Strategic Investment in Protection in Networked Systems

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Forthcoming in Network Science

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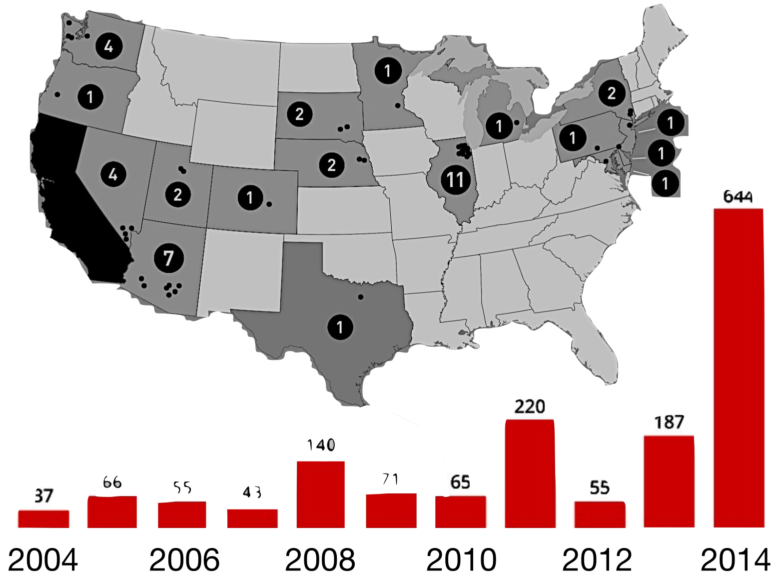
IIASA International Institute for Applied Systems Analysis

Stanford

INSEAD The Business School for the World®

EXAMPLES OF NETWORKED SYSTEMS IN WHICH INDIVIDUAL INCENTIVES MATTER

Measles outbreak in US 2014-2015



"While I think it's a good idea to take the vaccine, I think that's a ***personal decision*** *for individuals"*

Senator Rand Paul of Kentucky

"There is absolutely ***no reason to get the shot****. I said, 'I'd rather you miss an entire semester than you get the shot.' "*

Mother of a 16-year-old student

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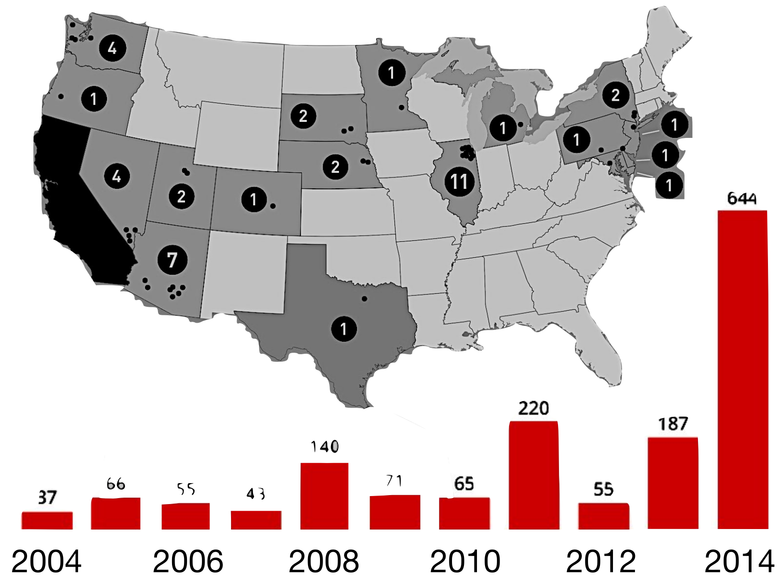

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*"There is absolutely **no reason to get the shot**. I said, 'I'd rather you miss an entire semester than you get the shot.' "*

Mother of a 16-year-old student

Paris Attacks, Nov 2015

"The European Union will step up checks on its citizens traveling abroad, tighten gun control and collect more data on airline passengers"

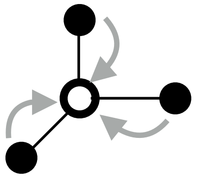
"David Cameron is to respond to the escalation in terror attacks around the world by making provisions for 1,900 extra security and intelligence staff and doubling funds for aviation security."

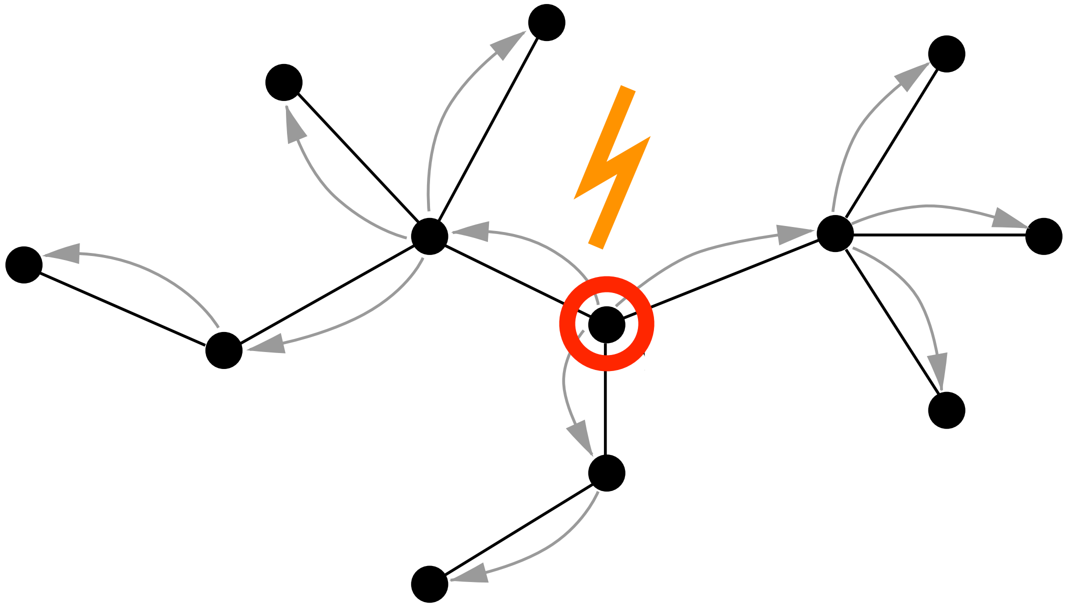
RESEARCH QUESTION

2 ways to fail:

intrinsic failure

cascade of failures





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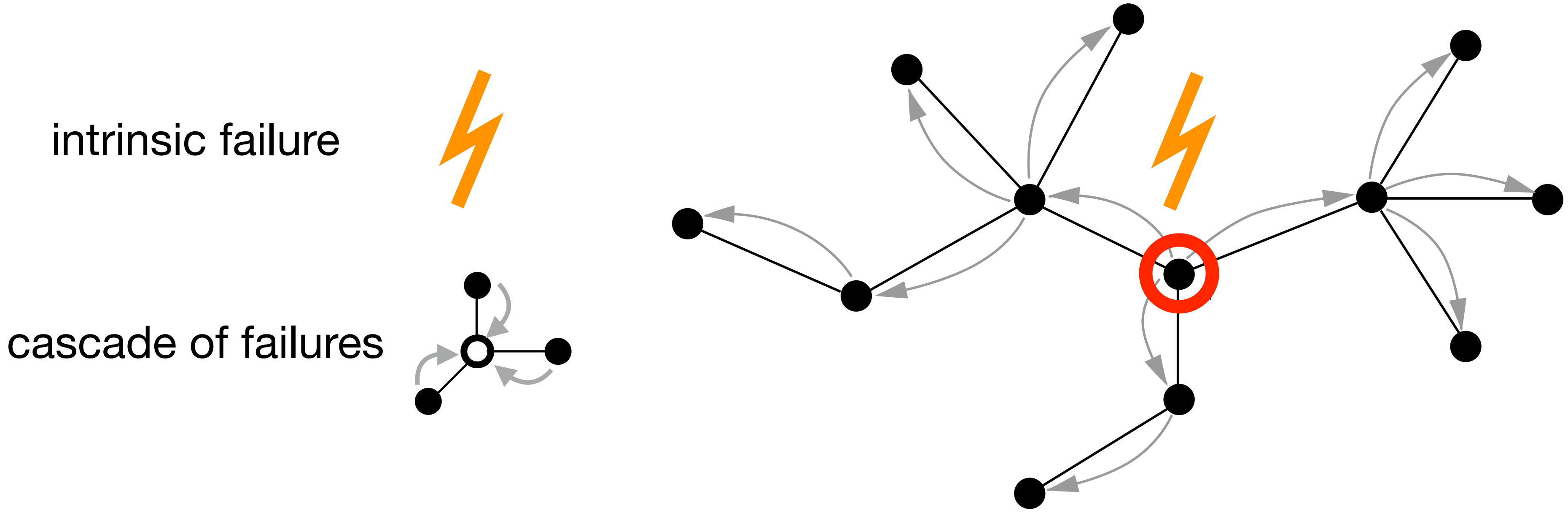
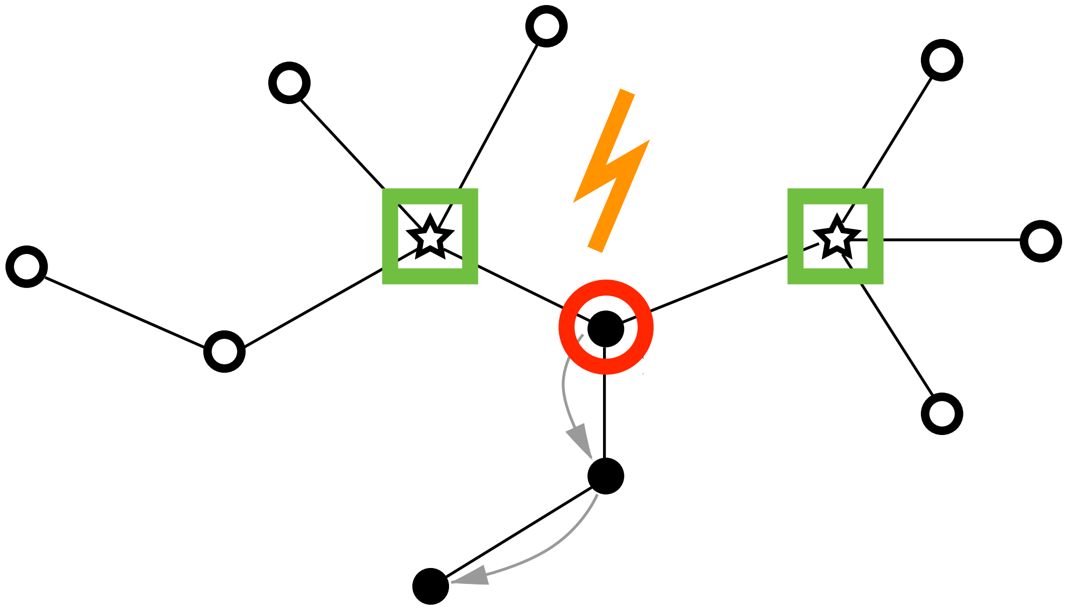


intrinsic failure

cascade of failures

Agents can invest in costly protection



*What are the strategic incentives of **agents** to invest in **costly protection**?*
*How does the **network structure** influence these decisions?*

LITERATURE

Network Games

- *Galeotti et al., 2010*
- *Jackson and Yariv, 2007*
- *Kearns, 2007*
- *Jackson and Zenou, 2014*

Interdependent Security (IDS)

