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A MULTIOBJECTIVE APPROACH TO ALLOCATING WATER RESOURCE FOR MUNICIPAL, AGRICULTURAL AND RECREATIONAL USES

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PREFACE

Water resource systems have been an important part of resources and environment related research at IIASA since its inception. As demands for water increase relative to supply, the intensity and efficiency of water resources management must be developed further. This in turn requires an increase in the degree of detail and sophistication of the analysis, including economic, social and environmental evaluation of water resources development alternatives aided by application of mathematical modelling techniques, to generate inputs for planning, design, and operational decisions.

During the year of 1978 it was decided that parallel to the continuation of demand studies, an attempt would be made to integrate the results of our studies on water demands with water supply considerations. This new task was named "Regional Water Management" (Task 1, Resources and Environment Area).

This paper is oriented towards the application of systems analysis techniques to water management problems in Western Skåne, Sweden. These problems concern the allocation of scarce water and related land resources among several mutually conflicting uses, e.g., municipal, industrial, agricultural and recreational water use.

The paper is part of a collaborative study on water resources problems in Western Skåne, Sweden, pursued by IIASA in collaboration with the Swedish National Environment Protection Board and the University of Lund. The paper describes a methodological proposal concerning allocation of water resources to different and mutually conflicting uses. This proposal is illustrated by a numerical example concerning water resources management in Western Skåne.

Janusz Kindler
Task Leader

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ABSTRACT

A water resource allocation problem in Western Skåne, Sweden, is formulated as a two-level multiobjective program, which reflects a decentralized institutional framework of the region. The upper level model deals with the region as a whole and seeks for technically feasible alternatives and their associated costs. The lower level models are concerned with activities of different water users which often conflict each other.

Both the upper and the lower level problems are solved in a stepwise manner using reference objective methods. Advantages of this class of multiobjective methods as a tool for aiding decision-making and conflict resolution are noted. Uses of the model and further extensions are also mentioned.

CONTENTS

I. INTRODUCTION 1

II. INSTITUTIONAL FRAMEWORKS AND MULTI-OBJECTIVE METHODS 3

III. SWEDISH CASE STUDY 5

IV. GENERAL FORMULATION OF WATER RESOURCE ALLOCATION MODEL 8

Upper Level Problem 9
Lower Level Problem 13

V. REGIONAL WATER ALLOCATION MODEL 17

Objectives
Constraints

VI. KÄVLINGE RIVER BASIN MODEL 20

A Lower Level Problem
Objectives
Constraints

VII. NUMERICAL EXAMPLE 26

Upper Level Problem - Regional Water Allocation Model

Data
Solution and Results

Lower Level Problem - Kävlinge River Basin Model 31

Data
Solutions and Results 35

Summary and Conclusions 40

VIII. USES OF THE MODEL AND POSSIBLE EXTENSIONS 41

REFERENCES 45

A MULTIOBJECTIVE APPROACH TO ALLOCATING WATER RESOURCE FOR MUNICIPAL, AGRICULTURAL AND RECREATIONAL USES

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I. INTRODUCTION

It has been recognized in a past decade that water resource planning problems are almost inherently of multi-objective nature. This is certainly true when the planning takes place in decentralized institutional systems in which each decision-maker acts more or less independently of other decision-makers to attain his objectives, as is the case in Sweden that is studied later. Even if there exists a central planning authority, water resource planning may take a form of multi-objective problem, if there are a number of functions which have to be fulfilled by water resource systems and if all the functions cannot be satisfied to the fullest extent at the same time.

Naturally many methodologies have been proposed for solving multi-objective problems and applied to water resource planning problems (for a survey, see Cohon and Marks, 1975, or Haith and Loucks, 1976). The two most popular of these methods are the multi-attribute utility method applied by Keeney and Wood (1977) and others, and the Surrogate Worth Trade-off method which has been applied extensively by Haimes and others (1975, 1977). There are also variants of these methods which are based on evaluation of decision-makers' utility and preferences by means of trade-off ratios or functions among objectives (see, for example, Seo and Sakawa 1979, Neuman and Krzysztofowics, 1977). All of these methods, of course, are based in one way or another on interactions with decision-makers. Differences are, when and how extensively the interactions are made and also the kinds of questions asked to reveal decision-makers' preferences.

