NOT FOR QUOTATION
WITHOUT PERMISSION
OF THE AUTHOR

A NONLINEAR MULTISECTORAL MODEL FOR HUNGARY: GENERAL EQUILIBRIUM VERSUS OPTIMAL PLANNING APPROACH

Ernö Zalai

October 1980
WP-80-148

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

Dr. Ernö Zalai is Associate Professor at the Institute for National Economic Planning, Karl Marx University of Economics in Budapest, Hungary.

FOREWORD

Roughly 1.6 billion people, 40 percent of the world's population, live in urban areas today. At the beginning of the last century, the urban population of the world totaled only 25 million. According to recent United Nations estimates, about 3.1 billion people, twice today's urban population, will be living in urban areas by the year 2000.

Scholars and policy makers often disagree when it comes to evaluating the desirability of current rapid rates of urban growth and urbanization in many parts of the globe. Some see this trend as fostering national processes of socioeconomic development, particularly in the poorer and rapidly urbanizing countries of the Third World; whereas others believe the consequences to be largely undesirable and argue that such urban growth should be slowed down.

This paper is the product of collaboration between the Human Settlements and Services (HSS) and the System and Decision Sciences (SDS) Areas at IIASA on the topic of urbanization-development modeling. Professor Zalai was brought to Laxenburg by both Areas to explore commonalities between the concepts and techniques incorporated in nonlinear computable general equilibrium models and the methodologies of planning models currently being used in centrally planned economies. The first results of that exploration are set out in this paper. The author's principal conclusion is that, despite their fundamental conceptual differences, both classes of models exhibit close technical similarities, which make transfer of modeling techniques possible.

Recent papers in the Population, Resources, and Growth Series of the HSS Area and related papers in the SDS Area are listed at the end of this report.

Andrei Rogers
Chairman
Human Settlemets
and Services Area

Andrzej Wierzbicki
Chairman
System and Decision
Sciences Area

ACKNOWLEDGMENTS

I wish to thank all of those at IIASA and in Hungary who read the first draft for their useful comments. I am especially grateful to Lars Bergman, Urban Karlström, and András Pór for the many discussions and their valuable suggestions from which I benefited a great deal. I would also like to acknowledge with thanks the encouragement and constant interest of Andrei Rogers and Andrzej Wierzbicki.

Maria Rogers edited and Susie Riley typed the paper. I thank them for their efficient job and patience.

ABSTRACT

Recent years have witnessed a shift in nation-wide economic modeling techniques. Parallel to the use of traditional linear models has been the development of more sophisticated nonlinear models, under the name of applied (computable) general equilibrium models. Some of these models have been especially designed to capture the interrelationships of economic, demographic, and spatial processes. This paper investigates the possibilities and expected benefits of incorporating nonlinear multisectoral models of the computable general equilibrium type into the planning methodology of socialist countries.

Linear multisectoral models have become more or less integrated into the complex process of planning in most of the socialist countries. Despite their fundamental conceptual differences the optimal planning models and the computable general equilibrium models exhibit close technical similarities, which make the transfer of modeling techniques feasible. For illustration a tentative nonlinear model is developed for Hungary which combines the concepts and techniques of the above two modeling approaches. The model differs considerably from its Western counterparts and can be viewed as a natural extension of the planning models used in Hungary. The model takes explicitly into account the interaction of real and value variables, emphasizes the foreign trade flows and incorporates demographic and spatial aspects as well.

CONTENTS

INTRODUCTION 1

1. APPLIED GENERAL EQUILIBRIUM MODEL VERSUS OPTIMAL PLANNING MODELS 4

   1.1. A Simple Multisectoral General Equilibrium Model 5
   1.2. An Optimal Planning Model Version of the Problem 12
   1.3. Technical Similarities and Conceptual Differences 19
   1.4. The Use of Smooth Production Functions in Planning Models 24

2. A TENTATIVE MODEL FRAMEWORK FOR HUNGARY 29

   2.1. Commodities and Commodity Balances 30
   2.2. Import and Export Functions, and Trade Balances 33
   2.3. Final Demand and Regional Aspects 39
   2.4. Prices and Costs 42
   2.5. Production Technology and Decision-Making Rules 48
   2.6. Formal Statement of the Model 49

REFERENCES 58

A NONLINEAR MULTISECTORAL MODEL FOR HUNGARY: GENERAL EQUILIBRIUM VERSUS OPTIMAL PLANNING APPROACH

INTRODUCTION

Recent years have witnessed a shift in macroeconomic modeling techniques. Parallel to the use of traditional linear (input-output and programming) models that concentrate on the production sphere, has been the development of more complex, nonlinear models. These models are usually referred to as applied general equilibrium models. The purpose of this study is to investigate the possibilities and expected benefits of incorporating nonlinear and multisectoral models of the general equilibrium type into the planning methodology of socialist (centrally planned) economies. Such models, both static and dynamic, have been developed in increasing number for development planning and policy analysis purposes over the past few years.*

This paper concentrates on the *intratemporal* rather than intertemporal equilibrium and efficiency conditions of such models. Also, the models considered here possess a lower degree of closure

*The basic ideas of a multisectoral general equilibrium growth model were laid down by Johansen (1959). Full scale implementation of large, nonlinear models has become computationally feasible only lately. Recent applications include Adelman and Robinson (1978), De Melo (1978), Bergman (1978), Dervis and Robinson (1978), the IMPACT project (see, for instance, Dixon et al., 1977), McCarthy and Taylor (1980), and Kelly and Williamson (1980).

in their general equilibrium properties than most of the models in this field. As for the basis of discussion and comparison a model developed by Bergman and Pór (1980) at IIASA has been chosen both for its relatively simple structure and its close conceptual resemblance to the optimal resource allocation planning models used in some socialist countries, namely, Hungary.

The focus of this paper is on the *techniques* of *applied* general equilibrium models with special reference to the Hungarian planning modeling experience. It is organized into two main sections. Section 1 is a comparative modeling exercise intended partly to bridge the gap between model builders coming from different socioeconomic environments and partly to pave the way for the model specified in Section 2. Although the audience addressed in this paper is mainly planning modelers from socialist countries who are less familiar with applied general equilibrium modeling, it is hoped that some of the conclusions of this exercise will also be of some value to experts in this field. Section 2 describes a tentative general equilibrium model framework, reflecting to a large extent—but of course in a simplified manner—the existing planning theory and practice in Hungary. Different parts of the model are defined so that they can or could be incorporated into some partial investigations concerning, for example, price formation or physical resource allocation coordination in Hungary. There are, however, a few places where the mathematical formulation differs from the "traditional" form, and this is mostly due to the nonlinearity of the model. The novelty of the outlined model lies mostly in the fact that it integrates the above partial models into a consistent framework and directly takes into account the interdependence of real and value processes—a basic requirement not fully met by recently applied planning models in socialist countries.

It should be emphasized, that this paper is only a first step toward the use of some more advanced, applied general equilibrium modeling techniques in socialist economies. There are many issues not raised here which are left for further research. Planning in socialist countries is a complex social exercise. National planning itself is a highly decomposed and iterative information processing system with many informal elements. It is a system that involves several administrative and scientific institutions. Any model that is not intended to remain a pure academic exercise must be carefully designed against that background and find its proper place within that system. This means, that constraints on input and output data specifications must be recognized. It also means that one has to find the proper phase and stage of planning appropriate for a model and the proper issues to which the model can be fruitfully addressed. This task is not easy and a great deal remains to be done.

For the above reasons the model developed in Section 2 sets out only a tentative and general (non-issue specific) framework of a multisectoral, static model. Throughout the study we had in mind two possible planning applications for such a model. One area of possible applications is the so-called *coordination phase* of a medium term plan. It is well known that in Hungary, linear programming models* have been experimented with in this phase of planning (where the main aim is to establish an overall consistency and optimality of detailed partial plans). They concern, basically, the planned allocation of resources and various consistency requirements in a terminal year. The models are built on the detailed plan calculations and use *linear approximations* to represent the feasible movement around the planned levels of some crucial variables. The aim of the investigation is to check the consistency of the draft plan and indicate various possibilities for increasing the efficiency of the plan by a constrained reallocation of resources.

In this context, the proposed equilibrium model can be simply seen as a (partly) *nonlinear* version of the above models, in which most of the data are derived from the plan calculations or based on expert judgments. Another area of application could be the early (*forecasting*) *phase* of planning, when the data of the model are, to a large extent, based on statistical sources, and the model is used for generating conceivable directions for detailed planning activity.

---

*See Kornai (1974) for an account on the use and development of such models in the Hungarian medium-term planning calculations.

These are, however, just two possible areas. Taking into account the great flexibility of the equilibrium models (in terms of their size and structure, the choice of the endogenous and exogenous variables, the issues focused on, etc.), these models are probably worth experimenting with in other areas as well. For example, it is possible to simulate—either *ex post* or *ex ante*—the likely effects of changing exogenous (to the model) conditions. It is also possible to further develop the model for multiperiod forecasting purposes. This could be done either by the use of "snapshot" techniques or by "dynamizing" the static model. In the first case the values of the exogenous variables and the parameters of the model are independently forecasted for some future years and for each year a static model is solved. It seems to be promising in this context to experiment with reference path optimization techniques (see Wierzbicki 1979) by prescribing target values for some of the endogenous variables as well. In the case of dynamization some exogenous variables (investment and capital stocks) are endogenized through intertemporal relationships. These loosely defined alternative uses of equilibrium models would, however, require changes in their specifications from those used in the basic models in this paper.

1. APPLIED GENERAL EQUILIBRIUM MODELS VERSUS OPTIMAL PLANNING MODELS

This section is a rejoinder to the old theme of a fundamental equivalence existing between equilibrium solutions through a competitive mechanism and the optimal solutions of a centrally planned resource allocation problem. This topic has been formulated in many ways (e.g., in terms of welfare economics or a simple linear programming model). Here we will put it into a slightly different context. First, we will use it to gain better insight into the problem of how and where the analytical techniques used in multisectoral general equilibrium models could be fit into the current planning modeling methodology of centrally planned economies. At the same time the exercise will help us to better understand the working of the general equilibrium model (e.g., the determination of the consumption expenditure or the possibility of incorporating various economic policy goals into a general equilibrium framework). Last but not least, unlike most of the literature, we will put more emphasis on the conceptual differences that lie behind the technical similarities.

The organization of Section 1 is as follows. First we summarize the basic features of a general equilibrium formulation of the resource allocation problem in the framework of a simple model economy. Second, within the same framework the problem is then reformulated in a way familiar to the socialist planning modeling practice. Next, some of the fundamental technical similarities and the conceptual differences are analyzed. And, finally, we close this Section with some observations related to the problem of using smooth production functions in macro planning models.

1.1. A Simple Multisectoral General Equilibrium Model

For the purpose of comparison we have chosen a general equilibrium model developed by L. Bergman and A. Pór (1980) at IIASA. Its static character, relatively simple structure, and focus on allocational efficiency in the context of a small open economy makes it convenient to compare with linear programming models developed in Hungary for similar purposes. The underlying logic of multisectoral general equilibrium models and their relation to some structurally similar optimal planning models will be better understood if the resource allocation problem is stripped to its bare essentials. Therefore, we disregard some elements of the Bergman-Pór model, like foreign trade variables, government consumption, and taxes, and treat energy inputs in the same way as other intermediate inputs. (That is, energy is considered one of the intermediate commodities.) By doing this the above general equilibrium model is reduced to the following simple form.

First we define the various (endogenous) variables and extraneous parameters that appear in the model.

Variables

X_j gross output in sector $j = 1,2,\ldots,n$*

X_{n+1} total gross investment

K_j capital stock in sector $j = 1,2,\ldots,n$

N_j employment in sector $j = 1,2,\ldots,n$

C_i consumption of commodity $i = 1,2,\ldots,n$

P_i price of commodity $i = 1,2,\ldots,n$

P_{n+1} price of the composite capital good

P_i^* "net price" (value added per unit) of commodity $i = 1,2,\ldots,n$

W general index of the level of wages

W_j level of wages in sector $j = 1,2,\ldots,n$

R general index of the return on capital

R_j rate of return on capital in sector $j = 1,2,\ldots,n$

Q_j user cost of capital in sector $j = 1,2,\ldots,n$

E consumption expenditures

Data

N total labor force

K total capital stock

I total net investment

a_{ij} input of commodity $i = 1,2,\ldots,n$ per unit of output in sector $j = 1,2,\ldots,n$

$a_{i,n+1}$ input of commodity $i = 1,2,\ldots,n$ per unit level of gross investment

δ_j annual rate of depreciation in sector $j = 1,2,\ldots,n$

ω_j index of the relative wage rate in sector $j = 1,2,\ldots,n$

β_j index of the relative rate of return on capital in sector $j = 1,2,\ldots,n$

b_i, c_i parameters in the consumer's demand function for commodity $i = 1,\ldots,n$

---

*In the model each sector produces only one kind of commodity and each commodity is produced only by one sector. Thus there is a one-to-one correspondence between the sectors and the produced commodities.