- *Heal and Kunreuther, 2005*
- *Heal et al., 2006*
- *Johnson et al., 2011*

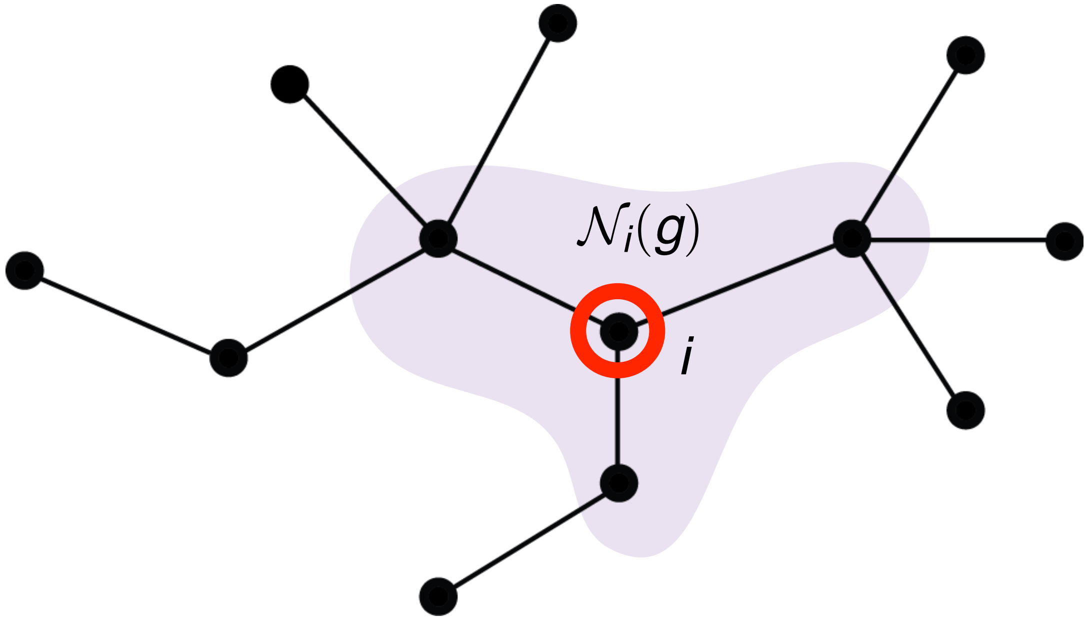
Cascade Risk in Networks

- *Lelarge, Bolot, 2008, 2009*
- *Galeotti, Rogers, 2013*
- *Dziubinski, Goyal, 2014*
- *Goyal, Vigier, 2014*
- *Blume et al., 2011*

Contribute to the literature on strategic investments in protection in complex interconnected systems.

MODEL OVERVIEW

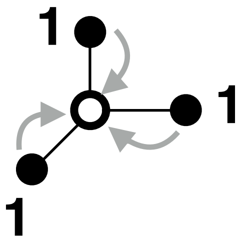
Network - **nodes** (agents) and **edges** (interconnections)



$\mathcal{N}_i(g)$ - neighborhood of agent i

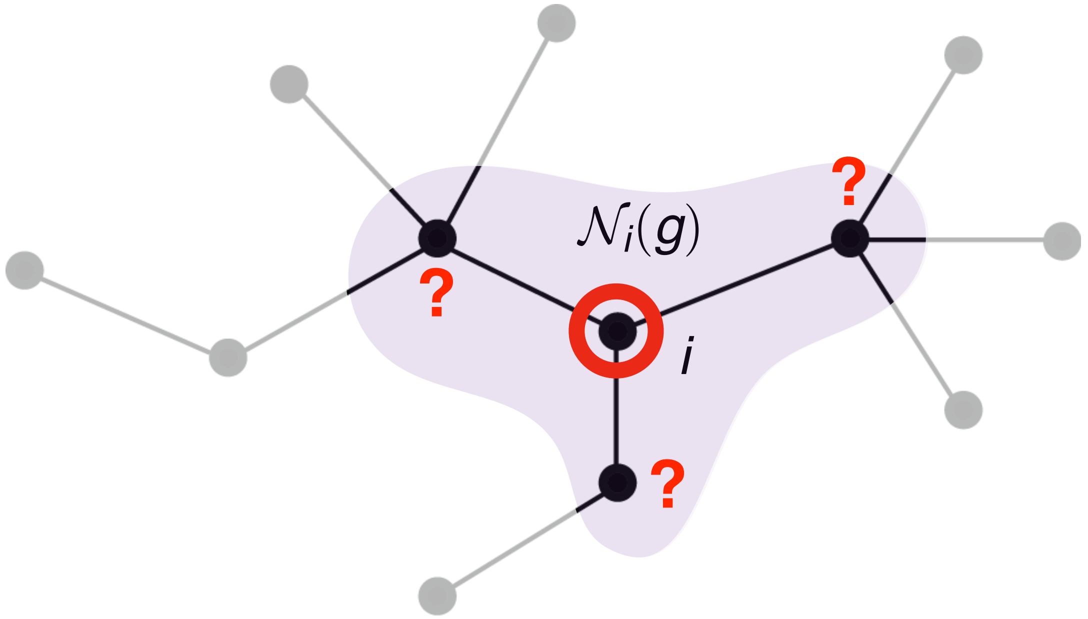
$d_i(g) = |\mathcal{N}_i(g)|$ - degree of agent i

Network - nodes (agents) and edges (interconnections)

agents can fail:	**intrinsically** (ext)	**cascade of failures** 
probabilities:	p	$\mathcal{H}_d : \{0,1\}^d \rightarrow [0,1]$ cascading failure function: vector of binary variables(**friend** failed/didn't fail) **->** agent's probability to fail

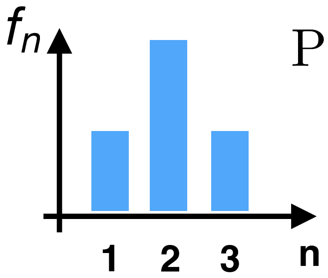
This model leads to BNE - hard to work with it. Can only prove existence of eq.

MEAN-FIELD MODEL



Agent i knows his **own** degree and:

doesn't know full network structure **but** knows the degree distribution it is drawn from



$\Pr[\text{an agent has degree } d] = f_d$

$\{f_1, f_2, \ldots\}$

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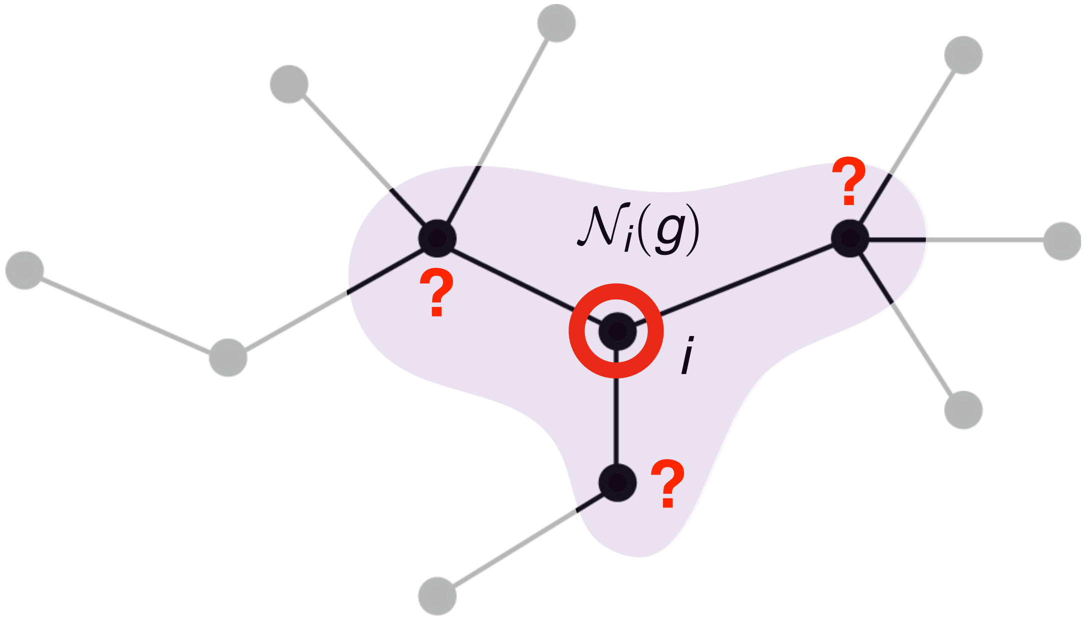

