A PROPOSAL FOR A DECISION FRAMEWORK IN THE SKÅNE PROJECT

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PREFACE

Water resources systems have been an important part of resources and environment related research at IIASA since its inception. As demands for water increase relative to supply, the intensity and efficiency of water resources management must be developed further. This in turn requires an increase in the degree of detail and sophistication of the analysis, including economic, social and environmental evaluation of water resources development alternatives aided by application of mathematical modeling techniques, to generate inputs for planning, design, and operational decisions.

During the year of 1978 it was decided that parallel to the continuation of demand studies, an attempt would be made to integrate the results of our studies on water demands with water supply considerations. This new task was named "Regional Water Management" (Task 1, Resources and Environment Area). It is concerned with the application of systems analysis techniques for planning and operational management of integrated regional water resources systems.

This paper by Professor M.B. Fiering from Harvard University was drafted during his short visit to IIASA in March 1979. It contains a methodological proposal for analysis of regional water resources management. A model which couples alternative water demand patterns with the long-term availability of water is formulated.

CONTENTS

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1. Availability of Water

Consider a matrix whose elements, i.e. x_{ij}, are the allocations of water from basin B_j to municipality M_i. The sources available to B_j might include stocks of water in storage and fluxes over which the M_i have jurisdiction. For example, in Figure 1 a simple stock and flux model is shown for a column of soil. There are 4 storages or stocks, 2 inputs, 3 outputs, and many internal fluxes. For simplicity the diagram shows connections between adjacent stocks, but in fact a more elaborate connection network exists in nature. Some of the connections make no hydrologic sense and can safely be ignored.

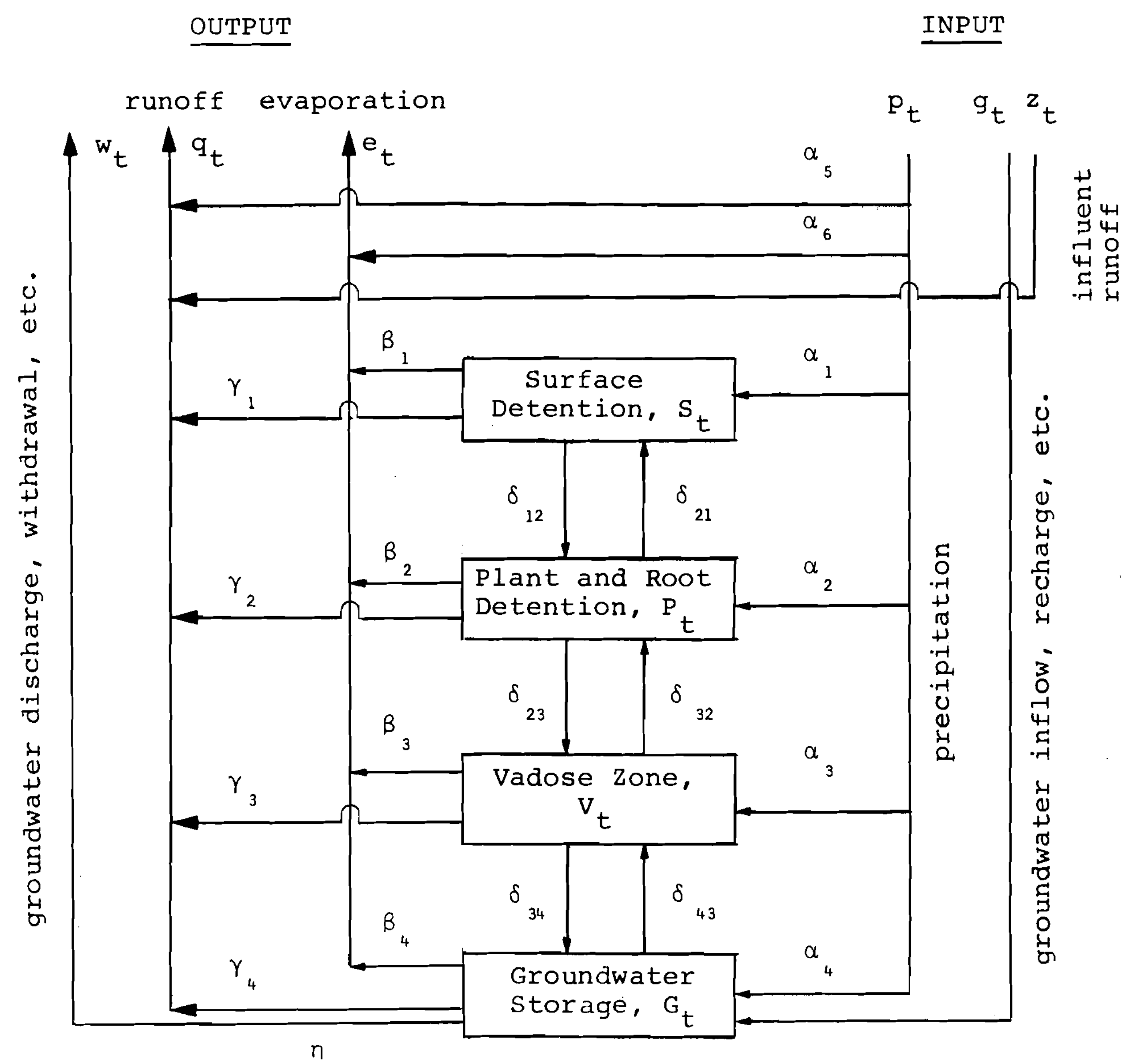
If we make a number of assumptions about prototype behavior and apply the law of continuity across each of the stocks, it is possible uniquely to find values for all or most of the parameters α_m, β_m, γ_m, δ_{mn}, etc. For example, some obvious constraints are