With the symbols defined above we now summarize the basic features of the general equilibrium model. We do not reproduce here the arguments supporting one or another specific formulation, for which the reader is referred to the original paper. However, we try to represent the model in a self-contained manner.

Commodities

There are n produced commodities in the model available for both intermediate and final use, one composite capital good (which is used only for investment), and two primary commodities (capital and labor).

Technology

The production technology is given for the sectoral commodities by the combined Leontief-neoclassical formulation, used by Johansen (1959). The amount of primary commodities needed to produce X_j unit of commodity j is described by a linear homogeneous, smooth production function, thus allowing for substitution possibilities:

$$X_j = F_j(N_j, K_j) \qquad j = 1,2,\ldots,n$$

The use of intermediate inputs is assumed to be proportional to the output level of the produced commodity, i.e.,

$$a_{ij}\ X_j \qquad \begin{array}{l} i = 1,2,\ldots,n \\ j = 1,2,\ldots,n \end{array}$$

The production of the composite capital good requires only intermediate commodities in amounts proportional to the level of gross investment (capital formation):

$$a_{i,n+1}\ X_{n+1} \qquad i = 1,2,\ldots,n$$

The technology defined above exhibits constant returns to scale, therefore, in equilibrium the nonprofit condition must hold for each producing sector.

Market Behavioral Rules for the Producers

Producers are assumed to maximize their net income (or profits), i.e., the difference of their gross income and total costs. Total cost is made up of the cost of intermediate inputs and primary inputs. Capital is reevaluated at the current price on capital goods in accordance with the following rule:*

$$P_{n+1} = \sum_{i=1}^{n} P_i \, a_{i,n+1} \tag{1'}$$

Therefore, the cost of using capital (evaluated at *base* price) in sector j is given by

$$Q_j = (\delta_j + R_j)\, P_{n+1} = (\delta_j + \beta_j\, R)\, P_{n+1} \tag{2'}$$

The introduction of different rates of return requirements on capital can be interpreted, for instance, as a reflection of lasting market imperfections. It will be shown that this solution has effects similar to individual bounds on sectoral capital inputs, which, in turn, can be interpreted as limited intersectoral mobility of capital.

Introducing $W_j = \omega_j W$ to represent the cost of labor, the net income earned by producing X_j can be defined by the following expression:

$$\Pi_j = P_j\, X_j - \sum_{i=1}^{n} P_i\, a_{ij}\, X_j - W_j\, N_j - Q_j\, K_j \tag{2a'}$$

which is to be maximized subject to the constraint given by the production function:

$$X_j = F_j(N_j, K_j)$$

Substituting X_j by $F_j(N_j,K_j)$ in equation (2a') and differentiating the net income function with respect to N_j and K_j will yield the following necessary first-order conditions for an optimal solution:

---

*The equation numbers in Section 1 of this paper are independent of those in Section 2.

$$P_j^* \frac{\partial F_j}{\partial N_j} = W_j = \omega_j W \tag{3'}$$

$$P_j^* \frac{\partial F_j}{\partial K_j} = Q_j \tag{4'}$$

where P_j^* is the value added per unit of output j: *

$$P_j^* = P_j - \sum_{i=1}^{n} P_i a_{ij} \tag{5'}$$

It can easily be seen that if we multiply equations (3') and (4') with N_j and K_j, respectively, and add them together, then, because of the assumed linear homogeneity of the production functions, we will have

$$P_j^* = W_j \frac{N_j}{X_j} + Q_j \frac{K_j}{X_j} = W_j n_j + Q_j k_j \tag{5a'}$$

which in turn implies that the net income must be zero in equilibrium (nonprofit—or more accurately—"nonextraprofit" condition).

If we insert equation (5a') in equation (5'), after rearrangment we get

$$P_j = \sum_{i=1}^{n} P_i a_{ij} + W_j n_j + Q_j k_j \tag{5b'}$$

The above price formation rule strongly resembles the form that is used to determine the so-called "two channel price system" known in the socialist price planning theory and practice. We will come back to this point later.

---

*Notice that if instead of substituting X_j by F_j in the net income function we utilized a Lagrange multiplier, then P_j^* would be the value of that multiplier.

Consumer Demand

The demand for consumer goods and services is represented by a demand function:

$$C_i = C_i(P_1, P_2, \ldots, P_n, E) \qquad i = 1,2,\ldots,n$$

where E is the total consumption expenditure, an endogenous variable of the model.* In most applied models a simple or extended Linear Expenditure System (LES) is used:

$$C_i = b_i + \frac{c_i}{P_i}\left(E - \sum_{j=1}^{n} P_j\, b_j\right) \qquad i = 1,2,\ldots,n \qquad (6')$$

where b_i is some times interpreted as the minimum ("subsistence") consumption of commodity i, which must be fulfilled before the remaining income is allocated between the various commodities depending on their relative prices and on the marginal propensities to consume different commodities (c_i). It is worth noting that such demand functions can be derived on the basis of utility maximization theory assuming a Cobb-Douglas utility function for the "surplus" consumption:

$$U = (C_1 - b_1)^{c_1} (C_2 - b_2)^{c_2} \cdots (C_n - b_n)^{c_n} \quad \text{and} \quad \sum_{i=1}^{n} c_i = 1$$

The Physical (Real) Conditions of an Equilibrium

In this simplified model the state of the economy can be fully described by the value of the endogenous variables. Among them, variables X_i, C_i, K_j, N_j (which can be called *real* variables) describe the production and the use of different commodities. Whether it is a centrally planned or a market economy (or the mixture of the two), the above variables must fulfill

*The total expenditure is, if I understand correctly, determined with no clear relationship to the wages. Therefore one will get, in fact, endogenously determined tax and savings rates out of the model, which may be quite absurd. The analysis of the linear programming model will shed some light on the endogenous determination of the consumption expenditure.

certain "physical" conditions of feasibility. These conditions incorporate commodity and resource balances and technological restrictions. We now list these conditions. The balance conditions will be given in the form of inequalities, which is more general than the equalities used in the Bergman-Pór model. However, we know that if the equilibrium price of a commodity is positive, then the corresponding balance inequality must be fulfilled as an equality. The special assumptions of the Bergman-Pór model guarantee that the prices of all commodities and resources will always be positive

$$\sum_{j=1}^{n+1} a_{ij}X_j + C_i \overset{(<)}{=} X_i \qquad i = 1,2,\ldots,n \tag{7'}$$

$$\sum_{j=1}^{n} \delta_j K_j + I \overset{(<)}{=} X_{n+1} \tag{8'}$$

$$\sum_{j=1}^{n} K_j \overset{(<)}{=} K \tag{9'}$$

$$\sum_{j=1}^{n} N_j \overset{(<)}{=} N \tag{10'}$$

$$F_j(N_j,K_j) \overset{(>)}{=} X_j \qquad j = 1,2,\ldots,n \tag{11'}$$

Equations (7') - (11') together with behavioral and pricing equations (1') - (6') define a simultaneous system of equations that must be fulfilled by all equilibrium solutions. It can easily be checked that all the equations are homogeneous in all prices (both gross and net prices), wage rate, and total consumption expenditure. Therefore, the general level of prices is indeterminate, i.e., it can be arbitrarily set. This can also be checked by counting the equations and variables (7n + 4 equations, 7n + 5 variables).

1.2. An Optimal Planning Model Version of the Problem

Now we come to the description of a rather typical planning model that would seek the optimal allocation of resources in the framework of the above model economy. An economy-wide planning model, built into and upon the traditional planning methodology of a socialist country, would differ from the above general equilibrium model in several respects. First, it would contain almost exclusively only "real" variables and relations reflecting physical allocational constraints. Second, because the prices used in a planning model are either constant or planned prices, forecasted more or less independently from "real" processes, the interdependence of the real and value (price, taxes, rate of return requirements, etc.) economic variables would not be taken explicitly into consideration in the model. Third, mathematical planning models in most cases closely relate to and rely on traditional or nonmathematical planning. This means, among other things, that the values of the exogenous variables and parameters and also certain upper and/or lower target values for some of the endogenous variables would not be directly derived from statistical observations, but would be based on figures given by traditional planners.* (This is not to say, however, that more or less sophisticated statistical estimation techniques would not be utilized, in combination with expert's "guesstimations," in traditional planning.) And, finally, planning modelers in socialist countries tend to concentrate more on the problems of how to fit their models into the actual process of planning and make them practically applicable and useful. Therefore, applied planning models tend to be simpler than those in the development planning literature both from economic theoretical and methodological points of view. The above list is, of course, far from complete, but nevertheless, these are some of the major characteristics common to many socialist planning models. These are also areas where the study of more sophisticated development planning models

---

*This is especially true for the nation-wide programming models used in Hungary, where the basic aim of the modelers is to check the feasibility and improve upon the efficiency of the plans elaborated by traditional planners (see Kornai, 1974).

(e.g., the general equilibrium models in question) may provide useful suggestions for further development of socialist economic model building.

To illustrate this point we will introduce a simplified planning model, which can be viewed as a representative example of how the above resource allocation problem would be modeled in a centrally planned economy. The variables in our case are the production levels of the various commodities (X_j; $j = 1,2,\ldots,n+1$), their consumption levels (C_i; $i = 1,2,\ldots,n$) and the amounts of labor and capital allocation for their production (N_j, K_j; $j = 1,2,\ldots,n$). All feasible resource allocation programs must satisfy the commodity (resource) balance requirements and the technological constraints given by inequalities (7') - (11'). Beyond that, as mentioned earlier, the planning model should reflect certain requirements set on the basis of the traditional plan calculations. We will consider here only a few representative solutions. For example, consumption of different commodities is limited from below:

$$C_i \geqq \bar{C_i} \qquad\qquad i = 1,2,\ldots,n$$

where $\bar{C_i}$ may be taken as the planned target level, or possibly somewhat lower. Despite a striking technical similarity of the assumed LES demand function in the outlined general equilibrium model and the "demand function" implied by the objective function of the planning model, there are basic, conceptual differences between the two approaches. In the former b_i's are usually interpreted as "subsistence" or, more accurately, "committed" consumption levels and assumed to reflect the preferences of the individual consumers. Their values are, in principle, based on reliable statistical estimates. In the latter, $\bar{C_i}$'s are more or less arbitrarily set minimum target levels, and thus they represent directly the planners' preferences—their "commitments."

The model builders would also take into consideration certain limitations concerning the possible intersectoral allocation of given primary resources. In the case of capital, for example, the already existing sectoral capacities may be taken as lower limits, while calculations concerning the capital absorptive capacities of the various sectors may indicate some upper limits for the amount of capital allocated to any given sector. In a similar way, lower and upper limits can be established for the number of workers employed in different sectors.

The first thing that the model builder would try to do with his model would be to check on the feasibility of the traditional plan and then to see if improvements could be made. For the sake of simplicity we assume that by improvement we mean an increase in consumption. More precisely, the level of performance of the economy is measured by the following objective (welfare) function:

$$g(C) = (C_1 - C_1^-)^{s_1} (C_2 - C_2^-)^{s_2} \cdots (C_n - C_n^-)^{s_n}$$

where $\sum_{i=1}^{n} s_i = 1$ by assumption. The chosen objective function is, thus, formally the same as the utility function underlying the LES. Introducing C_i^+ to represent the surplus (incremental) consumption instead of $C_i - C_i^-$, the above function can be rewritten in a simpler form

$$g(C^+) = C_1^{+\,s_1} C_2^{+\,s_2} \cdots C_n^{+\,s_n}$$

In most of the socialist planning models several different objective functions are used to find alternative ways of improving the efficiency of the plan. The objective function corresponding to consumption increase in a linear programming model is usually the surplus consumption (y) in a given structure. Thus, the consumption of commodity i is given by the following expression:

$$C_i = C_i^- + y\, c_i^+$$

where c_i^+ indicates the surplus consumption of commodity i in the case of a one unit increase in the general level of surplus consumption. This formulation may be interpreted in terms of consumer demand theory as a case where no substitutability between the different commodities exists. Thus in this case the demand function is:

$$C_i = C_i^- + \frac{c_i^+}{\sum_{j=1}^{n} P_j C_j^+} \left(E - \sum_{j=1}^{n} P_j C_j^- \right)$$

Using the above specifications the optimal plan would be determined as the solution of a nonlinear programming problem in which we maximize function $g(C)$ subject to the following constraints

$$(P_i) \qquad \sum_{j=1}^{n+1} a_{ij} X_j + C_i^- + C_i^+ \leqq X_i \qquad i = 1,2,\ldots,n \tag{7'}$$

$$(P_{n+1}) \qquad \sum_{j=1}^{n} \delta_j K_j + I \leqq X_{n+1} \tag{8'}$$

$$(R) \qquad \sum_{j=1}^{n} K_j \leqq K \tag{9'}$$

$$(R_j^-, R_j^+) \qquad K_j^- \leqq K_j \leqq K_j^+ \qquad j = 1,2,\ldots,n$$

$$(W) \qquad \sum_{j=1}^{n} N_j \leqq N \tag{10'}$$

$$(W_j^-, W_j^+) \qquad N_j^- \leqq N_j \leqq N_j^+ \qquad j = 1,2,\ldots,n$$

$$(P_j^*) \qquad X_j - F_j(N_j, K_j) \leqq 0 \qquad j = 1,2,\ldots,n \tag{11'}$$

$$X_j, C_j^+, K_j, N_j \geqq 0$$

where the meaning of the old variables and parameters is the same as before, C_i^+ stands for the amount of surplus consumption from commodity i, K_j^-, K_j^+ and N_j^-, N_j^+ represent, respectively, the lower (-) and upper (+) bounds of K_j and N_j. The symbols in parentheses denote the dual variables associated with the given constraints.

