MULTIREGIONAL METHODS FOR SUBNATIONAL POPULATION PROJECTIONS

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FOREWORD

Roughly 1.6 billion people, 40 percent of the world's population, live in urban areas today. At the beginning of the last century, the urban population of the world totaled only 25 million. According to recent United Nations estimates, about 3.1 billion people, twice today's urban population, will be living in urban areas by the year 2000.

Scholars and policy makers often disagree when it comes to evaluating the desirability of current rapid rates of urban growth in many parts of the globe. Some see this trend as fostering national processes of socioeconomic development, particularly in the poorer and rapidly urbanizing countries of the Third World; whereas others believe the consequences to be largely undesirable and argue that such urban growth should be slowed down.

As part of a search for convincing evidence for or against rapid rates of urban growth, a Human Settlements and Services research team, working with the Food and Agriculture Program, is analyzing the transition of a national economy from a primarily rural agrarian to an urban industrial-service society. Data from several countries selected as case studies are being collected, and the research is focusing on two themes: spatial population growth and economic (agricultural) development, and resources/service demands of population growth and economic development.

This paper focuses on urban-rural population projection methods and contrasts projections obtained by uniregional methods with those derived by biregional and multiregional techniques. Problems of bias and inconsistency are discussed, and the advantages of disaggregated multiregional projections are pointed out.

A list of related papers in the Population, Resources, and Growth Series appears at the end of this publication.

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ABSTRACT

Most projections of urban and rural populations are generated by models that are fundamentally nonspatial and uniregional in character. Migration streams are treated as net flows, and urban and rural populations are projected independently of each other. This paper argues for a multiregional spatial perspective that incorporates directional gross migration flows. Differences between the two approaches are identified and problems of bias and inconsistency are discussed.

CONTENTS

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Although methods for carrying out system-wide projections of interdependent subnational populations have been available for some time, they have not been widely used for reasons that are not at all self-evident. The multiregional methods are not difficult to grasp and, when programmed for execution on any of the current generation of computers, they can be used to summarize and analyze vast quantities of demographic data at a very modest cost. The information they provide is of central importance for spatial planning and comparable results cannot be obtained as effectively with more conventional uniregional (i.e., single-region) projections, such as are used, for example, by the United Nations (Appendix A).

This paper focuses on multiregional population projection methods and illustrates some of their advantages over uniregional techniques. Because a large number of different subnational projections are possible within an individual country, we cannot in this essay consider all of the combinations that might be relevant. Consequently, we shall focus our attention on a particular example of multiregional mathematical demography: the biregional urban-rural population model and its extensions. However, it should be clear that the methods described herein also are applicable to more than two regions, indeed even to regions that are not regions in the physical sense but that are states of existence, for example, the states of being married, divorced, healthy, sick, employed, and unemployed.

AGGREGATION AND DECOMPOSITION IN POPULATION PROJECTION

The large data and computational requirements of many current socioeconomic forecasting and projection efforts have stimulated a renewed interest among social scientists in techniques for reducing the dimensionality of large-scale models. Two of the most commonly used "shrinking" techniques to effect such reductions are aggregation and decomposition (Rogers 1976a).

Aggregation

Aggregation in demographic analysis may be carried out by consolidating population characteristics, time units, or spatial units. As in economic analysis (e.g., input-output modeling), the consolidation expresses a set of "new" variables as weighted averages of a set of original "old" variables, such that there are fewer new variables than old ones. And, as in economic analysis, aggregation generally introduces differences between the outputs of the disaggregated and the aggregated models--differences that are frequently referred to as biases.

Consider, for example, a projection of the 1970 USSR total population to 1980 on the then prevailing national rate of growth of .09 percent per year (Appendix B). The national total of 242 million would be expected to increase to 265 million. Now disaggregate the national population into urban and rural components and repeat the projection, applying the urban and rural rates of growth separately to each of the two constituent subpopulations. The urban rate in 1970 was 2.5 percent and the urban population stood at 136 million; a 10-year projection, therefore, gives a 1980 urban population of 174 million. A parallel computation for the rural population, using the rate of -1.1 percent and a base of 106 million, produces a 1980 rural population of 95 million. Thus, relative to the disaggregated projection, the aggregated model underprojects the total by 4 million in just 10 years.

Keyfitz (1977:14-18) shows that such an underprojection bias will always occur, proving that the sum of the separate projections of each of the parts of a heterogeneous population will always be greater than the projection of the sum of the parts at the average rate of increase prevailing at the start. The aggregation introduces a projection bias.

Decomposition

The fundamental idea behind decomposition is the notion that the analysis of a complex system containing a large number of interacting components can often be carried out by combining analyses of its relatively independent subsystems.

Perhaps the most common application of decomposition in demographic analysis is the representation of a multiregional population system by a collection of independent uniregional models. The multiregional system is modeled one region at a time. These single-region models may assume that each regional population is undisturbed by migration; more commonly, they account for the impact of migration by introducing the notion of net migration. In such a perspective outmigrants are treated as deaths and inmigrants as replacements for deaths in the calculation of survivorship proportions.

The process of decomposition generally proceeds in two stages. First, there is the search for disjoint subsystems, that is, subsets of relations that do not contain any common variables. If such subsystems exist, then each one can be analyzed independently of the others. The rearrangement and reordering that is part of this search is often referred to as partitioning.

