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DO CITIES GROW BY NATURAL INCREASE OR BY MIGRATION?

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RR-80-24
June 1980

Reprinted from *Geographical Analysis*, volume 12(2) (1980)

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
Laxenburg, Austria

Reprinted with permission from *Geographical Analysis* 12(2):142–156, 1980.

FOREWORD

Roughly 1.6 billion people, 40 percent of the world's population, live in urban areas today. At the beginning of the last century, the urban population of the world totaled only 25 million. According to recent United Nations estimates, about 3.1 billion people, twice today's urban population, will be living in urban areas by the year 2000.

Scholars and policy makers often disagree when it comes to evaluating the desirability of current rapid rates of urban growth in many parts of the globe. Some see this trend as fostering national processes of socioeconomic development, particularly in the poorer and rapidly urbanizing countries of the Third World; whereas others believe the consequences to be largely undesirable and argue that such urban growth should be slowed down.

As part of a search for convincing evidence for or against rapid rates of urban growth in developing countries, the Population, Resources, and Growth Task in the Human Settlements and Services Area initiated in 1977 a research project to study the process of structural transformation in nations evolving from primarily rural–agrarian to urban–industrial societies. Data from several countries selected as case studies are being collected, and the research is focusing on spatial population growth and economic development, and on their resources and service demands.

This paper analyzes the urbanization of a national population that at first is entirely rural. The population is subjected to fixed rates of natural increase and net migration and the evolution of its urban and rural subpopulations is studied by means of a pair of differential equations. The analysis identifies the relative contributions of these two components of change to the urbanization process.

A list of papers in the Population, Resources, and Growth Series appears at the end of this publication.

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Do Cities Grow by Natural Increase or by Migration?

It has been argued that cities grow mostly by net in-migration, and it has also been argued that they grow mostly by their own natural increase. Kingsley Davis finds that cities in the European industrial revolution grew mostly by in-migration, but cities of the contemporary less-developed countries are growing mostly by their own natural increase [*1*, *6*]. The case for their growth by migration was expressed as far back as Süssmilch, who showed that deaths exceeded births in the principal cities of Europe, so that without in-migrants cities would decline. The same view has been developed by Fischer [2] and Wrigley [*12*]. Most recently Sharlin [*8*] finds the contrary for the cities of Europe. He adduces evidence that the population native to the city did replace itself, and that the excess of deaths over births that appeared in the overall statistics was due to temporary residents, especially tradesmen and servants who had no chance to marry but whose deaths would be counted as urban if they died in the city.

This paper studies the contributions of the two components of city growth under various hypothetical conditions. It starts from the simple notion that when there is no city population there can be no natural increase, and during the time after a city is established but still small, its births cannot be numerous. At the other extreme, when the country is mostly urbanized there is little rural population left to migrate to cities. Between these two extremes there must be a moment in the course of urban evolution when natural increase begins to exceed in-migration. The model that follows establishes this moment in terms of three parameters: urban rate of natural increase u, rural rate of natural increase r, and rate of net out-migration from the countryside m. The discussion starts with a special case in which $u = r$, then goes on to arbitrary fixed values of u, r, and m, and finally presents a result for arbitrary functions of time, $u(t)$, $r(t)$, and $m(t)$. The first case, with $u = r$, can be solved in commonsense terms; distinguishing u from r, the urban rate of population increase from the rural, requires a pair of differential equations.

Most of what follows consists in finding what would happen if various combinations of fixed rates persisted over a period of time. The results provide a certain kind of knowledge, analogous to that provided by stable population theory. They answer such questions as What would happen to the ratio of urban to rural population, and to the migration and natural increase components of city growth, if we abstract from

*I am grateful to William Alonso, Joel Bergsman, Kazimierz Dziewonski, Allen C. Kelley, S. Krishnamoorthy, Roman Kulikowski, Gary Littman, Zbigniew Pawlowski, Dimiter Philipov, Samuel Preston, Andrei Rogers, Leon Tabah, and Frans Willekens for helpful comments. The work was done at the International Institute for Applied Systems Analysis in Laxenburg, Austria.

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0016-7363/80/0480-0142$00.50/0
GEOGRAPHICAL ANALYSIS, vol. 12, no. 2 (April 1980)
Submitted 4/79. Revised version accepted 6/79.

all other circumstances? Empirical correlations do not abstract from other circumstances, but estimate the relations when these vary as they do in the real world. Thus, if x is an independent variable, y dependent, and z other conditions that are not explicitly recognized, then both empirical correlation and the mathematics of this paper seek y as a function of x. In the empirical study the unknown z varies as it does in the real world; in stable population theory and in this paper, on the other hand, the unmentioned z is implicitly taken as fixed.