$$\Sigma\ \alpha_m = 1 \quad ,$$

$$0 \leq \alpha_m,\ \beta_m,\ \gamma_m,\ \eta \leq 1 \quad ,$$

$$0 \leq \delta_{mn} \leq 1 \quad ,$$

and an obvious approximation to a physically motivated system is



Notes: 1) δ_{mn} for other pairs of stocks are not shown.

2) Some δ_{mn} may not make hydrologic sense and are zero.

Figure 1. Stocks and fluxes.

the linear hypothesis coupled with continuity:

$$S_{t+1} = S_t + \alpha_1 p_t + \delta_{21} P_t + \delta_{31} V_t + \delta_{41} G_t$$

$$- S_t(\delta_{12} + \delta_{13} + \delta_{14} - \beta_1 - \gamma_1) \quad ,$$

$$P_{t+1} = P_t + \alpha_2 p_t + \delta_{12} S_t + \delta_{32} V_t + \delta_{42} G_t$$

$$- P_t(\delta_{21} + \delta_{23} + \delta_{24} - \beta_2 - \gamma_2) \quad ,$$

$$V_{t+1} = V_t + \alpha_3 p_t + \delta_{13} S_t + \delta_{23} P_t + \delta_{43} G_t$$

$$- V_t(\delta_{31} + \delta_{32} + \delta_{34} - \beta_3 - \gamma_3) \quad ,$$

$$G_{t+1} = G_t + g_t + \alpha_4 p_t + \delta_{14} S_t + \delta_{24} P_t + \delta_{34} V_t$$

$$- G_t(\delta_{41} + \delta_{42} + \delta_{43} - \beta_4 - \gamma_4 - \eta)$$

$$e_t = \alpha_6 p_t + \beta_1 S_t + \beta_2 P_t + \beta_3 V_t + \beta_4 G_t \quad ,$$

$$q_t = \alpha_5 p_t + \gamma_1 S_t + \gamma_2 P_t + \gamma_3 V_t + (\gamma_4 + \eta) G_t + z_t \quad .$$

Of course, the stocks in this model are of infinite capacity and drive linear fluxes through the frictionless system connectors. In any real applications, these assumptions would have to be relaxed and replaced with real system representations.

