OPERATION OF MULTIPLE RESERVOIR SYSTEMS:
A CASE STUDY OF THE UPPER VISTULA SYSTEM
(An Introduction)

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PREFACE

Water resource systems have been an important part of resources and environment related research at IIASA since its inception. As demands for water increase relative to supply, the intensity and efficiency of water resources management must be developed further. This in turn requires an increase in the degree of detail and sophistication of the analysisk including economic, social and environmental evaluation of water resources development alternatives aided by application of mathematical modeling techniques, to generate inputs for planning, design, and operational decisions.

During the year of 1978 it was decided that parallel to the continuation of demand studies, an attempt would be made to integrate the results of our studies on water demands with water supply considerations. This new task was named "Regional Water Management" (Task 1, Resources and Environment Area).

This paper is concerned with operational decision-making in the existing multiple reservoir systems. Following a short description of the case system, three different approaches to optimization of the system's operation are presented. First, the three-step stochastic implicit approach; second, the simulation approach; and third, the approach based on the concepts of hierarchical control systems. Distinction is made between the long-term reservoir operation rules and the short-term operational decisions using the real-time forecasts of reservoir inflows and water demands.

The paper is part of a collaborative study on the operation of the Upper Vistula multiple reservoir system in Poland, carried out by the Institute for Meteorology and Water Management, Warsaw, Poland and IIASA.

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1. INTRODUCTION

The purpose of this paper is to show, using as an example one of the existing multireservoir systems in Poland, how reservoir operation rules can be developed with the application of some theoretical tools. This will take into account the decisionmaking and information structures and their pattern. Therefore, attention is focused not only on methodological developments, but also on the kind of information necessary for the determination of operational rules, and on the informational structure of the preassumed form of the decision rules.

First, the system considered is described and the objectives of water management are specified. Next, two methods of determining the parameters of preassumed operation rules are presented, and finally the hierarchical approach to reservoir control in the system considered is described.

2. A SHORT DESCRIPTION OF THE UPPER VISTULA SYSTEM

A general layout of the Upper Vistula system is shown in Figure 1. The system includes five storage reservoirs. The Goczalkowice reservoir (1) is located on the Small Vistula River, while the Tresna (2) and Porabka (3) reservoirs are located on the Sola River. Immediately below Porabka is the small Czaniec reservoir which is not shown in Figure 1. The system can be easily expanded by the Swinna Poreba reservoir (4) to be located on the Skawa River. A decision concerning construction of this reservoir has already been made by the authorities concerned. Finally, there is the off-the-river Dzieckowice reservoir (5), which was built as a buffer (compensation) reservoir for one of the major water users in the area.

The major objectives of the Upper Vistula system are to secure water supply for the industrial and municipal water users referred to in Figure 1 as A, B, E; to supply the steel works D with water from the Sola reservoir via the Dzieckowice reservoir; and to supply water to chemical plant C, and fish farms R. At the same time, concentration of several pollutants, which are discharged mainly to the Vistula River downstream of the outlet of the Przemsza River, should be maintained at the levels compatible with water quality requirements. Reservoirs (2) and (3) are provided with hydroelectric power stations; however, this study focuses on water supply and water quality considerations. The flood control portions of storage capacities are not taken into account.

The principles of water resources management in the Small Vistula and Sola river basins have not changed for a long time, with the Goczalkowice reservoir being operated independently from the Sola reservoirs and vice versa. The Goczalkowice reservoir is operated for the constant release rate, and it supplies water mostly to user B. The Sola reservoirs equalize streamflows in that river, mostly for the purpose of supplying water to users A, B, and C. Water requirements of fish farms denoted in Figure 1 as "R", are not taken into account by the present operation rules of the Sola reservoirs. Decisions concerning the fulfillment of these requirements are made when the need arises; they are mostly based on the actual streamflow rate at the outlet of the Sola River. The operation of water transfer facilities from the Sola River to the Dzieckowice reservoir is under the control of user D. The transfer rates are decided upon by user C, depending mostly on the actual storage level of the Dzieckowice reservoir. Quite often the transfer rates are decided upon by the "intervention" way, depending on user C's requirements. Although there are a few general rules to follow, in principle operational decisions concerning Sola reservoirs are based on ad hoc agreements among all parties concerned. What is important, moreover, is that water quality requirements of the Vistula River, downstream of the Sola outlet, are not accounted for in the operational decisions concerning releases of water from the Sola reservoirs.

To summarize, storage reservoirs in the Upper Vistula system are not operated at present as a system of interrelated flow control facilities. It is felt that efficiency of the system performance could be considerably enhanced if the interdependencies among the reservoirs (and among some of the users) are more explicitly taken into account in the operation rules developed for the system as a whole.

The general layout of the complete system is shown in Figure 1, but for the modelling purposes only some selected elements of the system and interactions among these elements are considered.

3. THE MONTE CARLO APPROACH TO OPTIMIZATION OF THE OPERATION RULES

During the past few years, the first attempt to develop operation rules for the Upper Vistula multireservoir system was carried out by J. Kindler [1977], who has taken into account the system shown in Figure 2.

Water control objectives, in the (1,2-3,4) system were limited for the purpose of the study to:

a. water supply for five municipal and industrial centers of the region, and
b. maintenance of minimum acceptable flows (MAF) in some of the river reaches.

The mean monthly target water demands and MAF rates and the penalty functions describing economic losses due to not meeting the target demands and MAF rates have been determined by a separate study.

It was assumed that the system derives its supply from the randomly distributed natural inflows. The inflow into any branch of the system in any time period cannot be analyzed in isolation from the other branches or from inflows in other time periods. Since the explicit consideration of the multivariate inflow process poses a number of well-known difficulties in the case of a multireservoir situation, the method approaches the problem in a way consisting of the three following steps:

1. Development of a mathematical model of the multivariate (time and space) river flow process and generation of a synthetic trace of inflows to the system;
2. Development of a mathematical model of a water resources system and simulation of its operation over the long trace of synthetic inflows (simulation coupled with one of the mathematical programming techniques);
3. Statistical analysis of the results of the simulation-optimization computations and identification of the optimal operation rules for the system of storage reservoirs.

Such a procedure, generally known as the Monte Carlo or stochastic implicit approach, was first proposed by Young [1967], for the solution of a single reservoir problem.

3.1 MULTI-SITE FLOW GENERATION

For implementation of the first step of the proposed method, a multi-site flow generation model was developed, based in principle on the paper by Matalas [1967]. The model was based on the following assumptions:

1. The process is a cyclic one and the number of cycle elements correspond to the number of time intervals into which the year is divided.

2. The internal structure of the streamflow time series is described by a lag-one Markov process with a discrete time parameter.
3. The mean flows in each time interval of a year are log-normally distributed.
4. The normalization of marginal distributions, in this case by a logarithmic transformation, leads to normalization of the multivariate distribution [Kaczmarek 1963].

For the case system consisting of three reservoirs, for which mean seasonal (monthly) synthetic streamflows were desired, the subject of modelling was the sequence of three multi-dimensional random variables

$$\{\bar{Q}_2(i),\ \bar{Q}_{2-3}(i),\ \bar{Q}_4(i)\} \tag{1}$$

where the lower index denotes the site number (Figure 1 or 2) and i (i = 1,2,...,n) denotes the season number.

Following normalization of sequence (1), several statistics associated with these sequences were estimated. Using these statistics, the model for generating synthetic streamflow sequences was developed for each month of a year.

