A REVISIT TO THE PROTOTYPE WATER SYSTEM

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In M. Fiering's paper, "Mathematical Model of a Prototype Water System," he describes a system which has the following properties.

1) Upstream Reservoir with associated benefits (e.g. power).
2) Downstream Levees with benefits from flood damage reductions.
3) Independent inflows known on the first day of the season.
4) One-season model, i.e. yearly.
5) Technical functions between yearly inflow volume and the instantaneous flood peak downstream.

Fiering was aware that many of the assumptions do not hold in real cases. Furthermore, there are other extensions of the described prototype water system that would be useful to explore--for example, downstream supply by either the upstream reservoir or by an alternate ground water source.

Another set of questions is centered around the upstream/downstream division of costs and benefits. The area of the division of costs and benefits falls under such headings as 'game-theory', 'conflict resolution', 'bargaining', etc. and involves often side-payments from one user to another. In many cases, the role of the analyst is not to find an optimal solution to the bargaining procedure but to display various outcome sets to alternative actions.

In a framework similar to that used by Fiering, this working paper investigates the outcome sets (upstream and downstream benefit positions) in the following cases:

1) A one-season model for upstream water-supply and downstream supply, where the targets may be different;
2) A two-season model for upstream water supply and downstream supply, where the targets may be different; and
3) A two-season extension of upstream supply and flood control downstream.

From these results, further extensions to a real situation, like the Tisza, will be discussed in detail and will include a "where-do-we-go-with-this" section.

In the one-season model, the season will be represented by a year. Therefore, the inflow for season i, x_i, will enter the reservoir of capacity k from which a seasonal release r_i is made. Like Fiering's model, the x_i's, are in compatible units.

The reservoir services an upstream demand, and like Fiering assumed, this could be hydropower. After leaving the reservoir, the water services some downstream demands which may be irrigation. In this model, the downstream demands may also be serviced by pumping from an underground aquifer. It will be assumed that the aquifer has a sustained yield of 2 units, and its use will be supplementary to the river source. Furthermore, as long as the withdrawls from the groundwater aquifer is less than or equal to 2 units, the quantity of water is assured with certainty, and the cost of its water supply as a function of flow does not change.

Like the releases r_i, the groundwater 'release', g_i, are in seasonal units that are converted from some flow rate.

The vector of inflows into the reservoir x, represents a random process without serial correlation. The probability density of any particular flow is presented in Table 1. The capital cost for reservoir construction, $c_1(k)$, is also presented in Table 1. The capital costs plus the present value of the OMR costs for the groundwater system is as given in Table 1.

The economic characteristics, like the physical characteristics, are similar to those of Fiering's. A standard operating policy, as shown in Figure 1, is used, which is characterized by the reservoir capacity, k, and the target release, T. It is important to consider the constraints, reservoir empty and reservoir full, which defines the band of feasible operating policies. We will return to this later when a discussion of flood control, in a two-season model, is made.

The benefit functions are three-part linear functions which characterize a long-term component and two short-term components. In the irrigation model, the target is 3 and

Table 1.

flow x	0	1	2	3	4	5	6	7
P{x}	.05	.12	.15	.20	.20	.10	.10	.08

Flow Probabilities

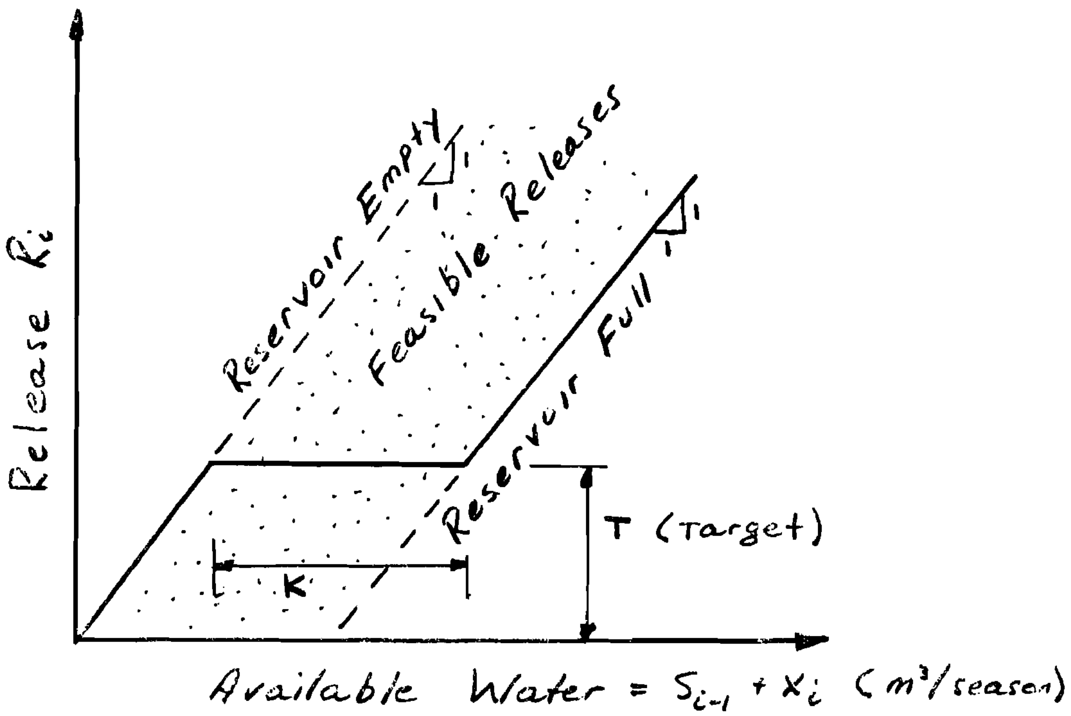
capacity k	0	1	2	3	4	5	6
C.(k)	2	4	8	14	25	40	57