The data are the time series q_t and p_t and perhaps some basin characteristics. The commonly used approach to calibrating a basin model is to aggregate or lump the parameters whenever possible, and to fit by least squares the observations on q_t and p_t. This often leads to quite good fits, but just as often, to chronic instabilities outside the range of observations. These failures of runoff models have led to much hand-wringing and to the promulgation of much foolishness; perhaps the following arguments can explain the problem.

In the U.S., about 28% of all precipitation becomes runoff, so that most rainfall/runoff models have an implied residual which is 72/28 = 2.6 times as large as the observed dependent variable.

Any small instabilities in this residual are levered into enormous errors in the runoff model, whereupon the fit collapses outside the range of observations.

It is proposed to accommodate more time series into the fitting procedure. In particular, if the observation at some time interval t is the vector $\{z,p,g,w,q,S,P,V,G\}_t$, which might require some innovative measurement techniques (all of which are feasible), and if some of the parameters are constrained to fall within ranges established by hydrological experience and experiment, the use of constrained least-squares techniques will lead unambiguously to a solution for the parameter set. The set may have some persistent lumpiness, and it is difficult to know *a priori* if good estimates of the lumped parameters imply equally good estimates of the individual constituent values. This will have to await empirical validation. This approach leads to a basin budget rather than a rainfall/runoff model; this budget is not nearly as detailed as the Stanford Watershed Model, which requires literally dozens of parameters but which suffers from a lack of uniqueness in parameter estimation. The proposed budget provides a direct and statistically stable linkage among the available resource elements so that various small natural and man-made system perturbations can be assessed in terms of their impacts on stocks and fluxes. It is not appropriate to discuss in this paper the many assumptions, shortcomings and applications of such a budget; the important point is merely to note that it connects the several potential sources of water in the basin and establishes an accounting framework for the basin's transient and retained resources.

This model serves principally to couple the long term availability of water with use patterns. For example, mining the groundwater will ultimately reduce the aquifer's contribution to surface runoff, which shift should be reflected by an updated availability constraint on surface water. The coupling mechanism suggests a damped response in that intervening random fluctuations tend to mask the interdependencies, to make the system respond sluggishly. But by simulating over long enough intervals, the deterministic mechanisms dominate random impulses and the relationships emerge.

It should here be noted that a time interval of the order of a week should be utilized to estimate the parameters and calibrate the model. This short interval suggests that data sequences will not be widely available and that some networking may have to be undertaken to develop a data base representative of the entire Skåne region. The short interval virtually guarantees, however, that enough data points can be collected from a few months or years of observations. The parameters are assumed to be invariant with time unless some specified development induces a change. Thus even though they are estimated from a brief period of observation, they can at least in principle be used to predict long term basin response over many seasons.

2. Matrix Formulation

Suppose we have a region with distinct hydrological subdivisions or basins B_j, with $j = 0,1,\ldots,m$, and users or municipalities M_i, with $i = 0,1,\ldots n$. The basins are sources of water; B_o is a generic exogenous source whose origin lies outside the subdivision. Each source can be subdivided into stocks and fluxes. A simple first approximation is to generate for each source a 3-dimensional vector whose elements are groundwater storage G_t, average basin precipitation p_t, and total channel flow or runoff q_t. Physical and institutional constraints limit the fractions of each supply element available to the municipality M_i in that portion of the basin B_j over which it has jurisdiction. Pumping limitations and permeability place a bound on groundwater withdrawal. Some of the incident precipitation evaporates or runs off, making it unavailable for utilization by crops. Water quality, fish and wildlife, and institutional constraints limit the withdrawal from surface fluxes by placing lower bounds on channel flow. These bounds reflect various use levels and reliabilities.

In any event, municipality M_i can divert its total supply, from whatever sources or combinations thereof, to competing uses such as water supply, industrial use and irrigation. Others might be added; these might be aggregated to simplify the problem, and other adjustments might be made to reflect the utilization of total supply. It is assumed that each use and user has a constant characteristic return factor so that the effects of return flow are introduced merely by appropriately modifying (i.e., reducing) the withdrawal.