NATURAL INCREASE EVERYWHERE THE SAME

Suppose natural increase r is the same in the city and countryside. Then the population $P(t)$ at time t in terms of the initial P_0 must be

$$P(t) = P_0 e^{rt}$$

for the country as a whole. Let the migration rate from rural to urban be m, taken as a fraction of the rural and compounded momently like r. Then the rural population is

$$P_r(t) = P_{r0} e^{(r-m)t} \tag{1}$$

and the urban population is the difference between these,

$$\begin{aligned} P_u(t) &= P_0 e^{rt} - P_{r0} e^{(r-m)t} \\ &= e^{rt}(P_0 - P_{r0} e^{-mt}) . \end{aligned} \tag{2}$$

The rate of increase of the urban population is

$$\frac{1}{P_u(t)} \frac{dP_u(t)}{dt} = d \ln P_u(t) = r + \frac{m}{(P_0/P_{r0})\, e^{mt} - 1} . \tag{3}$$

The first term of (3), r, is the natural increase of the city and the second is its increase through migration. By making the decomposition, and noting that for $m > 0$ the second term is a declining function of t, irrespective of r, we have shown that the part of the increase due to migration steadily declines, as common sense suggests.

A simple upper limit can be set on (3). Plainly P_0/P_{r0} must be at least unity, and e^{mt} must be at least $1 + mt$. Hence the migration term is at most

$$\frac{m}{(1 + mt) - 1} = \frac{1}{t} ,$$

and for m of the usual size will be considerably less than this. Thus if $P_0 = P_{r0}$ (i.e., the country starts wholly rural), for $m = 0.01$ the value of (3) at $t = 50$ is 0.0154 against $1/t = 0.02$. In short, even if the country starts entirely rural, by year t its rate of out-migration to cities will be less than $1/t$, provided r and m remain constant.

From (3) the expression for the ratio of migration to natural increase (still with $P_0 = P_{r0}$) is

$$R = \frac{m}{r(e^{mt} - 1)} , \tag{4}$$

whose steady decrease is shown in Table 1 for three values of m. It may or may not be obvious that the greater the value of the migration rate m the faster the migration component of city increase falls.

TABLE 1

Ratio R of a Rate of In-Migration to Rate of Natural Increase for Urban Population in a Country That Starts Wholly Rural, $r = 0.03$, from (4)

t	m		
	0.01	0.02	0.03
0	∞	∞	∞
10	3.17	3.01	2.86
20	1.51	1.36	1.22
30	0.95	0.81	0.68
40	0.68	0.54	0.43
50	0.51	0.39	0.29

THE CROSSOVER POINT

With fixed rates, a point exists where natural increase of the city becomes greater than in-migration. The natural increase of the city is $rP_u(t)$ and the in-migration is $mP_r(t)$, so the crossover is at the value of t for which

$$rP_u(t) = mP_r(t),$$

or from (1) and (2)

$$r\,(P_0e^{rt} - P_{r0}\,e^{(r-m)t}) = mP_{r0}\,e^{(r-m)t},$$

and if we assume that the entire population was initially rural so that $P_0 = P_{r0}$, then this is readily solved for t

$$t = (1/m)\ \ln\,[1 + (m/r)], \tag{5}$$

a value that can be denoted t_c, the crossover moment.

Table 2 shows some values of this expression. The more rapidly the population as a whole increases the sooner the crossover; more surprisingly, the larger the value of m, the fraction of the countryside migrating, the sooner natural increase exceeds migration as a factor.

The way in which t_c depends on the rate of migration can be found by differentiating in (5) with respect to m.

$$\frac{dt_c}{dm} = \frac{1}{m}\left[\frac{1}{r+m} - \frac{\ln\,[1+(m/r)]}{m}\right];$$

for $r = 0.03$, $m = 0.02$ this is

$$\frac{1}{0.02}\left[\frac{1}{0.05} - \frac{\ln 5/3}{0.02}\right] = -\ 277\ .$$

The derivative will almost always be negative, so the higher the migration rate the shorter the time to crossover, after which the natural increase of the city exceeds its in-migration. At the level of this illustration each 1 percent by which m is higher reduces the time to crossover by 2.77 years. The derivative, however, is sensitive to the value of the rate of natural increase.

TABLE 2

Values of the Function $t_c = (1/m)\ln[1 + (m/r)]$ for r, m from 0 to 0.04, Giving Years to the Crossover Point at Which Natural Increase Is Equal to Migration

	m				
r	0	0.01	0.02	0.03	0.04
0	—	—	—	—	—
0.01	—	69.3	54.9	46.2	40.2
0.02	—	40.5	34.7	30.5	27.5
0.03	—	28.8	25.5	23.1	21.2
0.04	—	22.3	20.3	18.7	17.3

Table 3 shows the trajectory for a country that starts with ten million people, all rural, using the values $r = 0.03$, $m = 0.02$. The natural increase comes to exceed migration in absolute terms where R falls below unity, which is after approximately the twenty-fifth year. This is the point where the ratio S of urban to rural is 0.65, 10 years before the urban population overtakes the rural and where $S = 1$, i.e., before the 50 percent point is reached in the scale of urbanization.

