SENSITIVITY TO UNCERTAINTY IN A PHYTOPLANKTON-OXYGEN MODEL FOR LOWLAND STREAMS

Gerrit van Straten
Bart de Boer

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INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

GERRIT VAN STRATEN is with Twente University of Technology, Enschede, The Netherlands. At present, he is a research scientist at the International Institute for Applied Systems Analysis, Schloss Laxenburg, 2361 Laxenburg, Austria.

BART DE BOER is with the Provincial Waterboard, Arnhem, The Netherlands.

PREFACE

During the past decade, there has been considerable interest in the development of models for river and lake ecological systems. Much of this interest has been directed towards the construction of progressively larger and more complex deterministic simulation models. However, while our experience is growing, we begin to realize that our computational capabilities exceed by far the level of our knowledge of the complex processes in the real world. Furthermore, in ecological modeling we seldom, if ever, encounter data of sufficient amount and accuracy to allow for a rigid calibration of our models. IIASA's Resources and Environment Area task on Models for Environmental Quality Control and Management recognizes the problems of coping with model, data, and parameter uncertainties as a challenging issue for future research. This paper can be seen as a preliminary and modest contribution in this direction.

The study was done at IIASA in a limited amount of time and was finalized during a short stay of the second author at the Institute. The results were also presented at the joint colloquium of the Commission for the Study of Water Management in the Province of Gelderland, the Netherlands, and the Special Research Area 79 of the Federal Republic of Germany (Hannover University of Technology) held at Wageningen, The Netherlands, from 14 - 16 February 1979.

ABSTRACT

The applicability of water quality models depends upon the quality of the parameter estimates. A phytoplankton-oxygen model developed for canalized lowland streams is tested against data from a limited plug following measurement program. The accuracy of the parameter estimates is limited by the inaccuracy of the BOD measurement in the presence of algae. Other sources of parameter uncertainties are: (i) site dependency of parameters lumping complex subsystems, such as the BOD decay rate coefficient, having a higher value directly after a waste discharge, and (ii) time dependency of lumped parameters, such as the algal death rate coefficient. A sensitivity analysis, based on the solution of the sensitivity equations of the model, is then performed to provide some insight into the effects of parameter uncertainties on model results. It appears that the growth and death rates of algae dominate the phytoplankton, BOD and oxygen behaviour, and that a separate estimate in the absence of accurate BOD measurements is difficult to obtain without additional information.

INTRODUCTION

The development of decision making tools in the field of water quantity as well as quality is the principal aim of the integrated regional water management study, conducted in recent years by the Committee for the Study of Water Management in the Province of Gelderland, the Netherlands (CWG). The inclusion of water quality problems in the study was in part due to the reasoning that a possible link exists between surface water quality and its suitability for drinking water purposes. But even if surface water is not to be used, ground water being in sufficient supply, there is still a strong incentive to study water quality in relation to quantity problems, since quality is affected by water management in several ways (van Straten, 1979).

The development of the water quality modelling element in the integrated study is based on several years of field research in the Gelderland area (mainly conducted as student projects). This work has finally led to the computer package GELQAM (Gelderland Water Quality Analysis Model), a major product of the water quality work. The package will be available for interested users. GELQAM focuses on dissolved oxygen as relevant characteristic of water quality. A detailed description of GELQAM and its unique numerical features is given elsewhere (de Boer, 1978).

This paper discusses some of the problems and difficulties met, and partly overcome, during the study. It is our aim to highlight uncertainties which are still associated with river water quality modelling nowadays. We develop the issue in two ways: first we deal with uncertainties in the evaluation of model parameters, and secondly we discuss the consequences of simplifications in the process description. The effect of inadequacy of the data is also illustrated. We then focus on sensitivity analysis as a helpful tool in the appreciation of model results under the uncertainties given.

SYSTEM CHARACTERISTICS

The rivers and brooks in the Gelderland area are lowland streams, which are typical of relatively flat areas. Most of them have been canalized for reasons of water level control, mainly for agricultural purposes. Only some of the upper branches have retained their original, more natural shape. From the hydrobiological point of view canalization brings considerable changes. Water plants and sessile algae characterize the shallow, relatively fast flowing natural brooks. In the canalized sections less favourable conditions prevail because of the higher light attenuation in the deeper water column. Instead, essentially higher residence times allow for a notorious development of planktonic algae, especially during dry summer periods. Also, a remarkable change of macrophauna has been observed (Tolkamp, 1975).

For several reasons the quality work has been restricted mainly to the canalized stream sections of brooks and rivers of moderate size. One important justification is that those sections are probably of more significance for potential water usage functions than the smaller, natural brooks.

In many respects canalized river sections behave similar to lakes. It should be pointed out, however, that the river system is more sensitive to hydrodynamical variations. During storm water periods the contents of the river may be refreshed within a few days or even hours, so that the planktonic algae are washed out. During subsequent drier periods a new development will start, but the population may be different due to a different seed flushed in from the upstream water courses.

SHORT MODEL DESCRIPTION

In general terms the results of our field studies did not deny the pattern usually reported for the oxygen behaviour in streams. Thus, processes influencing the dissolved oxygen content are: decay of both carbonaceous and nitrogeneous oxidizable matter, consumption by the sediment layer, consumption by algal respiration, reaeration and, during daylight, production by photosynthesis. Consequently, the state variables in the model are dissolved oxygen, C-BOD, N-BOD and algae (expressed as chlorophyll-a). In addition, soluble reactive phosphorus has been included in the state vector because of its dominant role in the control of algal blooms. The pathways followed in the development of this structure are outlined in van Straten (1977) and will not be repeated here.

