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A DEMOECONOMETRIC MODEL OF POLAND: DEMP 1

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#### PREFACE

Roughly 1.6 billion people, 40 percent of the world's population, live in urban areas today. At the beginning of the last century, the urban population of the world totaled only 25 million. According to recent United Nations estimates, about 3.1 billion people, twice today's urban population, will be living in urban areas by the year 2000.

Scholars and policymakers often disagree when it comes to evaluating the desirability of current rapid rates of urban growth in many parts of the globe. Some see this trend as fostering national processes of socioeconomic development, particularly in the poorer and rapidly urbanizing countries of the Third World; whereas others believe the consequences to be largely undesirable and argue that such urban growth should be slowed down.

As part of a search for convincing evidence for or against rapid rates of urban growth, a Human Settlements and Services research team, working with the Food and Agriculture Program, is analyzing the transition of a national economy from a primarily rural agrarian to an urban industrial-service society. Data from several countries selected as case studies are being collected, and the research is focusing on two themes: spatial population growth and economic (agricultural) development, and resource/service demands of population growth and economic development.

This paper is one of several focusing on one of five case studies: Poland. In it, Dr. Pawlowski uses a number of economic, technological, and demographic variables in order to explain by means of a demoeconometric model, the past growth of the Polish economy.

A list of related papers in the Population, Resources and Growth Series appears at the end of this publication.

#### ABSTRACT

This paper presents a demoeconometric model of Poland, i.e., a model that tries to explain the growth mechanism of the economy not only by analysing economic or technological factors, but also by making use of a number of demographic variables. The behavior of some important demographic phenomena is, in turn, presented as being a function of economic factors.

A general view of the model is given, as well as the endogenous and the predetermined variables used. The model concentrates on five blocks of phenomena: 1) employment, 2) investments, 3) national income formation, 4) consumption, and 5) demography. In its present form the model may be used to analyse the quantitative relations between the variables chosen, to compute counterfactual simulations, and to make predictions on future behavior.

#### A DEMOECONOMETRIC MODEL OF POLAND: DEMP 1

#### I. INTRODUCTION

The history of econometric macromodeling of the Polish economy has been a relatively short one, starting in the midsixties. Reference can be made here to a paper by Pawlowski, et al. (1964) in which a small, six-equation model of such highly aggregative variables as national income, total employment, investments, wage rate, and foreign trade were presented. One should also mention Pajestka's book (1961) in which its author uses the classical Cobb-Douglas-Tinbergen type of production function to estimate the influence of increased labor inputs, investments, and neutral technical progress on the growth of national income in Poland. In 1968 Pawlowski, together with Barczak, Ciepielewska, and Jakubczyk (1968), published a 17-equation model of the Polish economy.

In later years other authors contributed to this type of research, building new and larger econometric macromodels (see Maciejewski [1976], Maciewjewski, et al. [1974], Kanton [1975], and Welfe, W. and W. Debski [1976]). All these models were essentially of a short or mid-term character and concentrated mainly on three types of problems: productive activity and national income formation, foreign trade, and consumption. Provision for demographic phenomena or attempts at explaining them in the context of economic growth, to this author's knowledge, have so far not been made in the course of econometric modeling activities in Poland.

This paper presents a first version of a demoeconometric model of Poland: a model that takes into account not only the classic economic or technological factors of growth but also makes provision for the influence of demographic phenomena and tries to explain how some of the demographic coefficients are affected by economic variables.

Poland is an especially interesting country for demoeconometric modeling since many of its economic and especially demographic variables exhibit specific features unknown in other European countries. First, there exists a strong post-war urbanization process that induces significant migration from rural to urban areas. Second, there is the phenomenon of peasantworkers, i.e., of people who own small private farms and simultaneously take permanent jobs in state-owned industrial, construction, or transportation enterprises. Third, the Polish agriculture consists of two sectors, the majority of land being owned by small farmers and the remaining being composed of large state or cooperative farms. Finally, during the entire postwar period there has been no unemployment, in fact, there have been periods of serious shortages of manpower in non-agricultural The main way of coping with this problem has been to sectors. attract young people from agriculture to state-owned non-agricultural enterprises, thus stimulating urban immigration and creating the phenomenon of peasant-workers.

The model presented here has 30 endogenous variables that can be divided into five blocks, each block containing variables referring to a separate sphere of economic or demographic phenomena. Because the various economic and demographic factors are interdependent, these five blocks of endogenous variables are also interdependent

- 1) Employment
- 2) Investments
- 3) National income formation
- 4) Consumption
- 5) Demographic phenomena

As can be seen from this list, our model does not consider a number of phenomena which are usually included in econometric modeling. First, one should note that no provision is made for foreign trade. Second, the model does not deal with a price mechanism. Third, there are no financial flow variables in the model. Omission of foreign trade has been done purposively in order to keep the size of the model within the reasonable limits. Restriction of the model to the "real part" of economic flows is due to the fact that in planned economies prices are mostly determined by administrative decisions and, therefore, price equations would have no predictive value.\* The lack of a block of equations reflecting financial flows is partly due to the specific character of prices and partly to the fact that in a planned economy such flows are also to a large extent administratively determined.

While the foreign trade is meant to be included in a later version of the model one must note that an exception has been made to the rule excluding price equations. Among the endogenous variables of the model there is one defined as the consumer price index. Since such prices determine to some extent the level of the standard of living it was thought advisable to include the relevant equation in the model, although it does not explain entirely the underlying mechanism for the formation of consumer prices.\*\*

A detailed list of endogenous and exogenous variables of the model are presented in Sections II and III. Meanwhile, we wish to draw attention to another aspect of the model, namely the existence of a relatively large number of dummy variables. Introduction of such variables is chiefly due to the fact that a planned economy is not a self-expanding mechanism but undergoes stimuli and shifts induced by planners and administrative authorities. If such stimuli motivate changes of some known measurable decision variables, then they can be dealt with in

<sup>\*</sup>On the other hand, one might note that an ex-post modeling of prices gives some insight into the problem of how the relevant economic institutions are making their decisions about price levels.

<sup>\*\*</sup>Let us observe also that while the price of a single commodity is fully regulated by administrative decisions the situation is more promising when observing an aggregate composed of a large number of individual prices. In the second case one can expect the law of great numbers to enter into action, thus finding some regularities as to the behavior of the overall index of consumer prices.

the classical way, i.e., by introducing these variables in the appropriate equation(s) of the model. However, one also meets situations where there are shifts of economic policy of a more qualitative character. Such changes can be introduced as dummy variables, assuming their zero values in "normal" periods and unit values when a special policy is pursued.

