A GENERALIZED MODEL FOR MARKET SUBSTITUTION

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PREFACE

Market penetration by new technologies is an established fact. The form of the curves of penetration can be expressed by simple mathematical rules, and fit experience very well. However, it has not been able to argue rigorously that future market penetration will follow the same rules, because a theoretical basis for these rules is lacking.

V. Peterka has proposed such a basis. It is shown here that it follows from detailed considerations of the investment practices of centrally planned economies. Another model, heuristically reasonable for market economies, is needed. This report offers such a model. The mathematical structure of the new model is identical with that of Peterka, but the models differ in one significant parameter, as well as in applicable rules for specifying costs.

In spite of these differences, the two models each support the market penetration rules, and thus we can expect that prediction of future market penetrations can be more confidently expressed, <u>both</u> for centrally planned and for market economies.

SUMMARY

V. Peterka (1977) has proposed a theoretical economic framework from which the logistic model for market penetration may be derived. His basic equation is consistent with the use of capital charge rates equal to amortization rate plus industry growth rate, to determine total costs of a technology; and the use of a price which exactly recovers these costs on an industry-wide basis.

Recasting his original model in this form removes a central objection to the original work, since it is no longer implied that all technologies grow explosively in the revised form. Yet, the equations derived for market penetration are not changed by this recasting. This suggests that the model is specific for centrally planned economies, which use the cost, charge and price rules just set forth.

A companion model is proposed for market economies. It is based on the principle that each technology in an industry contributes to increased profit as the industry expands, the share of that contribution being a constant times the existing capacity of that technology.

The Peterka model and the market model can be expressed in identical mathematical form, so that their qualitative features must be similar. However, the parameters used are different. The differences in parameters suggest that rates of technological substitution could be different in centrally planned market economies.

The mathematical form of the combined model is:

$$\frac{\dot{f}_i}{f_i} = \gamma_i \sum_j W_j(d_j-d_i) \quad ,$$

$$W_i = (f_i\gamma_i)/\sum_j(f_j\gamma_j) \quad ,$$

where f_i is the market share of a particular technology, d_i is the total production cost, including capital charges and amortization, and γ_i is a constant of the particular technology. In the Peterka model,

$$\gamma_i = \frac{1}{\alpha_i} \quad ,$$

where α_i is the specific capital investment per unit of production capacity of technology i. For the market model,

$$\gamma_i = \rho/d_i \quad ,$$

where ρ is the logarithmic expansion rate of the industry.

Both models are pseudo-study state models, but all the parameters may be expressed as functions of time without violating the principles of the heuristics on which they are based.

CONTENTS

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B.I. Spinrad

PETERKA MODEL-VALIDITY FOR CENTRALLY PLANNED ECONOMIES

The fact of market substitution is well established (see, for example, Marchetti et al. (1978)). However, the theoretical basis of logistic substitution is not well established. For example, Peterka (1977) exhibits a model in which investment in a technology is made at a rate such that new facilities are financed by the marginal income from existing facilities of the same type. Mathematically, this is expressed as:

$$\alpha_i \dot{P}_i = P_i(p-c_i) \quad , \tag{1}$$

where P_i is capacity of plants exhibiting technology i, α_i is investment required for unit increase of that capacity, p is price of the commodity and c_i is operating cost per unit commodity. For example, in electrical generation P_i might be kilowatts, α_i dollars/kW and p and c_i\$/kW-yr, with $\dot{P}_i$ then being yearly capacity addition rate in kW/yr. The operating cost is defined, according to Peterka, so as to include charges against capital for amortization and taxes, but not charges for profit or for accumulation of new capital by the enterprise. These latter items are, rather, taken up in the term $p-c_i$.

As shown in Peterka (1977), this model can be manipulated to remove price, p, from consideration, and expressed in terms of fractional market shares, $f_i = P_i/P$, so that a substitution model independent of industry growth rate is generated when specific investments are made equal; and growth rate, $\frac{\dot{P}}{P} \equiv \rho$ for the industry remains a weak parameter otherwise.