The basic equations are given in Table 1. The model combines elements of usual dissolved oxygen river models (e.g., O'Connor and Di Toro, 1970), with elements from well-known algal dynamics models for lakes (e.g., Di Toro, O'Connor and Thomann, 1974).

The model according to Table 1 constitutes a set of second order partial differential equations. The longitudinal dispersion term is not shown but has been included, because dispersion effects can be significant especially during low flow conditions. A more detailed analysis of the role of dispersion can be found in van Straten (1979).

The numerical solution of the set of equations requires special attention. Discretization in both time and space is necessary. Usual difference schemes with fixed time and space grid appear less profitable for river systems. The reason is that the relatively low dispersion (as opposed to estuaries) requires an uneconomical number of grid points for accurate results. Therefore, for GELQAM, a moving cell method has been developed (de Boer, 1977, 1979). The basic feature of this is a coordination transformation in such a way that the system is solved along the stream-flow trajectories. In other words, the model follows a number of consecutive 'plugs' of water on their travel downstream.

PARAMETER ESTIMATION

Before the model can be used it is necessary to have estimates for the parameters. For most of them a first order estimate can be obtained from the literature. However, parameters for which the model is very sensitive have to be evaluated in the field. Sometimes it is possible to isolate part of the processes by designing special experiments. An example of this is the well-known dark and light bottle experiment. Here reaeration, vertical mixing and, in the dark bottles, photosynthesis are excluded. The vertical profile of the net oxygen production in the bottles enables the identification of the algal growth system and the estimation of its parameters (i.e., I_s or I_k and the product $Y_{ca}k_{pa}$. Note the uncertainty in the growth rate itself as a consequence of the wide range of oxygen yield Y_{ca}: from 50 to 250 mg oxygen per mg chlorophyll-a, cf. van Straten, 1978).

		algal growth	algal respiration	algal death	C-BOD decay	N-BOD decay	reaeration	sediment interaction
algae	$\dot{A} =$	$k_{pa}\ FA$	$-\ k_{ra}\ AC_m$	$-\ k_{da}\ A$				
C-BOD	$\dot{L} =$			$+\ XY_{la}\ k_{da}\ A$	$-\ k_b\ LC_m$			
N-BOD	$\dot{N} =$	$-\ Y_{na}\ k_{pa}\ FA$		$+\ XY_{na}\ k_{da}\ A$		$-\ k_n\ NC_m$		
dissolved osygen	$\dot{C} =$	$Y_{ca}\ k_{pa}\ FA$	$-\ Y_{ca}\ k_{ra}\ AC_m$		$-\ k_b\ LC_m$	$-\ Y_{cn}\ k_n\ NC_m$	$+\ k_r\ (C_s - C)$	$-S/H$
phosphorus	$\dot{P} =$	$-\ Y_{pa}\ k_{pa}\ FA$		$+\ XY_{pa}\ k_{da}\ A$				$-\ k_s\ X_{ps}\ P$

Oxygen limit function

$$C_m = \frac{C}{C_k + C}$$

Light function

$$F = P_m \frac{e}{k_e H} \left\{ \exp\left(-\frac{I_H(t)}{I_s}\right) - \exp\left(-\frac{I_o(t)}{I_s}\right) \right\}$$

<u>extinction and selfshading</u>:

$$k_e = k_o + \alpha A$$

$$I_H(t) = I_o(t) \exp(-k_e H)$$

<u>daily light pattern</u>:

$$I_o(t) = 0.5\ I_{tot} \frac{\sin(2\pi(t_{dl}-12)/48) + \sin(2\pi(t-6)/24)}{\frac{48}{2\pi}\cos(2\pi(12-t_{dl})/48) + t_{dl}\sin(2\pi(t_{dl}-12)/48)}$$

where t_{dl} = daylength (h)

and I_{tot} = daysum of total radiation (J/cm^2)

Phosphorus limit function

$$P_m = \frac{P}{P_k + P}$$

Reaeration

$$k_r = c_r\ D_m^{\frac{1}{2}}\ (Q/A)^{\frac{1}{2}}\ H^{-3/2}\ (1.024)^{T-20}$$

$$D_m = 7.5 \times 10^{-6}\ m^2/h$$

where Q = flow rate (m^3/h)
A = cross sectional area (m^2)
H = depth (m)

Temperature functions

$$k_{pa} = k'_{pa}\ T$$

$$k_{ra} = k'_{ra}\ T$$

$$k_b = k_{b\ 20}\ (1.047)^{T-20}$$

$$k_n = k_{n\ 20}\ (1.108)^{T-20} \qquad T < 20$$

$$= 1.5\ k_{n20} \frac{1}{\left(\frac{T-32}{17}\right)^2 + 1} \qquad 20 \leqslant T \leqslant 30$$

$$S = S_{20}\ (1.065)^{T-20}$$

$$C_s = 13.97 \exp(-\ 0.021\ T)$$

where T = temperature (C)

TABLE 1. *Model description*
List of symbols and parameter values given in Table 2

In addition, information about the processes and the parameters can be obtained by following the course of the state variables in a plug of water moving downstream, a method most profitable if dispersion is relatively low. If the system is simple, such as in the case of the classical BOD-DO model without algae, the evaluation of some of the parameters is straightforward. For instance, the BOD-balance reduces to

$$\frac{dL}{d\tau} = -k_b L$$

where τ is the travel time, and k_b follows simply from

$$L_\tau = L_o \exp(-k_b \tau)$$

However, if algae are present these simple relationships no longer hold firstly because the algae contribute to the measured BOD and secondly because detritus produced by the death of algae constitutes an additional term in the mass balance. Therefore, essentially, the ultimate parameter estimates are obtained by tuning the model to the measurements.