The statistical resemblance between the historic and synthetic streamflow sequences was analyzed by the t-test (mean values), z-Fisher test (variances) and log-transform test (correlation coefficients). At the significance level of $\alpha = 0.05$, the differences between the corresponding statistics proved to be insignificant for about 90 per cent of the analyzed parameters.

As a result of the streamflow generation, 100-year-long synthetic sequences of mean monthly inflows to the system were obtained.

3.2 SIMULATION-OPTIMIZATION MODEL OF THE SYSTEM

The next step of the approach presented here was to develop a simulation model of the system, where at each time step the vector of optimal controls is defined by application of one of the mathematical programming techniques. In the case of the multireservoir water resources system, this vector consists of optimal releases from individual reservoirs. The "optimality" of decisions on the reservoir releases depends to a large extent upon the forecast of future inflows to the system. The problem arises of how many future inflows influence the decisions concerning the reservoir releases at a particular moment. In other words, the question is what is the time horizon of significant future inflows when the reservoir operator must make his decision concerning the release.

Referring to the Upper Vistula situation where reservoir capacities are relatively small, it has been assessed that, for the monthly seasons, the "significant future" is equal to approximately six months [Hydroprojekt 1972]. Therefore the basic assumption which underlies the simulation-optimization analysis is that future inflows to the system--those which influence the decision on reservoir releases--are known. All inflow values are elements of the previously generated synthetic traces and the simulation-optimization process is carried out in the deterministic environment.

At the first step of the simulation procedure the following optimization problem is solved: for the given vector V(1) of initial storage volumes in all reservoirs, for the given sequence of vectors $\overline{Q}(i)$ of inflows to the reservoirs in the $i = 1,2,\ldots,$ 6 months, and for the given pattern of water demands in the water resources system served by these reservoirs (for $i = 1,2,\ldots,6$), define a sequence of vectors $\overline{W}(i)$ of final storage volumes in all reservoirs that minimizes the total cost of operating the system. The vector W(1) was next used as the initial storage volume for the solution of similar optimization problems (with appropriately changed reservoir inflow and demand data) for the next 6 months. Such simulation-optimization process was carried throughout the whole 100-year-long period for which synthetic sequences of mean monthly inflows to the system were generated earlier. The cost of operating the system was the function of water transfer pumping costs and penalties associated with not meeting the predetermined water demands as well as the minimum flow requirements in the system.

It should be noted here that the sequence of vectors W(i) describing optimal reservoir volumes at all time intervals of the simulated period correspond to the D(i) sequence of vectors describing optimal releases from the reservoirs. These releases are optimal from the point of view of the minimized objective function subject to a set of three reservoir balance equations and other constraints.

The optimization problem can be rewritten in a short, mathematical form as given below:

$$\min_{W} K(W) \tag{2}$$

subject to:

$$W(i) = V(i) + \bar{Q}(i) - D(i); \quad i = 1,2,\ldots,k \tag{3}$$

where:

V(i) - given initial vector of states of the reservoirs,

$$W(i) = V(i + 1) \tag{4}$$

and to a set of respective inquality-type constraints on decision variables. Function K(W) is the objective function of the model.

The simulation-optimization model employed the out-of-kilter algorithm (e.g., see Fulkerson [1961] and Barr et al. [1974]) which is a special purpose linear programming method designed for the solution of network allocation problems. The suitable implementation of the algorithm allows for dynamical generation of reservoir releases, in accordance with the current and forecasted water demand and inflow situation (the operation rules do not have to be specified a priori).

As a result of the second step of the computations, a set of optimal control vectors D(i) and W(i) was obtained for each month.

3.3 IDENTIFICATION OF THE OPERATION RULES

The sequences of optimal releases from the reservoirs, determined for each of the months and resulting from implementation of the first two steps of the procedure, were used next for estimation of the parameters of the operational rules. It was decided to describe the relationship between the state vector V(i) of the system at the beginning of the i-th time period, the vector Q(i) of forecasted inflows in the given season and the vector W(i) of final storage volumes at the end of the season by a set of linear equations:

$$W_j(i) = b_{o,j}(i) + b_{1,j}(i) \cdot Q_1(i) + b_{2,j}(i) \cdot Q_{2-3}(i) + b_{3,j}(i) \cdot Q_4(i) + b_{4,j}(i) \cdot V_1(i) + b_{5,j}(i) \cdot V_{2-3}(i) + b_{6,j}(i) \cdot V_4(i) \quad , \tag{5}$$

where j is the reservoir index.

The set of operation rules for the system was defined by estimating the sequence of parameters $\{b_{o,j}(i), b_{1,j}(i), \ldots, b_{6,j}(i)\}$ for each of the reservoirs. The parameters b have been estimated by the linear step-wise multiple regression using the observations on Q, W, and V obtained as a result of the simulation-optimization procedure.

The set of (12 x 3) regression equations constitutes the operation rules, the application of which can secure the long-term optimality of the reservoirs operation.

3.4 REMARKS TO SECTION 3

The fundamental assumption for this kind of operation (or decision) rule is the feedback between the state of the system expressed in terms of actual volume of water stored, forecasted inflows and resulting values of releases from the reservoirs. The relationship between information available, which is necessary for decision making, and the final decision on the desired releases from the reservoirs of the considered time interval (season), can be visualized as the implication formula:

$$\{i\} \cdot \{V(i), \overline{Q}(i)\} \rightarrow \{W(i) \equiv V(i+1)\} \rightarrow \{D(i)\} \quad . \qquad (6)$$

The decisions are taken in a centralized manner (even if some parameters of the operating rule are equal to zero) on the basis of the current state V(i) of reservoirs and forecasted inflows $\overline{Q}(i)$ to the system. It is worthwhile to stress the fact that the parameters b of the operating rule do not depend directly on the current state of or inflow to the system. They depend first of all on time but also hidden in this relationship is the dependence on statistical properties of the inflow process and the state of the system's reservoirs.

The centralization of the decision rule and high degree of its aggregation makes the decisions quite general and aggregated. Listed above are features of the method presented as well as the length of the time discretization interval, which is equal to one month. These make this approach more applicable for long-term operation planning or target storage volume determination than for real-time control or on-line control purposes. Therefore, the approach presented provides a rule for storage policy planning.

4. OPTIMIZATION OF MULTIRESERVOIR SYSTEM OPERATION RULES VIA THE SIMULATION METHOD

Another approach to development of the operation rule for the Upper Vistula system was presented recently by Slota et al. [1978].

For modelling and simulation purposes the system shown in Figure 1 was divided into three subsystems [Słota 1978]:

1. A subsystem of the distribution of water resources. The subsystem consists of storage reservoirs (1,2,3,4,5), man-made conduits delivering water to the users and the Sola River channel downstream from reservoir (3), cross-section H, and Skawa River (below reservoir (4) to cross-section N);
2. A subsystem of water use, including the most important water users in the system specified as A,B,C,D,E;
3. A subsystem of water quality which is composed of the Vistula River (the river reach between reservoir (1) and cross-section G), the Przemsza River along its main course, and their tributaries.

The relationships among all subsystems and their respective inputs and outputs are shown in Figure 3.

4.1 SUBSYSTEM OF WATER RESOURCES

Problems of water quantity and distribution predominate over problems of water quality which can be neglected when the model of the first subsystem is derived.

There are two vector inputs to the first subsystem:

- vector $\bar{Q}^{(1)}$ describing natural inflows, and
- vector $\bar{X}$ of control variables such as releases from the reservoir and flows in the conduits.

There are two vector outputs from the subsystem:

- vector $\bar{M}$ of water supply to the subsystem of water use and
- vector $\bar{U}^{(1)}$ of releases from the reservoirs or flows in river channels.