TABLE 3

Urbanization in a Hypothetical Country Starting with a Population of 10 Million, All Rural, in Which Population Increases at 3 Percent Per Year and 2 Percent of Rural Population Moves to a City Each Year (Hypothetical numbers in millions; $r = 0.03$, $m = 0.02$)

t Year	Total P_0e^{rt}	$P_r(t)$ Rural $P_0e^{(r-m)t}$	$P_u(t)$ Urban = Total Minus Rural	$S = \frac{P_u(t)}{P_r(t)}$ Ratio S = Urban/Rural	$\frac{mP_r(t)}{P_u(t)}$ Annual Rate of Urban In-Migration	Ratio R of Migration to Natural Increase in City Growth = Migration/0.03
0	10.0	10.0	0	0	0	0
5	11.6	10.5	1.1	0.11	0.190	6.34
10	13.5	11.1	2.4	0.22	0.090	3.01
15	15.7	11.6	4.1	0.35	0.057	1.91
20	18.2	12.2	6.0	0.49	0.041	1.36
25	21.2	12.8	8.4	0.65	0.031	1.03
30	24.6	13.5	11.1	0.82	0.024	0.81
35	28.6	14.2	14.4	1.01	0.020	0.66
40	33.2	14.9	18.3	1.23	0.016	0.54
45	38.6	15.7	22.9	1.46	0.014	0.46
50	44.8	16.5	28.3	1.72	0.012	0.39

In general, a statement on the ratio of urban to rural at the point where the natural increase becomes more important than in-migration in city growth can be readily obtained by taking the ratio of (2) to (1) and entering t_c from (5). If $P_0^r = P_{r0}$ we have

$$S = \frac{e^{rt_c}(1 - e^{-mt_c})}{e^{(r-m)t_c}} = e^{mt_c} - 1$$

$$= \exp\left\{m\left[\frac{1}{m}\ln\left(1 + \frac{m}{r}\right)\right]\right\} - 1,$$

which reduces to m/r. An alternative way of viewing this is that the natural increase of the city is rP_u, and its migration is mP_r, and at the crossover point these are equal, so $rP_u = mP_r$ and $P_u/P_r = m/r$. This is exemplified to a close approximation by the ratio of urban to rural of $8.4/12.8 = 0.65$ at time $t = 25$ of the illustration of Table 3. Here

$$m/r = 0.02/0.03 = 0.67,$$

close to the value 0.65 shown for $P_u(25)/P_r(25)$.

Table 3 is an only slightly stylized version of what actually happened in Mexico (Tables 4 and 5), where about 1950 in-migration to cities dropped below natural increase. This can be identified with the year 25 in Table 3; in both tables the ratio of urban to rural was still far less than unity. One could do a tighter fitting to Table 4, but this will serve for the present. The main conclusion is that in-migration ceases to dominate urban increase at a point where the urban population is still much less than the rural, both with the parameters of the simple model of Table 3, and in the actual case of Mexico.

TABLE 4

Urban Growth in Mexico (thousands), Showing Part Due to Natural Increase and Part Due to Migration

	Total Urban Growth	Natural Increase	Percent	In-Migration	Ratio R of In-Migration to Natural Increase
1940–50	2,822	1,167	41.3	1,655	1.42
1950–60	4,883	3,122	63.9	1,761	0.56
1960–70	8,433	5,684	67.4	2,749	0.48

Source: [9, pp. 44–46].

TABLE 5

Urban and Rural Population of Mexico (thousands)

	Urban	Rural	Total	Ratio S Urban/Rural
1910	1,783	13,377	15,160	0.133
1920	2,100	12,335	14,335	0.170
1930	2,891	13,662	16,553	0.212
1940	3,928	15,721	19,649	0.250
1950	7,210	18,569	25,779	0.387
1960	12,747	22,176	34,923	0.575
1970	22,004	27,046	49,050	0.803

Source: [9, p. 27].

How fast is the urban population growing at the crossover point? The answer is obtained by entering the value of time given by t_c in (5) in expression (3) with $P_0 = P_{0r}$:

$$r + \frac{m}{e^{mt} - 1} = r + \frac{m}{\exp[m\ (1/m)\ \ln(1 + (m/r))] - 1} = 2r\ .$$

This also can be seen directly: the rate of natural increase is r, and at the crossover point the migration is equal to it.

