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A Dismal Future?
Towards a New Model of Labor Market Dynamics Based on Evolving Demographic Variables

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Contents

Abstract

Standard labor market models assume that individuals work between ages 15-64 and then retire. Forecasted improvements in demographic variables such as life expectancy and health level of the population question the validity of those assumptions. Here we develop a model in which individuals decide optimally their retirement age according to changes in demographic and economic variables. Under this framework, individuals should naturally delay their retirement in the following decades. This implies a very different approach to policy, since the implied support ratios may not be as worrisome as the ones coming from the standard models. However, a word of caution is necessary, as the design of the pension system may give enough incentives for people to retire before what could be optimal on an equitable pension system.

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1 Introduction

In the last decades there has been a significant amount of discussion regarding the future of labor markets, in particular, when considering the fact that the life expectancy of individuals is continuously increasing over time. This is specially crucial for pensions systems, as most of the current schemes depend either directly or indirectly on the amount of time that individuals remain in the labor market. Now, standard labor market projections usually assume, for the sake of simplicity, that individuals work when they are between a certain age interval (the most common is between ages 15 and 65) and after that they retire. This, of course, would imply a significant stress on pensions systems, as increases in life expectancy would directly affect the number of years that people stay retired. However, it is not hard to see that this assumption is essentially flawed, as, specially in developed countries, individuals rarely start working at age 15 and, increasingly so, they don't retire at age 65.

Appropriately, retirement age has been a hot topic in economic studies for the last decades. Several studies analyze this decision in a life-cycle approach (e.g. Feldstein 1974; Hu 1979; Mitchell & Fields 1984; Gustman & Steinmeier 1986). Particularly interesting are the studies which include the increases in life expectancy (Chang 1991; Heijdra & Romp 2009; Kalemli-Özcan & Weil 2010; Haan & Prowse 2014). However, there is also another relevant, associated factor which is the health of the individuals. Indeed, a pure increase in life expectancy would not necessarily mean an increase in the retirement age as individuals should also have an appropriate health to be able to keep on working. In this line, we have French (2005); Galama et al. (2013); Bloom et al. (2007, 2014); Kuhn et al. (2015).

Another related literature is on the impacts of increasing longevity on the pensions systems (İmrohoroğlu et al. 1995; Bergstrom & Hartman 2005; Cooley & Soares 1996; De Nardi et al. 1999; Attanasio et al. 2007; Kitao 2014). Most of these papers paint a gloomy picture of a future in which social security systems will not be sustainable, either economically nor politically. As a matter of fact, when looking at the picture where retirement (and pension) ages are not moving over time, it is easy to foresee such consequences.

For our study, we take a novel approach. Instead of focusing on the change of economic variables over time (such as consumption, income, etc.) we analyze how the retirement age would evolve, first based solely on demographic changes that can be obtained from life tables, and second, on a life-cycle approach including demographic changes. To do that, we develop a theoretical model of endogenous retirement with exogenous health status and life expectancy to find conditions on retirement age based on some relevant parameters. Then, for the first model, we simplify the model by assuming the economic variables as constant, and for the second model, we allow individuals to optimize consumption and savings. Finally, we use as an input the projected life tables from the World Population Prospects (WPP) (United Nations, 2015) to find estimated paths for retirement ages for all the countries in the sample.

In Section 2 we show the theoretical model which is the foundation of the study. In Section 3 we calibrate the model from Section 2 to find retirement ages based only on changes in life tables. In Section 4 we relax the model to allow individuals to optimally choose consumption and savings. Section 5 shows the results of the model on different pension schemes. Finally, we conclude by analyzing some policy implications of the presented models.

2 Theoretical Model

In line with Kuhn et al. (2015), we follow a life cycle approach, were the individuals take decisions based on their income, health status and life expectancy. However, here we assume that the health of the individuals, $\mathcal{H}_a$ in our model, is exogenous (and obtained from life tables, as we will show in Section 3).

The lifetime of individuals can be divided in two broad periods: a period when individuals work and a period where they are retired. The length of these periods is endogenous: individuals decide when to optimally retire in order to maximize lifetime utility.

