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Alternative Approaches for Integration of Models

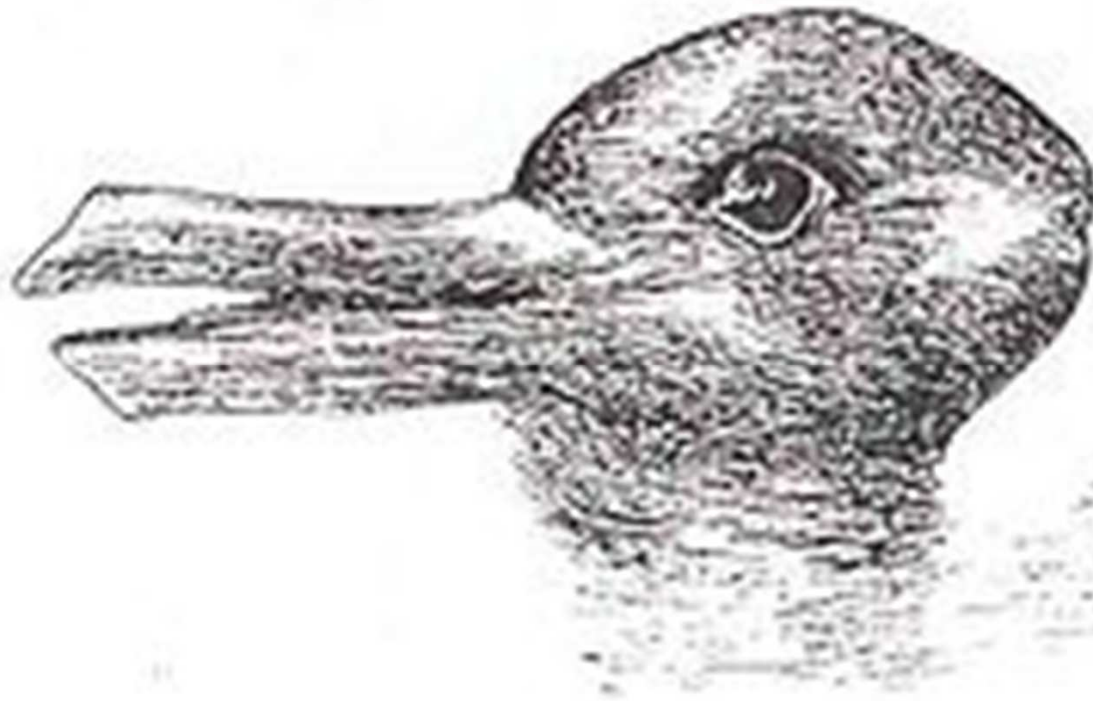
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IIASA, International Institute for Applied Systems Analysis

Sometimes multi-model approach is necessary...

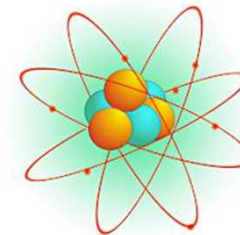
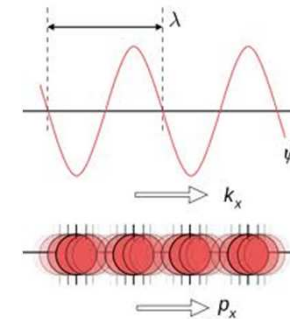


Paradigm shifts by Kuhn: successive change of one model
by another, rather than integration of different paradigms

Progress of science: from single- to multi-model approach

Some examples from natural science...