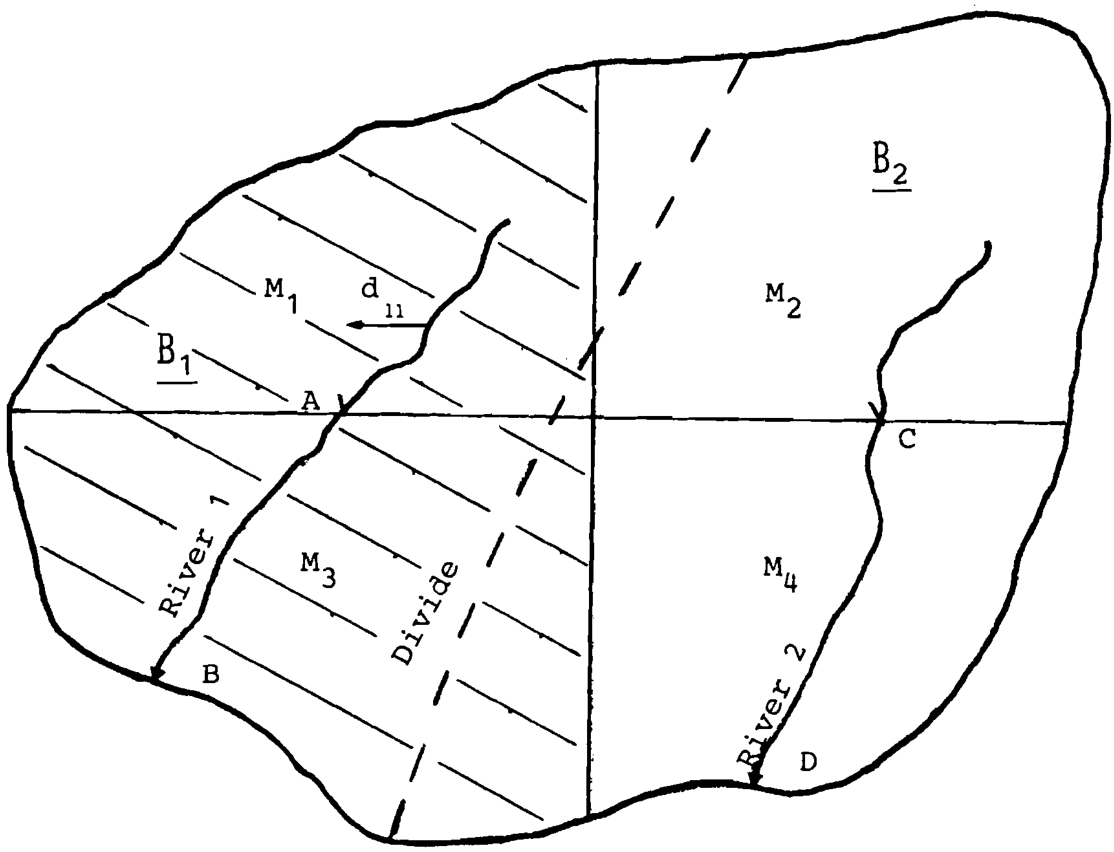
Figures 2 and 3 indicate how the x_{ij} are constituted. The basin model and a trivial nodal analysis are joined to produce the matrix M-I (see Fig. 4), with risk analysis implicit in selecting the limitations or constraints. For example, the analysis should be performed for several security levels, or flows (and precipitations) which correspond to a range of return intervals. An initial sample should include at least the 10, 50, and 90 percentile events, with follow-up analyses in the vicinity of the more interesting and critical results. Each such analysis would define another matrix, i.e. M-II, M-III, etc. as appropriate.

3. Constraints and Objectives

The current values of x_{ij} are obtained from existing data. The superscript notation x_{ij}^0 is introduced to indicate the current situation, whereupon the current utilization vector $(X_0, X_1 \ldots, X_n)^0$ is congruent to the current demand vector $(D_0, D_1, \ldots, D_n)^0$ and the current extraction vector $(E_0, E_1, \ldots, E_n)^0$ obeys the inequality constraint $E_j^0 \leq A_j$ for all $j = 0, 1, \ldots, m$. Continuity demands that the total extraction $E^0 = \sum_{j=0}^{m} E_j$ should equal the total utilization $X^0 = \sum_{i=0}^{n} X_i$.