Formally, we have:

$$\max_{c_a, h_a, s_{a+1}, a_R} \sum_{a=a_S}^{a_R-1} \beta^a S_a u\left(c_a, h_a, \mathcal{H}_a\right) + \sum_{a=a_R}^{\mathcal{A}} \beta^a S_a u\left(c_a, 0, \mathcal{H}_a\right) \tag{*}$$

s.t.:

$$\begin{aligned} c_a + s_{a+1} &= (1-\tau) w_a h_a + (1+r) s_a && a_S \le a < a_R \\ c_a + s_{a+1} &= p(a) + (1+r) s_a && a_R \le a \le \mathcal{A} \\ s_{a_S} &= 0, s_{\mathcal{A}} = 0 \end{aligned}$$

where a is chronological age; a_S is the age when individuals enter the labor market; a_R is the age when they retire; S_a is the probability of surviving up to age a; c_a, h_a and $\mathcal{H}_a$ are consumption, working hours and health at age a, respectively; s_a is the stock of savings (which are set to be zero at the first and last periods); w_a is labor income; $p(a)$ is the pension benefit; τ is a pension tax and $\mathcal{A}$ is the time horizon. As we can see, the difference between working and retirement periods in the utility function comes from the working hours h_a term, which becomes zero after retirement. Also, we assume that individuals start their working life with zero savings and end up their life also with no savings ($s_{a_S} = 0$ and $s_{\mathcal{A}} = 0$). We do this as a simplification to avoid dealing with bequest issues. Finally, it is important to realize that this model also includes the probability of dying before age $\mathcal{A}$ (given by the survival probabilities S_a). Therefore, even though the individual would optimize assuming that he lives up to age A, he is also accounting for the fact that he may not survive up to that age.

In the end, the retirement age comes implicitly from:

$$\frac{u\left(c_{a_R}, h_{a_R}, \mathcal{H}_{a_R}\right) - u\left(c_{a_R}, 0, \mathcal{H}_{a_R}\right)}{u_{c_{a_R}}} = p(a_R) - (1-\tau)w_{a_R}h_{a_R} \tag{1}$$

This means that the individual will stop working when the utility surplus from working that period over the marginal utility gain from the consumption at that period equals the difference between the pension income and the net labor income[1].

3 Forecasting Retirement Ages with Dynamic Demographic Variables

From the model above we have the conditions that set up the retirement age for individuals according to economic and demographic variables. Now, in this study, we are interested in analyzing how the retirement age would evolve based only on the change in demographic variables. To do so, then we assume everything in the model as fixed, except survival rates, life expectancy and health status.

First of all, following Sanderson & Scherbov (2013), we will define a health-based index $\mathcal{H}_a$ as:

$$\mathcal{H}_a = \frac{S_{a+3}}{S_a} = \frac{l_{a+3}}{l_a}$$

that is, the probability of survival up to three years ahead of the current age. This gives us a rough estimate of the health level of the population at various periods of time.

We propose an utility function of the form:

$$u(c_a, h_a, \mathcal{H}_a) = \frac{c_a^{1-\psi}}{1-\psi} + \nu \ln\left(1 - (1-\mathcal{H}_a)h_a\right)$$

a function increasing in consumption and health, while decreasing on working hours. In this function, the health of the individual only affects the disutility of working. This form allows us to isolate the disutility of labor and include the detrimental effect of health while working. Hence, (1) is:

$$c_{a_R}^{\psi} \cdot \nu \cdot \ln\left(1 - \left(1 - \mathcal{H}_{a_R}\right) h_{a_R}\right) = p(a_R) - (1-\tau)w_{a_R}h_{a_R}$$

As stated, to analyze the pure effect of demographic improvements, we will assume that, when working, all individuals work full time ($h_1 = 1,\ \tilde{a} \leq a_R$) and that both the wages and consumption are constant at the age of retirement on all different periods (let's say $\bar{w}$ and $\bar{c}$ respectively):

$$\bar{c}^{\psi} \cdot \nu \cdot \ln\left(\mathcal{H}_{a_R}\right) = p(a_R) - (1-\tau)\bar{w}$$