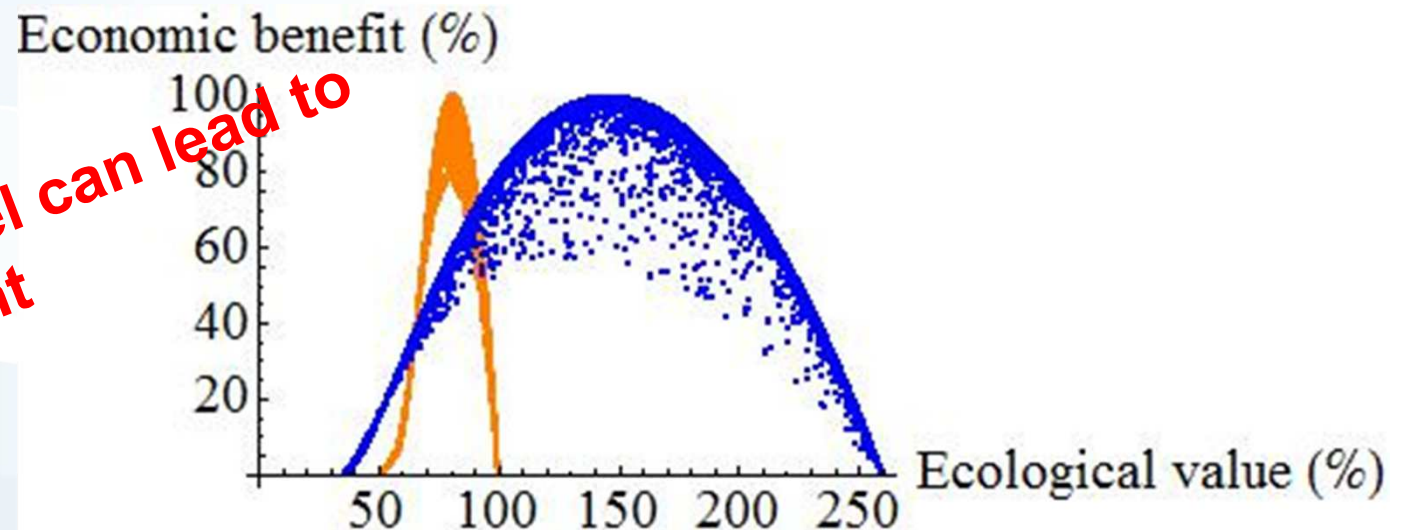
- **theory of light:** from vibration of ether to wave-particle duality
- **laws of motion:** from Newton's dynamics to Schrödinger's and Heisenberg's formalism



In social and environmental sciences appreciation of the multi-model approach is to be obtained

Example: multi-model approach for sustainable forest management

**use of one model can lead to
mismanagement**



Orange area is the Pareto area for the PPA model, blue area is the Pareto area for the model with no feedback (IIASA project on optimization of forest management)

The relationship between economic benefit and ecological value is rather different in two similar models

Evolution of modeling paradigm

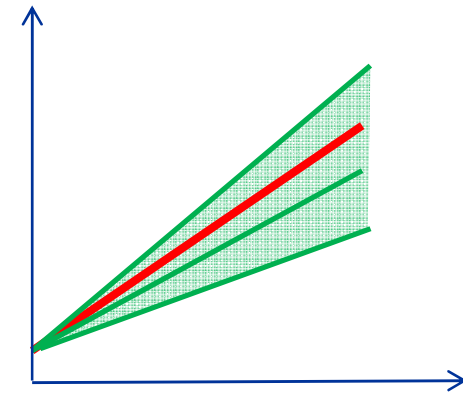
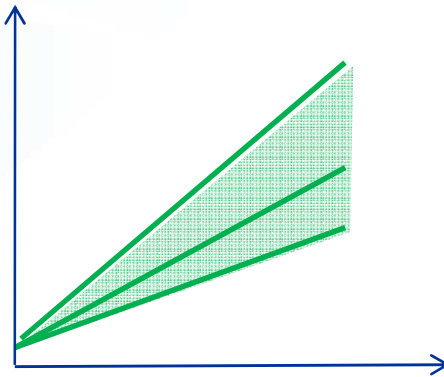
single-model approach