River Berkel Example

A data set is available for the Berkel River in the eastern Gelderland area where a plug following program was conducted from 21st-25th June, 1976 (Hiemstra, 1978). Starting from known value ranges for the parameters, obtained from earlier measurements in other Gelderland regions, and with additional information from a dark and light bottle experiment, a parameter set was found that could explain the experimental results. Table 2 lists the full parameter set, with the appropriate changes to accommodate slight differences in model description between Table 1 and Hiemstra's original model.

The River Berkel is a canalized river. The study reach covered two sections between weirs and was about 5.8 km long. A discharge of mechanically treated sewage water (approx. 15 kg c-bod/h) is located 1 km upstream from the study reach. Table 3 summarizes the main characteristics. For reasons pointed out below, travel time and distance are

No	Parameter	Symbol	Nominal Value	Unit
1	growth rate algae	k'_{pa}	0.0077	1/(hC)
2	Michaelis-Menton coefficient for P	P_k	0.01	mg P/l
3	extinction coefficient	k_o	1.83	1/m
4	self-shading coefficient	α	0.17	1/mg dw,m
5	optimal light intensity	I_s	60	J/cm^2h
6	algal respiration rate coefficient	k'_{ra}	0.00039	1/(hC)
7	algal death rate coefficient	k_{da}	0.04	1/h
8	C-BOD decay rate coefficient	$k_{b\ 20}$	0.01	1/h
9	Michaelis-Menton coefficient oxygen	C_k	0.5	mg O_2/l
10	detritus production efficiency	X	0.9	-
11	BOD/algae yield	Y_{la}	1.4	mg O_2/mg dw
12	nitrification rate	$k_{n\ 20}$	0.004	1/h
13	N/algae yield	Y_{na}	0.123	mg N/mg dw
14	reaeration factor	c_r	1	-
15	O_2/algae yield	Y_{ca}	1.4	mg O_2/mg dw
16	O_2/ammonium yield	Y_{cn}	4.3	mg O_2/mg $N\text{-}NH_4$
17	sediment oxygen comsumption	S_{20}	0.05	g O_2/m^2h
18	P/algae yield	Y_{pa}	0.01	mg P/mg dw
19	P recycling efficiency	X_p	0.9	-
20	P sedimentation rate coefficient	k_s	0.0	1/h
21	fraction P that might sediment	X_{ps}	1.0	-

TABLE 2. *Parameter description and nominal values for Berkel River simulation*

counted from the beginning of the discharge section, i.e. the section *before* the study reach.

location	cross section (m^2)	depth (m)	travel distance (m)	travel time (h)	real time	
					date	time
weir 2/3	36	1.5	0	0	19-6	18.20
STP			2050	26.5	20-6	20.50
weir 3/4	53 55	2.0 1.9	3090	42.7	21.6	13.00
weir 4/5	70 14	2.3 1.2	5380	84.7	23-6	7.00
weir 5/6	29	2.0	8920	105.9	24-6	4.20

TABLE 3. *Characteristics River Berkel/plug following program*

A qualitative idea of the goodness of fit obtained in the parameter calibration phase gives Fig.1 for algae and Fig.2 for dissolved oxygen. Clearly, quite large discrepancies between model and measurements exist, see for instance time 86h for algae and time 68h for dissolved oxygen. It is interesting to note that very low flow conditions prevailed during the measurement week, suggesting the possibility of only limited vertical mixing especially in stretches far away from the weir. This might explain the high oxygen content observed at 68h, i.e. 14.00h clock time, because the sample is taken from the surface where during the day the highest oxygen production takes place.

Parallel to the plug following program an independent input-output program was also done for 24 hours. This enabled the testing of the parameter set by comparing the observed output time series with the one predicted by the model using the input series as initial conditions. The result was reasonable, although in the end the predicted chlorophyll-a, and consequently the dissolved oxygen, were too high.

UNCERTAINTY

The Berkel River example is also suitable for illustrating some uncertainties in river quality modelling. For this purpose we take a look at both model

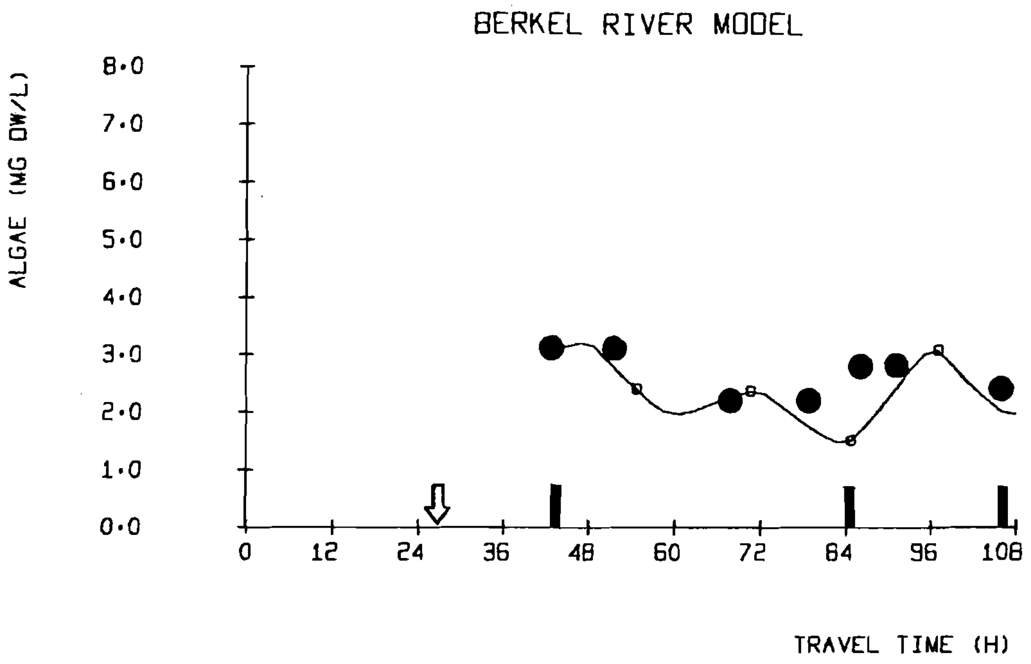

Figure 1. *Measurements (dots) and simulation for phytoplankton*
Arrow: waste discharge point
Bars : weirs