Finally, we model the pension income in two ways. First, we assume a pay-as-you-go, defined contribution style pension system where people can retire whenever they want

[1] Details of the derivation can be found in the Appendix.

and received an annualized pension income according to their savings up to the age of retirement. Therefore:

$$\bar{c}^{\psi} \cdot \nu \cdot \ln\left(\mathcal{H}_{a_R}\right) = \frac{1}{e_{a_R}} \left[\sum_{\tilde{a}=a_S}^{a_R-1} \tau \cdot \bar{w} \cdot (1+r)^{a_R-\tilde{a}} \right] - (1-\tau)\bar{w} \tag{2}$$

were e_{a_R} is the remaining life expectancy at the age of retirement a_R.

We calibrate the model according to some parameters. First, we assume $r = 2\%$, $\tau = 0.1$. We also take a more updated age[2] for entering the labor market: $a_s = 20$. With these, we obtain the constant factor $\bar{c}^{\psi}\nu$ in such a way that, according to (*), people decide to retire at age 65 in the year 2002[3].

With all these parameters set, we only need the input from life tables to find the optimal retirement age at any period. For this illustration, we use the life tables for males[4] from the World Population Prospects (WPP) (United Nations, 2015). As these life tables are abridged, we also extrapolated the missing ages to have a complete life table. To do that, and following the results from Baili et al. (2005), we decide to use the method developed by Elandt-Johnson & Johnson (1999) to complete the life tables[5].

Additionally, as a benchmark, we will compare these retirement ages with the intergenerationally equitable alpha normal pension ages, α_p proposed by Sanderson & Scherbov (2015). The intergenerationally equitable normal pension age is derived from three assumptions. First, each cohort receives as much in pension benefits as it contributes in pension taxes. Second, the generosity of the pension system is the same for all cohorts. The generosity of the pension system is defined as the ratio of the average annual pension receipt to the average income of workers, net of their pension contributions. The third assumption is that the pension tax rate is the same for all cohorts.

Given these assumptions, for each cohort, the ratio of the number of person-years receiving a pension to the total number of person-years lived from labor market entry onwards is fixed. In particular, we fix that ratio as the one observed in the countrys life table at age 65 in year 2002, when 20 is the age of labor market entry. We denote this ratio as $\bar{k}$. In order to compute the intergenerationally equitable pension age, let $C_t(a)$ be the ratio of the number person-years receiving a pension to the total number of person-years lived from labor market entry onwards derived from the life table of year t. Then the intergenerationally equitable pension age in year t can be computed from the equation $\alpha_p = C_t^{-1}(\bar{k})$.

Table 1 presents the optimized retirement ages and alpha normal pension ages for the G8 countries[6]. As expected, if individuals of the represented countries take their increase

[2]Considering that, especially in developed countries, practically all individuals finish at least their high school education before entering the labor market. Therefore we choose a middle point between high school education and college education, that is age 20.

[3]This is the year that the US Social Security uses as a base for setting the Full Retirement Age.

[4]Even though the model we present can be applied to any individual who works full time between ages a_s and a_r, in the empirical part of this paper we focus on males, since the females labor market participation behavior is much more complex in real life.

[5]This method consists in a six-point Lagrangian interpolation between ages 1-9 and 10-74 and a Gompertz curve fitting for ages 75 and above.

[6]Results for the countries in the sample can be given by request.