What is the ratio of migration to natural increase at the point where the population is 50 percent urban? Here the total is equal to twice the rural, or

$$P_0e^{rt} = 2P_{r0}e^{(r-m)t},$$

so that if $P_0 = P_{r0}$ we have

$$t = (\ln 2)/m;$$

entering this value of time in the ratio of migration to natural increase gives

$$R = \frac{m}{r(e^{mt} - 1)} = \frac{m}{r(e^{\ln 2} - 1)} = \frac{m}{r},$$

which has to be right, since when the rural part of the population is equal to the urban part the gain through migration is m (as a factor of either the rural or the urban figure), and the gain through natural increase is r.

To find a time-invariant relation between the ratio S of urban to rural population and the ratio R of urban in-migration to urban natural increase we eliminate t between

$$P_u(t)/P_r(t) = e^{mt} - 1 = S,$$

and the ratio of $m/(e^{mt} - 1)$ to r from (3), which is

$$R = m/[r(e^{mt} - 1)] .$$

From the first of these $e^{mt} - 1 = S$; from the second $e^{mt} - 1 = m/rR$, and equating the two gives

$$S = m/rR,$$

so the ratio in which we are interested, R, is m/r times the reciprocal of S. This is equivalent to saying that m times the rural population divided by r times the urban gives R.

We would not expect a theory as simple as the one sketched to be verified by data. Nor is its algebra interesting, since most of the results can be seen directly. It has a place here only as an introduction to more realistic models. The first step towards realism will be allowing urban natural increase to differ from rural, both still being constant; after that the constancy will be dropped and all rates allowed to change arbitrarily.

DIFFERENT RATES OF NATURAL INCREASE FOR CITY AND COUNTRYSIDE

Instead of taking r for the natural increase of the urban as well as the rural populations, we now allow a lower rate of increase, say u, for urban; the rate of increase of the rural parts is still r. A pair of differential equations represents the situation:

$$\begin{aligned} dP_r(t)/dt &= (r - m)P_r(t), \\ dP_u(t)/dt &= uP_u(t) + mP_r(t) . \end{aligned} \tag{6}$$

The solution of the first of these is $P_0e^{(r-m)t}$, where we will suppose $P_0 = P_{r0}$, i.e., the country starts entirely rural. Entering this in the second gives

$$dP_u(t)/dt = uP_u(t) + mP_0e^{(r-m)t} . \tag{7}$$

Multiplying by e^{-ut} we have

$$d[e^{-ut}P_u(t)]/dt = mP_0e^{(r-m-u)t},$$

and hence by integration,

$$P_u(t) = \frac{mP_0e^{(r-m)t}}{r - m - u} + Ke^{ut} .$$

We determine the constant K by setting the urban population to zero at time zero:

$$0 = \frac{mP_0}{r - m - u} + K,$$

so

$$K = - \frac{mP_0}{r - m - u},$$

and hence

$$P_u(t) = \frac{mP_0(e^{(r-m)t} - e^{ut})}{r - m - u} . \tag{8}$$

Note that putting $r = u$ in this gives

$$P_u(t) = P_0e^{rt} (1 - e^{-mt}),$$

which is the same as previously obtained in (2). If one avoids assuming that the whole population was rural at the start, the more general result is

$$P_u(t) = \frac{mP_{r0} (e^{(r-m)t} - e^{ut})}{r - m - u} + P_{u0}e^{ut} . \tag{9}$$

To see what part of the increase of $P_u(t)$ is due to in-migration and what part to natural increase, revert to the second member of (6), and divide by $P_u(t)$ from (8):

$$\begin{aligned} \frac{1}{P_u(t)}\frac{dP_u(t)}{dt} &= u + \frac{e^{(r-m)t}(r - m - u)}{(e^{(r-m)t} - e^{ut})} \\ &= u + \frac{u + m - r}{e^{(u+m-r)t} - 1}, \end{aligned} \tag{10}$$

of which u is the natural increase of the urban part and the second term is the migration part. The second term steadily decreases for the values of u, r, and m that we have in mind, starting with infinity for $t = 0$, as corresponds to the assumption that the population starts out wholly rural.

MORE GENERAL CROSSOVER POINT

The crossover point where natural increase and in-migration are equal is given by the value of t obtained by equating the two terms of (10),

$$u = \frac{u + m - r}{e^{(u+m-r)t} - 1},$$

whose solution is

$$t_c = \ln\left(2 + \frac{m - r}{u}\right)/(u + m - r), \tag{11}$$

which reduces to (5) for $r = u$. As one would expect, a small value of u in comparison with r makes for a later crossover. In particular, with a value of $u = 0$, t_c is infinity, i.e., the crossover never occurs. We saw that with $r = u = 0.03$ and $m = 0.02$ the crossover is just over 25 years (Table 3). If now u drops to 0.02, the crossover from (11) is later at

$$t_c = [\ln\left(2 + \frac{0.02 - 0.03}{0.02}\right)/(0.02 + 0.02 - 0.03)] = 40.5 \text{ years}.$$

The drop of natural increase in the city by 0.01 defers the crossover by 15 years. If the urban increase u plus the migration rate m is just equal to the rural increase, the value of t in (11) is indeterminate, with a limiting value of $1/(r - m)$ or $1/u$. In the present numerical illustration, if the urban increase drops to 0.01, while $m = 0.02$ and $r = 0.03$, we have for the limiting value of (11) the quantity $t = 1/(r - m) = 1/0.01 = 100$. The crossover point comes 100 years after the start of the process if the urban rate of increase is as low as 1 percent.