Once a large system of variables and relations has been either completely or partially decomposed into subsystems that cannot be decomposed further, additional simplification can only be achieved by a separation, called tearing, that simply deletes variables from one or more of the relations in which they appear. If some adjustment is introduced to take into account the effects of the deletion, then we have an instance of compensated tearing. Net migration is a form of compensated tearing inasmuch as it compensates for the separation that occurs when two spatially interdependent populations are separately and independently projected into the future.

The growth and change of a total population that is the sum of several recognizable subpopulations can be projected with or without an explicit recognition of directionally-specific transfers among the subpopulations. Our two simple uniregional projections of the urban and rural populations of the Soviet Union did not explicitly recognize directional gross migration flows between urban and regional populations. Thus they may be viewed as arising out of a decomposition of a biregional model that explicitly defines rates of transfer between these two subpopulations. Ignoring the two rates of transfer would be a consequence of tearing; reintroducing their net impact by means of a net migration rate is an example of compensated tearing. As in the case of aggregation, such a decomposition introduces a projection bias.

UNIREGIONAL VERSUS BIREGIONAL PROJECTIONS OF URBAN AND RURAL POPULATIONS

There are three principal ways of incorporating internal migration in regional population projections. The first focuses on net migration, the other two on gross migration. Net migration totals reveal only the "tip of the iceberg" because they describe a difference; they are difficult to model behaviorally because there is no such individual as a net migrant, and they generally introduce a bias into the projection process because both the numerator and the denominator of the net migration rate are changing.

Gross migration may be entered into the projection process in a simple way by considering only inflows and outflows (a biregional perspective) or, in a more complex manner, by keeping track of the various origins and destinations (a multiregional perspective). In either case, one obtains a considerable increase in useful information over the net migration projection (a uniregional perspective).

Bias and Inconsistency

A uniregional perspective of population growth and change can easily introduce biases and inconsistencies into a regional population projection. The problems arise because all migration flows are assessed only with respect to the population in the region of destination. Thus changes in the size of the destination population, arising out of changes in the patterns of natural increase for a given year, will produce a higher net migration total in the following year and introduce a bias into the projection.

Changes in the population at the region of origin are totally ignored, an omission that can produce serious inconsistencies in the projection process. For example, the origin population ultimately may be reduced to zero, but a fixed and positive net migration rate in the destination region will nevertheless continue to generate a flow of net inmigrants from that region.

The conditions leading to bias and inconsistency in a uniregional model may be illuminated by a return to our numerical illustration based on data for the Soviet Union. For ease of exposition, fixed rates of fertility, mortality, and migration will be assumed throughout this paper.

The urban population of the Soviet Union was increasing by about 2.5 percent a year during the early 1970s. The urban growth rate, r_u, was the outcome of a birth rate of 17 per 1000, a death rate of 8 per 1000, an inmigration rate of 27 per 1000, and an outmigration rate of 11 per 1000 (Rogers 1976b, and Appendix B). Expressing these rates on a per capita basis leads to the fundamental identity

$$r_u = b_u - d_u + i_u - o_u$$

$$= .017 - .008 + .027 - .011$$

$$= .025 \quad .$$

The corresponding identity for the rural population was

$$r_r = b_r - d_r + i_r - o_r$$

$$= .019 - .009 + .014 - .035$$

$$= -.011 \quad .$$

The total national population of the USSR in 1970 was about 242 million, of which roughly 136 million (56 percent) was classified as urban. Multiplying this latter total by the urban growth rate gives

$$136(.025) = 3.40 \text{ million}$$

as the projected increase for 1971. An analogous calculation for the rural population yields

$$106(-.011) = -1.17 \text{ million}$$

for the corresponding projected decrease in the rural population. These changes imply, for 1971, an urban population of 139 million, a rural population of 105 million, and a rate of national population increase of

$$\frac{136(.025) + 106(-.011)}{242} = .56(.025) + .44(-.011) = .009 \quad ,$$

i.e.,

$$r_T = .56\, r_u + .44\, r_r = .009 \quad .$$

A uniregional perspective of urban growth in the Soviet Union would describe the dynamics of urbanization by focusing on the natural increase and net migration components of the .025 rate of urban growth, i.e., .009 and .016, respectively. On the assumption that these rates remain fixed, a 10-year projection gives a 1980 urban population of

$$P_u(1980) = 136(1.025)^{10} = 174.09 \text{ million} \quad .$$

Subtracting this quantity from the corresponding projected national total of

$$P_T(1980) = 242(1.009)^{10} = 264.68 \text{ million}$$

gives, as a residual, a projected rural population of

$$P_r(1980) = 264.68 - 174.09 = 90.59 \text{ million} \quad .$$

An alternative uniregional formulation is one that projects the rural population directly and obtains the national total as the sum of the urban and rural projections. In this perspective, the projected rural population in 10 years is

$$106(.989)^{10} = 94.90 \text{ million} \quad ,$$

and the national total is

$$174.09 + 94.90 = 268.99 \text{ million} \quad .$$

These calculations simply set out explicitly what was implicit in our earlier discussion of aggregation. And, as was pointed out there, the aggregation of the urban and rural growth rates before projection introduces a bias--an underprojection of more than 4 million over a decade.