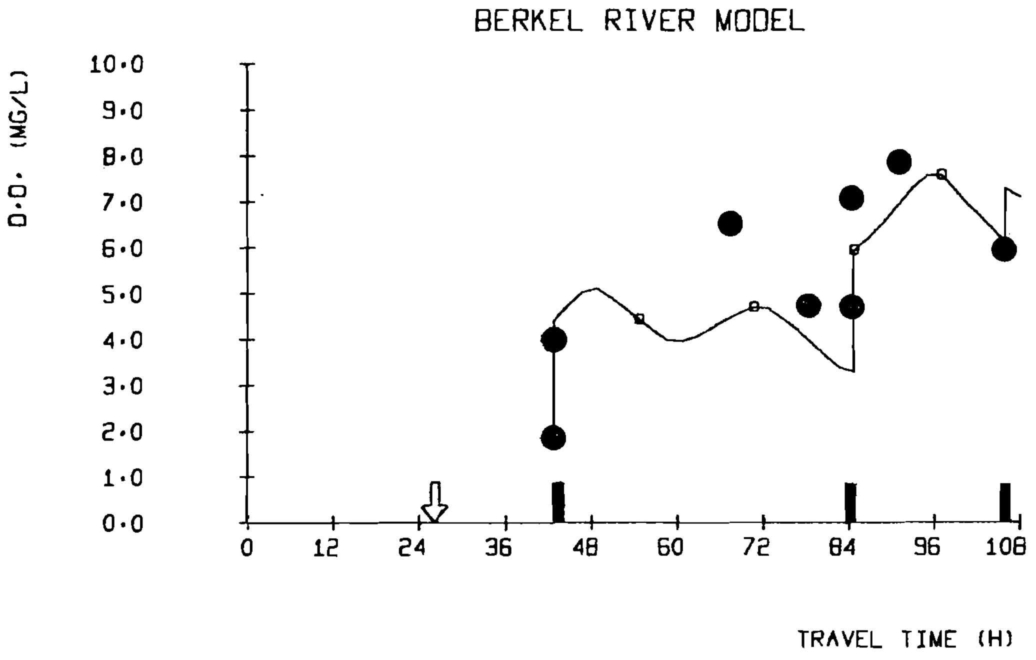

Figure 2. *Measurements (dots) and simulation for dissolved oxygen*

and measurement results for carbonaceous BOD (Fig.3). An immediate difficulty arises: what is (C-)BOD? The model requires the ultimate BOD of all oxidizable organic matter except living algae. In practice a 20-day BOD-test was done. A period of 20 days is sufficient to approach the BOD ultimate, but the result includes both the ammonium oxidation and, perhaps only partly, the oxygen demand resulting from algae dying and respiring during the test. In order to exclude the algae a BOD_{20} for filtered samples was also done. However, this procedure also excludes non-algal particulate oxidizable organic matter. Thus, all that can be said is that the actual BOD must be somewhere between the values for the filtered and unfiltered samples, as indicated by the open circles in Fig.3. A correction for the nitrification was made by multiplying the measured ammonium nitrogen concentration with 4.3 (mg O_2 per mg N) and subtracting this value from the BOD test result (again introducing some uncertainty).

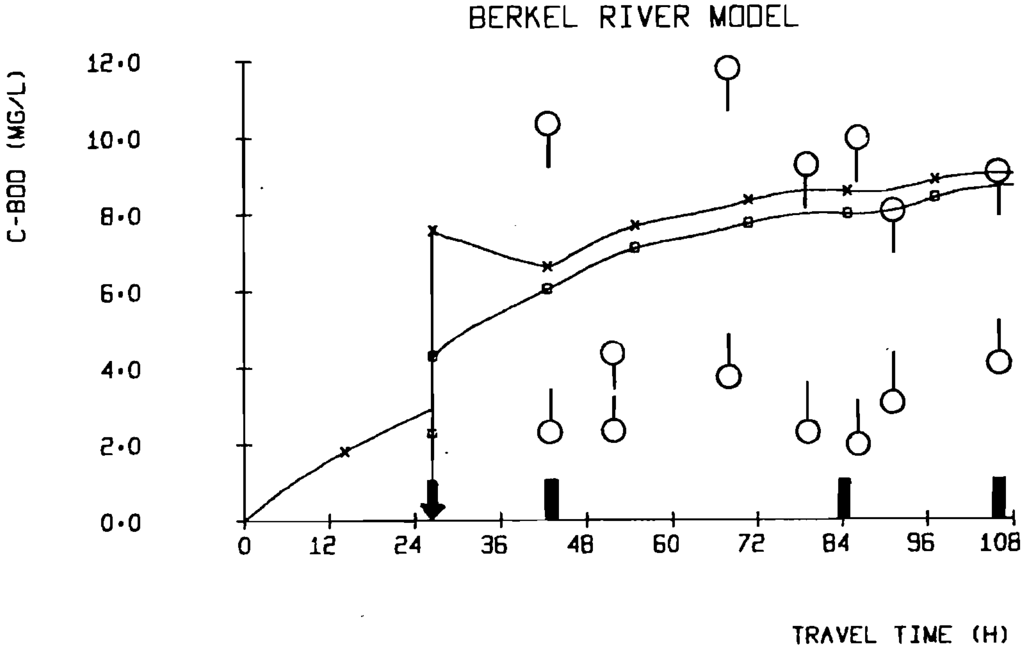

Figure 3. *Measurements for filtered (lower circle) and total carbonaceous BOD (upper circle) and two simulation results (see text)*

