WORKING PAPER

THE REGIONAL PLANNING OF HEALTH CARE SERVICES: RAMOS AND RAMOS^{-1}

L.D. Mayhew

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FOREWORD

The principal aim of health care research at IIASA has been to develop a family of submodels of national health care systems for use by health service planners. The modeling work is proceeding along the lines proposed in the Institute's current Research Plan. It involves the construction of linked submodels dealing with population, disease prevalence, resource need, resource allocation, and resource supply.

The work presented in this paper, based on RAMOS (Resource Allocation Model Over Space), addresses the theme of the geographical allocation of health care resources. Whereas an earlier paper (Mayhew and Taket, 1980) examined the empirical basis for such a model, material presented here suggests how RAMOS may be used in a decision-making role.

Related publications in the Health Care Systems Task are listed at the end of this report.

Andrei Rogers
Chairman
Human Settlements
and Services Area

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ABSTRACT

This working paper is the second in a series on RAMOS (Resource Allocation Model Over Space), dealing with the effects on in-patient hospitalization rates of changing resource levels, population trends, and accessibility costs in heavily populated or congested regions. Whereas the first paper emphasized and detailed the empirical basis, calibration, and validation of RAMOS, the material presented here examines its application in a decision-making context. For simplicity, two levels in the planning process are identified: the tactical and the strategic. For each level a different approach for using RAMOS is recommended. In the first case, it is argued that the planning problems are relatively self-contained, and they can be analyzed in only a few computer runs of the model. In the second case, the possibilities are unrealistically many and so must be narrowed down. To tackle this a new version of the model $RAMOS^{-1}$ is developed and tested in detail. The objective in $RAMOS^{-1}$ is to pick resource configurations such that the relative needs of the population in each place of residence in a region are met. However, so that other objectives in the health care system do not conflict in the process, upper and lower bounds on permissible resource allocations are introduced in each treatment-district zone. The problem is solved using a quadratic programming technique and applying it to four, hypothetical planning scenarios using data based on London, in south-east England. A sensitivity analysis is also conducted on the model parameter, before some conclusions are drawn and recommedations for further developments are made.

CONTENTS

THE REGIONAL PLANNING OF HEALTH CARE SERVICES: RAMOS AND RAMOS^{-1}

1. INTRODUCTION

RAMOS (Resource Allocation Model Over Space) is a behavioral model that explores the geographical interactions between the supply of, and demand for, acute in-patient hospital services. It is formed from the hypothesis that in a region the number of patients generated in origin zone i (a place of residence) and treated in destination zone j (a hospital district) is in proportion to the morbidity or "patient generating potential" of i and the resources available in j, but is in inverse proportion to the accessibility costs of getting from i to j.

The model provides a simple method for choosing between different resource configurations in congested regions (very large urban areas, industrial agglomerations, and so forth) when the size and structure of the population and the resource availability are changing over space and time. It can be applied to one clinical category, or more likely to several in conjunction, depending on the hierarchical structure and geographical distribution of services. In its present form the model assumes that there are not enough resources to satisfy demand, and that patients are not restricted by their places of residence to use only certain hsopitals. The health care systems for which it is best suited are:

- -- Payment-free systems or systems operating comprehensive health insurance schemes where there are few market signals to regulate supply and demand
- -- Systems with national, regional, or local health care planning machinery, and a commitment to the effective territorial planning of health care services
- -- Systems in which there is a historical tendency to over-allocate health resources in some regions, and under-allocate them in others,and in which there is a growing desire by statutory authorities to redress these imbalances
- -- Incipient systems in developing countries undergoing rapid urbanization

The extension of the model to a more market-based health care system presents problems which are not insurmountable, and some research priorities in this direction are outlined in the conclusions (section 8).

In a previous paper the empirical basis and assumptions underlying the model for over twenty acute clinical specialties were considered and tested in detail (Mayhew and Taket, 1980). It was shown how to calibrate the model by fitting it to real data, to what extent it reasonably described the behavior of the actual pattern of patient flows between origins and destinations, and what improvements still seemed necessary.

In this paper we are concerned with developing systematic methods for applying RAMOS in a decision-making environment. A broad distinction is drawn between the tactical and strategic level of planning, while ways are also discussed for connecting the model to different submodels dealing with trends in population, service utilization, and resource availability. The most important results center on the subject of strategic level modeling. A method, based on RAMOS, is developed and tested to show how resources may be distributed in each treatment zone j so that the relative needs of each place of residence are met. What emerges is essentially a new model called RAMOS^{-1}. This model is applied in the London region in England to four, hypothetical planning scenarios (changing resource and population levels), and the preliminary results are given. The sensitivity of the model to parameter changes are then considered, before some conclusions are drawn.

2. THE BASIC MODEL*

Mathematically the model is stated as follows:

$$T_{ij} = B_j \, D_j \, W_i \, \exp\,(-\beta \, c_{ij}) \qquad (1)$$

where

T_{ij} = the predicted patient flow from zone i to treatment zone j

D_j = the caseload capacity in j for treating patients in a specialty or group of specialties

W_i = the patient generating factor (pgf), which is an index of the propensity of the population in i to generate patients in the same group of specialties

c_{ij} = the accessibility costs from i to j

β = a parameter to be determined empirically

and

$$B_j = \left[\sum_i W_i \, \exp\,(-\beta \, c_{ij}) \right]^{-1} \qquad (2)$$

Equation (2) is a constraint, known as a balancing factor, which ensures that

$$\sum_i T_{ij} = D_j \qquad (3)$$

In words, the model assumes that all acute in-patient resources in each place of treatment are used to capacity. Realistically, some fluctuation of say ±5 percent about this assumption is likely because of increases in hospital throughput or slack in the system. This can be built into forecasts as desired. The behavioral basis of the model entails finding a value for β based on existing population, resource levels, patient flows, and accessibility costs and then assuming this parameter is more or less unchanged over a typical forecasting period.

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*For a glossary of all the terms used in this paper see Appendix A.

3. PLANNING LEVELS IN THE HEALTH CARE SYSTEM (HCS)

When planning in-patient hospital services, decisions are taken on a variety of levels in the administrative and managerial structure. Some decisions are relatively minor affecting few resources in only one location—for example, the decision to close temporarily a hospital ward for modernization. Others are of far greater significance involving, say, the commissioning or closing of a large hospital. At a higher level still, plans are formulated to determine the direction of the in-patient health care services in all zones of a region for a period of time. Typically such plans will take into account trends in the population structure, the current availability of physical resources, the manpower supply, capital development programs, and so on (Pelling, forthcoming).

Although it depends on the country, a hypothetical planning region may contain one city, several towns, and a population of several million, served by over one hundred variously sized hospitals. Finding the right blend of resources to provide for such a region is very difficult. There are few easily expressed objectives to guide the development of the in-patient services, while the benefits obtained from allocating resources to one activity in one location, as opposed to another somewhere else, are not readily quantifiable. In this decision-making environment it is useful to identify two broad levels in which RAMOS can be helpful: the tactical level and the strategic level (Shigan, Hughes, and Kitsul 1979). For current purposes tactical planning can be defined as decisions involving particular projects—such as the building of a new hospital. The strategic level of planning is concerned with the broad direction of the entire system and its long-term resource requirements (DHSS 1976).

4. RAMOS: ABILITIES AND LIMITATIONS

RAMOS is not intended to provide ready made plans at either of the levels described. Rather, it is a tool for helping to decide between alternatives, for examining the implications of a particular decision or set of assumptions, or for simply observing where the system is heading. It allows the decision maker to test a variety of scenarios without committing him to any particular one. This flexibility is both an advantage and a disadvantage. It permits in theory the evaluation of many competing alternatives, yet it cannot tell the user which is best. For a relatively small problem (say, at the tactical level) these alternatives will be few, and it is probable that they can be judged for their suitability in only a few computer runs of the model. In the formulation of a strategic plan, however, both population and resource levels are changing over time and space. The alternatives here will generally be too many to evaluate, and the decision maker will need to direct his search.

It is reasonable to ask therefore whether methods can be found to use RAMOS in narrowing down the choices to those which in some sense are best and which can be achieved in the planning period. To do this RAMOS must be directed to pick resource configurations that satisfy a particular objective. The problem is which objective to choose and how to express it in a way that can be used by a mathematical model. Some guidance is offered by the National Health Service in England and Wales formed in 1948. Its expressed aim is

> ...to ensure that every man and woman and child can rely on getting all the advice and treatment and care they need in matters of personal health...[and]... that their getting these should not depend on whether they can pay for them (Feldstein 1963:22, quoting from HMSO 1944).

The assumption in 1944 that all needs can be provided for has proved unrealistic: health care expenditures and the consumption of health care services in general continues to rise not only in England and Wales but also in most other countries. A realistic alternative is that in-patient health care resources be rationed instead according to the relative (and not to the absolute) needs in each part of a region (RAWP 1976). This is the objective that we shall begin with here.

In striving for this objective, the decision maker will inevitably conflict with others. For example, in addition to treating patients the hospital sector of the health care system also trains doctors. However, the teaching and service needs of a region's health care system may differ significantly (LHPC 1979). Also, the decision maker is faced with the general inertia of a system in which finance is scarce and physical resources (buildings and equipment) cannot be made perfectly mobile. Finally, factors such as physician availability or economies of scale can also set upper or lower limits on the resources it is possible to reallocate. These and related considerations will act as constraints on the changes in resource allocation the decision maker is willing or able to allow. Such constraints must therefore be incorporated into RAMOS in a way which allows it to move in the direction of its principal objective but with due regard to the operating environment.

Two Types of Models

Two variants of RAMOS are therefore suggested: one, functioning at a tactical level, takes as inputs a set of relative needs and a given resource configuration,and then predicts the consequent service levels in each place of residence; the other, functioning at a strategic level, takes as inputs a set of relative needs, the total of available resources,and the constraints on change. It then predicts the required resource allocations in each treatment zone that are necessary to come as close as possible to the objective that has been set. The first model will be termed RAMOS and the second RAMOS^{-1}. The minus one is a convention to indicate that the second model is essentially the inverse of the first. All the results shown below will be from RAMOS^{-1}, since much of the groundwork for the first model is already contained in Mayhew and Taket (1980).

5. CONNECTING SUBMODELS

The three main input variables in the applications of RAMOS are resource availability, relative need,and accessibility costs. Of these the HCS essentially exercises control over only one--resource availability. The other variables may, in certain respects, be controlled by the HCS, but for all practical purposes they are really exogenous. Nevertheless, each variable in turn is a function of others which must be estimated separately and incorporated in the proposed scheme. Two flow diagrams -- one for RAMOS and one for $RAMOS^{-1}$--show one possible set of linked submodels and their interconnections (Figures 1 and 2). The particular structure shown here is illustrative only and is not meant to exhaust the possibilities. It does, however, imply approaches which have already proved useful in another context (LHPC 1980). In the first diagram, the outputs from RAMOS relate mainly to the service levels in each origin; in the second, they are the resource allocations in each destination.

5.1. Patient Generating Factors (pgfs)

The submodels in both figures fall into three categories. The first looks at population trends, mortality, and national patterns of hospital utilization in different clinical specialities. This category has the purpose of estimating the patient generating factors for the forecast period. For example, hospital utilization is increasing in many specialties but at different rates in each age-sex group. Such trends should be reflected in the measures of patient potential along with the expected demographic changes, (see also Appendix B).

A patient generating factor (pgf) has been defined previously by Mayhew and Taket (1980) as follows

$$W_{it} = \sum_{m} \sum_{l} P_{ilt} U_{lmt} \tag{4}$$

where P_{il} is the population in i in age-sex category l, and U_{lm} is the national utilization rate in l for specialty m. The subscript t is there to indicate the time horizon of the planner. A pgf is hence simply the expected number of patients a zone would produce if the national pattern of hospital usage in different specialties were to apply to the local age-sex structure. As a measure of relative need, it can be criticized on several grounds. For example, it ignores those differences between zones of a socio-economic kind which can be variously important in deciding the eventual usage of hospital services. One solution to this is to use a measure which is strictly related to absolute morbidity (Kitsul 1980); another is to modify the current measures of pgfs in another way such as by multiplying W_i by the

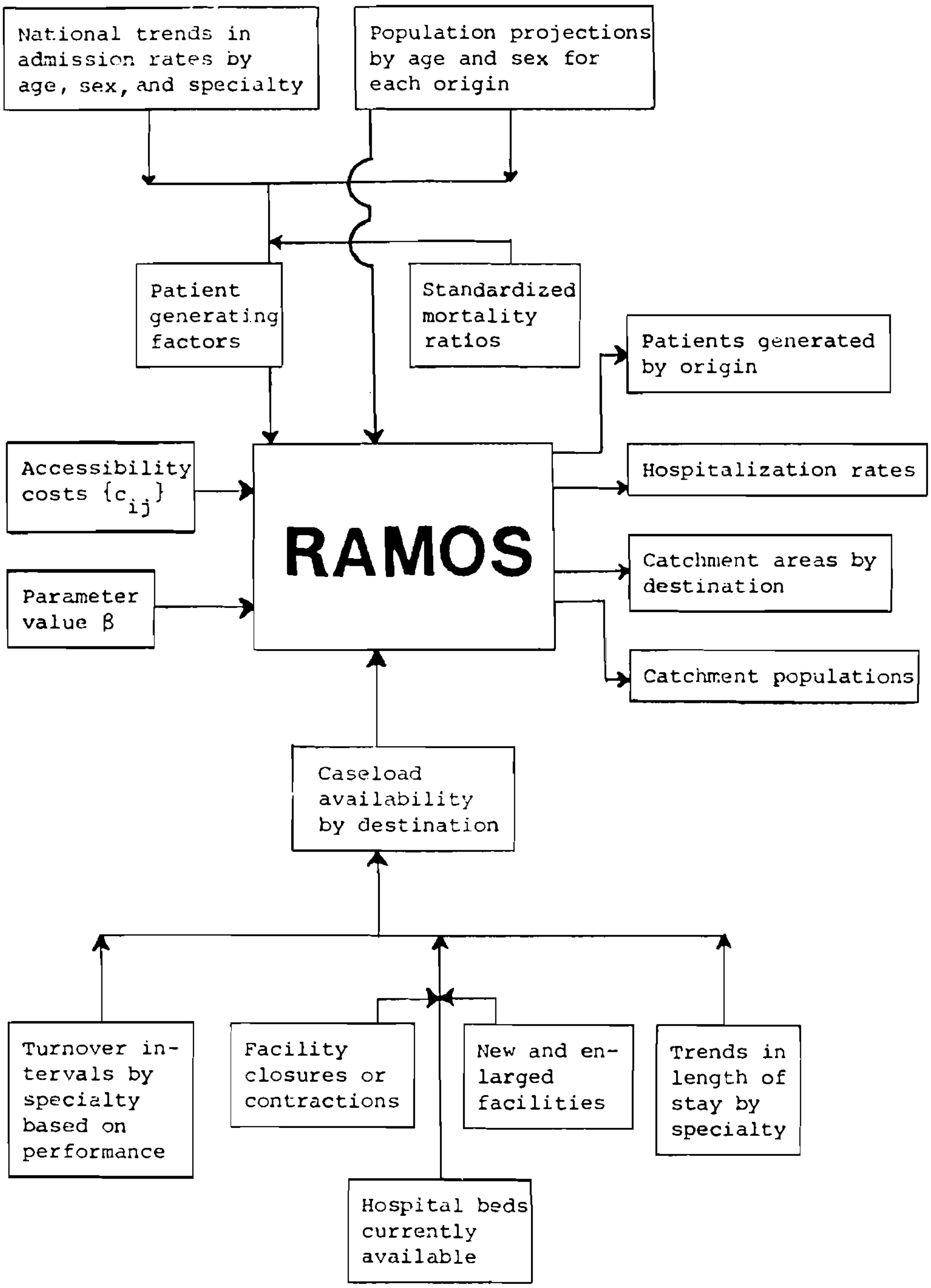


Figure 1. Planning acute in-patient hospital services using RAMOS. This model predicts the service levels in each origin zone.