Table 1: Retirement Ages and Pension Ages for G8 Countries

Period	CA		FR		DE		IT		JP		RU		GB		US	
	a_R	α_P	a_R	α_P	a_R	α_P	a_R	α_P	a_R	α_P	a_R	α_P	a_R	α_P	a_R	α_P
2000-2005	65	65	65	65	65	65	65	65	65	65	65	65	65	65	65	65
2005-2010	66	66	66	66	66	66	66	66	66	66	66	67	66	66	66	66
2010-2015	67	67	68	67	67	67	67	67	66	66	68	68	67	67	67	66
2015-2020	68	67	69	68	68	67	68	67	67	66	68	69	68	67	68	67
2020-2025	69	68	69	68	69	68	69	68	68	67	68	69	68	68	68	67
2025-2030	70	69	70	69	70	69	69	69	69	67	68	69	69	68	69	68
2030-2035	70	69	70	69	70	69	70	69	69	68	68	69	70	69	69	68
2035-2040	71	69	71	69	71	70	71	69	70	68	68	70	71	70	70	69
2040-2045	71	70	71	70	71	70	71	70	70	69	68	70	71	70	70	69
2045-2050	72	70	72	70	72	71	72	70	71	69	68	70	72	70	71	70
2050-2055	72	71	72	71	72	71	72	71	71	69	69	71	72	71	71	70
2055-2060	73	71	72	71	73	72	73	71	72	70	69	71	73	71	72	71
2060-2065	73	71	73	71	73	72	73	72	72	70	69	72	73	72	72	71
2065-2070	73	72	73	72	74	72	74	72	73	71	70	72	73	72	73	71
2070-2075	74	72	74	72	74	73	74	72	73	71	70	73	74	72	73	72
2075-2080	74	72	74	72	74	73	75	73	74	71	71	73	74	73	73	72
2080-2085	75	73	74	73	75	73	75	73	74	72	71	74	75	73	74	72
2085-2090	75	73	75	73	75	74	76	74	75	72	71	74	75	73	74	73
2090-2095	75	73	75	73	76	74	76	74	75	72	72	75	75	74	74	73
2095-2100	76	74	76	74	76	75	76	74	76	73	72	75	76	74	75	73

Notes: CA: Canada; FR: France; DE: Germany; IT: Italy; JP: Japan; RU: Russia; GB: United Kingdom; US: Unites States of America in life expectancy in consideration for their retirement decision, they should naturally delay their retirement. This pattern can be seen in all the countries represented in the WPP life tables. Therefore, the first conclusion is that, unless the social security based pension age sets a strong psychological barrier, individuals should choose to prolong their stay in the labor market, easing as a consequence the burden of the pension system.

Also, it is quite interesting to notice that these "optimized" retirement ages are, in almost all cases, above the calculated alpha normal pension ages (the exception being Russia in the presented table). This seems to be a natural result, individuals would like to maximize their own consumption, regardless of aggregate notions of *ex-ante* intracohort equality. Nevertheless, it is important to remember that these results are strongly dependent on the assumed pension system. For different schemes (e.g. defined benefit) individuals may have incentives to retire earlier (see discussion on Section 5).

4 Retirement Ages on an Life-Cycle Optimization Framework

As a second exercise, we add more flexibility to the model by allowing individuals to optimize their consumption and savings in conjunction with their retirement age. At first, we solve problem (*), that is, we assume that individuals decide optimally at the time they enter the labor market how much they will consume and how much they save in each period and when they will retire. In this model, which we denote as *M2*, individuals do not update their survival probabilities during their lifetime, which means that whatever they decided optimally at the beginning of their working life remains being their optimal choice during the rest of their lifetime. Under this framework individuals tend to increase their retirement age on a much faster way than in the purely demographic driven model (Table 2).

Second, we propose to adjust problem (*) to find a more realistic way of setting the age of retirement. We know that people learn more information as time passes by. In this case, as people get older, the probabilities of surviving up to later ages increase. Therefore, adaptive expectations seem to be an appropriate setting for this case. This means, at each age $\tilde{a}$ people readjust problem (*) according to the new survival probabilities:

$$\max_{c_a, h_a, s_{a+1}, a_R} \sum_{a=\tilde{a}}^{a_R-1} \beta^a S_a u\left(c_a, h_a, \mathcal{H}_a\right) + \sum_{a=a_R}^{\tilde{a}+e_{\tilde{a}}} \beta^a S_a u\left(c_a, 0, \mathcal{H}_a\right)$$

s.t.:

$$\begin{aligned} c_a + s_{a+1} &= (1-\tau) w_a h_a + (1+r) s_a && \tilde{a} \leq a < a_R \\ c_a + s_{a+1} &= p(a) + (1+r) s_a && a_R \leq a \leq \tilde{a} + e_{\tilde{a}} \\ & s_{\tilde{a}} \textit{ given}, \; s_{\tilde{a}+e_{\tilde{a}}} = 0 \end{aligned}$$

where $e_{\tilde{a}}$ is the remaining life expectancy at age $\tilde{a}$ and $s_{\tilde{a}}$ is the stock of savings at age $\tilde{a}$.