The model simulation results (solid lines Fig.3; the origin of the two curves is explained below) suggest a clear increase in carbonaceous BOD, caused by production of detritus due to algal death. Although the increase is not very clear in the measurements the trend is also observed in the input-output results, where the C-BOD at the output is significantly higher than at the input. Simultaneously there is a remarkable decline in chlorophyll-a (Fig.1) in spite of very favourable weather conditions for algal growth, likewise pointing towards an unusually active death process.

The accuracy of the measurements does not allow for a precise evaluation of the parameters. Let us, therefore, for the time being, adopt the parameter set of Table 2. Now, if this is the valid set, it must be possible to extrapolate the simulation backward in time to see whether acceptable values are obtained in the discharge section before the measurement sections. Doing this it appears that the C-BOD just before the waste discharge must have been negative in order to produce the results in the measurement sections. This, of course, is impossible, demonstrating that an acceptable parameter set can become non-acceptable if extrapolations are made to other sections or other times.

An explanation can be found by noting that the BOD decay rate coefficient represents a lumped process, covering a complex of bacterial subprocesses. Observations in different rivers in the Gelderland area indicate that the rate constant is BOD dependent, as illustrated in Table 4.

Brook or River		BOD_{20} filtered	BOD_{20} total	C-BOD calculated	k_6 day^{-1}
Groenlose Slinge	1972		40-180	15-130	1.5-2
Oude IJssel	1974	20-40	20- 60	5- 25	<0.05
Oude IJssel	1975	8-15	20- 35	10- 25	0.1-0.6
Berkel	1974	10-20	20- 40		0.3
Berkel	1978	15-20	20- 25	0- 8	0.1-0.2
Eem (Utrecht)	1977	20-25	25- 40		0.1-0.2

TABLE 4. *BOD decay rate and associated BOD-ranges (ref. van Straten and de Boer, 1979)*

Obviously, the BOD decay rate coefficient is higher in more polluted streams because untreated waste water is more easily degradable than effluents from sewage treatment plants. Therefore, it may be hypothesised that the decay rate coefficient has been higher in the section directly after the waste discharge than in the rest of the river stretch.

Similar arguments apply to the death rate of algae. Again this coefficient lumps a variety of underlying processes such as respiration, natural death and death by zooplankton grazing. The latter process especially, introduces a high variability of the death rate in time. In fact, it is known that zooplankton was present during the measurement period. Fortnightly routine measurements revealed that the measurement period coincided with the lowest chlorophyll-a (and the highest pheophytine) concentration in the entire summer period, and that a sharp decline had occurred from about 140 µg chlorophyll-a/l to 50 µg/l within the two preceeding weeks. Combined with the low flow rate it is most likely that the zooplankton reached its peak concentrations in or after the measurement week, resulting in a still increasing death rate coefficient during the measurement period.

For demonstration purposes a simulation was run based on these two hypothesis. It was assumed that the algal death rate was only half of its nominal value during the first 27 hours, and that the BOD decay rate was four times its nominal value in the river section between the discharge point and the first weir. The result for C-BOD is given in Fig.3, where the zero initial condition is the ultimate possible, though still not a satisfactory assumption. In Fig.4 the effect on the algal pattern is shown. The hypothesis assumptions seriously affect the behaviour before the measurement section. Whether or not this is realistic could only be said if data had been available, although the new initial condition for algae is more likely. Note that in the absence of data, numerous other assumptions can be made (for instance, a lower efficiency of algae into BOD transfer, parameter X, would lead to essentially less BOD produced by algal decay).

The problem of inappropriate model description as compared to the complex reality, demonstrated by the rather academic exercises above, is a fundamental problem of any modelling activity. Since the model is always a simplification one can expect that the model parameters are functions