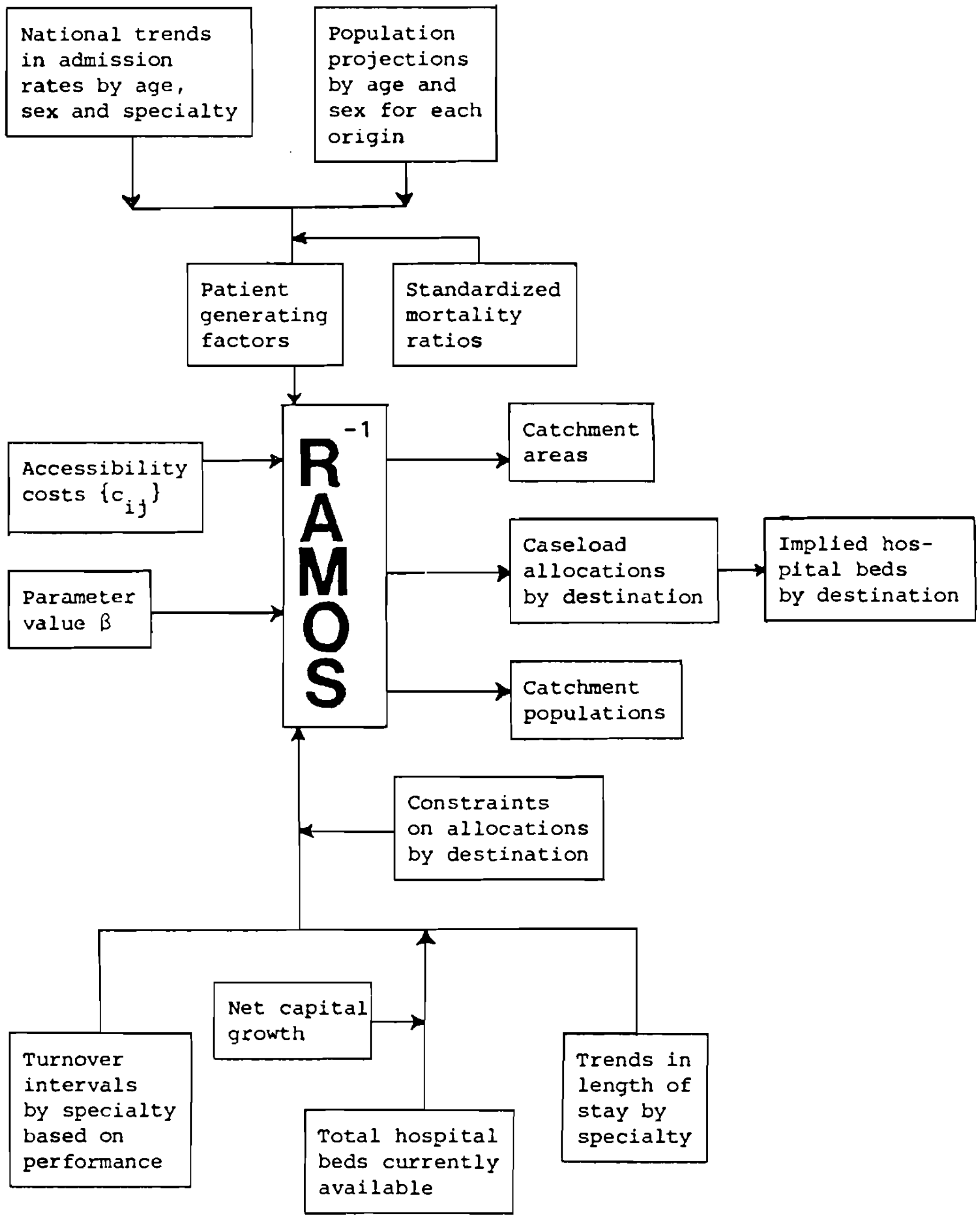


Figure 2. Planning acute in-patient hospital services using $RAMOS^{-1}$. This model selects the resources in each destination zone.

by the standardized mortality ratio (SMR_i) in each region zone. An SMR in i is defined as

$$SMR_i = \sum_l M_{il} \Big/ \sum_l r_l P_{il} \qquad (5)$$

where M_{il} is the actual number of deaths in age-sex category l, P_{il} is the population in i and r_l is the national age-sex specific death rate. If an SMR_i is greater than one, the number of deaths are greater than expected, and therefore,the response will be to introduce a greater patient-generating potential in zone i. This approach has been tested with initial success in Mayhew and Taket (1980), and more research in in progress.

5.2. Resource Availability

The second category of submodels concerns resources as measured in terms of caseloads. The number of cases that can be treated in a destination zone is a function of the available beds and the efficiency with which patients can be treated. From Figures 1 and 2 it is seen that the resource variable is treated slightly differently in each case. Both RAMOS and $RAMOS^{-1}$, however, rely on estimates of the average length of stay in each specialty and the turnover interval (defined as the average length of time between discharge and admission of a new patient). There is a decrease in lengths of stay in some specialties, and therefore it is desirable to introduce this trend into the caseload estimates (see Appendix B). Turnover intervals are also not constant; these, too, will require simple analysis (LHPC, 1979, and Appendix B).

The fundamental relationship in a specialty between cases, beds, and throughput is

$$B_{mt} = \frac{d_{mt}(\ell_{mt}+t_{mt})}{365} \qquad (6)$$

where B_{mt} is the number of beds in specialty m in time t, d_{mt} the number of cases, ℓ_{mt} the average length of stay between admission and discharge, and t_{mt} the turnover interval. In RAMOS an estimate of total caseloads D_{jt} is required by destination in the specialties of interest. This is

$$D_{jt} = 365 \sum_{m} \frac{B_{jmt}}{(\ell_{mt}+t_{mt})} \tag{7}$$

Both ℓ_{mt} and t_{mt} are assumed to be based on national trends, but if local conditions predominate or medical experts disagree with the statistical forecasts, then the estimates can be adjusted accordingly. At this point proposed expansions or contractions in bed availability are also introduced to give a revised measure of D_{jt}.

In RAMOS^{-1} the same procedure applies except that the resource availability is estimated over the whole region and not for each destination

$$Q_t = 365 \sum_{m} \frac{B_{mt}}{(\ell_{mt}+t_{mt})} \tag{8}$$

Here B_{mt} is taken as the aggregate regional bed forecast in each specialty. The problem in RAMOS^{-1} is to divide Q_t into destinations, but in a way that unreasonable changes in D_j do not result. Constraints on input into the model are therefore placed on the practicable changes that can be made. For example, suppose that in j an increase of over ±p percent in resource levels is regarded by the decision maker as unmanageable in a planning period, then the constraints are set as

$$D_{jt}(1 + p) \geq D_{jt} \geq D_{jt}(1 - p) \tag{9}$$

where t is the time horizon.

5.3. Accessibility Costs

Accessibility costs express the difficulty of someone in zone i being admitted as a patient in treatment zone j. In any HCS the route by which a patient chooses, or is referred to, a particular destination may be complex. In some cases the decision to use one place rather than another will be simple and based wholly on convenience; in others, it may be the result of a series of referrals from a general practitioner and from specialists lower in the HCS hierarchy. In still other cases, the patient may be taken in an emergency to a destination unrelated to his place of residence. This whole process is thus extremely difficult to model as one measure. Nevertheless, research has shown (Mayhew and Taket 1980) that simple journey time, or modified distance, act as good surrogates, indicating that convenience of access is still the dominant consideration in most instances. It is not difficult, however, to state occasions for which this is always untrue or for when some modification is appropriate. For instance, in health care systems where a mixed public-private system of hospitals exists, some destinations will be "closed" to part of the population. Accessibility costs for the "closed" facilities are, in this instance, effectively infinite. There are, however, simple ways of handling this in RAMOS which will be of value when applying the model in countries with more market-based health care systems, or for representing other types of accessibility behavior. These will be discussed at a later date.

In the applications of RAMOS^{-1} shown below, the accessibility measures developed in Mayhew and Taket (1980) are retained for exposition purposes. These measures and their derivation are fully discussed in this reference. The UK in-patient sector that is private is regarded as negligible in this application and is not considered further.

6. RAMOS^{-1}: THE STRUCTURE AND A SOLUTION METHOD

The objective of RAMOS^{-1} is to choose resource configurations such that the patients generated in each i are in proportion to the relative needs of i. Restating the basic model we have,

$$T_{ij} = B_j D_j W_i e^{-\beta c_{ij}} \qquad \begin{array}{l} i = 1,\dots,m \\ j = 1,\dots,n \end{array} \qquad (10)$$

The predicted number of patients generated by i is hence,

$$\sum_j T_{ij} = W_i \sum_j B_j D_j e^{-\beta c_{ij}} \qquad (11)$$

Since W_i, an index of patient generating potential, is in this case the expected number of patients, the expression

$$\sum_j \frac{T_{ij}}{W_i} = \sum_j B_j D_j e^{-\beta c_{ij}} \qquad (12)$$

is therefore the ratio in i of the predicted to the expected. More importantly, it is also the ratio of the predicted service levels to the relative need, and, as we have defined it, the objective is to ensure that this ratio is constant in all origins i by choosing the appropriate values for D_j. However, this quantity cannot be calculated directly without *a priori* knowledge of the service prediction, $\sum_j T_{ij}$. Fortunately, it is completely analogous to base the estimation of this ratio on the total resources available in the system, Q, and W_i. Thus, a new term α is defined which is given by

$$\alpha = \frac{Q}{\sum_i W_i} \qquad (13)$$

This is simply the total resources divided by the total relative needs in the region of interest. If Q reflects resource availability over the whole country,and if the generating factors are based on the expected number of patients, then α will be one. If W_i is calculated in another way this result will not follow automatically.

Taking into account the constraints on change permitted at each destination, the reformulated problem can now be written as

$$\underset{D_j}{\text{Min}} \sum_i \left(\sum_j B_j D_j e^{-\beta c_{ij}} - \alpha \right)^2 = F \qquad (14)$$

subject to

$$D_{j(max)} \geq D_j \geq D_{j(min)} \qquad \forall_j \qquad (15)$$

and

$$\sum_{j \in L} D_j = Q \qquad (16)$$

This says: Choose D_j to minimize the square of the differences over all origins between the two ratios. The use of the "square" is to avoid (as in ordinary least squares regression) the problems with mixed negative and positive signs. The constraints are on each destination,and they are fixed as appropriate. The total resources,Q,can apply to the whole region, or to a subset L of it (see also section 7.3). If it is only a subset then the quantity $\sum_i W_i$ should apply over an equivalent subset. Putting,

$$B_j \, e^{-\beta c_{ij}} = \gamma_{ij} \qquad (17)$$

where

$$B_j = \frac{1}{\sum_i W_i \, e^{-\beta c_{ij}}} \qquad (2)$$

expanding (14), and ignoring the constant term $m\alpha^2$, where m is the number of origins, we obtain

$$F = \frac{1}{2} \underline{D}^T A \, \underline{D} - \underline{b}^T \underline{D} \qquad (18)$$

where $\underline{D}$ and $\underline{D}^T$ is the vector of resources and its transpose.

$$\underline{D} = \begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ D_j \\ \vdots \\ D_n \end{bmatrix} \quad \text{and} \quad \underline{D}^T = \left[D_1 \; D_2, \ldots, D_j, \ldots, D_n \right] \qquad (19)$$

A is a symmetric matrix composed of the following elements,

(18) is A itself. This matrix is positive definite in all but a few very special cases that are unlikely to be met in practice. This indicates that global minima are obtainable for the range of use anticipated.

There are a number of methods for solving quadratic programming problems based on linear programming techniques (Beale, 1967; Dantzig, 1963). The present method uses an algorithm by Fletcher (1970, 1971) which avoids this dependence. The second reference provides further details. Briefly, however, the algorithm operates by retaining a basis of inequality constraints (active constraints) which are treated as equalities. The current approximation of $\underline{D}$ is the minimum subject to these constraints and is a feasible point. A systematic adjustment of the basis then follows until $\underline{D}$ becomes the required solution.

A geometric depiction gives an intuitive feel for the problem in the simplest of cases: 2 origins and 2 destinations. In Figure 3a, D_1 and D_2 are plotted on the horizontal and vertical axes. Values of F are represented as contours in the plane. AB is the resource constraint Q. A miniumum of F is found when the vector of first derivatives disappears. In the general case, this is

$$g = \nabla(\tfrac{1}{2}\underline{D}^T A\underline{D} - \underline{b}^T\underline{D}) = A\underline{D} - b = \underline{0} \tag{24}$$

For the 2 x 2 case, it is

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} - \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{25}$$

$$A = \begin{bmatrix} 2\sum_i \gamma_{i1}^2 & 2\sum_i \gamma_{i1}\gamma_{i2} & \cdots & \cdots & 2\sum_i \gamma_{i1}\gamma_{in} \\ 2\sum_i \gamma_{i2}\gamma_{i1} & 2\sum_i \gamma_{i2}^2 & \cdots & \cdots & 2\sum_i \gamma_{i2}\gamma_{in} \\ \vdots & & & & \vdots \\ 2\sum_i \gamma_{ij}\gamma_{i1} & \cdots & 2\sum_i \gamma_{ij}^2 & \cdots & 2\sum_i \gamma_{ij}\gamma_{in} \\ \vdots & & & & \vdots \\ 2\sum_i \gamma_{in}\gamma_{i1} & \cdots & \cdots & \cdots & 2\sum_i \gamma_{in}^2 \end{bmatrix} = \left\{a_{ij}\right\} \tag{20}$$

$\underline{b}^T$ is the transpose of the vector $\underline{b}$ in which the elements are

$$\underline{b} = \begin{bmatrix} 2\alpha\sum_i \gamma_{i1} \\ 2\alpha\sum_i \gamma_{i2} \\ \vdots \\ 2\alpha\sum_i \gamma_{ij} \\ \vdots \\ 2\alpha\sum_i \gamma_{in} \end{bmatrix} = \left\{b_j\right\} \tag{21}$$

Similarly (15) and (16) can be written in matrix notation

$$\underline{D}_{min} \geq \underline{D} \geq \underline{D}_{max} \tag{22}$$

and

$$\underline{C}^T\underline{D} = Q \tag{23}$$

where $\underline{C}^T$ is a 1 x n vector transpose with all the elements set equal to one. Equations (18), (22), and (23) have now been put into the standard form expected by a general quadratic programming algorithm. The matrix of second derivatives or Hessian of

Matrix equation (25) is represented in the diagram by two straight lines EF and GH along which $\partial F/\partial D_1 = 0$ and $\partial F/\partial D_2 = 0$. Where they intersect is the absolute minumum of F and hence the required solution. Through this point, too, must pass $D_1+D_2=Q$, the resource constraint. Solving this minimum in the two destination case, we find

$$D_1' = \frac{(a_{22}b_1 - a_{12}b_2)}{(a_{11}a_{22} - a_{12}a_{12})} \qquad (26)$$

$$D_2' = \frac{(a_{11}b_2 - a_{12}b_1)}{(a_{11}a_{22} - a_{12}a_{12})} \qquad (27)$$

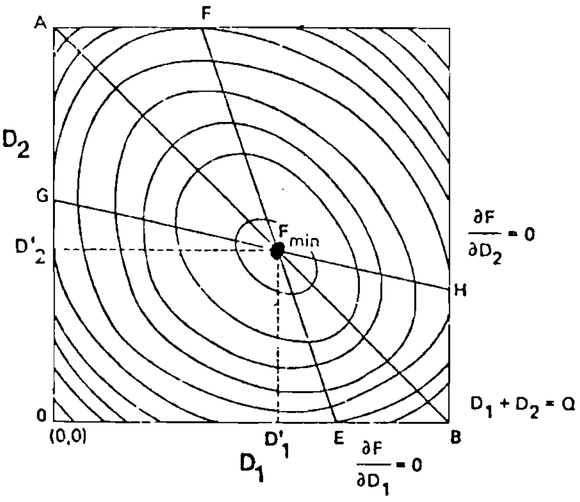
where D_1' and D_2' are the coordinate values of F_{min} in Figure 3a. For the equality of resource allocation in each destination (i.e. $D_1 = D_2$), it is seen that this occurs when

$$\frac{b_1}{b_2} = \frac{(a_{11} + a_{12})}{(a_{22} + a_{12})} \qquad (28)$$

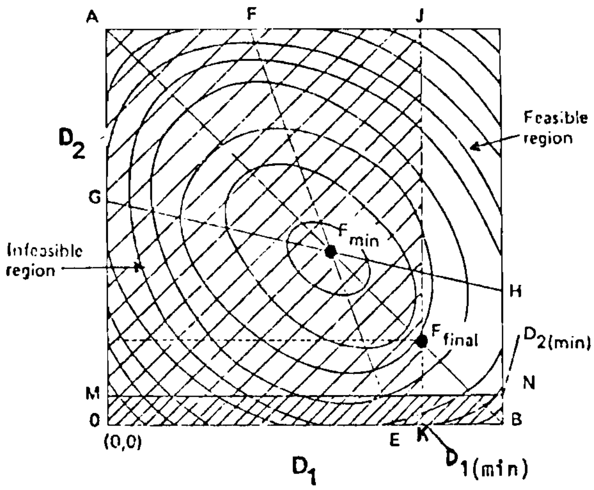
On the other hand, for a value of D_1 or D_2 equal to zero, F_{min} is simply a corner point (either B or A). A further result is that, if there are no active constraints on D_1 or D_2 other than (23), proportionate increases/decreases in Q cause proportionate increases/decreases in D_1 and D_2 (Figure 3a). This may be verified by writing equations (26) and (27) explicitly, and observing that Q acts in both cases as a scaling factor.

In the case when lower bounds apply to each destination value, the plane is divided into a feasible and an infeasible region (shaded) as shown in Figure 3b. It is seen that only the constraints fixing the lower bound of D_1 and the total resources Q are active. Here, the constraints on destination allocations are represented by the vertical and horizontal lines $JK(D_1)$ and $MN(D_2)$. The minimum still lies along AB (the Q equality constraint), but it cannot go lower than F_{final} since at this point $D_{1(min)}$ becomes active. The required resource allocations are thus the coordinates (D_1', D_2') of F_{final}.

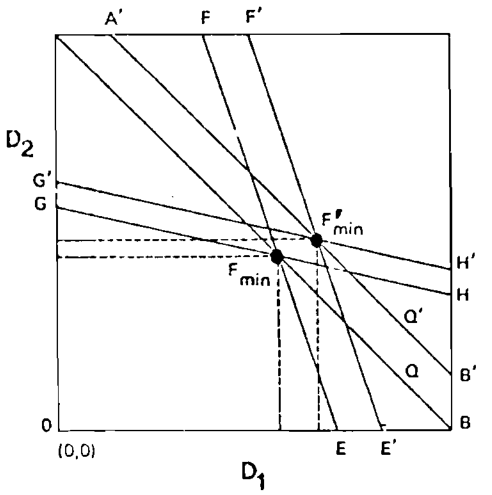
a) Unconstrained solution



b) Constrained solution



c) Solution when Q increases to Q'



KEY

Q = Total resource constraint;

F_{min} = Absolute minimum of objective function;

F_{final} = Constrained minimum of objective function;

D'_1, D'_2 = Solution values to the destination case allocations

Figure 3. Three solutions for the 2-origin, 2-destination problem.