In the end, the problem turns out to be very simple: people will retire at the age when the utility of working one more period and retire afterwards is lower than the utility of retiring at the moment. We call this model *M3*.

Finally, we include an expansion of model *M3* where labor income is also changing over periods and lifetime. In this model, *M4*, we include an income profile for the US coming from the National Transfer Accounts Project (NTA) (Lee & Mason, 2011) for the period 2000-2005 and adjust it to be increasing by 1% on all the consecutive periods[7].

We keep the parameter values from Section 3 and we add two more parameters: $\beta = 0.98 \approx \frac{1}{1+r}$ and, following Havránek (2015), $\psi = 0.3$. Again, we calibrate the remaining parameter ν to have male individuals for the US retire at age 65 in the year 2002.

Table 2: Retirement Ages for US Males Coming from Different Models

Period	*M1*	*M2*	*M3*	*M4*
2000-2005	65	65	65	65
2005-2010	66	66	66	66
2010-2015	67	68	67	66
2015-2020	68	68	68	66
2020-2025	68	69	68	66
2025-2030	69	71	69	66
2030-2035	69	70	70	67
2035-2040	70	72	70	67
2040-2045	70	72	71	67
2045-2050	71	73	72	67
2050-2055	71	75	72	68
2055-2060	72	74	72	68
2060-2065	72	75	73	68
2065-2070	73	76	73	68
2070-2075	73	75	74	68
2075-2080	73	77	74	68
2080-2085	74	77	74	68
2085-2090	74	77	75	69
2090-2095	74	78	75	69
2095-2100	75	78	75	69

Notes:
M1: Model only with changes from life tables
M2: Model with first period optimization
M3: Model with adaptive expectations
M4: Model with adaptive expectations and increasing labor income

In Table 2 we see the results of the models *M1* (the model presented in section 3), *M2*, *M3* and *M4* for the US (similar results can be seen for the remaining countries in the sample). It is interesting to see that, even with fixed income, the retirement ages obtained from the life cycle optimization models (*M2* and *M3*) are even higher than in the purely demographic driven model (*M1*).

Finally, model (*M4*) presents a very different picture. In this case, if labor income starts declining around mid 50s, an additional extra year of work after 65 does not affect too much the potential pension income nor the retirement decision. This explains the slow increasing pattern of the retirement age, only reaching up to 69 by the end of the century. This could imply a different policy approach: instead on focusing on a pension reform,

[7] We are aware that this income profile does not truly represents the income profile of an individual working full time. However, we use it for illustration purposes: what happens if the income starts decreasing after a certain point in life due to exogenous labor market frictions

it could be more effective to focus on the labor conditions and income of people at ages close to retirement. In that way, policymakers could give enough incentives to people to remain in the labor force for a longer time, and therefore reduce the stress on the pension system.

5 Retirement According to Different Pension Schemes

As we discussed in Section 3, the optimal decisions of the individuals heavily depend on the pension scheme. Defined contribution systems as the one we used so far are easy to analyze on a life-cycle consumption and savings decisions model as, in this case, the pension can be seen as a special case of savings. Therefore, it provides a clean insight of the problem in individual terms, as, in a way, the money the individual saves on the pension fund is the money that he will receive when he retires. But when we look at other systems, for example, defined benefit systems, this kind of analysis becomes blurrier.

To illustrate this, we will show two different schemes. The first one, a simplified defined benefit scheme where individuals receive 60% of their post-tax income as a pension benefit. For the second scheme, let's assume that we have a system in where individuals retire at the time that their calculated pension income in the defined contribution scheme we proposed in Section 3 is again 60% their income before retiring[8]. We estimate both models in the adaptive expectations framework that we presented in Section 5 (model *M3*). As we can see in Table 3 (which also include alpha normal pension ages for a reference), the story changes drastically. In a defined benefit system, individuals at the starting periods would retire considerable earlier than in a defined contribution system. However, and also very interesting, as time goes by, and as life expectancy increases, the retirement age from the defined benefit model converges to the alpha normal pension age. For the second case, which we call the "target income" model, we see an opposite story, where the retirement age starts above alpha normal pension age and then also converges to it.