The model presented in this paper has been estimated on the basis of 17 yearly observations pertaining to the years 1960-1976. The choice of this particular period was due primarily to the fact that structural parameters can reasonably be expected to remain constant in time. However, the author is aware of the fact that to begin the analysis with 1960 as the first sample year means to leave out the first post-war years when migrations from rural to urban areas were especially strong, giving rise to a number of completely new towns. Therefore, a new second version of the model will be based on a larger sample covering the period from 1950 to 1976, or perhaps even to 1978. Besides giving a better opportunity to study migrations and the interdependence of economic and demographic variables, the model based on an extended sample will give an insight into some probable changes of structural parameters.

In its present form the model may be used for three types of activities. First, it provides an insight into the observed structure, thus revealing the quantitative relations between This, in turn, makes it possible to analyse the variables. extent to which the corresponding regularities were blurred by random effects and the order of magnitude and pattern of behavior of such disturbances. Second, the model can be used for counterfactual simulations, i.e., for computing the expected values of endogenous variables under the assumption that the exogenous variables followed by a determined path, different however from the one they actually took in the past. Finally, one must point out that the model may also be used for making predictions into the future. From the nature of the model and judging by past experience in Poland, one would expect the

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model to give reasonable predictions for short or mid-term (not longer than five years) inference into the future.

#### II. THE ENDOGENOUS VARIABLES OF THE MODEL

The following is a list of the endogenous variables appearing in the model.

- Y<sub>1</sub> national income computed according to the material product concept from non-agricultural sectors in billions zlotys, constant prices (Whenever referred to, constant prices denote prices of 1971.),
- Y<sub>2</sub> employment in non-agricultural productive sectors in millions, peasant-workers excluded (All data referring to employment and population size are yearly averages.),
- Y<sub>3</sub> employment of peasant-workers in non-agricultural productive sectors\*, in millions,
- Y<sub>4</sub> employment in agriculture, both in private, cooperative, and state farms, part-time work on private farms by family members included, in millions,
- $Y_5$  urban population, in millions,
- $Y_{c}$  rural population, in millions,
- $Y_7 = Y_2 + Y_3$ ,
- Y<sub>8</sub> national income from agriculture in billions zlotys, constant prices,
- Y<sub>9</sub> employment in services, in millions (Here the sector of services corresponds to all non-productive sectors, i.e., state administration, health care, education, culture and science, trade, individual craftsmanship.),

 $Y_{10} \equiv Y_2 + Y_3 + Y_4 + Y_9$ ,

- Y<sub>11</sub> endogenous investment in non-agricultural productive sectors, constant prices,
- Y<sub>12</sub> endogenous investment in agriculture, constant prices,
- Y<sub>13</sub> endogenous investment in services, constant prices,

<sup>\*</sup>There are virtually no peasant-workers employed outside the sectors of industry, building, and transportation.

<sup>Y</sup> 14	-	total investment in non-agricultural productive sectors, constant prices,
<sup>Y</sup> 15	-	total investment in agriculture, constant prices,
<sup>Y</sup> 16	-	total investment in services, constant prices,
<sup>Y</sup> 17	-	total investment in industry, constant prices,
<sup>Y</sup> 18	Ξ	<sup>Y</sup> 1 <sup>+</sup> <sup>Y</sup> 8 ′
<sup>Y</sup> 19	-	labor productivity (ratio of national income stemming from given sector to employment in that sector) in non-agricultural productive sectors,
		10 <sup>°</sup> zlotys/person,
<sup>Y</sup> 20	-	labor productivity in agriculture, 10 <sup>3</sup> zlotys/person,
<sup>Y</sup> 21	Ξ	$\alpha_{1}(t) \cdot Y_{19} + \alpha_{2}(t) \cdot Y_{20}$ , $\alpha_{1}(t) + \alpha_{2}(t) = 1^{9}$ ,
v		
-22	-	consumption out of private funds in constant prices, i.e., personal consumption stemming from individual incomes (wages, old-age pensions, scholarships, sales of agricultural products by farmers to the state, etc.),
<sup>1</sup> 22 <sup>Y</sup> 23	-	consumption out of private funds in constant prices, i.e., personal consumption stemming from individual incomes (wages, old-age pensions, scholarships, sales of agricultural products by farmers to the state, etc.), overall index of consumer prices,
<sup>Y</sup> 23 <sup>Y</sup> 24	-	<pre>consumption out of private funds in constant prices, i.e., personal consumption stemming from individual incomes (wages, old-age pensions, scholarships, sales of agricultural products by farmers to the state, etc.), overall index of consumer prices, urban birth rate, per 1000 inhabitants,</pre>
<sup>Y</sup> 23 <sup>Y</sup> 23 <sup>Y</sup> 24 <sup>Y</sup> 25		<pre>consumption out of private funds in constant prices, i.e., personal consumption stemming from individual incomes (wages, old-age pensions, scholarships, sales of agricultural products by farmers to the state, etc.), overall index of consumer prices, urban birth rate, per 1000 inhabitants, rural birth rate, per 1000 inhabitants,</pre>
<sup>Y</sup> 23 <sup>Y</sup> 24 <sup>Y</sup> 25 <sup>Y</sup> 26		<pre>consumption out of private funds in constant prices, i.e., personal consumption stemming from individual incomes (wages, old-age pensions, scholarships, sales of agricultural products by farmers to the state, etc.), overall index of consumer prices, urban birth rate, per 1000 inhabitants, rural birth rate, per 1000 inhabitants, urban death rate, per 1000 inhabitants,</pre>
<sup>Y</sup> 23 <sup>Y</sup> 24 <sup>Y</sup> 25 <sup>Y</sup> 26 <sup>Y</sup> 27		<pre>consumption out of private funds in constant prices, i.e., personal consumption stemming from individual incomes (wages, old-age pensions, scholarships, sales of agricultural products by farmers to the state, etc.), overall index of consumer prices, urban birth rate, per 1000 inhabitants, rural birth rate, per 1000 inhabitants, urban death rate, per 1000 inhabitants, rural death rate, per 1000 inhabitants,</pre>
<sup>Y</sup> 23 <sup>Y</sup> 24 <sup>Y</sup> 25 <sup>Y</sup> 26 <sup>Y</sup> 27 <sup>Y</sup> 28		<pre>consumption out of private funds in constant prices, i.e., personal consumption stemming from individual incomes (wages, old-age pensions, scholarships, sales of agricultural products by farmers to the state, etc.), overall index of consumer prices, urban birth rate, per 1000 inhabitants, rural birth rate, per 1000 inhabitants, urban death rate, per 1000 inhabitants, rural death rate, per 1000 inhabitants, urban net inmigration rate, per 1000 inhabitants,</pre>
<sup>Y</sup> 23 Y24 Y25 Y26 Y27 Y28 Y29		<pre>consumption out of private funds in constant prices, i.e., personal consumption stemming from individual incomes (wages, old-age pensions, scholarships, sales of agricultural products by farmers to the state, etc.), overall index of consumer prices, urban birth rate, per 1000 inhabitants, rural birth rate, per 1000 inhabitants, urban death rate, per 1000 inhabitants, rural death rate, per 1000 inhabitants, rural death rate, per 1000 inhabitants, rural net inmigration rate, per 1000 inhabitants, rural net inmigration rate, per 1000 inhabitants,</pre>