Both uniregional projections ultimately lead to a fundamental inconsistency. The fixed rates eventually empty out the rural areas, but net inmigration to the urban areas continues unchanged! In the long run, the entire national population is urban, and is growing at a rate of .025 a year, even though the annual rate of natural increase in urban areas is still assumed to be .009.*

A logical escape from this built-in inconsistency of the uniregional perspective is the adoption of a biregional model in which rates of migration reflect populations at risk. The urban rate of natural increase is the same as before, but the urban net migration rate of .016 now is decomposed into its in- and outmigration components:

$$m_u = i_u - o_u = .027 - .011 = .016 \quad ,$$

and the inmigration rate is then expressed as the product of a rural outmigration rate of .035 and the ratio of rural to urban population, .78. Thus

$$m_u = o_r\left(\frac{P_r}{P_u}\right) - o_u = .035(.78) - .011 = .016 \quad ,$$

and urban population growth may be described by**

$$P_u(t + 1) = (1 + b_u - d_u - o_u)P_u(t) + o_rP_r(t) \quad . \qquad (1)$$

Equation (1) states that next year's urban population total may be calculated by adding to this year's urban population (a) the increment due to urban natural increase, (b) the decrement due to urban outmigration to rural areas, and (c) the increment

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*We are not, of course, suggesting that anyone would carry the projections forward to such an extreme. Nevertheless, the assessment of a model's reasonableness by reference to its asymptotic properties is a well-established procedure for evaluating, for example, alternative models of comparative statics and dynamics in economics.

**The crude migration rates are assumed to also include the contribution of births to migrants and migrating births. This contribution is more readily measured with the age-disaggregated model, where it appears in the top rows of the off-diagonal submatrices in the multiregional projection matrix.

due to rural to urban migration. Substituting in the rates for the Soviet Union gives the accounting identity

$$P_u(1971) = (1 + .017 - .008 - .011)P_u(1970) + .035\ P_r(1970)$$

$$= .998(136) + .035(106)$$

$$= 139.44 \text{ million} \quad .$$

An analogous equation for the rural population yields

$$P_r(1971) = .011\ P_u(1970) + (1 + .019 - .009 - .035)P_r(1970)$$

$$= .011(136) + .975(106)$$

$$= 104.85 \text{ million} \quad .$$

Assuming, once again, that the various rates remain unchanged over a 10-year period, gives

$$P_u(1980) = 168.53$$

$$P_r(1980) = 97.21$$

$$P_T(1980) = 168.53 + 97.21 = 265.74 \quad .$$

Apart from minor errors due to rounding, the biregional model generates the same 1971 projection as the second uniregional model; its 10-year projection, however, differs from the two uniregional projections. Moreover, its long-run projection does not locate the entire national population into urban areas. Its stable distribution accords rural areas about a fourth (.2431) of the total stable national population.

The differences between the uniregional and biregional projections are due to bias, and the principal cause of this bias may be shown to be a consequence of treating gross migration flows as a net flow. To see this more clearly, consider how the migration specification is altered when the biregional model is transformed into a uniregional model. The accounting relationship in equation (1) is replaced by the expression

$$P_u(t + 1) = (1 + b_u - d_u - o_u)P_u(t) + \left[o_r \frac{P_r(t)}{P_u(t)}\right] P_u(t)$$

$$= (1 + b_u - d_u - o_u + i_u)P_u(t)$$

$$= (1 + b_u - d_u + m_u)P_u(t) = (1 + r_u)P_u(t) \quad , \qquad (2)$$

where

$$i_u = o_r \frac{P_r(t)}{P_u(t)} = o_r\left[\frac{1 - U(t)}{U(t)}\right] \quad , \qquad (3)$$

$$m_u = i_u - o_u \quad ,$$

and U(t) is the fraction of the total national population that is urban at time t. In the 10-year projection, all annual rates are assumed to be fixed. But i_u, and therefore also m_u, depend on U(t), which varies during the 10 years and creates a bias.

Bias and inconsistency may result from viewing biregional (and, by extension, multiregional) population systems through a uniregional perspective. Expressing migration's contribution to regional population growth solely in terms of the population in the region of destination, can lead to over- or underprojection and introduce inconsistencies in long-run projections. These problems of bias and inconsistency become even more important when age composition is taken into account.

Age Composition Effects

Crude rates are weighted combinations of age-specific rates; changes in age composition alter the weights and produce changed crude rates. Aggregation of age groups, therefore, creates a bias of its own, quite apart from the bias introduced by viewing gross migrations flows as net flows. Table 1 illustrates the extent of this aggregation bias for our biregional illustration based on data for the Soviet Union.

A biregional cohort-survival projection of the 1970 USSR population to the year 2000, on the assumption of unchanging age-specific rates of fertility, mortality, and internal migration, gives a total population of 313.57 million, with 73 percent of that total residing in urban areas. The annual growth rate of the urban population is .013 and that of the rural population is almost zero [$r_r(2000) = .0006$]. The national population at that moment in time is projected to be increasing at the rate of .009 per year.

