Control of Diffusion Processes in Multi-Agent Networks

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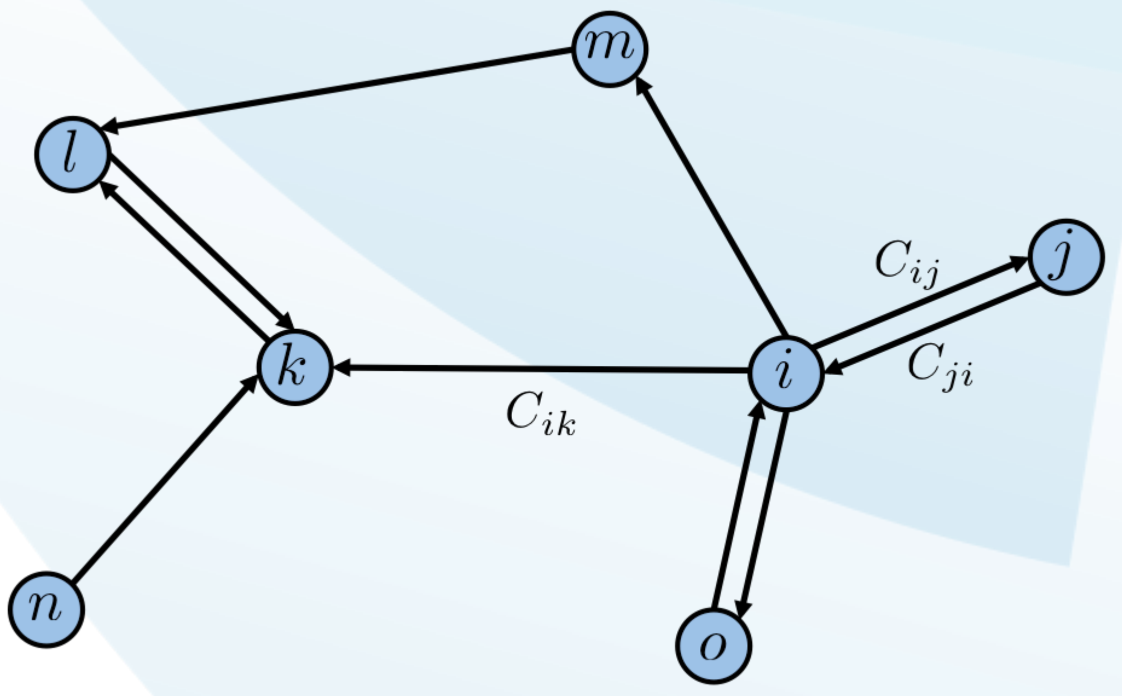
Contribution

Diffusion processes are instrumental to describe the movement of a continuous quantity in a generic network of interacting agents. Diffusion processes are relevant to describe the dynamical behavior of large-scale networks, for instance in the case of opinion dynamics and epidemic propagation, or in the context of trade and financial networks.

Our main contributions can be listed as follows:

i. We describe the network dynamics for asymmetric networks with stochastic updating.
ii. We introduce a classification of agent interactions according to two protocols where the total network quantity is conserved or variable.
iii. We capture network control by allowing external time-varying input functions or by considering network structure modifications.

The proposed framework is relevant in the context of group coordination, herding behavior, distributed algorithms, and network control.



System model and update protocols

Conservative networks

The total amount of the considered quantity is conserved over time. Conservative networks can represent stylized instances of hydraulic, financial, or trade networks.

The agents obey the following protocol in conservative networks for the considered quantity $\mathcal{S}_i(t)$ at node *i*

$$\mathcal{S}_i(t+\Delta t) = \mathcal{S}_i(t) + \mathcal{C}_{ij}\mathcal{S}_j(t)$$
$$\mathcal{S}_j(t+\Delta t) = (1-\mathcal{C}_{ij})\mathcal{S}_j(t)$$

with edge weight $C_{ij} \in (0,1]$. The dynamics of the expected value of the network properties can be written as

$$\dot{\bar{\mathcal{S}}}(t) = \mathcal{Q}\bar{\mathcal{S}}(t), \quad \mathcal{S}(t) = [\mathcal{S}_1(t)\ldots\mathcal{S}_n(t)]$$

where the effects of the network topology, the weights, update rates, and the followed protocol are captured in $\mathcal{Q}$, the negative of the weighted in-degree Laplacian.