multi-model approach

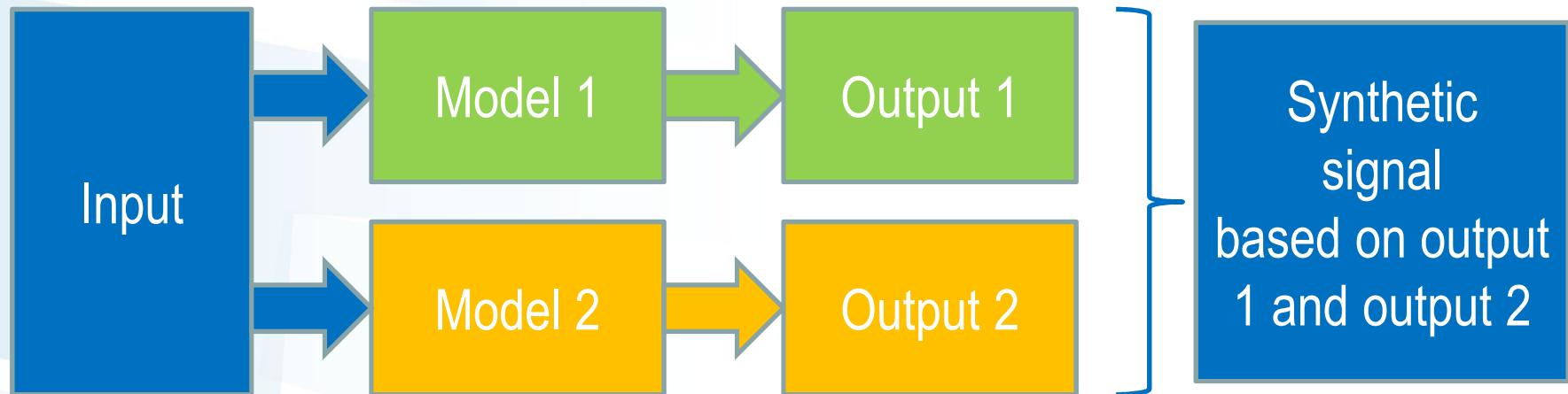
Belief in
one model

Comparison
of models

Integration of
models



Models integration: formalization



- Output 1 and output 2 represent the model results for the **same real quantity**
- Output 1 **does not coincide** with output 2
- Output 1 and output 2 can be either **deterministic** or **stochastic**, either scalar or vector, either finite or infinite dimensional variable

Basing on the past approach

- **Approximate** the **past history** by two models' outcomes and **extrapolate** the obtained approximation **into the future**

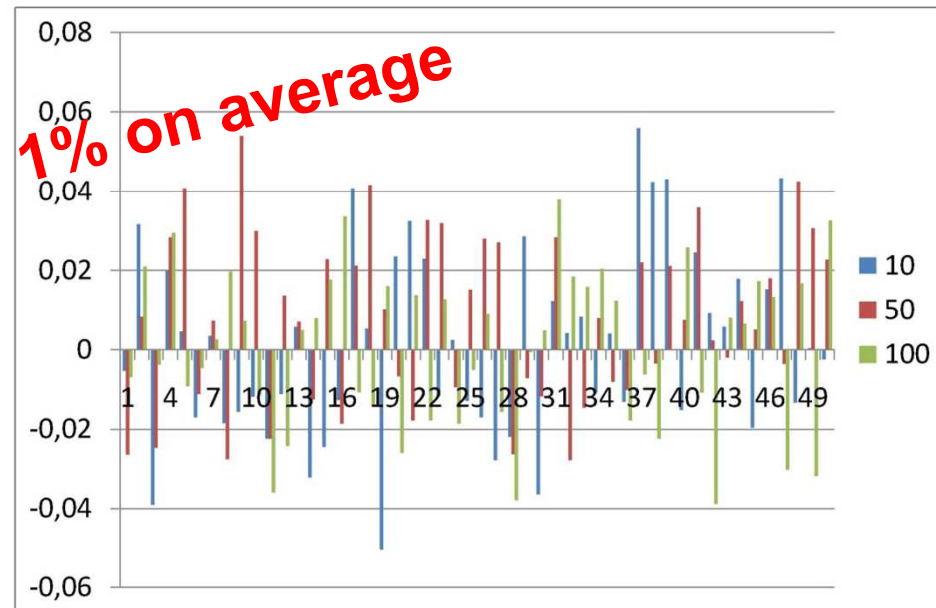
$$C_1^*, C_2^* = \underset{C_1, C_2}{\text{Arg min}} \|x - C_1 x_1 - C_2 x_2\|$$