All of these results may be generalized to many destinations. Their main value is to show precisely how the model is working. More important,however, are the empirical results and whether the solutions to equation (14) are demonstrably more efficient than those obtained by trial and error (i.e. by using RAMOS).

7. RAMOS^{-1}: THE APPLICATION

In this section we apply RAMOS^{-1} to four, completely hypothetical and somewhat exaggerated planning scenarios and then analyze the results. Other scenarios which stretch the performance of the model either more or less have been chosen and solved with comparable effectiveness. Also discussed is the important issue of the sensitivity of the model's predictions to changes in the parameter value β. This is examined below (section 7.6.). It should be stressed, however, that despite very encouraging results in all applications to date, these are still the early stages in the development of RAMOS^{-1}. A sample of the computer output is shown in Appendix C.

7.1. The London Problem

The inner parts of London, as in many cities, have been experiencing gradual depopulation for many years. The impact that this has had on hospital services can be gauged from Figure 4, which shows the change in the distribution of hospital beds between 1951 and 1971. It is true to say that until recently few of the plans for reallocations which have taken place in this period have paid proper regard to complex interaction effects between the supply of in-patient hospital services and their demand in each part of the city (RAWP, 1976), so that according to the principle of 'relative need' at least, the resultant pattern of services has left much to be desired. On the other hand, London is a national and international center for medical and dental education and research which depends very heavily on the existing hospital system for teaching needs. This is a most important consideration. In spite of this and other reasons, there is substantial pressure on the Regional Health Authorities responsible for the city to reduce the level of acute services, and to develop instead services in the areas surrounding

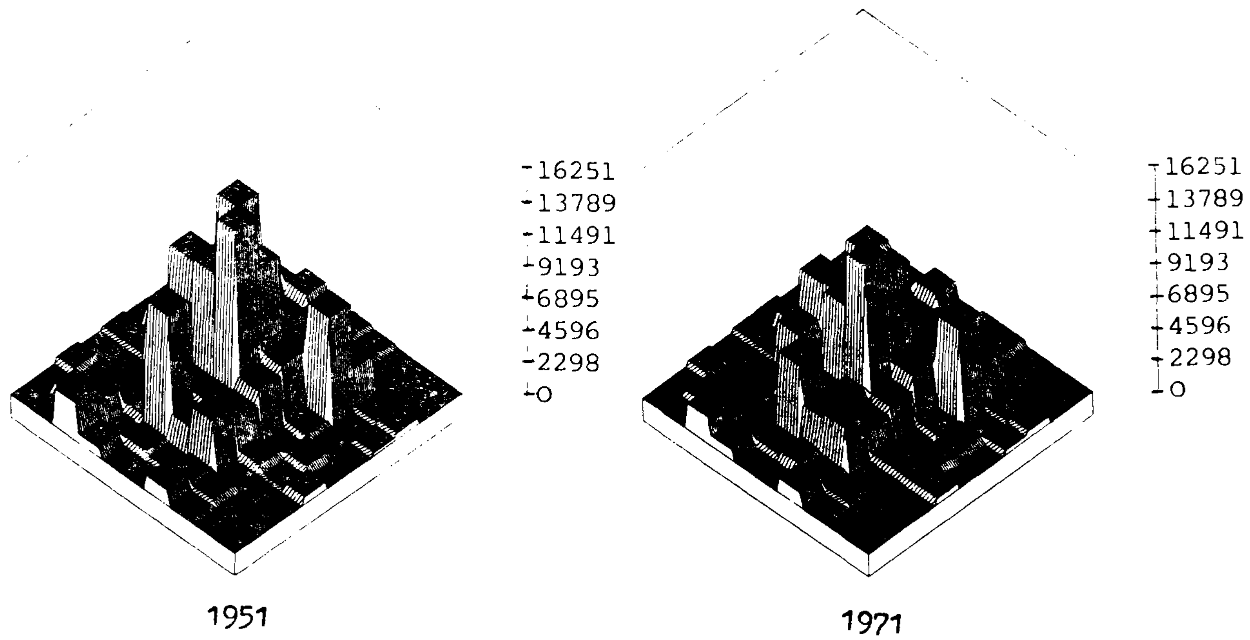


Figure 4. The distribution of hospital beds in 10^2 sq.km. cells over the London region, 1951 and 1971. The scales are in units of hospital beds. (Source: Mayhew, 1979:308)

London and elsewhere. Nevertheless, it should be noted that despite this pressure small increases in total resource availability, Q, still cannot be entirely ruled out because of the throughput effect caused by falling lengths of stay. These considerations are broadly reflected in the chosen scenarios.

7.2. The Scenarios

The four scenarios assume the following:

1. No change in the patient generating potentials or the resource level Q; a permissible maximum loss of resources not exceeding 25% in each destination; and no upper bounds on the destination gains
2. No change in patient generating potential; a 10% decrease in Q; a permissible loss of resources not exceeding 25% in each destination; and no upper bounds on gains
3. No change in patient generating potential; a 10% increase in Q; a permissible loss of resource not exceeding 5% in each destination; and no upper bounds on gains
4. A large increase ($+10^4$) in the patient generating potential in outer parts of the city; a 10% increase in Q; a permissible loss of resources not exceeding 25% in each destination; and no upper bounds on gains

Scenario *1* is designed to test the current resource configuration against the outputs of the model. The 25% maximum permissible reduction in each destination is arbitrary and used simply for test purposes. Scenarios *2* and *3* look at the implications of a resource decrease or increase, while in *3* the constraints have been made more stringent than before to see how much improvement results when the permitted change is small. In *4* the pfg (W_is) and the resource levels are readjusted (by a particularly large amount for the pgfs to see what the model does when it is "stretched"). In none of the scenarios has the upper bound been fixed. This is in order to find where the maximum shortfalls in resources exist.

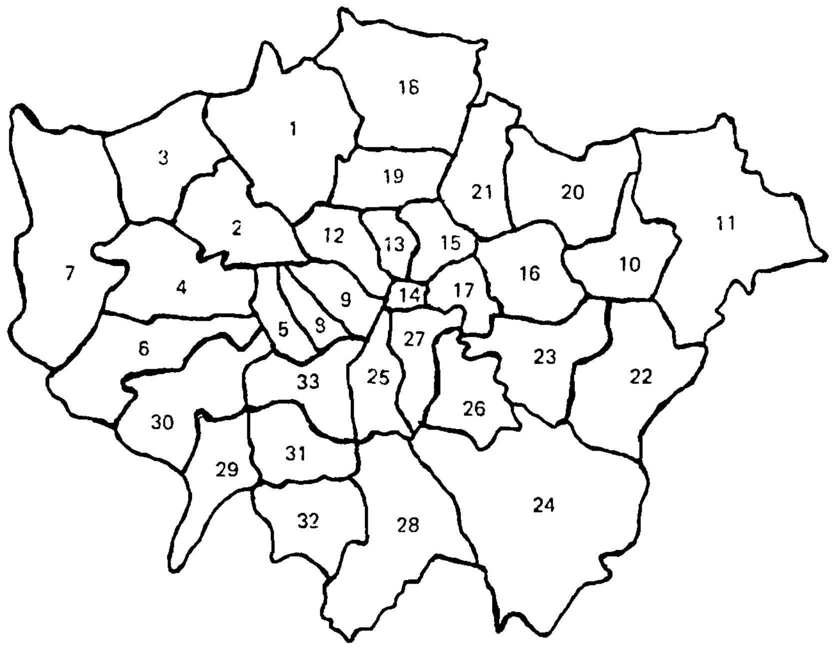
7.3. Interpretation of the Outputs

The outputs from RAMOS^{-1} will be interpreted in the forms of a map, two scattergrams and a bar-chart of which the scattergrams are the most important for validating the solution. These graphs show the relationship -- both before and after the application of RAMOS^{-1} -- between the patients generated in i (i.e., $\sum_j T_{ij}$) and the relative need in i scaled by (i.e., αW_i). If the method is successful, a linear equation fitted to the scatter should always return a slope of one with a zero intercept. This indicates that a one to one correspondence has been obtained in all origin zones as a result of the resource reallocation process. If the coefficient explanation R^2 is one too, then the objective function we have set has been met exactly. Experience has shown that the required parameters of the equation nearly always result to within the acceptable limits of error. Slope bias sometimes occurs in cases where a particular constraint -- say on large external zones -- dominates the others to a large degree. The solution to this problem has been to redefine Q so that it applies only to over a subset of the region, such as the study area of interest (see below, section 7.5). The value of R^2 obtained depends on the severity of the constraints. If they are not particularly stringent, it will be close to one. Nonetheless, it is interesting that even with stringent restrictions (as in scenario 3), worthwhile results are still obtained.

7.4. The Results

The model was applied to 33 origin zones (administrative boroughs) and 36 destination zones (Health Districts) in the area covered by the Greater London Council (GLC) (Figure 5). The destination zone numbering system differs from that used in Mayhew and Taket (1980) and a new key is attached (Table 1). The area outside the GLC is now represented by one very large origin and destination zone. This reduction was found to be necessary in developmental runs of the model to reduce the

A) Origin zones



B) Destination zones

Figure 5. The Greater London Council: definition of zones.

Table 1. Key to Figure 5.

Origin		Destination	
1 Barnet	24 Bromley	1 Barnet	24 West Roding
2 Brent	25 Lambeth	2 Edgware	25 Bexley
3 Harrow	26 Lewisham	3 Brent	26 Greenwich
4 Ealing	27 Southwark	4 Harrow	27 Bromley
5 Hammersmith	28 Croydon	5 Hounslow	28 St.Thomas†
6 Hounslow	29 Kingston	6 South Hammersmith	29 Kings
7 Hillingdon	30 Richmond	7 North Hammersmith	30 Guys
8 Kens & Chelsea	31 Merton	8 Ealing	31 Lewisham
9 Westminster	32 Sutton	9 Hillingdon	32 Croydon
10 Barking	33 Wandsworth	10 KCW Northwest*	33 Kingston
11 Havering	34 Other	11 KCW Northeast	34 Roehampton
12 Camden		12 KCW South	35 Wandsworth/East Merton
13 Islington		13 Barking	36 Sutton
14 City		14 Havering	37 Other
15 Hackney		15 North Camden	
16 Newham		16 South Camden	
17 Tower Hamlets		17 Islington	
18 Enfield		18 City	
19 Haringey		19 Newham	
20 Redgridge		20 Tower Hamlets	
21 Waltham Forest		21 Enfield	
22 Bexley		22 Haringey	
23 Greenwich		23 East Roding	

*K/C/W = Kensington, Chelsea, and Westminster

†Destinations 28, 29, 30 are named after teaching hospitals within the districts.

computational size of the problem. The parameter value β used in the experiments described is calibrated from 'Matrix 3', the details of which are also in the last reference. It equals 0.367. Figure 6, which is a plot from the same reference of predicted flows $\{T_{ij}\}$, on actual flows $\{N_{ij}\}$, shows the goodness-of-fit obtained with this matrix during the calibration stage of RAMOS.

The GLC has a population of about seven million, and the hospitals within it treated 923,618 in-patient cases in 23 acute specialties in 1977. The ratio α of Q to the total patient generating potential in this area is 1.57. (The external zone, 34, is not included in the definition of α.) This figure is used in the no-change scenario (scenario *0*) and as a scaling factor in the 'before' graph which is the same in all cases. Also presented in the results are the final values of the objective function and the theoretical minima. Because we ignore the constant term, the function [see equation (18)] has a theoretical minimum of not zero but $-m\alpha^2$, where m is the number of origins. In scenario *1*, therefore

$$F_{min} = -m\alpha^2 = 34(1.57)^2 = -83.81 \qquad (29)$$

7.5. Scattergrams: Predicted Patients versus Relative Need

Table 2 shows the main results of the model. As is seen, a clear improvement over the current resource configuration (scenario *0*) is obtained in each of the four scenarios. The slope values b (the associated "t"-test statistics are in brackets underneath) are all extremely close to the desired value of one; while the intercept values are not statistically significantly different from zero. In other words, the model has performed as it should in each case. The associated scattergrams for each scenario shown in Figures 7 to 11 give added confirmation of this. From scenario *3*, it is seen that, as expected, the more stringent the constraints, the lower the value of R^2. Also the objective function F_{final} is more distant from its theroetical minimum. Finally, the reallocations

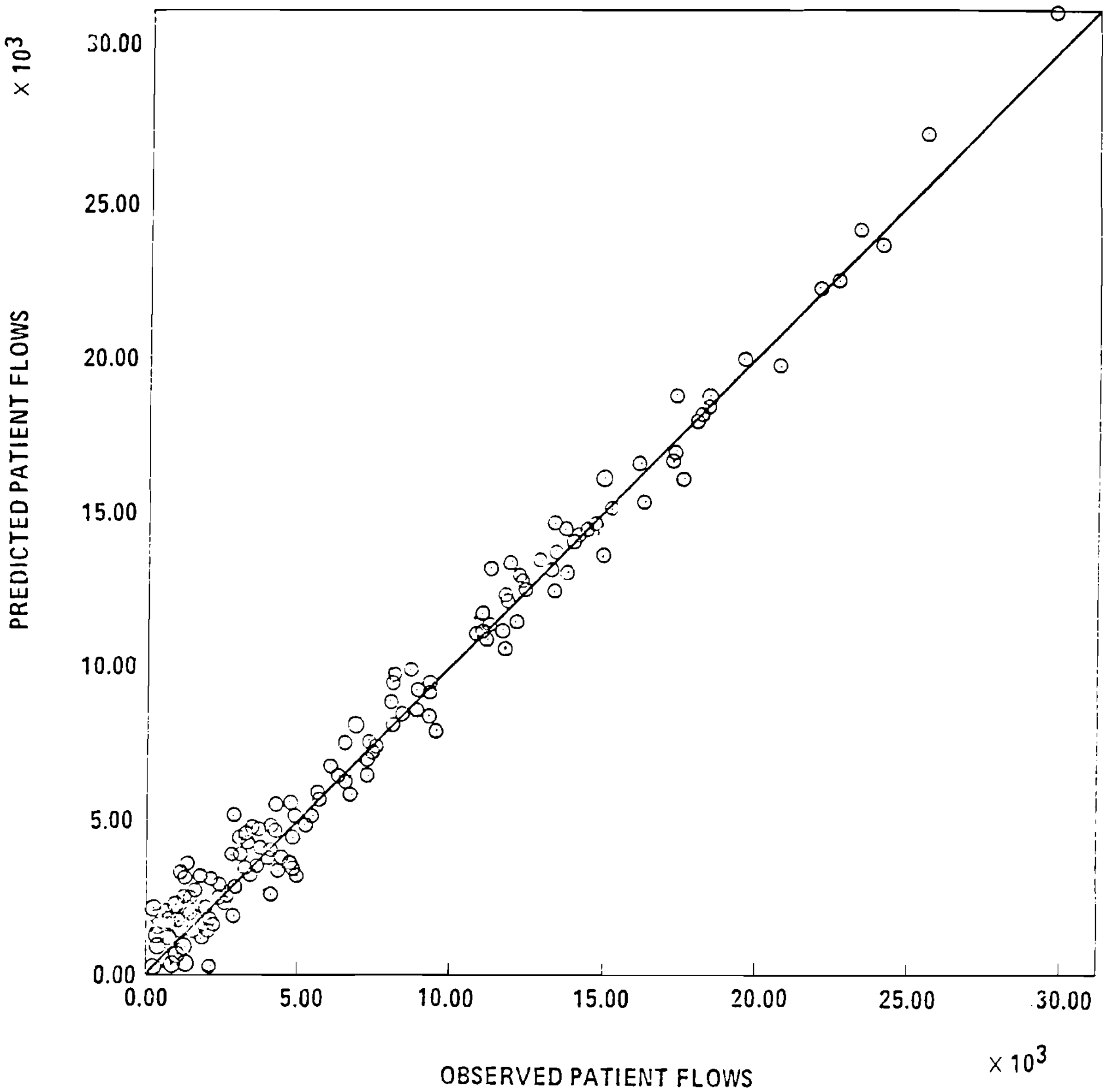


Figure 6. "Four Thames Regions" Model: A plot of prediction on expected patient flows for each origin-destination pair.
(Source: Mayhew and Taket, 1980:38)

Table 2. Results from $RAMOS^{-1}$: the scenarios.

Scenario	Q	α	F_{final}	F_{min}	$\hat{b}$	$\hat{a}$	R^2	$\bar{c}$
0	923618	1.57	-61.02	- 83.81	0.892 (9.4)	3029.8 (0.9)	0.70	6.73
1	923618	1.57	-74.61	- 83.81	0.993 (52.2)	207.9 (0.38)	0.99	6.38
2	831256	1.41	-57.73	- 67.60	0.962 (29.2)	963.2 (1.1)	0.96	6.44
3	1015979	1.73	-84.32	-101.76	0.937 (29.3)	1926.8 (1.86)	0.96	6.50
4	1015979	1.52	-70.02	- 78.55	0.994 (90.4)	191.5 (0.52)	1.00	6.33

KEY

$\beta = 0.367$ $\bar{c}$ = mean accessibility costs

$y = \hat{b}x + \hat{a}$ where $\hat{b}$ = slope and $\hat{a}$ = intercept

$F_{min} = -m\alpha^2$ the absolute minimum of the objective function

R^2 = coefficient of explanation

Figures in brackets are "t"-test statistics.