Non-conservative networks

The total amount of the considered quantity can vary over time. Non-conservative networks are of interest to describe opinion dynamics or preference dynamics in cultural theory.

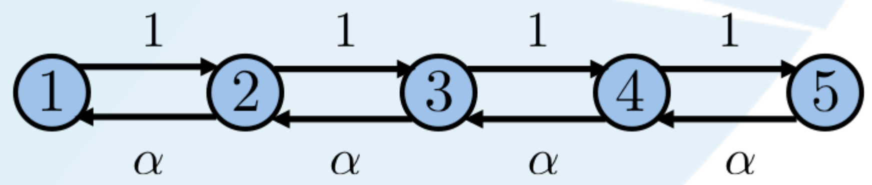
The agents obey the following convex update-rule in non-conservative networks

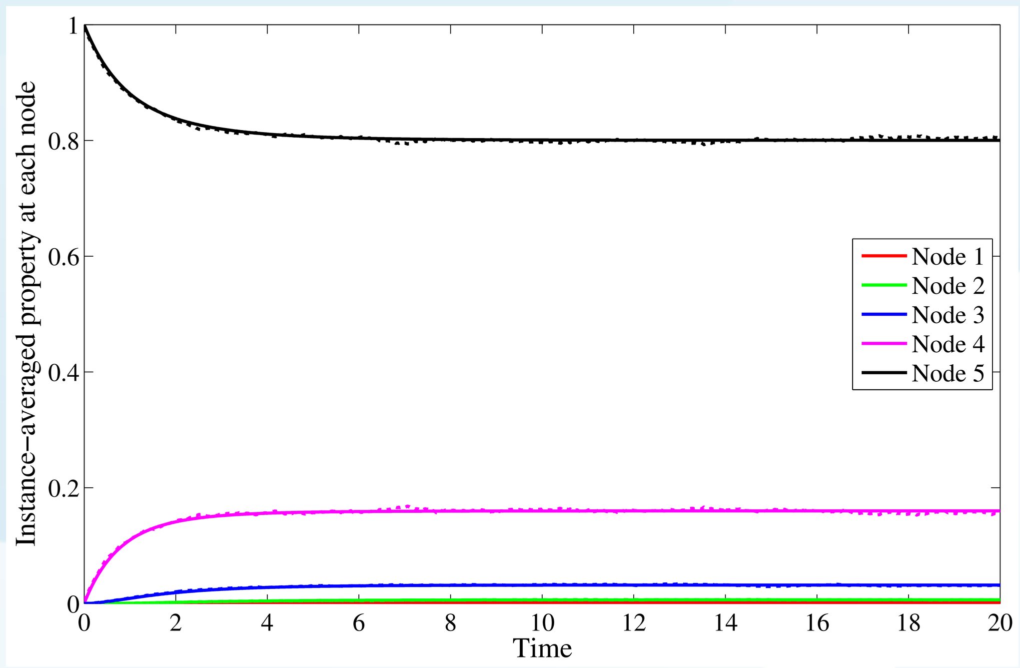
$$\mathcal{S}_i(t+\Delta t) = \mathcal{C}_{ij}\mathcal{S}_j(t) + (1-\mathcal{C}_{ij})\mathcal{S}_i(t)$$