Now suppose a new set of demands $\underline{D}^1 = (D_0, D_1, \ldots, D_n)^1$ is introduced as the result of an independent locational (or other form of) analysis. The first step in assessing $\underline{D}^1$ is to test its feasibility, or to verify that the scalar sum of demands $\Sigma D^1{}_i = D^1 \leq A$. If this constraint is not met, the individual magnitudes or reliabilities (or both) must be changed. The achievement of macro-feasibility is not a guarantee that internal consistency or micro-feasibility can be attained. Linkages between sources and sinks may have inherent capacity constraints which make it impossible to move requisite volumes of water to their use-points.

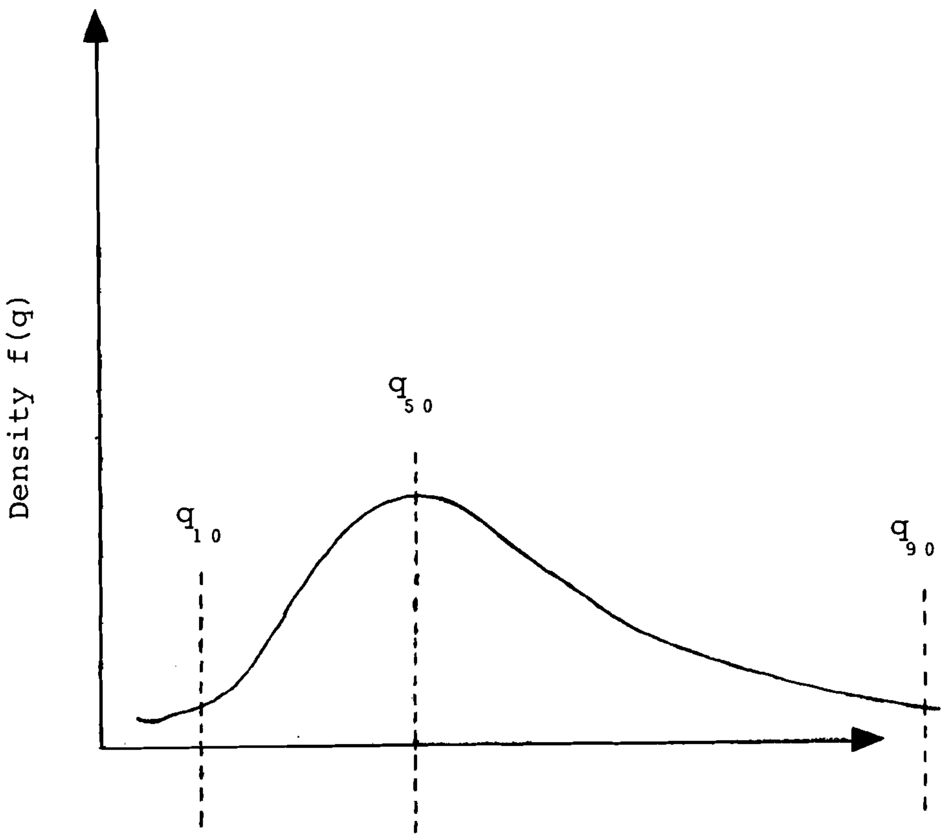
We define a <u>solution</u> to be the optimal $\underline{x}^1$ which meets some or all of the following linear constraints:



Municipality M_1 draws water from B_1 and the western part of B_2.
M_2 draws from B_2 and the northeastern corner of B_1.
M_3 draws from B_1 and the western part of B_2.
M_4 draws only from B_2.

x_{11} is the (flow and gw and ppn) at point A, expressed at some specified fractile. If d_{11} is withdrawn, the entire downstream regime is changed. In particular, surface and groundwater supplies will shift. x_{31} includes the water generated in that part of B_1 which lies within the jurisdiction of M_3, while x_{30} includes the unutilized flow from that portion of B_1 which lies within the jurisdiction of M_1.

Figure 2. Labelling flows.