Number of observations = 33

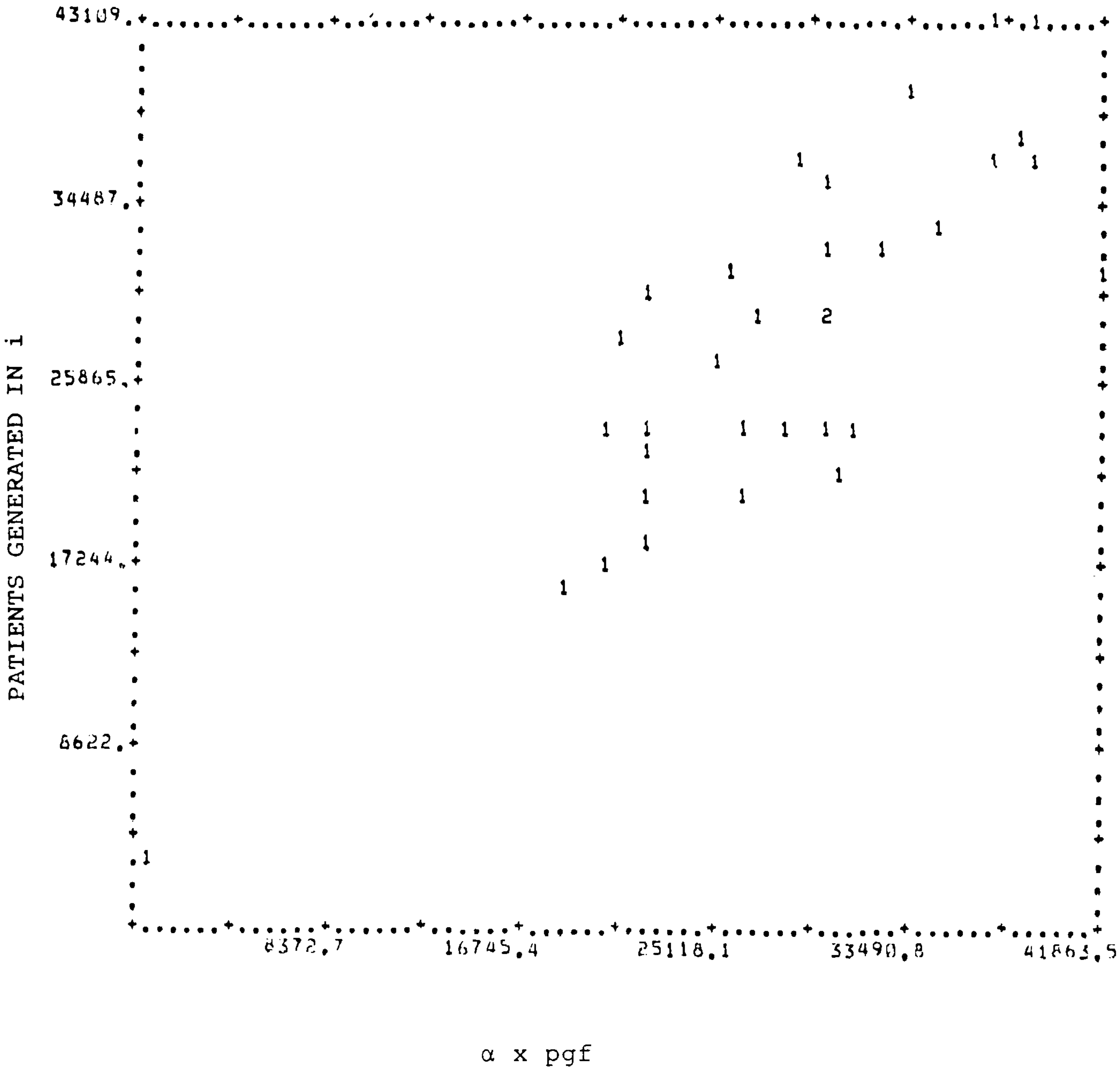


Figure 7. Scenario *0*: Existing allocations before the application of RAMOS^{-1}.

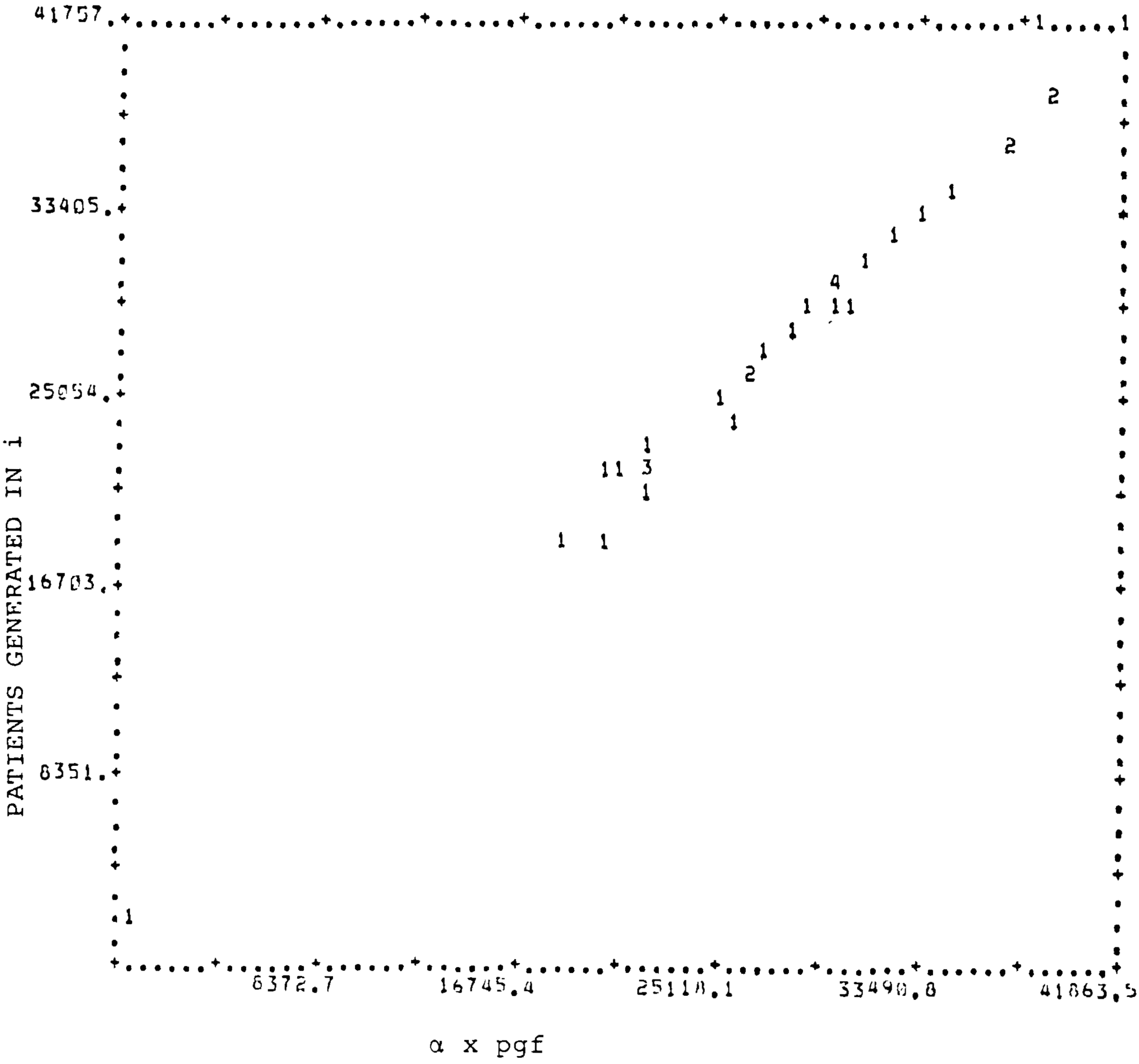


Figure 8. Scenario 1: Existing allocations revised by RAMOS^{-1}.

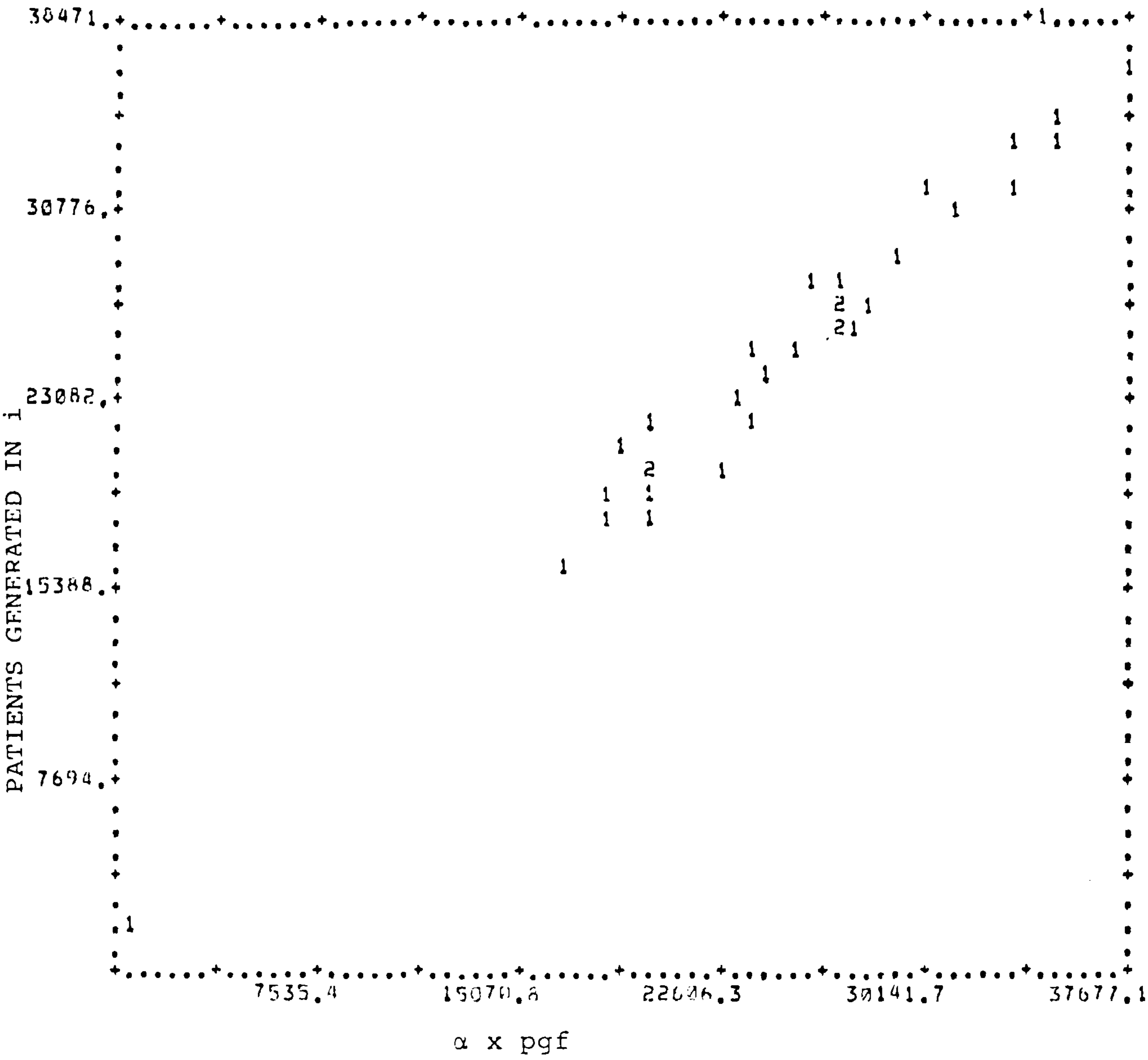


Figure 9. Scenario 2: A decrease in resources applying $RAMOS^{-1}$.

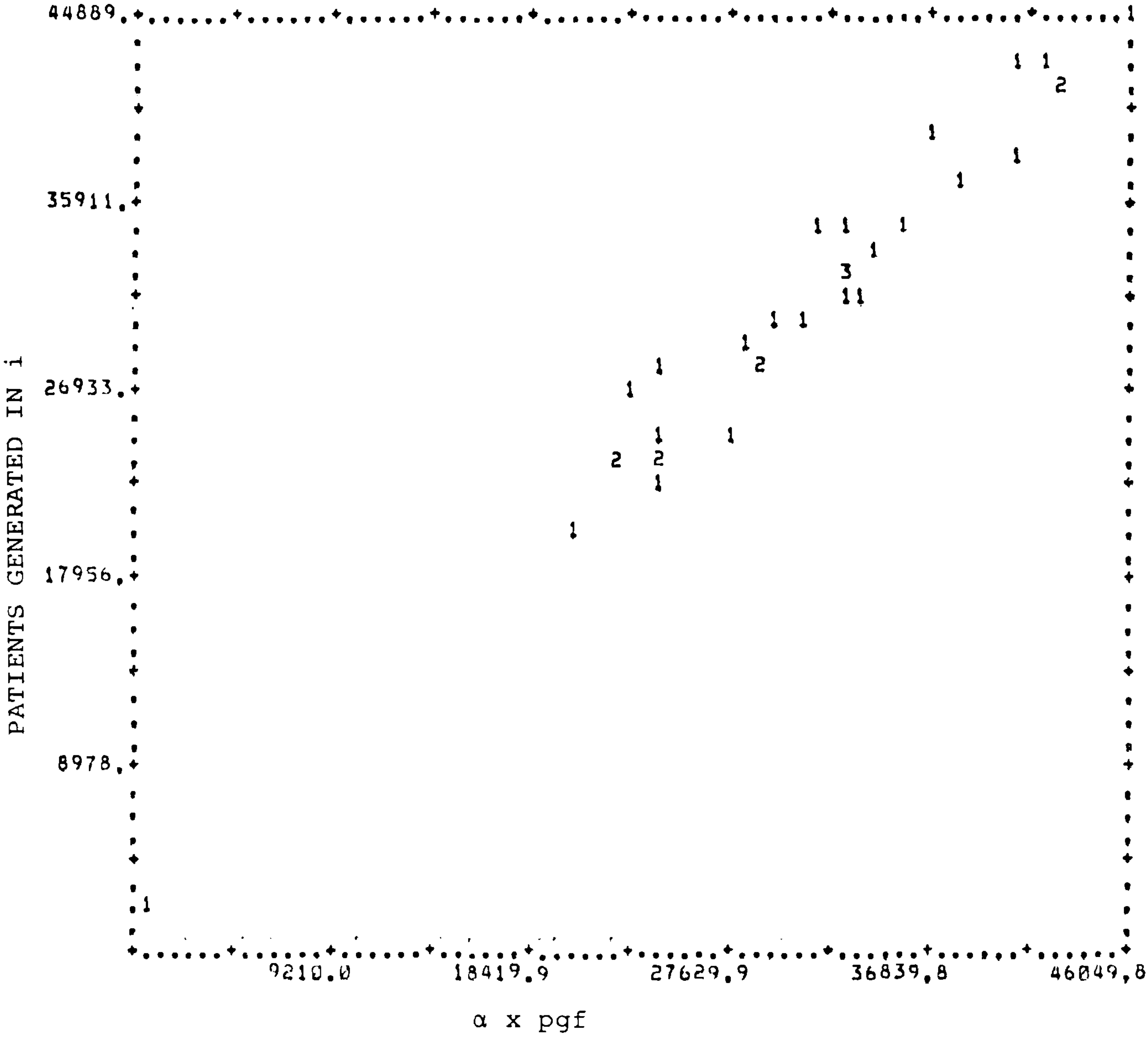


Figure 10. Scenario 3: An increase in resources applying $RAMOS^{-1}$.