These latter streams are inputs to the water quality subsystem.

Elements of another vector $\bar{V}$, express volumes of water stored in a system's reservoirs.

All vectors mentioned above are functions of time but for brevity's sake this dependence is not indicated in the equations which follow.

The flow balance equations which have been formulated for the specified hydrologic cross-sections or for the specified nodes of the system, as well as the reservoir balance equations, are used to describe the processes taking place in the subsystem of water resources.

The relationships among vector variables describing a subsystem's water balance and outputs from the subsystem are expressed by means of the operator $\overline{F}^{(1)}$ given formally as:

$$\{\overline{M},\overline{U}\} = \overline{F}^{(1)}(\overline{Q}^{(1)},\overline{X},\overline{V}) \tag{7}$$

4.2 SUBSYSTEM OF WATER USE

Major difficulties arose when the model of water use subsystem was derived.

A model of this subsystem was used to describe the transformation of its inputs:

- $\overline{M}_o$ = water supply to the users from their own sources or from the system's environment;
- $\overline{M}$ = water supply from the resources subsystem,

into outputs such as:

- $\overline{Z}_o$ = wastewater discharge outside the system;
- $\overline{Z}$ = amount of water discharged to the subsystem of water quality, and
- $\overline{C}_z$ = concentration of selected water quality indices in wastewater discharged to the subsystem of water quality.

Operator $\overline{F}^{(2)}$ of the subsystem describes very complicated processes associated with water treatment, flows in the pipeline network, municipal and industrial water use, wastewater and sludge treatment, precipitation on--and outflow from urbanized areas, etc.

Due to the lack of sufficient information, the model of a water use subsystem is rather general; the amount of sewage and wastewater discharged by water users was evaluated as a function of time-dependent water supply.

The relationship between water supply and wastewater discharge is modelled from the quantitative point of view only. It was assumed that the water quality indices are constant and equal to the mean concentrations which have been evaluated on the basis of measurements performed in 1973 and 1974.

The total amount of wastewater discharged from each particular water user to a particular river basin was derived by the formula:

$$Z_{l,k} = \beta_{l,k} \cdot \alpha_l \cdot (M_{o,l} + M_l) \tag{8}$$

where:

l = index of water user (l = A,B,C,D,E);
k = index of wastewater discharge point;
$\beta_{l,k}$ = coefficient describing partitioning of wastewater amount among separate points of discharge ($\sum_k \beta_{l,k} = 1$);
α_l = reduction coefficient evaluating water losses (including water discharge outside the system);
$M_{o,l}$ = water supply from out-of-the-system sources;
M_l = water supply from the water resources subsystem.

Coefficients α and β were evaluated on the basis of data collected in 1973 and 1974, when all points of water intake and wastewater discharge had been identified.

The identification of the coefficients in the model describing subsystem of water use, stationarity of these coefficients and their dependence on the amount of water delivered to the user--these are the crucial problems encountered in the model development. But such problems are caused mostly by the lack of or inaccessibility to suitable and sufficient data.

4.3 SUBSYSTEM OF WATER QUALITY

The model of the third subsystem, where water quality phenomena dominate, attempts to describe qualitative as well as quantitative processes taking place in these river reaches which have been incorporated into the subsystem.

Inputs to this subsystem are given as outputs from the water resources and water use subsystems; they are also given as vectors $\overline{Q}^{(3)}$ and $\overline{C}_{Q^{(3)}}$ characterizing the quality and quantity of uncontrolled inflows to the subsystem, water intakes and wastewater discharges of water users belonging to the system environment (system-environment interactions are not shown in Figure 1 because of their considerable number).

Vectors $\overline{U}^{(3)}$ and $\overline{C}_{U^{(3)}}$, describing streams at the chosen cross-sections from the quantitative and qualitative points of view, are the subsystem outputs. In the face of the lack of possibilities to determine directly vectors $\overline{C}_{U^{(1)}}$ and $\overline{C}_{Q^{(3)}}$, the dependence between flows and concentration of selected water quality indices was expressed in the model by means of operators Φ_1 and Φ_2 (see Figure 3). These operators have been derived on the basis of historical data. Relationships among processes taking place in the subsystem of water quality can be briefly described by the following equations:

$$\overline{U}^{(3)} = \overline{F}_u^{(3)} (\overline{U}^{(1)}, \overline{Z}, \overline{Q}^{(3)}) \tag{9}$$

$$\overline{C}_{U^{(3)}} = \overline{F}_C^{(3)} (\overline{U}^{(1)}, \overline{Z}, \overline{Q}^{(3)}, \overline{C}_{U^{(1)}}, \overline{C}_Z, \overline{C}_{Q^{(3)}}) \quad .$$

The vector function $\overline{F}_U^{(3)}$ describes by means of balance equations the quantitative inter-relationships among inflows, flows, amounts of water withdrawn or wastes discharged. Successively, function $\overline{F}_{C^{(3)}}$ is concerned with two fundamental processes taking place in the river flow--dilution and selfpurification--which are described by the classical Streeter-Phelps equation [Adamczyk et al. 1978]. It was assumed that the qualitative processes in the Vistula and Przemsza Rivers can be described with sufficient accuracy by a model consisting of a series of several nodes and elementary intervals representing separate reaches of the river.

Water quality was described using seven following indices such as BOD_5, dissolved oxygen, oxygen consumption, phenols, chlorides, sulphates and suspended solids.

4.4 OBJECTIVE OF THE SYSTEM OPERATION

The objective of the optimal control in the system is equivalent to determination of such a sequence of control variables:

$$\{\overline{X}\} = \{\overline{X}(1), \overline{X}(2), \ldots, \overline{X}(i), \ldots, \overline{X}(N)\} \quad , \tag{10}$$

where N is a total number of discrete time invervals during the control period, which satisfy all constraints and secure minimization (or maximization) of the system's performance index (objective function). It was assumed that for each discrete time interval the elements of control variables vector $\overline{X}$ are determined on the basis of fixed and a priori rules of water distribution, with parameters of these decision rules evaluated by means of simulation-optimization techniques.

The operation rule can be represented as an operator $\overline{H}$ defined on the vectors $\overline{V}, \overline{Q}^{(1)}$, $\overline{P}$ and $\overline{V}$:

$$\overline{X}(i) = \overline{H}(\overline{V}(i), \overline{Q}^{(1)}(i), \overline{P}(i), \overline{V}) \tag{11}$$

where:

$\overline{V}(i)$ = state variables vector equivalent to volumes of water stored in each of the reservoirs at the beginning of the i-th time interval;

$\bar{Q}^{(1)}(i)$ = forecasts of natural inflow during the i-th time interval;

$\bar{P}(i)$ = vector of water demands in the system in the same time interval;

$\bar{\nu}$ = vector of unknown parameters of operation rules.

Simulation of the control process in the system, when the decision variables are determined using formula (11), allows selection of parameters $\bar{\nu}$ which secure optimal operation of the system.

Therefore, the results of the simulation of the system operation provide a basis for rational selection of parameters of the decision rules.

4.5 SYSTEM SIMULATION AND OPTIMIZATION OF THE OPERATION RULES

Regulation capacity of the water resources subsystem does not allow 100% certainty in meeting all of the total water demands in the system [Slota and Wawro 1979]. It is obvious that periodic water deficits in the system can occur; therefore the purpose of optimization and control is to minimize and properly distribute in time these deficits. Towards this aim, three groups of users were distinguished, according to their relative importance. This classification has been established arbitrarily but several variants were considered, for example, such as follows:

- group I - of the highest priority; this group includes minimum acceptable flows and 75% of the total municipal and industrial water demands;
- group II - the remaining 25% of municipal and industrial water demands;
- group III - all other water users.

