A DEMOECONOMIC MODEL OF INTERREGIONAL GROWTH RATE DIFFERENCES

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FOREWORD

Declining rates of national population growth, continuing differential rates of regional economic activity, and shifts in the migration patterns of people and jobs are characteristic empirical aspects of many developed countries. In several instances, they have combined to bring about a relative (and in some cases absolute) population decline of highly urbanized areas, e.g., New York, Tokyo, and Stockholm. In other cases, they have brought about rapid metropolitan growth, e.g., Houston, Miami, and Moscow.

The objective of the Urban Change Task in IIASA's Human Settlements and Services Area is to bring together and synthesize available empirical and theoretical information on the principal determinants and consequences of such urban growth and decline.

This paper argues for a demoeconomic modeling of multiregional systems. It proposes a model that accounts for interregional growth rate differences by means of an endogenous and simultaneous determination of labor force participation, migration, and unemployment.

A list of related publications in the Urban Change Series appears at the end of this paper.

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A Demoeconomic Model of Interregional Growth Rate Differences

One of the most interesting models of interregional growth is that of Dixon and Thirlwall [2] (hereafter referred to as DT). They attempt to formalize Kaldor's thoughts on development along non-neoclassical lines. Their formal model includes a price markup equation, in place of a marginal-cost-determined competitive price, as well as a positive feedback between the region's rate of technical innovation and regional economic growth rates (the Verdoorn effect). Competition between a pair of regions is accommodated by a relationship between relative regional prices and export demand.

The DT model is useful for studying the possibilities of income divergence or convergence between regions over the long term. Yet the model is linear in the rates of change of all included variables and, not surprisingly, yields an outcome of stable growth rates in the long run. DT cite this as an example of equilibrium characterized by an absence of divergence or convergence. Their conclusion actually describes just one special sort of equilibrium. Recent debates on regional convergence and divergence look at long-term trends in income levels rather than their growth rates [*1*]. Since stable growth rates for a pair of regions can easily be associated with an ever-widening divergence of incomes, we do not necessarily expect the DT result to be a long-term equilibrium: given enough of an income gap, people will move from the poor to the rich region.

This brings us to the second point, which has to do with the secondary equilibrating *and* disequilibrating effects of migration. Simple models of factor price equilization cite the migration response as an equilibrating force that puts a brake on interregional income divergence. Yet, over shorter time spans, migration may well have an agglomerative effect (for example, only the most skilled and non risk averse may migrate) that accelerates income divergence. Thus, we claim that the stable growth equilibrium that DT cite is due not only to the linearity of their model, but also to the omission of a demographic sector.

In order to put this assertion into focus, we suggest the following: first, an interesting model of interregional development might be demoeconomic (i.e., include both demographic and economic aspects of development); second, such a

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demoeconomic model cannot be totally linear in the rates of change; and third, nonstatic long-term rates of change should emerge from the simulation of such a model. This means that, as a consequence of the introduction of migration, regional fluctuations (cycles) accompanied by cycles of divergence and convergence of incomes should appear.

To recapitulate,

1. DT should not be surprised that their *linear* model leads to constant growth rates in the long run.
2. They should not confuse steady growth with an absence of divergence or convergence of incomes.
3. The implausibility of the DT result (steady growth by a pair of regions over the long term) reflects the absence of migration; this suggests an approach such as the demoeconomic approach.
4. The migration response would eventually dampen the implied income divergence.
5. The proper modeling of demoeconomic effects introduces nonlinearities.

Our objective in this paper is to demonstrate these points with the help of an interregional demoeconomic extension of the DT model, which constitutes a useful reference point from which to proceed along the non-neoclassical path.

Beyond the specific model that is developed in the following pages, we also hope to outline the demoeconomic approach. Because economic and demographic variables interact, regional models that are either purely economic *or* demographic in nature may not be satisfactory. Yet the demoeconomic synthesis is not trivial. Looking at the labor market in spatial terms, we treat the decision to migrate as endogenous; this extends the notion of job search [*4*]. The central idea is that labor force participation, migration, and unemployment rates are endogenous and simultaneously determined. Yet it has been shown [*3*] that a model including variables of this sort is likely to generate preposterous unemployment and/or labor force participation rates unless the relationship between comparable variables of the economic and demographic sides (employment and labor force, respectively) is properly modeled. This is referred to as the consistency problem, which is particularly acute if unemployment and labor force participation rates are defined as residuals. Also, when these variables are dependent, a linear model eventually develops population and labor force dimensions that imply unrealistic unemployment and labor force participation rates. This suggests that a demoeconomic model must be nonlinear.