Reservoir Construction

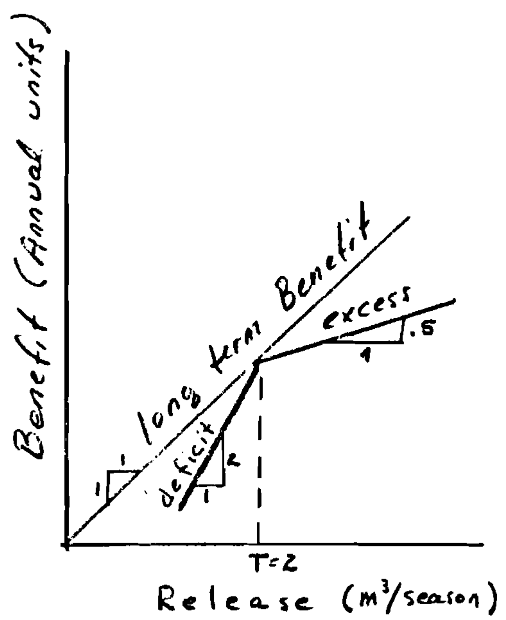
Yield G	0	1	2
C(G)	1	5	15

Ground Water Development plus OMR

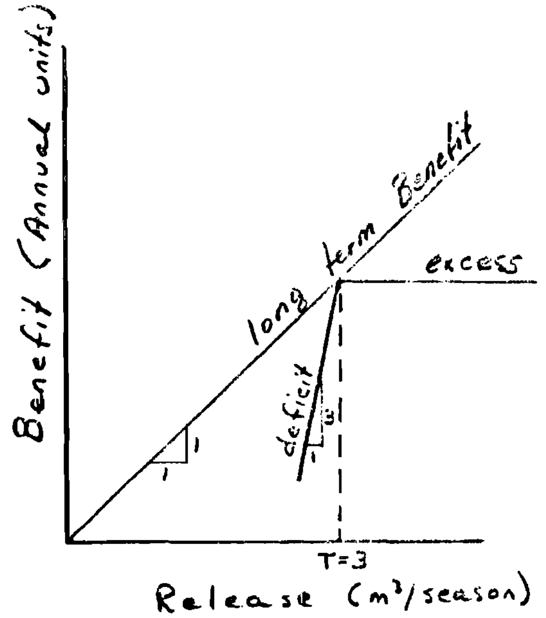
FIGURE 1.



OPERATING POLICY



UPSTREAM USER'S BENEFIT FUNCTION



DOWNSTREAM USER'S BENEFIT FUNCTION

water delivered in excess of 3 does not contribute to increased benefits since commitments for acreage, etc. for a particular year have already been made. This is not true for hydro-power where a market for 'dump' power exists--at a lower price than for 'firm' power. For the power demand, a target of 2 is assumed. This target will be accepted as given, and the institutional or economic situation that established the target will not be addressed--except to note that in an investment model of water resource systems both the reservoir capacity K and the targets T are decision variables.

Results

Assuming that the upstream target is 2 and the downstream target is 3, the matrix of the reservoir storages, $\underline{S}$, and the vectors of the steady state storages, P(s), the release P(R) will be the same as were found in Fiering's paper. There are six possible pairs of actions that the upstream and downstream decision makers can participate in--for the upstream decision maker, to build or not to build the reservoir, and for the downstream decision maker, to develop groundwater to 0,1, or 2 units. The gross annual benefits from these six action pairs are given below.

		G = 0	G = 1	G = 2
K = 4	U/S	2.733	2.733	2.733
	D/S	1.725	2.988	2.997
K = 0	U/S	0	0	0
	D/S	1.48	2.475	2.87

Note: U/S ≡ upstream user
D/S ≡ downstream user

If the appropriate costs from Table 1 are used, and if an interest rate of 4% with a 25 yr. is applied then the present value of each strategy is

		G = O	G = O	G = 2
K = 4	U/s	17.694	17.694	17.694
	D/s	25.947 1	41.678 2	31.819 3
K = O	U/s	O	O	O
	D/s	22.12 4	33.664 5	29.835 6

The six strategies are numbered in the lower right hand corner.

Another scenario that may be offered to the upstream-downstream decision makers is the following "if the reservoir operator sets a target of 3, which is not compatible to the upstream uses, how much will the downstream user pay."

Under such an operating policy, the upstream user may forego a small amount of benefits while the downstream user may gain substantially. This scenario was analyzed and the probability matrix of the reservoir storage, $\underline{S}$, was, for the same probability density function of the inflows,

	S_{i-1}					
		0	1	2	3	4
S_i	0	.52	.32	.17	.05	.00
	1	.20	.20	.15	.12	.05
	2	.10	.20	.20	.15	.12
	3	.10	.10	.20	.20	.15
	4	.08	.18	.28	.48	.68

The steady state probabilities are:

$$P\{S\} = \{.14,.12,.14,.15,.45\}$$

The probability vector of the releases was found to be:

$$P\{R\} = \{.01,.02,.04,.63,.13,.07,.06,.04\}$$

Three additional action pairs, to the six given earlier, arise from this analysis. The present value of these strategies, under a 25 year life and 4% discount rate, are as follows:

		G = 0		G = 1		G = 2	
K = 4	U/s	16.28		16.28		16.28	
	D/s	40.61	7	39.99	8	31.39	9

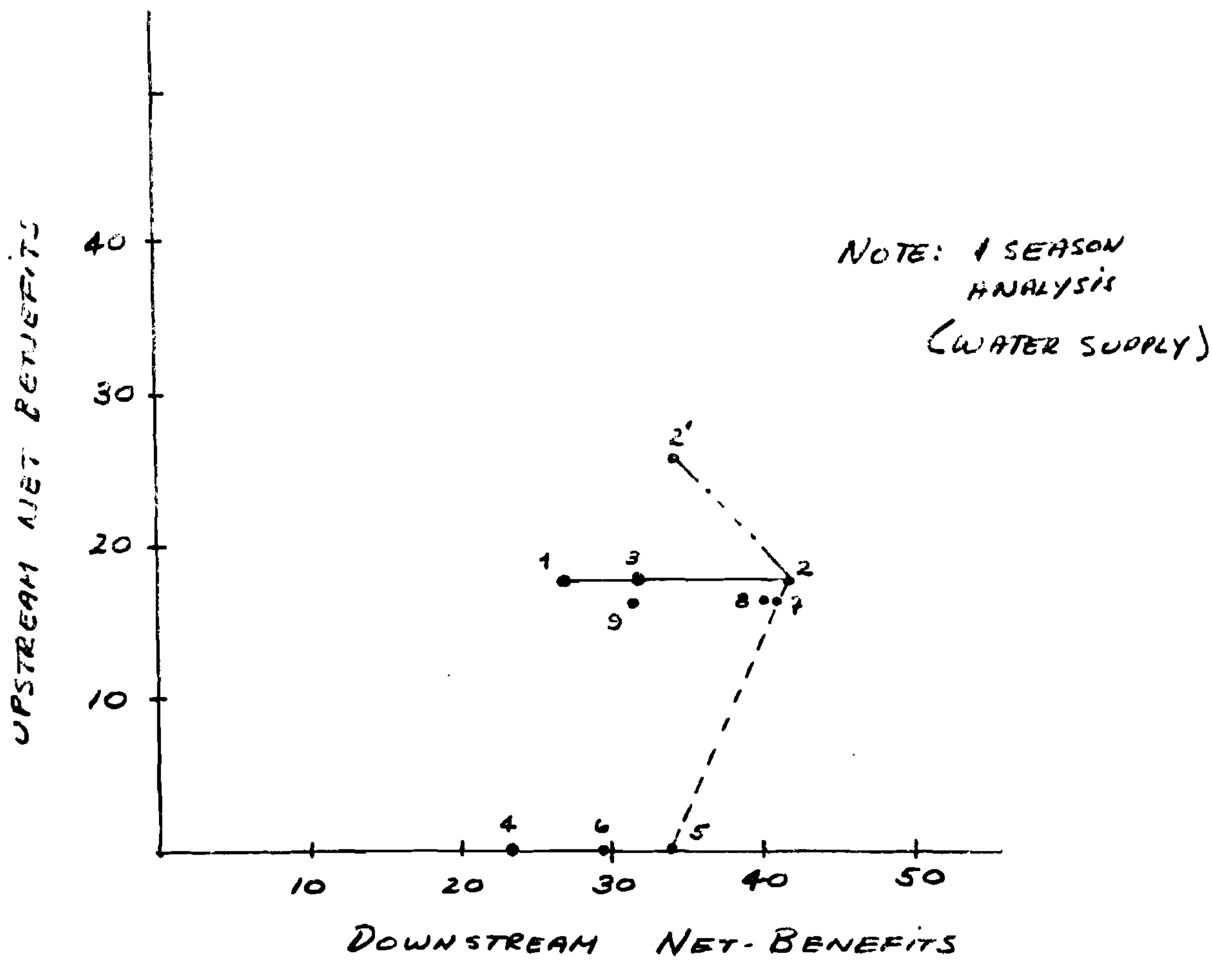
Bargaining Positions

The nine action pairs, analyzed above, can now be plotted as shown in Figure 2. This figure displays some very interesting bargaining positions. Before discussing these in detail, let's look at the general structure.

The upstream net benefit level can be at one of three positions, depending upon the upstream strategies of no reservoir, a reservoir of capacity 4 and target 2 or a reservoir of capacity 4 and target 3. For each upstream strategy, the downstream benefits can be determined for its strategies of groundwater development.

It can be seen immediately that the strategy of an upstream reservoir with a target of 3 (which would only be

FIGURE 2


NOTE: 1 SEASON ANALYSIS (WATER SUPPLY)
UPSTREAM NET BENEFITS
DOWNSTREAM NET-BENEFITS
10
20
30
40
50
1
2
2'
3
4
5
6
7
8
9

realized through co-operation) is completely dominated by the strategy of an upstream reservoir and a target of 2.

Thus, the upstream decision maker is faced with the strategy of not to build, or to build to a capacity of 4 with a target of 2, and the downstream decision maker will always choose to develop groundwater to 1 unit.

If the reservoir is built, then the downstream decision maker will gain an additional 8.02 units of benefits--without a change in strategy. Will the downstream user pay a side payment? There is no clear answer to this but some of the considerations are:

1) If the benefits to the upstream user (from building) are quite small, then the downstream may make a side payment as an inducement. If the upstream benefits are quite large, then the downstream user may 'gamble' that the reservoir will be built.

2) A side payment moves the position of strategy set 2, in Figure 2, towards 2'. In a certain world the downstream user may go up to 2', i.e. a side payment equal to 8.02 units. In a stochastic world, it is not clear how large a certain side payment would be indifferent to expected additional benefits of 8.02. Here utility theory would be helpful in determining how risk adverse the downstream user is. The downstream users adversion to risk determines this tradeoff.

Two-Season Model for Water Supply

The model presented in the first section considered each season to be a year. In many ways this is not reasonable--especially for the downstream irrigation demand. Conceptually, the same model can be used to analyze a two-season configuration.

Season one will be the 'wet'season, when the reservoir can fill up to help service the 'dry' season, season two which corresponds to the growing season and irrigation demands for water.

The probability density functions for volumes of water in each season are given to be

$$P\{X_1\} = \{.05,.12,.15,.20,.20,.10,.10,.08\}$$

$$P\{X_2\} = \{.10,.28,.22,.15,.12,.08,.04,.01\}$$

It will be assumed that flows between seasons are independent.

The economic assumptions of the one season model will be used, for the most part, in the two season analysis. For the upstream user, the three piece linear benefit function given in Figure 1 will be valid for both seasons. For the down-stream user, the three piece linear benefit function used in the one-season model will apply to benefits in season two only. No irrigation benefits are obtained in season one (the non-growing season).

Analysis

The analytical analysis of the two season model is fairly straight forward. Given the operating rules for season 1, the probability matrix of releases, dependent upon the available water (inflow plus storage from season 2), $P[\underline{R}_1|\underline{A}_1]$ is easily generated. Given the probability vector of season 1 inflows, $P\{\underline{X}_1\}$, the matrix of the available water, depending upon the last season's (season 2) storage, $P[\underline{A}_1|\underline{S}_2]$ is also easily generated. Matrix multiplication yeilds $P[\underline{R}_1|\underline{S}_2]$, the probability matrix of releases in season 1 given the last season's storage. Season 1's operating rule can also be used to generate the probability matrix of season 1 storage conditional upon the available water in season 1, $P[\underline{S}_1|\underline{A}_1]$. Matrix multiplication with $P[\underline{A}_1|\underline{S}_2]$ yields the storage transition matrix for the reservoir, $P[\underline{S}_1|\underline{S}_2]$ which gives the probability that the storage this season (season 1) will be at a particular level, given that last season (season 2) was at some specified level.

The exact same approach, using season two's operating rules and the inflow probabilities, can be used to generate the probability matrix of releases in season 2 conditional upon the storage in season 1, $P[\underline{R}_2|\underline{S}_1]$, and the storage transition matrix for season 2, $P[\underline{S}_2|\underline{S}_1]$.

The two storage transition matrices are then used to generate the storage transition matrix for the <u>current</u> season 2 conditional upon the storage level in the <u>last</u> season 2.

This leads directly to the steady state probability storage vector for season 2. Similarly, the two storage transition matrices can be used to find the steady state probability storage vector for season 1. With the steady state storage probabilities, through a vector multiplication with the probability of the releases conditional upon the storage, the probability vector of the releases is obtained directly.

Results

The two season analysis was performed with the reservoir targets in both season 1 and season 2 set at 2. The probability matrices, $P[\underline{R}_1|\underline{S}_2]$, $P[\underline{R}_2|\underline{S}_1]$, $P[\underline{S}_1|\underline{S}_2]$, and $P[\underline{S}_2|\underline{S}_1]$ are given in Table 2.

The vectors of steady state probabilities for the reservoir in each season were found to be

$$P\{\underline{S}_1\} = \{.03,.05,.10,.14,.68\}$$

$$P\{\underline{S}_2\} = \{.05,.06,.14,.24,.51\}$$

and the probability vectors of the releases were found to be

$$P\{\underline{R}_1\} = \{0.,.01,.47,.17,.15,.09,.07,.01\}$$

$$P\{\underline{R}_2\} = \{0.,.01,.66,.13,.10,.06,.03,.01\}$$

There exists six action pairs that the upstream and downstream decision makers may engage in. The gross annual benefits from these action pairs are as follows:

Table 2.

