# Dynamics of optimal growth: economic drivers and long-term scenarios

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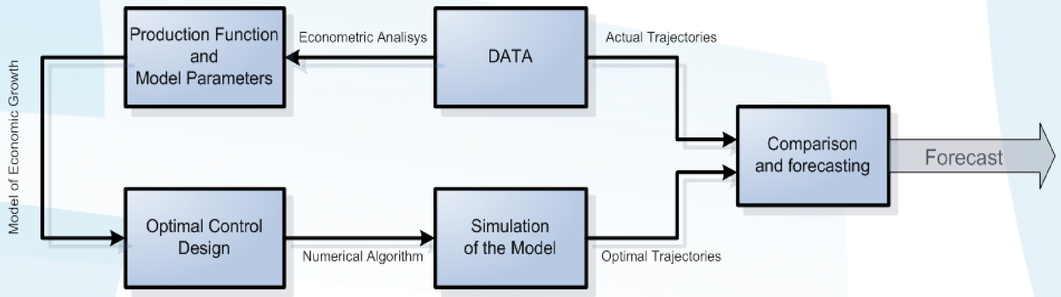
## Introduction

The poster is devoted to studies in the framework of the Economic Growth Project at IIASA.

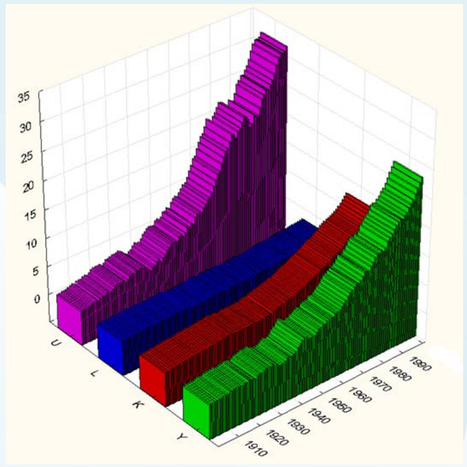
The research deals with the interdisciplinary approach for constructing optimal trajectories of growth based on the analysis of real time-series. The background of the study is the following:

- economic growth theory (Arrow, Solow, Shell);
- optimal control theory (Pontryagin's maximum principle for problems on infinite horizon);
- econometric analysis of the model;
- numerical simulation and forecasting of future scenarios.

## Methodology of the Research




## Application to Country-Specific Data




Normalized Data for the US Economy

| 1900 | | | |
|---|---|---|---|
| GDP | Capital | Labor | Useful work |
| $ 354 billion | $ 2012 billion | Index of hours worked | 0.64 EJ |

### Production Functions

- Cobb-Douglas Production Function
- Linear-Exponential Production Function

developed by Prof. Robert U. Ayres and Dr. Benjamin Warr

"*Accounting for growth*: The role of physical work." (2005).

### Optimal Control Problem

$$J = \int_0^{+\infty} \left[\ln f(k(t)) + \ln(1 - s(t))\right] e^{-\delta t} dt \to \max$$

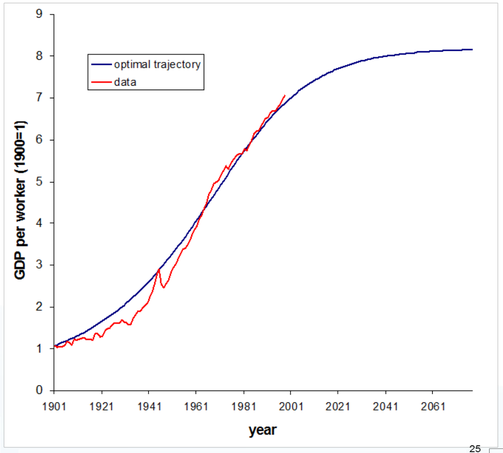
$$\dot{k} = s\, f(k) - \lambda k$$

$$k(0) = k^0 \qquad s \in [0, a] \qquad a < 1$$

$\delta$ is a discount parameter

The problem is to find the optimal investment level $s^0(\cdot)$ and the corresponding trajectory of the capital per worker stock $k^0(\cdot)$ for maximizing the consumption per worker functional.

## Results and Verification



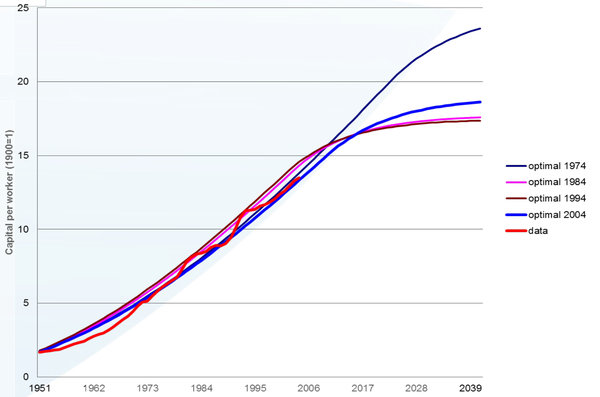

Forecast of optimal growth for the US

The optimal trajectory is constructed as the solution to the corresponding optimal control problem on infinite horizon using the elaborated numerical algorithm.

The synthetic trajectory is scaled in real time for comparison with historical data.

Using the ASA developed STARK Pilot toolbox the user can obtain scenarios of optimal growth for various data sets, time periods and input parameters.

The methodology is verified by a sequential analysis of historical data.




Sequential forecasts for the UK