Classification of water users is equivalent to the succession of water supply reduction when the amount of water stored in the system's reservoirs is decreasing and insufficient to satisfy the total demands.

The first two coordinates of vector $\bar{\nu}$ set up the limitations on the summarized volumes of water stored and predicted inflows to the reservoirs during the nearest month. According to the limitations water supply to the users is restricted. Therefore parameters ν_1 and ν_2 define three states of the system. For state No.1 water demands of all users in the system are satisfied; at the second state (No.2) only users of the I and II groups are taken into account, and at the third (No.3) state of the system only users of the highest priority, i.e., those which belong to group I can be supplied.

The other coordinates of vector $\bar{\nu}$ are equivalent to the parameters of functions defining releases from the reservoirs, flows in the conduits and transfers of water among river basins. The following general assumptions have been introduced:

-- the proportions among outflows from the reservoirs supplying the common balance node in the system is determined by the ratio between volumes of water stored in those reservoirs;
-- the amount of water transferred among river basins is the linear function of the flow at the outlet cross-section of the river and the volume of water stored in the reservoir supplying this cross-section (in case of reservoir (5) the amount of water transferred is also a function of the volume of water stored in this reservoir);
-- for all states of the system, the form of operational rules is the same, the only differences are in the values of parameters;
-- water demands are to be successively satisfied according to the predetermined hierarchy of the users and according to the number of existing sources of water (in the first order minimal flows in the rivers are maintained, then water users supplied from one source of water, afterwards users supplied from two sources, etc.).

The number of parameters $\bar{\nu}$ results from the degree of complexity of the assumed form of the operation rules and functions describing the resource allocation process. For the system presented in Figure 1 there were are 22 elements of vector $\bar{\nu}$.

The previously formulated objective of the optimal water resources distribution was treated as the polioptimization problem.

Vector $\overline{M}$ characterizing water users supply and vector $\overline{C}_{u(3)}$ describing the quality of water in the system have been used to evaluate the results of the system operation. Values of the elements of these vectors have been settled by the choice of operation rule parameters which belong to vector $\overline{\nu}$.

The consequences and effects of the operation have been estimated based upon some statistical characteristics of $\overline{M}$ and $\overline{C}_{u(3)}$ vectors obtained from a computer simulation of the system's operation over a 45-year-long sequence of historical data. Results of the operation are expressed in terms of the following performance indices:

(1) performance index evaluating control in the system from the point of view of meeting water demands:

$$\max_{\overline{\nu}} \left[K_1(\overline{\nu}) = \sum_{j=1}^{3} \alpha_j \cdot G_j(\overline{\nu}) \right] , \qquad (12)$$

where:

j = index of the group (category) of water users;

$G_j(\overline{\nu})$ = warranted frequency of meeting water demands of the user belonging to the j-th group;

α_j = the weighting coefficient. Values of those coefficients have been assumed fairly arbitrarily: $\alpha_1 = 99999$, $\alpha_2 = 680$, $\alpha_3 = 2$. These values give preference to solutions which secure a 100% guarantee of meeting water demands of the first group of users and simultaneously maximizes the guarantee of satisfying demands of the II group of water users.

(2) performance index evaluating system operation from the water quality point of view:

$$\min_{\bar{\nu}} \left\{ K_2(\bar{\nu}) = \left[\sum_{p=1}^{4} \beta_p \left(\frac{1}{w} \sum_{q=1}^{w} J_q(\bar{\nu}) \right) \right] \frac{1}{\sum_{p=1}^{4} \beta_p} \right\} \tag{13}$$

where:

p = index of the control cross-section ($p = 1$ for cross-section L, $p = 2$ for cross-section M, $p = 3$ for cross-section F and $p = 4$ for cross-section G);

q = water quality index;

w = number of water quality indices considered;

β_p = weighting coefficient for the p-th cross-section (it was assumed $\beta_1 = \beta_2 = 1$; $\beta_3 = \beta_4 = 3$);

$J_q(\bar{\nu})$ = mean value of the q-th water quality index in the considered period of time.

Values of water quality indices have been determined in accordance with the proposal of Prati et al. [1973] who developed functions for converting incompatible (between each other) concentrations of pollutants to the comparable values of water quality indices.

The number of control variables and the relatively long time of computer simulations have caused several simplifications to be introduced to the optimization procedure. It was decided to perform the so called stage-optimization. At first, with respect to the performance index (12), simulation was focused only on the problems of resources distribution in subsystem I. The relaxation method was used to search for the maximum value of performance index and during the optimization procedure the range of parameters' variability and the lengths of the searching step have been limited based on observation of results obtained.

As a result, the optimal solution, as well as the set of feasible solutions which assure that water demands are satisfied with the same tolerance, were determined.

At the next stage of optimization, the simulation of the system operation over the 15-year sequence of daily mean flows has been performed with respect to the qualitative and quantitative processes. This way, the value of performance index (13) has been evaluated.

The optimal values of parameters of the decision rules have been chosen according to the compromise approach where the sum of proportional deviations of the performance indices (12) and (13) from their optimal values (obtained from the first two stages of the optimization-simulation procedure) was minimized.

4.6 REMARKS TO SECTION 4.

The method described in this section differs considerably from the Monte-Carlo approach presented in the previous section despite the similar form of the operation rule which is defined on the basis of a state vector V and forecasts of natural inflow $\overline{Q}$. First of all, the parameters $\overline{\nu}$ of the operation rule (11) are defined on the basis of the current state of the reservoirs, while in the Monte-Carlo approach the vector of parameters b is determined depending on the season number and does not depend on the current state of the system. The approaches differ also because of the less general, and therefore more detailed character of decisions which directly result from the operation rule (11). Another difference is caused by the fact that the planning rule (5) was derived without taking into account the problems related to water quality. One of the objectives of the method presented in this section was explicitly expressed in terms of water quality control. The lengths of the discretization time interval (one month) and the preassumed form of the planning rule (5) and the operation rule (11) mean that there is no possibility of using short-term forecasts and it is impossible to introduce relatively frequent modifications of the storage policy. It makes the planning or operation rules, described previously, relatively inflexible.

5. HIERARCHICAL-STRUCTURE FOR THE MULTIRESERVOIR SYSTEM OPERATION

The approach which is presented in this section is based on the concepts of hierarchical control systems (see Findeisen, Malinowski [1978]). The general idea of the approach is to introduce a two-layer structure for the control of systems operation. The upper layer of the control structure is responsible for the determination of the storage policy of the reservoirs over the long time horizon, while the lower layer accomplishes operating rules (to be applied for on-line control) using short-term forecasts.

In the following sub-sections the model of the case system is briefly described, then in a more systematic manner the particular elements of the control problem are discussed.

5.1 MODEL OF THE UPPER VISTULA WATER RESOURCES SYSTEM

The simplified version of the case system, which was presented in Figure 1, is shown in Figure 4 (Salewicz [1978]. Three storage reservoirs are distinguished in the model:

(1) Goczalkowice, (2)Tresna, and (3) Czaniec which in this model comprises two reservoirs: Porabka and Czaniec.

The streamflow rates, their quality, and their mutual relationships are described by means of the reservoir balance equations, the flow balance equations formulated for the selected cross-sections, the pollutants balance equations and the model of the selfpurification process in the Vistula River reach between the control cross-sections F and G_1.