Shadow Valuation System and Shadow Behavioral Rules

Under reasonable assumptions the above problem will have a solution and all the variables will have positive optimal values. In such a case the *dual variables* associated with the various constraints (in parentheses) will, in the optimal solution, satisfy certain conditions. These conditions may be derived by differentiating the Lagrange function with respect to the primal variables, as indicated after each equation in parentheses (positivity of the primal variables is assumed):

$$P_{n+1} = \sum_{i=1}^{n} P_i a_{i,n+1} \qquad \left(\frac{\partial L}{\partial X_{n+1}}\right) \tag{1'}$$

$$P_j = \sum_{i=1}^{n} P_i a_{ij} + P_j^* \qquad j = 1,2,\ldots,n \qquad \left(\frac{\partial L}{\partial X_j}\right) \tag{5'}$$

$$P_i = \frac{\partial g}{\partial C_i^+} \qquad i = 1,2,\ldots,n \qquad \left(\frac{\partial L}{\partial C_i^+}\right) \tag{12'}$$

$$P_j^* \frac{\partial F_j}{\partial N_j} = W + (W_j^+ - W_j^-) \qquad j = 1,2,\ldots,n \qquad \left(\frac{\partial L}{\partial N_j}\right) \tag{13'}$$

$$P_j^* \frac{\partial F_j}{\partial K_j} = P_{n+1}\, \delta_j + S + (S_j^+ - S_j^-) \qquad j = 1,2,\ldots,n \qquad \left(\frac{\partial L}{\partial K_j}\right) \tag{14'}$$

Now it can easily be shown that the shadow prices given by the optimal dual solution (satisfying the above equations) are, in fact, of the same nature as the equilibrium prices and rates of return in the equilibrium model that has been examined. Also, we will show that one can formulate behavioral equations from this model similar to those of the general equilibrium model. Then we will comment on the interpretational differences of the two models and derive some conclusions.

To see the formal identities of the valuation and behavioral rules in the two cases, notice first that equations (1') and (5') of the equilibrium model appear in identical forms in the dual of the otpimal planning model. Equations (3') and (4'), which represent the necessary conditions for profit maximization, have equations (13') and (14') as their counterparts. At first glance they seem to be quite different, but closer examination reveals some essential similarities. Let us take equations (3') and (13') first. Their left-hand sides are identical, whereas on the right-hand sides we find different forms. In the literature concerning the design of (linear) programming models for development planning (see, for example, Taylor 1975 and Ginsburgh and Waelbroeck 1979), the use of individual bounds (like L_j^-, L_j^+) is often criticized because they "pick up shadow prices which have no clear meaning and which, since all dual prices are independent, distort the dual solution" (Ginsburgh and Waelbroeck 1979:5-6). In our case, however, the shadow prices of the individual bounds can be given reasonable meaning in the light of the equilibrium model. Variable W can be interpreted as the general level of optimal rate of return on labor. Next, we define

$$\omega_j = \frac{W + W_j^+ - W_j^-}{W} = \left(1 + \frac{W_j^+ - W_j^-}{W}\right) \qquad j = 1,2,\ldots,n$$

where the derived variable, ω_j may be interpreted as an endogenously determined index of the relative optimal rate of return on labor in sector j.

Similarly, we may interpret the dual variable S as the general level of the optimal (shadow) rate of return on capital at base price. Thus we can calculate $R = \frac{S}{P_{n+1}}$ so as to get the same rate of return at current (shadow) prices, and

$$\beta_j = \frac{S + S_j^+ - S_j^-}{S} \qquad j = 1,2,\ldots,n$$

can be interpreted as an index of relative rate of return requirement on capital in sector j.

So far we have not yet shown that the solution of the optimal planning problem would also imply the emergence of a set of special "demand" equations. We now look at this. Observe that the partial derivative of the primal objective function in equation (12') can be substituted by the term $\frac{s_i}{C_i^+} g(C^+)$ where $g(C^+)$ is the value of the objective function. Thus

$$P_i = \frac{s_i}{C_i^+} g(C^+) \qquad i = 1,2,\ldots,n \qquad (14a')$$

Multiplying the above equations by the respective C_i^+ and adding them together yields

$$\sum_{i=1}^{n} P_i \, C_i^+ = g(C^+) \qquad (14b')$$

On the other hand, total consumption expenditure is determined by

$$\sum_{j=1}^{n} P_j \, C_j^- + \sum P_j \, C_j^+ = E \qquad (14c')$$

Incidentally, this indicates how the level of total expenditure is endogenously determined in the general equilibrium model. Since we have only one consumer, the Pareto-optimal solution will be simply that which maximizes the utility function. The expenditure level will be determined by the value of this consumption bundle evaluated at the equilibrium prices.

From equations (14b') and (14c') we get

$$g(C^{+}) = \left(E - \sum_{j=1}^{n} P_j\ C_j^{-}\right) \qquad j = 1,2,\ldots,n$$

Finally, substituting $g(C^{+})$ in (14a') by the above value and solving the equation for C_i^{+} yields

$$C_i^{+} = \frac{s_i}{P_i}\left(E - \sum_{j=1}^{n} P_j\ C_j^{-}\right) \qquad i = 1,2,\ldots,n$$

Thus, the total consumption of commodity i is

$$C_i = C_i^{-} + \frac{s_i}{P_i}\left(E - \sum_{j=1}^{n} P_j\ C_j^{-}\right) \qquad i = 1,2,\ldots,n$$

which is the demand function implied by the specifications of the optimal planning model. The parameters of this are, however, evaluated on the basis of information provided by traditional plan calculations.

1.3. Technical Similarities and Conceptual Differences

We have illustrated the *technical similarities* of the programming models and equilibrium models. There is only one point where the two models are not formally identical. This is the "mechanism" by which the allocation of primary resources is exogenously controlled. This is, in fact, the only deviation from the standard literature in which primary factors are assumed to be perfectly homogeneous, with no constraints on their intersectoral (re)allocation. It is even tempting to interpret these different formulations as two alternative ways of reflecting the limited intersectoral mobility of the primary factors. In an otherwise perfect market economy this immobility would be indirectly expressed, by varying rates of return on the primary factors. In a centrally planned economy, on the other hand, this immobility would be directly accounted for, in terms of physical constraints. The planners would separate in advance the sectorally committed (immobile) part of the primary factors from their mobile one.

Beside the word similarity (or identity) the adjective technical also deserves attention. The formulation of a general equilibrium model is strongly influenced and directed by abstract theoretical considerations. Both the structure of the model and the numerical evaluation of its parameters depend heavily on, and should be consistent with, theoretical assumptions, e.g., individual optimization behavior and marginal productivity pricing. These are retained even though the model is usually built upon macro aggregates, to which the postulated micro behavioral rules cannot be mechanically applied. Socialist planning model building, on the other hand, tends to be more pragmatic. Linear programming, for instance, is considered one available technical device or framework that may help planners to generate additional information by numerical thought experiences. The term optimal planning model is even misleading in this context. The main role of programming models in planning, as we have indicated earlier, is in the coordination phase of planning, where it serves the purpose of checking the consistency and the efficiency of the proposed resource allocation. Based on the available planning information the model is used for generating more efficient programs by allowing a limited reallocation of resources and by formulating alternative objective functions.

Nevertheless, the main point of this exercise is that an optimal resource allocation framework can be substituted by a simultaneous equation system, i.e., by a system common to most applied general equilibrium models. At this point, however, the question arises: what are the possible benefits of such a transformation, therefore justifying the adaption of more complicated solution techniques? The answer is in the greater flexibility of their formulation. A general equilibrium model can do almost everything that a programming model can do, plus it incorporates considerations that are not possible in a programming model. Of course, the usefulness of such a transformation depends, to a large extent, on the overall specification and intended use of the model. In Section 2 we will try to demonstrate that in some aggregated nation-wide modeling exercises a general equilibrium framework allows for, among other things, a much greater flexibility in defining the relationships of the model variables and also a more realistic description of existing price formation rules, taxes, subsidies, etc.

One of the outstanding advantages of the equilibrium framework is that it may provide ways for planners to achieve a better linkage between planning the real and the value processes. These two main planning functions are usually quite separate from each other both in traditional planning and in modeling. Changes in relative prices, costs, tariffs, etc., are not reflected properly in physical allocation models, while the effects of production, export-import, and consumption decisions are not always taken into consideration in price planning models. Planning models in the form of a simultaneous nonlinear equation system might prove to be especially useful in aggregate comparative statics analyses. These models are useful because they can accommodate substitution possibilities and prevent overspecialized solutions by means of a relatively small number of parameters, unlike the linear programming models.

We would like to be more specific on two of the abovementioned issues. One of them concerns the possibility of having alternative economic policy goals to measure efficiency gains in a general equilibrium resource allocation model in a way similar to alternative objective functions in a programming model. It should be clear from the specification of the equilibrium conditions explained above, that the model is not a completely closed equilibrium system. Thus, for example, the distribution and redistribution of income does not appear in the model. At the same time the total household expenditure and consumption is endogenously determined. The programming reformulation sheds some light on the nature of such a solution. Since every other possible policy issue, such as net investments, government consumption, levels of primary input usage, and current account balances are exogenously determined, practically all gains (resulting from increased allocational efficiency) will show up as an increase in the level of consumers' utility. In the light of this consideration it becomes obvious that the same kind of general equilibrium model can be made to reflect various other possible economic policy goals, e.g., increasing government consumption or net investment, or decreasing deficit on current account, etc. (The reverse case is also interesting, i.e., when exogenous changes cause a decrease in overall efficiency of the given economy. In such a case one could estimate losses in various terms.) It is also possible to build into the model some weighted sum of the improvements. The incorporation of "objective functions" other than consumption would need changes only in the structure of endogenous and exogenous variables or perhaps the introduction of some new variables and equations into the model. By such simple modifications one can make the equilibrium model capable of handling alternative policy objectives in the same way as the programming models (also see Section 2 on this issue).

The second issue is related to the price formation rules. An equilibrium approach is strongly favored here. The zero or nonprofit condition has appeared in both the equilibrium model and in the programming model. We will discuss the validity and usefulness of this assumption in more detail in Section 2, however, we would like to point out here that with constant returns to scale technology, the optimal programming model will always generate shadow prices that fulfill the zero profit condition. On the other hand, the equilibrium framework—with slight reinterpretation—gives room for taking into account positive profits even with constant returns to scale. The prices generated in this way can reflect more accurately the real price formation rules.

Finally, a few words to indicate our understanding of the term "techniques of applied general equilibrium models." General equilibrium *theory*, especially its theoretical *models* possessing a high degree of closure and a rather narrow, mathematically oriented scope of investigation, has been criticized from several points of view and by many Marxist and non-Marxist authors.* It is not always clear what the boundaries of general equilibrium theory are, since it is capable of incorporating many partial models and techniques that have been originally developed independently. Economists understand and relate to these problems differently, therefore it seems to be useful to indicate our understanding of general equilibrium theory and models as well as their place among the analytical tools available to economists. We would like to make two distinctions, namely, between general equilibrium *theory* and general equilibrium *techniques* on the one

---

*See, for instance, Kornai (1971) for a systematic exposition of the most common critiques.

hand and between *pure* and *applied* general equilibrium models on the other. These distinctions are rather tentative.

General equilibrium theory, in our understanding, is an abstract representation of the law of supply and demand placed in the framework of a simplified model of a much more complex economic system. By general equilibrium modeling techniques we mean the more or less standard, analytical tools that can be used either in defining the elements of a general equilibrium model (supply and demand functions, production functions, programming models, etc.) or in the definition of, or the search for, an equilibrium (e.g., complementary slackness criteria, fixed point algorithms). A model using general equilibrium modeling techniques can be completely outside of the theory. Let us only refer here to our earlier discussion, where we have tried to demonstrate that there is only a formal, technical identity between an optimal planning model and a neoclassical general equilibrium theoretical model.