Mean annual flow, q
or Mean annual ppn (precipitation)
or Mean growing season ppn

Figure 3. Certainty equivalents

Sinks or Municipalities	Sources or Basins: Exogenous 0	Endogenous Supplies → 1	2 ...	m	Current Utilization	Demand Under a Given Scenario
0 (The Sea)	x_{00}	x_{01}	x_{02}	x_{0m}	X_0 (to the sea)	D_0
1	x_{10}	x_{11}	x_{12}	x_{1m}	$X_1 = \sum_{j=0}^{m} x_{1j}$	D_1
2	...	...	$x_{ij} = \sum_{k=1}^{3} \sum_{\ell=1}^{3} x_{ij} \begin{array}{cc} k & \ell \\ \end{array} \left[\begin{array}{cc} 1{:}gw & 1{:}ws \\ 2{:}sw & 2{:}ind \\ 3{:}ppn & 3{:}irr \end{array}\right]$		$X_i = \sum_{j=0}^{m} x_{ij}$	D_i
⋮						
n	x_{n0}			x_{nm}	$X_n = \sum_{j=0}^{m} x_{nj}$	D_n
Extraction	$E_0 = \sum_{i=0}^{n} x_{i0}$			$E_m = \sum_{i=0}^{n} x_{im}$	$E = X$	D
Absolute Limits	A_0			$A_m = \sum_{i=0}^{n} \max(x_{im})$		$A = \sum_{j=0}^{m} A_j$

Note:

Source	Sink
k = 1 gw	ℓ = 1 ws
2 sw	2 ind
3 ppn	3 irr

Figure 4. Matrix M-I.

$$x_{00}{}^1 + x_{01}{}^1 + \ldots + x_{0m}{}^1 = X_0{}^1 \geq D_0{}^1 \quad \left.\vphantom{\Big|}\right\} \text{(maintain quality at the boundary of the basin and other in-stream constraints elsewhere)}$$

$$\left.\begin{aligned} x_{10}{}^1 + x_{11}{}^1 + \ldots + x_{1m}{}^1 &= X_1{}^1 \geq D_1{}^1 \\ x_{n0}{}^1 + x_{n1}{}^1 + \ldots + x_{nm}{}^1 &= X_n{}^1 \geq D_n{}^1 \end{aligned}\right\} \text{(local demands)}$$

$$\left.\begin{aligned} x_{00}{}^1 + x_{10}{}^1 + \ldots + x_{n0}{}^1 &= E_0{}^1 \leq A_0 \\ x_{01}{}^1 + x_{11}{}^1 + \ldots + x_{n1}{}^1 &= E_1{}^1 \leq A_1 \\ x_{0m}{}^1 + x_{1m}{}^1 + \ldots + x_{nm}{}^1 &= E_m{}^1 \leq A_m \end{aligned}\right\} \text{(regional supplies)}$$

$$\left.\begin{aligned} x_{ij}{}^1 - x_{ij}{}^0 &\leq \Delta_{ij} x_{ij}{}^0 \\ x_{ij}{}^0 - x_{ij}{}^1 &\leq \Delta_{ij} x_{ij}{}^0 \end{aligned}\right\} \text{(controlled gradient)}$$

where Δ_{ij} is the allowable fractional change in $x^0{}_{ij}$ and may be systematically varied,

$$x_{ij}{}^1 \geq 0 \qquad \text{(non-negativity)} \quad .$$

The Δ_{ij} reflect local political and institutional constraints, and it is anticipated that they are known before the solution x_{ij} is attained. However, if on *post hoc* analysis it happens that a community is unwilling to accept its share $x_{ij}{}^1$, a new set of Δ_{ij} may be tried. This continues until an acceptable allocation array is found. The solution confers minimal value on a linear objective function of the form

$$Z = \sum_i \sum_j \lambda_{ij} \, (x_{ij}{}^1 - x_{ij}{}^0) \quad , \qquad \text{(cost of adjustment)}$$