A relatively new approach which has been advocated by Wierzbicki (1979a, 1979b) is the reference objective methods. The basic idea of this approach is to let decision-makers to specify reference or target levels (called "aspiration levels" or "utopia points" as the case may be) for all the objectives and to find such a Pareto efficient solution that is as close as possible to this reference point, where the closeness is measured in some appropriate way. This is in contrast with the more conventional methods described above which in essence depend on evaluation of trade-off ratios among objectives to reveal decision-makers' preferences. An advantage of this approach is that individual decision-makers are apt to think in terms of goals or desirable levels of objectives rather than in terms of utility and preferences (Wierzbicki, 1979b).

In case where multiple decision-makers (aggregated or not) are involved, the reference objective approach has an additional and significant advantage that aggregation of decision-makers' "preferences" can be done in easier and more natural way. For instance, the minimum of reference values of multiple decision-makers may be taken as the aggregated reference point for some objectives which are to be maximized. Or for some other objectives, the sum of all the reference values specified by individual decision-makers may give an appropriate reference value for the group. On the other hand, in the case of multi-attribute utility method or other methods based on trade-off functions (e.g. Surrogate Worth Trade-off method), group assessment of utility or preferences may be more difficult. Application of the reference objective approach to water resource planning, however, is still very limited (Kindler, Zielinski and deMare, 1980).

In this paper, water resource allocation problem in Western Skåne, Sweden is formulated as a two-level multiobjective program. The formulation reflects the institutional framework of the region, which is an important consideration as discussed in Section II. Section III describes the case study area with respect to its institutional, geographical, physical and other background features as they relate to water resource planning, and the problem that it faces. In Section IV, the general formulation of water resource allocation model is presented and solution procedures based on reference objective methods are described. The following two sections present specific and detailed models for the water resource allocation problem in Western Skåne region. In Section V, the upper level problem which deals with the region as a whole is presented, and Section VI offers description of one of the lower level problems - Kävlinge river basin subproblem. In Section VII, these problems are solved in a stepwise manner using data obtained for the region and the results are analyzed. The example illustrates how such a multiobjective model can aid in solving decision-making problems concerning water resource planning with multiple decision-makers. Finally Section VIII contains suggestions for potential uses of the model and also for its possible extensions.

II. INSTITUTIONAL FRAMEWORKS AND MULTI-OBJECTIVE METHODS

Obviously how to formulate a water resource planning problem as a multi-objective program and which solution methods should be used depend very much on institutional framework of a particular planning site. If there exists a central planning authority, the first step to formulate the water resource planning as a multi-objective problem is to aggregate objectives of each water-user into different accounts which will serve as separate objectives for this planning authority. For instance, as done by Haimes et al (1977), soil loss from different agricultural sites in the planning region may be aggregated to define total soil loss of the region which may constitute one objective to be minimized; or sum (weighted as appropriate) of net benefits of all agricultural water-users derived from irrigation may be used as another objective.

In more decentralized institutional systems, this kind of aggregation often is not justified, since each decision-maker may be concerned about objectives of other decision-makers only to the extent that they affect attainment of his own objectives. In this case, water resource planning problem takes in general the form of multi-objective, multiple decision-maker problem, which often is intractable. One way to alleviate this problem is to decompose it into more tractable subproblems. A question is how - geographically (e.g. by river basin), functionally (e.g. municipal use sector, agricultural use sector) or jurisdictionally (e.g. by municipalities)? The answers to the questions again depend on the institutional framework of the region of concern.