In the next section, we present an augmented DT model, along demoeconomic lines. We then specify reasonable parameter values for the two-region case and suggest that the results of a long-term simulation of the expanded model are much more plausible than the growth equilibrium of DT. Finally, we comment on some of the costs and benefits of our approach to regional analysis.

FORMULATION OF THE MODEL

In what follows, we present a two-region model that extends the DT model by allowing migration between the two regions.

It consists of three blocks, which describe successively: (a) the impact of demographic forces on regional income growth rates, (b) the impact of economic forces on regional population growth rates, and (c) the relationships linking employment and labor force variables, ensuring the consistency between the economic and demographic sides of the model.

The first equation of the first block relates a two-element vector of regional income growth rates to the growth in the region's exports as well as in the region's population and labor force. The export-base approach was suggested by DT. We add the other elements to bring in the impact of demographic factors on growth, emphasizing the role of households as consumers as well as of suppliers of labor. Thus

$$(g_t) = \Gamma(x_t) + \phi^1(n_t) + \phi^2(l_t + n_t), \tag{1}$$

where

(g_t) = vector of regional growth rates
(x_t) = vector of export growth rates
(l_t) = vector of labor force participation rate changes
(n_t) = vector of population growth rates
Γ, ϕ^1, and ϕ^2 = diagonal matrices of coefficients.[1]

The second relationship expresses the growth of exports in terms of changes in relative prices and world demand:

$$(x_t) = \eta(p_t) + z\epsilon(i), \tag{2}$$

where (p_t) is the vector of regional export price changes, (i) is a two-element vector of ones, and z is the change in world demand. Note that ϵ is a diagonal matrix of coefficients, unlike η, whose off-diagonal elements represent the impact of a region's price change on the growth of the other region's exports.

Prices are explained by a cost markup equation, just as in the DT paper:

$$(p_t) = (w_t) - (r_t) + (\tau), \tag{3}$$

where (w_t) is the vector of regional wage rate changes, (r_t) is the vector of regional rates of technological change, and (τ) is the exogenous vector of regional rates of change of cost markup.

Equation (4) explains regional technical innovation in terms of an endogenous and exogenous element.

$$(r_t) = (\bar{r}) + \lambda(g_t), \tag{4}$$

where $(\bar{r})$ is the vector of the exogenous elements and λ is a diagonal matrix of coefficients. Just as in the DT paper, the second term represents the Verdoorn effect.

At this point, it may be noted that substituting (4) into (3) and the result into (2) reveals a particular impact of one region's growth on the other region's export growth. This reflects a competitive effect, in that growth in region i diminishes the export demand growth of region j through an impact on relative export prices. Another growth effect on export demand growth could be included with a positive impact via the traditional income-consumption linkage. Clearly, the two effects work in opposite directions and are of different magnitudes. In the former case we emphasize competition between regions and in the latter case we emphasize trade. The two cases are probably differentiable in terms of the sizes of the regions vis-à-vis rest-of-the-world demand. For the sake of continuity and simplicity, we retain the (implicit) small-but-competitive region example of the DT model. Also,

[1]Because all the variables are expressed in their growth rates, the coefficients are elasticities.

we wish to highlight the demoeconomic effects and it makes no difference which case is studied to make that point.

Equation (5) concerns the wage rate, which, unlike DT, we chose to make partially endogenous:

$$(w_t) = (\bar{w}) + \psi_t(l_t) . \tag{5}$$

A time subscript is attached to the diagonal matrix ψ_t because its elements, representing each region's wage elasticity with respect to labor force participation rate (LFPR) are not taken as constants. It is hypothesized that the absolute value of each element ψ_{it}, which has a negative sign, increases with the value of the beginning-of-period LFPR. Thus, supposing in addition that each region's labor force participation rate can take on values within a range of (ρ^1, ρ^r)—where ρ^1 is a LFPR low enough to have no impact on wage rate change and ρ^r is a LFPR high enough to have an infinite impact on wage rate change—we have

$$\psi_{it} = d_i[(\rho_{it} - \rho^1)/(\rho_{it} - \rho^r)]; \tag{6}$$

or, in compact form,

$$\psi_t = \mathbf{D}(\rho_t - \rho^1\mathbf{I})(\rho_t - \rho^r\mathbf{I})^{-1}, \tag{6'}$$

where ρ_t is a diagonal matrix of the beginning-of-period LFPR, **I** is the two-by-two identity matrix, and **D** is a diagonal matrix of coefficients.