Vector V is used to describe the state of reservoirs. Natural inflows to the system are represented as Q_1, Q_2, Q_p, Q_L and Q_H. The decision (control) variables are water supply rates for the specified users: $M_A, M_{B1}, M_{B2}, M_D, M_C, M_R$ and also releases from three reservoirs: U_1, U_2, U_3. The wastewater discharges, denoted by Z with the respective subscript, are handled as the external variables (forecasts). The simplifications of the model do not reduce the generality of the control scheme which is described below, and the extended, more detailed model of the investigated water system can be also adopted to this scheme.

5.2 THE BASIC CONCEPTS OF PROPOSED CONTROL SCHEME

The control system has, as was stated previously, a two-layer structure (see Kaczmarek et al. [1978], Malinowski, Terlikowski [1978], Terlikowski [1978]). The upper layer determines the storage policy $\hat{V}(t)$, $t \varepsilon [t_o, t_f]$ ($[t_o, t_f]$ is the optimization horizon), of the system on the basis of the following information:

- measured, current state $V(t_o) = [V_1(t_o), V_2(t_o), V_3(t_o)]$ of the reservoirs;
- long-term forecasts of natural inflows to the system (vector Q (t)) and water demands described by means of the time dependent vector function P(t) characterizing water demands of specified users. These vector functions are defined over the planning horizons $[t_o, t_f]$, e.g., 3 or 6 months) and when the discretized version of the model is considered, vectors Q(i) and P(i) denote the mean weekly value of the flows' intensity.

The planned trajectory $\hat{V}$ (t) results from the solution of the following, general form of the dynamic optimization problem:

$$\min_{M,U} \int_{t_o}^{t_f} K(M(t), U(t), P(t), Q(t), Z(t))dt + K_V(V) \qquad V\ (14)$$

subject to:

- so called "local constraints" given as

$$(M(t), U(t)) \ \varepsilon \ MU, \qquad (15)$$

- "global constraints":

$$v(t) \ \varepsilon \ V(t),\ t \ \varepsilon \ [t_o, t_f] \quad , \qquad (16)$$

where MU and V(t) are specified sets of constraints (given for example as the balance equations, inequality constraints, etc.).

The objective function K is expressed in terms of the penalties associated with unsatisfied water demands, minimal acceptable flows and desired water quality standards. It can be decomposed with respect to individual water demands and objectives of the system operation:

$$K(M,U,P,Q,Z) = K_A(M_A,P_A) + K_B(M_{B1},M_{B2},P_B) + K_C(M_C,P_C) + K_D(M_D,P_D) + K_R(M_R,P_R) + K_{U_1}(U_1) + K_{U_2}(U_2) + K_{U_3}(U_3) + K_q(U_1,U_2,U_3,M_C,M_D,M_R,Z_C,Z_P,Q_H,Q_L,Q_P) \quad . \tag{17}$$

The latter component, K_q, expresses the "losses" associated with exceeding the desirable concentration of the pollution indices of the control cross-sections E and G_1.

The simplified static formulas, describing water flow balance and selfpurification process are included in the expression defining function K_q. The local constraints (15) can be decomposed analogously to the objective function decomposition.

One of the most relevant questions is concerned with defining the function $K_v(V)$ [see (14)] and global constraints (16). These questions can be relatively easily answered as far as intermediate values of V(t) (i.e., for $t < t_f$) are determined; however, the key question concerns the value of the final state $V(t_f)$ which should be given as a fixed target point (stiff constraint) or as the desired one, introduced to the function $K_v(V)$ as the penalty-type function of the deviations of the final state from the target value.

This value is the most important parameter with regard to the dynamics of the system ("the further future"). If the optimization horizon $[t_o,t_f]$ is relatively short, then the value of $V(t_f)$ could be defined, for example by simulation-optimization techniques; by application of some ideas which lead to the determination of the planning rules described in Sections 3 and 4.

The formal difference between upper layer activity and methods presented in Sections 3 and 4 consists in an explicit definition, as a given time dependent function $\hat{V}(t)$, of some trajectory over the planning (optimization) horizon $[t_o, t_f]$. The practical difference results from the fact that the upper layer of the hierarchical control structure has a more elastic construction which allows change in the priorities of control (optimization), constraints and parameters in a relatively simple manner.

5.3 THE LOWER LAYER OF THE CONTROL STRUCTURE

The objective of the lower layer is to generate direct control decisions M(t), U(t) according to the storage policy determined by the upper layer at the beginning of the time interval (optimization horizon) $[t_o, t_f]$. The information which is necessary for controls determination is the following:

- an actual (measured) state of the reservoirs V(t), and
- short-term forecasts (e.g. twenty-four hours, one week) of uncontrolled phenomena such as natural inflow, water demands, etc.,

The operational purpose of the lower layer is to make rational current decisions on water resources allocation with regard to the information used (such as was mentioned above) and given long-time horizon storage policy. Thus, the mechanism of modifying the storage policy is not incorporated in the operating rule. The lower layer is constructed only to improve on-line system operation without the necessity of repeating the long-horizon optimization. The structure of the lower layer allows to use the current information, (i.e., short-term forecasts) in a very elastic way. First of all, the existing structure of the controlled system (with respect to available information and/or decision competences) is considered.

The following decomposition of the considered model of the system was assumed:

- -- subsystem I - which consists of water user B and the upstream Vistula River from the reservoir (1);
- -- subsystem II - two Sola River reaches: between reservoir (2) and (3), and upstream from the reservoir (2);
- -- subsystem III- water user A;
- -- subsystem IV - water users C,D,R, reach of the Sola River downstream from the reservoir (3) and Vistula River downstream from the reservoir (1).

This decomposition implies the existence of four local decision units (LDU), I to IV, associated with the respective subsystems. Each of the local decision units has at its disposal current (the most precise) information concerning the respective subsystem and the set of local decisions. The information pattern and authority range of the particular LDU is shown in Table 1.

Table 1. Information pattern and authority range of local decision units.

LDU	Local information	Local decisions
I	Q_1, P_B	M_{B_1}, M_{B_2}
II	Q_2	U_2
III	P_A	M_A
IV	$P_C, P_D, P_R, Q_E, Q_H, Q_P, Z_C, Z_P$	M_C, M_D, M_R, U_1, U_3

It is worthwhile to notice that the last subsystem (and the corresponding LDU IV) is the most extended since it includes the river reaches where water quality requirements should be met. Water quality is influenced directly by all control variables and inflows mentioned in Table 1, thus there is a common local objective concerned with all these variables together. Each of the LDU's takes into account its local information and the so called coordination variables, which are the decisions settled by the coordinator--a central decision unit which instantaneously influences all LDU's.

5.4 COORDINATION OF LOCAL DECISION UNITS

Assignment of the coordinating variables (denoted by p) relates the planning policy derived by solving the long-horizon optimization problem with the on-line control performed by local decision units (in the approach presented in Section 4 this relationship was accomplished directly). Coordinator influences all LDU's in such a way that the whole, controlled system follows the long-horizon storage policy. The LDU operation rule consists of performing an independent, rational allocation of water resources inside the subsystem with regard to local objectives, local short-term forecasts and coordinator decisions p. The local objectives (formally-respective components of (17)) should be properly modified by the coordinator decisions which should be also chosen properly (i.e., an adequate coordinating rule has to be determined as mentioned at the end of this sub-section).