To investigate how much the crossover point is deferred when urban increase is appreciably below rural, let us differentiate expression (11) with respect to u:

$$\frac{dt_c}{du} = \left[\frac{(u + m - r)(r - m)}{(2u + m - r)(u)} - \ln\left(2 + \frac{m - r}{u}\right)\right]/(u + m - r)^2,$$

and entering $u = 0.02$, $m = 0.02$, $r = 0.03$ we find

$$dt_c/du = -2388.$$

This tells us that a drop of u by 0.001 defers the time to crossover by (0.001)(2388) = 2.4 years.

Again using hypothetical figures rather than data, we calculate Table 6 with levels of increase and migration that might have prevailed in an industrializing country of the nineteenth century. The upper limit of the rural population's rate of increase would have been $r = 0.02$, and of this we suppose that half migrated to cities: $m = 0.01$. We also take it that deaths were higher than births in the city, so natural increase was negative, $u = -0.01$.

The difference from Table 3 is striking. At the end of 50 years the total population has multiplied by 2.72 rather than by 4.48; the ratio of urban to rural is 0.65 rather than 1.72; the annual urban in-migraton as a fraction of urban population is at all points higher than before. The last column R is negative and, for what it is worth, also shows a decrease in absolute value, though of course no crossover point.

The assertion that in contemporary less-developed countries migration is a smaller element than the natural increase of the cities rests on the conditions (1) that urban and rural natural increases are both high and not very different from one another, and (2) that the cities have attained at least 30 percent of the national population. These conditions apply in most of the contemporary world. They did not apply in the earlier history of urbanization.

TABLE 6

Urbanization in a Hypothetical Country Starting with a Population of 10 Million, All Rural, in Which Rural Natural Increase Is $r = 0.02$, Urban Natural Increase Is $u = -0.01$, Rural-Urban Net Migration Is $m = 0.01$

	Millions of persons					
Year	Total P_0e^{rt}	$P_r(t)$ Rural $P_0e^{(r-m)t}$	$P_u(t)$ Urban = Total Minus Rural	$S = \frac{P_u(t)}{P_r(t)}$ Urban/ Rural	$\frac{mP_r(t)}{P_u(t)}$ Annual Rate of Urban In-Migration	Ratio R of Migration to Natural Increase in City Growth = Migration/−0.01
0	10.0	10.0	0	0	0	0
5	11.1	10.5	0.5	0.05	0.195	−19.5
10	12.2	11.1	1.2	0.11	0.095	−9.5
15	13.5	11.6	1.9	0.16	0.062	−6.2
20	14.9	12.2	2.7	0.22	0.045	−4.5
25	16.5	12.8	3.6	0.28	0.035	−3.5
30	18.2	13.5	4.7	0.35	0.029	−2.9
35	20.1	14.2	5.9	0.42	0.024	−2.4
40	22.3	14.9	7.3	0.49	0.020	−2.0
45	24.6	15.7	8.9	0.57	0.018	−1.8
50	27.2	16.5	10.7	0.65	0.015	−1.5

The ratio of urban to rural population at the crossover point can be calculated as before, now by entering the value of t from (11) in the ratio $P_u(t)/P_r(t)$, when $P_r(t)$ is given by $P_{r0}e^{(r-m)t}$ and $P_u(t)$ is given by (8):

$$S_c = \frac{P_u(t)}{P_r(t)} = \frac{m[e^{(r-m)t} - e^{ut}]}{r - m - u} \Big/ e^{(r-m)t}, \tag{12}$$

and the reader who wishes to work out the algebra will see that this is m/u at time t_c, which agrees with the simple case where $r = u$.

Neither here nor elsewhere in the present paper is age taken into account. Yet migrants are selected by age, and a model in which this is recognized would be more realistic. Since migrants to the city have a high proportion in marrying and child-bearing age, their entry could well raise the rate of natural increase of the city. It is a handicap that our u and r are crude rates of natural increase, yet to recognize age would make both the mathematics and the interpretation too difficult. Age is omitted throughout the paper.

DATA FOR EIGHT CONTINENTAL REGIONS

The cross-sectional material for eight regions does not show simple evolution (Table 7). Europe is highly urbanized, yet also shows a high ratio of migration to natural increase, as do the USSR and East Asia, the latter being very low in urbanization. For all other regions migration is less important than natural increase; some of these have crossed the equality point, and others have not.