S_2

	0.05	0.00	0.00	0.00	0.00
	0.12	0.05	0.00	0.00	0.00
	0.75	0.77	0.72	0.52	0.32
	0.08	0.10	0.10	0.20	0.20
$\underline{R}_1$	0.00	0.08	0.10	0.10	0.20
	0.00	0.00	0.08	0.10	0.10
	0.00	0.00	0.00	0.08	0.10
	0.00	0.00	0.00	0.00	0.08

$P[\underline{R}_1|S_2]$

S_1

	0.10	0.00	0.00	0.00	0.00
	0.28	0.10	0.00	0.00	0.00
	0.61	0.85	0.87	0.75	0.60
	0.01	0.04	0.08	0.12	0.15
R_2	0.00	0.01	0.04	0.08	0.12
	0.00	0.00	0.01	0.04	0.08
	0.00	0.00	0.00	0.01	0.04
	0.00	0.00	0.00	0.00	0.01

$P[R_2|S_1]$

S_2

	0.32	0.17	0.05	0.00	0.00
	0.20	0.15	0.12	0.05	0.00
S_1	0.20	0.20	0.15	0.12	0.05
	0.10	0.20	0.20	0.15	0.12
	0.18	0.28	0.48	0.68	0.83

$P[\underline{S}_1|\underline{S}_2]$

S_1

	0.60	0.38	0.10	0.00	0.00
	0.15	0.22	0.28	0.10	0.00
S_2	0.12	0.15	0.22	0.28	0.10
	0.08	0.12	0.15	0.22	0.28
	0.05	0.13	0.25	0.40	0.62

$P[S_2|S_1]$

		G = 0	G = 1	G = 2
K = 4	U/s	4.92	4.92	4.92
	D/s	.96	2.97	3.0
K = 0	U/s	0	0	0
	D/s	-.24	1.56	2.70

Using the costs of the one season analysis, namely

$K_4 = 25$

$G_0 = 1$

$G_1 = 5$

$G_2 = 15$

and a 25 year life discounted at 4%, the present value of the six action pairs are:

		G = 0		G = 1		G = 2	
K = 4	U/s	51.85		51.85		51.85	
	D/s	14.00	1	41.39	2	31.86	3
K = 0	U/s	0		0		0	
	D/s	-4.75	4	19.37	5	27.17	6

Three additional scenarios were analyzed by considering a reservoir target of 2 units for season 1 and a target of 3 units for season 2. It will be remembered that the downstream user had a target of 3 during season 2. The probability matrices, $P[\underline{R}_1|\underline{S}_2]$, $P[\underline{R}_2|\underline{S}_1]$, $P[\underline{S}_1|\underline{S}_2]$ and $P[\underline{S}_2|\underline{S}_1]$ are given in Table 3.

The seasonal steady state storage probabilities were found to be

Table 3.

S_2

	0.05	0.00	0.00	0.00	0.00
	0.12	0.05	0.00	0.00	0.00
	0.75	0.77	0.72	0.52	0.32
R_1	0.08	0.10	0.10	0.20	0.20
	0.00	0.08	0.10	0.10	0.20
	0.00	0.00	0.08	0.10	0.10
	0.00	0.00	0.00	0.08	0.10
	0.00	0.00	0.00	0.00	0.08

$P[\underline{R}_1|\underline{S}_2]$

S_1

	0.10	0.00	0.00	0.00	0.00
	0.28	0.10	0.00	0.00	0.00
	0.22	0.28	0.10	0.00	0.00
	0.40	0.61	0.85	0.87	0.75
R_2	0.00	0.01	0.04	0.08	0.12
	0.00	0.00	0.01	0.04	0.08
	0.00	0.00	0.00	0.01	0.04
	0.00	0.00	0.00	0.00	0.01

$P[\underline{R}_2|\underline{S}_1]$

S_2

	0.32	0.17	0.05	0.00	0.00
	0.20	0.15	0.12	0.05	0.00
S_1	0.20	0.20	0.15	0.12	0.05
	0.10	0.20	0.20	0.15	0.12
	0.18	0.28	0.48	0.68	0.83

$P[\underline{S}_1|\underline{S}_2]$

S_1

	0.75	0.60	0.38	0.10	0.00
	0.12	0.15	0.22	0.28	0.10
S_2	0.08	0.12	0.15	0.22	0.28
	0.04	0.08	0.12	0.15	0.22
	0.01	0.05	0.13	0.25	0.40

$P[\underline{S}_2|S_1]$

$$P\{S_1\} = \{.10,.10,.13,.15,.52\} ,$$
$$P\{S_2\} = \{.20,.15,.22,.16,.27\} ,$$

and the probability vector of the releases were found to be

$$P\{R_1\} = \{.01,.03,.60,.14,.10,.06,.04,.02\}$$
$$P\{R_2\} = \{.01,.04,.06,.73,.08,.05,.02,.01\} .$$

This analysis adds three additional action pairs for the upstream and downstream decision makers. The present value from these strategies (assuming a 25 year life and a 4% interest rate) are as shown in Table 3.

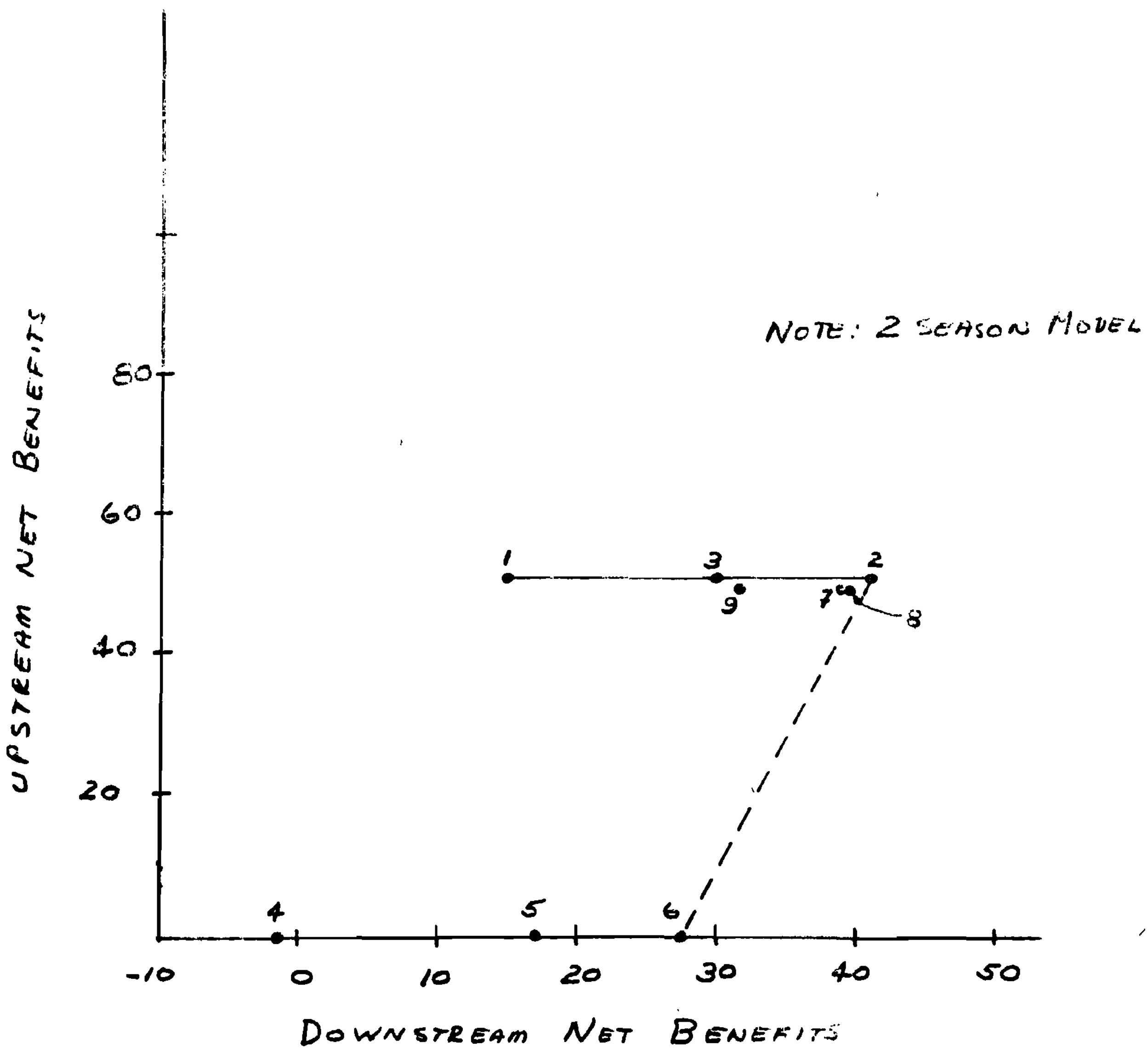
		G = 0		G = 1		G = 2	
K = 4	U/S	49.51		49.51		49.51	
	D/S	37.89	7	39.05	8	31.39	9