The last equation of the first block relates a region's rate of income growth to its rate of change in employment level.

$$(e_t) = \mu(g_t), \tag{7}$$

where (e_t) is the vector of regional employment growth rates, and μ is a diagonal matrix of coefficients. Note that the rationale for this equation is the availability of an economic variable directly comparable with a variable from the demographic side (labor force) to ensure the aforementioned consistency.

The next block of the model describes the impact of economic forces on population growth through migration. The demographic model underlying this block is the so-called components-of-change model of population growth and distribution [7].

$$N_{i,t+1} = N_{it} + b_i N_{it} - m_{it} N_{it} + m_{jt} N_{it}, \tag{8}$$

where N_{it} is population in region i at time t, b_i is region i's exogenous rate of natural increase, and m_{it} is the migration rate from region i to region j in period $(t, t + 1)$. Rewritten, this relationship yields

$$n_{it} = \frac{N_{i,t+1} - N_{it}}{N_{it}} = b_i - m_{it} + m_{jt}\frac{N_{jt}}{N_{it}}, \tag{8'}$$

or, in a more compact form,

$$(n_t) = (b) - \mathbf{N}_t^{-1}\mathbf{P}\mathbf{N}_t(m_t), \tag{8''}$$

where (n_t) is the vector of regional population growth rates, $\mathbf{P}$ is the matrix $\left[\begin{smallmatrix} 1 & -1 \\ -1 & 1 \end{smallmatrix}\right]$, and (b), (m_t), and $\mathbf{N}_t$ are vector or matrix equivalents of previously defined variables.

To assure a demoeconomic model, it is necessary to specify the way in which economic forces cause migration rates to change. We suggest that

$$m_{it} = \alpha_i \frac{N_{it}}{N_{it} + N_{jt}} \left[1 + \beta_i \left(\frac{e_{jt}}{u_{jt}} - \frac{e_{it}}{u_{it}} \right) \right]. \tag{9}$$

That is, the migration rate out of each region is proportional to the attractiveness of the other region—measured by the part of the total population living in this region—and is related to the difference in the economic opportunities offered by the two regions. Note that the index of regional economic opportunities used here is a slight variation of Todaro's probability that a migrant finds a job [8]; it is the ratio of employment growth rate e_{it} to the beginning-of-period unemployment rate u_{it} (defined below). Equation (9) can be rewritten in a more compact form as

$$(m_t) = \frac{1}{N_{\cdot t}} \alpha \mathbf{N}_t \left[(i) - \beta \mathbf{P} \mathbf{u}_t^{-1} (e_t) \right], \tag{9'}$$

where $N_{\cdot t}$ is the total population of the system at time t, α and β are diagonal matrices of coefficients, and $\mathbf{u}_t$ is the matrix of regional unemployment rates at time t.

The last block of the model defines the labor force and unemployment variables. The first equation of this block posits a behavioral basis for the change in LFPR

$$(1_t) = \gamma_t (\mathbf{I} - \mathbf{u}_t)^{-1} \left[(u_{t+1}) - (u_t) \right], \tag{10}$$

in which γ_t is a diagonal matrix introducing further nonlinearity into the model. It is hypothesized that the value of each element γ_{it}, which, by the way, has a negative sign, is smaller when the unemployment rate takes on extreme values, either low or high, and much larger for unemployment rate values intermediate between those extremes:

$$\gamma_{it} = a_i (u_{it} - u^1)(u_{it} - u^r), \tag{11}$$

where u^1 and u^r are the extreme values of the range in which u_{it} falls, and, in more compact form,

$$\gamma_t = \mathbf{A} \, (\mathbf{U}_t - u^1 \mathbf{I})(\mathbf{U}_t - u^r \mathbf{I}), \tag{11'}$$

where $\mathbf{A}$ is a diagonal matrix of coefficients.