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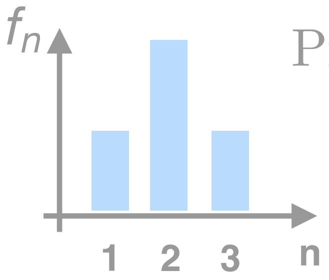

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doesn't know each friend's degree **but** knows edge-perspective degree distribution

$\mathrm{Pr}[\text{a } \textbf{friend} \text{ has degree } d] = \widetilde{f}_d$

$\{\widetilde{f}_1, \widetilde{f}_2, \ldots\}$

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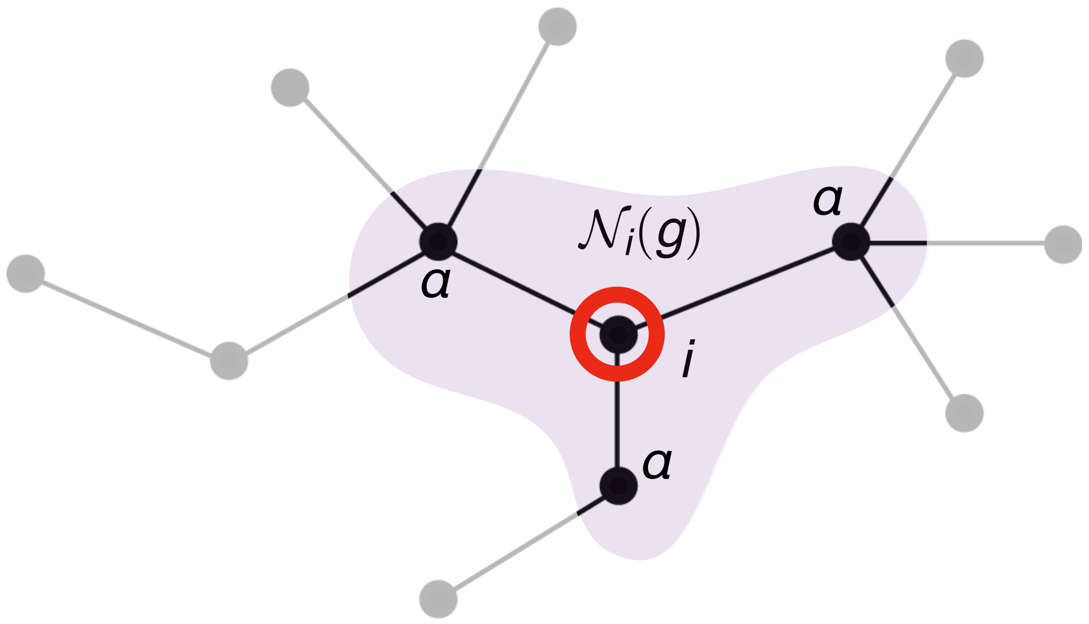

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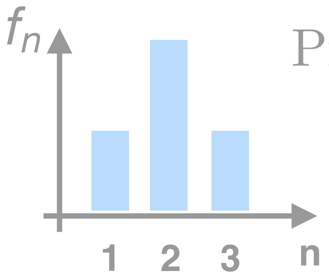

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$$\{f_1, f_2, \ldots\}$$

doesn't know each friend's degree **but** knows edge-perspective degree distribution

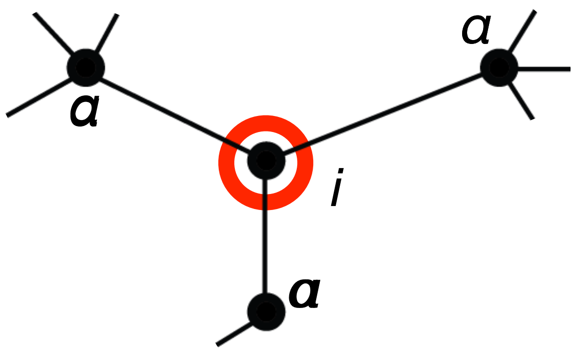
$$\Pr[\text{a \textbf{friend} has degree } d] = \widetilde{f}_d$$

$$\{\widetilde{f}_1, \widetilde{f}_2, \ldots\}$$

doesn't know each friend's failure probability **but** conjectures that each friend fails with the same probability (*bounded rationality*)

$$\Pr[\text{a \textbf{friend} fails}] = a$$

TOTAL PROBABILITY TO FAIL



Model of accumulative risk:

agent's cascading failure probability: $q_d : [0, 1] \to [0, 1]$

$$q_{d'}(a) > q_d(a), \; \forall d' > d$$

more connections - **higher** risk

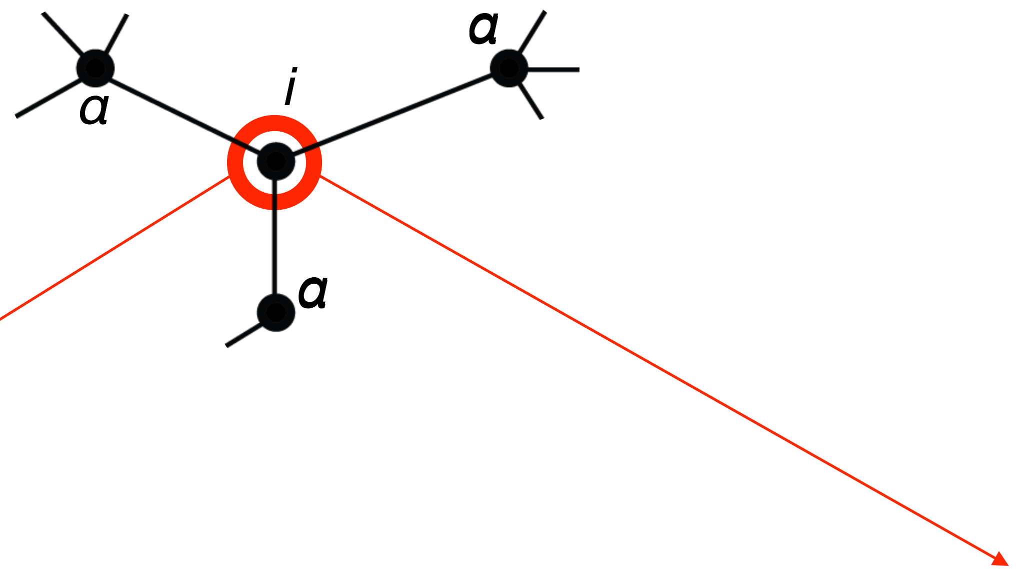
<u>example</u>: malware or virus spread

$$q_d(a) = 1 - (1 - ra)^d$$

virus is transmitted with r probability

Total probability to fail: $\beta_d = p + (1 - p)q_d$

DECISIONS



invest in protection

VS

don’t invest in protection

$$a_i = 1$$

$$U(a_i = 1, \alpha) = -V \cdot \mathcal{B}(p, q_d(\alpha), a_i = 1) - C$$

cost of protection

$$a_i = 0$$

$$U(a_i = 0, \alpha) = -V \cdot \mathcal{B}(p, q_d(\alpha), a_i = 0)$$

effective probability to fail

mean-field strategy

for each degree-type specifies probability to invest in protection

$$\mu : \mathbf{N}^+ \to [0, 1]$$

MEAN-FIELD EQUILIBRIUM

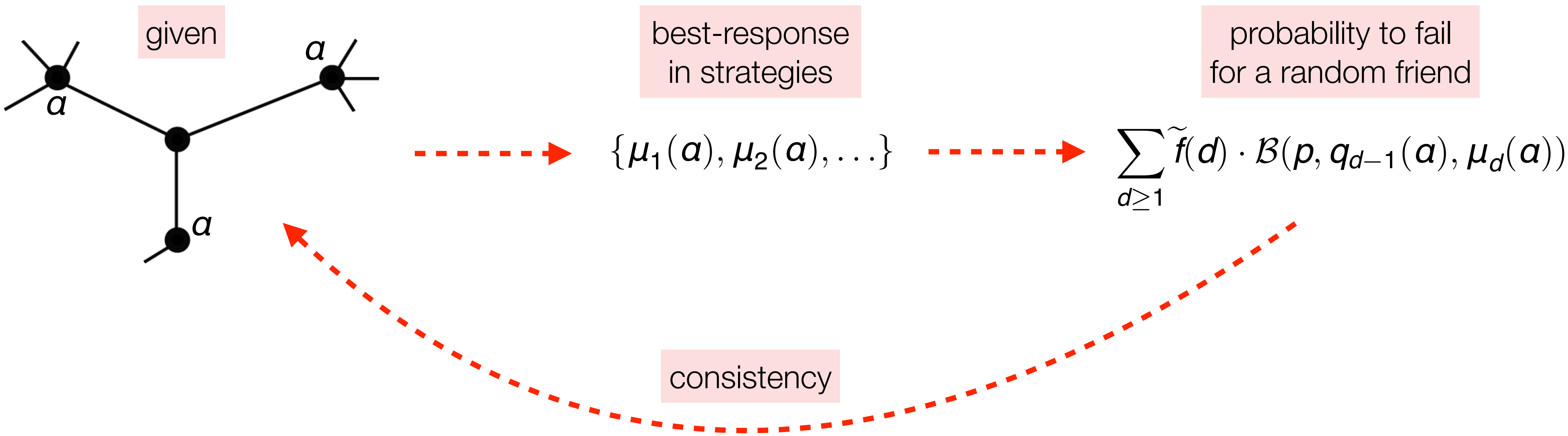
We are searching for:

- mean-field local probabilities to fail
- set of strategies for each degree-type

$$(a^*, \mu^* = \{\mu_1^*, \mu_2^*, \ldots\})$$

Fixed point argument:

a^* must be induced by the mean-field strategies μ^* that are BR to a^*



best-response in strategies

$$\{\mu_1(a), \mu_2(a), \ldots\}$$

probability to fail for a random friend

$$\sum_{d\geq 1} \tilde{f}(d) \cdot \mathcal{B}(p, q_{d-1}(a), \mu_d(a))$$

consistency

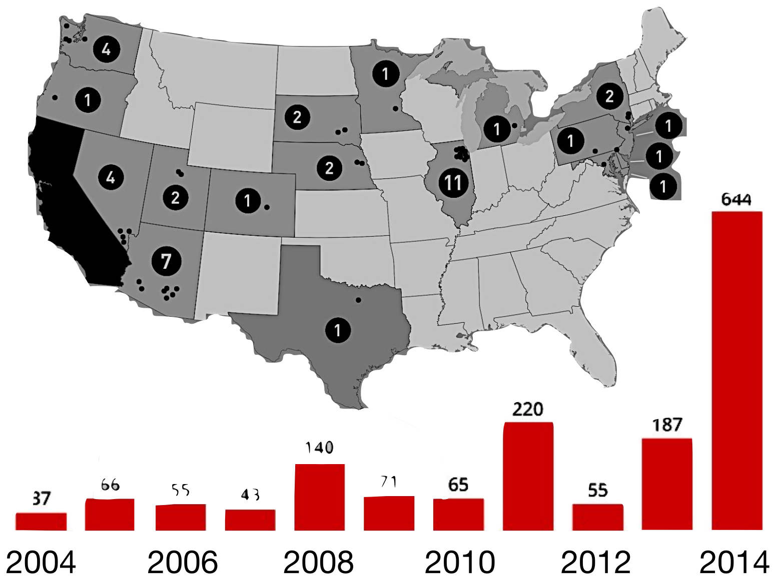
Th: there ***exists*** *a mean-field equilibrium*

WHAT DOES PROTECTION DO?

insulates against **total** risk

games of total protection

$$\mathcal{B}(p, q_d(a), a) = \Big(p + (1-p)q_d(a)\Big) \cdot (1 - ka)$$



Examples:

- computer antivirus software
- vaccination agains measles

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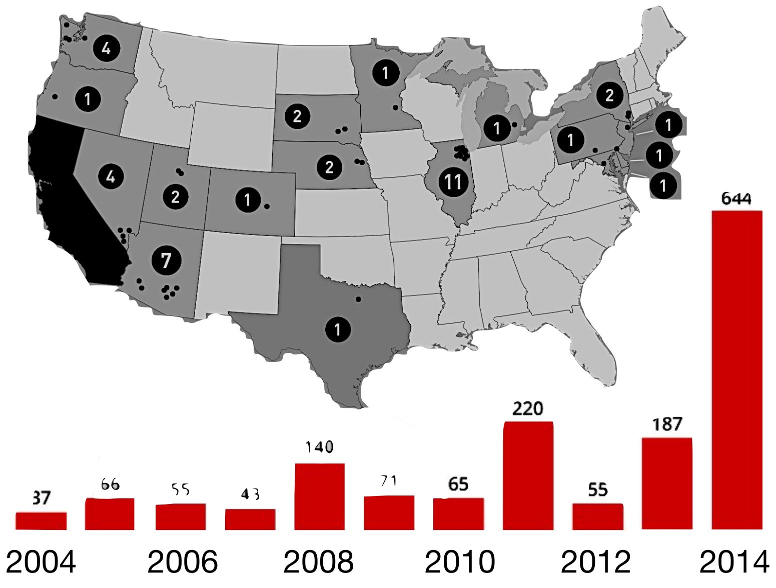


Examples:

- computer antivirus software
- vaccination agains measles

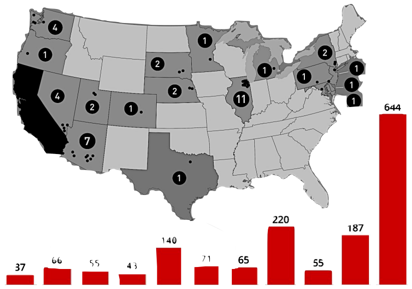
insulates against **intrinsic** risk **only**

games of self protection

$$\mathcal{B}(p, q_d(a), a) = p \cdot (1-ka) + (1 - p \cdot (1-ka))q_d(a)$$

Examples:

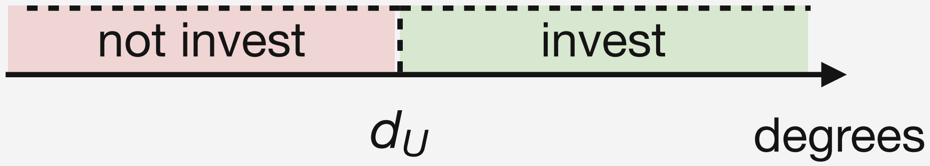
- investing in airport security
- investing in national security within EU

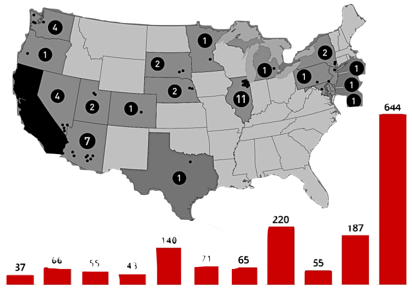


EQUILIBRIUM: TOTAL PROTECTION

submodular game (**strategic substitutes**)

Th: equilibrium is **unique** and only sufficiently connected agents invest in protection (**upper-threshold strategy**).





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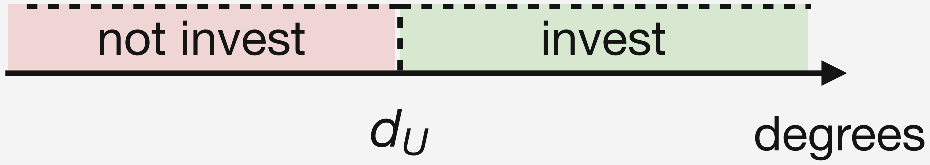

not invest

invest

d_U

degrees

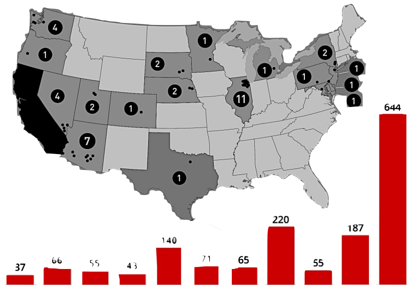
low connected agent

d_L **<** d_H

high connected agent

intrinsic risk **=** intrinsic risk

cascading failure risk **<** cascading failure risk



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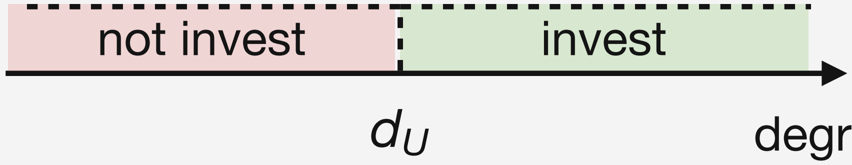
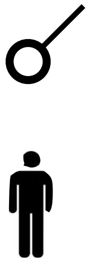


not invest | invest

d_U degrees

low connected agent



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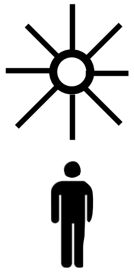


intrinsic risk = intrinsic risk

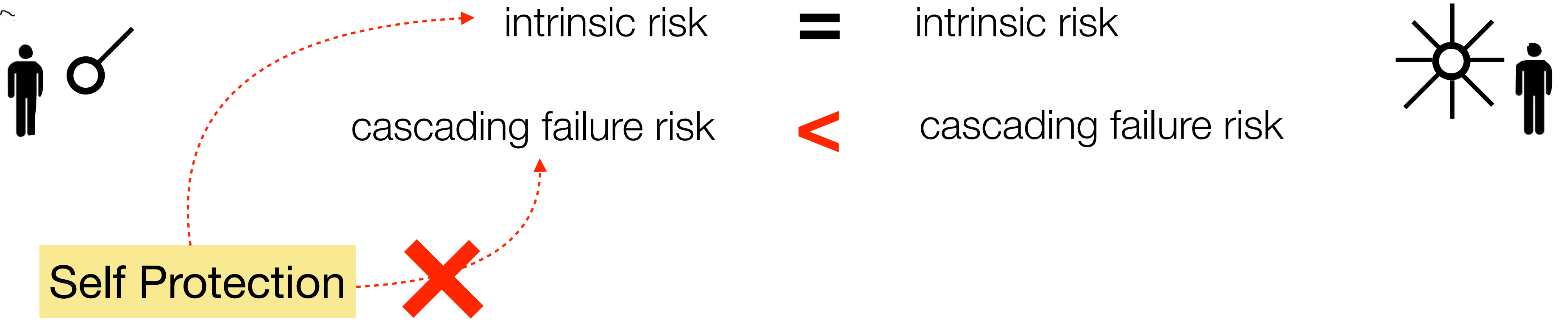
cascading failure risk < cascading failure risk