Figure 3 shows the nine strategies. It is of great interest to note that the optimal strategy for the downstream user is to develop groundwater to 1 unit regardless of what the upstream user does. Clearly this must have implications to the bargaining aspects. The other bargaining issues are similar to those discussed earlier for the one season model.

Flood Control in a Two-Season Model

The one-season flood control model of Fiering's can be expanded into two seasons exactly the same way that the water supply model was extended.

FIGURE 3


NOTE: 2 SEASON MODEL
UPSTREAM NET BENEFITS
80
60
40
20
1
3
2
9
7
8
4
5
6
-10
0
10
20
30
40
50
DOWNSTREAM NET BENEFITS

Like Fiering's model, enough is known about the hydraulic configuration of the system to assert that a seasonal release from the reservoir will result in a known flood peak at the downstream location. Everything will be expressed in seasonal flows, and it is assumed that the resulting downstream consequences can be evaluated. The channel capacity, D, will be expressed in the units of volume per season which will be consistent with the units of the reservoir releases. From our hydrologic knowledge the units of volume per season can be converted to peak stage or peak discharge. It will be assumed that the 'capacity' of the channel in the unimproved system is 4 units. Furthermore, dikes can be built to increase the channel capacity to 5, 6, or 7 units. The costs for this improvement are:

D	5	6	7
C(D)	5	10	20

Flood control benefits can be realized by the downstream user either from the reservoir or from the dikes (or both). The reservoir provides benefits by reducing the probability of large flows. Since the capacity of the unimproved channel was 4 and the maximum release will be 7, then flood damages will occur with releases of 5, 6, or 7. The probabilities of these releases should decrease with the construction and reasonable operation of the reservoir.

If dikes are constructed, then flood benefits are derived from having more water flow down the channel (higher capacity) and having less overflow. A channel capacity of 5 will have 1 unit of overflow from a release of 6 units as opposed to 2 units from the unimproved channel. This procedure was also followed by Fiering.

The construction of the reservoir reduces the probability of large flows while the construction of dikes reduces the amount of overflow. The damages for overflow that we will use are as follows:

Loss (F)	2	6	8
F	1	2	3

The interesting scenario to look at in the two-season flood control problem is: "how much will the downstream user pay for a specified amount of flood storage". That is, during the 'winter'season the reservoir is never filled above some specified level and when the flood comes, part of the water will go into storage reducing the release.

Analytical Procedures

To clearly understand the flood storage operation, consider the standard operating policy shown in Figure 4a. The standard operating policy is characterized by the storage capacity K and the target release T. There exists two constraints, reservoir empty and reservoir full, between which fall all feasible releases.

The release pattern for season 1 (the winter season) is shown in Figure 4b along with the operating rule for season 2.