The last equation of this block is the relationship

$$(e_t) = (l_t) - (\mathbf{I} - \mathbf{U}_t)^{-1} \left[(u_{t+1}) - (u_t) \right] + (n_t), \tag{12}$$

obtained by differentiating (logarithmically) the identity relating employment levels (E_t) and population levels (N_t); i.e.

$$(E_t) = \rho_t (\mathbf{I} - \mathbf{U}_t)(N_t) \,. \tag{13}$$

As shown in the Appendix, various substitutions permit one to reduce each of the three blocks of the system to a single equation in three variables (e_t) [or (g_t)], (l_t), and (n_t). This leads to a simple model of three equations in three unknowns that can be analytically solved in spite of the nonlinearities introduced into the model: the derivation of the reduced-form equations of the model here is tractable because the coefficients of the endogenous variables are known variables (either constant or depending on lagged variables).

It is clear, from these reduced-form equations, that the introduction of the equations of population change have added difference equations that make the model much more dynamic than the DT model. Also, a radical departure from linearity has been introduced in the process.

SIMULATION OF THE MODEL

From the three reduced-form equations concerning (e_t), (l_t), and (n_t), it is easy to develop a simulation of the time paths of these variables and then of all the other variables. For maximal policy interest, the simulation was conducted for a hypothetical pair of regions where one is economically advanced and the other is developing. As already mentioned, these are competing regions whose primary trade is with the rest of the world.

It will be seen that the resulting time paths of growth rate changes fluctuate over patterns of convergence *and* divergence. As suggested at the outset, since nonlinearities and a migration response have been added to the DT model, we would not expect anything like steady-state growth rates and the associated diverging regional income levels. Though our results simply indicate a simulation result, we have based the simulation on reasonable assumptions and parameter choices. In defending this sort of approach to model building, Nelson and Winter assert that "Simulation . . . can be a useful adjunct to an analytical approach. It can establish, with the same finality as a theorem, the logical consistency of the model's assumptions with a set of proportions about its behavior. And while it offers a way around the tractability constraints of analytic methods, it imposes its own constructive discipline of modeling dynamic systems: the program must contain a complete specification of how the system at $t + 1$ depends on that at t and exogenous factors, or it will not run" [5, p. 272]. The earlier discussion on labor force participation rates reflects precisely this point. The problems cited were not evident in the original DT model and only became apparent once the long-term demoeconomic interactions were modeled and simulated.

Our results, as indicated, follow from defensible values of the parameters. Table 1 provides a summary of these values, many of which are similar in order of magnitude to those employed by DT. The export elasticity with respect to regional income growth is lower in the developing region (region 2) because a younger region is usually more trade dependent, causing smaller internal foreign trade multiplier effects. The elasticity of regional population growth with respect to income growth is slightly larger in the developing region, suggesting that the developing region has greater (dynamic) opportunities for import substitution.

All price elasticities of export demand are greater, in absolute value, than unity. In fact, DT invoke values of 1.5 for these, as we do. The justification for a price elasticity in the elastic part of the demand curve rests on the small region (vis-à-vis the rest of the world) assumption: as the region's export price rises by 1 percent, the demand for its exports falls by about 1.5 percent. Yet, since the cross-elasticities are also elastic, this assumption must be tempered. Since any price increase is met by a fall in "own" demand and an almost equivalent rise in the competing region's demand, we have the case of close substitutability of the export, most of which is supplied by these two regions.