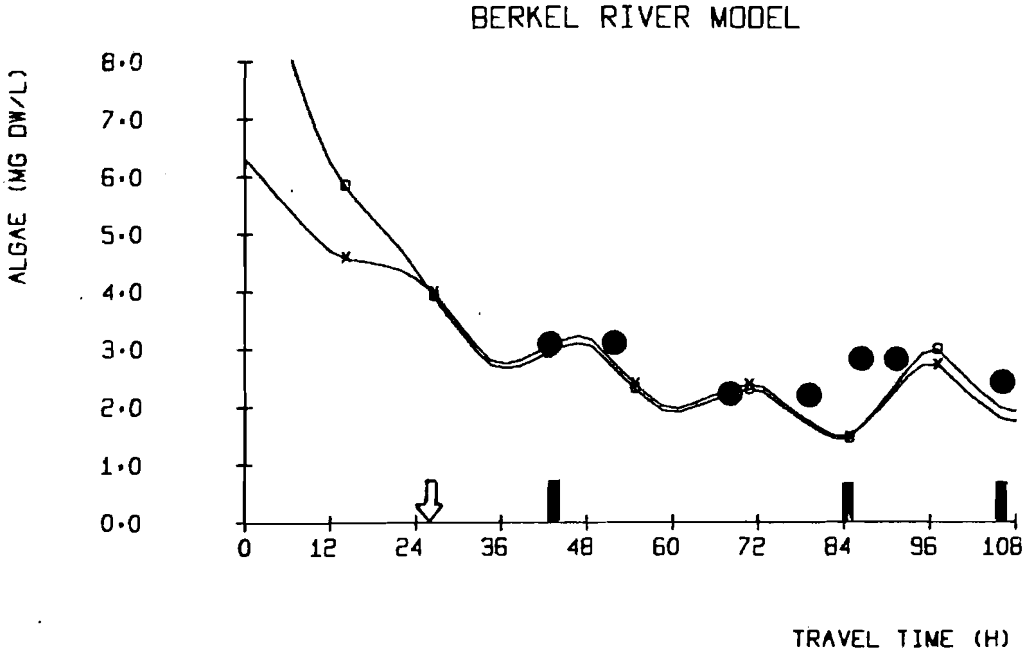

Figure 4. *Effect of parameter variability on phytoplankton simulation*

—□—□—□—□—□— *constant parameters*

—×—×—×—×—×— *variable decay and death rate coefficients (see text)*

of time and space due to the variability in the unobserved underlying processes.

A natural way to solve this difficulty would be to detail the description further, as soon as knowledge permits one to do so, in the hope that the new parameters are less time and space variant. However, this implies at the same time more state variables and a considerable increase in parameters to be estimated. For this reason such an approach is not always the desirable one, especially not if management problems have to be solved. For instance, modelling zooplankton would certainly improve the prediction of algal minima, at the expense of a considerable measurement effort, but would do little for the algal peaks. If algal peaks are the main concern of the management a model with a conservative estimate for the death rate, or an empirical time function, could be very well acceptable.

Generally, it is very useful for further model development to examine the consequences of parameter uncertainties for the certainty or uncertainty of model predictions. One way of doing this is by a sensitivity analysis.

SENSITIVITY ANALYSIS

An interesting way of analyzing the model sensitivity is based on the solution of the sensitivity equations. Let the system equations be written in state space form as

$$\dot{\underline{c}} = \underline{f}\ (\underline{c}, \underline{p}, \underline{u}, \tau)$$

where $\underline{c}$ is the n-dimensional state vector (vector of system variables), $\underline{p}$ the r-dimensional parameter vector, $\underline{u}$ the m-dimensional input vector (forcing functions) and τ the time. The initial conditions are

$$\underline{c}\ (t_o) = \underline{c}_o$$

Let

$$\underline{s}_j = \left(\frac{\partial \underline{c}}{\partial p_j}\right)_{\underline{p}^o}$$

be defined as the trajectory sensitivity vector of the state vector $\underline{c}$ with respect to the parameter p_j around the nominal parameter set $\underline{p}^o$. Note that the elements of $\underline{s}_j$ are time funcions. Then taking the total derivative to p and interchanging the order of differentiation leads to the so-called sensitivity equations (note: vector differential equations):

$$\dot{\underline{s}}_j = \left.\frac{\partial \underline{f}}{\partial \underline{c}}\right|_{p_j^o} \underline{s}_j + \left.\frac{\partial \underline{f}}{\partial \underline{p}_j}\right|_{\underline{c}^o}$$

with

$$\underline{s}(o) = o \qquad j = 1,2, \ldots ,r$$

cf. Frank (1978). It is interesting to note that the sensitivity system is always linear, even if the model f is nonlinear. Therefore, in the case of relatively simple models such as the Streeter-Phelps model, analytical solutions for the sensitivity system can often be obtained (e.g. Rinaldi and Sonsini-Sessa, 1977). For more complex models a numerical solution is necessary but not difficult to achieve.

The method was applied to the River Berkel example from the previous section. A manageable state space form is obtained, as before, by considering only one plug of water and neglecting dispersion effects. In this application the Jacobian matrices $\partial\underline{f}/\partial\underline{c}$ and $\partial\underline{f}/\partial\underline{p}$ were derived analytically. The solution of the sensitivity system is done simultaneously with the system equations (involving the numerical solution of a system of $n * (r + 1)$ simultaneous difference equations).

The results are shown for algae in Fig.5. The sensitivity on the vertical axis has been transformed such that the absolute change of the state variable to a relative increase of 10% in the parameter value is shown. In this way the readings have the dimension of concentration and can be compared directly with Figs. 1-4. The curves selected represent the most significant sensitivity functions.

From Fig.5 it is seen that both the growth rate (Curve 1) and the death rate (Curve 7) govern the algal system. The day and night pattern is also reflected in the sensitivity. The jump in the death-rate sensitivity at 27 h is caused by the doubling of this parameter beyond this time as mentioned previously. Note that an increase of only 10% in the death rate would cause the algal concentration to drop by 0.8 mg dw/l at the end of the study reach, i.e. ± 40% of the actual concentration. However, the effect could be counterbalanced by an increase in growth rate of about 12%. Because the sensitivity pattern of the growth rate and the death rate coefficients are similar, apparently more than one combination of both parameters will describe the overall algal pattern. This is to be expected since the algal equation reads