This is the general idea of the proposed on-line control scheme (Malinowski, Terlikowski [1978]). In the current investigations (Kaczmarek et al. [1978], Terlikowski [1978]) based on the optimizing scheme of the control structure, the price mechanizm was used as the method of coordination. The coordination variables have been expressed in this scheme as the vector:

$$p = (p_1, p_2, p_3)$$

where p_1, p_2, p_3 are scalar variables, so called <u>prices</u>, corresponding to reservoirs (1), (2), and (3) respectively.

Modified, local objectives (denoted by $L_I, L_{II}, L_{III}, L_{IV}$) for each of the subsystems result from the decomposition of Lagrangian

$$L(p,M,U,P,Q,Z) = K(M,U,P,Q,Z) + \langle p, \dot{V} \rangle \qquad (18)$$

where $\dot{V}$ is the right hand side of the reservoir state equation (for example, the state equation of the reservoir (3) is the following):

$$\dot{V}_3(t) = (U_2(t) - M_A(t) - M_{B2}(t) - U_3(t))$$

It is clear, that

$$L = L_I + L_{II} + L_{III} + L_{IV} \quad ,$$

where $L_I, \ldots, L_{IV}$ depends only on decisions and forecasts related to the respective subsystem; it depends also on p. For example, the modified performance index of subsystem I has the following form:

$$L_I(p_1, M_{B1}, M_{B2}, Q_1, P_B) = K_B(M_{B1}, M_{B2}, P_B) + $$
$$+ p_1 \cdot (M_{B1} - Q_1) + p_3 \cdot M_{B2} \quad .$$

Thus, the LDU's operating rule consists in optimizing the local objectives:

$$\min_{M_{(i)}, U_{(i)}} L_{(i)} \quad ; \quad (i) = I, \ldots, IV \qquad (19)$$

subject to given prices (coordination variables) and actual, local forecasts.

Prices p should be chosen in such a way, that controls M(t), U(t) defined by the respective LDU according to (19) yield some trajectory $V_r(t)$ of the reservoirs' state, which approximately (according to specific requirements and conditions) follow the storage policy $\hat{V}(t)$ defined at the upper layer.

Based on the measured, actual state of the reservoirs $V_r(t_j)$ $(t_j \varepsilon [t_o, t_f])$ the prices are modified subject so some formula A (see Malinowski, Terlikowski [1978])

$$V_r(t_j) \xrightarrow{A} \tilde{p}_j \qquad (19a)$$

The coordinating rule (19a) and the LDU operating rule (19) constitute the whole operating rule as accomplished by the lower layer of the considered control scheme.

5.5 REMARKS TO SECTION 5

At the end of this section some additional properties of the described method may be discussed.

Modifications of the local objectives introduced by the specific [see (19)] choice of the prices enable to balance, in a rational manner, current water demands and other requirements within the possibilities concerned with the local short-term policy of the system.

It is worth observing that the LDU rule (19) has the following <u>optimality property</u>:

-- if short-term forecasts prove to be fully consistent with reality, then the controls M(t),U(t) assigned by (19) are strictly optimal for the performance index (17), subject to constraint $V(t) = V_r'(t)$, where $V_r'(t)$ is a state trajectory occurring in the real, controlled system.

Hence:

- -- local decision units accomplish a rational (optimal) current water distribution in a system;
- -- the coordinator sees that this distribution is realized according to the storage policy;
- -- the upper layer (planning layer) aims for a proper choice of this policy.

The scheme presented is quite general and elastic. This very natural structure can be adjusted to specific real conditions. It is possible to introduce some additional elements (which may exist in reality) into the operating rule and use various methods of long-horizon policy planning.

At the present stage of research (when the numerical experiments are extensively performed), the proposed control scheme has not yet been entirely adapted to the system, especially from the practical point of view. However, if seems that its appealing features (as far as control in drought and normal flow conditions is considered) provide considerable incentive to carry on this research.

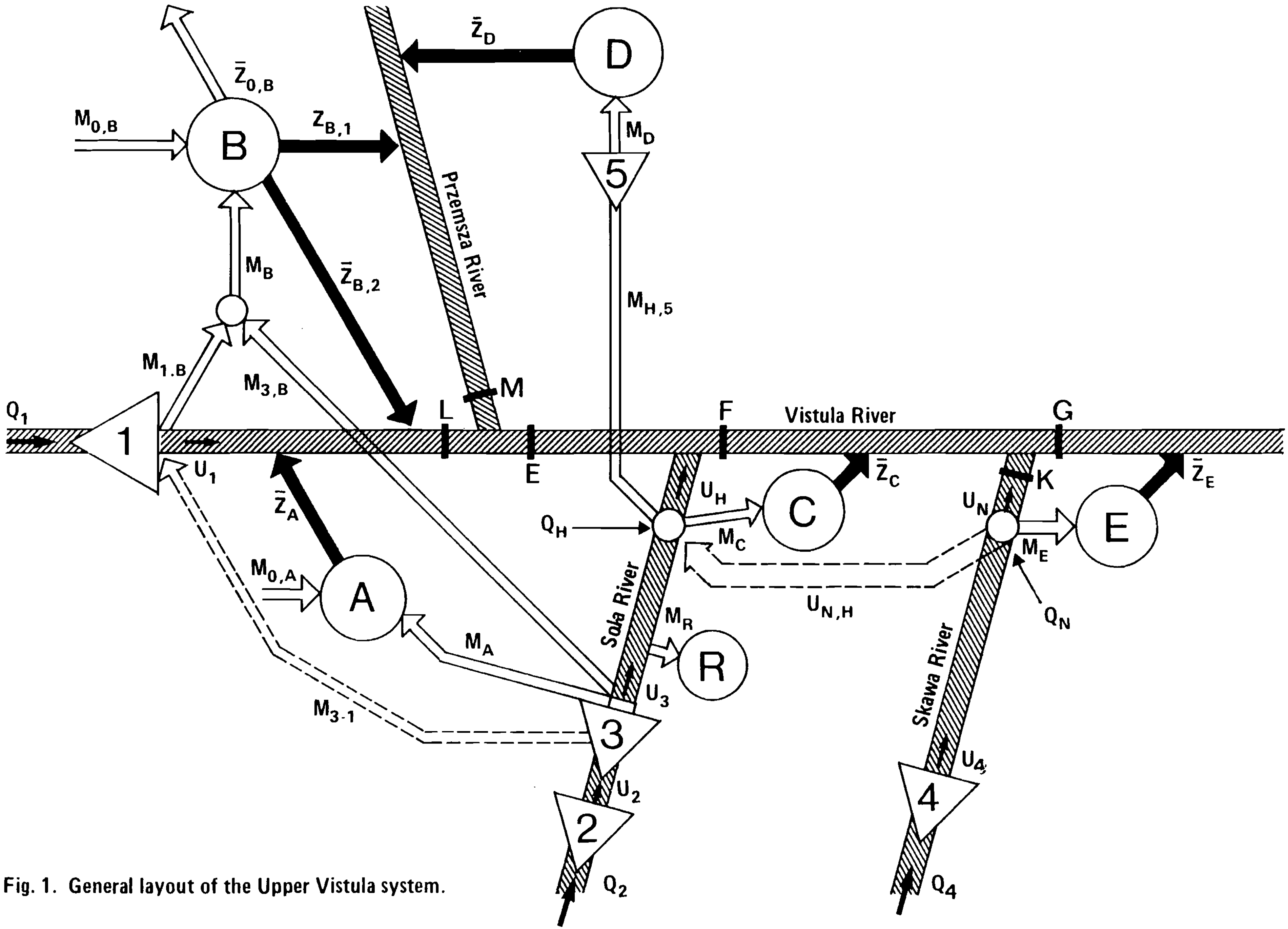

Fig. 1. General layout of the Upper Vistula system.