In spite of these facts, the model still rests on equation (1), and this equation is quite vulnerable. Of greater concern is the fact that commodity price, p, is always greater than unit operating cost, c_i, for virtually every competing technology in a capital-intensive industry. Thus, even when, for example, high operating costs make a technology relatively uneconomic, equation (1) predicts exponential growth for that technology. Indeed, equation (1) is best understood as a mathematical formulation of Libermanism. For if we sum equation (1) over all i, the left hand side becomes equal to the total rate of system investment for new facilities and the right hand side becomes the rate of capital accumulation in existing facilities. Their equality is then a statement that the price should be set at such a level that the growth of the industry is entirely self-financed. But this is the basic economic principle only of centrally planned economies. Market economies, on the other hand, can show strong flows of capital into and out of an industry, even without incentives or disincentives due to matters of policy. The main influence is simply the profitability of investment in the particular industry as compared with that of alternative investments.

Notwithstanding this special applicability of the model, it is internally consistent to recast Peterka's model into one which does not require explosive growth. This recasting is done by noting that, if we define system growth rate as an extrinsic parameter,

$$\frac{\dot{P}}{P} \equiv \rho \quad , \tag{2}$$

then ρ becomes the rate at which capital costs are charged, and we can express total planned costs of system i as

$$d_i = \alpha_i \rho + c_i \quad . \tag{3}$$

Then, we can express (1) as

$$\alpha_i \dot{P} = \alpha_i \rho P_i + P_i (p - d_i) \quad . \tag{4}$$

In this form, the system expansion rate, ρ, becomes an explicit forcing parameter for the system. That is to say, the addition and subtraction of the term $\alpha_i \rho P_i$ to the right hand side of (1) permits incorporation of capital charges (e.g., investors' profits) into costs while also exhibiting the influence of system expansion rate, ρ, on the growth rate of a specific P_i according to the model.

We can also solve equation (1) to find the price, p. If we divide both sides of (1) by α_i and then sum over all i, we may derive, with the help of (2) and (3)

$$p = \sum_i \frac{P_i}{\alpha_i} d_i / \sum_i \frac{P_i}{\alpha_i} \quad . \tag{5}$$

Finally, the basic model equation can be expressed in terms of market shares,

$$f_i \equiv P_i/P \quad , \tag{6}$$

as

$$\alpha_i \frac{\dot{f}_i}{f_i} = p - d_i \quad . \tag{7}$$

Equations (5) and (7) lead to the same development as exhibited by Peterka.

GENERALIZATION TO INCLUDE REPLACEMENT OF AMORTIZED PLANT

The previous development did not include replacement of amortized plant. Yet the rate of new construction is not $\dot{P}$, but $\dot{P} + \sum a_i P_i$, where a_i is the retirement rate of new facilities of type i; and the rate of new construction of type "i" $\dot{P}_i + a_i P_i$. If we use the principle that all new construction is to be self-financed in detail, equation (1) then gets corrected to

$$\alpha_i [\dot{P}_i + a_i P_i] = P_i (p - c_i) \quad . \tag{8}$$

Converting to fractional shares, we get, after some manipulation,

$$\alpha_i \frac{\dot{f}_i}{f} = p - (c_i + [\rho + a]\alpha_i) \quad . \tag{9}$$

We preserve the form of the basic equations, (5) and (7), if we define

$$d_i \equiv c_i + [\rho + a_i]\alpha_i \quad . \tag{10}$$

Equation (10) demonstrates that the inclusion of amortization charges, $a_i\alpha_i$, in the operating costs is not merely permitted, but required; for a little reflection will show that the rhs of (8), representing the actual cash flow, rests on a definition of c_i in this particular instance as consisting of operating costs only.

MARKET ECONOMIES

In a market economy, price is, in principle a function of demand. We can define

$$\rho = \frac{\dot{P}}{P} = \rho_o - \beta \frac{\dot{p}}{p} \quad , \tag{11}$$

when ρ_o is the growth rate that would pertain for the industry if price were constant and β is the price elasticity of demand. As used here, β is a positive number, expected to be between 0 and 1; demand goes down as price goes up.