The table does not seem to justify any simple statement as to the relative amounts of the two components of city growth. Its regions are not by any means homogeneous, and more detail, preferably by country or by areas within countries, would be needed for satisfactory interpretation.

TABLE 7

Urbanization in Eight Regions, 1960

	Urban		Rural				
	Natural Increase	Migration	Natural Increase	Migration	Ratio Urban to Rural Population	Ratio R (Migration / Natural Increase in Urban Growth)	R as Calculated by
	as Fraction of Urban		as Fraction of Rural				
	$1000u$		$1000r$	$1000m$	$S = P_u(t)/P_r(t)$		$R = m/uS$
East Asia	17.1	29.8	17.6	8.7	0.292	1.74	1.74
South Asia	23.0	14.5	24.5	3.2	0.220	0.63	0.63
Europe	7.7	10.4	11.7	16.0	1.392	1.35	1.49*
USSR	14.4	20.7	18.2	20.3	0.980	1.44	1.44
Africa	24.0	21.9	23.0	4.8	0.219	0.92	0.91
Northern America	15.4	9.2	15.6	17.4	2.295	0.60	0.49*
Latin America	24.6	20.3	32.0	19.3	0.938	0.83	0.84
Oceania	13.7	12.9	23.6	10.3	1.899	0.94	0.40*

Source: [11].
*Discrepancy due to international migration.

LONGITUDINAL AND CROSS-SECTIONAL COMPARISONS

The inability of cross-sectional data to show the tendency of the migration fraction to decline contrasts with longitudinal data, at least for certain countries. Mexico has shown steady decline in the fraction due to in-migration; and it passed the crossover point about 1950, at a time when it was less than 30 percent urban (Table 4). By the 1960s only one-third of the city increase was due to immigration, roughly resembling Table 3, where rural and urban natural increase were both 3 percent per year and 2 percent of the rural transferred to urban each year.

CONDITIONS OF INCREASE OF S AND R

With given rates of increase in rural and urban parts, designated r and u respectively, and given rate m of outflow from rural, the rural-urban constitution of a country proceeds through a certain evolution. Its most important common features for the magnitudes of r, u, and m are increasing fraction urban and increasing fraction of urban growth due to natural increase in the cities. What are the limits on r, u, and m for which these features apply? For the simple case of r, u, and m constant, the answer is easy. Evidently S is increasing as long as $dS/dt > 0$, i.e., from (12),

$$d\,\frac{m(e^{(u-r+m)t}-1)}{u-r+m}\Big/ dt > 0,$$

which reduces to $m > 0$, since the exponential is always positive. The signs of u and r do not matter, nor does it matter which is greater, as long as m is positive.

The condition that R be decreasing is similarly that $dR/dt < 0$, or that $d(1/R)/dt > 0$, which is, from (12)

$$d \; \frac{u(e^{(u-r+m)t} - 1)}{u - r + m} \Big/ dt > 0,$$

or $u > 0$. As long as natural increase of the city is positive, the r and m can be fixed at any values whatever, and the ratio of migration to natural increase in the city will steadily decline.

At any time when the parameters m and u apply to migration from the countryside and natural increase of the city respectively, the ratio R is $mP_r(t)/uP_u(t)$, and this is the same as m/uS. Like any other mathematics, the relation $R = m/uS$ is a tautology and easier to see through than some. It tells us that as urbanization (measured by S) increases with given m and u, the migration component of city growth will go down.

Such a theory throws the spotlight on what actually happens—especially, in this case, the fact that highly urbanized Europe has a much higher ratio R of migration to natural increase than has either Africa or South Asia. Writing the expressions for $R = m/uS = 1000m/1000uS$ with South Asian numbers, we have

$$\frac{3.2}{(23.0)\,(0.220)} = 0.63,$$

and with European numbers,

$$\frac{16.0}{(7.7)\,(1.392)} = 1.49.$$

Evidently Europe's greater ratio of urban to rural S is more than offset by the fact that its m is much higher and its u is much lower.

There are three very different sets of circumstances in which the type of theory here developed might be applied: (1) In the presently less-developed countries rural and urban natural increase are both high and approximately equal, i.e., $r \doteq u$, and m is very high, as corresponds to our first and simplest mode. (2) In European and American cities up to recent times differential mortality and fertility operated to make $u < r$, with m still fairly high, all three quantities being positive. (3) In parts of Europe and America now, the current of migration has reversed, and more people are moving to the countryside than to the city.[1]

MIGRATION FLOWS IN BOTH DIRECTIONS

The last case needs a further distinction. When most of the migration is cityward, then the net movement m serves all purposes. But once out-migration from the city becomes substantial, we must recognize both currents. Each year about 3 percent of the urban population of the United States moves to rural parts and 2 percent of the rural moves to urban. Changing preferences on where to live and improved communication—especially the interstate highway network—are among the influences that act on migration rates. These also need exploration, and in the final section a more general theory is sketched that deals with changing rates. But even with fixed rates a model can be devised for the part of the trajectory beyond the halfway point $S = 1$. Ledent [3], following [6, 7], takes up the two-way movement in some detail. Suppose that m is the fraction of the rural population that moves to the city, now a gross figure, and n is the gross fraction of the urban population that moves to the countryside.