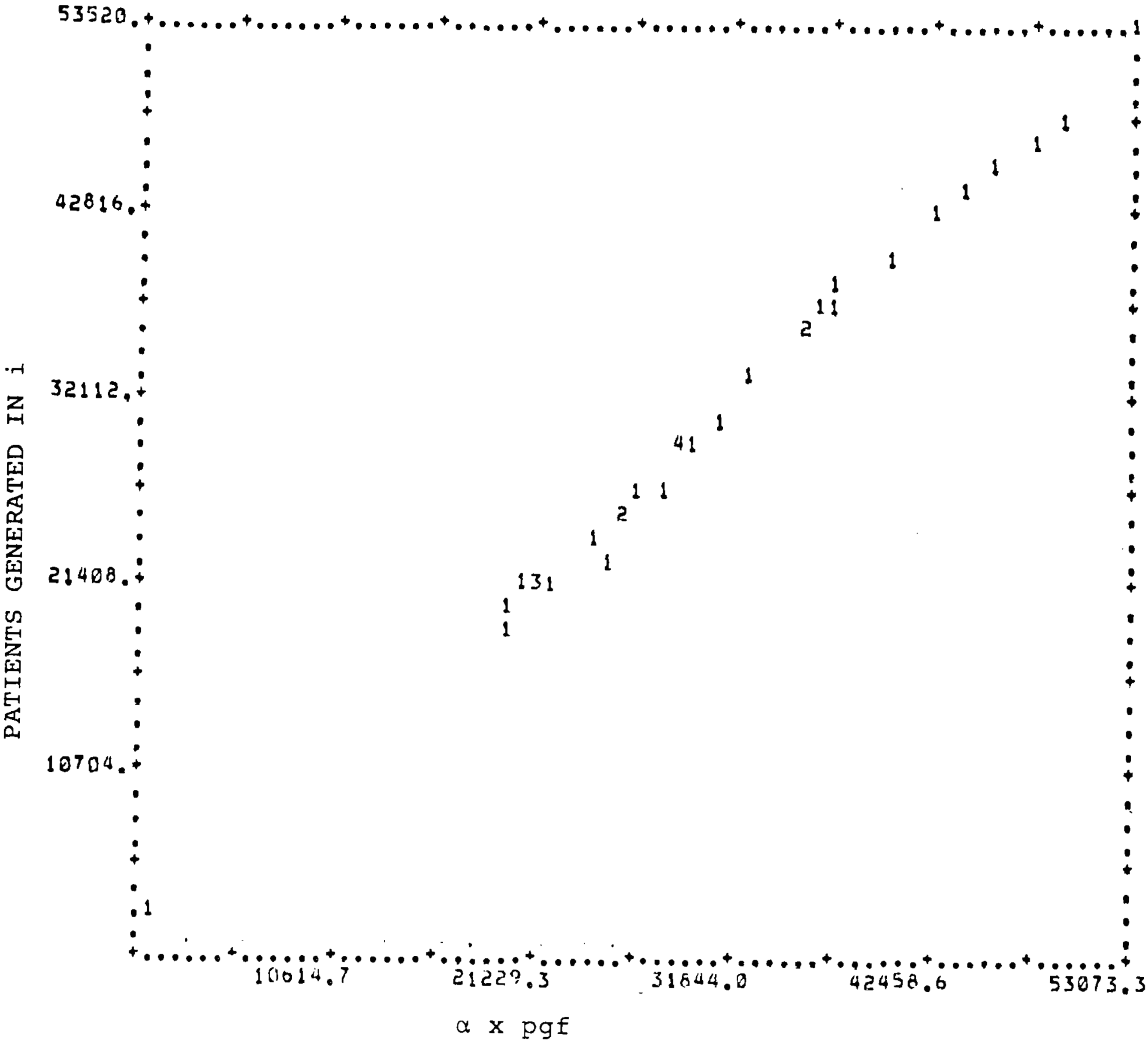


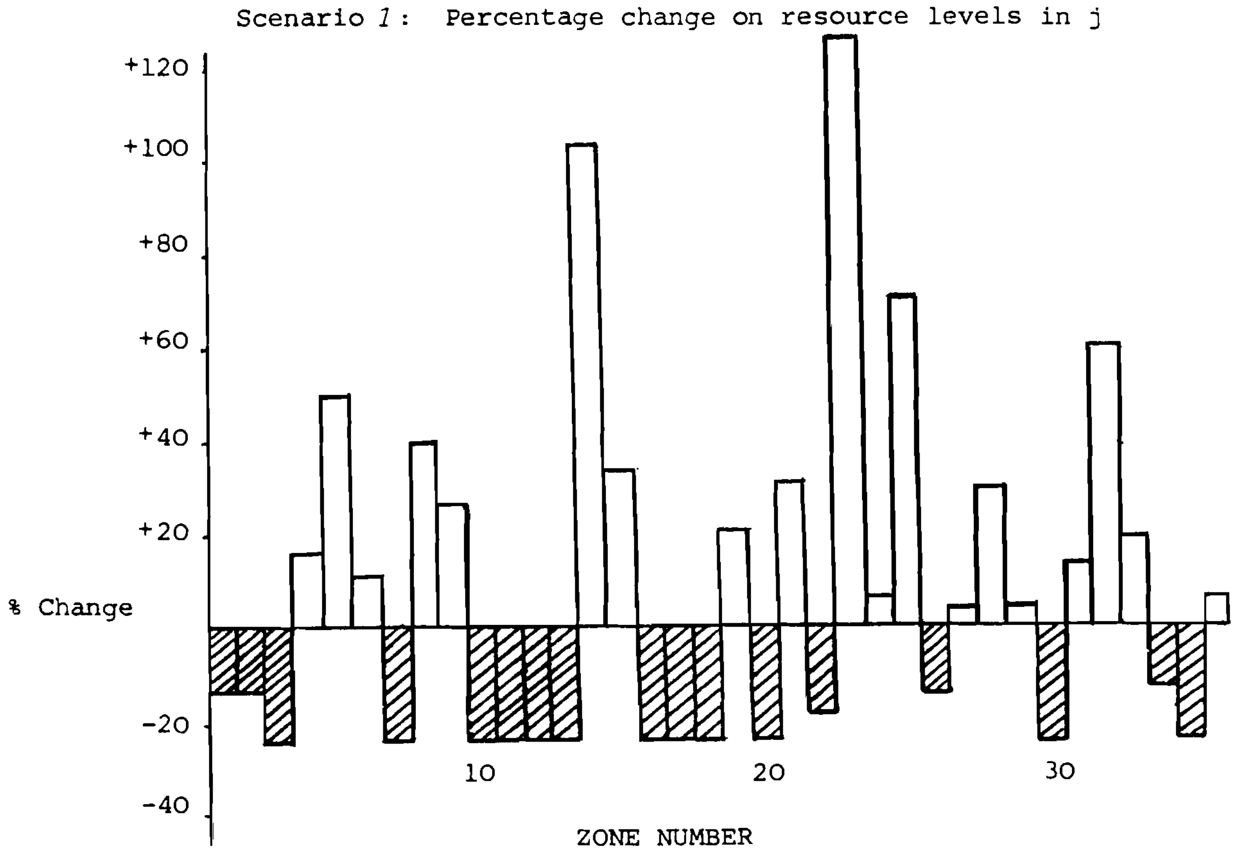
Figure 11. Scenario *4*: An increase in resources and patient generating potential applying $RAMOS^{-1}$.

recommended by the model result in small decreases in the average accessibility costs to the population in all these examples.

7.6. Bar Charts: Percentage Changes in Origins and Destinations

Figures 12 to 15 are bar charts showing the implied percentage change in the resources allocated to each destination recommended under the four scenarios, together with the predicted percentage change in the patients generated in each origin.

A comparison with the maps in Figure 5 shows where the main changes are taking place. In all scenarios (regardless of an increase or a decrease in Q) a loss of resources up to the permitted maximum occurs in the inner-most destination zones: 10, 11, 12, 16, 17, 18, 20, and 30, but interestingly not in 28 which is elongated in shape and has one foot almost in the suburbs. The maximum gains occur in the outer-most zones, particularly in 5, 14, 23, 24, and 32. The picture presented, therefore, is one of resource transferral from the center to the periphery of the city. There are two exceptions to this, however, which are evident from comparisons with the no-change scenario (Figure 10). These are zones 1 and 2 which experience small decreases. A similar set of exchanges takes place with regard to patients generated, but in percentage terms the adjustments are generally much smaller. For recipient zones where the resource gains are very large, there would appear to be a *prima facie* case for a substantial enlargement of the existing hospital facilities. In scenario *1*, for instance, increases of over 60% are given to zones 14, 23, 25, and 32. These trends are emphasized when the resource level Q is decreased (scenario *2*) and increased (scenario *3*) but in reverse directions. In scenario *3*, it is interesting to note that substantial improvements are possible, when zones are asked to give up a maximum of only 5% of their resources assuming a 10% increase in Q overall. In scenario *4*, very large ($+10^4$) additions are made to the pgfs in origin zones 1, 2,7,11,18,22,29,and 32.All of these except 2 are at the edge of the



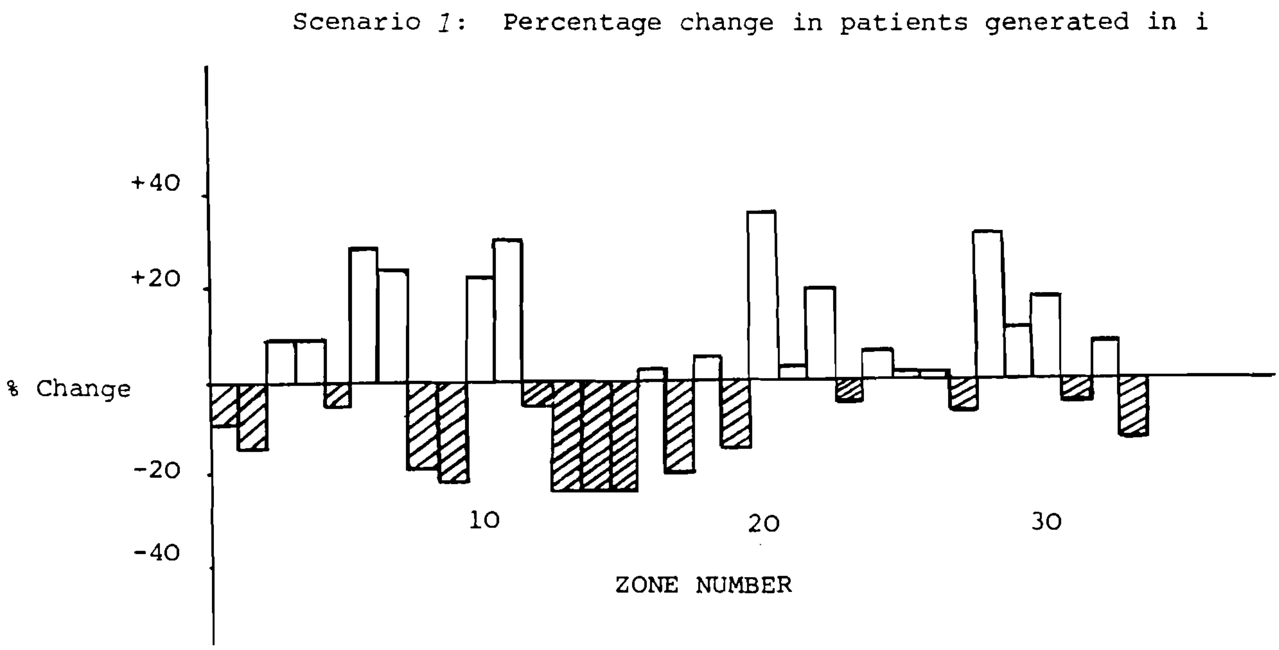
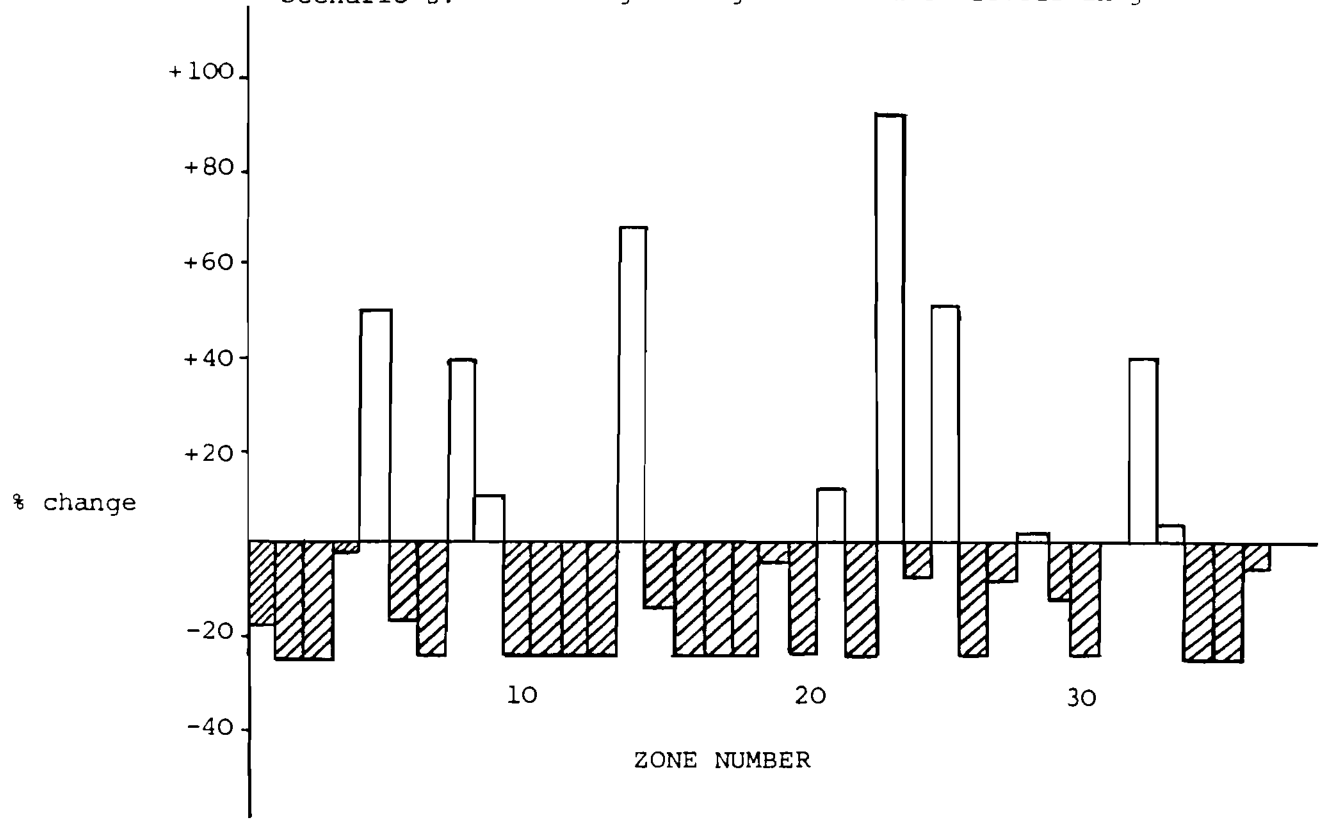


Figure 12. Predicted changes in resource levels and patients generated: Scenario *1*.

Scenario 2: Percentage change in resource levels in j



Scenario 2: Percentage change in patients generated in i

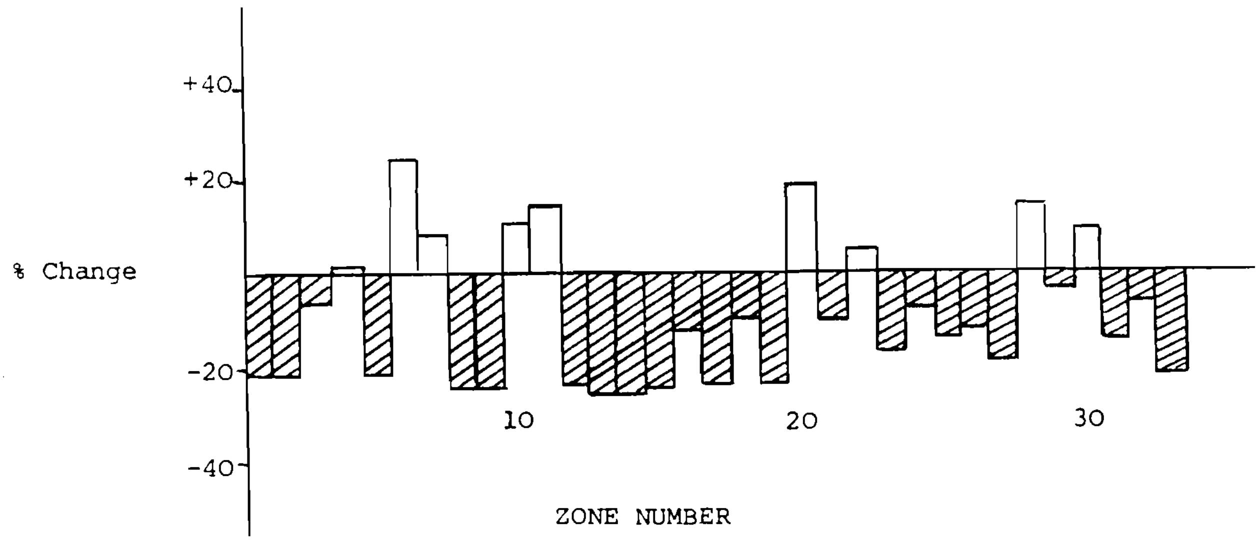
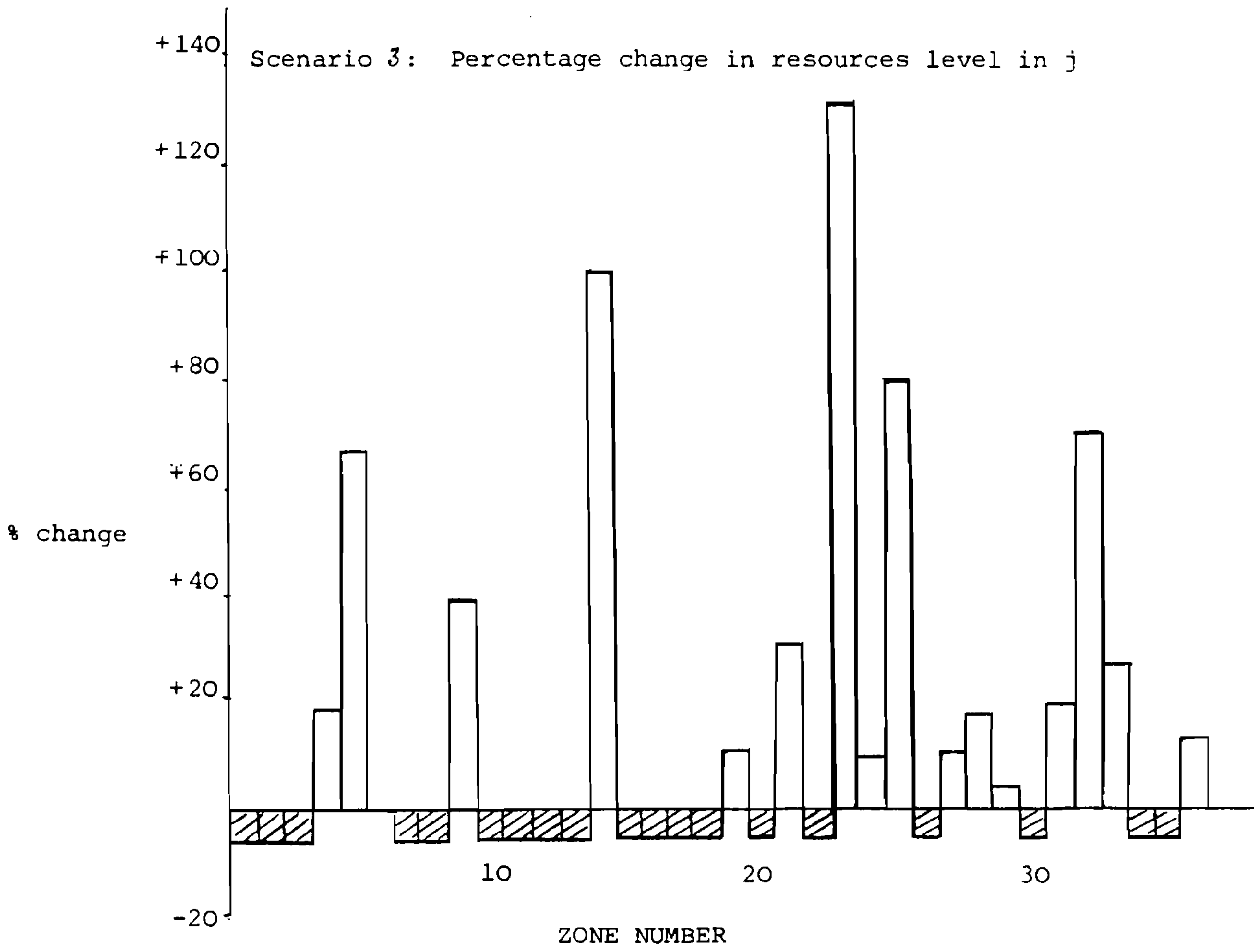


Figure 13. Predicted changes in resources and patients generated: Scenario 2.