We can define total system profit, or gain, as

$$G \equiv \sum_j (P_j p - P_j d_j) = Pp - \sum_j P_j d_j \quad . \tag{12}$$

The rate of change of profit can then be found as

$$\dot{G} = \dot{P}p + P\dot{p} - \sum_j \dot{P}_j d_j \quad . \tag{13a}$$

Substituting (11) and eliminating $\dot{p}$,

$$\dot{G} = \left(\rho + \frac{\rho_o + \rho}{\beta}\right)Pp - \sum_j \frac{\dot{P}_j}{P_j} P_j d_j \quad , \tag{13b}$$

or finally

$$\dot{G} = \sum_j P_j \left[\left(\rho + \frac{\rho_o - \rho}{\beta} \right) p - \frac{\dot{P}_j}{P_j}\, dj \right] \quad . \tag{13c}$$

A strategic principle based on the concept that each technology contributes proportionally to increase in profit is:

$$d_i \frac{\dot{P}_i}{P_i} = \lambda \left(\rho + \frac{\rho_o - \rho}{\beta} \right) p \quad , \tag{14a}$$

or

$$\frac{\dot{f}_i}{f_i} = \lambda \left(\rho + \frac{\rho_o - \rho}{\beta} \right) \frac{p}{d_i} - \rho \quad . \tag{14b}$$

This principle is entirely arbitrary. It has been selected because it corresponds most closely to the strategy of Peterka's principle as translated to market economies, a matter which is illustrated in the next section.

Multiplying (14b) by f_i and summing enables us to solve for λ:

$$\lambda \left(\rho + \frac{\rho_o - \rho}{\beta} \right) p = \frac{\rho}{\sum_j \frac{f_j}{d_j}} \quad , \tag{15}$$

yielding finally,

$$\frac{\dot{f}_i}{f_i} = \frac{\rho}{d_i} \left[\frac{1}{\sum_j \frac{f_j}{d_j}} - d_i \right] \quad . \tag{16}$$

We can think of this by analogy to (7) if we define a "ghost price", p':*

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*We use the term "ghost price" because it is an entirely illusory concept. It can not be confused with "shadow price", the price a technology would command if deployed.

$$p' \equiv \frac{1}{\sum_j \frac{f_j}{d_j}} \quad . \tag{17}$$

Then, (16) becomes

$$\frac{\dot{f}_i}{f_i} = \frac{\rho}{d_i} (p'-d_i) \quad . \tag{18}$$

It is straightforward to show that this is a gainful strategy (positive $\dot{G}$) whenever

$$p > \frac{p'}{1 + \frac{\rho_o - \rho}{\beta\rho}} \quad . \tag{19}$$

This is normally the case; for, if the industry is profitable,

$$p > \sum_j f_j d_j > p' \quad , \tag{20}$$

and ρ is normally close to ρ_o. This latter is a statement that prices (in constant-value currency) change slowly in a stable industry.

For market economies, amortization charges are incorporated into d_i ab initio.

THE TWO MODELS AS STRATEGIC PRINCIPLES

Peterka's model states that the rate of increase in investment in a technological option is a product function of two factors: the rate of positive cash flow per unit output, $p-c_i$; and the existing deployment P_i. For a society intent on achieving production increases with a minimum of capital requirements, this is a suboptimal choice. Instead, all new investment would be concentrated in that technology which exhibited the largest value of $(p-c_i)/\alpha_i$.

Similarly, the market model proposed here is not optimum for the purpose of maximizing increased profits from systems expansion. In that case, one would construct all new plants according to the technology for which d_j is minimum.

It is nevertheless usually the case that technologies with clear economic advantage do not immediately take over the market for expansion of a product. Both Peterka's model and the one offered here prescribe, instead, a situation in which all competent technologies share in the market expansion. This can only be considered as a strategy which has non-economic justifications. What are they?

Both models exhibit a relation in which $\dot{P}_i$ is proportional to P_i. In Peterka's model, elimination of price leads to

$$\dot{P}_i = P_i\left(\rho + \frac{\sum f_j (d_j - d_i)}{\alpha_i}\right) . \qquad (21a)$$

In the "market" model the corresponding equation is

$$\dot{P}_i = P_i\left(\frac{\rho}{d_i \sum \frac{f_j}{d_j}}\right) . \qquad (21b)$$

What does the factor P_i tell us about the hidden strategy?

It is here proposed that the factor P_i signifies a safety factor. It is safe to build what has been built before, and the factor P_i ensures that this is given weight in the strategy. It may also be argued that the validity of the economic projection is proportional to experience--the more existing plant there is, the better the economics are known. This gives safety a slightly different meaning: the inverse of uncertainty in the economic factor.