The allocation of the endogenous variables to the five endogenous blocks already referred to in Section 1 is as follows:

Employment Block A =  $\{Y_2, Y_3, Y_4, Y_7, Y_9, Y_{10}, Y_{19}, Y_{20}, Y_{21}\}$ Investments Block B =  $\{Y_{11}, Y_{12}, Y_{13}, Y_{14}, Y_{15}, Y_{16}, Y_{17}, Y_{30}\}$ National income formation Block C =  $\{Y_1, Y_8, Y_{18}\}$ Consumption Block D =  $\{Y_{22}, Y_{23}\}$ Demographic phenomena Block E =  $\{Y_5, Y_6, Y_{24}, Y_{25}, Y_{26}, Y_{27}, Y_{28}, Y_{29}\}$ .

#### III. THE PREDETERMINED VARIABLES OF THE MODEL

The set of predetermined\* variables of the model is composed of 32 variables, out of which 14 are genuinely exogenous, 11 are lagged endogenous, 6 are dummy, and one is the unit variable introduced in order to allow constant terms in linear equations. Below are the definitions of all predetermined variables:

 $Z_1 - Y_1$  lagged one year,  $Z_2 - Y_{14}$  lagged two years,  $Z_3 - Y_{14}$  lagged three years, Z<sub>n</sub> - fixed assets in agriculture in billions zlotys, constant prices, Z<sub>5</sub> - real wage per capita in socialized non-agricultural sectors,  $Z_6 - Y_5$  lagged one year,  $Z_7 - Y_6$  lagged one year,  $z_8 \equiv z_5 - z_{10}$ , Z<sub>o</sub> - use of artificial fertilizers in non-agricultural productive sectors (100 kg/ha), Z<sub>10</sub> - index of real agricultural incomes, i.e., real income per capita derived from private farming,  $Z_{11}$  - time variable equals 1 in 1960, equals 2 in 1961, etc.,  $Z_{12}$  - weather dummy variable;  $Z_{12} = 1$  in bad years when agriculture suffered from exceptionally dry weather or from an unusually wet one, Z<sub>13</sub> - exogenous investment in non-agricultural producductive sectors, billions zlotys, constant prices,  $Z_{1\mu}$  - exogenous investment in agriculture, billions zlotys, constant prices, Z<sub>15</sub> - exogenous investment in services, billions zlotys, constant prices, Z<sub>16</sub> - exogenous investment in industry, billions zlotys, constant prices, . . . . . . . . . .

<sup>\*</sup>We use here the terminology introduced by T.C. Koopmans. The set of predetermined variables is composed of exogenous and of lagged endogenous variables.

Z <sub>17</sub> - void,	
Z <sub>18</sub> - flats constructed without residential construction	
(state or private) in rural areas, in 10 <sup>5</sup> rooms,	
<sup>Z</sup> <sub>19</sub> - balance of foreign trade (export minus import), current zlotys,	
$Z_{20} \equiv 1$ , unit variable,	
$z_{21}$ - heavy investment dummy variable, $z_{21}$ = 1 for years	
when investing was especially favored,	
Z <sub>22</sub> - fast economic growth dummy variable, Z <sub>22</sub> = 1 for 1971-1976,	
$Z_{23}$ - bad agricultural production variable, $Z_{23+} = 1$	
when $Y_{8t} < Y_{8,t-1}$	
$Z_{24}$ - demographic echo dummy variable ( $Z_{24}$ =1 in years	
when large generations, born during the post-war baby-boom, came to maturity and started breeding children themselves.),	
<sup>Z</sup> <sub>25</sub> - <sup>Y</sup> <sub>19</sub> lagged one year,	
<sup>Z</sup> <sub>26</sub> - <sup>Y</sup> <sub>15</sub> lagged one year,	
<sup>Z</sup> <sub>27</sub> - Y <sub>15</sub> lagged two years,	
<sup>z</sup> <sub>28</sub> - void,	
<sup>2</sup> 29 - void,	
$z_{30}$ - squares of time variable, i.e., $z_{30} = z_{11}^2$ ,	
Z <sub>32</sub> - Y <sub>14</sub> lagged one year,	
Z <sub>33</sub> - Y <sub>18</sub> lagged one year,	
Z <sub>34</sub> - Y <sub>16</sub> lagged one year,	
$Z_{35}$ - dummy variable; $Z_{35}$ = 1 for every year from 1970	
until 1976 (This variable is connected with a variant of economic policy arrived at by giving more weight to the expansion of the services sector especially those services dealing with trade, health, education, culture, and to ser- vices provided by individually working craftsmen.),	
Z <sub>36</sub> - Y <sub>17</sub> lagged one year.	

As will be seen from later sections of this paper, not all of the predetermined variables listed above were included in the adopted version of the model. Their listing, however, reflects the variety of experiments performed before finding the final structural form of the model--at least the final one at this stage of research.

#### IV. THE METHOD ADOPTED FOR CHOOSING THE EXPLANATORY VARIABLES

Building the structural form is one of the crucial tasks of any econometric modeling and it also becomes one of the most difficult tasks when there is no well-developed economic quantitative theory underlying the system subject to modeling. This was, to a large extent, true of the situation met while building the present model. Economic theory provided guidelines, but a restricted number of variables should be included-as explanatory ones--into various equations and these varibles in most cases were not enough to assure an adequate degree-offit of the respective equations with the statistical data. It was necessary, therefore, to look for additional explanatory variables as well, although, because of the short time series used for estimation, the number of explanatory variables included in the structural form equations could not be large.

To cope with this situation the following procedure was adopted. For every endogenous variable of the model (excluding variables explained by means of definition identities and investment variables for which a different approach was used\*) a list of potential explanatory variables were drawn. Such a list usually included three types of variables: a) variables suggested by the existing economic theory, b) variables advocated by experience, practical knowledge of economic mechanism, or good common sense, and c) variables accounted for on the basis of working hypotheses describing their possible impact on the endogenous variable to be explained. Once such a list was completed there arose the need for a rule for the final adoption or rejection of potential explanatory variables.

