Pi^i\left[\varpi^i\left(\frac{l_i}{m_i}\right) - \zeta - \delta\right]dj. \qquad (73)
$$

Because the expected utilities Π^i for $i \in [0,1]$ are random variables, then, given (73), the only possible stationary state that corresponds to a technology-invariant control ε is

$$
\varpi^i\left(\frac{l_i}{m_i}\right) = \zeta + \delta \ \text{ for } i \in [0,1]. \qquad (74)
$$

The equilibrium conditions (65) for the agents $i \in [0,1]$ as well as those (74) for the central planner can be written as

$$
\varpi^i\left(\frac{l_i^P}{m_i^P}\right) = \zeta + \delta, \qquad \frac{(a-1)\lambda_i l_i^P}{\rho + (1-a)\lambda_i(L_i - l_i^P)} = \alpha_i - \zeta - \delta, \qquad (75)
$$

where the superscript P denotes the Pareto optimum equilibrium.

The comparison of (75) with (67) shows that the introduction of a benevolent central planner increases the parameter ζ up to $\zeta + \delta$ in the system. Thus, Proposition 7 has the following corollary:

Proposition 8 *A shift from laissez-faire to the Pareto optimum decreases emissions, $m_i^P < m_i^L$, and increases R&D, $z_i^P > z_i^L$.*

The introduction of a benevolent central planner internalizes the negative externality through emissions. This increases incentives to perform R&D. With the uniform proportionality rule ε, all agents face the same marginal benefits from pollutants via allocation in subsequent periods. In contrast to Böringer and Lange (2005), the regulatory cap m^P is not exogenous but endogenously determined.

3.4.6 Regulation

In this section, agents $i \in [0,1]$ lobby the central planner over the policy parameter ε. Following Grossman and Helpman (1994), it is assumed that the central planner has its own interests and collects political contributions R_i from agents $i \in [0,1]$. This is a common agency game, the order of which is then the following (cf. Grossman and Helpman 1994, and Dixit et al. 1997). First, the agents $i \in [0,1]$ set their political contributions R_i conditional on the central planner's prospective policy ε. Second, the central planner sets its technology-invariant policy variable ε and collects the contributions from the agents. Third, the agents maximize their utilities. This game is solved in reverse order.

Optimal program

Agent i pays its political contributions R_i to the central planner. It is assumed, for simplicity, that the central planner consists of civil servants who inhabit agents $i \in [0,1]$ evenly. Thus, the agents gets an equal share R of total contributions,

$$R \doteq \int_0^1 R_i dj \bigg/ \int_0^1 dk = \int_0^1 R_i dj. \tag{76}$$

Noting the production function (58), consumption in agent i is then

$$c_i = y_i + R - R_i = a_i^{\gamma_i} f^i(l_i, m_i) m_i^{-\zeta} + R - R_i, \tag{77}$$

where y_i is income from production and $R - R_i$ net revenue from political contributions in agent i. Noting (77), the expected utility of agent i starting at time T, (62), becomes

$$\Theta^i = E \int_T^\infty \left[a_i^{\gamma_i} f^i(l_i, m_i) m_i^{-\zeta} + R - R_i\right] m^{-\delta} e^{-\rho(\theta - T)} d\theta. \tag{78}$$

Agent i maximizes its expected utility (78) by its labor devoted to production, l_i, subject to technological change in the agent, (61), given the emission cap m_i, pollution m and political contributions R_i and R. Using stochastic dynamic programming, Palokangas (2014) proves the following result:

Proposition 9 *The solution for the optimal program* $\max_{l_i} \Theta^i$ *is the function*

$$\Theta^i(m_i, m, R, R_i, \gamma_i), \quad \frac{\partial \Theta^i}{\partial m_i} = m^{-\delta} \frac{\Gamma^i(\gamma_i, m_i, T)}{m_i} \left[\varpi^i \left(\frac{l_i^*}{m_i} \right) - \zeta \right],$$

$$\frac{\partial \Theta^i}{\partial m} = -\delta m^{-\delta-1} \left(\Gamma^i + \frac{R - R_i}{\rho} \right), \quad -\frac{\partial \Theta^i}{\partial R_i} = \frac{\partial \Theta^i}{\partial R} = \frac{m^{-\delta}}{\rho}, \tag{79}$$

where the random variable Γ^i *is the expected value of the flow of output for agent* i *and* l_i^* *is the optimal labor input in production for which*

$$\frac{(a_i - 1)\lambda_i l_i^*}{\rho + (1 - a_i)\lambda_i (L_i - l_i^*)} = \alpha_i - \varpi^i \left(\frac{l_i^*}{m_i} \right). \tag{80}$$