How does this projection compare with one produced by the aggregate biregional model of the preceding section? The corresponding totals provided by that model are

$$P_u(2000) = 226.58 \qquad r_u(2000) = .013$$

$$P_r(2000) = 93.37 \qquad r_r(2000) = .001$$

$$P_T(2000) = 319.95 \qquad r_T(2000) = .009$$

$$U(2000) = .71 \quad .$$

The very slight differences between the above totals and rates of growth and those set out in Table 1 are a consequence of aggregation bias, which in this illustration results from an aggregation across age groups.

Differences in projected totals introduced by aggregation have their counterparts in differences introduced by decomposition, i.e., by a decoupling of the population of interest from

Table 1. Uniregional and biregional cohort-survival projections of the 1970 urban and rural populations of the USSR to the year 2000.

Age, x	Uniregional projections[1]				Biregional projection[2]			
	Population (in thousands)		Age composition		Population (in thousands)		Age composition	
	Urban	Rural	Urban	Rural	Urban	Rural	Urban	Rural
0-4	20,494	5,607	0.0719	0.0855	16,350	7,370	0.0710	0.0884
5-9	19,058	5,994	0.0669	0.0914	16,027	7,500	0.0696	0.0900
10-14	18,795	6,606	0.0659	0.1008	16,307	7,897	0.0708	0.0947
15-19	20,081	6,080	0.0705	0.0927	16,701	7,285	0.0725	0.0874
20-24	22,462	4,033	0.0788	0.0615	17,069	5,159	0.0741	0.0619
25-29	22,083	2,780	0.0775	0.0424	16,568	3,862	0.0720	0.0463
30-34	18,297	2,834	0.0642	0.0432	15,619	3,801	0.0678	0.0456
35-39	22,310	3,176	0.0783	0.0484	18,617	4,623	0.0809	0.0555
40-44	23,349	2,984	0.0819	0.0455	18,588	4,712	0.0807	0.0565
45-49	24,181	2,060	0.0848	0.0314	16,006	4,035	0.0695	0.0484
50-54	16,655	1,849	0.0584	0.0282	11,946	3,165	0.0519	0.0380
55-59	10,071	2,602	0.0353	0.0397	8,576	3,090	0.0373	0.0371
60-64	13,399	4,425	0.0470	0.0675	11,886	4,947	0.0516	0.0594
65-69	8,523	3,839	0.0299	0.0586	7,920	4,062	0.0344	0.0487
70+	25,262	10,695	0.0886	0.1631	22,029	11,847	0.0957	0.1421
Total	285,019	65,563	0.8130	0.1870	230,208	83,355	0.7342	0.2658
USSR	350,583		1.0000		313,564		1.0000	
Mean			35.66	36.74			35.83	35.97
Age			35.86				35.86	
Annual Growth Rate	0.0242	-0.0133			0.0127	0.0006		
	0.0172				0.0094			

[1]Source: Uniregional cohort-survival projections using the data in Appendix B.

[2]Source: Biregional cohort-survival projection using the data in Appendix B.

the rest of the national population in order to permit its study within a uniregional perspective. Differences arising as a consequence of such decouplings will be attributed to decomposition bias.

Table 1 also sets out the projected USSR population in the year 2000 that is obtained with a uniregional cohort-survival model (one for the urban population and another for the rural population), and the data in the Appendix B. Differences between the uniregional and the biregional results may be attributed to decomposition bias. Note that the significance of this bias is much greater than the aggregation bias alluded to earlier.

Table 2 presents a uniregional cohort-survival projection to the year 2000 for the total national population of the Soviet Union. The projected national total of just under 323 million is somewhat higher than that of the biregional projection and so is the corresponding annual rate of increase. This is a consequence of the aggregation bias due to the consolidation of the urban and rural populations before the projection. Subtracting either the urban or the rural uniregional projections in Table 1 from the corresponding national totals in Table 2 gives the other projection as a residual, as in the U.N. manual's composite methods (Appendix A).

The profile of a schedule of age-specific net migration rates, such as those used in generating the uniregional projections in Table 1, is defined by the profiles of the corresponding age-specific inmigration and outmigration rates:

$$m_u(x) = i_u(x) - o_u(x) \quad . \tag{4}$$

The relationship in equation (3) still applies, but in this case age, denoted by x, is explicitly a part of the argument:

$$i_u(x) = o_r(x) \frac{P_r(x,t)}{P_u(x,t)} = o_r(x) \left[\frac{1 - U(x,t)}{U(x,t)}\right] . \tag{5}$$

Table 2. Uniregional cohort-survival projection of the total population of the USSR to the year 2000.