All cases have different policy implications. If we analyze this on a pure actuarial fairness basis, the defined contribution system is the one that evolves closer to the alpha normal pension ages. This is expected: as we said, here individuals save, retire optimally and receive ad-hoc pensions in a way that maximizes their lifetime utility. Now, in the first case, since the pension that the individuals receive is independent of how much they contributed to the system, there are all sorts of imperfections. Indeed, a 60% replacement rate is a relatively high number, so individuals have enough incentives to try to retire before what an actuarially fair system would propose[9]. This is certainly implied in the second model, as, to be able to perceive such a high percentage of their post tax income, individuals would have had to save for a much longer period.

[8] That is, when:

$$p(a_R) = \frac{1}{e_{a_R}} \left[\sum_{\tilde{a}=a_S}^{a_R-1} \tau \cdot (1+r)^{a_R-\tilde{a}} \right] = 0.6\,[(1-\tau)\bar{w}]$$

[9] And this is what we see in real life, a considerable amount of people start taking their pensions as soon as they can.

Table 3: Retirement Ages for US Males According to Different Pension Schemes using Adaptive Expectations (*M3*) and Alpha-Normal Pension Ages

Period	Defined Contribution	Defined Benefit	Target Income	α-normal Pension Age
2000-2005	65	59	68	65
2005-2010	66	60	69	66
2010-2015	67	60	69	66
2015-2020	68	61	69	67
2020-2025	68	62	70	67
2025-2030	69	63	70	68
2030-2035	70	65	70	68
2035-2040	70	66	71	69
2040-2045	71	67	71	69
2045-2050	72	68	71	70
2050-2055	72	69	72	70
2055-2060	72	69	72	71
2060-2065	73	70	72	71
2065-2070	73	71	72	71
2070-2075	74	71	73	72
2075-2080	74	72	73	72
2080-2085	74	72	73	72
2085-2090	75	73	73	73
2090-2095	75	73	73	73
2095-2100	75	74	74	73

6 Conclusions

A labor market model with dynamic entry and retirement ages are a must if we want to do analyses of labor markets and related topics such as tax and pensions systems. Here we show that, when including dynamic demographic variables, the outlook of labor markets for the upcoming years change considerably in terms of retirement age. It means that if people consider the changes in health and life expectancy they will naturally prolong their stay in the labor force. Even though this preliminary model is not considering changes in entry ages to the labor market, the results seem more adequate than models with fixed retirement age. Indeed, on a simple look, entry ages depend heavily on education: young people normally enters the labor force when they complete their desired level of education, that is, after a certain number of years which is independent on demographic variables[10]. Therefore retirement ages seem to be the main key to create a more realistic model of labor market.

The forecasted postponements in retirement immediately imply that support ratios should not be as bad as expected with the standard models. In fact, as Table 4 shows, the adaptive expectations model imply fairly stable support ratios, completely opposite to what standard models show[11].

[10]For example, at age 18 if the individual would only want to finish high school education, or age 22 if he chooses to have 4 years of college education.

[11]As the ratio of people between ages 15-64 and 65 and older.

Table 4: Estimated Support Ratios from Standard Models vs Adaptive Expectations (*M3*)

Period	Standard Models	Adaptive Expectations
2000-2005	5.4	4.8
2005-2010	5.3	5.1
2010-2015	4.8	5.1
2015-2020	4.2	5.0
2020-2025	3.6	4.2
2025-2030	3.1	4.0
2030-2035	2.9	3.9
2035-2040	2.8	3.6
2040-2045	2.8	3.8
2045-2050	2.7	4.1
2050-2055	2.7	4.0
2055-2060	2.6	4.0
2060-2065	2.5	4.1
2065-2070	2.4	3.9
2070-2075	2.4	4.0
2075-2080	2.3	3.9
2080-2085	2.3	3.8
2085-2090	2.2	4.0
2090-2095	2.2	3.9
2095-2100	2.1	3.8

This effectively changes the focus of many policy approaches. Should countries really take extreme measures designed to deal with the support ratio problem? If we consider the results of this paper, then the answer depends on many factors, but most importantly on the configuration of the pension system. A more flexible system, such as the one presented in this paper, seems to give enough incentives to individuals to delay retirement on their own, without much need for government intervention. But also, it is clear that in other systems the incentives point to the opposite direction.