<sup>\*</sup>The way the investment equations were built is presented in Section VII.

Variables listed under (a) were usually assumed to enter the corresponding variables without further considerations. If these variables alone were not sufficient to ensure an adequate fit of equation to statistical data\*, then a special algorithm was used (Pawlowski, 1973). From the list of potential explanatory variables, subsets were considered assuming the following constraints.

- All variables suggested by economic theory must be included in the considered subsets of explanatory variables.
- 2) The explanatory variables forming the subset must guarantee an admissible degree-of-fit of the equation with statistical data.
- 3) The explanatory variables must be as little intercorrelated among themselves as possible.
- 4) The number of explanatory variables must be small, i.e., from among all possible subsets of explanatory variables obeying the conditions (1), (2), and (3), the subset containing the least number of variables was finally chosen as the vector or explanatory variables of the equation to be estimated.

Let us note that condition (4) is of importance only when the sample size is small. Were a longer time series used for the estimation, there would have been no need to restrict the number of explanatory variables. In cases when the sample is small, however, it usually pays to also keep the number of  $e_{x}$ planatory variables small, otherwise their standard errors of estimation would assume values that would be too high.

<sup>\*</sup>One can adopt different measures of the degree-of-fit; the essential thing being, however, to have some a priori idea as to the admissible degree-of-fit. Let us also note that this level of admissible degree-of-fit may not be the same for all endogenous variables, but may vary according to practical needs and other considerations.

When speaking about standard errors of estimation one should bear in mind that the approach adopted here may result sometimes in arriving at statistically non-significant estimates of structural parameters, especially in the case of explanatory variables suggested by economic theory. This stems from the fact that in the case of small samples even when an explanatory variable has a genuine impact on the endogenous one, the probability of making a second-type error\* may still be large. Therefore, the explanatory variable in question should be retained on the basis of the underlying economic theory (or strong empirical evidence) while at the same time a larger sample would prove to be significant from the statistical point of view.

It seems worthwhile to point out that in the process of building the present model a rather extensive use of dummy variables has been made. Roughly speaking, the dummy variables introduced into the model can be split into two categories.

The first category includes dummy variables that account for the exceptional impact of some exogenous natural conditions. For example, we have a dummy variable which assumes the value of one in the years when very bad weather conditions prevail. The Polish agriculture is very sensitive to strong deviations from the normal climatic conditions and therefore, this variable is important to our analysis. Also in the first category we have the  $Z_{24}$  dummy variable. This variable was introduced in order to account for the "demographic echo" phenomenon, i.e., the short-run rise of births in the early 1970s to parents who themselves were born during the post-war baby-boom.

The second category consists of dummy variables that represent those shifts of economic policy that cannot be seen

<sup>\*</sup>In this case a second-type error means rejecting a variable from the set of explanatory variables when it actually exerts an influence on the endogenous variables. (In other words, it is the acceptance of a null hypothesis when this hypothesis is false.)

as a natural projection of the present national economy and that are too substantial to be represented by mere quantitative changes in other explanatory variables. Two such variables are typical examples. One of them is the Z<sub>22</sub> variable, which assumes the value of 1 in the years 1971-1976 and the value of 0 for all previous years. The reason for introducing Z<sub>22</sub> (which, as can be seen from estimation results, proved to be highly significant) is that a new policy of fast economic growth, coupled with a significant rise in the standard of living, was initiated by Polish authorities in 1971. This policy generated substantial changes not only in economics-fast rate of growth, greater efficiency, better management -but also in social relations and attitudes--higher labor productivity, new consumption patterns, attaching new value to family life. A second example of this category of dummy variables is variable Z<sub>21</sub> which is equal to 1 in years when the policy of heavy productive investment is especially pursued and which is equal to 0 in all other years. Since the policy of heavy investing had several side-effects on phenomena represented by a number of endogenous variables in the model, it was necessary to make use of Z<sub>21</sub> in order to account for this exogenous impact.

To conclude this argument it should be noted that in general, the part of the model that is composed of dummy variables and their coefficients has no predictive meaning. Although the coefficient associated with a dummy variable represents the (average) past level of impact of factors summarily represented by this dummy, it does not necessarily follow that the size of such an impact would be the same in the future. Therefore, when making predictions for a future time period, T, one must have some additional information. This information must be sufficient in order to decide whether the dummy variable should assume the value of 1 or 0 for time T and, second, this information must provide the answer to the question, what in time T will be the value of the coefficient associated with the dummy variable in question?

#### V. THE STRUCTURAL FORM OF THE MODEL

The structural form of the present version of the model was reached in steps, after having experimented with several variants of alternative equations, such variants being characterized by different sets of explanatory variables. Even so, some of the equations still seem to need improvements. Their refinement, however, requires additional statistical data.

The model is predominantly linear; the only non-linearities appear in equations explaining the endogenous variables  $Y_5$  and  $Y_6$  (urban and rural population, respectively) and in that block of the model which generates endogenous investments.

Since there are 30 endogenous variables in the model there are as many equations explaining the variations of these variables, 10 of them being identities corresponding to the used system of definitions of variables.\* All the equations which are not identities are of stochastic character. Thus, in the estimation process of the model, not only the structural parameters were found from collected statistical data, but also some characteristics of probability distribution of the random components of such equations were computed.

The numerical results of estimation are found in Section VII and a more detailed explanation of how the endogenous investment was computed appears in Section VI. We present now the general shape of the structural form, but restrict ourselves to the enumeration of the explanatory variables connected with the endogenous ones. The symbol L appearing in an equation stands for a linear relationship while N denotes a non-linear one. The symbol  $Z_{20}$  denotes a constant term while  $\xi$  stands for a random component.

$$Y_{1} = L(Y_{7}, Z_{1}, Z_{2}, Z_{3}, Z_{22}, Z_{20}, \xi)$$
$$Y_{2} = L(Y_{1}, Y_{5}, Z_{11}, Z_{20}, \xi)$$

<sup>\*</sup>The model does not contain equilibrium identities.