[1] I am grateful to William Alonso for identifying these three different sets of circumstances.

Then the population evolves according to the equations

$$dP_r(t)/dt = (r - m)P_r(t) + nP_u(t)$$
$$dP_u(t)/dt = mP_r(t) + (u - n)P_u(t), \tag{15}$$

or expressed in matrix form,

$$d\{\mathbf{P}_t\}/dt = \mathbf{M}\{\mathbf{P}_t\}, \tag{16}$$

whose solution in each of $P_u(t)$ and $P_r(t)$ is the weighted sum of two exponentials. The exponentials may be called e^{xt} and e^{yt}, where x and y are the roots of the characteristic equation in g

$$|\mathbf{M} - g\mathbf{I}| = 0, \tag{17}$$

or

$$g^2 - g(m + n) + nr + mu - ru = 0\ . \tag{18}$$

The weights to be attached to the two exponentials depend on the initial conditions. Suppose that they give $P_u(t) = u_1e^{xt} + u_2e^{yt}$ and $P_r(t) = r_1e^{xt} + r_2e^{yt}$. Then the crossover point where the net migration component of city growth ceases to exceed natural increase is the solution in t of

$$uP_u(t) = mP_r(t) - nP_u(t), \tag{19}$$

or

$$(u + n)\,(u_1e^{xt} + u_2e^{yt}) = m(r_1e^{xt} + r_2e^{yt})\ .$$

Solving for t gives the value

$$t_c = \frac{1}{y - x}\, ln \left(\frac{(u + n)u_1 - mr_1}{mr_2 - (u + n)u_2}\right)\ . \tag{20}$$

In the same way, the time when total rural and urban populations are equal is the solution of

$$P_u(t) = P_r(t), \tag{21}$$

or

$$u_1e^{xt} + u_2e^{yt} = r_1e^{xt} + r_2e^{yt},$$

i.e.,

$$t_e = \frac{1}{y - x}\ \ln\ \frac{u_1 - r_1}{r_2 - u_2}\ . \tag{22}$$

From (20) and (22) the crossover precedes the point where 50 percent of the population is urban, i.e., $t_c < t_e$, if

$$\frac{(u + n)\, u_1 - mr_1}{mr_2 - (u + n)u_2} < \frac{u_1 - r_1}{r_2 - u_2},$$

which comes down to either

$$u < m - n \text{ and } u_1/u_2 > r_1/r_2$$

or (23)

$$u > m - n \text{ and } u_1/u_2 < r_1/r_2 .$$

Again, we require explicit values of the weighting coefficients to interpret these.

A REALISTIC MODEL

What makes the preceding work artificial is the use of constant rates of natural increase and migration. In the history of any country there are fluctuating rates of increase and migration, say $u(t)$, $r(t)$, and $m(t)$. In terms of this history we may find the present numbers of urban and rural populations $P_u(t$ and $P_r(t)$. Again we have a pair of simultaneous differential equations

$$\begin{aligned} dP_r(t)/dt &= [r(t) - m(t)]\, P_r(t), \\ dP_u(t)/dt &= u(t)P_u(t) + m(t)\, P_r(t) . \end{aligned} \tag{24}$$

The solution of the first is

$$P_r(t) = P_{r0} \exp\left(\int_0^t [r\,(a5) - m(a)]\, da\right), \tag{25}$$

and entering this in the second, applying the integration factor $\exp(-\int_0^t (t)dt)$, then integrating and evaluating the constant, we obtain

$$P_u(t) = P_{r0} \exp\left[\int_0^t u(a)da\right] \int_0^t \exp\left[\int_0^a [r(b) - u(b) - m(b)]db\right] m(a)da$$
$$+ P_{u0} \exp\left[\int_0^t u(a)da\right], \tag{26}$$

which reduces to (9) for $r(a) = r$, $u(a) = u$, and $m(a) = m$ (i.e., all three parameters constant), and to (8) if, in addition, $P_0 = P_{r0}$ (i.e., the entire population is rural at the start).

Dividing the $P_u(t)$ by the $P_r(t)$ provides the trajectory of the S through time, say $S(t)$. It is not simple, but nonetheless can be called a closed-form solution, available for any particular case by numerical integration.