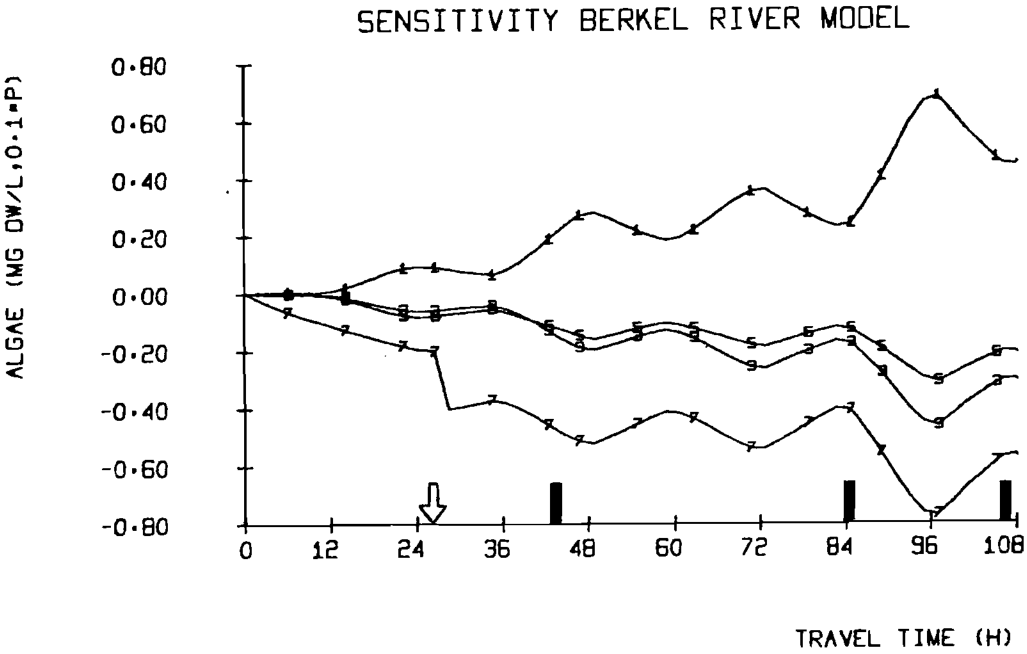

Figure 5. *Sensitivity functions for phytoplankton*

1 = growth rate coefficient k'_{pa}
3 = extinction coefficient k_o
5 = saturation light intensity I_s
7 = death rate coefficient k'_{da}

$$\frac{\partial A}{\partial t} = (k_{pa}F - k_{da})A$$

so that in fact $K = k_{pa}F - k_{da}$ is the rate constant of interest, and a separation of k_{pa} and k_{da} on the basis of observations of the algae concentration alone will be practically impossible. This demonstrates how sensitivity analysis can help in indicating the principal components in parameter space, which is extremely useful for parameter estimation procedures.

Fig.6 shows the sensitivity plot for dissolved oxygen. The mitigating effect of the weirs on the deficit is reflected in the curves as a reduction in sensitivity directly after the weir. Not surprisingly dissolved oxygen is sensitive to essentially the same parameters as the algae. The sensitivity to the BOD decay rate and nitrification rate coefficients is in the order of -0.1 mg O_2/l per 10% increase, and to the reaeration rate approximately 0.1 mg O_2/l per 10% increase (not shown). This demonstrates the dominant role of the algal system in the overall oxygen behaviour.

By comparing Figs.5 and 6 it can be seen that a simultaneous 10% increase of both the growth and death rate coefficients will lead to somewhat lower algae concentrations and somewhat higher dissolved oxygen content. This is because the growth of algae and the production of oxygen are directly coupled, whereas the death of algae results in BOD production first, with only retarded oxygen consumption. In this connection a comparison with Fig.7, showing the C-BOD sensitivity is interesting. A higher algal death rate coefficient (Curve 7) would initially lead to a higher BOD, but in the long run to less algae and thereby to a lower BOD contribution from detritus production. Thus, a change in the death coefficient effects the shape of the BOD curve in time. A similar argument applies to the effect of the growth rate (Curve 1): initially there is no effect, as expected. but later the higher algae concentration leads to more death and thereby to more BOD production.

The BOD is also quite sensitive to the amount of BOD released per unit dying algae (X and Y_{1a}, Curve A; they have the same sensitivity because they appear as a product). The effect on the dissolved oxygen balance is in the same order of magnitude as the effects of the BOD decay rate coefficient (not shown). The jump in the sensitivity to the BOD decay rate coefficient (Fig.7, Curve 8) is caused by the assumption of a higher rate in the section directly after the waste discharge point.

From all graphs it is apparent that the sensitivity has still an increasing trend at the end of the measurement reach. This implies that small errors in parameter estimates obtained from this limited measurement period may lead to considerable prediction errors if extrapolations are being made. This suggests that the measurement period should have been longer for more accurate parameter estimates.