Year 2000		
Age, x	Population (in thousands)	Age Composition
0-4	26,181	0.0811
5-9	25,563	0.0792
10-14	25,931	0.0803
15-19	25,379	0.0786
20-24	23,226	0.0719
25-29	20,828	0.0645
30-34	19,421	0.0602
35-39	23,243	0.0720
40-44	23,307	0.0722
45-49	20,050	0.0621
50-54	15,120	0.0468
55-59	11,673	0.0362
60-64	16,847	0.0522
65-69	11,993	0.0371
70+	34,092	0.1056
Total USSR	322,855	
Mean Age		35.20
Annual Growth Rate		0.0108

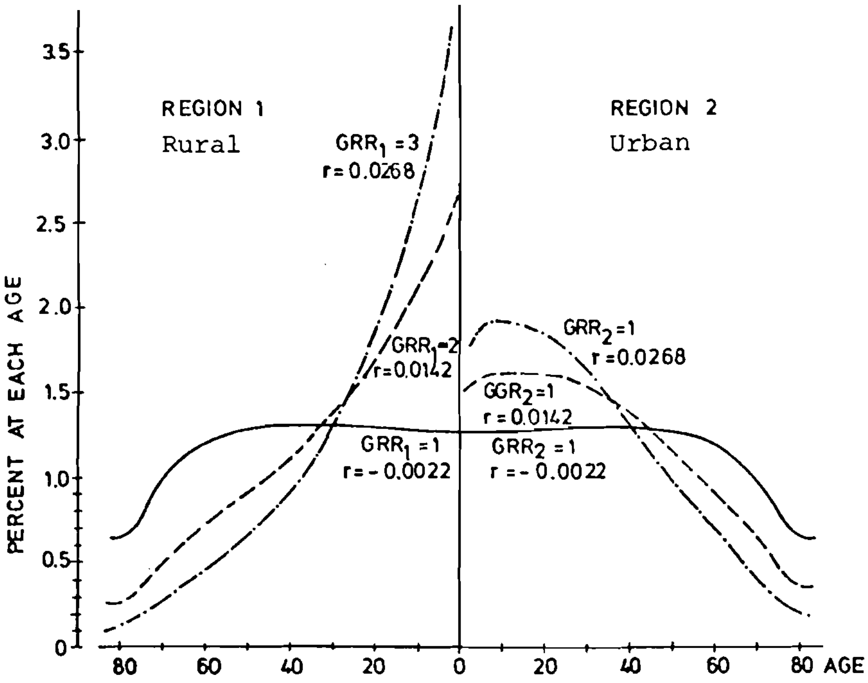
Source: Cohort-survival projection using the fertility and mortality data set out in Appendix B.

Two important observations are suggested by the relationships set out in equations (4) and (5). First, since the age profile of net migration is defined as a difference, it is clear that different combinations of age-specific in- and outmigration schedules can give rise to the same age-specific net migration schedule. Second, since the inmigration schedule is itself a weighted outmigration schedule, changes in the weights over time will change the age-specific inmigration rates. Thus the assumption of a fixed inmigration (or net migration) schedule will produce a bias.

The age compositions of gross migration flows influence not only the projected population totals at origins and destinations, but also <u>their</u> age compositions and, therefore, their crude rates of birth, death, and growth. To emphasize the importance of this contribution to population change, consider a hypothetical urban-rural population system with <u>identical</u> migration schedules for both directional flows, <u>identical</u> mortality schedules in both regions, and <u>different</u> fertility levels (but with identical age profiles). Further, to remove the possibly confusing influences of initial conditions, consider this population only after it has entered a state of <u>stable</u> growth and therefore has "forgotten" its past. How are the changes in fertility levels in the rural population transmitted to the urban population?

Figure 1 illustrates the age compositions of the hypothetical urban and rural populations for three different levels of fertility in the rural region (gross reproduction rates of 1, 2, and 3, respectively), and a fixed gross reproduction rate of unity in the urban region. In all three cases, the expectation of life at birth is 70 years in each region, and 70 percent of that lifetime is assumed to be lived in the region of birth (for further details, see Rogers and Castro 1976:47-54).

The most important conclusion indicated by Figure 1 is that the age composition of the hypothetical urban population changes even though its fertility and mortality schedules do not. The change occurs solely as a consequence of migration from the rural areas. The higher the level of fertility in rural areas, the



Source: Rogers and Castro (1976:53).

Figure 1. Hypothetical rural-urban projections.

larger is the flow of migrants into urban areas and, because most migrants are young, the younger (and, of course, the larger) is the urban population. Note that these dynamics could not occur in a uniregional model; once that model was calibrated for a particular moment in time, any subsequent changes in rural fertility would not affect the urban populations. This suggests that inferences made about fertility levels in the context of a perspective that ignores the fundamental notion of populations at risk in the regions of outmigration can be seriously in error.

Disaggregated Projections

The discussion until now has revolved around uniregional versus biregional models as alternative means for accomplishing the same end: a projection, disaggregated by age, of a nation's urban and rural populations. We now turn to a consideration of ends that can only be achieved with the use of biregional and multiregional models: projections disaggregated by age and residence-duration status. Because uniregional models do not focus on gross migration flows, they cannot follow the migration paths of particular population groups over time; biregional and multiregional models, on the other hand, can identify the life histories of such groups, and this gives them a decisive advantage over uniregional models.

Projections disaggregated by age and residence-duration status have both a retrospective and a prospective aspect. For example, given our earlier projections to the year 2000 of the urban and rural populations of the Soviet Union, we may wish to identify how many of the projected urban residents were living in rural areas at the start of the projection period (i.e., in 1970) or 5 years ago. Or we may be interested in determining what fraction of the projected urban dwellers were born in rural areas (i.e., are "alien" residents), and what proportion are urban "natives". And among urban natives, how many never migrated at all, i.e., were "stayers"? How many were "returners"?