What about the trajectory of the *R*? We can think of it as immediately determined by the values at time *t* of the quantities already located. We would not have to go back in time again, but could simply take it that

$$R(t) = m(t)/[u(t)S(t)] . \tag{27}$$

Now we have a model that is realistic, but in its general form is not readily interpreted. If we had complete data on the rate functions through time we could indeed reconstruct the trajectory of the $P_u(t)$. This would be no great achievement, but only a bookkeeping reconciliation of the known $Pu(t)$, $S(t)$, and $R(t)$ with the history of the rates that determined them.

A further generalization consists in recognizing out-migration from the cities as well as in-migration. Instead of taking $m(t)$ as the net migration from the countryside we would have two functions: $m(t)$ for the gross movement out of the countryside and $n(t)$ for the gross movement out of the city, expressed as fractions of the rural and urban populations, respectively. This is the same as (17) except that the parameters are now functions of time:

$$\begin{aligned} dP_r(t)/dt &= [r(t) - m(t)]P_r(t) + n(t)P_u(t), \\ dP_u(t)/dt &= m(t)P_r(t) + [u(t) - n(t)]P_u(t) . \end{aligned} \tag{28}$$

This has been suggested by William Alonso, Andrei Rogers, and Jacques Ledent, who have in mind advanced countries, where most of the population is urban, and significant amounts of migration occur in each direction.

Samuel H. Preston, in a major work on urbanization to be published by the United Nations, [*4*], provides a pair of differential equations similar to (28). He applies them to examine the trajectory of the difference between urban and rural growth rates and shows that under quite general conditions the urban-rural growth difference will decline as the ratio of urban to rural population increases. This is true both with constant coefficients such as $r(t)$ and with declining coefficients.

Having attained the maximum of realism and complexity we can backtrack a short distance, suppose some hypothetical trajectories for the rates, and see how the S and the R would be affected.

Among the obvious questions to be asked of the more general model is what happens when the rural and urban rates of increase decline, or when the rate of out-migration from the countryside accelerates. Also of interest is the consequence of a step function in either of these—sharp discontinuities in the rates do actually occur, especially in migration, due to sudden economic changes. Expressions (26) and the corresponding solution of (28) are eminently suited to experimentation on all of these and other matters.

A DYNAMICAL FORMULATION

An adequate theory of the evolution of urban population is what everyone would like to have. The simplest possible theory has been explored in detail in this article: the nature of the evolution that takes place with constant natural increase of urban population u, and natural increase r and fraction out-migrating m of the rural population. Such a constancy does not accord with what we know of the tendency of natural increase to decline as populations urbanize. A more realistic version would have it that $u(t)$ declines through time in a degree that offsets the inevitable rise of $S(t)$, the ratio of urban to rural population. If the product $u(t)S(t)$ remains constant, and so does $m(t)$, then from (27) the ratio $R(t)$ of in-migration to natural increase in the cities will be constant. If $u(t)S(t)$ remains constant, $m(t)$ would have to decline for the ratio $R(t)$ to decline. One can imagine that happening as populations matured —on the approach to equilibrium $m(t)$ (which is a net figure) could well drop towards zero while urban increase $u(t)$ was still positive and $u(t)S(t)$ was approximately constant. In such circumstaces $m(t)/u(t)S(t)$ would steadily decline, and this means the decline of in-migration relative to natural increase as a contributor to city growth. But one can easily imagine other courses.

The arithmetic considerations of this paper might be fitted into an analysis of the dynamics of urbanization. If the rural increase $r(t)$ remains high, so is $m(t)$ likely to be high. A possible evolution is high and constant $r(t)$ and $m(t)$ along with low $u(t)$—this supposes that only as people move into the city does their birth rate fall faster than their death rate. If $u(t)$ drops towards zero all further increase of the city would be through migration, and national growth would stop when, and only when, the country was entirely urbanized. This might be called the theory of the undeveloped countryside: it supposes that there is little development outside the cities; major investment and improvement are confined to the cities, and the countryside is left to fend for itself.

The equally simple alternative to such a trajectory is the decline of rates of natural increase both in the city and the countryside, with $m(t)$ falling faster than $u(t)$. This could well occur as a result of massive investment in the countryside, in industry as well as in agriculture, which ought simultaneously to slow the rural rate of natural increase $r(t)$ and the migration rate $m(t)$. In fact, the action on $m(t)$ would follow two paths—economic through the creation of opportunities for remunerative work and demographic through reduction of the rate of increase of rural population.

On the first route, concentration of economic advance in the cities causes the natural increase of cities to fall, and in-migration stays high, so $m(t)/u(t)$ remains high. On the second route economic advance occurs in the countryside, and $R(t)$ and $m(t)$ fall as a result. If the choice is between these two routes of development, then the suggested demographic-economic linkages make the question of whether the cities grow by in-migration or by natural increase answerable in terms of where investment mostly takes place: the more it is concentrated in the cities the more these will grow by in-migration; the more it takes place in the countryside, the more the growth of cities will be limited to their own natural increase.

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