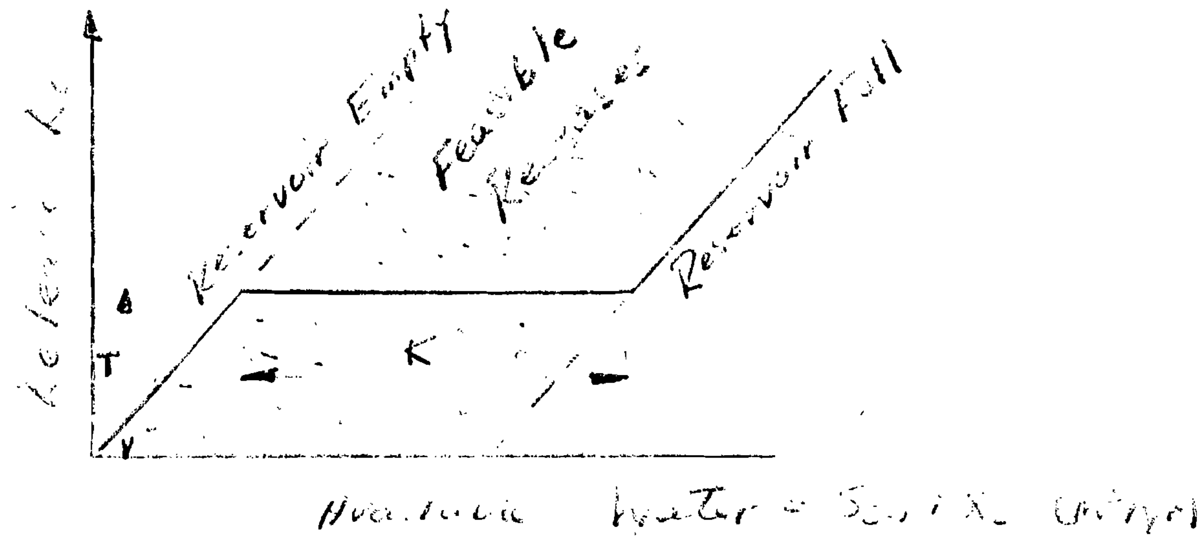


Figure 4a

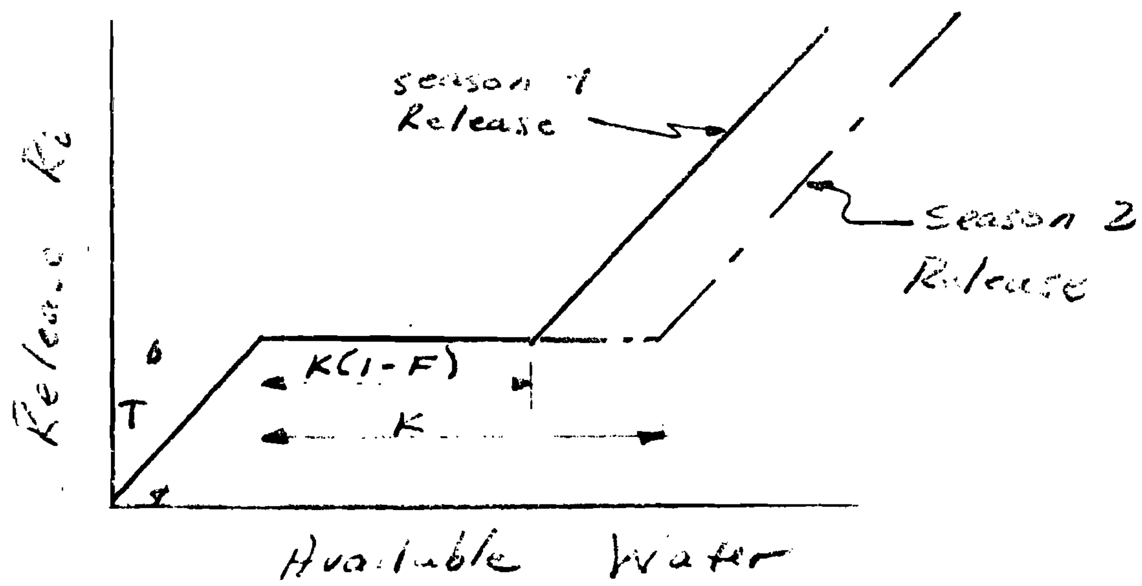
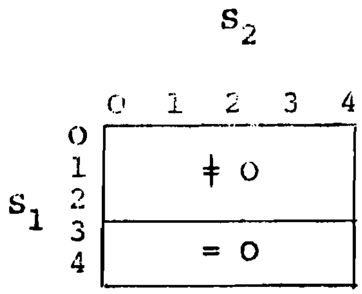


Figure 4b

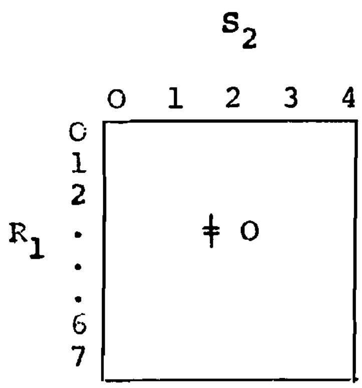
Season 1's operation is as if the capacity of the reservoir was K(1-F), where F is the fraction of the storage that is held for flood storage.

The analytical procedures are similar to those of the two season water supply analysis. Given the operating rules for each season and the probabilities of the inflows for that season, we can genuate the matrices $P[\underline{R}_i|\underline{S}_j]$ and $P[\underline{S}_i|\underline{S}_j]$.

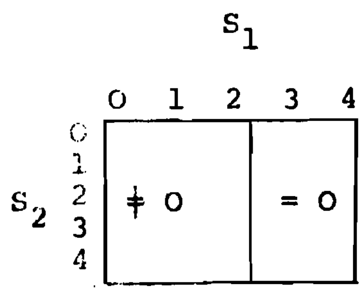
If F is .5 in season 1 and if there are 5 levels for the storage reservoir and 8 inflow and release levels, then the qualitative structure of the matrices will be



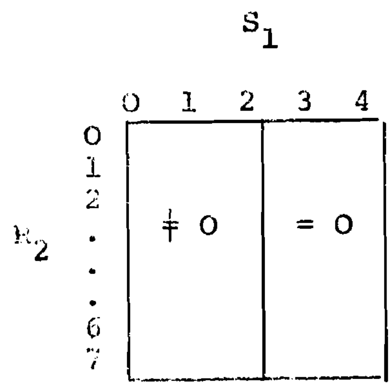
$P[S_1|S_2]$



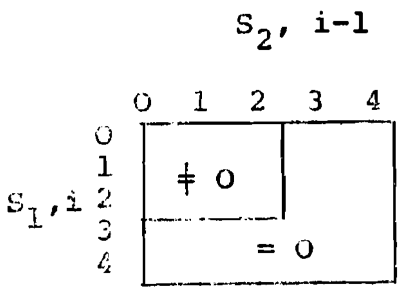
$P[\underline{R}_1|\underline{S}_2]$



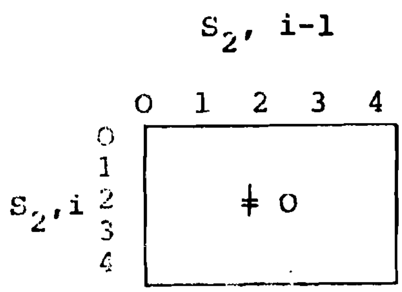
$P[S_2|S_1]$



$P[\underline{R}_2|\underline{S}_1]$



$P[\underline{S}_1, i|\underline{S}_1, i-1]$



$P[\underline{S}_2, i|\underline{S}_2, i-1]$

Let the seasonal inflows have the following probability density function:

$$P\{X_1\} = \{.15,.40,.25,.11,.06,.02,.01,0.\}$$

$$P\{X_2\} = \{.05,.10,.15,.24,.22,.12,.07,.05\}$$

then from the operating rule the steady state probabilities for each season can be found and that the probabilities of the releases for each season.

For a reservoir of size 4 and F = 0 (no flood storage) the steady state storage probabilities and the probability vector of the releases are,

for Season 1

$$P\{\underline{S}_1\} = \{.101,.095,.193,.297,.314\} \quad ,$$

$$P\{\underline{R}_1\} = \{.01,.03,.82,.08,.04,.014,.006,0\} \quad ,$$

and for Season 2

$$P\{\underline{S}_2\} = \{.054,.073,.120,.155,.598\} \quad ,$$

$$P\{\underline{R}_2\} = \{.005,.015,.560,.175,.120,.070,.040,.015\} \quad .$$

For a reservoir of size 4 and F = .25 (1 unit of flood storage) the steady state storage probabilities and the vector of releases can be calculated to be

for Season 1

$$P\{\underline{S}_1\} = \{.125,.113,.197,.565,0.\} \quad ,$$

$$P\ \underline{R}_1 = \{.01,.04,.66,.16,.07,.035,.01,.005\} \quad ,$$

and for Season 2

$$P\{\underline{S}_2\} = \{.064,.095,.141,.172,.528\} \quad ,$$

$$P\{\underline{R}_2\} = \{.01,.02,.64,.16,.09,.05,.03,0.\} \quad ,$$

and finally for a reservoir of size 4 and F = .50 (2 units of flood storage) the steady state storage probabilities and the vector of releases can be calculated to be,

for Season 1

$$P\{\underline{S}_1\} = \{.19,.15,.66,0.0,0.0\} \quad ,$$

$$P\{\underline{R}_1\} = \{.02,.06,.46,.23,.13,.06,.03,.01\} \quad ,$$

and for Season 2

$$P\{\underline{S}_2\} = \{.112,.134,.177,.214,.363\} \quad ,$$

$$P\{\underline{R}_2\} = \{.01,.03,.78,.10,.05,.03,0,0\} \quad .$$

Economic Analysis

Using the probability of the releases in each season, the benefits to the upstream user and to the downstream user can be calculated directly. The benefit function for the downstream user will be the same three-piece linear function that was used in the water-supply analysis. This function is presented in Figure 1. The cost of the reservoir is taken as 40 units for a capacity of 4. Table 4 gives the upstream power benefits.