The political equilibrium

Because each agent i affects the central planner by its contributions R_i, its contribution schedule depends on the central planner's policy ε [cf. (76)]:

$$R_i(\varepsilon) \text{ for } i \in [0, 1], \quad R(\varepsilon) \doteq \int_0^1 R_k(\varepsilon) dk. \tag{81}$$

The central planner maximizes present value of the expected flow of the political contributions R from all agents $i \in [0, 1]$:

$$G(R) \doteq E \int_T^\infty R e^{-\theta(\theta - T)} d\theta = \frac{R}{\rho}. \tag{82}$$

Each agent i maximizes its expected utility Θ^i [cf. (79)].

According to Dixit at al. (1997), a subgame perfect Nash equilibrium for this lobbying game is a set of contribution schedules $R_i(\varepsilon)$ and a policy ε such that the following conditions $(i) - (iv)$ hold:

(i) Contributions R_i are non-negative but no more than the contributor's income, $\Theta^i \geq 0$.

(ii) The policy ε maximizes the central planner's welfare (82) taking the contribution schedules $R_i(\varepsilon)$ as given,

$$\varepsilon = \arg\max_{\varepsilon} G\big(R(\varepsilon)\big) = \arg\max_{\varepsilon \in [0,1]} R(\varepsilon). \tag{83}$$

(iii) Agent i cannot have a feasible strategy $R_i(\varepsilon)$ that yields it a higher level of utility than in equilibrium, given the central planner's anticipated decision rule (70),

$$\varepsilon = \arg\max_{\varepsilon} \Theta^i\big(m_i, m, R, R_i(\varepsilon), \gamma_i\big) \text{ with } m_i = \varepsilon\widehat{m}_i \text{and } m = \varepsilon\widehat{m}. \quad (84)$$

Because the agent is small, it takes the total contributions of all agents, R, as given. However, the agent observes the dependency of pollution m on environmental policy ε [cf. (70)].

(iv) Agent i provides the central planner at least with the level of utility than in the case it offers nothing ($R_i = 0$), and the central planner responds optimally given the other agents contribution functions,

$$G\big(R(\varepsilon)\big) \geq \max_{\varepsilon} G\big(R(\varepsilon)\big)\Big|_{R_i=0}.$$

The stationary state

Noting (79), the conditions (84) for agents $i \in [0,1]$ is equivalent to

$$0 = \frac{d\Theta^i}{d\varepsilon} = \frac{\partial\Theta^i}{\partial R_i}\frac{dR_i}{d\varepsilon} + \frac{\partial\Theta^i}{\partial m_i}\underbrace{\frac{\partial m_i}{\partial\varepsilon}}_{=\widehat{m}_i} + \frac{\partial\Theta^i}{\partial m}\underbrace{\frac{\partial m}{\partial\varepsilon}}_{=\widehat{m}} = \frac{\partial\Theta^i}{\partial R_i}\frac{dR_i}{d\varepsilon} + \frac{\partial\Theta^i}{\partial m_i}\widehat{m}_i + \frac{\partial\Theta^i}{\partial m}\widehat{m}$$

$$= -\frac{m^{-\delta}}{\rho}\frac{dR_i}{d\varepsilon} + m^{-\delta}\Gamma^i\left[\varpi^i\left(\frac{l_i}{m_i}\right) - \zeta\right]\frac{\widehat{m}_i}{m_i} - \delta m^{-\delta}\left(\Gamma^i + \frac{R - R_i}{\rho}\right)\frac{\widehat{m}}{m}$$

and

$$\frac{1}{\rho}\frac{dR_i}{d\varepsilon} = \Gamma^i\left[\varpi^i\left(\frac{l_i}{m_i}\right) - \zeta\right]\frac{\widehat{m}_i}{m_i} - \delta\left(\Gamma^i + \frac{R - R_i}{\rho}\right)\frac{\widehat{m}}{m} \quad \text{for } i \in [0,1]. \quad (85)$$