TABLE 1
Summary of Parameter Values and Initial Conditions

Parameter	Advanced Region (Region 1)	Developing Region (Region 2)
Elasticities		
Elasticity of export growth wrt income growth (1)*	$\gamma_1 = 0.60$	$\gamma_2 = 0.55$
Elasticity of population growth wrt income growth (1)	$\phi_1^1 = 0.65$	$\phi_2^1 = 0.70$
Elasticity of labor force growth wrt income growth (1)	$\phi_1^2 = 0.10$	$\phi_2^2 = 0.10$
Price change elasticity wrt export growth (2)	$n_{11} = -1.50$	$n_{12} = 1.50$
	$n_{21} = 1.50$	$n_{22} = -1.50$
Elasticity of world demand change wrt export growth (2)	$\varepsilon_1 = 1.00$	$\varepsilon_2 = 1.10$
Elasticity of income growth wrt technological change (4)	$\lambda_1 = 0.50$	$\lambda_2 = 0.70$
Elasticity of income growth wrt employment growth (7)	$\mu_1 = 0.30$	$\mu_2 = 0.40$
Other Coefficients		
Coefficient in determination of elasticity of labor force participation rate change wrt wage rate change (6)	$d_1 = 3.00$	$d_2 = 2.00$
Coefficients in determination of the migration rates (9)	$\alpha_1 = 0.0700$	$\alpha_2 = 0.0725$
	$\beta_1 = 0.25$	$\beta_1 = 0.30$
Coefficient in determination of elasticity of unemployment rate change wrt labor force participation rate change (11)	$a_1 = 6000$	$a_2 = 3000$
Other Parameters		
Price markup factor (3)	$\tau_1 = 0.015$	$\tau_2 = 0.015$
Exogenous rate of technological change (4)	$\bar{r}_1 = 0.025$	$\bar{r}_2 = 0.025$
Exogenous element of the wage growth rate (5)	$\bar{w}_1 = 0.015$	$\bar{w}_2 = 0.015$
Rate of natural increase (8)	$b_1 = 0.01$	$b_2 = 0.013$
Initial Conditions		
Initial population (in thousands)	$N_{10} = 7{,}500$	$N_{20} = 2{,}500$
Initial unemployment rate	$u_{10} = 0.05$	$u_{20} = 0.035$
Initial labor force part. rate	$\rho_{10} = 0.35$	$\rho_{20} = 0.37$
Nonregionalized Parameters		
Bounds on labor force part. rate (6)	$\rho^l = 0.30$ $\rho^r = 0.42$	
Bounds on unemployment rate (11)	$u^l = 0$ $u^r = 0.10$	
Rate of change of world demand (2)	$z = 0.04$	

*The numbers refer to the equations in which the parameters are used.

The next difference in parameter values involves the elasticity of world demand change with respect to export growth. This parameter is larger for the growing region, showing a greater orientation to external demand. Also, regional growth has a stronger effect on induced innovation in the younger region, which has far less durable capital to depreciate before innovation can proceed.

Employment growth is more sensitive to economic development in region 2 ($\mu_2 > \mu_1$), since it is entirely plausible that growth in that region would include labor-intensive processes.

The coefficient d_i in equation (6) has a greater value for the advanced region. This means that the elasticity of wage rate change with respect to labor force participation rate change is *more* sensitive to fluctuations in the levels of the LFPR in the advanced region. At the same time, market institutions in the advanced economy may be more developed, permitting greater scope in these wage adjustments or less wage rigidity than in the traditional but emerging region.

Perhaps the most important of these institutional differences is in information channels that underlie the labor market and aid the job search process.

The outmigration rates from the developing region are thought to be slightly more sensitive to economic conditions, since the younger population of that region is probably made up of more economic opportunity seekers. Thus, $\alpha_2 > \alpha_1$ and $\beta_2 > \beta_1$.

Turning to equation (11), the coefficient a_i is significantly larger for the first region. This is because the labor force participation rate varies more in a region where pensions and other nonlabor incomes are possible. In other words, the more advanced region is thought to have a social service apparatus that makes leaving the labor force more plausible. The rate of natural increase is, of course, slightly larger in the developing region with its younger population. The remaining regional parameters are common to the two regions.

Turning to the initial conditions, the older region has three times the population of the developing region. Its initial unemployment rate is larger and its labor force participation rate is lower because its population contains more older people. The bounds on the labor force participation, and unemployment rates used in the formulation of the nonlinear equations (6) and (11) are the same in the two regions.

Finally, the rate of change of world demand that drives the model is taken equal to 4 percent, as in the DT model. Results of the simulations are shown in Figures 1 through 4. In discussing these results of the simulation, it is difficult to identify simple cause-and-effect relationships because of the large number of second-order effects. Most important among these are the interregional feedback effects. Also, since migration and population levels appear as independent as well as dependent variables throughout the model, it is almost impossible to isolate the causal influences on net migratory flows; while migration is responding to economic conditions, it is also fostering many of these conditions.

It is important to note that the model does generate oscillations in many of the important growth rates (such as output, employment and population). The same applies to the growth rates of the labor force participation rate, which peaks in region 1 between the fifth and the eleventh time periods and hits lows in region 2 between 75 and 90, and again at the end of the simulation.