And this is what is shown in most studies of pension systems. In the end, if the system is not designed in a way that individuals receive something that is consistent with what they are contributing, then problems will definitely arise. In this way, it is especially crucial to recognize the psychological bias that a pension age produces on an individual. Certainly, we cannot pretend to assume that all individuals conduct a full analysis on when to retire optimally according to evolving demographic variables[12]. On the contrary, it is safe to assume that a significant group of people would actually retire when they reach their pension age. For these cases, setting a demographically indexed pension age ends up being a critical issue, otherwise, the "demographic time bomb" will explode only as the result misshaped policies.

[12] Although individuals' subjective life expectancy it is closely related to their objective life expectancy (e.g. Hurd & McGarry 2002), actual retirement does not really coincide with planned retirement ages (e.g. Khan et al. 2014.)

7 References

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8 Appendix

There are two Lagrangians representing working and retirement periods:

$$\begin{aligned}
\mathcal{L}_w =& S_a u\left(c_a, h_a, \mathcal{H}_a\right) + \lambda_a^w\left[(1-\tau)w_a h_a + (1+r)s_a - c_a - s_{a+1}\right] \\
\mathcal{L}_r =& S_a u\left(c_a, 0, \mathcal{H}_a\right) + \lambda_a^r\left[p(a) + (1+r)s_a - c_a - s_{a+1}\right]
\end{aligned}$$

Taking first order conditions on consumption we have:

$$\begin{aligned}
\frac{\partial \mathcal{L}_w}{\partial c_a} =& S_a \frac{\partial u(c_a, h_a, \mathcal{H}_a)}{\partial c_a} - \lambda_a^w = 0 \\
\frac{\partial \mathcal{L}_r}{\partial c_a} =& S_a \frac{\partial u(c_a, 0, \mathcal{H}_a)}{\partial c_a} - \lambda_a^r = 0
\end{aligned}$$

Now, because of continuity, it has to be that at the age of retirement both Lagrange multipliers $\lambda_{a_R}^w$ and $\lambda_{a_R}^r$ have identical value:

$$\begin{aligned}
\lambda_{a_R}^w &= \lambda_{a_R}^r \\
\Rightarrow S_{a_R} \frac{\partial u(c_{a_R}, h_{a_R}, \mathcal{H}_{a_R})}{\partial c_{a_R}} &= S_{a_R} \frac{\partial u(c_{a_R}, 0, \mathcal{H}_{a_R})}{\partial c_{a_R}} = S_{a_R} u_{c_{a_R}}
\end{aligned}$$

Also, again because of continuity, the Lagrangians have identical values at the age of retirement:

$$\begin{aligned}
\mathcal{L}_w(a_R) &= \mathcal{L}_r(a_R) \\
\Rightarrow S_{a_R} u\left(c_{a_R}, h_{a_R}, \mathcal{H}_{a_R}\right) + &\lambda_{a_R}^w\left[(1-\tau)w_{a_R} h_{a_R} + (1+r)s_{a_R} - c_{a_R} - s_{a_R+1}\right] \\
= S_{a_R} u\left(c_{a_R}, 0, \mathcal{H}_{a_R}\right) &+ \lambda_{a_R}^r\left[p(a_R) + (1+r)s_{a_R} - c_{a_R} - s_{a_R+1}\right]
\end{aligned}$$

After simplifying we end up with condition (1):

$$\frac{u\left(c_{a_R}, h_{a_R}, \mathcal{H}_{a_R}\right) - u\left(c_{a_R}, 0, \mathcal{H}_{a_R}\right)}{u_{c_{a_R}}} = p(a_R) - (1-\tau)w_{a_R} h_{a_R} \tag{1}$$