The flood control benefits from a particular decision can be taken to be the reduction in the expected damages. Table 5 gives the expected annual damages for the four reservoir strategies with the four like decisions.

Table 4. Upstream Power Benefits

Expected Annual Gross Benefits

Flow	Benefits	K = 4 K = 0	K = 4 F = .25	K = 4 F = .50
0	-2	-.03	-.04	-.06
1	0	0	0	0
2	2	2.76	2.6	2.48
3	2.5	.64	.8	.83
4	3.0	.48	.48	.54
5	3.5	.29	.30	.32
6	4.0	.18	.16	.12
7	4.5	.07	.02	.05
Present value of expected net benefits:		28.57	27.48	26.85

(25 years life and 4% discount factor)

Table 5

No Reservoir

Flow	D = 4 level	E[Dam]	D = 5 level	E[Dam]	D = 6 level	E[Dam]	D = 7 E[Dam]
5	1	.28	0	-	0	-	
6	2	.48	1	.16	0	-	
7	3	.40	2	.30	1	.10	-
		1.16		.46		.10	0

Reservoir F = 0

Flow	D = 4 level	E[Dam]	D = 5 level	E[Dam]	D = 6 level	E[Dam]	D = 7 E[Dam]
5	1	.22	0	-	0	-	
6	2	.32	1	.11	0	-	
7	3	.17		.13	1	.04	-
		.71		.24		.04	0

Reservoir F = .25

Flow	D = 4 level	E[Dam]	D = 5 level	E[Dam]	D = 6 level	E[Dam]	D = 7 E[Dam]
5	1	.17	0	-	0	-	
6	2	.24	1	.08	0	-	
7	3	.04	2	.03	1	.01	
		.45		.11		.01	0

Reservoir F = .50

Flow	D = 4 level	E[Dam]	D = 5 level	E[Dam]	D = 6 level	E[Dam]	D = 7 E[Dam]
5	1	.18	0	-	0	-	
6	2	.18	1	.06	0	-	
7	3	.08	2	.06	1	.02	
		.44		.12		.02	0

E[Dam] ≡ expected damages
D ≡ channel capacity
level ≡ amount of overflow

The present value of these 16 action sets can be quickly tabulated. Assuming a 25 year life and a 4% discount rate, the present values for the action sets were calculated and are presented in Table 6. Each action set has two calculations of the downstream flood benefits. The first one (top row) has the 'marginal benefits' which are calculated conditional to the reservoir being built and operated as indicated. The bottom row presents the flood benefits due to the joint decision of reservoir construction, reservoir operation, and dike construction. Figure 5 presents the 16 action sets showing the expected net benefits to each group. It is from the points presented here that bargaining takes place.

Some Bargaining Issues

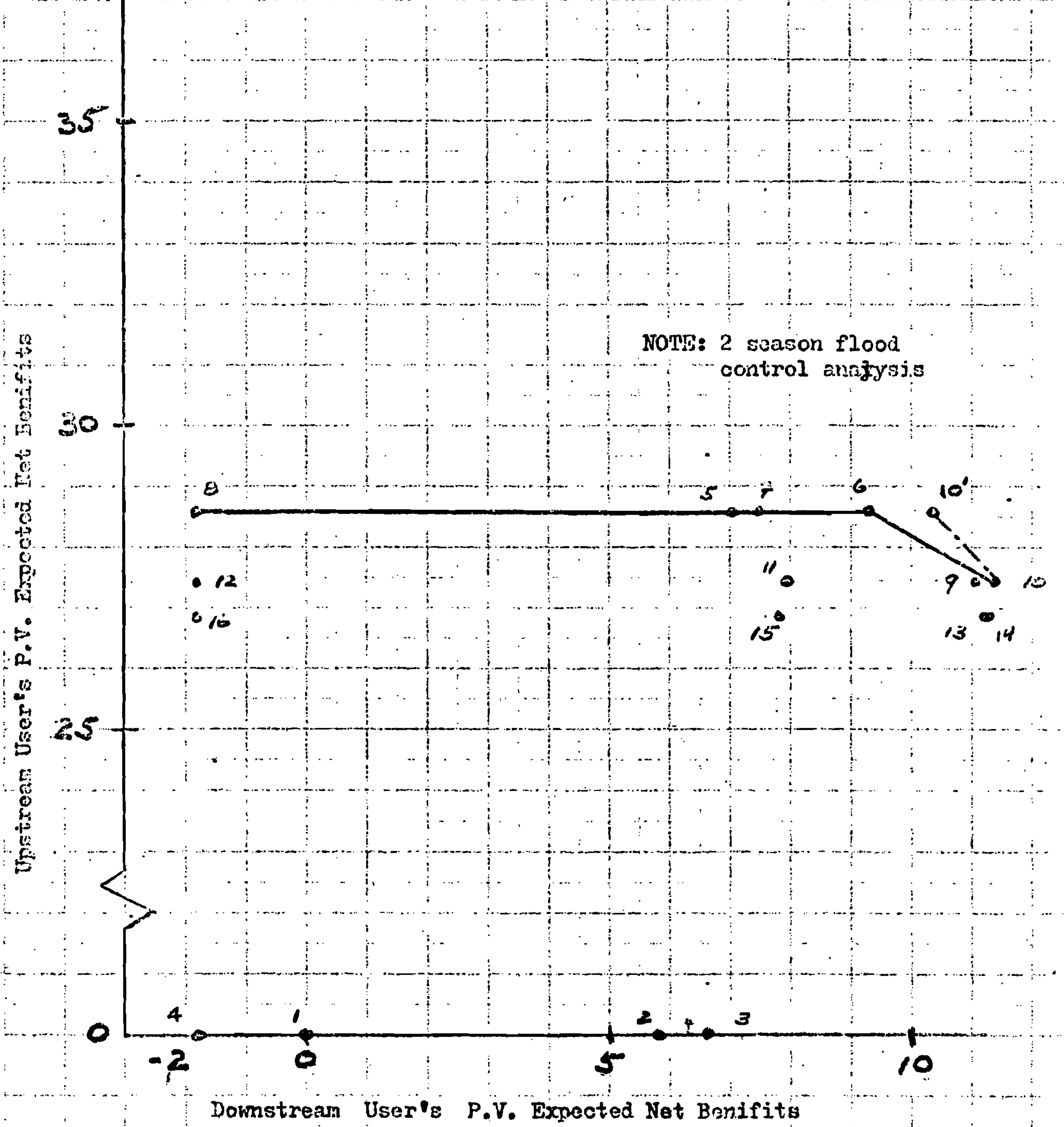
1. In the water-supply analysis, the downstream user had a pure strategy that consisted of developing groundwater to 1 unit regardless of the decision the upstream user made. The upstream decision affected the benefits that the downstream user realized but the downstream user may not bargain in the hopes that the reservoir would be built. In the flood control example presented here, the downstream user's optimal strategy is partially affected by the upstream decision concerning the construction of the reservoir (for example if $K = 0$, then $D^* = 6$; if $K = 4$ $F = 0$, then $D^* = 5$) and partially by the operation of the reservoir (for example if $K = 4$ $F = 0$ then $D^* = 5$; if $K = 4$