Figures 3 and 4 show that the actual labor force participation and unemployment rate levels for region 2 fluctuate. Moreover, both regions' rates stay within ranges of values that are entirely reasonable and also consistent. Thus, although

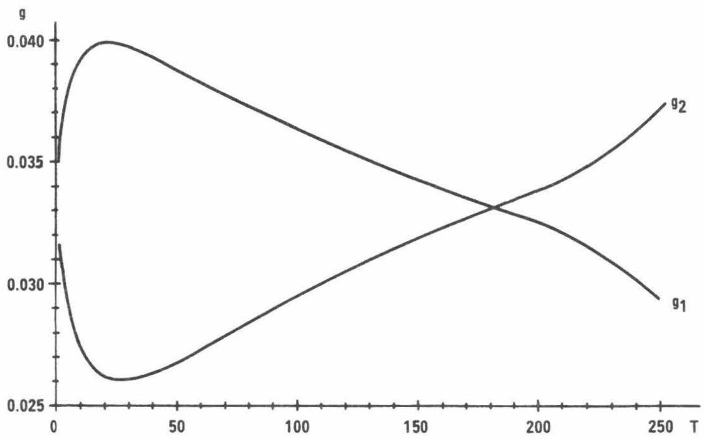

Fig. 1. Evolution of the Annual Income Growth Rates

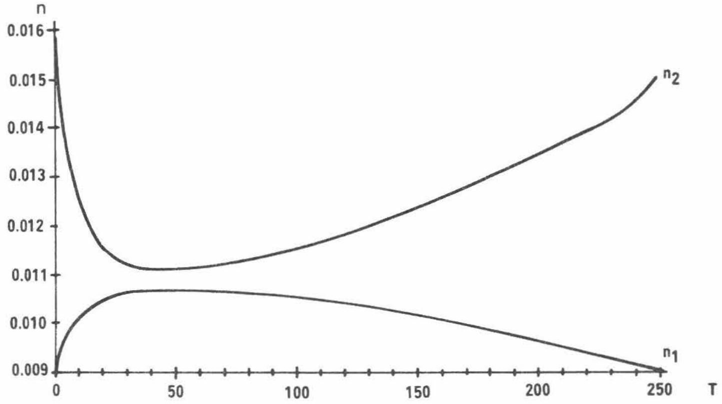

Fig. 2. Evolution of the Annual Population Growth Rates

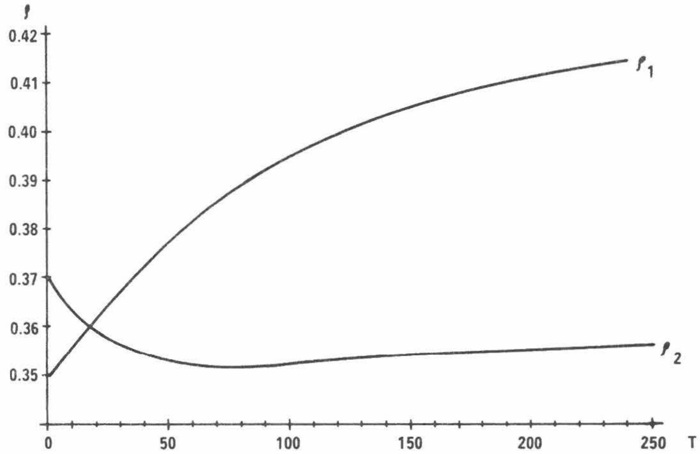

Fig. 3. Evolution of the Labor Force Participation Rates

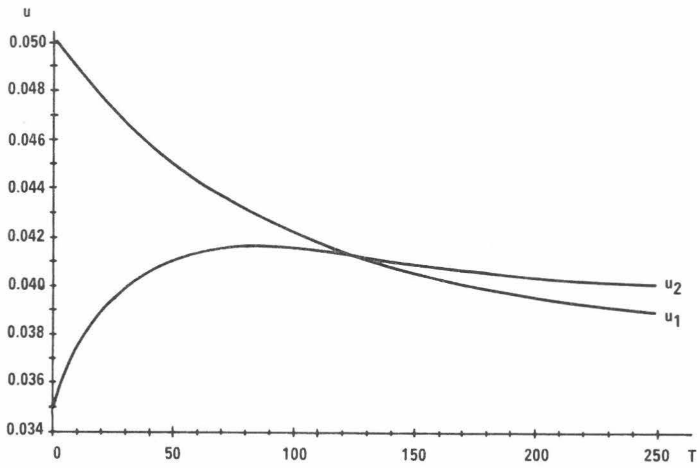

Fig. 4. Evolution of the Unemployment Rates

actual levels of population, employment, and labor force increase regularly, labor force participation and unemployment rates do not take on implausible values.