wherein λ_{ij} is the weighted cost of transferring a unit of water from basin B_j to municipality M_i. Other linear terms may be added as appropriate; under certain conditions the objective function might be extended to include quadratic terms. The point to recall is that the $x_{ij}{}^1$ consist of waters derived from several sources (e.g., gw, sw, ppn) and directed at several uses (e.g., ws, ind, irr) and that these may embed additional constraints within the simple set given above. For example, if we laboriously and inelegantly add 2 more indices so that $y_{ijk\ell}$ is that portion of x_{ij} taken from stock k and directed at use ℓ, and if we can parcel the total available resource A_j into stocks A_{jk} such that $\sum_k A_{jk} = A_j$ for all j = 0, 1, ..., m, then we can impose the further linear constraint set

$$\sum_{i=o}^{n} \sum_{\ell} y_{ijk\ell} \leq A_{jk}$$

to guarantee internal consistency with respect to mass balance and simply redefine the x_{ij} in terms of new decision variables $y_{ijk\ell}$. The objective function weights, λ_{ij}, now reflect the fractions of x_{ij} owing to each of the (k, ℓ) couples and the relative price of each.

The objective function chosen for this system is cost minimization. This precludes any *a priori* discussion of benefits, the evaluation of which is now relegated to the acceptance by the parties of the components of the demand vector. Thus the objectives are in a sense converted into constraints.

4. Further Explorations

Clearly a solution can be attained for every feasible demand vector--, i.e., for every demand vector $\underline{D}^r$ that arises from an exogenous analysis, random sample, or whatever. Associated with each is a minimal scalar cost Z^r and a vector of dual variables Π which reflect the shadow prices or the values of relaxing the several constraints. Where the dual variable is zero the associated constraint does not bind, so from the duals the several municipalities can learn the importance of retaining the various constraints.

One important purpose which might be served by the methodology is to identify stable components of the solution, or components which remain in the final basis almost independent of the preferences Δ_{ij} and the demand $\underline{D}^r$. That is, we would like to find at least several elements $x_{ij}^{\ r}$ which are so clearly influential that they appear in the solution to virtually every problem. This suggests that the hardware and structures associated with $x_{ij}^{\ r}$ can confidently be installed because any changes in the plan are not likely to involve $x_{ij}^{\ r}$.

More to the point, each municipality is likely to have a different view of the "true value" associated with the solution ($\underline{x}^r$ and Z^r) to the problem posed by specifying $\underline{D}^r$. That is, under the $\underline{r}^{th}$ scenario the common regional interest might be best served by exporting from a particular municipality as much water as possible and reducing its water-dependent activities in another municipality.

If decisions in Skåne were to be made by a monolithic decision-making authority whose preference function across different demand vectors could be represented by the scalar Z^r associated with each $\underline{D}^r$, it would be a simple matter to propose a large number of scenarios and to implement that solution which minimizes over $Z^0, Z^1, \ldots, Z^r$. That is, only the scalar is of consequence, not the allocations, to the authority. But if a consensus is to be reached among the municipalities, that minimal Z^r might be associated with a decision unacceptable to at least one participant. Solution by Paretian Analysis is then indicated to eliminate a large number of proposals $\underline{D}^r$ and to identify a negotiation frontier among the few undominated alternatives. This form of analysis is shown in Figure 5, which is a 2 dimensional decision space (only 2 decision-makers, but the concept generalizes immediately). For example, in a trivial case the 2 decision-makers might be parties whose span of control encompasses several communities with similar objectives. Each participant considers all the options and calculates the perceived benefits; these are plotted as (X_i, Y_i) for the $\underline{i}^{th}$ option. Any point which lies to the south or west of another point is said to be dominated because either player (or both) could do better by moving to the northeasterly point.