The factors in parentheses (21a) and (21b) are, on the other hand, measures of economic merit. For (21a), this merit measure is the ratio of cash income to specific investment. For (21b) it is essentially the reciprocal of total unit cost, including all capital charges.

In summary, both models weight new plant additions as being proportional to two factors. One is a measure of economic merit, and the other is a measure of the reliability of the economic measure. Only the economic merit measures are different between the two models. And, indeed the measures of economic merit that are exhibited reflect the differences in philosophy between the centrally planned and non market systems.

THE TWO MODELS--COMPARISON AND CORRELATION

It is interesting to compare the two models: the Peterka model as reworked here, with d_i defined by equation (10), then p by equation (5), and finally the model equation (7); and the market model just constructed, with ghost price, p', defined by (17) and the model itself exhibited as equation (18).

The definitions of d_i in the two systems are entirely compatible if we think of ρ as a capital charge rate for planned economies. However, we must replace the real price, p, of the planned economy model with the ghost price, p', in the market model. With this change, both models are of the form:

$$\frac{\dot{f}_i}{f_i} = \gamma_i \sum_i (q-d_i) \quad . \tag{22}$$

For the Peterka model

$$\left.\begin{aligned} \gamma_i &= \frac{1}{\alpha_i} \quad , \\ q &= p \quad . \end{aligned}\right\} \tag{23a}$$

For the market model

$$\left.\begin{aligned} \gamma_i &= \frac{\rho}{d_i} \quad , \\ q &= p' \quad . \end{aligned}\right\} \tag{23b}$$

The common mathematical structure guarantees that the two models will have qualitative features in common. However, the substitution orders and rates could be quite different.

The mathematical structure similarity can be emphasized by writing the two models in a slightly more transparent form which takes advantages of the definitions of p and p'. This form is

$$\frac{\dot{f}_i}{f_i} = \gamma_i \sum_j W_j (d_j - d_i) \quad . \tag{24}$$

For both models,

$$W_i = \gamma_i f_i / \sum_j \gamma_j f_j \quad . \tag{25}$$

For the Peterka model,

$$\gamma_i = \frac{1}{\alpha_i} \quad , \tag{26a}$$

and for the market-economy model

$$\gamma_i = \frac{\rho}{d_i} \quad . \tag{26b}$$

It should also be noted that the costs, d, and the values of γ can be renormalized by any factor that is independent of i (renormalized in opposite senses, of course, so that what multiplies γ divides d). This device may make numerical comparisons more transparent.

REFERENCES

Peterka, V. (1977) Macrodynamics of Technological Change: Market Penetration by New Technologies. RR-77-22. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Marchetti, C., N. Nakicenovic, V. Peterka, and F. Fleck (1978) The Dynamics of Energy Systems and the Logistic Substitution Model. AR-78-1A, -1B, -1C. Laxenburg, Austria: International Institute for Applied Systems Analysis.

APPENDIX

AN EXACT SOLUTION FOR A SPECIAL CASE

Peterka has demonstrated that certain features of the solutions to his equations are quite insensitive to the values of the α_i used. From this observation, one derives some interest in the case where γ_i are replaced by constant values, $\overline{\gamma}$. The situation is of even greater interest for the market model, as it is even more likely that the $\frac{1}{d_i}$ values will be close than is that $\frac{1}{\alpha_i}$ will be close--at least, for situations where substitution is slow.

If we replace γ_i by $\overline{\gamma}$, the model equations become:

$$\frac{\dot{f}_i}{f_i} = \overline{\gamma} \sum_j f_j (d_j - d_i) \quad . \tag{A-1}$$

This set of equations has a solution in closed form. It is:

$$f_i = \frac{c_i}{\sum_j c_j e^{-\overline{\gamma}(d_j - d_i)t}} = \frac{c_i e^{-\overline{\gamma} d_i t}}{\sum_j c_j e^{-\overline{\gamma} d_j t}} \quad . \tag{A-2}$$

(A-2) applies for constant $\overline{\gamma}$, d_i, but it is even more generally

$$f_i = \frac{c_i \exp - \int_o^t \overline{\gamma} d_j dt'}{\sum_j c_j \exp - \int_o^t \overline{\gamma} d_j dt'} , \qquad \text{(A-3)}$$

when $\overline{\gamma}$ and the d_i vary with time. The c_i are determined, of course, by conditions at the reference time, $t = 0$.