Net migration oscillates too (Fig. 5). Initially, there is a net flow of migrants from the advanced to the developing region in which employment opportunities are better (higher employment growth, lower unemployment rate). But as employment opportunities worsen in the developing region this flow tends to diminish, leading to a reversal in the direction of the net flow of migrants between the two regions. But toward the end of the simulation, the developing region regains a better position and the direction of the net migration flow is once more reversed.

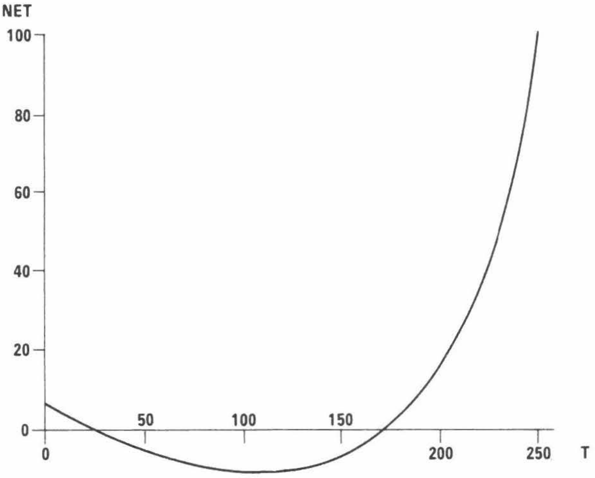

Fig. 5. Evolution of the Net Migration Flow from Region 2 to Region 1

In order to show that the direction of the net flow of migrants depends on the relative economic conditions of both regions, we can, from equation (9), formulate an expression for the net migratory flow from region 1 to region 2. Substituting (9) into the identity $NET_t = m_{it}N_{it} - m_{jt}N_{jt}$ leads to

$$RNET_t = \frac{N_{it}N_{jt}}{N_{\cdot t}}\left[\alpha_i - \alpha_j + (\alpha_i\beta_i + \alpha_j\beta_j)\left(\frac{e_{jt}}{u_{jt}} - \frac{e_{it}}{u_{it}}\right)\right]. \tag{9a}$$

Thus, there is a net flow of migrants from the advanced region to the developing region as long as the difference between the two regional indices appearing in (9a) remains higher than $[(\alpha_j - \alpha_i)/(\alpha_i\beta_i + \alpha_j\beta_j)]$, i.e., 0.064. Yet it must be recalled that, because of the effect of migration on regional population growth and because of its effect on regional output growth (equation (1)), we have a more complex situation than (9a) might imply. In fact, as we have seen, the oscillation of net migration is a response to, as well as a cause of, other fluctuations.

To summarize, the results of our simulation show that the introduction of migration into the DT model—which has been the impetus for a nonlinear formulation—has released us from the inexorable divergence of the DT model. Thus, the demoeconomic approach suggests the possibility of fluctuations. As shown above, these fluctuations are of a particularly long-run periodicity, which agrees with Richardson's claim [6] that regional growth trends change much more slowly than planners had heretofore anticipated.

There is a final point worth noting in this connection. The simulation of our extended DT model was performed over a 250-year period, thus covering less than a full period of oscillation and disregarding long-term behavior. However, this single and incomplete simulation has provided us with the desired counter-example to the DT result of steady-state growth that we sought to establish in this paper.

ADVANTAGES AND DISADVANTAGES OF THE DEMOECONOMIC APPROACH

Linearity and tractable reduced-form results, as obtained by DT, are unlikely. That is, we should not expect any two regions to settle on steady-state growth rates over the long term. Our demoeconomic model shows that this will not occur. We have seen that it is useful to relax some of the linearity of the DT model because steady-state growth of employment and population could distort the labor force participation rate, which is often defined as a residual quantity. As usual, we pay for an increment in realism by surrendering an amount of simplicity.

In addition, the inclusion of a transition matrix from interregional demography necessarily introduces difference equations; any demoeconomic model would have to be dynamic. This is surely a benefit, as is the notion that, rather than taking migration rates as fixed, we make them endogenous. In fact, the model allows us to observe how migration rates and labor force participation rates interact with each other and with unemployment rates. This allows for a superior analysis of labor markets (it makes them spatial) and job search.