$$\begin{split} \mathbf{Y}_{3} &= \mathbf{L}(\mathbf{Y}_{1}, \mathbf{Y}_{14}, \mathbf{Z}_{5}, \mathbf{Z}_{18}, \mathbf{Z}_{20}, \mathbf{\xi}) \\ \mathbf{Y}_{4} &= \mathbf{L}(\mathbf{Y}_{6}, \mathbf{Y}_{15}, \mathbf{Z}_{22}, \mathbf{Z}_{20}, \mathbf{\xi}) \\ \mathbf{Y}_{5} &\equiv \left(1 + \frac{\mathbf{Y}_{24} - \mathbf{Y}_{26} + \mathbf{Y}_{28}}{1000}\right) \mathbf{Z}_{6} \\ \mathbf{Y}_{6} &\equiv \left(1 + \frac{\mathbf{Y}_{25} - \mathbf{Y}_{27} - \mathbf{Y}_{29}}{1000}\right) \mathbf{Z}_{7} \\ \mathbf{Y}_{7} &\equiv \mathbf{Y}_{2} + \mathbf{Y}_{3} \\ \mathbf{Y}_{8} &= \mathbf{L}(\mathbf{Z}_{4}, \mathbf{Z}_{9}, \mathbf{Z}_{12}, \mathbf{Z}_{22}, \mathbf{Z}_{20}, \mathbf{\xi}) \\ \mathbf{Y}_{9} &= \mathbf{L}(\mathbf{Y}_{1}, \mathbf{Y}_{5}, \mathbf{Y}_{22}, \mathbf{Z}_{20}, \mathbf{\xi}) \\ \mathbf{Y}_{10} &\equiv \mathbf{Y}_{2} + \mathbf{Y}_{3} + \mathbf{Y}_{4} + \mathbf{Y}_{9} \\ \mathbf{Y}_{11} &= \mathbf{N}(\mathbf{Y}_{14}, \mathbf{Y}_{18}, \mathbf{Z}_{22}, \mathbf{Z}_{32}, \mathbf{Z}_{33}, \mathbf{Z}_{20}, \mathbf{\xi}) \\ \mathbf{Y}_{12} &= \mathbf{N}(\mathbf{Y}_{15}, \mathbf{Y}_{18}, \mathbf{Z}_{11}, \mathbf{Z}_{26}, \mathbf{Z}_{33}, \mathbf{Z}_{20}, \mathbf{\xi}) \\ \mathbf{Y}_{13} &= \mathbf{N}(\mathbf{Y}_{16}, \mathbf{Y}_{18}, \mathbf{Z}_{33}, \mathbf{Z}_{34}, \mathbf{Z}_{35}, \mathbf{Z}_{20}, \mathbf{\xi}) \\ \mathbf{Y}_{14} &\equiv \mathbf{Y}_{11} + \mathbf{Z}_{13} \\ \mathbf{Y}_{15} &\equiv \mathbf{Y}_{12} + \mathbf{Z}_{14} \\ \mathbf{Y}_{16} &\equiv \mathbf{Y}_{13} + \mathbf{Z}_{15} \\ \mathbf{Y}_{17} &\equiv \mathbf{Y}_{30} + \mathbf{Z}_{31} \\ \mathbf{Y}_{18} &\equiv \mathbf{Y}_{1} + \mathbf{Y}_{8} \\ \mathbf{Y}_{19} &= \mathbf{L}(\mathbf{Z}_{1}, \mathbf{Z}_{25}, \mathbf{Z}_{20}, \mathbf{\xi}) \\ \mathbf{Y}_{20} &= \mathbf{L}(\mathbf{Z}_{26}, \mathbf{Z}_{27}, \mathbf{Z}_{12}, \mathbf{Z}_{20}, \mathbf{\xi}) \\ \mathbf{Y}_{21} &\equiv \alpha_{1}(\mathbf{t}) \cdot \mathbf{Y}_{19} + \begin{bmatrix}1 - \alpha_{1}(\mathbf{t})\end{bmatrix} \mathbf{Y}_{20} \\ \mathbf{Y}_{22} &= \mathbf{L}(\mathbf{Y}_{8}, \mathbf{Y}_{20}, \mathbf{Y}_{23}, \mathbf{Z}_{27}, \mathbf{Z}_{23}, \mathbf{Z}_{20}, \mathbf{\xi}) \\ \mathbf{Y}_{23} &= \mathbf{L}(\mathbf{Y}_{8}, \mathbf{Y}_{10}, \mathbf{Z}_{19}, \mathbf{Z}_{20}, \mathbf{\xi}) \end{split}$$

$$Y_{24} = L(Y_{22}, Z_{22}, Z_{24}, Z_{20}, \xi)$$

$$Y_{25} = L(Y_{22}, Z_{22}, Z_{20}, \xi)$$

$$Y_{26} = L(Y_1, Z_{11}, Z_{30}, Z_{20}, \xi)$$

$$Y_{27} = L(Y_8, Z_{11}, Z_{30}, Z_{20}, \xi)$$

$$Y_{28} = L(Y_{14}, Z_8, Z_{21}, Z_{20}, \xi)$$

$$Y_{29} = L(Y_{14}, Z_8, Z_{21}, Z_{20}, \xi)$$

$$Y_{30} = N(Y_{17}, Y_{18}, Z_{22}, Z_{33}, Z_{20}, \xi)$$

As can easily be verified, if identities are put apart, the model contains 20 equations to be estimated. Of these, 7 are interdependent linear equations, 9 are linear equations, either recursive or such that the endogenous variable depends only on predetermined variables, and 4 are non-linear relations connected with endogenous investment formation.

#### VI. THE DATA

The model has been estimated by using Polish official statistics yearly data, the basic period being that of 1960-1976. Although most of the data were provided by consecutive Statistical Yearbooks of Poland (Rocznik Statystyczny Polski, 1960-1975), in several cases the data appearing there had to be corrected by the author either in order to express them in the same prices or to remedy some changes of classification and definition used by the Central Statistical Office of Poland.

The data collected are shown on graphs in Appendix A. In order to facilitate comparisons of different rates of growth and variation of variables, all data (except in the case of dummy variables and variables  $Z_8$  and  $Z_{19}$  which take positive as well as negative values) are expressed as indices with 1960 as the base year, i.e., the year for which the value of the index number is 100.

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#### VII. ESTIMATION OF ENDOGENOUS AND EXOGENOUS INVESTMENT

In macroeconometric modeling one often comes to the probshould investment variables be treated endogenously or lem: exogenously? Both types of solutions can be found in econometric literature and both approaches can be shown to have some advantages as well as disadvantages. In studies of growth patterns of countries with centrally planned economies, it is usually thought more convenient to treat investment as an exogenous decision variable since its behavior in time can be considered as reflecting planned actions and shifts in economic policy. While agreeing that an important part of investment outlays can be treated as exogenous, one should also point out that past investments determine to a certain degree the level of investment outlays in the next years. This is due to the 1) investment activity is usually a long one--in fact that: most known cases its cycle being longer than one year, and 2) investment activities which were started in the past must be carried on until their completion.

The above arguments lead to the conclusion that one should try to split total investment outlays into two components: the endogenous part and the exogenous one. To this scope the author has devised some simple models which have enabled him to proceed with the disaggregation of the following four total investment variables:

- a) total investment in non-agricultural productive sectors - variable Y<sub>14</sub>,
- b) total investment in agriculture variable Y<sub>15</sub>,
- c) total investment in services variable Y<sub>16</sub>,
- d) total investment in industry variable  $Y_{17}$  .