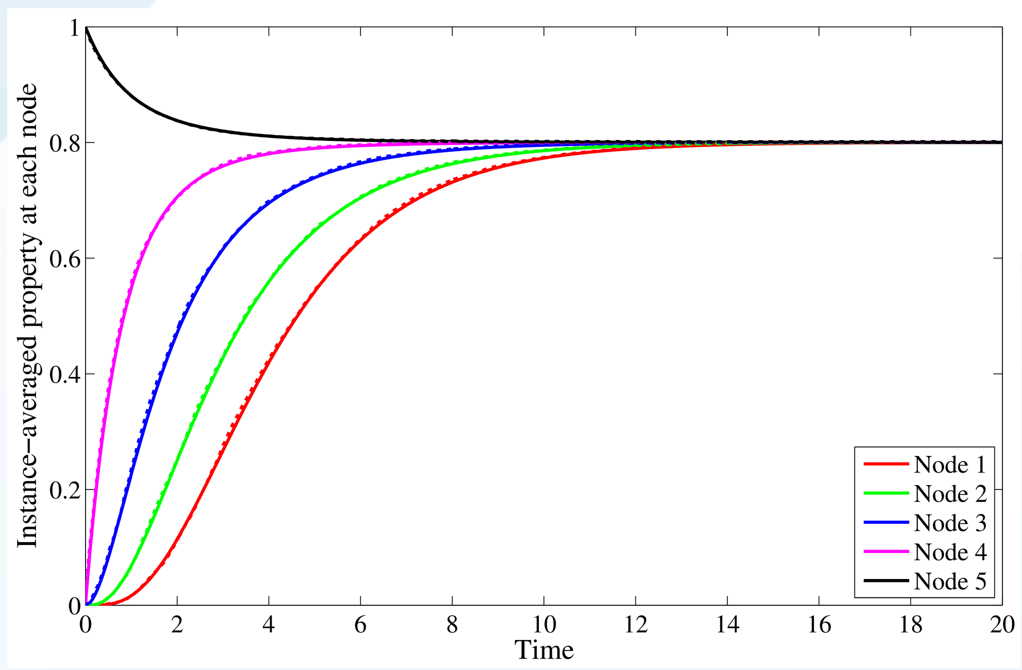
with the trust parameter $C_{ij} \in (0,1]$. The dynamics of the expected value of the network properties can be written as

$$\dot{\bar{\mathcal{S}}}(t) = \mathcal{Q}\bar{\mathcal{S}}(t)$$

where $\mathcal{Q}$ is the negative of the weighted out-degree Laplacian







Network Control

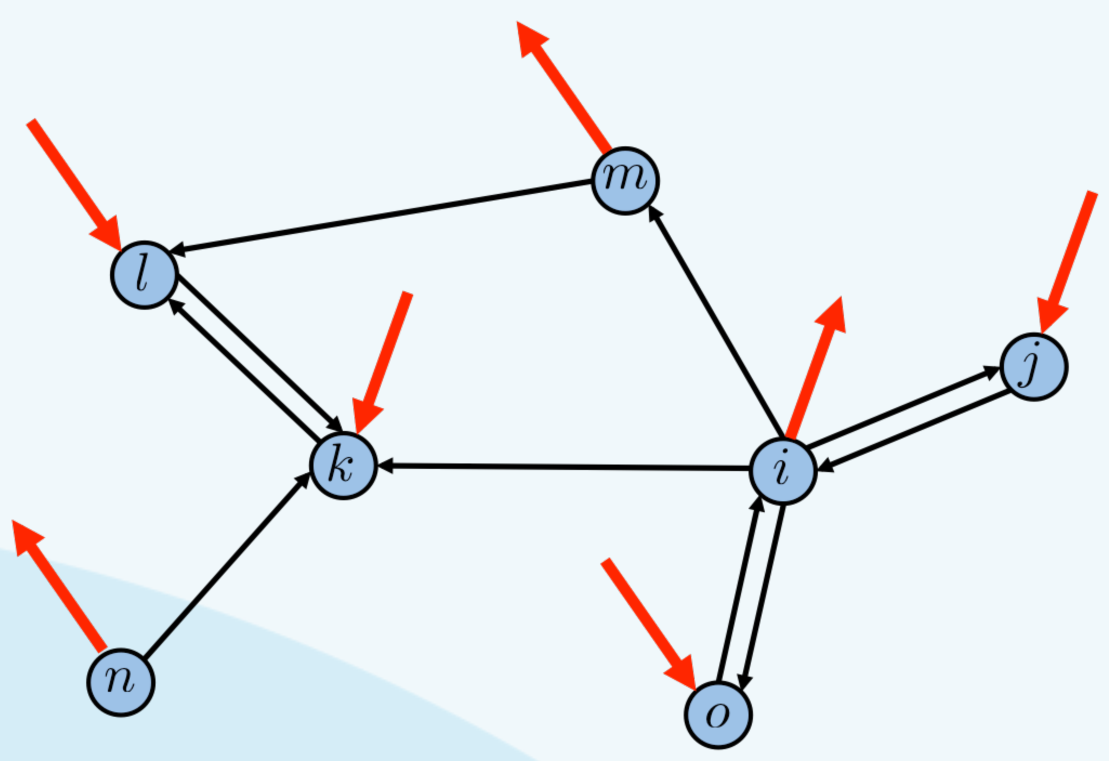
Network control by exogenous excitation

In order to model the addition and subtraction of property to and from the network at individual nodes, the inhomogeneous equation can be written as

$$\dot{\bar{\mathcal{S}}}(t) = \mathcal{Q}\bar{\mathcal{S}}(t) + \mathcal{U}(t)$$