Total Protection

incentive to invest in total protection < **incentive to invest** in total protection

EQUILIBRIUM: SELF PROTECTION

supermodular game (**strategic complements**)



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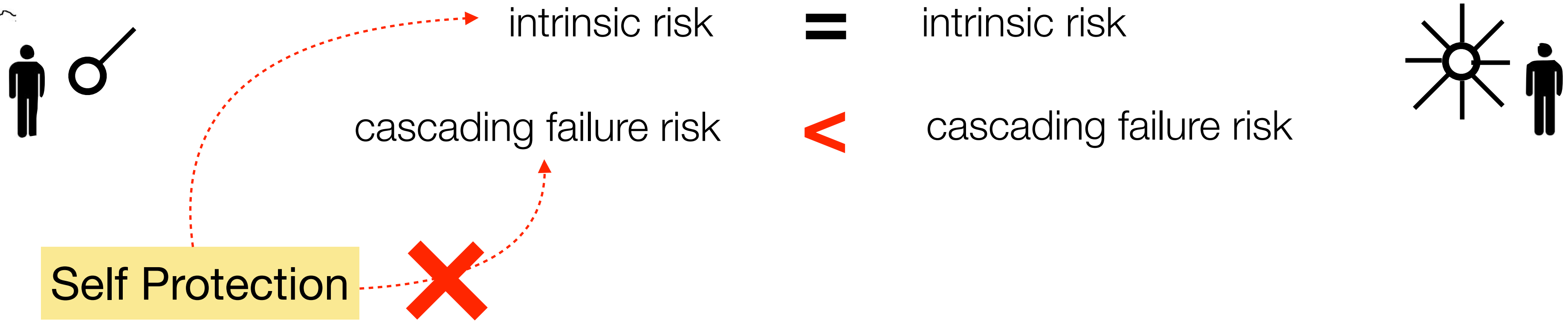

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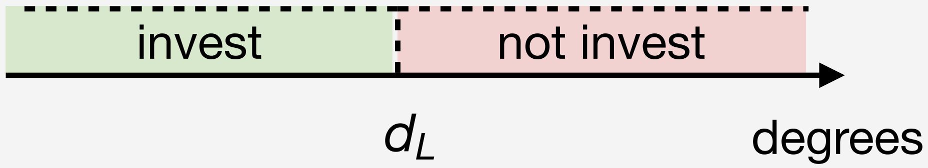
Self Protection

incentive to invest in self protection > **incentive to invest** in self protection

Th: in equilibrium only low connected agents invest in protection (**lower-threshold strategy**).



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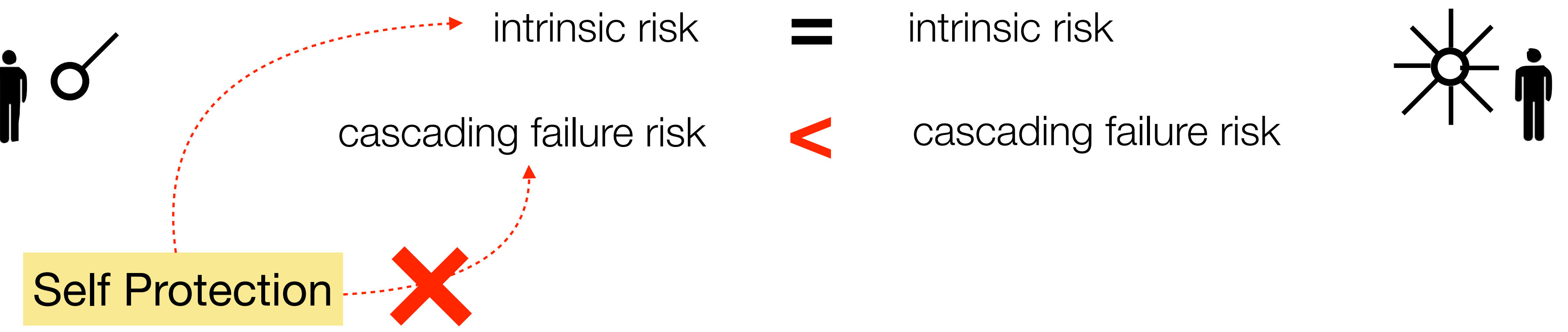

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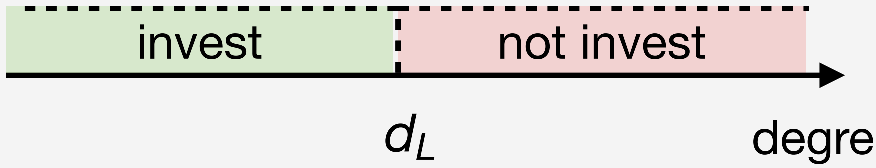


Proposition:

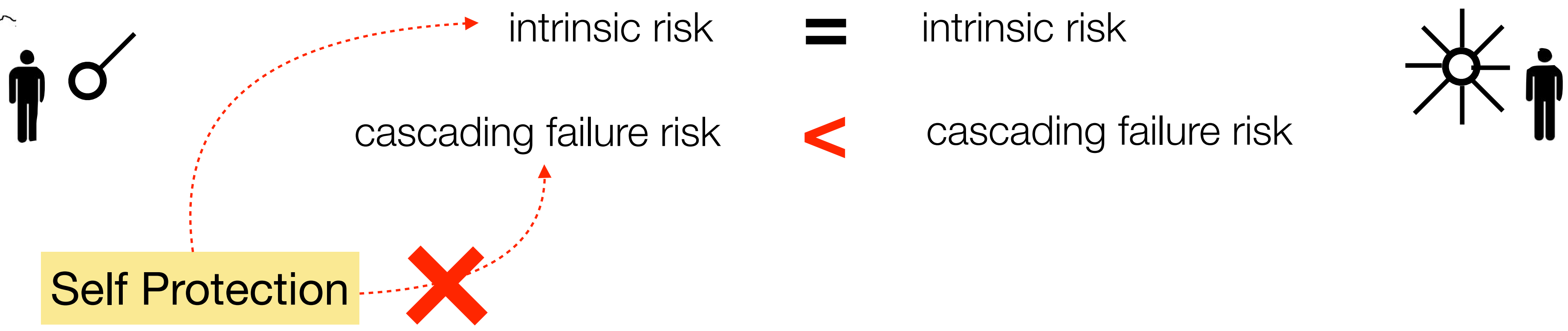
equilibrium effective failure probability $\mathcal{B}(p, q_d(\alpha^*), \mu^*(d))$ $\leq$ equilibrium effective failure probability $\mathcal{B}(p, q_d(\alpha^*), \mu^*(d))$