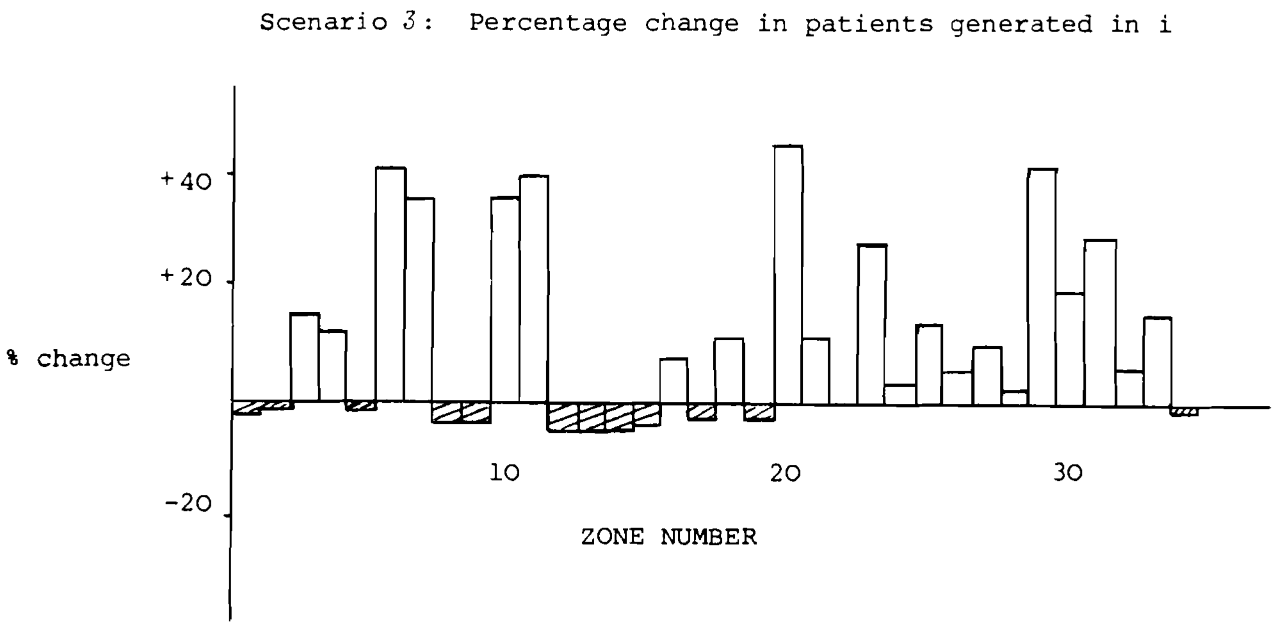
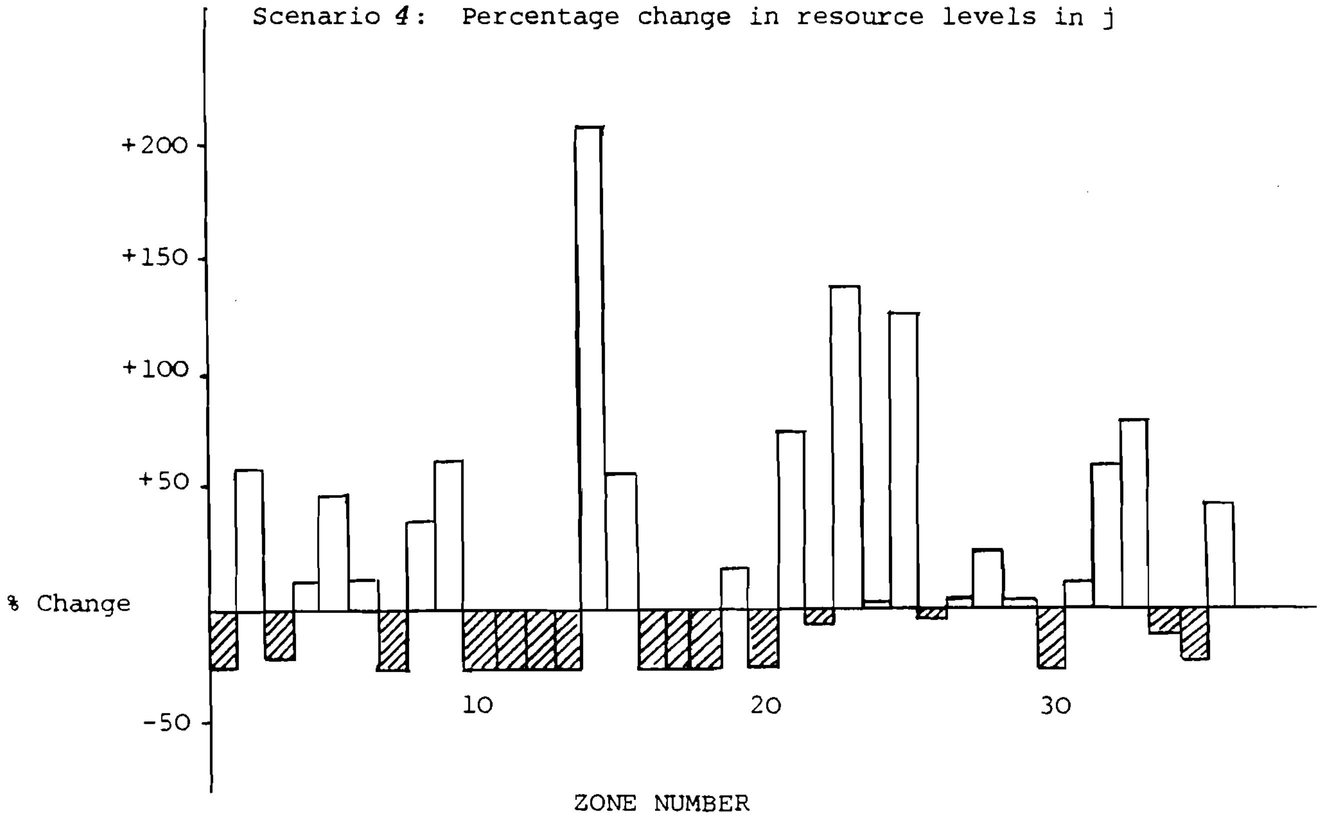


Figure 14. Predicted changes in resource levels and patients generated: Scenario 3.



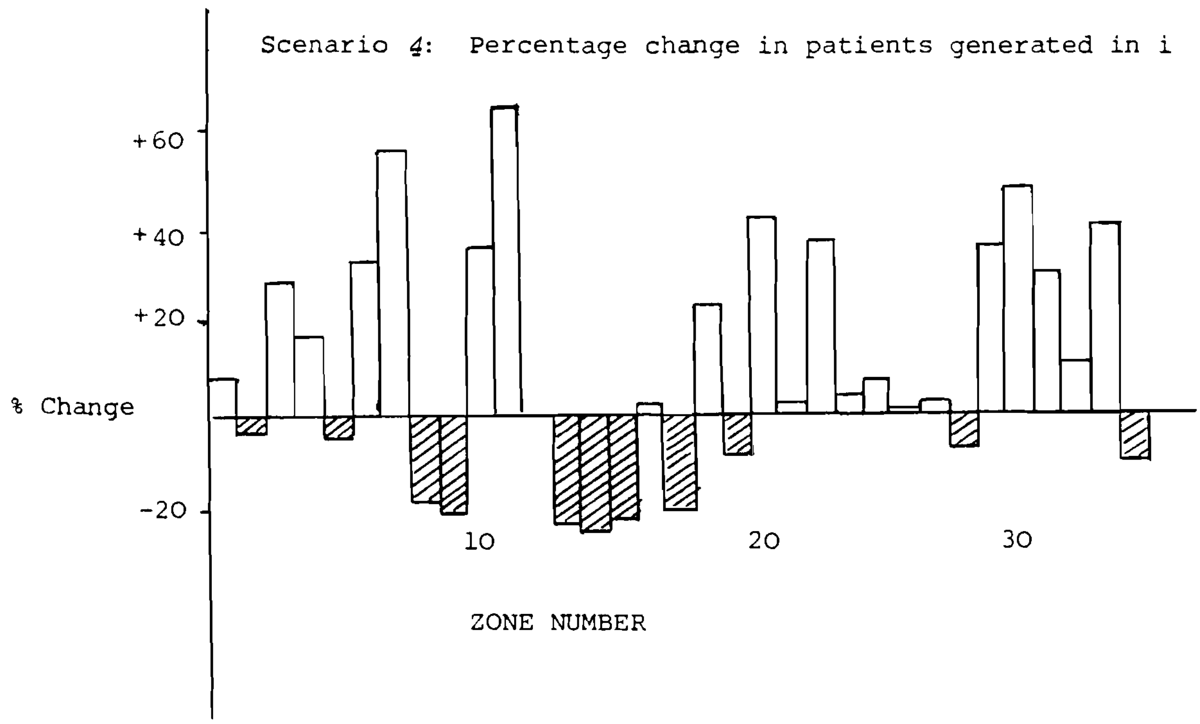


Figure 15. Predicted changes in resource levels and patients generaged: Scenario 4.

region. Assuming again a 10% increase in the total resource levels, the results from the model are again reasonable. Very large reallocations go to destination zones in the area of these origins -- notably in destination zones 2,9,14,21,23,25,32,and 33. Exceptions are zones 3 and 1. The impression drawn from these scenarios, therefore, is that the method is well-behaved, and its results are logically consistent and intuitively reasonable.

7.7. Catchment Populations

The revised resource allocations under each scenario naturally cause changes in the sizes of the populations dependent on each destination as well. These dependent populations are called "catchment populations", and they have been defined as follows (Mayhew and Taket, 1980:22):

$$C_j = \sum_i E_{ij} P_i \qquad (30)$$

where C_j = the population dependent on hospitals in zone j

P_i = the resident population of i

and

$$E_{ij} = T_{ij} / \sum_i T_{ij} \qquad (31)$$

Figure 16 is a bar chart of the percentage changes in catchment populations that result from the resource reallocations in scenario 3. A comparison with Figure 14 shows that increased catchment populations are usually associated with increased resource allocation, but that the proportionate change is generally less. Conversely, decreases in resource allocation (maximum -5%) can cause proportionately larger decreases in catchment populations. For example, destination zone 13 loses much of its catchment population to zone 14 in which a large increase in resources is proposed. In all these scenarios, however, the main trend is towards

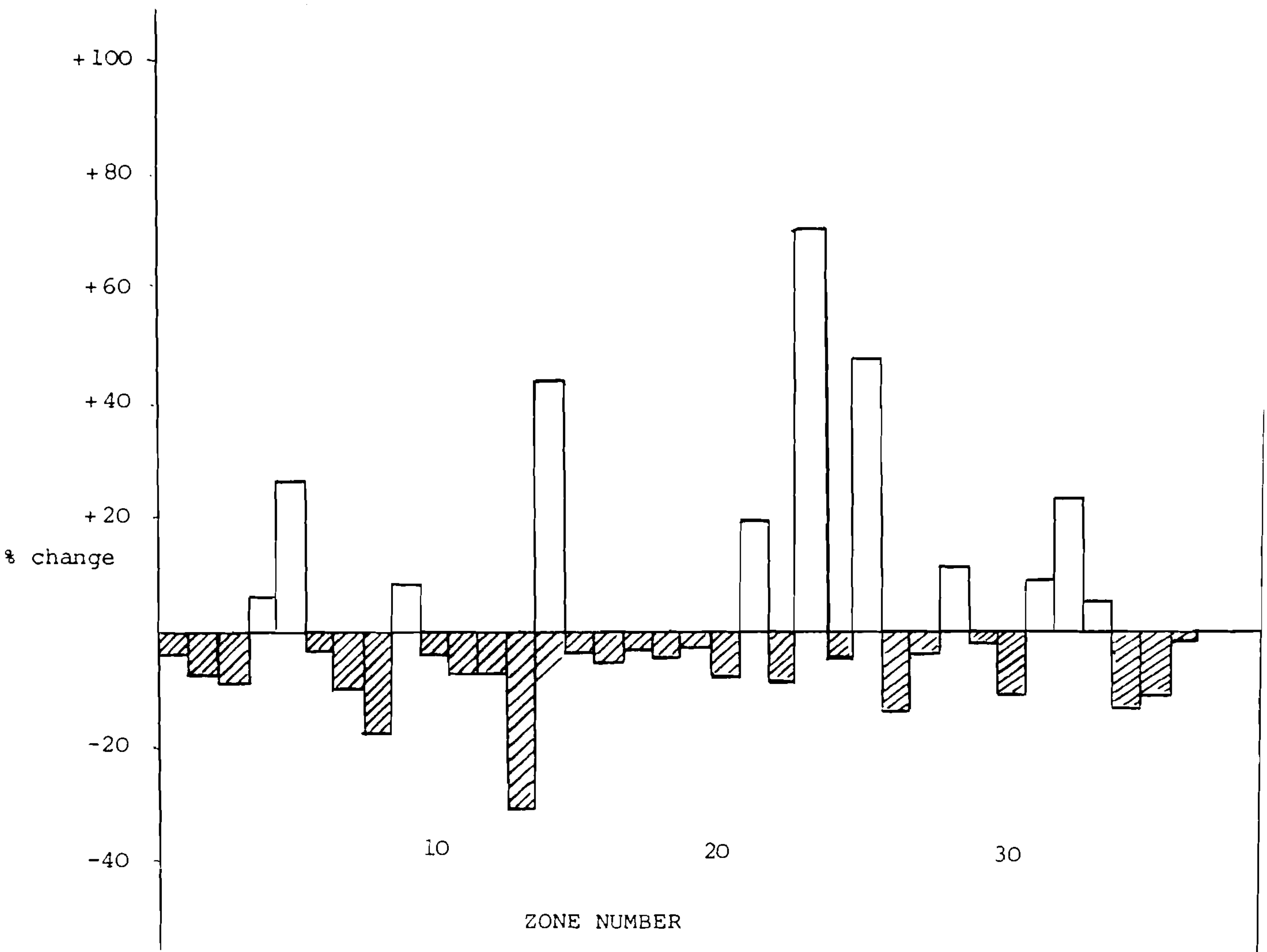


Figure 16. Changes in catchment populations resulting from Scenario 3.

the redirection of many patients, who would previously have gone to the central hospitals for treatment, to the expanded facilities in the suburbs. This tendency is reflected in the geographical distribution of catchment population changes which generally show an increase in the peripheral destinations.

7.8. Sensitivity Analysis

In this section the sensitivity of the resource allocations is examined with respect to β. This is the parameter on which the model is calibrated. In a typical forecasting period β is assumed to change little or not at all. Here, however, we introduce changes over a large range of values to see how the model performs when it is taken to the limits of feasibility. If β is very small, for instance, it means that accessibility costs are relatively unimportant, and that facilities can be concentrated into fewer zones. If β is very large (accessibility costs are relatively important), we would expect facilities to be opened in nearly all zones, otherwise the objective function cannot be met. In this exercise, only three general constraints are observed: first, zone allocations must not be negative; second, resources in external zone 37 are held at present levels; and third, Q is set to the value in scenario *1*. Table 3 presents the results for values of β ranging from 0.005 to 8.0. A black dot means that all the facilities in a zone have been closed. The regression statistics and values for F are presented in Table 4.