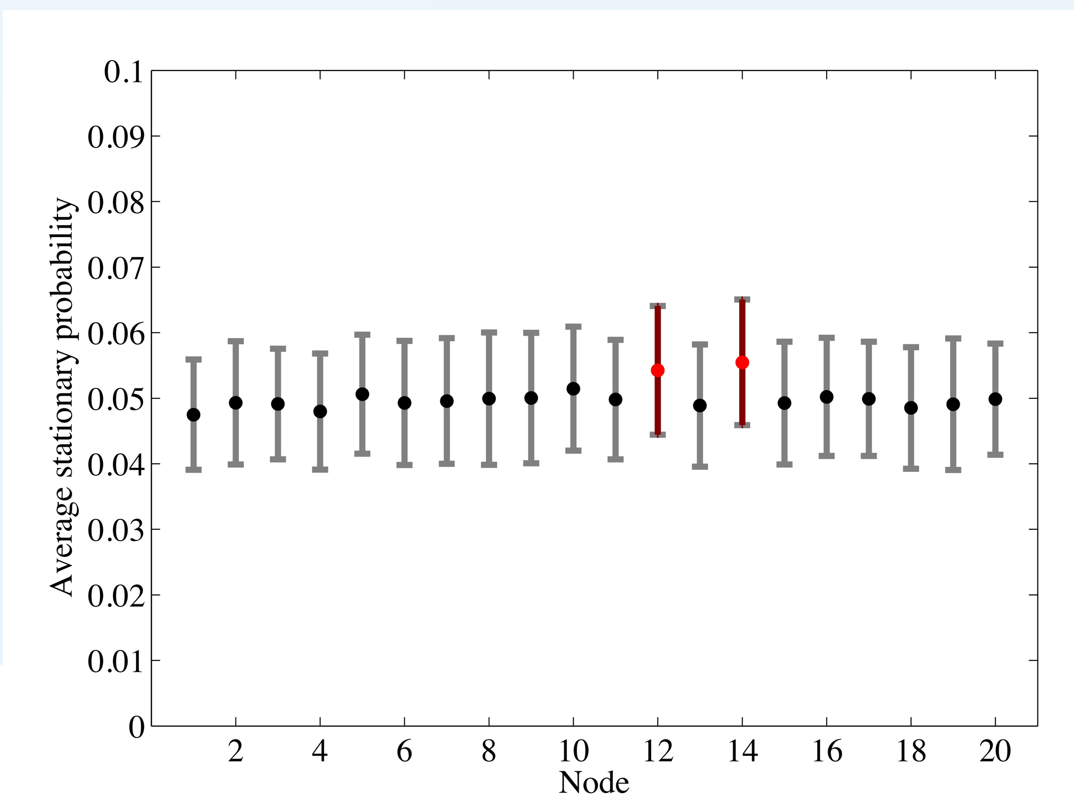
The inhomogeneous equation is of interest in the following scenarios:

- Stubborn agents: Opinion dynamics where a set of agents has constant opinion result in a static input.
- Dynamic learning: Information of the neighborhood is augmented with time-varying measurements.
- Control schemes with feedback enable us to steer the dynamical and stationary network behavior.



Network control by structure modification

- Network structure modification to change the dynamical network behavior
- Network structure modification through adaptive control
  - Learning the appropriate network structure through reinforcement learning
  - Tradeoff between exploration and exploitation
  - Store reward for different network structures to inform decision-making
- Example: selection from randomly generated networks with 20 nodes, where nodes 12 and 14 get higher reward.



Future work

The framework to model diffusion over networks can be applied in the context of distributed optimization, network control, and group coordination. Two promising examples are subject of ongoing research:

- Sequential decision making: Consider the decision making problem where several options are available with uncertain rewards. The framework can be used to extend the traditional multi-armed bandit problem to sequential decision making in networks. In this scenario, the measurements of individual agents are complemented with information from the agent's neighborhood.

- Economic migration: Consider a network of countries with different levels of labor force and capital. Due to the differences in economic and social parameters, a fraction of the active population will migrate to increase the expected future earnings. The migration of labor can be modeled based on the conservative network model.