The main idea underlying the models that are used as tools for finding the endogenous and the exogenous part of investment is that the share of a given sector of total investment in national income be dependent on a similar share in previous years and, eventually, on other variables as well. This leads to models of the following type:  $\frac{\text{Total investment in year t, sector j}}{\text{National income in year t}} = \alpha_1 \frac{\text{Total investment in year t-1, sector j}}{\text{National income in year t-1}}$ 

+ 
$$f(v_1, v_2, ..., v_k) + \alpha_2 + \eta$$
, (1)

where  $f(v_1, v_2, ..., v_k)$  stands for an (eventual) function of other variables,  $\alpha_2$  is a constant term, and  $\eta$  is a random component. Once the model (1) has been estimated and thus the estimate of  $\alpha_1$  is known, the endogenous part of investment in year t in sector j is computed using the formula:

Endogenous investment year t, sector  $j = \alpha_1$  Total investment in year t-1, sector j, (2)

where  $a_1$  stands for the estimate of  $\alpha_1$ . Below, we present the four empirical versions of the model (1) by means in which investment totals were disaggregated into the endogenous and exogenous components. Obviously once the endogenous investment has been found, the exogenous one is obtained by subtracting the endogenous investment from the total investment.

$$\frac{Y_{14}}{Y_{18}} - 0.229 = 0.6063 \left( \frac{Z_{32}}{Z_{33}} - 0.222 \right) + 0.0435 (Z_{22} - 0.294)$$
(3)

$$\frac{Y_{15}}{Y_{18}} - 0.045 = 0.5758 \left( \frac{Z_{26}}{Z_{33}} - 0.0434 \right) + 0.001174 Z_{11}^{*}$$
(4)

<sup>\*</sup>In this section variable  $Z_{11}$  is defined in such a way that it is equal; -7.5 in 1961, -6.5 in 1962, -5.5 in 1963, etc.

$$\frac{Y_{16}}{Y_{18}} - 0.236 = 0.8272 \left( \frac{Z_{34}}{Z_{33}} - .226 \right) + 0.0042 (Z_{35} - 0.4375)$$
(5)

$$\frac{Y_{17}}{Y_{18}} - 0.143 = 0.6371 \left( \frac{Z_{36}}{Z_{33}} - 0.137 \right) + 0.0338(Z_{22} - 0.3125)$$
(6)

It is worthwhile noting that the equations (3)-(6) gave quite a good fit with empirical data. The  $\varphi^2$  coefficients of fit were equal (0.02, 0.03, 0.02, 0.02, respectively) while the computed values of the coefficient of random variation were 4.2%, 4.0%, 3.5%, and 4.7%. The coefficient of random variation is defined as the percentage ratio of s (square root of residual variance) to  $\overline{y}$ , i.e., the observed mean of the dependent variable.

Using models (3)-(6) and formula (2) it was possible to disaggregate total investment outlays. An interesting experiment is provided by plotting the shares of exogenous investment on graphs, together with data referring to national income growth.

Formula (2) can be used to calculate the average length of investment cycle A, namely by putting:

$$A \approx \frac{1}{1 - a_1} \qquad (7)$$

As can be checked, this gives 2.5 years in non-agricultural productive sectors, 2.3 years in agriculture, 5.7 years in services, and 2.8 years in industry.

### VIII. ESTIMATION OF THE STRUCTURAL FORM OF THE MODEL

The present section is devoted to the presentation of estimation results of the structural form of the model. As is evident from the arguments of the previous sections, the structural stochastic equations are of three different types. There are equations pertaining to endogenous investment mechanisms explained in the previous section. There are also 16 "classical" linear stochastic equations and these, in turn, can be split into two groups, the first one including equations describing the mechanism of interdependent endogenous variables and the second group being composed of recursive or simple equations. The structural parameters of the first group were estimated by the two-stage least squares method while the estimation of the second group was performed by the classical least-squares method.

In order to get an idea of the degree of fit, two goodnessof-fit parameters were computed for each estimated equation, namely s - the standard error, i.e., the square-root of the residual variance, and c - the coefficient of random variation.\* While the first parameter measures the average level of deviations of an endogenous variable's observed values from its "theoretical" values, the second parameter expresses that level of such deviation in percentage of observed mean value of the endogenous variable explained by the estimated equation.

Standard estimation errors of structural parameters were also computed and are given in brackets under the corresponding parameter estimates. These standard errors, however, have only a limited information value since they were computed under the lack of autocorrelation hypothesis. In fact, for many of the estimated equations the value of the Durbin-Watson statistic fell below 2.0, thus suggesting the existence of positive first-order autocorrelation.

<sup>\*</sup>For definition of this coefficient see formula (1) in the previous section.

The equation-after-equation results of the estimation are given below. Goodness-of-fit parameters are presented in Table 1. Variables which do not appear in the listing are those explained by means of definition identities.