As is often the case, the abstract general economic equilibrium theory differs in many ways from applied models based on that theory. We want to emphasize one important point of departure. The abstract theory of general equilibrium postulates the *a priori* knowledge of the external environment (production and consumption sets, preference orderings, relative profit shares of various households, etc.) which is assumed to be independent of the endogenous variables (prices, production, and consumption decisions). The equilibrium solution—if there is any—is then determined by the parameters of the external environment and by the also *a priori* postulated behavioral rules. We could characterize this approach as a "global" or "absolute" one. Applied general equilibrium models follow a more or less reverse order and take a "local" or "relative" point of view. What one can observe in reality is mostly the values of the endogenous variables. Whether they represent an equilibrium or not, and more importantly, whether there is any such mechanism behind the determination of these values, is not really known. As a matter of fact, one determines many of the assumed but directly unobservable external parameters by assuming that the observed data were generated as equilibrium. (It seems to be almost impossible to test this assumption or to estimate the errors caused by this assumption in our analysis.) The aim is to estimate the likely consequences of alternative external environment changes in terms of *relative* changes, i.e., by comparing the "base equilibrium solution" with the calculated one. The equilibrium framework is, therefore, used only as one of those tools that present economic theory can offer for the complex analysis of such issues. One could also say, that general equilibrium theory has empirical values, in fact, only in the above "relative sense."

The general ideas expressed by the abstract theory of general equilibrium (basically the dependence of economic decisions on relative prices and costs and on resource constraints) have certainly more empirical relevance when one tries to explain relative changes rather than absolute magnitudes. The main advantage of these models is that they provide a framework in which the envisioned partial changes can be evaluated in a consistent and coordinated way, taking into account the interdependence of some crucial variables.

1.4. The Use of Smooth Production Functions in Planning Models

The simplified planning model that has been examined in Section 1.2 differs from the typical applied models in one point—it is nonlinear. This nonlinearity is only due to the use of smooth production functions, because the other nonlinear relationship, the objective function, could be easily linearized. It should be clear that the production function serves only one purpose in the model.* If one had fixed labor and capital input coefficients—as in the case of the intermediate inputs—then there would be no choice between more or less labor (capital) intensive techniques, in fact, there would be no technological alternatives for the different sectors at all. Linear planning models usually do allow for alternative technologies but, of course, in a different way. In rather aggregated macro planning models one would usually see two or three alternative technologies, but, in addition, upper and/or lower bounds would restrict

*The term "production function" is not quite appropriate here since it defines only a composite primary factor.

the output levels. The range of choice between more or less labor (capital) intensive technologies will, therefore, be very limited.

To illustrate this point we take a rather typical example. Suppose the model builder has information on two technologies available in the future for a specific sector: production with old and new equipments. For the sake of simplicity, it will be assumed that these technologies differ only with respect to their labor and capital intensities. Let k^o, n^o, and k^1, n^1 represent the capital and labor coefficients in the "old" and "new" technology, respectively. Let us also take into account the bounds imposed on the production levels

$$\underline{X}^o \leqq X^o \leqq \overline{X}^o$$

and

$$X^1 \leqq \overline{X}^1$$

where we have assumed that the production with old equipment is bounded both from above ($\overline{X}^o$) and below ($\underline{X}^o$), while the production with new equipment is bounded only from above ($\overline{X}^1$). (The upper bounds can be taken as the planned capacities.)

We illustrate the result of these restrictions in Figure 1. It can easily be checked that only the points in the parallelogram ABCD will give actually feasible combinations of capital and labor. It is also apparent that the substitution possibilities vary with the level of production. At levels $\underline{X}^o$ and $\overline{X}^o + \overline{X}^1$ there is no possibility for substitution between the production factors, but in between these levels the substitution possibility first increases, then decreases with the level of output. The feasible capital and labor coefficients will also be limited by k^o, $\hat{k}$ and n^o, $\hat{n}$, where $\hat{n}$ and $\hat{k}$ are determined by point B. [From the above observations it also becomes apparent how misleading it would be to identify the feasible technological set with the isoquant map derived from the two basic linear activities in a "text-book" fashion (Figure 1).]

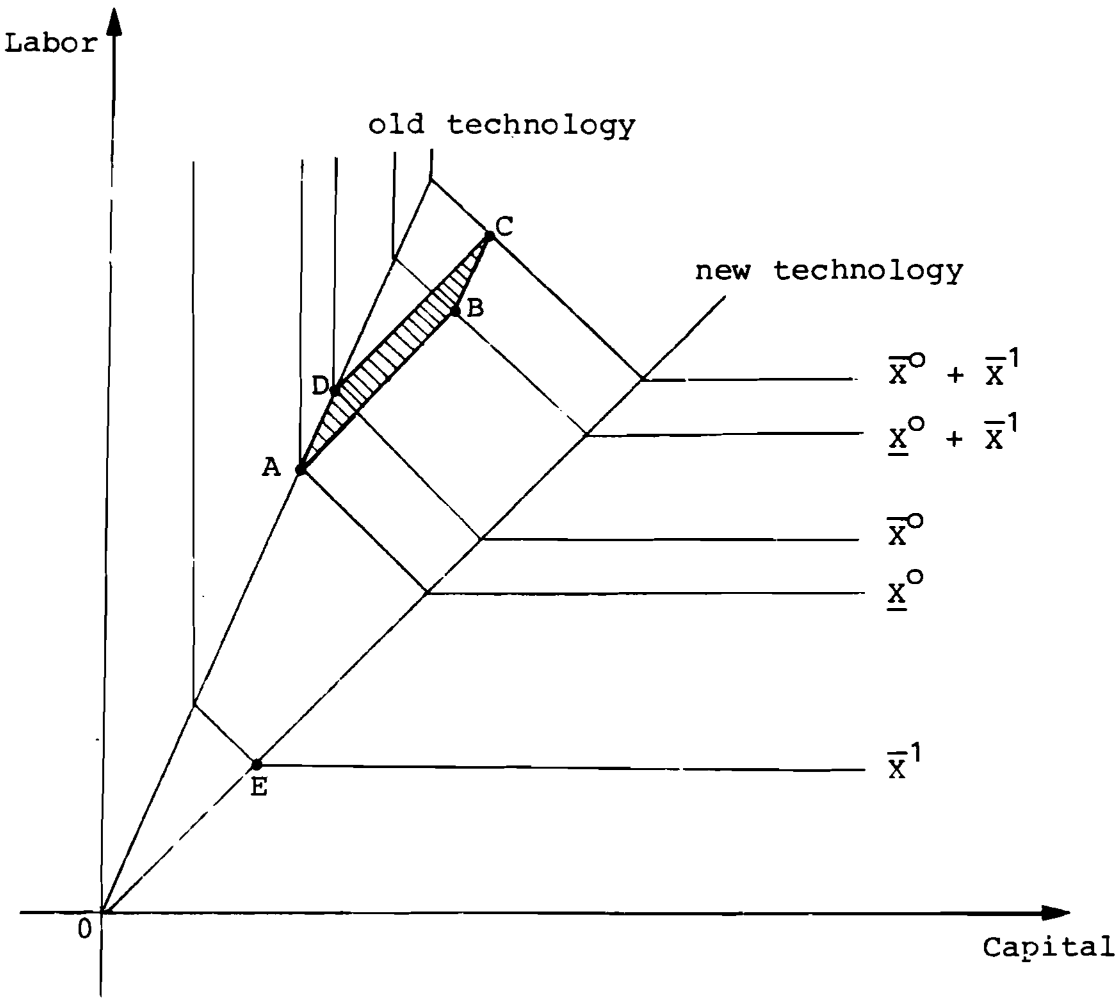

Figure 1. The theoretical isoquant map and the real substitution possibilities in a linear programming model.

For a more realistic description of the sectoral production possibilities in a linear model one has to give up the macro character of the model. Each sector has to be broken down into subsectors and individual bounds should be introduced to the subsectoral activities. If, however, one wants to keep the size of the model small (to have, say, 15-20 sectors only), and still represent a reasonable technological choice for each sector, smooth production functions seem reasonable to use. Here again, the real considerations behind this choice are pragmatic rather than theoretical. What one makes use of is the information-condensing power of the production functions, as simple statistical devices (regressional schemes).

Of course, in a planning model, which is based on the detailed (traditional) plan calculations, the estimates of parameters of the macro production functions should also be based on the above information. This is unlike the case of the cited applied general equilibrium models. In these models the parameters of the sectoral (macro) production functions are either econometrically estimated or simply "guesstimated" on the basis of similar econometric estimates. Beyond the well known statistical estimation problems (see, for instance, Berndt 1976 or Caddy 1976) these empirical estimates are more severely biased by the neoclassical marginal productivity pricing assumption widely used in the (indirect) estimation procedures. One would be rather reluctant to use such estimation techniques in socialist planning.

The estimation of the parameters of a short term macro production function could be based on the following or similar procedure.* One defines first a set of activities in terms of the total level of their output, and labor and capital inputs ($\overline{X}$, $\overline{N}$, $\overline{K}$). These activities in an *ex post* analysis could be identified by actual enterprise (or subsectoral) data. In an *ex ante* (planning) model such data could be generated on the basis of enterprise calculations concerning their future development plans. Next, upper and lower bounds have to be assigned to the individual

*This method bears an obvious resemblence to Johansen's (1972) treatment of the sectoral production functions and also to the way in which Rimmler, Dániel, and Kornai (1972) estimated macro functions on the basis of programming models.

output levels, within which they are allowed to vary $(\underline{\alpha}_k \overline{X}^k, \overline{\alpha}_k \overline{X}^k)$. Finally, fixing the sectoral total output at some level, say at $\overline{X}$, one could generate a reasonable number of alternative *intrasectoral* production structures yielding the same amount of total sectoral output. The different production structures will imply different labor/capital combinations. These combinations, in turn, can be treated as points lying on or around the same isoquant. Thus, they make it possible to estimate the parameters of a linear homogeneous LES production function.

The formal procedure that generates the above alternative labor/capital combinations can be based on the solution of the following linear inequality system:

$$\sum_{k=1}^{N} X^k = \overline{X}$$

$$\underline{\alpha}_k \, \overline{X}^k \leq X^k \leq \overline{\alpha}_k \, \overline{X}^k \qquad\qquad k = 1,2,\ldots,N$$

$$N = \sum_{k=1}^{N} n_k \, X^k$$

$$K = \sum_{k=1}^{N} k_k \, X^k$$

The above system could be solved by parametric solution techniques, i.e., by fixing the amount of, say, labor at different levels and calculating the corresponding values for capital. It is important to note here that we do not necessarily want to look for efficient capital/labor combinations only. (Such solutions could be achieved if instead of simply solving the inequality system we minimized the amount of capital at each level of labor.)

It should also be clear that the substitution of basic linear technologies by smooth production functions in a planning model is, in fact, a way to decompose the problem. For each sector one solves first separately a constrained linear activity model. Then by production functions, one condenses this production information into a few (three or less) parameters. By doing so one can reduce the size of the core model to a considerable degree, which may be quite useful in the case of many repeated runs. It also keeps the model more transparent. One would suspect that the dual solutions of such aggregated models would become more stable and easier to interpret than the shadow prices of large linear systems with many individual bounds.

Finally, we view the linear homogeneity assumptions simply as convenient assumptions. By using smooth functions instead of fixed capital and labor coefficients, we merely incorporate substitution possibilities into an otherwise linear model.

2. A TENTATIVE MODEL FRAMEWORK FOR HUNGARY

The model developed here reflects to a large extent—but of course in a simplified manner—the existing planning theory and modeling practice in Hungary. There are, however, a few places where the mathematical formulations differ from the "traditional" ones, and this is mostly due to the nonlinearity of the model. The novelty of the outlined model lies mostly in the fact that it integrates the partial models into a consistent framework and directly takes into account the interdependence of real and value processes. This is a basic requirement not met by the recently applied planning models in socialist countries.

The model differs in many aspects from other applied general equilibrium models, including the Bergman-Pór (1980) model, which had the strongest influence on its specifications. For example, several modifications in the treatment of foreign trade can be found. First of all, the two main trading areas (the rouble and dollar regions) are represented separately in our model. There is also a distinction made with respect to the competitive and noncompetitive nature of imported commodities. A third difference in the treatment of foreign trade is that we use export supply functions rather than export demand functions, which are more in line with the basic assumptions that small economies are price-takers on the world market. We have also taken into consideration that, even if a small country cannot influence the world market prices on the whole, it may face decreasing returns on exports. Another important difference can be found in the way we take wage determination into account. Instead of using the marginal productivity assumption, we treat wages as exogenously determined variables and introduce the concept of the user cost of labor, which is made up of the wage rate and a general net return requirement. We have also eliminated the zero profit assumption without foregoing the linear homogeneity assumptions. A further deviation from the Bergman-Pór model is that instead of treating net investment exogenously, we introduce a fixed consumption-investment ratio. We have also incorporated some demoeconomic elements to take into account the difference in the demand structure of urban and rural households.

In Sections 2.1 - 2.5 the structural equations of the model are presented and discussed. Section 2.6 contains the list of variables and parameters as well as a condensed mathematical statement of the model.