For $\beta = 0.005$ the only facilities open are those in the city center itself (zone 18). This seems to be a most logical result because this zone is also the focus for the whole region. As β increases other facilities in zones bordering the center begin to open. The first facilities in outer zones appear when $\beta = 0.1$. When $\beta = 0.2$, facilities in the center close because as costs get higher needs are better served locally instead of centrally. As β increases, still more facilities open than close until a maximum of 32 out of 36 zones have resources

Table 3. Sensitivity of facility behavior with respect to β. (Zones where all facilities have been closed are indicated by black dots.)

Zone j \ β	0.005	0.01	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	1	3	5	8
1	●	●	●	●	●	●	●		●							●
2	●	●	●	●	●										●	
3	●	●	●	●	●	●			●	●	●	●				
4	●	●	●													
5	●	●	●	●												
6	●	●	●													
7	●	●	●	●	●	●	●	●					●	●	●	
8	●	●	●	●	●	●	●	●								
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34	●	●	●	●	●									●	●	
35	●	●	●	●	●	●										
36	●	●														

Table 4. Sensitivity analysis of β.

Results	β 0.005	0.01	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	1	3	5	8
$F_{(initial)}$	-72.4	-72.1	-82.2	-81.4	-81.8	-82.0	-80.8	-76.4	-66.3	-46.9	-12.7	43.5	3.5×10^3	9.6×10^3	1.4×10^4	2.3×10^5
$F_{(final)}$	-73.0	-73.5	-83.3	-83.2	-83.4	-83.5	-83.5	-83.5	-83.4	-83.2	-83.0	-82.8	-80.0	-76.2	-75.8	-75.8
R^2	1.00	0.99	0.96	0.95	0.96	0.98	0.98	0.98	0.9	0.95	0.93	0.91	0.76	0.67	0.67	0.67
b	0.64	0.64	1.06	1.07	1.06	1.03	1.02	1.00	1.01	1.02	1.03	1.04	1.16	1.28	1.29	1.30
a	-18.2	-3.15	-1734	-2018	-1552	-884	-424	-122	-256	-427	-698	-1022	-4367	-7895	-8158	-8385
Total zones with open facilities	1	2	11	20	25	28	30	31	32	31	32	31	32	31	30	32

KEY

$\alpha = 1.57$ $F_{(minimum)} = -83.81$

R^2 = coefficient of explanation

$\hat{b}$ = slope

$\hat{a}$ = intercept

allocated to them. For these values changes are less frequent. The statistics in Table 4 show that the slope and intercept $\hat{b}$ and $\hat{a}$ are well-behaved over the range found in practice, with current measurement units, but they deviate more from one and zero as values of β become extreme. R^2 is highest for low β and lowest when β is high. Over the middle range of β the theoretical minimum of F is almost reached (F_{min} =-83.81). Finally, we should note the special case when $\beta = 0$ (i.e., no accessibility costs at all). From equations (2) and (17) we see that the coefficients γ_{ij} become constant and that the ob-objective function reduces to

$$\sum_i \left(\frac{\sum_j D_j}{\sum_i W_i} - \alpha \right)^2 = F \qquad (32)$$

Since $\sum_j D_j = Q$, F will be a minimum no matter how the resources are allocated.

The contours of F would be represented in the 2 x 2 example in Figure 3 as straight lines parallel to the resource constraint Q(AB). The equations of the derivatives are superimposed on AB indicating that all solutions are a minimum and that they all satisfy the objective function.

8. CONCLUSIONS

The application of RAMOS to issues affecting health care resource allocations have been described for two broad levels in the decision-making process: the tactical and the strategic. At the tactical level it was shown that RAMOS can test the outcomes of alternative resource configurations. At the strategic level it was suggested that the alternatives may be too many to test. A new model, $RAMOS^{-1}$, was therefore proposed. This model strives to allocate resources, so that the service levels (patients generated) in areas of residence correspond to their relative needs (pgfs). Because the HCS has other objectives as well, constraints on the permissible allocations were also introduced. It was then shown how RAMOS and RAMOS^{-1} could be connected to submodels concerning population trends, and patterns of hospital utilization, resource availability and treatment efficiency. The proposed model, RAMOS^{-1}, was then developed in detail and tested on data based on the London region in England. The preliminary results obtained were generally encouraging. The model behaved well when applied to four different planning scenarios, and gave logically consistent results during a sensitivity analysis on the model parameter β.

The technique presented in RAMOS^{-1} has been applied here to one objective: that of equating health resource allocations with relative need. It is probable that similar objectives stated slightly differently, apply in activities other than health care. In regional planning for example, it is often a goal of governments to influence patterns of industrial investment which smooth out the unemployment rates in different areas, subject to a variety of constraints. The model required is essentially the same as that presented here with, for example, the generating variables (pgfs) redefined in terms of expected labor participation rates. A next step, if preliminary tests prove successful, could be to link this model with another dealing with the provision of different services (shopping centers, health care facilities, etc.) to produce a composite model of activities of the Lowry type (Lowry, 1964; Wilson, 1974; Leonardi, 1980) .

In health care applications there are several avenues of advance. On the applied front the following seem important:

- The further refinement and testing of the input variables: resources, patient generating potentials, and accessibility costs

- Disaggregation of the model into local, regional or individual specialties, and a new application to outpatient services

- More validation tests based on correctly predicting past hospitalization rates and patient flows
- The development of a new model in another region of another country

On the theoretical front the following suggest themselves:

- An extension of the principles to apply to more market-based health care systems
- Further refinements in the linking of the submodels
- An investigation into the possibility of making endogenous the lengths of stay, turning the model into a spatial form of DRAM (Gibbs,1978;Hughes and Wierzbicki,1980)
- Designing a special version with different behavioral assumptions for application to the locations of emergency medical centers
- A general write-up of the mathematical connections between the various proposals outlined above

Because the model is generally producing very useful results in all the tests and experiments carried out to date, it is difficult to attach priorities to the list shown above since all the suggestions appear to be viable and so worth doing. On the applied front, it is true that the Health Care Task is constrained by its data sources to work at present only on the United Kingdom case study. Further advances in testing the model in other countries and on different health care systems are dependent on the collaboration with institutes outside IIASA which have access to the appropriate information. In contrast, the theoretical front is more easily developed, at least in the short run, providing that certain of the proposals are tested at each step and that they are not allowed to outstrip the availability of data by a wide margin. In view of the growing importance of health care resource allocation problems throughout the world, and because of the versatile model RAMOS is proving to be, the case for continuing with the development and further refinement of this approach seems strong.

APPENDIX A: Glossary of Main Terms used in RAMOS

Variable name	Notation[a]	Remarks
Accessibility costs	c_{ij}	Expresses the difficulty of someone in c obtaining treatment in j
Age/sex category	l	Used for pgf (W_i) calculation
Alpha	α	The ratio of total resources to total relative need: $\frac{Q}{\sum_i W}$
Beds	B_{jmt}	Acute beds in j in specialty m and time t
Caseloads (i)	Q_t	The total available resources in time t: $Q_t = \sum_m \frac{B_{mt}}{(\ell_{mt} + t_{mt})}$
Caseloads (ii)	D_{jt}	Case capacity in j in time t: $D_{jt} = 365 \sum_m \frac{B_{jmt}}{(\ell_{mt} + t_{mt})} = \sum_m d_{jmt}$
Caseloads (iii)	d_{jmt}	Cases in specialty m in time t: $d_{jmt} = \frac{365B_{jmt}}{\ell_{mt} + t_{mt}}$
Catchment population	C_j	Resident population dependent on j: $\sum_i E_{ij} P_i$
Deterrence function	$f(c_{ij})$	eg. exp $(-\beta c_{ij})$, $c_{ij}^{-\beta}$
Elasticity	E_{ij}	Sensitivity of R_i to changes in D_j: $T_{ij}/\sum_j T_{ij}$
Hospitalization rate	R_i	Patients per head of population in i: $\sum_j \frac{T_{ij}}{P_i}$
Length of stay	l_{mt}	The time from admission to discharge in specialty m, time t
National death rate	r_l	Used for SMRs
National utilization rate	U_{lmt}	National death and discharge rate in m, l and t used in pgfs
Parameter value	β	Behavioral parameter calibrated from actual patient flows
Objective function	F	$\min_{D_j} \sum_i (\sum_j D_j B_j e^{-\beta c_{ij}} - \alpha)^2$
Patient generating factor	W_{it}	Expected number of patients: $\sum_m \sum_l P_{ilt} U_{lmt}$ (A measure of relative need in i)
Place of residence	i	Zone i, i = 1,m
Place of treatment	j	Zone j, j = 1,n
Predicted patient flow	T_{ij}	Predicted flow of patients between i and j
Region of interest	L	Subset of a country
Specialty	m	Clinical specialty or disease category
Standardized mortality ratio	SMR_i	Further measure of relative need in i: $\sum_l M_{il}/\sum_l r_l P_{il}$, where M is the mortality rate in i
Time	t	Planning horizon
Turnover interval	t_{mt}	Average time between patient discharge and new admittance in specialty m and time t

[a]Where not indicated, the time dimension t is assumed to be implicit in the definitions

APPENDIX B: Forecasting Lengths of Stay and Utilization Rates, and Analyzing Turnover Intervals

1. Statistical projections for utilization rates and lengths of stay form integral parts of the resource allocation process. They are represented in section 5 as linked submodels whose outputs provide inputs for both RAMOS and RAMOS^{-1}. This appendix briefly describes one method that has been used, and found to be satisfactory (LHPC, 1979).

In the method, the projections are based on curves fitted for each of the age-sex specialty categories to a historical series of observations at the national level using ordinary least squares regression techniques. The curves best employed are of the saturation type: that is they assume a finite lower or upper bound. Just as the utilization rates in some specialties cannot continue to increase indefinitely, so the lengths of stay cannot go on falling. One example out of many such trend curves is

$$y = \frac{x}{bx + a} \qquad \text{(A1)}$$

where y = length of stay or the square root of the discharge rate

x = calendar year

a,b = estimated coefficients

The estimating ordinary least squares equation is derived from the simple transformation

$$\frac{1}{y} = \frac{a}{x} + b \qquad \text{(A2)}$$

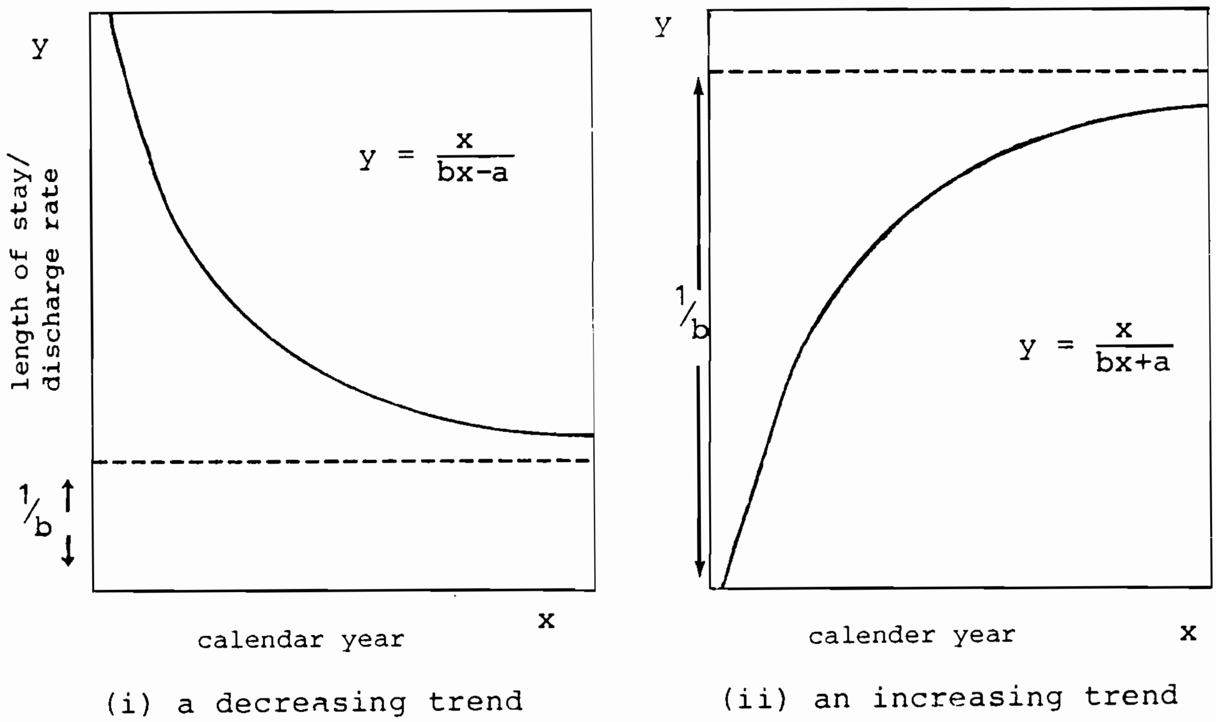


Figure A1. General slopes of the trend curves.

The use of the square-root of the discharge rate is a method for correcting heteroscedasticity, and it permits the construction of more reliable confidence intervals (Kelejian and Oates, 1974). Figure A1 summarizes the possibilities based on the signs of coefficient a.

In the London work referred to above increasing (or decreasing) projections were not accepted unless the regression model explained a statistically significantly proportion (at the 95% level) of the variation; otherwise the trend was taken as a constant, varying about the arithmetic mean of the series. For lengths of stay, for example, a significant increasing trend was not observed in any category. As an added precaution, however, length of stay projections were tested against current performance and were not considered valid unless at least one in three hospitals currently achieved the extrapolated level. For hospitalization rates increasing trends were obtained in the majority of categories. Here, too, precautions were taken when it was clear, on inspection, that the results were inappropriate or in contradiction to recent experience or medical opinion.

2. The choice of suitable turnover intervals t_m follows a different approach. In the London work it was recognized that there are many factors -- including the pressure on beds, the proportion of emergency admissions, and working practices -- which influence the time between the discharge and the admission of a new patient. An analysis of the available data showed a range of values that depended both on the specialty and on the individual hospital. Although the eventual criteria adopted in a study will vary from country to country, it was considered reasonable in this case, having taken all the relevant factors into consideration, to base judgements on the following procedures, which are shown here as an example.

All specialties (except "infectious diseases") were divided into two groups: those with a "high" potential for reduction and those with a "low" potential. The "high" category applied to specialties for which at least 60% of all hospitals had a turnover interval of more than 4 days. The "low" category applied when more than 40% were achieving less than 4 days. Clearly, the exact proportions and choice of days are matters for careful judgement. Within these two defined divisions, however, the following levels were finally designated for forecasting purposes.

Category 1 — "high potential":

An interval equivalent to the level achieved by 25% of hospitals at the time of study in specialties in which 60% of hospitals had turnover intervals of more than 4 days

Category 2 — "low potential":

An interval equivalent to the level achieved by 40% of hospitals in 1976 for the remaining specialties

Clearly, these are simple and pragmatic conventions, and other users of RAMOS may prefer more or less sophisticated methods. Certainly, turnover intervals are too important to ignore.

APPENDIX C: An Overview of the Computer Output for $RAMOS^{-1}$

The following sections give an explanation and an example of the computer output from $RAMOS^{-1}$. The problem solved in the example assumes no change in Q (the total resource level or the resources allocated to the external zone) or in the patient generating factors, no constraints on the resource allocations made in each destination (except the condition that $D_j \geq 0 \forall_j$), and a parameter value of 0.367. The upper bounds on D_j have been set at an arbitrarily high number.

The output comes in five sections (p.53-57) described briefly below:

1. Section 1 gives the values of the main constants and the parameter value. It also gives the initial and final values of F, the objective function. In the example shown (p.53), the theoretical minimum of F is -83.81; the realized value, -83.33, comes very close because the constraints ($D_j \geq 0$) are so lax. The value of alpha (at the old resource levels) is shown against the new value. Both of them depend on Q and $\sum_i W_i$. Since neither Q nor $\sum_i W_i$ changes, here the two values for alpha are the same.
2. Section 2 gives the results of the allocation procedure by destination, the balancing factor B_j, and the initial and final values of the first derivatives of F with respect to $D_j (\partial F/\partial D_j)$. Because the constraints are lax, the resource allocation changes selected by $RAMOS^{-1}$ are very large and clearly impractical in the real world.

This illustrates vividly the importance of giving careful attention to these constraints, particularly in terms of what else the decision maker must take into consideration before running the model. The example also shows the convergence and shrinkage in the values of the derivatives after optimization.

3. Section 3 gives the results of changes in the catchment populations, the lower and upper bounds set on the D_j, and two coefficients a_{kk} and b_k from equations (20) and (21), where a_{kk} is the k-th diagonal element. These coefficients are sometimes useful for diagnosing the results obtained, but it is difficult to give them a simple interpretation. For the bounds it is seen that the upper levels have all been set to an arbitrarily high number, while the lower levels have been fixed at zero.

4. Section 4 gives the results for each place of residence. Column one gives the patients generated before optimization; and column two gives those generated after. Column four gives the input values of W_i, the pfg or measure of relative need, while the last two columns show the predicted average accessibility costs (measured in 'modified' kilometers) of the patients to their places of treatment.