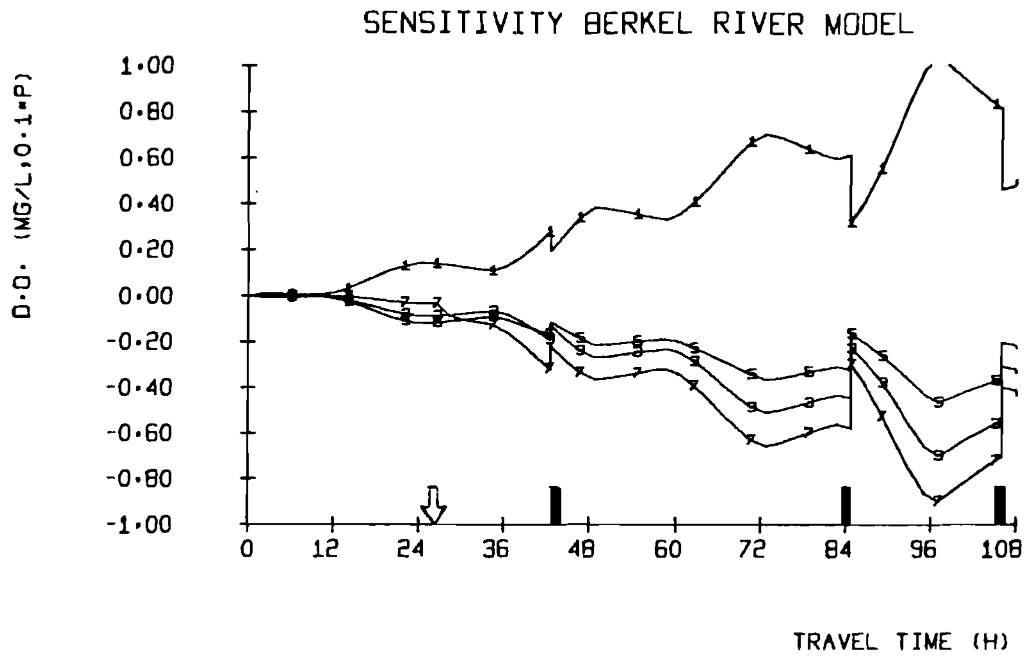

Figure 6. *Sensitivity functions for dissolved oxygen parameters as in Figure 5.*

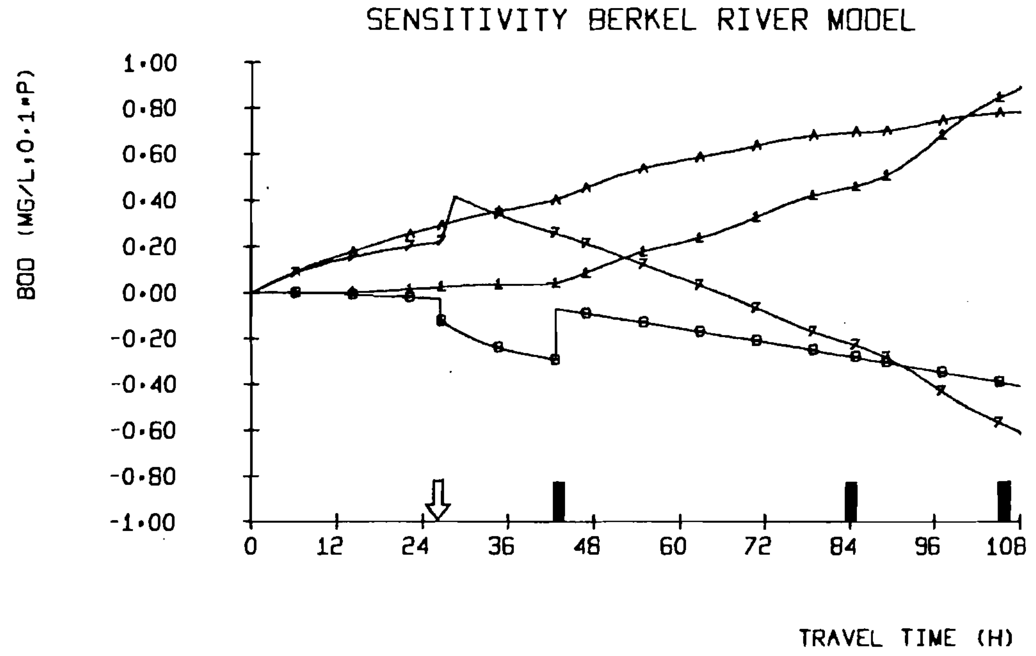

Figure 7. *Sensitivity functions for C-BOD*

Additional parameters: 8 = *BOD decay rate coefficient* k_{b20}

A = *BOD released per unit dying algae X or* Y_{la}

Limitations

Clearly, sensitivity analysis leads to considerable insight into the systems behaviour. However, we should like to point out some of the limitations of the application above. One of these is that the effects of parameter changes on the state variables are compared on the basis of equal parameter variations. Therefor, for uncertainty analysis the method must be extended to cope with different degrees of uncertainty in the parameters involved. For example, the effect of variations in the extinction coefficient on algae and dissolved oxygen (Figs.5 and 6, Curve 3) can be even more dramatic than suggested in the graphs, because the extinction coefficient may vary by 50-100% rather than 10%. Similar arguments apply, of course, to every parameter. It should also be noted that the linear extrapolation of the state according to $\Delta c = s_j \Delta p_j$ is only allowed for small excursions from the nominal parameter values, because the systems model is non-linear. Finally, the system is also sensitive to initial conditions and forcing functions; however, these could be handled in a similar fashion without difficulty.

CONCLUSIONS

- Models such as GELQAM can produce a fair description of the oxygen and phytoplankton behaviour in (canalized) lowland streams.
- Due to uncertainty in the underlying processes it may be expected that parameters are site dependent (e.g. BOD decay rate constant) or time dependent (e.g. rate coefficient for algal death). It is not possible to detect these functional relationships by measurements in a limited river stretch and for a short time.
- An accurate measurement of one of the model state variables, carbonaceous BOD, is not possible. This uncertainty reduces the precision of parameter estimates.
- A sensitivity analysis is a useful element in the evaluation of the effects of process and parameter uncertainty. In situations with extensive algal growth and moderate organic waste discharge, the growth and death processes of algae govern the oxygen behaviour. The sensitivity analysis also reveals the difficulty of estimating the growth and death rate coefficients independently, if no accurate data are available.

- The inadequacy of the model caused by lumping complex processes poses the problem of predictive power, a fundamental difficulty of any model activity. It is worthwhile to investigate the use of sensitivity analysis in connection with a parameter variability study in assigning a 'reliability factor' to the effectiveness of management decisions expected from the model.

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