$$x \cong C_1^* x_1 + C_2^* x_2$$

Example

- Nordhaus's DICE-model (nonlinear!) as a generator of “real” data with the terminal GDP as a model's output
- Two one-dimensional linear models of the global GDP

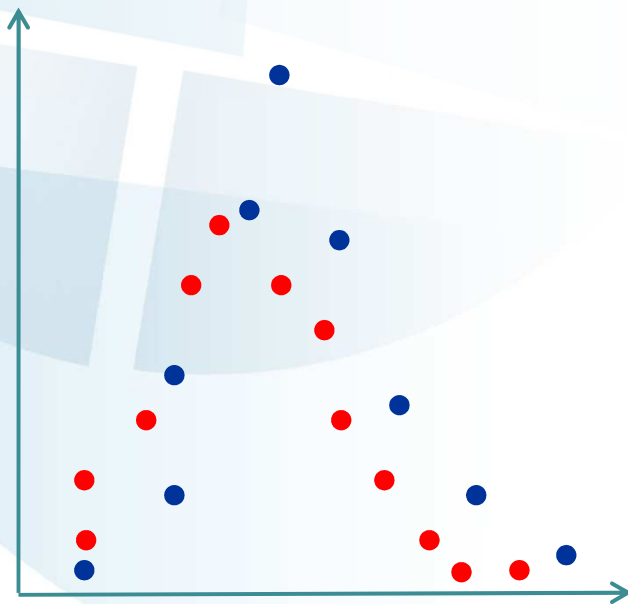
The error of approximation is 1% on average and does not exceed 6%!



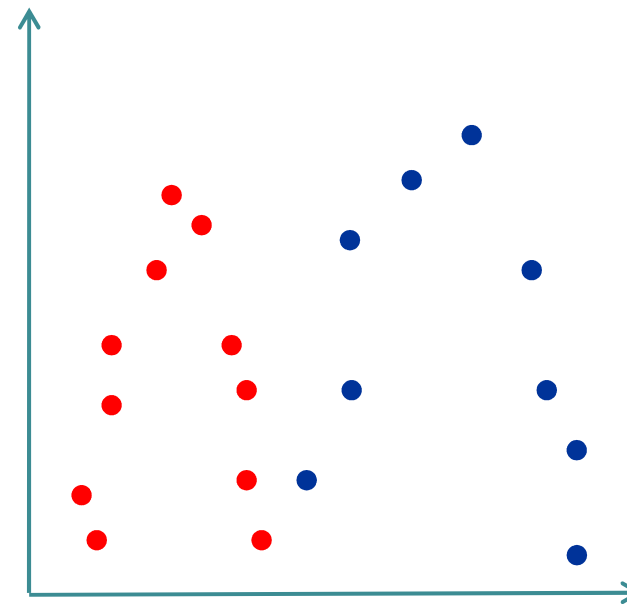
The blue, red and green bars represent relative errors in terminal GDP for 50 testing controls in case the learning database consists of 10, 50 and 100 controls correspondingly (IIASA project on integration of models)

Distribution-based approach

- Compare the distributions of models' outputs with the joint distribution => in case the **joint distribution** has **lower variance**, use its expectation