2.1. Commodities and Commodity Balances

Primary Resources and Factors of Production

There will be two *primary factors of production* taken into consideration at this stage of the research: labor and capital. Their available quantity is assumed to be exogenously given, while their intersectoral allocation will be determined within the model. It is required that the total use of these production factors be equal to their available amounts. This requirement can be expressed by the following resource balance equations*

$$\sum_{j=1}^{n} K_j + K_g = K \tag{4}$$

$$\sum_{j=1}^{n} N_j + N_g = N \tag{5}$$

*Equation numbers in this section correspond with the mathematical statement in Section 2.6.

where K stands for total capital stock, N for total labor available, K_g and N_g denote capital stock and labor used in public (governmental) services (all of them exogenously determined), while K_j and N_j represent the amount of capital and labor used in different sectors ($j = 1,2,\ldots,n$).

We are aware that these are very simplistic treatments of labor and especially capital, but this simplicity makes it easier to understand the general structure of the model. If one wants to use a static model such as ours, a distinction must be made at least between sectorally committed and uncommitted primary factors in order to constrain their intersectoral mobility. In a planning context some combination of *ex post* and *ex ante* production functions might provide a more realistic description of the resource allocation possibilities.

Apart from labor and capital there are n *noncompetitive import* commodities that are treated as primary resources in the model. These will be discussed later.

Intermediate Commodities and Their Balances

Production in the economy will be classified into n producing sectors, each of them producing a sector-specific commodity (or, rather, a commodity group). We will adopt the usual input-output modeling framework and assume that the sectoral outputs are homogeneous commodities. Also, when competitive imports are taken into consideration, they are assumed to consist of the same homogeneous commodities as the sectoral outputs. These are rather binding but necessary concomitant assumptions of the convenient input-output modeling framework.

The number of sectors and the character of the sectoral classification depends to a large extent on the specifics of the whole model, but in order to control the size of the model we intend to have not more than 20 - 30 sectors.

To each sector there belongs a commodity balance equation. More precisely, there are two commodities belonging to each sector: one composed of the *noncompetitive imports* of the same sectoral classification. We use these terms in a slightly unusual way. By noncompetitive imports we mean not only those imported commodities that are not and cannot be produced within the country, but also those imports that are deemed by planning experts to be totally unsubstitutable by domestic production in the given period.

The balance equation for noncompetitive imports takes into account their use in different areas, i.e., in production ($\overline{M}_{ij}$, $j = 1,2,\ldots,n$), in investment ($\overline{M}_{i,n+1}$), and in private ($\overline{C}_i$) and public ($\overline{G}_i$) consumption. The sum of these different uses must be equal to the total available amount ($\overline{M}_i$):

$$\overline{M}_i = \sum_{j=1}^{n+1} \overline{M}_{ij} + \overline{C}_i + \overline{G}_i \qquad i = 1,2,\ldots,n \qquad (3)$$

The balance equations for the commodities that are regarded homogeneous with (and perfect substitutes for) the domestic production will have the following form:

$$X_i + M_{ir} + M_{id} = \sum_{j=1}^{n+1} X_{ij} + C_i + G_i + Z_{ir} + Z_{id} \qquad (1)$$

$$i = 1,2,\ldots,n \; .$$

Total source is, thus, made up of domestic production (X_i), and competitive import from rouble (M_{ir}) and dollar (M_{id}) trading regions, whereas total use is the sum of intermediate usage (X_{ij}, $j = 1,2,\ldots,n$), capital accumulation ($X_{i,n+1}$), private consumption (C_i), consumption in public services (G_i, exogenously given), and exports into rouble (Z_{ir}) and dollar (Z_{id}) regions.

The (n+1)th sector represents gross investment.* It is a so-called book-keeping sector that creates homogeneous capital goods from the sectoral commodities, which is, in turn, used for replacing old capital and for net investment. The balance equation for this (n+1)th sector will thus have the following form

*Stock formation will, in general, be treated as part of the gross investment. In some calculations, however, it might be more appropriate to treat it exogenously.

$$X_{n+1} = \sum_{j=1}^{n} \delta_j K_j + \delta_g K_g + I \qquad (2)$$

where X_{n+1} is real gross investment, I is real net investment, and δ_j ($j = 1,2,\ldots,n$) is the depreciation rate of capital in sector j.*

2.2. Import and Export Functions, and Trade Balances

Imports are classified according to four criteria (but not in all cases):

- sectoral character of the imported commodity
- trading area (rouble or dollar region)
- competitive or noncompetitive (complementary) character of the imported commodity
- area of use of the imported commodity

Noncompetitive imports, used in production and investment, are determined as fixed proportions of the output levels:

$$\bar{M}_{ij} = \bar{m}_{ij} X_j \qquad \begin{matrix} j = 1,2,\ldots,n+1 \\ i = 1,2,\ldots,n \end{matrix} \qquad (10)$$

Government (public) consumption of noncompetitive imports are treated as exogenous parameters in the model, while the use of noncompetitive imports in private consumption is determined by demand functions.

The total noncompetitive imports of a given commodity are split into two parts: imports from rouble and dollar regions, assuming a finite but rather small elasticity of substitution between imports from the two trading areas:

$$\bar{M}_{ir} = \alpha_i \bar{M}_i \qquad i = 1,2,\ldots,n \qquad (14)$$

$$\bar{M}_{id} = (1 - \alpha_i) \bar{M}_i \qquad i = 1,2,\ldots,n \qquad (15)$$

---

*We do not distinguish here between depreciation and replacement rate, which may be quite different. In some cases, especially in short-run calculations, such a distinction may be desirable.

$$\alpha_i = \alpha_i^o \left(\frac{\overline{\theta}_{id} \; V_d \; \overline{P}_{id}^{WI}}{\overline{\theta}_{ir} \; V_r \; \overline{P}_{ir}^{WI}} \right)^{\delta_i} \qquad i = 1,2,\ldots,n \qquad (13)$$

Competitive imports are treated as perfect substitutes for the sectoral output, therefore, there is no need to specify their area of use. The competitive imports of a given commodity from rouble and dollar regions are treated separately. In both cases imports are determined by the total domestic use of the sectoral output ($X_i - Z_i$) and variable proportions of the imports* according to the following rules:

$$M_{ir} = m_{ir} \; (X_i - Z_i) \qquad i = 1,2,\ldots,n \qquad (18)$$

$$M_{id} = m_{id} \; (X_i - Z_i) \qquad i = 1,2,\ldots,n \qquad (19).$$

where

$$Z_i = Z_{ir} + Z_{id} \qquad i = 1,2,\ldots,n \qquad (20)$$

The proportions of the imports are determined in accordance with the following import functions

$$m_{ir} = m_{ir}^o \left(\frac{P_i}{\theta_{ir} \; V_r \; P_{ir}^{WI}} \right)^{\mu_{ir}} \qquad i = 1,2,\ldots,n \qquad (16)$$

---

*In some cases one might want to take into consideration the fact that imports from rouble regions are, as a rule, determined by long-term trade agreements from which it is rather difficult to deviate. In such a case, therefore, it seems more appropriate to determine the amount of imports from the rouble area independently from the domestic output, in accordance with import functions of the following type:

$$M_{ir} = M_{ir}^o \left(\frac{P_i}{\theta_{ir} \; V_r \; P_{ir}^{WI}} \right)^{\mu_{ir}} \qquad i = 1,2,\ldots,n$$

$$m_{id} = m_{id}^{o} \left(\frac{P_i}{\theta_{id} \; V_d \; P_{id}^{WI}} \right)^{\mu_{id}} \qquad i = 1,2,\ldots,n \qquad (17)$$

The interpretation of these import functions in a planning context could be based on the following observations. If, say, μ_{id} in equation (17) were set equal to zero, then the dollar imports would be determined as fixed proportions of the total domestic use of the sectoral output (i.e., as fixed proportions of total output less exports). This kind of import determination is quite commonly used in applied input-output models, where m_{id}^{o} can be taken as the planned proportion of the imports. The modifying term in the import function is intended to reflect substitution effects in the following sense. If, *ceteris paribus*, the domestic price (P_i) of the given commodity increases relative to its import price (θ_{id} V_d P_{id}^{WI}), then the relative share of imports is going to increase. The elasticity parameter μ_{id} is intended to reflect whether one assumes a larger or a smaller shift in the import share (more or less "friction" in the adjustment process). It should be emphasized that although import functions of this form are commonly used in general equilibrium models, the interpretation attached to them is often quite different from ours.* This difference in interpretation provides an illustration for our earlier discussion concerning the distinction between theory and techniques.

Exports are determined by export functions. There are essentially two ways to define export functions: export supply or export demand functions. From a technical point of view the

*The theoretical justification of such functions is based on the assumption of the same type of CES preference function for each user and their optimizing behavior. (See, for instance, Bergman, 1980). It has been pointed out to me by Lars Bergman that if one wants to be consistent with these assumptions, then the domestic output and the competitive imports could not be treated as perfect substitutes (they cannot be added together). It should be clear that our understanding of "limited substitution possibility" is different from the one implied by the above assumptions. We do assume the perfect substitutability of competitive imports and domestic production, but an imperfect substitution mechanism. Whereas a neoclassical interpretation would assume imperfect substitutability in use but a perfect substitution mechanism in theory.

export function has identical forms in both cases (for the sake of simplicity in this general discussion we disregard taxes and trade region specifications):

$$Z_i = Z_i^o \left(\frac{P_i}{V\ P_i^{WE}}\right)^{\varepsilon_i} \qquad\qquad i = 1,2,\ldots,n$$

The interpretation of this form, however, is quite different in the two cases, also, the choice of interpretation will affect the treatment of exports in the trade (current account) balance equation.

In the case of export supply interpretation, the basic assumption is that the world market (export) price (P_i^{WE}) and the rate of exchange (V) determine the price that the seller obtains for his commodity if he sells it abroad. Thus the export supply is determined on the basis of the domestic seller price (P_i) and the export seller price ($V\ P_i^{WE}$). In this case the exports in the trade balance must be evaluated at world market (export) prices. Note also, that in this case it is assumed that the given country is a perfect price-taker in the world market.

In the second case, there is an implicit assumption that the given country cannot influence the *general* world market price level (P_i^{WE}), but it can set the export price for its commodity. This latter price is determined by the domestic price (P_i) and the exchange rate (V). It is assumed that the foreign purchasers will increase or decrease their demand in accordance with the relative change of the general world market price (P_i^{WE}) and the offered price level (P_i/V). In this case the exports in the trade balance must be evaluated at prices P_i/V.

In the case of small, open economies, it is reasonable to assume that they are, in general, price-takers rather than price-makers on the world market. This, however, should not mean that they cannot influence their export prices at all. It is generally assumed that the volume of the country's exports does have an effect on the export price that it can achieve, due to the limited absorptive capacities of the supplied markets. The combination of these two assumptions gives rise to the following export specification. The export price is basically following the general world market export price (P_i^{WE}), but it is modified by a term reflecting the effect of the size of the export on the actual export price (P_i^{AE}):

$$P_i^{AE} = P_i^{WE} \left(\frac{Z_i^o}{Z_i}\right)^{\lambda_i}$$

In a planning modeling context, Z_i^o can be the planned amount of exports and P_i^{WE} the forecasted unit export price.* Therefore, in this case, the export supply function will have the following form:

$$Z_i = Z_i^o \left[\frac{P_i}{V\ P_i^{WE}\left(\frac{Z_i^o}{Z_i}\right)^{\lambda_i}}\right]^{\varepsilon_i}$$

The solution of this equation for Z_i yields

$$Z_i = Z_i^o \left(\frac{P_i}{V\ P_i^{WE}}\right)^{\frac{\varepsilon_i}{1-\lambda_i\varepsilon_i}} = Z_i^o \left(\frac{P_i}{V\ P_i^{WE}}\right)^{\varepsilon'_i}$$

where

$$\varepsilon'_i = \frac{\varepsilon_i}{1 - \lambda_i\varepsilon_i}$$

The export function is, therefore, formally the same in this case as above. There is only a change in the absolute value of the elasticity parameter; it will be somewhat lower than in the case of a "pure" export supply function. Note, however, that in the trade balance the exports must be evaluated in this case at prices P_i^{AE}.

---

*In a static model like ours a change in the export volume of one country can be interpreted as if its share in the total world export had changed. In the light of this observation one can generalize the above price form by substituting the export volume (Z_i) by the country's share in the total world export (s_i). Such an export price function could then be used in a dynamic model as well. (I owe thanks to Urban Karlstrom for calling my attention to this interpretational possibility).