Table 6. Expected Flood Damages Assuming a 25 Year Project Life and 4% Interest Rate

Reservoir decision	Dike Decision			
	D = 4	C = 5 D = 5	C = 10 D = 6	10 = 20 D = 7
K = O	O O 1	5.93 5.93 2	6.56 6.56 3	-1.88 -1.88 4
K = 4, F = O	O 7.03 5	2.34 9.37 6	.47 7.49 7	-8.91 -1.88 8
K = 4, F = .25	O 11.09 9	.31 11.40 10	-3.13 7.96 11	-12.97 -1.88 12
K = 4, F = .50	O 11.25 13	O 11.24 14	-3.44 7.81 15	-13.13 -1.88 16

Note: Top row: marginal benefits conditional upon the reservoir decosion
Bottom row: benefit due to both reservoir and dike level decision.

FIGURE 5


NOTE: 2 season flood control anajysis
Upstream User's P.V. Expected Net Benifits
35
30
25
0
8
5
7
6
10'
11
9
10
12
16
15
13
14
4
1
2
3
-2
0
5
10
Downstream User's P.V. Expected Net Benifits

F = .50 then D* = 4). The overall efficient solution is D = 5 and K = 4, F = .25, which is obtained from the construction and operation of the reservoir with some dike construction. In the flood control example, the downstream decision maker must bargain with the upstream decision maker if he is to move to a better position.

2. Where does the bargaining begin? If the downstream user feels that the reservoir will be built, then maybe at the operating policy. Then action sets 6 and 10 are the two bargaining positions. A side payment of 1.06 units to the upstream user would move 10 to 10'. The downstream user would be better off than if he were at 6 and the upstream user should be indifferent between 6 and 10'. 1.06 is the minimum side payment that the downstream user can pay so that the benefits to the upstream user do not decrease.

3. We have been talking about certain costs, certain side payments and uncertain benefits. Are the utilities for these equal--I feel not. This will affect the evaluation of the action pairs to the extent that expected utilities instead of expected net benefits will be calculated.

Some of the bargaining issues will be addressed more directly in a forthcoming working paper (Ostrom and Wood [1]).

A Visit to Reality

Leaving the prototype water system, the question remains about how the procedures relate to 'real-world' case studies such as the Tisza River.

The procedure presented here worked towards finding the probabilities of the releases, P(R), which were used in finding the expected benefits from a set of strategies. From the ranking of the strategies, 'efficient' pairs can be identified and where conflict exists hopefully bargaining could lead to mutually satisfying positions.

Conceptually this procedure of identifying the outcome sets is the way to go. The analysis of the prototype system presented here is a very simple simulation model to achieve the impacts of various strategies. Such a model has many deficiencies, some of them are:

1. The technological relationships of the simple model are inadequate. In the flood control analysis, the resulting stages from a release depends not only upon the release but upon the flood levee construction at all locations upstream to the location being evaluated. Thus, if these are two downstream users, the lower downstream user must decide his strategy by considering the strategy of the other downstream user and and the upstream user.
2. Considering the year as one or two seasons does not adequately represent the hydrologic events. There

often exists correlation between river discharges on both an annual level and at an intra-season (monthly, for example).Within the Markov structure that this working paper is cast in, such correlations would explode the matrices to very large levels. This explosion is especially true if many reservoirs are considered.

3. The analysis presented in the paper investigated various scenarios. The reservoir capacity and the reservoir target were set prior to the analysis, therefore we cannot determine whether the combination is on the efficient frontier. This is true for all simulation modelling.

   To overcome this problem, we either simulate exhaustively all combinations (not a very feasible procedure) or an optimization model should be constructed--the later is obviously the best procedure. There is a whole host of optimization models (mostly LP) for water resource systems but they suffer from their inability to richly describe the physical system--especially the stochastic aspects. What has often been done (for example, the Argentina Study by MIT) is to build a deterministic LP model to find efficient configurations and then to simulate these configurations to 'redesign' them to better account for issues that do not lend themselves to optimization (stochasticity, for example). In a subsequent working paper Wood [2] will address the issues of optimization models for water resource systems. Spofford [3] has put forward one proposal to consider the flood control optimization problem.

Conclusion

For all the difficulties of applying the simple model, the conceptual nature of the solution should not be lost. We mustidentify feasible action sets from which to bargain from. The working paper by Wood [4] on the Tisza identifies the issues which affect the Tisza and which must be modelled. This first step should be started immediately.

Once feasible sets are established, bargaining positions can be identified. Ostrom is putting together a group of IIASA personnel who are interested in conflict resolution--initially around the Pulgia-Basilicata problem and then the Tisza. Belyaev is, I understand, also starting to get into game theory. The bargaining aspects of these projects interfaces the Water Project with other projects very well, and the methodological aspects can be addressed within a realistic setting.

Wood, Spofford, and Koryavov are all trying to establish modelling procedures to find the efficient set of possible strategies. This involves simulation modelling and optimization modelling. The relevant decision attributes and measures of effectiveness must be identifed. Some of these problems, example the optimization modelling, will utilize the skills of the methodology project. Keeney and Wood plan to collaborate on applying utility theory to water resource problems of this nature.

So, the revisit to the prototype water system was useful. It re-affirmed our ideas of where we want to go, unfortunately the vehicle that got us to the prototype system cannot get us to the real system. But knowing where we want to go is half the battle.

References

[1] Ostrom, T. and Wood E. "Some Bargaining Issues from the Prototype Water Model," WP-74-

[2] Wood, E. "Some Issues Affecting Optimization Models in Water Resources Planning," WP-74-42, August 1974.

[3] Spofford, W.O. "An Approach to the Flood Protection Investment Problem," August 20, 1974.

[4] Wood, E. "An Approach to the Tisza Basin Study," WP-74-30, July 1974.