$$\begin{array}{l} \mathbf{Y}_{1} = -61.097 + 9.274 \, \mathbf{Y}_{7} + 1.147 \, \mathbf{Z}_{1} + 0.516 \, \mathbf{Z}_{2} - 1.217 \, \mathbf{Z}_{3} + 21.496 \, \mathbf{Z}_{22} \\ (102.6) & (22.2) & (0.29) & (0.74) & (0.51) & (12.4) \end{array} \\ \mathbf{Y}_{2} = 13.287 + 0.0024 \, \mathbf{Y}_{1} + 0.557 \, \mathbf{Y}_{5} + 0.326 \, \mathbf{Z}_{11} \\ (4.5) & (0.007) & (0.33) & (0.07) \end{array} \\ \mathbf{Y}_{3} = 1.024 + 0.0013 \, \mathbf{Y}_{1} + 0.0015 \, \mathbf{Y}_{14} - 0.0092 \, \mathbf{Z}_{5} + 0.0005 \, \mathbf{Z}_{18} \\ (0.62) & (0.0005) & (0.0013) & (0.007) & (0.0006) \end{array} \\ \mathbf{Y}_{4} = 27.080 - 0.974 \, \mathbf{Y}_{6} + 0.044 \, \mathbf{Y}_{15} - 0.538 \, \mathbf{Z}_{22} \\ (4.4) & (0.28) & (0.005) & (0.20) \end{array} \\ \mathbf{Y}_{8} = 140.826 + 0.076 \, \mathbf{Z}_{4} + 0.391 \, \mathbf{Z}_{9} - 21.224 \, \mathbf{Z}_{12} - 12.473 \, \mathbf{Z}_{22} \\ (18.5) & (0.0058) & (0.13) & (3.9) & (6.8) \end{array} \\ \mathbf{Y}_{9} = -1.694 + 0.00045 \, \mathbf{Y}_{1} + 0.209 \, \mathbf{Y}_{5} + 0.00051 \, \mathbf{Y}_{22} \\ (0.43) & (0.0007) & (0.03) & (0.0009) \end{array} \\ \mathbf{Y}_{11} = 0.6063 \, \mathbf{Z}_{32} \\ \mathbf{Y}_{12} = 0.5758 \, \mathbf{Z}_{26} \\ \mathbf{Y}_{13} = 0.8272 \, \mathbf{Z}_{34} \\ \mathbf{Y}_{9} = 26.492 + 0.079 \, \mathbf{Z}_{1} - 0.115 \, \mathbf{Z}_{25} \\ (8.5) & (0.02) & (0.33) \end{array} \\ \mathbf{Y}_{19} = 26.492 + 0.079 \, \mathbf{Z}_{1} - 0.115 \, \mathbf{Z}_{25} \\ (8.5) & (0.02) & (0.33) \end{array} \\ \mathbf{Y}_{20} = 10.079 - 1.914 \, \mathbf{Z}_{12} - 0.088 \, \mathbf{Z}_{26} + 0.220 \, \mathbf{Z}_{27} \\ (0.48) & (0.51) & (0.13) & (0.12) \end{array} \\ \mathbf{Y}_{22} = -1711.2 + 0.076 \, \mathbf{Y}_{8} - 13.084 \, \mathbf{Y}_{20} + 24.253 \, \mathbf{Y}_{23} + 112.314 \, \mathbf{Z}_{22} - 7.125 \, \mathbf{Z}_{23} \\ \mathbf{Y}_{12} = 0.076 \, \mathbf{Y}_{13} + 0.076 \, \mathbf{Y}_{13} - 0.0776 \,$$

$$Y_{23} = -30.493 - 0.070 Y_8 + 6.431 Y_{10} - 0.235 Z_{19}$$
(14.3) (0.04) (0.8) (0.2)  

$$Y_{24} = 16.266 + 0.00019 Y_{22} + 0.042 Z_{22} + 2.446 Z_{24}$$
(1.7) (0.0004) (0.8) (1.12)  

$$Y_{25} = 24.339 - 0.0122 Y_{22} + 5.407 Z_{22}$$
(3.3) (0.08) (3.2)  

$$Y_{26} = 8.297 - 0.0041 Y_1 - 0.0053 Z_{11} + 0.0175 Z_{30}$$
(0.7) (0.002) (0.04) (0.007)  

$$Y_{27} = 10.244 - 0.018 Y_8 - 0.0132 Z_{11} + 0.0058 Z_{30}$$
(0.9) (0.008) (0.07) (0.003)  

$$Y_{28} = 5.587 + 0.020 Y_{14} + 0.006 Z_8 - 0.139 Z_{21}$$
(0.6) (0.005) (0.03) (0.72)  

$$Y_{29} = 4.543 + 0.031 Y_{14} + 0.011 Z_8 - 0.027 Z_{21}$$
(0.7) (0.006) (0.03) (0.83)

Table	1.	Some	goodness-of-fit	parameters.
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Endogenous variable	Parameters of fit		Endogenous	Parameter	s of fit
	S	С	variable	s	e
1	10.92 a)	1.7%	22	24.27 a)	4.5%
2	0.13 b)	2.3%	23	1.67 d)	1.7%
3	0.03 b)	4.5%	24	1.56 e)	9.3%
4	0.20 b)	2.0%	25	2.08 e)	10.5%
8	5.69 a)	4.4%	26	0.18 e)	2.3%
9	0.04 b)	2.0%	27	0.28 e)	3.3%
19	0.82 c)	1.2%	28	1.11 e)	12.9%
20	0.83 c)	6.5%	29	1.13 e)	12.0%
		1			1

a) billions zlotys 1971b) millions people

- c) thousands zlotys 1971/person
- d) index points
- e) persons/1000 inhabitants

Although the structural equations themselves provide the best information about the relations which exist between various variables of the model, it seems worthwhile to comment briefly on some of them.

As exhibited by the equation explaining  $Y_1$ , national income stemming from non-agricultural productive activities depends strongly on labor inputs and on lagged investments. The negative sign of the coefficient connected with  $Z_3$ , i.e., investment lagged three years, can probably be explained by the frequent shifts in economic policy with respect to the intensity of investments. Of interest is the positive coefficient connected with the dummy variable  $Z_{22}$  which assumes the value of 1 for the years 1971-76, i.e., for the period of intensive growth due not only to economic but also to social and psychological factors.

In the equation explaining the behavior of  $Y_2$  all the coefficients have the expected signs. There is obviously a positive feed-back of production (represented here by  $Y_1$ ) on employment and in fact the coefficient associated with  $Y_1$  is positive. The positive coefficient of  $Y_5$  reflects the policy of full employment which causes about 55% of an urban population increase to be absorbed by non-agricultural productive sectors. Finally, the equation of  $Y_2$  contains also a time trend, whose introduction can be explained by the fact that during the past years the work participation coefficient of women has been steadily increasing.

The equation explaining the behavior of Y<sub>3</sub> shows that the inflow of peasant-workers depends on the level of current and investment activity taking place in non-agricultural productive sectors.\* Also, to some extent, the amount of peasantworkers depends positively on the intensity of housing construction. This can be explained by the fact that some of the

<sup>\*</sup>Peasant-workers are predominantly employed in industry, transportation or building sectors.

peasant-workers start working in non-agricultural sectors while having in mind a future possibility to leave their farm and to emigrate to urban areas. The negative sign of the coefficient connected with Z<sub>5</sub> is less evident. It may be that such a sign is due to the fact that periods of fast growth of wage-rates were also periods when incomes of private farmers increased very substantially and the general outlook for agricultural activity was bright, thus reducing the number of people who were willing to work both in agriculture and in other sectors.

The equation pertaining to  $Y_n$  shows that agricultural employment was affected by the amount of investment in that sector and by the general level of economic activity. Establishment, at the beginning of 1971 of a new policy of fast economic growth created many new jobs, particularly in industrial and building sectors. Owing to a lower birth-rate, the size of new generations in towns has always been noticeably smaller than in the countryside and since there were no reserves of manpower in urban areas, except for the natural reserves obtained by the maturity of new generations, the additional workers had to come from rural areas. The negative coefficient of the variable Y<sub>6</sub> provides an insight into the autonomous mechanism of emigration to towns--with better investments, agriculture does not need as many people to work in fields and to raise cattle.