In determining *dollar exports* we will use the above *modified* export supply function, with one additional change. In order to increase the export supply the government can subsidize the exporter or impose taxes on the income from exports as a means of curtailing exports. This factor has a direct effect only on the export supply function and not on the trade balance. For this reason, we use the following export supply functions

$$Z_{id} = Z^{o}_{id} \left(\frac{P_i}{\phi_{id} \; V_d \; P^{WE}_{id}} \right)^{\varepsilon_{id}} \qquad i = 1,2,\ldots,n \qquad (22)$$

In determining *rouble exports* we use similar export supply functions without taking into account the price modifying effect of the export size*

$$Z_{ir} = Z^{o}_{ir} \left(\frac{P_i}{\phi_{ir} \; V_r \; P^{WE}_{ir}} \right)^{\varepsilon_{ir}} \qquad i = 1,2,\ldots,n \qquad (21)$$

Since roubles and dollars are not exchangeable in general it is therefore more appropriate to have two trade balances in the model, rather than one aggregated current account. In accordance with the export-import specification the trade balances (current accounts) have the following forms

$$\sum_{i=1}^{n} \left(\frac{Z^{o}_{id}}{Z_{id}} \right)^{\lambda_i} P^{WE}_{id} \, Z_{id} - \sum_{i=1}^{n} P^{WI}_{id} \, M_{id} - \sum_{i=1}^{n} \overline{P}^{WI}_{id} \, \overline{M}_{id} = D_d \qquad (6)$$

$$\sum_{i=1}^{n} P^{WE}_{ir} \, Z_{ir} - \sum_{i=1}^{n} P^{WI}_{ir} \, M_{ir} - \sum_{i=1}^{n} \overline{P}^{WI}_{ir} \, \overline{M}_{ir} = D_r \qquad (7)$$

*Alternatively, Z_{ir} could be treated as a free variable assuming infinitely absorptive export market capacities in rouble relation. Such a solution could be supplemented with export capacity restrictions. For each sector we could define export as fixed proportions of the output. Thus the rouble export could be determined as the difference between total exports and dollar exports.

where D_d and D_r are the target surplus or deficit levels on dollar and rouble foreign trade balance.

2.3. Final Demand and Regional Aspects

Public (government) consumption is exogenously determined as well as foreign trade balances. If total net investments were also exogenously given as in the Bergman-Pór model, then, in a model that examines allocative efficiency, all gains that result from the reallocation of resources would show up as an increase in private consumption.* In our model we plan to investigate several alternatives. Here we will discuss only one possible way to endogenize net investments. Suppose we maintain a given (real) consumption-investment ratio (σ).** This gives rise to the following equation

$$G + C - \sigma \cdot I = 0 \tag{36}$$

The efficiency gains in this case show up as an increase in both private consumption and net investments.

In another alternative solution the real value of consumption (C) and that of net investment (I) could be fixed exogenously and one of the foreign trade balance targets, for example, could be freed and made endogenous. In this way the efficiency gains would appear as improvements in the trade balance.

Household (private) consumption is endogenously treated. Depending on its intended use this part of the model may become more or less crucial, especially the interpretation and the estimation of the assumed consumers' respond (demand) function parameters. For simplicity we intend to make use of the Linear Expenditure System. In a static model applied in the final coordination phase of national economic planning, the use of such a specification can be justified on pragmatic grounds. It is

---

*On this point see our earlier discussion in Section 1.3.

**The ratio can be determined, for example, by the planned values of the variables: $\sigma = \frac{G^o + C^o}{I^o}$, if the model is used for analyzing a draft plan.

assumed that one can rely on the detailed plan calculations and use the planned consumption, expenditure, and price level to be a more or less consistent forecast of the future consumers' preferences. The model, however, would generate price levels and a total expenditure level that would vary from this. Therefore, one would like to incorporate into the model the likely effects of such changes on the level and structure of consumer demand. One could set the constant terms in the LES demand functions equal to the planned consumption levels and determine the elasticity parameters, relying on expert estimates of the desire structure of excess consumption. This solution would be basically equivalent to the one commonly used in the linear programming models applied to planning (see the corresponding discussion in Section 1.2).

If one wants a more reliable forecast of structural adjustment, especially in a longer time horizon, then some crucial demographic and spatial aspects cannot be neglected in the model. In traditional national economic planning models, such aspects are reflected only implicitly. (The data depict a given spatial, demographic structure and the changes are thought to be consistent with this assumed structure.) The need for demoeconomic formal models, integrating economic, demographic, and spatial variables into a consistent model framework, has been articulated mostly in the context of developing countries (see, for instance, Rogers, 1977). A few economic-demographic simulation models have recently appeared as a result of such efforts (for a critical review of these models see Sanderson, 1980). Although none of these models offers a satisfactory way for integrating the above mentioned aspects into a planning model, they may suggest useful points of departure for future research in this direction.

For illustration we will describe a simple device that captures the interaction of economic and demographic factors within the framework of our static model. The urban-rural distribution of economic activities and households and their interaction through final demand will be incorporated into the model. The basic idea, then, is to roughly assimilate shifts in the production structure, which imply changes in the urban-rural distribution of the economic activities and consequently, in the household distribution and consumption demand as well. In each sector, therefore, we estimate the proportion of urban and rural employment (s_{uj} and $s_{rj} = 1 - s_{uj}$), and we assume these proportions to be exogenous parameters. In the general equilibrium models of dualistic development (see, for instance, Kelley and Williamson, 1980 and Karlström, 1980) these proportions are either 1 or 0, so our solution may be seen as a generalization of this concept. Thus in each solution we can calculate the total urban and rural employment:

$$N_u = \sum_{j=1}^{n} s_{uj} N_j + s_{ug} N_g \tag{33}$$

$$N_r = N - N_u = \sum_{j=1}^{n} (1 - s_{uj}) N_j + (1 - s_{ug}) N_g \tag{34}$$

Next we assume that we have two different demand functions, one each for urban and rural households, and that the distribution of urban and rural households changes in accordance with urban and rural employment distribution. The aggregate household demand for commodity i will thus be given by the following sum:

$$C_i = \frac{N_u}{N_u^o} C_{iu} + \frac{N_r}{N_r^o} C_{ir} \qquad i = 1,2,\ldots,n \qquad (31)\ (32)$$

Rural and urban household expenditures are assumed to increase (decrease) in the same proportions as if there were no change in the distribution of rural and urban households. Thus if E is the general level of household expenditure, then the rural and urban household expenditures will be determined by

$$E_u = \frac{E_u^o}{E^o} E \tag{26}$$

and

$$E_r = \frac{E_r^o}{E^o} E \tag{30}$$

where E^o, E_u^o, and E_r^o are the planned (base) total, urban and rural expenditure levels, which fulfill the following identity:

$$E^o = E_u^o + E_r^o$$

The final demand for intermediate commodities and noncompetitive imports by the individual household sectors is determined by relative prices and total expenditure in accordance with LES demand functions:

$$C_{ik} = b_{ik} + \frac{c_{ik}}{P_i^D} \cdot EE_k \qquad (23)\ (27)$$

$$\bar{C}_{ik} = \bar{b}_{ik} + \frac{\bar{c}_{ik}}{\bar{P}_i^{DI}} \cdot EE_k \qquad \begin{matrix} i = 1,2,\ldots,n \\ k = u,r \end{matrix} \qquad (24)\ (28)$$

where

$$EE_k = E_k - \sum_{j=1}^{n} \left(P_j^D b_{jk} + \bar{P}_j^{DI} \bar{b}_{jk} \right) \qquad k = u,r \qquad (25)\ (29)$$

2.4. Prices and Costs

The price of commodity i is determined in accordance with the homogeneity assumption. There are three different sources of the same commodity: domestic production, rouble imports, and dollar imports. Except for exports, the user's price, i.e., the change of its level*, can be determined as the weighted average

*It should be noted that we are dealing with price indices (or levels) and not with actual prices that do not even exist for the commodity aggregates. The same is true for most of the other values or financial variables such as exchange rates and import tariffs. Their values are taken to be 1.00 at the base year (or base solution), which also means that the corresponding "real" variables are measured in these constant base-year prices.

of the three different price levels.* The domestic price level of the imported commodities changes if, *ceteris paribus*, there is a change in the import tariff-subsidy multiplier (Θ), in the exchange rate (V), or in the commodity's world market (import) price level (P^{WI}). Thus the average change of the domestic user's price level of commodity i (P_i^D) can be calculated from the following relationship

$$P_i^D (X_i - Z_i + M_{id} + M_{ir}) = P_i (X_i - Z_i) + \Theta_{id} V_d P_{id}^{WI} M_{id}$$

$$+ \Theta_{ir} V_r P_{ir}^{WI} M_{ir} \qquad i = 1,2,\ldots,n$$

Dividing by ($X_i - Z_i$) and solving the above equation for P_i^D yields

$$P_i^D = \frac{1}{1 + m_{id} + m_{ir}} P_i + \frac{m_{id}}{1 + m_{id} + m_{ir}} \Theta_{id} V_d P_{id}^{WI}$$

$$+ \frac{m_{ir}}{1 + m_{id} + m_{ir}} \Theta_{ir} V_r P_{ir}^{WI} \qquad i = 1,2,\ldots,n \qquad (42)$$

In socialist countries, the price of the domestic output is most often measured by the so-called producer's (factor cost) price, which does not contain turnover taxes and other *ad valorem* taxes or subsidies. The consumer's (market) price of the same commodity can be different depending on the purchasing area. It would make the model overly complicated if we took all these variations into consideration. Therefore, we assume a rather simple correspondence between the producer's and the consumer's price of the sectoral output, namely,

*Recent pricing policy in Hungary (competitive pricing) connects the change of domestic prices more closely to foreign trade (either exports or imports) price changes. This part of the model needs revision in the light of the new price formation principles. The model could also be used for the assessment of the likely overall impact of this change in price policy.

$$P_j^C = (1 + \tau_j)\ P_j^P \qquad\qquad j = 1,2,\ldots,n$$

where τ_j is the net turnover tax rate on commodity j and P_j^C and P_j^P are the consumer's and producer's prices, respectively. Also, since we have competitive import in our model, the data concerning the production and use of domestic commodities are assumed to be given at consumer prices. The price of the domestic output is measured by the consumer price index, thus it reflects the changes of both the producer's price and the net turnover tax rate. We use the following basic price calculation scheme to determine the (consumer's) price of domestic output (omitting superscript C, since only the consumer's price is used in the formal model)

$$P_j = \left(\sum_{i=1}^{n} P_i^D a_{ij} + \sum_{i=1}^{n} \overline{P}_i^{DI} \overline{m}_{ij} + W_j n_j + Q_j k_j \right) (1 + \pi_j)(1 + \tau_j)$$

$$j = 1,2,\ldots,n$$

where the expression in parentheses indicates the producer's cost of producing one unit of output j (measured at base year prices!), and π_j and τ_j are exogenously given profit and turnover tax rates, respectively.

The above price formation rule is inconsistent with some concepts of the neoclassical general equilibrium theory. It is well known from this theory, that if technology exhibits constant returns to scale then in equilibrium no producer can make positive profits, i.e., the production activities that are used in the equilibrium solution must break even at equilibrium prices. The use of constant return technology in an applied model is, in our view, only a convenient assumption, and one should not take it too seriously. When one speaks about nonprofit in the real world, one hides profit in "normal" (or "abnormal") rates of return on different factors of production. This treatment of profit cannot be regarded only as a matter of taste or ideology. The real problem with it is that these returns are treated as micro-level factor costs. Wages in a market economy can be treated as cost for the producers, but it seems to be highly inappropriate to interpret the rest of the value added as the users cost of capital. It is also hard to believe that these returns are equal to the marginal products of labor and capital, unless one is a faithfull advocate of the neoclassical income distribution theory. Mark-up pricing behavior, wide variations in the rates of return on capital, and other alternative theoretical considerations clearly do not support such hypotheses.

One way to bridge the gap between theory and reality in empirical general equilibrium models (see, for instance, Johansen, 1959 or Bergman-Pór, 1980 and also the related discussion in Section 1 of this paper) is to incorporate (sector specific) relative rate of return requirements into the model. For example, if β_j is the relative level of net return on capital in sector j and R is the endogenous general net rate of return on capital, then the net rate of return in sector j, in effect, will be $R_j = \beta_j R$.

Looking at the problem in a different way it is obvious that the theoretical (computational) simplicity of using linear homogenous production functions is due to their nice behavioral property; namely, that the conditions for profit maximization can be brought to terms with a demand determined output specification. Given the relative factor prices one can determine the cost-minimizing factor proportions independently from the level of the output. If one assumes constant returns to scale, then the theoretical requirement for a meaningful profit maximization is that the unit (net) price is equal to the unit cost. In such a case, however, the output level is indeterminate and can be determined simply by taking into account the demand requirement.

In general, one might assume that demand and other constraints (such as institutional constraints that require the full utilization of capacities) have in the *short run* a more decisive impact on the output level than mere profit maximizing rules. Thus, for example, one might assume that producers do *minimize cost*, but the level of their output is determined by supply and demand relations, i.e., given the demand for the products of a sector, the output level of that sector would simply be adjusted to meet that demand. If one starts with these assumptions then the existence of (excess) profits, even in the case of linear homogeneous production functions, could also be taken into consideration. In terms of the general equilibrium theory such a solution would imply that there is a pressure on the side of the producers to increase the output level in each sector, which may be constrained either by effective demand in a market economy, or by factor availability in a centrally planned one. This, in fact, is not at all in contrast with actual experience. In the planned economies, for instance, there has been constant excess demand for both labor and capital, but, of course, for more complex reasons than the one implied above.