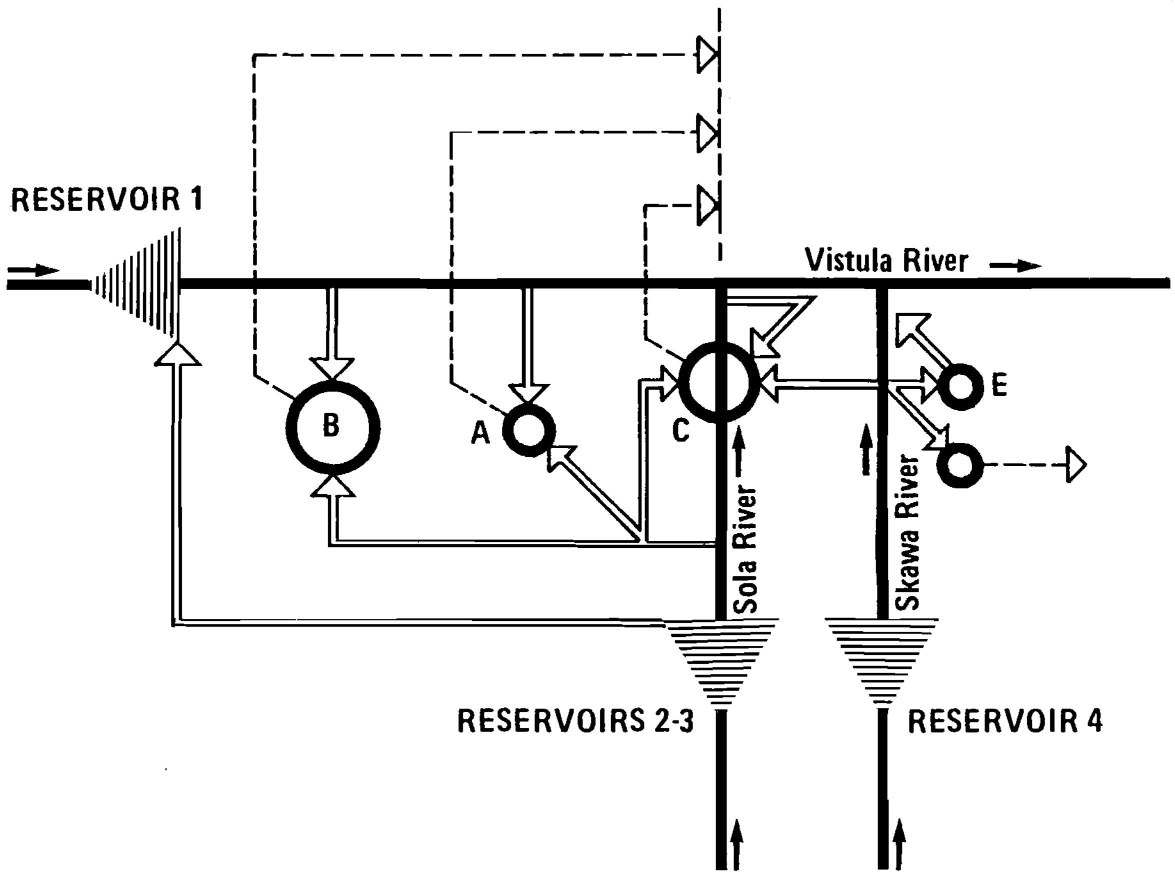

Fig. 2. Simplified layout of the Upper Vistula system (Kindler, 1977).

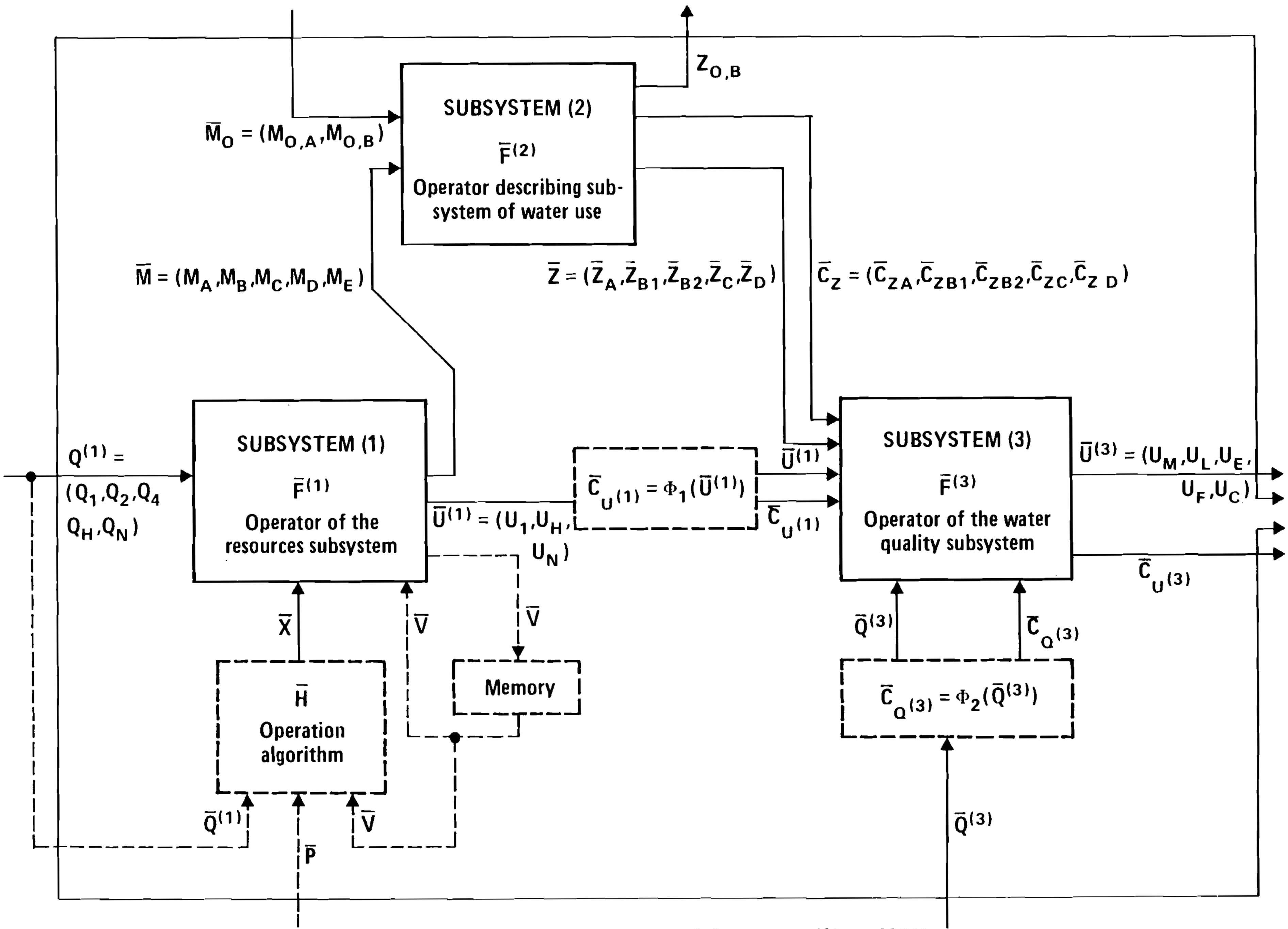

Fig. 3. Structural and functional scheme of the system. (Slota, 1978) (Informational inputs and non-physical elements of the system are marked by the dotted line.)