Once the economy attains the stationary state, the emissions under the previous and current technology become equal: $\widehat{m} = m$ and $\widehat{m}_i = m_i$ for $i \in [0,1]$. Plugging these conditions into (85) yields

$$\frac{1}{\rho}\frac{dR_i}{d\varepsilon} = \left[\varpi^i\left(\frac{l_i}{m_i}\right) - \zeta\right]\Gamma^i - \delta\left(\Gamma^i + \frac{R - R_i}{\rho}\right) \quad \text{for } i \in [0,1].$$

Noting these equations and (81), the government's equilibrium condition (83) is equivalent to

$$0 = \frac{dR}{d\varepsilon} = \int_0^1 \frac{dR_i}{d\varepsilon}dj = \rho\int_0^1 \left\{\left[\varpi^i\left(\frac{l_i}{m_i}\right) - \zeta\right]\Gamma^i - \delta\left(\Gamma^i + \frac{R - R_i}{\rho}\right)\right\}dj$$

$$= \rho \Big\{ \int_0^1 \Big[\varpi^i \Big(\frac{l_i}{m_i} \Big) - \zeta - \delta \Big] \Gamma^i dj - \frac{\delta}{\rho} \underbrace{\int_0^1 (R - R_i) dj}_{=0} \Big\}$$

$$= \rho \int_0^1 \Big[\varpi^i \Big(\frac{l_i}{m_i} \Big) - \zeta - \delta \Big] \Gamma^i dj. \tag{86}$$

In the stationary state corresponding to the technology-invariant control ε, all inputs (l_i, m_i) for all agents $i \in [0,1]$ must be constant. Because the expected value of the flow of output, Γ^i is a random variable for all agents $i \in [0,1]$, then, given (86), the only possible stationary state in the economy of agents $i \in [0,1]$ is

$$\varpi^i \Big(\frac{l_i}{m_i} \Big) = \zeta + \delta \ \text{ for } i \in [0,1]. \tag{87}$$

This means that if agent $i \in [0,1]$ has confidence on stable development, then it expects that its expenditure share of energy, ϖ^i, will be equal to $\zeta + \delta$ in the long run. From the equilibrium conditions (80) of the agents $i \in [0,1]$ as well as those (87) of the central planner, one obtains

$$\varpi^i \Big(\frac{l_i^G}{m_i^G} \Big) = \zeta + \delta, \qquad \frac{(a_i - 1)\lambda_i l_i^G}{\rho + (1 - a_i)\lambda_i (L_i - l_i^G)} = \alpha_i - \zeta - \delta, \tag{88}$$

where the superscript G denotes grandfathering of emissions.

Comparing the systems (75) and (88) yields the following result:

Proposition 10 *Regulation leads to the Pareto optimum,* $(l_i^G, m_i^G) = (l_i^P, m_i^P)$ *for* $i \in [0,1]$.

The introduction of a self-interested central planner has the same impact as that of a benevolent central planner: it internalizes the externality of emissions through pollution, leading to the Pareto optimum.

4 Conclusions

Different definitions of the sustainability concern the environmental impacts of current economic activities on future activities as well as the availability of resources for future generations. Factors of production include not only labor, physical capital, and land, but also limited resources. Moreover, production activities have external effects for the current as well as future generations. In order to discuss those issues, ideally, as Arrow et al (2012) argue, a broader measure of societies's wealth, including for example also human capital and health, need to be developed. Since this is still at the beginning, we here have restricted ourself to a more restricted measure of wealth and discuss the modelling of extraction of resources starting from the basic model and then go on to extensions including aspects of an open economy, backstop technology, and climate change effects, constraints of carbon dioxide emissions and the change of global temperature. Major macroeconomic problems of internal or external type of resource-rich countries are discussed. These include the economic impact of volatility and trend changes in world prices of the resources that can cause fluctuations in export revenues, income and external imbalances. We also review literature on resource depletion, peak production, "Dutch" disease, and overborrowing and the threats from extensive external debt. We further consider models and topics related to economic growth, abatement of environmental damages, and the transition to renewable energy. We also discuss decision making and policy options with multiple agents and more complicated settings where there is strategic dependence between the decisions of agents as is studied in game theoretical set ups. As shown strategic decision making can give rise to complicated issues arising with respect to resource extraction, environmental and mitigation policies and policy options on the micro as well macro level.