Thus the nonprofit conditions should and could be relaxed. This would cause the model to lose some of its general equilibrium character, but at the same time it would gain some empirical relevance. We therefore assume cost minimizing behavior and mark-up pricing behavior, combined with demand determined supply assumption, instead of simple profit maximization. Further, the model in its present form treats the profit rate as an exogenous parameter. If, however, one changed the price formation rules (for instance, let some prices be determined directly by world market prices), then the corresponding profit rates would become endogenous variables.

Returning to the price equations and their remaining variables, we have the price index for noncompetitive imports

$$\bar{P}_i^{DI} = \alpha_i \; \bar{\Theta}_{ir} \; V_r \; \bar{P}_{ir}^{WI} + (1 - \alpha_i) \; \bar{\Theta}_{id} \; V_d \; \bar{P}_{id}^{WI} \qquad (40)$$

$$i = 1,2,\ldots,n$$

The other cost elements that enter into price determination are the user's cost of labor (W_j) and capital (Q_j).

The user cost of labor is made up of two elements: the sectoral wage rate and a general tax rate on wages

$$W_j = (1 + W) \; w_j \qquad j = 1,2,\ldots,n \qquad (37)$$

This formation of the user cost of labor is in accordance with the actual Hungarian practice. The only difference here is in the determination of the wage tax rate, which in practice is given exogenously and is intended to correct wages such that the resulting cost expresses the actual social cost of labor. Here W will be an endogenous variable and its role will be discussed later.

The user cost of capital will be determined in the following way. First, the existing capital stock is reevaluated using the price index of the capital goods, which is given by

$$P_{n+1} = \sum_{i=1}^{n} P_i^D a_{i,n+1} + \sum_{i=1}^{n} \bar{P}_i^{DI} \bar{m}_{i,n+1} \tag{39}$$

Then, the user cost of the reevaluated capital will be made up of two parts: depreciation and a general tax on capital use. This solution is again not alien to the Hungarian practice where such tax rates have been applied and can be interpreted as minimum rate of return requirements. The taxes on labor and capital are assumed to fulfill the role of regulating the enterprises' demand for these factors of production in accordance with their availability and social costs. Thus, in our case, the user cost of capital (evaluated at base price level) will be determined by

$$Q_j = (\delta_j + R)\ P_{n+1} \qquad j = 1,2,\ldots,n \tag{38}$$

Finally, W and R will be set at such levels by the model that the cost minimizing sectors' demand for labor and capital matches their available amounts. They will therefore serve the same goals as their empirical counterparts. The determination of the labor and capital input coefficients (n_j and k_j) and the factors' cost will be discussed in the following section.

2.5. Production Technology and Decision-Making Rules

Production technology is described by the Johansen specification discussed earlier. Intermediate inputs and noncompetitive import inputs are assumed to change in proportion to the level of output

$$X_{ij} = a_{ij} X_j \qquad i = 1,2,\ldots,n \quad j = 1,2,\ldots,n+1 \tag{9}$$

$$\bar{M}_{ij} = \bar{m}_{ij} X_j \qquad i = 1,2,\ldots,n \quad j = 1,2,\ldots,n+1 \tag{10}$$

The use of the two primary factors of production are determined by the assumption that producers minimize their cost at any given output level. The feasible choice of factor combination is described by linear homogeneous production functions

$$X_j = F_j(N_j,K_j) \qquad j = 1,2,\ldots,n \tag{8}$$

Minimization of the total cost of the primary factors, $W_j N_j + Q_j K_j$, subject to the production function condition, yields the following necessary first order conditions:

$$W_j = S_j \frac{\partial F_j}{\partial N_j} \qquad j = 1,2,\ldots,n \tag{11}$$

$$Q_j = S_j \frac{\partial F_j}{\partial K_j} \qquad j = 1,2,\ldots,n \tag{12}$$

where S_j refers to the Lagrange multiplier. Further investigation reveals that S_j is the minimal user cost of the primary factors per unit of output in sector j:

$$S_j = W_j \frac{N_j}{X_j} + Q_j \frac{K_j}{X_j} = W_j n_j + Q_j k_j \qquad j = 1,2,\ldots,n$$

This can easily be checked by multiplying the necessary first order conditions by N_j and K_j, respectively, and adding them together. On the right-hand side, as a result of the Euler theorem, one obtains $S_j X_j$. Division of the resulting equation by X_j yields the desired form. Therefore, the determination of prices can be rewritten as follows:

$$P_j = \left(\sum_{i=1}^{n} P_i^D a_{ij} + \sum_{i=1}^{n} \bar{P}_i^{DI} \bar{m}_{ij} + S_j \right) (1 + \pi_j)(1 + \tau_j) \quad (41)$$

and consequently, the labor and capital coefficient (n_j and k_j) can be omitted from the model. And this completes the description of our model.

2.6. Formal Statement of the Model

Endogenous Variables

X_j	gross output in sector j = 1,2,...,n (real)*
M_{ir}, M_{id}	competitive rouble and dollar import of commodity** i = 1,2,...,n (real)
X_{ij}	use of domestic-import composite commodity i = 1,2,...,n in sector j = 1,2,...,n,n+1 (real)
C_i, C_{iu}, C_{ir}	total, urban, and rural household consumption of composite commodity i = 1,2,...,n (real)
Z_i, Z_{ir}, Z_{id}	total, rouble, and dollar export of commodity i (real)
X_{n+1}	total gross investments (real)
I	total net investments (real)
$\bar{M}_i, \bar{M}_{ir}, \bar{M}_{id}$	total, rouble, and dollar noncompetitive import of commodity i = 1,2,...,n (real)

*"Real" in brackets indicates that the given (real) variable is measured at base year (constant) prices. Also note that the meaning of some symbols in Section 2 differs from that in Section 1.

**See the footnote on page 6.

$\overline{M}_{ij}$ use of noncompetitive import commodity $i = 1,2,...,n$ in sector $j = 1,2,...,n,n+1$ (real)

$\overline{C}_i, \overline{C}_{iu}, \overline{C}_{ir}$ total, urban, and rural household consumption of noncompetitive import commodity $i = 1,2,...,n$ (real)

K_j capital used in sector $j = 1,2,...,n$ (real)

N_j labor employed in sector $j = 1,2,...,n$

S_j (optimal) user cost of labor and capital per unit of output in sector $j = 1,2,...,n$

W_j user cost of labor in sector $j = 1,2,...,n$

W net rate of return requirement (tax) on labor

Q_j user cost of capital in sector $j = 1,2,...,n$

R net rate of return requirement (tax) on capital

α_i share of rouble import in total noncompetitive import of commodity i

m_{ir}, m_{id} proportions of competitive rouble and dollar imports of commodity $i = 1,2,...,n$

P_j seller price of commodity $j = 1,2,...,n$ produced domestically (index)*

V_r, V_d exchange rate of roubles and dollars (index)

$\overline{P}_i^{DI}$ average domestic price of noncompetitive import of commodity $i = 1,2,...,n$ (index)

P_i^D average price of domestic-import composite commodity $i = 1,2,...,n$ (index)

E, E_u, E_r total, urban, and rural household expenditure level

---

*"Index" in brackets indicates that the corresponding variable in the base case has a value of 1.00.

EE_u, EE_r urban and rural excess expenditure level

N_u, N_r urban and rural employment

C total household consumption (real)

Extraneous Parameters (Variables or Data)

G, G_i total government (public) expenditure, governmental consumption of domestic-import composite commodity i = 1,2,...,n (real)

K_g capital used in public services (real)

δ_j depreciation rate in sector j = 1,2,...,n,g

$\overline{G}_i$ public consumption of noncompetitive import commodity i = 1,2,...,n (real)

K total capital stock (real)

N total labor

N_g labor usage in public services

$\left.\begin{matrix} Z^o_{id}, Z^o_{ir} \\ \lambda_i, \varepsilon_{ir}, \varepsilon_{id} \end{matrix}\right\}$ parameters in the export functions

$\left.\begin{matrix} P^{WI}_{id}, P^{WI}_{ir} \\ \overline{P}^{WI}_{id}, \overline{P}^{WI}_{ir} \end{matrix}\right\}$ world market import prices of commodity i (rouble-dollar, competitive-noncompetitive)(index)

D_d, D_r target surplus or deficit on dollar and rouble foreign trade balance

a_{ij} input coefficient of domestic-import composite commodity i = 1,2,...,n in sector j = 1,2,...,n,n+1

α^o_i, δ_i parameters in the determination of the area composition of the noncompetitive import of commodity i = 1,2,...,n

m^o_{ir}, m^o_{id} μ_{ir}, μ_{id}	parameters in the import functions, $i = 1,2,...,n$
$b_{iu}, \bar{b}_{iu}$ $c_{iu}, \bar{c}_{iu}$ $b_{ir}, \bar{b}_{ir}$ $c_{ir}, \bar{c}_{ir}$	parameters in the demand functions, $i = 1,2,...,n$
E^o, E^o_u, E^o_r	base total, urban and rural consumption expenditure
N^o_u, N^o_u	base employment distribution
s_{uj}	relative share of urban employment in sector $j = 1,2,...,n$
σ	real consumption-net investment ratio
w_j	wage coefficient in sector $j = 1,2,...,n$
Θ_{ir}, Θ_{id} $\bar{\Theta}_{ir}, \bar{\Theta}_{id}$	net import subsidy-tax factor on commodity $i = 1,2,...,n$ (competitive-noncompetitive, rouble-dollar) (index)
ϕ_{ir}, ϕ_{id}	net export subsidy-tax factor on commodity $i = 1,2,...,n$ (rouble-dollar) (index)
π_j	profit rate in sector $j = 1,2,...,n$
τ_j	net turnover tax-subsidy on commodity $j = 1,2,...,n$

Balancing Equations

Intermediate Commodities

$$X_i + M_{ir} + M_{id} = \sum_{j=1}^{n+1} X_{ij} + C_i + G_i + Z_{ir} + Z_{id} \qquad (1)$$

$$i = 1,2,...,n$$

$$X_{n+1} = \sum_{j=1}^{n} \delta_j K_j + \delta_g K_g + I \qquad (2)$$

Noncompetitive Imports

$$\bar{M}_i = \sum_{j=1}^{n+1} \bar{M}_{ij} + \bar{C}_i + \bar{G}_i \qquad i = 1,2,\ldots,n \qquad (3)$$

Primary Factors

$$K = \sum_{j=1}^{n} K_j + K_g \qquad (4)$$

$$N = \sum_{j=1}^{n} N_j + N_g \qquad (5)$$

Trade Balances

$$\sum_{i=1}^{n} \left(\frac{Z_{id}^{o}}{Z_{id}}\right)^{\lambda_i} P_{id}^{WE} Z_{id} - \sum_{i=1}^{n} P_{id}^{WI} M_{id} - \sum_{i=1}^{n} \bar{P}_{id}^{WI} \bar{M}_{id} = D_d \qquad (6)$$

$$\sum_{i=1}^{n} P_{ir}^{WE} Z_{ir} - \sum_{i=1}^{n} P_{ir}^{WI} M_{ir} - \sum_{i=1}^{n} \bar{P}_{ir}^{WI} \bar{M}_{ir} = D_r \qquad (7)$$

Technological Choice

$$X_j = F_j(N_j, K_j) \qquad j = 1,2,\ldots,n \qquad (8)$$

$$X_{ij} = a_{ij} X_j \qquad \begin{array}{l} i = 1,2,\ldots,n \\ j = 1,2,\ldots,n,n+1 \end{array} \qquad (9)$$

$$\bar{M}_{ij} = \bar{m}_{ij} X_j \qquad \begin{array}{l} i = 1,2,\ldots,n \\ j = 1,2,\ldots,n,n+1 \end{array} \qquad (10)$$

$$S_j \frac{\partial F_j}{\partial N_j} = W_j \qquad j = 1,2,\ldots,n \qquad (11)$$

$$S_j \frac{\partial F_j}{\partial K_j} = Q_j \qquad j = 1,2,\ldots,n \qquad (12)$$

Import and Export Functions

Noncompetitive Imports

$$\alpha_i = \alpha_i^o \left(\frac{\overline{\Theta}_{id} \; V_d \; \overline{P}_{id}^{WI}}{\overline{\Theta}_{ir} \; V_r \; \overline{P}_{ir}^{WI}} \right)^{\delta_i} \qquad i = 1,2,\ldots,n \qquad (13)$$

$$\overline{M}_{ir} = \alpha_i \; \overline{M}_i \qquad i = 1,2,\ldots,n \qquad (14)$$

$$\overline{M}_{id} = (1 - \alpha_i) \; \overline{M}_i \qquad i = 1,2,\ldots,n \qquad (15)$$