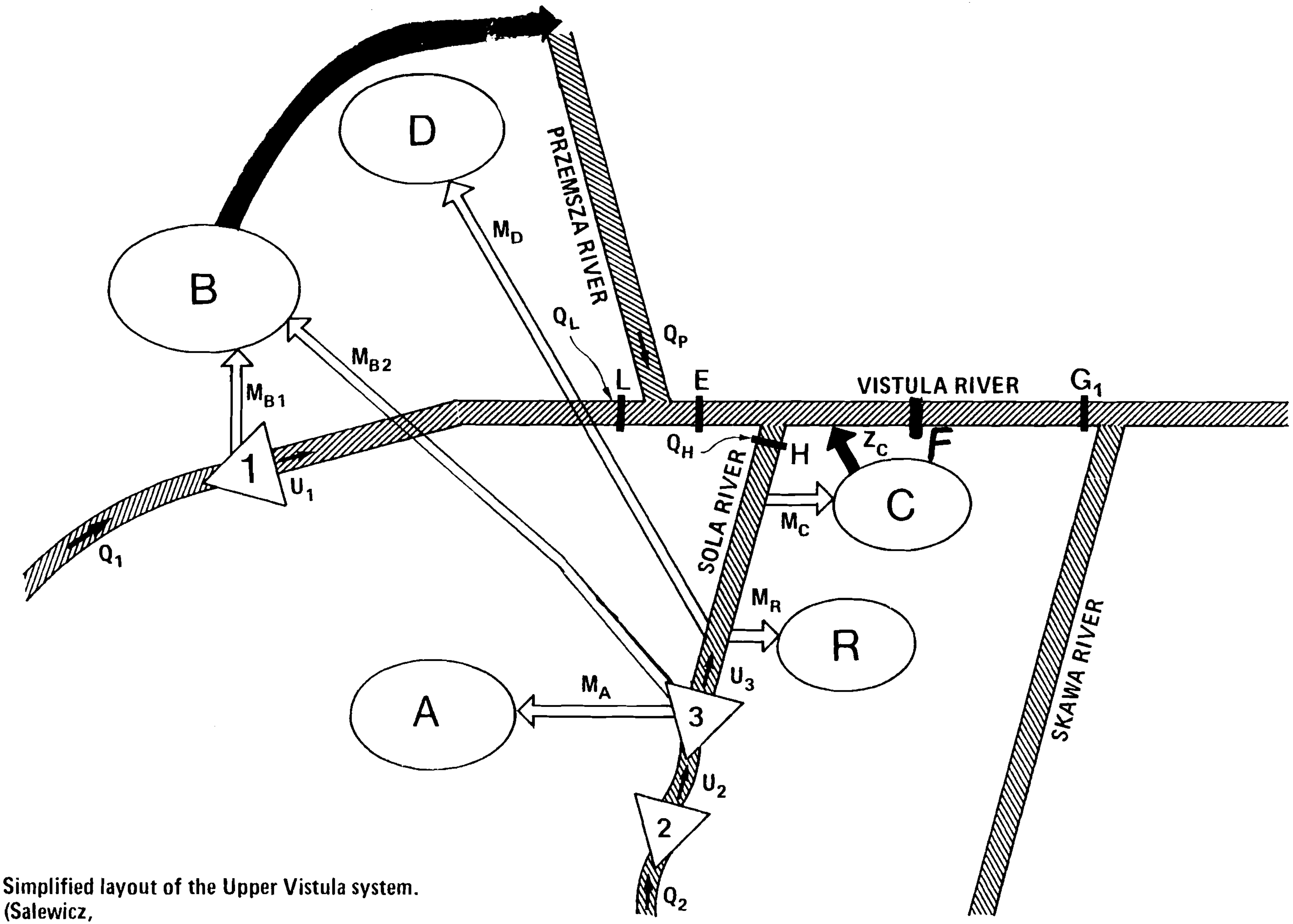

Fig. 4. Simplified layout of the Upper Vistula system.
(Salewicz,

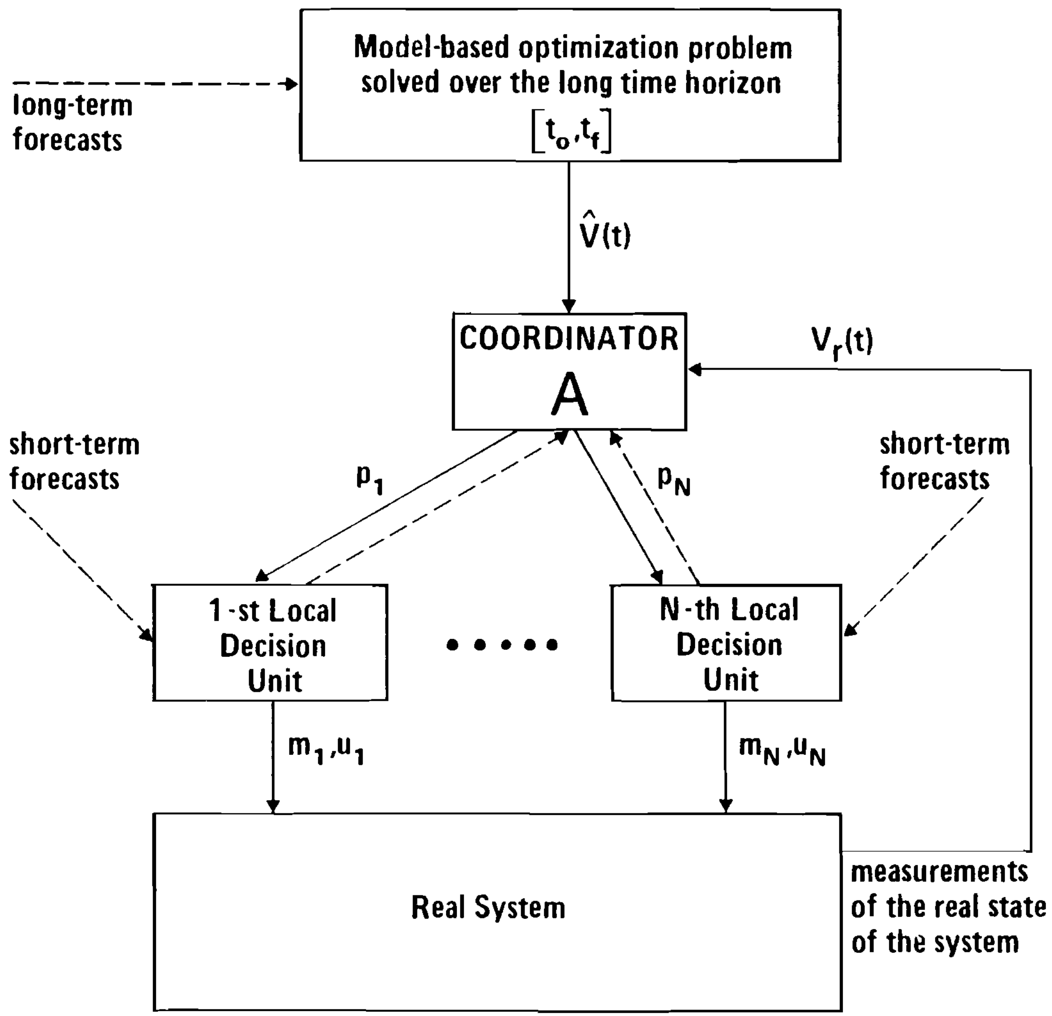

Fig. 5. Scheme of the control structure.

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