The model did not deal in terms of age-sex specific breakdown of cohorts, and we did not model the effect that changes in the age composition would have on the economic variables. That would be the obvious next step. The population does age inexorably and this momentum has well-known economic consequences. The demoeconomic approach has the potential for introducing age-sex detail into regional economics. Just as regional economists prize the sectoral detail of input-output model results, so ought they to value demographic detail. For example, such detail can give policymakers some idea of how formidable a task regional development or revival are likely to be in specific regions.

Finally, by the proper choice of regions, even the parameters of natural population growth can be made endogenous. What this means is that, since the demographic transition seems to be a function of urbanization and since urbanization is endogenous in a demoeconomic model that happens to deal with an urban and rural region (or regions), the natural rate of increase could be made endogenous.

All of this appears to be an important break with the type of regional modeling that has been done heretofore. We hope that the next few years will witness increasing interest in regional and interregional demoeconomics.

APPENDIX

Derivation of the Solution of the Model

Combining equations (1) through (7) of the first block leads to

$$\mathbf{E}(e_t) = (h) + \mathbf{F}(n_t) + \mathbf{G}_t(l_t), \tag{A1}$$

in which

$$\mathbf{E} = [\mathbf{I} + \Gamma\eta\lambda]\, \mu^{-1}$$

$$\mathbf{F} = \phi^1 + \phi^2$$

$$\mathbf{G}_t = \Gamma\eta\mathbf{D}(\rho_t - \rho^l\mathbf{I})(\rho_t - \rho^r\mathbf{I})^{-1} + \phi^2$$

$$(h) = \Gamma\{\eta[(\bar{w}) - (\bar{r}) + (\tau)] + z\mathbf{E}(i)\}\ .$$

In the second block [equations (8″) and (9′)], by substituting (9′) into (8″), we have

$$(n_t) = (k_t) + \mathbf{J}_t(e_t), \tag{A2}$$

in which

$$\mathbf{J}_t = \frac{1}{N_{\cdot t}}\ \mathbf{N}_t^{-1}\ \mathbf{P}\mathbf{N}_t\alpha\mathbf{N}_t\beta\mathbf{P}\mathbf{u}_t^{-1}$$

$$(k_t) = (b) - \frac{1}{N_{\cdot t}}\ \mathbf{N}_t^{-1}\ \mathbf{P}\mathbf{N}_t\alpha\mathbf{N}_t(i)\ .$$

Finally, the third block [equations (10), (11′) and (12)] yields

$$(e_t) = (n_t) + \mathbf{M}_t(l_t), \tag{A3}$$

in which

$$\mathbf{M}_t = \mathbf{I} - [\mathbf{U}_t - u^r\mathbf{I})(\mathbf{U}_t - u^l\mathbf{I})^{-1}\mathbf{A}^{-1}\ .$$

Thus, our demoeconomic model reduces to a three-equation system in three unknowns such that the coefficients of the endogenous variables are known in each period: they are either constant (independent of time) or depend on lagged variables. Then, by combining (A1) through (A3), it is simple to obtain the three reduced-form equations of the model concerning (e_t), (n_t), and (l_t):

$$(e_t) = [\mathbf{E} - \mathbf{F}\mathbf{J}_t - \mathbf{G}_t\mathbf{M}_t^{-1}(\mathbf{I} - \mathbf{J}_t)]^{-1}[(h) + (\mathbf{F} - \mathbf{G}_t\mathbf{M}_t^{-1})(k_t)] \tag{A4}$$

$$(n_t) = \mathbf{J}_t[\mathbf{E} - \mathbf{F}\mathbf{J}_t - \mathbf{G}_t\mathbf{M}_t^{-1}(\mathbf{I} - \mathbf{J}_t)^{-1}$$
$$[(h) + (\mathbf{E} - \mathbf{G}_t\mathbf{M}_t^{-1})\mathbf{J}_t^{-1}(k_t)] \tag{A5}$$

$$(l_t) = \mathbf{M}_t^{-1}[\mathbf{I} - \mathbf{J}_t][\mathbf{E} - \mathbf{F}\mathbf{J}_t - \mathbf{G}_t\mathbf{M}_t^{-1}(\mathbf{I} - \mathbf{J}_t)]^{-1}$$
$$[(h) + (\mathbf{F} - (\mathbf{E} - \mathbf{F}\mathbf{J}_t)(\mathbf{I} - \mathbf{J}_t)^{-1})(k_t)]\ . \tag{A6}$$

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