Variable  $Y_8$  denotes national income stemming from agriculture. As could be expected, such income depends positively on fixed assets and on the amount of fertilizers used. Also, it should be noted that the coefficients associated with  $Z_{12}$ and  $Z_{22}$  are negative. The first of these variables is a dummy, taking the value of 1 (in years of unfavorable natural conditions) and therefore, its coefficient should be negative. The second variable is also a dummy and assumes a value of 1 for all years of the 1971-1976 period. Unfortunately, half of these years were years of definitely bad weather conditions and perhaps this is the reason for the negative coefficient.

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As is seen from the list of the estimated equations, the variable  $Y_9$ , i.e., employment in services, depends on  $Y_1$ ,  $Y_5$  and  $Y_{22}$ . The first of these variables is justified by the fact that services is a sector which planners consider as subordinate to industrial activity and which therefore, can be expanded only in relation to the level of non-agricultural productive activity. The coefficient of  $Y_5$  shows that about 20% of the urban population increase is used as additional labor force in the services sector. Finally, the coefficient of  $Y_{22}$  reflects the situation when an increase in the consumption fund induces an expansion of services.

The equation explaining Y<sub>19</sub> is of a simple, autoregressive character. Labor productivity depends on its previous level, but since the corresponding coefficient is negative, one infers that productivity tends to oscillate when all other factors remain constant. Also appearing in the equation is another explanatory variable: Z1, which really is a proxy for one-year-lagged investment in non-agricultural productive sectors. The positive and statistically significant coefficient of Z<sub>1</sub> shows that such an investment plays an active role and increases the labor productivity. Investment is also seen to be the principal factor determining labor productivity in agriculture. This productivity is affected, however, by abnormal climatic conditions, as shown by the high negative coefficient associated with the dummy variable Z12.

Of interest is the equation reflecting the mechanism of  $Y_{22}$ . We find here that the level of private consumption is very much dependent on the level of agricultural production, i.e., on the domestic supply of food. The large positive coefficient connected with  $Z_{22}$  reflects the fact that the new economic policy pursued from 1971 onward has resulted in a very significant increase of private consumption.

The level of consumer prices (the equation of variable  $Y_{23}$ ) depends on agricultural production, on the balance of trade, and on the variable  $Y_{10}$ : total employment in socialized

sectors. Since the total amount of money earned by the population depends on  $Y_{10}$ , this equation reflects the mechanism used for equilibrating the purchasing power of the population with the supply of consumer goods, with food always the most important private consumption item.

The demographic variables of the model do show some dependence on the economic factors, although one would have expected it to be stronger. It is interesting to note that the birth rate reacts differently to a rise in private consumption, whether in urban or in rural areas. While in the towns an increase in private consumption stimulates births, the contrary takes place in rural areas where a better standard of living means less babies. Positive coefficients connected in both cases with the variable  $Z_{22}$  can be understood in the light of the new socioeconomic policy which aims at protecting families and encourages larger numbers of children.

The equations pertaining to death rates show these rates to depend negatively on economic growth which in turn provides better living and health care conditions. However, each of these equations has a quadratic trend and the coefficient of  $Z_{30}$ , i.e., of the squared time variable, is positive. This means that one can expect the average death rates to increase in the future, mainly because of an increase in the fraction of old people in the total population of the country and also to air pollution and other industrial side-effects.

Finally, the migration equations exhibit the existence of an investment-generated propensity to migrate to urban areas. This migration is motivated to some extent by the earnings differential (variable  $Z_8$ ). The negative coefficient of  $Z_{21}$ is not surprising. In the past years of heavy investments the labor force was attracted to non-agricultural sectors but such an attraction was not necessarily coupled with housing opportunities. Therefore, people took jobs in urban areas without actually migrating but commuting. The degree-of-fit of the equations is relatively good, especially when one takes into account that the model is based on 17 observations only and that, on the average, the number of explanatory variables used in one stochastic equation is slightly less than 3, not including the constant term.

### IX. THE REDUCED FORM

Once the structural form of the model has been estimated, it then becomes possible to find the reduced form which plays, in turn, a basic role in prediction and counterfactual simulation. The matrix-P of reduced-form coefficients premultiplied by -1 is reproduced at the end of this paper as Appendix B.

It should be noted, however, that because of the two nonlinearities involving variables  $Y_5$  and  $Y_6$  (see Section V) the reduced form could not be obtained in the standard way. Rows 5 and 6 of the matrix-P are composed of zeros only. In fact, to find the reduced-form coefficients for variables  $Y_5$  and  $Y_6$ , these zeros should be substituted by the expressions

$$\left(1 + \frac{Y_{24} - Y_{26} - Y_{28}}{1000}\right) Z_{6} , \text{ in the 5-th row}$$

$$\left(1 + \frac{Y_{25} - Y_{27} - Y_{28}}{1000}\right) Z_{7} , \text{ in the 6-th row}$$

and after substituting for  $Y_{24}, Y_{25}, Y_{26}, Y_{27}, Y_{28}$  and  $Y_{29}$ , their expressions are provided by the appropriate rows of matrix-P. Because of multiplicity factors  $Z_6$  and  $Z_7$  the coefficients of the reduced-form equations for  $Y_5$  and  $Y_6$  will not be constant but will be functions of  $Z_6$  (reduced-form equation for  $Y_5$ ) or of  $Z_7$  (reduced-form equation for  $Y_6$ ).\*

<sup>\*</sup>To stabilize the reduced-form parameters for  $Y_5$  and  $Y_6$ , one could alternately set  $Z_6$  and  $Z_7$  at their observed mean values, equal to 16.0 and 15.6 respectively.

To avoid misunderstandings and mistakes the reader must note, however, that the listing of rows of matrix-P does not directly correspond to the listing of endogenous variables. This is because the variable  $Y_{21}$ , as being defined by means of an identity with parameters changing in time, does not appear in P. Therefore, for  $i \ge 21$  the i-th row of matrix-P represents variable  $Y_{i+1}$ .

To conclude our remarks about the reduced form let us note that matrix-P has dimensions  $28 \times 30$ , which is less than the total number of endogenous and predetermined variables presented in Sections II and III, respectively. In the case of endogenous variables, this is due to the fact that variable  $Y_{30}$  has been dropped from consideration as being a fraction of  $Y_{11}$ . The smaller number of predetermined variables is due to the fact that the variables  $Z_{31} - Z_{35}$  only entered equations used for splitting total investments into their endogenous and exogenous parts, but otherwise do not appear in the structural equations of the model.

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# APPENDIX A



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# APPENDIX B

MATRIX -P OF REDUCED-FORM COEFFICIENTS (MULTIPLIED BY -1)

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