Appendix: numerical solution

Grüne and Pannek (2011) introduced the NMPC which can be used for solving the dynamic problems of the basic growth models and its variations with exhaustible resources. Nonlinear system's feedback control, which is based on optimization, is described as a NMPC. As described in Grüne and Pannek (2011) and shown in Nyambuu and Semmler (2014), there is a predicted process, where at each time instant we choose a control input, u_n. Future behavior of the state of the system with discrete time instants, x_n, is affected by the control inputs, u_n. The state of the system is tracked and control inputs are defined. It ensures to reduce the distance to a predetermined reference, x_n^{ref} (Grüne and Pannek, 2011). Then, a stabilization problem is defined when the reference is zero[38]

Greiner et al. (2012a) point out that the procedure also works if the steady state is not predicted. When the periods are very large the NMPC provides a good approximation of the infinite time horizon solution. "Instead of computing the optimal value function for all possible initial states, NMPC only computes single (approximate) optimal trajectories" (Greiner et al., 2012a, 16).

For the description the NMPC, we follow the sketch used in the Greiner et al. (2012a, pp. 16-17). The optimal control problem can be described as

$$\max \int_0^\infty e^{-\rho t} l(x_t, u_t) dt,$$

where x_t satisfies $\dot{x_t} = f(x_t, u_t)$, $x(0) = x_0$. A discrete time problem with the number of iteration, i, becomes:

$$\max \textstyle\sum_{i=0}^{\infty} \beta^i l(x_i, u_i).$$

The optimization has a finite horizon N where $x_{k+1,i} = \Phi(h, x_{k,i}, u_{k,i})$ (see Greiner et al., 2012a).

In solutions of the basic and extended growth models, a discounted variant of the MATLAB routine *nmpc.m*[39] with MATLAB's fmincon NLP solver can be used (see Greiner et al., 2012a). As N increases to infinity, the solution,

[38] See Grüne and Pannek (2011)

[39] The MATLAB routine nmpc.m is available from $www.nmpc - book.com$

(x_i, u_i), of the problem converges to the optimal solution. However, Grüne (2013) indicates that for the solution to converge to the optimal solution it does not require to find the equilibrium values.

NMPC can be applied to economics, in particular for dynamic decision problems. Grüne et al. (2013) show that NMPC can efficiently and quite accurately solve not only stochastic problems, but also models with "multiple domains of attraction and thresholds" and "multiple equilibria and regime switches in the dynamics."[40] They point out that, based on the solution method[41] of NMPC, "it is well suited to track the solution paths for information constrained agents in the sense of Sims (2005, 2006)."[42]

They present one dimensional (1d) optimal control problems, including basic growth models, basic DSGE models, as well as two dimensional (2d)[43] models such as a 2d stochastic growth model, a 2d model with multiple domains of attraction, and a 2d growth model with non-renewable resources. In addition, three dimensional[44] (3d) models of growth with non-renewable resources and backstop technology and models with credit market frictions are explored. These models are solved using NMPC algorithms and both MATLAB and C++.

Grüne et al. (2013) stress the advantages of NMPC in comparison to other methods such as Dynamic Programming and show how state and control variables can be computed. Numerical solution by NMPC does not require the steady states and linearization (for details see Grüne et al., 2013).

[40] Grüne et al. (2013, p. 24).

[41] Grüne et al. (2013) state that NMPC solves finite horizon dynamic decision problems by approximating the corresponding infinite horizon models.

[42] Grüne et al. (2013, p. 24).

[43] While 1d models have one state variable, 2d models have two state variables (Grüne et al., 2013)

[44] In Grüne et al. (2013), 3d models have three state variables and two and three decision variables.

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