Lower joint variance => compatible models

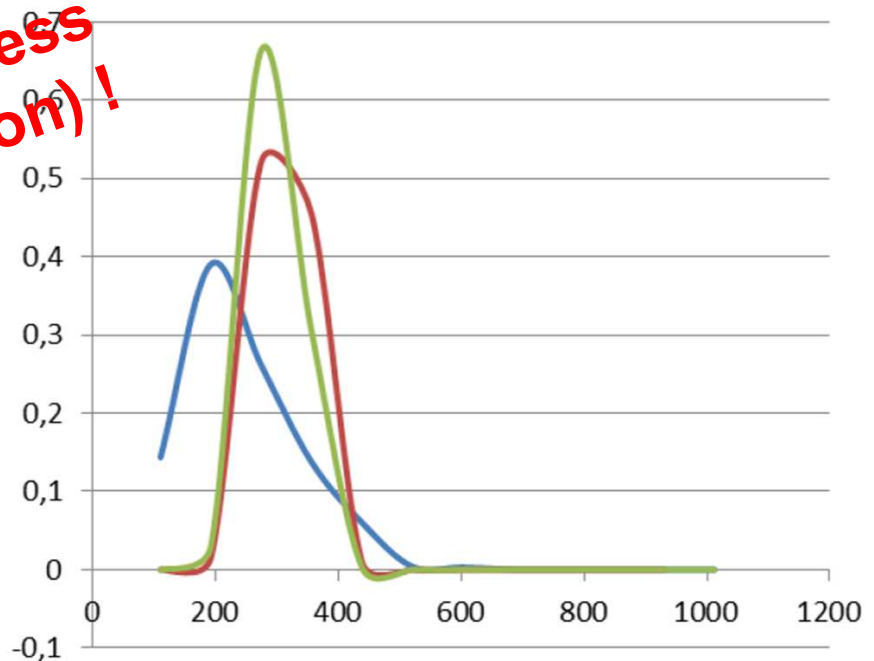


Higher joint variance => incompatible models

Example

- Integration of the Landscape Ecosystems Approach (LEA) and Stochastic Modeling Approach (SMA) of net primary production of the Russian forest-tundra

The integrated distribution is less uncertain (has smaller variation)!



The blue and red curves show the NPP distributions (in grams of carbon per square meter per year) given by LEA and SMA, respectively. The green curve shows the integrated distribution formed using the posterior integration analysis technique (**IIASA YSSP project on integration of models**)

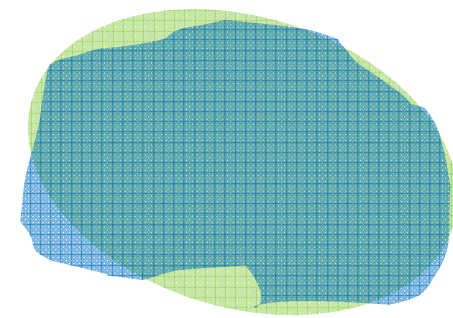
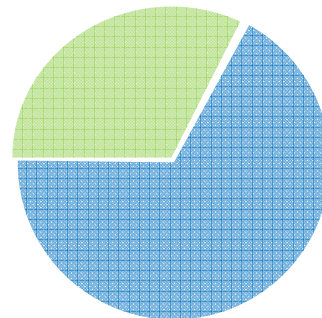
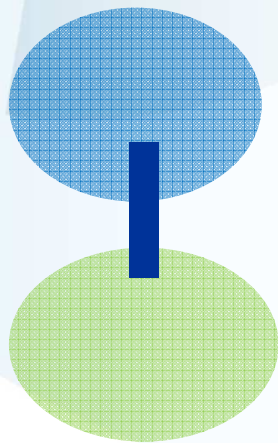
“Calculus of models”

- Objects:

models

- Actions:

linking (IAM), integration, approximation,...



THANK YOU FOR YOUR ATTENTION!

**I welcome your comments,
suggestions, ideas...
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