5. Section 5 consists of two graphs, depicting a typical situation 'before' and 'after' reallocation. The vertical axes in both measure the predicted patients generated in each origin ($\sum_j T_{ij}$); the horizontal axes show the relative need (W_i) scaled by alpha 1 ('before') and alpha 2 ('after'). Interestingly, the 'after' graph shows what can sometimes occur when the constraints are too lax as in this case. Here it is the existence of an 'outlier' in the observations far removed from the otherwise straight-line trend. Fortunately, this does not seem to occur in more realistic applications (see Figures 7 to 11). Under each graph is shown the results of the regression analysis. In the 'after' set of results, the user of RAMOS^{-1} is looking for first, a slope with a gradient as close as possible to one; second, a constant term which

does not differ significantly from zero; and third, a high value of R^2, the coefficient of explained variation, which has a range from zero to one. The standard errors of the coefficients are included for constructing the appropriate statistical hypotheses.

Computer Output

Section 1

```
results
*******
initial value of objective function =        -0.61019e+02
final value of objective function =         -0.83332e+02
number of origin zones =   34
number of destination zones=   37

parameter value beta =    0.3670

total current resources available =   923618. cases
total revised resources internal region =   923618. cases
total patient generating factors =   587707.00
ratio of existing resources to expected cases (alpha1) =      1.57
ratio of revised resources to pgfs (alpha2) =       1.57

----------------------------------------------------------------------
```

Section 2

revised and existing caseload resource
allocations by destination

zone	name	predicted val	current val	pc change	bal factor	deriv 1	deriv 2
1	barnet+	-0.01	13257.00	-100.00	0.108e-03	0.115e-04	0.946e-06
2	edgware	39711.35	20467.00	94.03	0.882e-04	0.770e-05	0.940e-06
3	brent	0.01	19068.00	-100.00	0.491e-04	0.156e-04	0.944e-06
4	harrow	16630.45	21315.00	-21.98	0.595e-04	-0.557e-05	0.940e-06
5	hounslow	34862.60	23024.00	51.42	0.525e-04	-0.335e-04	0.940e-06
6	s hamm	22906.89	23453.00	-2.33	0.904e-04	0.121e-04	0.940e-06
7	n hamm	38077.96	15439.00	146.63	0.124e-03	0.149e-04	0.940e-06
8	ealing	1503.76	7149.00	-78.97	0.378e-04	-0.658e-05	0.940e-06
9	hilligdn	45090.47	34643.00	30.16	0.215e-03	-0.250e-04	0.940e-06
10	kcw nw	34370.21	33012.00	4.11	0.139e-03	0.396e-04	0.941e-06
11	kcw ne	22245.94	33437.00	-33.47	0.476e-03	0.138e-03	0.941e-06
12	kcw s	4133.79	45941.00	-91.00	0.348e-03	0.356e-04	0.941e-06
13	barking	11650.41	24566.00	-52.58	0.105e-03	-0.358e-04	0.941e-06
14	havering	27736.67	11343.00	144.53	0.553e-03	-0.377e-04	0.941e-06
15	n camden	0.01	21935.00	-100.00	0.118e-03	0.197e-04	0.276e-05
16	s camden	94574.66	46414.00	103.76	0.325e-03	0.130e-03	0.941e-06
17	islingtn	25784.04	31479.00	-18.09	0.882e-04	0.496e-04	0.941e-06
18	city +	-0.01	40112.00	-100.00	0.133e-03	0.111e-02	0.309e-05
19	newham	19952.89	18587.00	7.35	0.433e-04	0.493e-06	0.940e-06
20	t hamlet	35977.03	39285.00	-8.42	0.114e-03	0.894e-04	0.940e-06
21	enfield	19867.75	16876.00	17.73	0.697e-04	-0.195e-05	0.940e-06
22	haringey	22181.57	25087.00	-11.58	0.698e-04	0.157e-04	0.941e-06
23	e roding	33867.25	13262.00	155.37	0.130e-03	-0.369e-04	0.940e-06
24	w roding	29856.15	28948.00	3.14	0.550e-04	-0.101e-04	0.940e-06
25	bexley	23199.14	13162.00	76.26	0.191e-03	-0.182e-04	0.941e-06
26	greenwch	33457.18	38156.00	-12.31	0.116e-03	-0.236e-05	0.941e-06
27	bromley	28237.59	26362.00	7.11	0.151e-03	-0.840e-05	0.941e-06
28	st thoms	33354.14	26153.00	27.53	0.521e-03	0.277e-05	0.942e-06
29	kings	43639.42	33096.00	31.86	0.180e-03	0.345e-05	0.942e-06
30	guys	14859.72	28887.00	-48.56	0.479e-03	0.372e-05	0.942e-06
31	lewisham	24324.06	20626.00	17.93	0.612e-04	-0.306e-05	0.942e-06
32	croydon	32119.36	19558.00	64.23	0.465e-04	-0.269e-04	0.941e-06
33	kingston	21612.95	18176.00	18.91	0.115e-03	-0.241e-04	0.941e-06
34	roehamtn	15009.53	13758.00	9.10	0.172e-03	0.163e-05	0.941e-06
35	wands+em	36684.35	43573.00	-15.81	0.113e-03	0.756e-05	0.941e-06
36	sutton +	36141.03	34012.00	6.26	0.164e-03	-0.889e-05	0.941e-06
37	others	3269161.00	3269161.00	0.	0.192e-05	-0.349e-06	-0.349e-06

reallocated resources = 923620. internal: 4192782. whole region

Section 3

current and predicted catchment populations
coeffs of obj function and upper and lower resource bounds

zone	name	current	predicted	pc change	coefficient	b	lwr bnd	upr bnd
1	barnet+	91062.	-0.	-100.00	0.2611e-08	.130e-03	. e+00	.100e+08
2	edgware	145889.	296539.	103.26	0.1326e-08	.145e-03	. e+00	.100e+08
3	brent	132104.	0.	-100.00	0.2046e-08	.146e-03	. e+00	.100e+08
4	harrow	166619.	124768.	-25.12	0.2372e-08	.166e-03	. e+00	.100e+08
5	hounslow	211520.	261627.	23.69	0.2752e-08	.187e-03	. e+00	.100e+08
6	s hamm	170132.	171012.	0.52	0.2899e-08	.206e-03	. e+00	.100e+08
7	n hamm	110556.	285371.	158.12	0.9890e-09	.174e-03	. e+00	.100e+08
8	ealing	56795.	11313.	-80.08	0.2339e-08	.138e-03	. e+00	.100e+08
9	hilligdn	313752.	342036.	9.01	0.2313e-08	.162e-03	. e+00	.100e+08
10	kcw nw	205610.	254441.	23.75	0.1711e-08	.182e-03	. e+00	.100e+08
11	kcw ne	225920.	170007.	-24.75	0.8283e-09	.216e-03	. e+00	.100e+08
12	kcw s	310026.	30931.	-90.02	0.1072e-08	.187e-03	. e+00	.100e+08
13	barking	234130.	88224.	-62.32	0.3170e-08	.190e-03	. e+00	.100e+08
14	havering	113314.	212003.	87.09	0.3741e-08	.168e-03	. e+00	.100e+08
15	n camden	149676.	0.	-100.00	0.1317e-08	.170e-03	. e+00	.100e+08
16	s camden	315182.	725669.	130.24	0.9997e-09	.214e-03	. e+00	.100e+08
17	islingtn	199704.	196441.	-1.63	0.1635e-08	.187e-03	. e+00	.100e+08
18	city +	252194.	-0.	-100.00	0.2996e-07	.556e-03	. e+00	.100e+08
19	newham	144056.	151596.	5.23	0.3115e-08	.178e-03	. e+00	.100e+08
20	t hamlet	272592.	280828.	3.02	0.3109e-08	.229e-03	. e+00	.100e+08
21	enfield	129112.	148090.	14.70	0.2649e-08	.144e-03	. e+00	.100e+08
22	haringey	176318.	168189.	-4.61	0.1810e-08	.158e-03	. e+00	.100e+08
23	e roding	127478.	252572.	98.13	0.2471e-08	.185e-03	. e+00	.100e+08
24	w roding	236837.	223008.	-5.84	0.2574e-08	.166e-03	. e+00	.100e+08
25	bexley	113397.	176276.	55.45	0.2944e-08	.170e-03	. e+00	.100e+08
26	greenwch	297972.	253151.	-15.04	0.2570e-08	.175e-03	. e+00	.100e+08
27	bromley	214180.	212579.	-0.75	0.2446e-08	.131e-03	. e+00	.100e+08
28	st thoms	196600.	251074.	27.71	0.1006e-08	.142e-03	. e+00	.100e+08
29	kings	247670.	327397.	32.19	0.1255e-08	.149e-03	. e+00	.100e+08
30	guys	215397.	111249.	-48.35	0.1534e-08	.159e-03	. e+00	.100e+08
31	lewisham	159533.	182232.	14.23	0.2569e-08	.152e-03	. e+00	.100e+08
32	croydon	194590.	244537.	25.67	0.2358e-08	.125e-03	. e+00	.100e+08
33	kingston	150670.	160514.	6.53	0.6671e-08	.250e-03	. e+00	.100e+08
34	roehamtn	101321.	112119.	10.66	0.1642e-08	.174e-03	. e+00	.100e+08
35	wands+em	317436.	275119.	-13.33	0.1248e-08	.156e-03	. e+00	.100e+08
36	sutton +	270758.	269188.	-0.58	0.3825e-08	.210e-03	. e+00	.100e+08
37	others	39381200.	39381200.	0.	0.1874e-12	.962e-06	. e+00	.327e+07

total internal catchment population before 6970100.
total internal catchment population after 6970100.

Section 4

a comparison of predicted and expected patients
generated by each origin zone

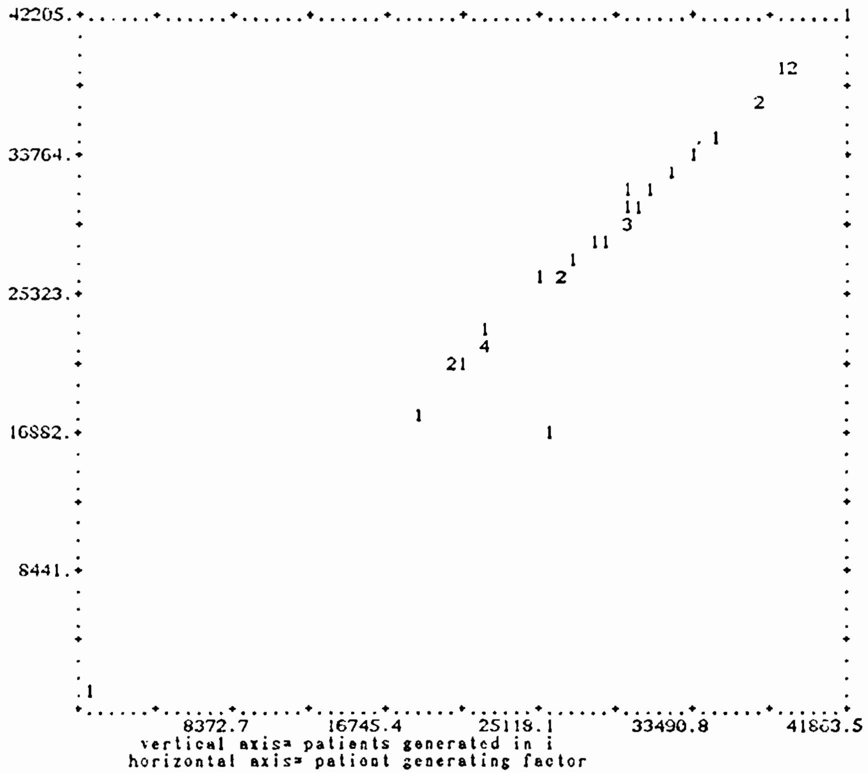
zone	name	current	predicted	pc change	expected	av cost1	avcost2
1	barnet+	43109.13	39374.20	-8.66	24880.07	7.2	8.0
2	brent	39652.76	33951.71	-14.38	21465.72	6.4	8.3
3	harrow	24010.06	26578.63	10.70	16827.54	5.4	5.8
4	ealing	37924.86	38840.53	2.41	24522.90	7.3	8.4
5	hammersh	22782.60	22025.17	-3.32	13960.01	6.2	5.4
6	hounslow	20097.40	26348.11	31.10	16729.07	5.9	5.0
7	hillingn	23612.23	29994.58	27.03	18976.30	6.1	5.9
8	kens+chl	27797.12	21139.57	-23.95	13404.71	8.1	8.0
9	westmstr	36882.16	28883.33	-21.69	18310.72	8.4	8.0
10	barking	17503.74	20604.13	17.71	13050.71	5.9	6.6
11	havering	23730.49	31287.82	31.85	19793.54	5.5	6.7
12	camden	26559.59	25853.39	-2.66	16065.83	8.3	9.1
13	islingtn	29757.48	22948.47	-22.88	14189.17	7.1	8.0
14	city	2775.47	763.03	-72.51	455.86	2.2	9.0
15	hackney	30801.04	17138.39	-44.36	16412.04	7.2	9.8
16	newham	29544.48	29818.92	0.93	19150.99	6.0	5.7
17	twr hams	24141.31	21438.16	-11.20	12751.12	4.6	4.4
18	enfield	33226.16	34834.55	4.84	22148.84	6.1	5.9
19	haringey	35695.71	31579.63	-11.53	18978.82	6.7	7.0
20	redbridg	21373.01	30785.02	44.04	19465.80	7.4	6.3
21	wal fort	29442.82	30023.45	1.97	18860.03	5.2	5.1
22	bexley	23528.22	28140.22	19.60	17810.06	6.9	6.2
23	greenwch	28763.08	27267.85	-5.20	17260.97	5.7	5.8
24	bromley	36520.86	38964.21	6.69	24612.24	7.1	6.8
25	lambeth	36421.84	37446.43	2.81	23639.07	9.0	8.5
26	lewisham	31821.52	32626.85	2.53	20632.64	6.3	5.6
27	sothwark	32346.12	30107.41	-6.92	19054.02	8.1	7.7
28	croydon	31708.69	42204.52	33.10	26638.13	5.9	4.3
29	kingston	16586.23	18292.69	10.29	11616.24	4.9	4.3
30	richmond	18667.46	22494.92	20.50	14195.69	6.6	5.7
31	merton	23344.07	22590.51	-3.23	14251.80	7.1	6.9
32	sutton	20734.84	22302.19	7.56	14166.02	5.0	4.9
33	wandswth	42755.44	36971.71	-13.53	23430.24	7.5	7.3
34		3269161.00	3269161.00	0.	3266630.00	5.0	5.0

average access costs internal region before 6.73

average access costs internal region after 6.65
total patients generated in internal before 923618.
total patients generated in internal after 923620.

Section 5

regression of predicted patients generated on pgfs



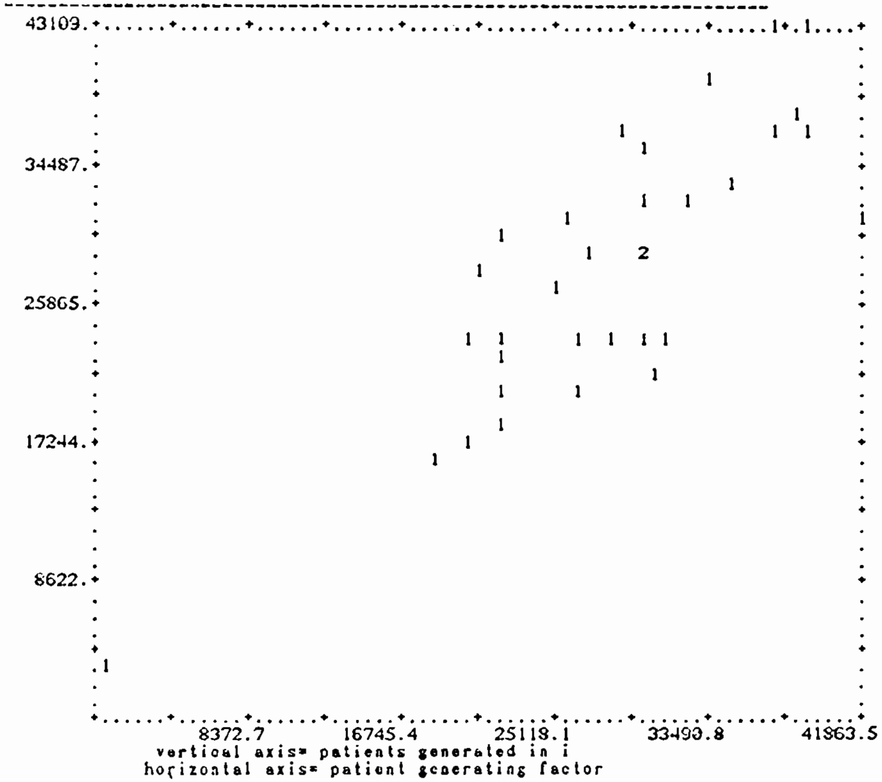
regression results

constant= -300.813 slope= 1.011
st error = 1047.212 st error = 0.036

r-squared = 0.962

number of observations = 33

regression of current patients generated on pgfs



regression results

constant= 3029.837 slope= 0.892
st error = 3074.477 st error = 0.106

r-squared = 0.696

number of observations = 33

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