Prospectively, we may ask, what proportion of the 1970 Soviet rural population will be living in urban areas in the year 2000? Or we may wish to calculate, on 1970 rates, the fraction of an average lifetime that a baby just born in a rural village in the Soviet Union can expect to live in the urban settlements of that nation.

To answer these and related questions, let us begin by dividing the resident urban population of a nation into natives and aliens:

$$\text{residents} = \text{natives} + \text{aliens} \quad ,$$

$$P_u(t) = {}_uP_u(t) + {}_rP_u(t) \quad , \qquad (6)$$

where the additional subscript on the left of the population variable, P, denotes the region of birth (the right subscript denotes the region of residence, as before).*

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*Although we speak of "region of birth", it should be clear that we instead could consider "region of residence in 1970" (or at some other past moment in time).

The accounting relationship for calculating urban residents was given earlier as equation (1). The same equation may be used for obtaining urban natives simply by introducing the place of birth subscript and allocating the new births during the year to the "native" population:*

$$ {}_{u}P_{u}(t) = (1 + b_{u} - d_{u} - o_{u})\,{}_{u}P_{u}(t - 1) + o_{r}\,{}_{u}P_{r}(t - 1) + b_{u}\,{}_{r}P_{u}(t - 1) \quad . \qquad (7) $$

Analogous relationships may be defined for ${}_{r}P_{u}(t)$, ${}_{u}P_{r}(t)$, and ${}_{r}P_{r}(t)$. It is assumed that natives and aliens experience the same fertility, mortality, and migration rates, i.e., those prevailing at their region of residence, and all births to alien migrants are added to the alien population stock.

For illustrative purposes, assume that a half of the 1970 Soviet Union's urban population was born in rural areas and that one tenth of its rural population was born in urban areas. Then

$$ {}_{u}P_{u}(1971) = .998(68) + .035(10.6) + 0.017(68) = 69.39 $$

$$ {}_{r}P_{u}(1971) = .981(68) + .035(95.4) = 70.05 $$

$$ {}_{u}P_{r}(1971) = .011(68) + .956(10.6) = 10.88 $$

$$ {}_{r}P_{r}(1971) = .011(68) + .975(95.4) + 0.019(10.6) = 93.96 \quad . $$

Reexpressing the above equations in matrix form gives

*In a forthcoming paper which considers the age-disaggregated model, a fraction of the births to alien migrants is added to the native population in the destination region, transforming four of the zeroes in the matrix below into positive quantities.

$$\begin{bmatrix} {}_uP_u(1971) \\ {}_rP_u(1971) \\ {}_uP_r(1971) \\ {}_rP_r(1971) \end{bmatrix} = \begin{bmatrix} 0.998 & 0.017 & 0.035 & 0 \\ 0 & 0.981 & 0 & 0.035 \\ 0.011 & 0 & 0.956 & 0 \\ 0 & 0.011 & 0.019 & 0.975 \end{bmatrix} \begin{bmatrix} 68 \\ 68 \\ 10.6 \\ 95.4 \end{bmatrix}$$

and, repeating the multiplication 30 times, yields

$${}_uP_u(2000) = 123.63 \qquad {}_ur_u(2000) = .018$$

$${}_rP_u(2000) = 102.95 \qquad {}_rr_u(2000) = .006$$

$${}_uP_r(2000) = 21.06 \qquad {}_ur_r(2000) = .021$$

$${}_rP_r(2000) = 72.31 \qquad {}_rr_r(2000) = -.004$$

$${}_uU = .39 \qquad {}_rU = .32$$

The same result may be obtained in a single matrix multiplication if the matrix of growth rates is first raised to the 30th power, as is demonstrated in Figure 2A.

The growth matrix of that multiplication may be aggregated to produce the previous urban-rural projections without reference to place of birth (Figure 2B). An analogous matrix, with the effects of fertility deleted (Figure 2C), gives, for example, the proportion of the initial rural population that one may expect to find in urban areas in the year 2000 (i.e., 0.45).

A number of interesting conclusions may be drawn at this point. First, on 1970 rates, roughly one fifth of the urban population in the year 2000 will consist of people who lived in rural areas in 1970; and, given our earlier assumptions regarding place of birth, almost one-half of the urban population will be comprised of rural born. Second, although the national population

$$\begin{bmatrix} 123.63 \\ 102.95 \\ 21.06 \\ 72.31 \end{bmatrix} = \begin{bmatrix} 1.0570 & 0.4145 & 0.6256 & 0.1775 \\ 0.0168 & 0.6593 & 0.1342 & 0.5823 \\ 0.1885 & 0.0476 & 0.3374 & 0.0150 \\ 0.0503 & 0.1912 & 0.2370 & 0.5594 \end{bmatrix} \begin{bmatrix} 68 \\ 68 \\ 10.6 \\ 95.4 \end{bmatrix}$$

2A. Projection of the urban and rural populations to the year 2000, disaggregated by place of birth.

$$\begin{bmatrix} 226.58 \\ 93.37 \end{bmatrix} = \begin{bmatrix} 1.0738 & 0.7598 \\ 0.2388 & 0.5745 \end{bmatrix} \begin{bmatrix} 136 \\ 106 \end{bmatrix}$$

2B. Projection of the urban and rural populations to the year 2000.

$$\begin{bmatrix} 47.50 \\ 34.21 \end{bmatrix} = \begin{bmatrix} 0.6425 & 0.4481 \\ 0.1408 & 0.3227 \end{bmatrix} \begin{bmatrix} 0 \\ 106 \end{bmatrix}$$

2C. Survivors of the 1970 rural population in the year 2000.