Proposition:

equilibrium expected utility $U_d(\mu^*(d), \alpha^*)$ $\geq$ equilibrium expected utility $U_d(\mu^*(d), \alpha^*)$

EQUILIBRIUM: SELF PROTECTION

supermodular game (**strategic complements**)



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Self Protection ✗

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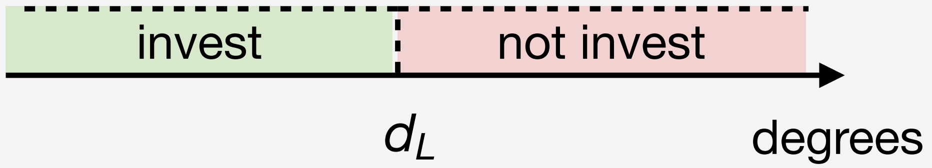

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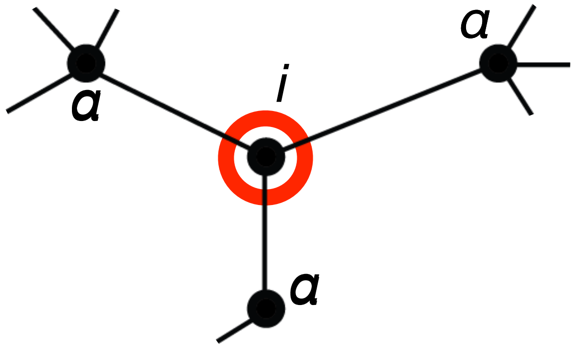
Proposition: under FOSD increase in $\tilde{f}_d$ incentives to invest in costly protection are lower.

GLOBAL & LOCAL EXTERNALITIES



$$U(a_i = 1, \alpha) = -V \cdot \mathcal{B}(p, q_d(\alpha), a_i = 1) - C$$

cost of protection

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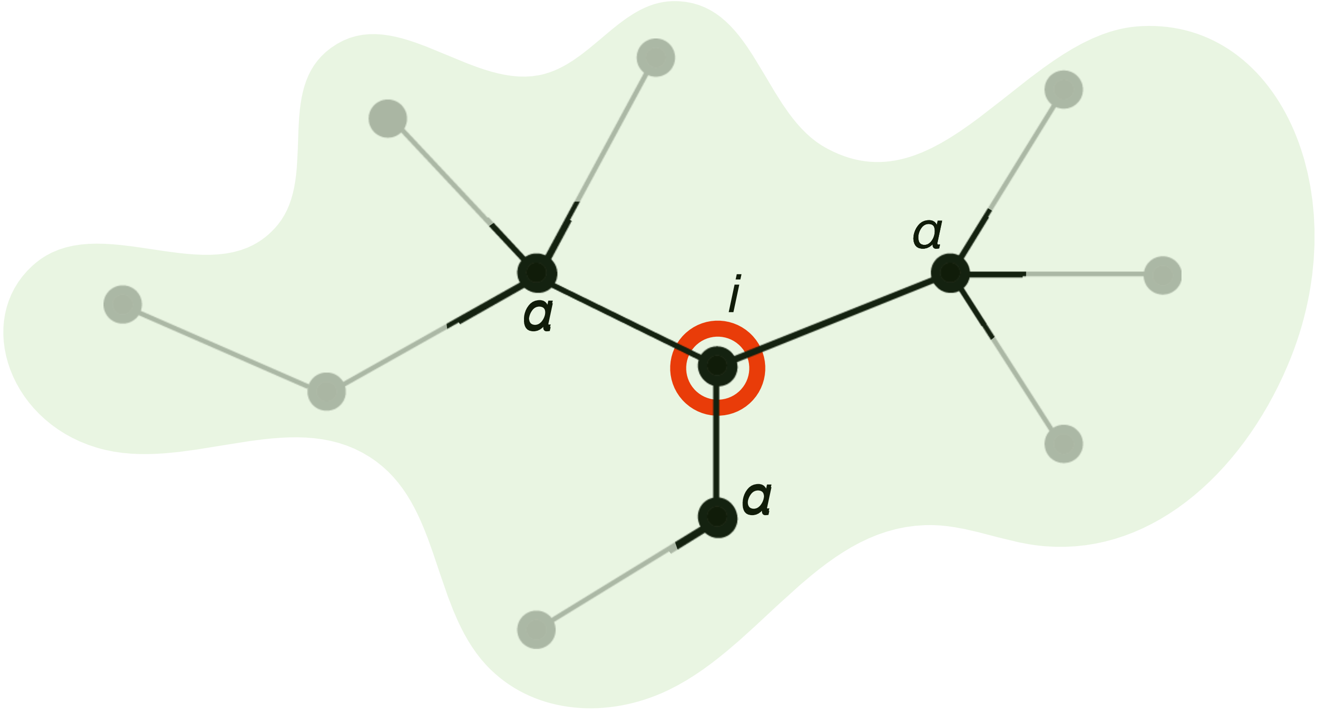

~ Total Demand

$$U(a_i = 1, \alpha) = -V \cdot \mathcal{B}(p, q_d(\alpha), a_i = 1) - C(\text{Demand})$$

cost of protection

Th: The threshold characterization of equilibria is robust to the introduction of a global price feedback

Th: In a game of total protection with global price feedback, the mean-field equilibrium is unique if C is an increasing function.

CONCLUSIONS

- Incentives to protect depend on **both** the _*type of protection*_ **and** _*network structure*._

- Market failure is more severe in case of self-protection (EU security, airport security) than in case of total protection (vaccination, malware)

    ➡ Incentives of the agents are aligned with the system's efficient outcome

- We employ a mean-field equilibrium concept that places a reasonable cognitive burden on the agents.

- Model is flexible and allows for:

    - comparative statics in the structure of the network
    - introduction of global externalities.