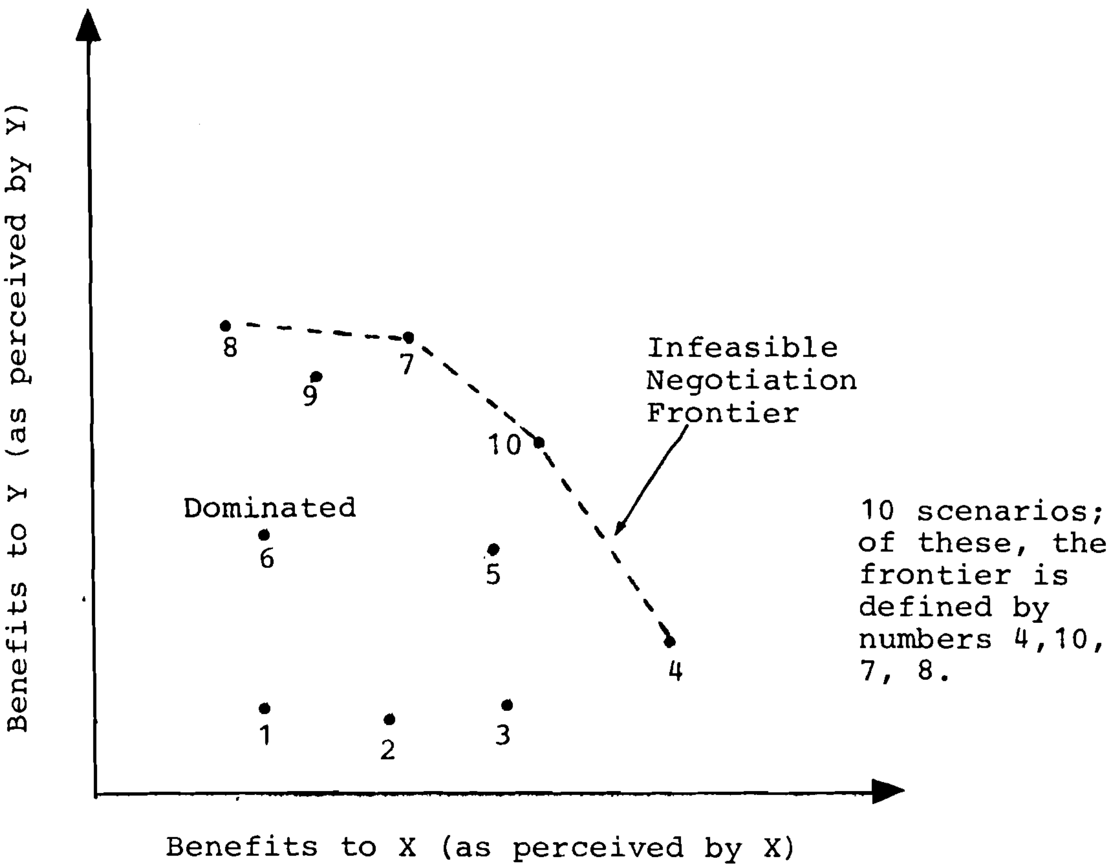


Figure 5. Paretian analysis.

If we assume that the players are not malevolent so that they do not purposefully obstruct one another unless it is to their own advantage to do so, only the undominated options need to be considered. These form a negotiation frontier along which the solution should lie. The closeness of that solution to one axis or another is determined by the bargaining strength of that participant. Thus if X is more influential it is reasonable to predict that the solution will prefer X to Y and that the equilibrium point will be closer to the X-axis than to the Y-axis.

It is also clear that the benefit values (X_i, Y_i) can be independent of an affine linear transformation under which the magnitudes but not the ranking of the outcomes are altered. A set of effective side payments can be deduced from the marginal benefits defined along the negotiation frontier.

5. Water Quality

Water quality considerations have been explicitly excluded from the proposed program; this should be remedied. Introduction of these issues should follow institutional paths appropriate to Skåne. For example, downstream users traditionally bear the effects and costs of upstream polluters, whereupon some incentive (or regulatory) process might be implemented to encourage (or require) economically efficient and equitable schemes for cost sharing. Techniques for establishing such schemes appear in many articles, and are not detailed here. The point to note is that water quality degradation should be accounted for by the linear allocation model, perhaps to the extent of identifying chance constraints, on the assumption of off-line calculations which are based on mixing and transfer properties of the stream system.

6. Implementation

It is urged that implementation of the algorithm be undertaken before a major data program is undertaken. When the staff is familiar and comfortable with this material a meeting in Sweden, and real data tabulation, can be arranged.