Figure 2. Alternative projections of the 1970 urban and rural populations of the USSR to the year 2000.

will be growing at an annual rate of just under one percent, the growth rate of "aliens" in the rural regions will be about twice as high, and that of the "aliens" in the urban region will be about 50 percent lower. Finally, as is to be expected in a national population experiencing high levels of rural to urban migration, the only declining rate of growth is exhibited by the rural "native" population.

Equation (6) disaggregated residents into natives and aliens. A further disaggregation of natives into stayers and returners may be useful in certain circumstances. Stayers are natives who have never migrated out of their region of birth; returners are natives who have but who have returned.

A disaggregation of aliens into "old" and "new" aliens also may be of interest, e.g., in studies of the assimilation of migrants. Let new aliens be all aliens who inmigrated during the most recent unit interval of time. All other aliens are old aliens. This then suggests the following disaggregated population projection:

residents = stayers + returners + old aliens + new aliens.

A number of complications arise as a consequence of such a decomposition, and the introduction of age brings with it additional problems. A discussion of these difficulties and their resolution is beyond the scope of this paper, but an overview of the kind of results that emerge may be sketched out as follows. Imagine a biregional urban-rural life table with each region receiving a radix of a single birth per year. These infants are exposed to age-region-specific regimes of mortality and migration to generate a life table and its stationary urban and rural populations, disaggregated by regions of birth and residence, and by status (i.e., stayers, returners, old aliens, and new aliens). What is the proportional distribution of each region's stationary population between the four statuses, and what is the mean age of these four populations? Finally, what are the respective place- and status-specific expectations of life at birth, say? Table 3

Table 3. Biregional life table population for the USSR: 1970 mortality and migration schedules and unit radices.

Region of Residence	Residents	Natives		Aliens	
		Stayers	Returners	Old Aliens	New Aliens
Urban					
Population (or Life Expectancy)	69.87	49.55	9.84	7.90	2.58
Share of Total	1.00	.71	.14	.11	.04
Mean Age	37.54	32.81	51.92	50.34	34.33
Rural					
Population (or Life Expectancy)	69.96	21.94	6.84	36.00	5.19
Share of Total	1.00	.31	.10	.51	.07
Mean Age	37.55	16.75	52.98	48.99	25.81

Source: Calculated using the migration and mortality data set out in Appendix B.

presents such information for the urban-rural life table populations of the USSR, calculated using the 1970 mortality and migration schedules set out in Appendix B.

On 1970 rates, a baby born in a rural village could expect to live almost 60 percent of its lifetime in an urban area (i.e., as an "alien"). An urban-born baby, on the other hand, could expect to live about 15 percent of its lifetime in rural areas. The 1970 rates generate an urban population 71 percent of whom never leave urban areas, more than twice the corresponding percentage for the rural population. Finally, although the average ages of the urban and rural stationary populations are identical, those of their constituent subcategories vary considerably. Returning and old aliens exhibit mean ages of about 50 years, whereas stayers and new aliens show mean ages that are much younger.

CONCLUSION

This paper has focused on alternative methods for projecting subnational populations, such as urban and rural populations. It has argued that multiregional models are superior to uniregional models in a number of respects. First, uniregional models can introduce a bias into the projections, and they may produce inconsistent results in long-term prognoses. Second, the effects of changes in age compositions on migration patterns can be very important, yet a uniregional approach fixes these effects at the start of the projection and thereby can introduce a potentially serious bias into the projection. Finally, multiregional models have a decisive advantage over uniregional models in that they alone can follow subcategories of a population over time. Thus they can produce disaggregated projections that are impossible to obtain with uniregional models.

The principal advantage of uniregional models over multiregional models lies in their more modest data requirements. Age-specific gross migration data are frequently unavailable, whereas survival methods may be used to infer net migration flows. Thus uniregional models are more readily applicable to most subnational population projections. However, recent developments in the construction and application of "model" migration schedules promise a relaxation of the data problem in the future (Rogers, Raquillet, and Castro 1978).

Finally, we have considered only the nonbehavioral "measurement and dynamics" aspects of the projection problem. A logical next step is to connect the demographics with the socioeconomics to examine whether the net-migration-based uniregional perspective may indeed be a perfectly adequate specification when used in a forecasting model and in the context of a particular theory of population and development. Such an approach is adopted by Kelley and Weiss (1969), for example, who conclude, on theoretical grounds, that a biregional model of the kind described in this paper is inconsistent with a wage adjustment model of migration and reflects instead the behavior of a stock-adjustment model. They argue that inasmuch as the former perspective seems to have received broader empirical support, the biregional (Markovian) model is likely to understate the population changes required for an equilibrium to arise.

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APPENDIX A: U.N. METHODS FOR PROJECTING URBAN AND RURAL POPULATIONS

The current "state of the art" in urban and rural population projection, using essentially uniregional methods, appears in a recent United Nations (1974) manual entitled: *Methods for Projections of Urban and Rural Population.* For all but one method reviewed, the manual assumes that national population projections have already been carried out. Thus, the task envisioned is one of projecting only the urban (or rural) population and then obtaining the corresponding rural (or urban) projection by subtracting the former from the national projection.