Competitive Imports

$$m_{ir} = m_{ir}^o \left(\frac{P_i}{\Theta_{ir} \; V_r \; P_{ir}^{WI}} \right)^{\mu_{ir}} \qquad i = 1,2,\ldots,n \qquad (16)$$

$$m_{id} = m_{id}^o \left(\frac{P_i}{\Theta_{id} \; V_d \; P_{id}^{WI}} \right)^{\mu_{id}} \qquad i = 1,2,\ldots,n \qquad (17)$$

$$M_{ir} = m_{ir} \; (X_i - Z_i) \qquad i = 1,2,\ldots,n \qquad (18)$$

$$M_{id} = m_{id} \; (X_i - Z_i) \qquad i = 1,2,\ldots,n \qquad (19)$$

Exports

$$Z_i = Z_{ir} + Z_{id} \qquad i = 1,2,\ldots,n \qquad (20)$$

$$Z_{ir} = Z^{o}_{ir}\left(\frac{P_i}{\phi_{ir}\ V_r\ P^{WE}_{ir}}\right)^{\varepsilon_{ir}} \qquad i = 1,2,\ldots,n \qquad (21)$$

$$Z_{id} = Z^{o}_{id}\left(\frac{P_i}{\phi_{id}\ V_d\ P^{WE}_{id}}\right)^{\varepsilon_{id}} \qquad i = 1,2,\ldots,n \qquad (22)$$

Final Demand Equations

$$C_{iu} = b_{iu} + \frac{c_{iu}}{P^D_i} \cdot EE_u \qquad i = 1,2,\ldots,n \qquad (23)$$

$$\bar{C}_{iu} = \bar{b}_{iu} + \frac{\bar{c}_{iu}}{\bar{P}^{DI}_i} \cdot EE_u \qquad i = 1,2,\ldots,n \qquad (24)$$

$$EE_u = E_u - \sum_{j=1}^{n}\left(P^D_j\ b_{ju} + \bar{P}^{DI}_j\ \bar{b}_{ju}\right) \qquad (25)$$

$$E_u = \frac{E^o_u}{E^o}\ E \qquad (26)$$

$$C_{ir} = b_{ir} + \frac{c_{ir}}{P^D_i} \cdot EE_r \qquad i = 1,2,\ldots,n \qquad (27)$$

$$\bar{C}_{ir} = \bar{b}_{ir} + \frac{\bar{c}_{ir}}{\bar{P}^{DI}_i} \cdot EE_r \qquad i = 1,2,\ldots,n \qquad (28)$$

$$EE_r = E_r - \sum_{j=1}^{n}\left(P^D_j\ b_{jr} + \bar{P}^{DI}_j\ \bar{b}_{jr}\right) \qquad (29)$$

$$E_r = \frac{E_r^o}{E^o} \; E \tag{30}$$

$$C_i = \frac{N_u}{N_u^o} C_{iu} + \frac{N_r}{N_r^o} C_{ir} \qquad i = 1,2,\ldots,n \tag{31}$$

$$\bar{C}_i = \frac{N_u}{N_u^o} \bar{C}_{iu} + \frac{N_r}{N_r^o} \bar{C}_{ir} \qquad i = 1,2,\ldots,n \tag{32}$$

$$N_u = \sum_{j=1}^{n} s_{uj} \; N_j + s_{ug} \; N_g \tag{33}$$

$$N_r = N - N_u = \sum_{j=1}^{n} (1 - s_{uj}) \; N_j + (1 - s_{ug}) \; N_g \tag{34}$$

$$C = \sum_{i=1}^{n} C_{ir} + \sum_{i=1}^{n} \bar{C}_{ir} + \sum_{i=1}^{n} C_{iu} + \sum_{i=1}^{n} \bar{C}_{iu} \tag{35}$$

$$G + C - \sigma \cdot I = 0 \tag{36}$$

Prices and Costs

$$W_j = (1 + W) \; w_j \qquad j = 1,2,\ldots,n \tag{37}$$

$$Q_j = (\delta_j + R) \; P_{n+1} \qquad j = 1,2,\ldots,n \tag{38}$$

$$P_{n+1} = \sum_{i=1}^{n} P_i^D \; a_{i,n+1} + \sum_{i=1}^{n} \bar{P}_i^{DI} \; \bar{m}_{i,n+1} \tag{39}$$

$$\bar{P}_i^{DI} = \alpha_i \ \bar{\Theta}_{ir} \ V_r \ \bar{P}_{ir}^{WI} + (1 - \alpha_i) \ \bar{\Theta}_{id} \ V_d \ \bar{P}_{id}^{WI} \tag{40}$$

$$i = 1,2,\ldots,n$$

$$P_j = \left(\sum_{i=1}^{n} P_i^D \ a_{ij} + \sum_{i=1}^{n} \bar{P}_i^{DI} \ \bar{m}_{ij} + S_j \right) (1 + \pi_j)(1 + \tau_j)$$

$$j = 1,2,\ldots,n \tag{41}$$

$$P_i^D = \frac{1}{1 + m_{id} + m_{ir}} \ P_i + \frac{m_{id}}{1 + m_{id} + m_{ir}} \ \Theta_{id} \ V_d \ P_{id}^{WI}$$

$$+ \frac{m_{ir}}{1 + m_{id} + m_{ir}} \ \Theta_{ir} \ V_r \ P_{ir}^{WI} \tag{42}$$

$$i = 1,2,\ldots,n$$

REFERENCES

Adelman, I., and S. Robinson (1978) *Income Distribution Policy in Developing Countries: A Case Study of Korea.* Stanford, California: Stanford University Press.

Bergman, L. (1978) *Energy Policy in a Small Open Economy: The Case of Sweden.* RR-78-16. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Bergman, L. (1980) *A Model of Multisectoral Economic Growth in a Small Open Economy.* Mimeograph. Sweden: Stockholm School of Economics.

Bergman, L., and A. Pór (1980) *A Quantitative General Equilibrium Model of the Swedish Economy.* WP-80-4. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Berndt, E.R. (1976) Reconciling Alternative Estimates of the Elasticity of Substitution. *Review of Economics and Statistics*, February:59-68.

Caddy, V. (1976) *Empirical Estimation of the Elasticity of Substitution: A Review.* WP-OP-09. Melbourne, Australia: IMPACT Project, Industries Assistance Commission.

De Melo, J. (1978) *A Simulation of Development Strategies in an Economy-Wide Policy Model.* Mimeograph. Washington, D.C.: World Bank (IBRD)

Dervis, K., and S. Robinson (1978) *The Foreign Exchange Gap, Growth and Industrial Strategy in Turkey: 1973-1983.* Working Paper No. 306. Washington, D.C.: World Bank (IBRD).

Dixon, P.B., B.R. Parmenter, G.J. Ryland, and J. Sutton (1977) *ORANI, A General Equilibrium Model of the Australian Economy: Current Specification and Illustrations of Use for Policy Analysis.* First Progress Report of the IMPACT Project, Vol. 2. Canberra: Australian Government Publishing Service.

Ginsburgh, V., and J. Waelbroeck (1979) *On the Design of Linear Programming Models for Open Economies.* Mimeograph. Belgium: University of Brussels.

Johansen, L. (1959) *A Multisectoral Study of Economic Growth.* Amsterdam: North-Holland Publishing Company.

Johansen, L. (1972) *Production Functions; An Integration of Micro and Macro, Short Run and Long Run Aspects.* Amsterdam: North-Holland Publishing Company.

Karlström, U. (1980) *Urbanization and Industrialization: Modeling Swedish Demoeconomic Development from 1870 to 1914.* WP-80-54. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Kelley, A.C., and J.G. Williamson (1980) *Modeling Urbanization and Economic Growth.* RR-80-22. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Kornai, J. (1971) *Anti-Equilibrium.* Amsterdam: North-Holland Publishing Company.

Kornai, J. (1974) *Mathematical Planning of Structural Decisions.* 2nd Edition. Amsterdam: North-Holland Publishing Company.

McCarthy, F.D., and L. Taylor (1980) Macro Food Policy Planning: A General Equilibrium Model for Pakistan. *Review of Economics and Statistics*, February:107-121.

Rimmler, J., Zs. Dániel, and J. Kornai (1972) Macrofunctions Computed on the Basis of Plan Models. *Acta Oeconomica* 8:375-406.

Rogers, A. (1977) *Migration, Urbanization, Resources, and Development.* RR-77-14. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Sanderson, W.C. (1980) *Economic-Demographic Simulation Models: A Review of Their Usefulness for Policy Analysis.* RR-80-14. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Taylor, L. (1975) Theoretical Foundations and Technical Implications. In *Economy-Wide Models and Development Planning* edited by C.R. Blitzer, P.C. Clark, and L. Taylor. Oxford: Oxford University Press.

Wierzbicki, A. (1979) *The Use of Reference Objectives in Multiobjective Optimization: Theoretical Implications and Practical Experience.* WP-79-66. Laxenburg, Austria: International Institute for Applied Systems Analysis.

RECENT PAPERS IN THE POPULATION, RESOURCES,
AND GROWTH SERIES

1. Donaldo Colosio, *Urbanization and Economic Development in Mexico.* WP-79-19.

2. Andrei Rogers and Dimiter Philipov, *Multiregional Methods for Subnational Population Projections.* WP-79-40.

3. Clark Reynolds, *A Shift-Share Analysis of Regional and Sectoral Productivity Growth in Contemporary Mexico.* WP-79-41.

4. Jacques Ledent and Andrei Rogers, *Migration and Urbanization in the Asian Pacific.* WP-79-51.

5. Bruce Johnston and William C. Clark, *Food, Health, and Population: Policy Analysis and Development Priorities in Low Income Countries.* WP-79-52.

6. Bruce Johnston, *The Choice of Technology in Strategies for Agricultural Development: Mechanical Innovations in East Africa.* WP-79-92.

7. Hiromitsu Kaneda, *Issues in Policy Analysis of Agricultural Development and Internal Migration.* WP-79-109.

8. Jacques Ledent, *Rural-Urban Migration, Urbanization, and Economic Development.* WP-80-19.

9. Jacques Ledent, *Comparative Dynamics of Three Demographic Models of Urbanization.* RR-80-01.

10. Urban Karlström, *Urbanization and Industrialization: Modeling Swedish Demoeconomic Development from 1870 to 1914.* WP-80-54.

11. Ahmed Seifelnasr, *Urbanization Level and Urban Concentration: Comparative Paths and a Performance Index.* WP-80-70.

12. Warren Sanderson, *Economic-Demographic Simulation Models: A Review of Their Usefulness for Policy Analysis.* RR-80-14.

13. Allen Kelley and Jeffrey Williamson, *Modeling Urbanization and Economic Growth.* RR-80-22.

14. Nathan Keyfitz, *Do Cities Growth by Natural Increase or by Migration?* RR-80-24. Also published in *Geographical Analysis* 12(2):142-156.

15. Zbigniew Pawlowski, *A Demoeconometric Model of Poland and Its Application to Counterfactual Simulations.* RR-80-35.

RELATED PAPERS OF THE SYSTEM AND DECISION SCIENCES AREA

1. D. Nyhus, *A Three Country Comparison of the Impact of Slower Growth on Three Industries Using Input-Output Analysis.* WP-79-021.

2. S. Littlechild, R. Webber, and P. Young, *Strategic Models of World Food Production and Trade.* WP-79-058.

3. A. Wierzbicki, *The Use of Reference Objectives in Multiobjective Optimization--Theoretical Implications and Practical Experience.* WP-79-066.

4. M. Stoto, *The Accuracy of Population Projections.* WP-79-075.

5. L. Klein, *A Global Economics Model.* WP-79-076.

6. R. Svento, *An Application of Complete Demand System Analysis in an Input-output Framework.* WP-79-114.

7. A. Wierzbicki, *A Methodological Guide to Multiobjective Optimization.* WP-79-122.

8. L. Bergman, C. Clemenz, and A. Hoelzl, *An Input Output Approach to the Analysis of Intercountry Differences in Per Capita Energy Consumption.* WP-80-03.

9. L. Bergman, and A. Pór, *A Quantitative General Equilibrium Model of the Swedish Economy.* WP-80-04.

10. R.M. Coen and B.G. Hickman, *A Disaggregated Annual Model of Labor Supply and Unemployment, 1951-2000.* WP-80-15.

11. D.E. Nyhus, *The Inforum-IIASA Trade Model: An Interim Report.* WP-80-24.

12. M. Kallio, A. Lewandowski, and W. Orchard-Hays, *An Implementation of the Reference Point Approach for Multiobjective Optimization.* WP-80-35.

13. A. Wierzbicki, *A Mathematical Basis for Satisficing Decision Making.* WP-80-90.

14. L. Bergman, *The Economic Impact of Nuclear Power Discontinuation in Sweden.* WP-80-97.

15. D. Snower, *Global Modeling and Target Optimality.* WP-80-99.

16. H.J. Barnett and G.M. van Muiswinkel, *Minerals Scarcity and Economic Change; Design and Discussion.* WP-80-100.

17. L. Bergman and L. Ohlsson, *Changes in Comparative Advantages and Paths of Structural Adjustment and Growth in Sweden.* WP-80-105.

18. W.B. Arthur, *The Analysis of Causal Linkages in Demographic Theory.* WP-80-110.

19. D.E. Nyhus and C. Almon, *Linked Input-output Models for France, Germany, and Belgium.* WP-80-122.

20. R. Coen and B. Hickman, *The Natural Growth Path of Potential Output.* WP-80-132.

21. J.A. Parker, *A Vintage Capacity Model of Production.* WP-80-143.