Four types of methods are distinguished in the U.N. manual, namely (U.N. 1974:3-4):

> a) Global methods; in these neither the sex-age composition of the population nor the component trends of urban and rural growth (fertility, mortality, migration and area reclassification) form part of the basic calculation;
>
> b) Composite methods; here, the calculation considers the sex-age composition of the urban and rural population, but not the separate effects of component trends (fertility, mortality, migration and area reclassification);

c) Crude component methods: urban and rural trend components (fertility, mortality, migration and area reclassification) are considered separately, but not the sex-age composition of the population; and

d) Cohort-survival methods, where use is made of the sex-age detail of the urban and rural population and the incidence of fertility, mortality and migration (and possibly area reclassification) by groups of sex and age.

These four methods may be conveniently located in a two-by-two table, whose rows refer to the presence or absence of components-of-change in the model, and whose columns differentiate models with regard to the presence or absence of age-sex details:

	Age-sex details	
Component details	No	Yes
No	a) Global methods	b) Composite methods
Yes	c) Crude component methods	d) Cohort-survival methods

Global methods consider only population totals and their respective rates of growth. Since a national population projection is assumed to be available, an enumerated current urban population and its assumed future growth rate are sufficient inputs for a projection of the urban population and, by subtraction, of the corresponding rural population.

An alternative global method is one that focuses on observed and anticipated ratios (or differences) between subnational (e.g., urban) and national populations (or growth rates). The application of such ratios to a given national projection, for example, can produce the corresponding urban and rural projections.

Composite methods depend on the availability of age-sex compositions for populations in two consecutive censuses. They use age-sex-specific urban residence ratios to project either fixed age-sex groups or cohorts. The former perspective assumes that the propensity of a national population to reside in urban areas varies by age and sex groups and changes in a predictable manner; the latter follows a group of individuals of a given age and sex over time, applying to them assumed trajectories of urban residence ratios. A variant of the latter method focuses instead on the rural population and uses migration-survival ratios, resembling thereby a cohort-survival approach with survival ratios that include the combined effects of mortality and net migration.

Crude component methods do not produce age-sex disaggregated projections, but they do allow the possibility of varying assumptions about patterns of future mortality, fertility, and migration. Thus they may be used to show to what extent urban and rural population growth is influenced by these three fundamental components of change. However, since they do not consider how changes in age composition might affect these crude rates, the projections that they lead to may be seriously in error in the long run.

Cohort-survival methods combine the best features of both composite and component methods, but they require the most detailed data. They trace the impact of each component of urban and rural population change, by age and sex. The initial urban and rural populations are survived forward through time by age-sex-location specific ratios of fertility, mortality, and migration. The consequences of a change in any single component may be assessed, and alternative "scenarios" of growth may be developed. Thus (U.N. 1974:4):

> ...if a theoretical model is required, the method will have to be of type (d), for only then can the precise effects of alternatives in fertility, mortality and migration trends upon the size and structure of the resulting populations be followed through.

None of the four U.N. methods described above considers gross (directional) migration flows. Internal migration is viewed as net migration, and all of the population models are therefore fundamentally uniregional in character: they analyze a multiregional population system one region at a time.

APPENDIX B. Population Data for the USSR, 1970.

URBAN				
Age	Population (in thousands)	Death rate	Fertility rate	Outmigration rate
0-4	9,876	0.0071		0.0093
5-9	11,712	0.0007		0.0029
10-14	12,132	0.0006		0.0023
15-19	13,737	0.0010	0.01450	0.0255
20-24	11,922	0.0016	0.07370	0.0379
25-29	8,830	0.0023	0.05590	0.0164
30-34	13,423	0.0029	0.03577	0.0117
35-39	9,783	0.0038	0.01549	0.0077
40-44	11,435	0.0048	0.00399	0.0056
45-49	7,110	0.0062	0.00069	0.0045
50-54	5,301	0.0089		0.0035
55-59	6,614	0.0120		0.0035
60-64	5,436	0.0185		0.0045
65-69	3,618	0.0282		0.0042
70+	5,062	0.0777		0.0036
TOTAL	135,992	0.0076	0.01657	0.01124
RURAL				
Age	Population (in thousands)	Death rate	Fertility rate	Outmigration rate
0-4	10,657	0.0069		0.0207
5-9	12,792	0.0007		0.0063
10-14	12,884	0.0006		0.0051
15-19	8,286	0.0010	0.01637	0.1015
20-24	5,202	0.0016	0.10364	0.2091
25-29	4,955	0.0022	0.08452	0.0704
30-34	7,745	0.0028	0.06238	0.0489
35-39	6,829	0.0037	0.03875	0.0264
40-44	7,589	0.0047	0.01494	0.0203
45-49	5,159	0.0060	0.00340	0.0150
50-54	3,787	0.0087		0.0116
55-59	5,413	0.0117		0.0102
60-64	4,912	0.0180		0.0120
65-69	3,649	0.0275		0.0101
70+	5,869	0.0756		0.0075
TOTAL	105,729	0.0091	0.01865	0.03475

Source: Rogers (1976b).

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