One important consideration in this case is to decompose the entire planning problem in such a way that will facilitate assessment of preferences and values of decision-makers. In this respect it is meaningful to distinguish two different characteristics of multi-objective problems. First, many problems are characterized by multiple physical attributes (e.g. quantity of water distributed in different parts of the system) which affect objectives of decision-makers and which can be varies only according to a range of technical feasibility. Thus search for technically feasible alternatives constitutes a multi-objective problem. Secondly, individual decision-makers' objectives depend on one or more of these attributes and may interact (and often conflict) each other through these attributes as media. That is, these physical attributes have different values for decision-makers. This kind of conflicts on value judgements constitute another aspect of multi-objective problems with multiple decision-makers.

In reality, the distinction described above is not so clear. Two variants can be considered, each of which has definite implications for institutional framework. Suppose first, a "single" decision-maker (or agency) can be identified for each attribute in the first type of multi-objective problem. For instance, water yield from a reservoir to serve public water supply may fall under the control of one decision-maker (or agency); release from the reservoir for downstream low-flow augmentation may be the responsibility of another decision-maker, and so on. In this case, the multiobjective problem can be solved based on assessment of preferences of these aggregated decision-makers.

If a "single" decision-maker cannot be identified for each attribute in the first type multi-objective problem, preference assessment procedure will be more involved. This situation arises if, for instance, several independent decision-makers are interested in release from a reservoir in relation to their own objectives (e.g. of maximizing crop yield by irrigation or securing water supply for municipal uses). A decomposed multi-objective program then has to provide some means to aid conflict resolution. First, a device is required to interpret values of the attributes or objectives in the first type multi-objective problem in terms of each decision-maker's objectives; secondly, a method or a procedure is necessary to aggregate decision-makers' preferences for each of these attributes so that the entire water resource system can be evaluated.

It is not intended in this paper to cover all the possibilities described above, nor to develop a general framework for analyzing decision-making problems with multiple goals (see, Wierzbicki, 1980). Rather a particular case in Western Skåne, Sweden is studied to show how the water resource allocation problem can be formulated as a multi-objective program, reflecting the institutional framework of this particular region and to illustrate how such a model can aid decision-making problems concerning water resource planning with multiple decision-makers.

The paper is devoted to a particular application rather than development of general theories or a planning framework. Therefore, data as specific and real for the region as possible are used and particular solution methods are adopted. However, the way the problem is formulated reflecting institutional arrangements of the study area, also suggests a general procedure which may be applicable to other cases.

III. SWEDISH CASE STUDY

The study area is Southwestern part of Sweden called Western Skåne, which coincides with Malmöhus County consisting of 20 municipalities (Figure 1). The area, which covers approximately 5,000 km^2, includes two major river basins - Kävlinge and Rönne - and several smaller ones.

At present most of the municipal water supply is drawn from three sources: local groundwater and two pipeline systems which distribute water from two lakes Vombsjön and Ringsjön located respectively in Kävlinge and Rönne river basins. Five major municipalities taking water from these lakes are shown in Figure 1. In addition, a major project to obtain water from a lake located north of the region (Lake Bolmen) via an 80 km tunnel was proposed in late 1960's to meet the future demand which was projected to more than double by the year 2000. However, after the initiation of the project, it became apparent that the expected increase in demand for municipal water would not materialize.

On the other hand, water use for supplementary irrigation, which was non-existent until a few years ago, has increased rapidly. The increase is expected to continue,since the climate, soil conditions, and crop structure of the area are favorable for irrigated agriculture (Andersson et al, 1979). Also demand for water-born recreation has been quite high in the area, but some concerns about adverse factors - e.g. lake level fluctuation in Vombsjön or water quality problem in Ringsjön - have been expressed recently. However, point source pollution has been largely controlled in the area to a high level, following a stringent Environmental Protection Act passed in late 1960's, and many existing wastewater treatment plants have excess capacity to cope with any foreseeable increase in wastewater of point source origin. A major water pollution problem will be caused by non-point sources, typically by agricultural runoffs, if irrigation should increase significantly.

Another factor that plays a major role in water resource planning of the region is the Swedish system of governance which is characterized by a high degree of decentralization. The basic decision-making unit concerning the use of land, water and other natural resources is the municipality. The existing local water supply systems are owned and operated by different

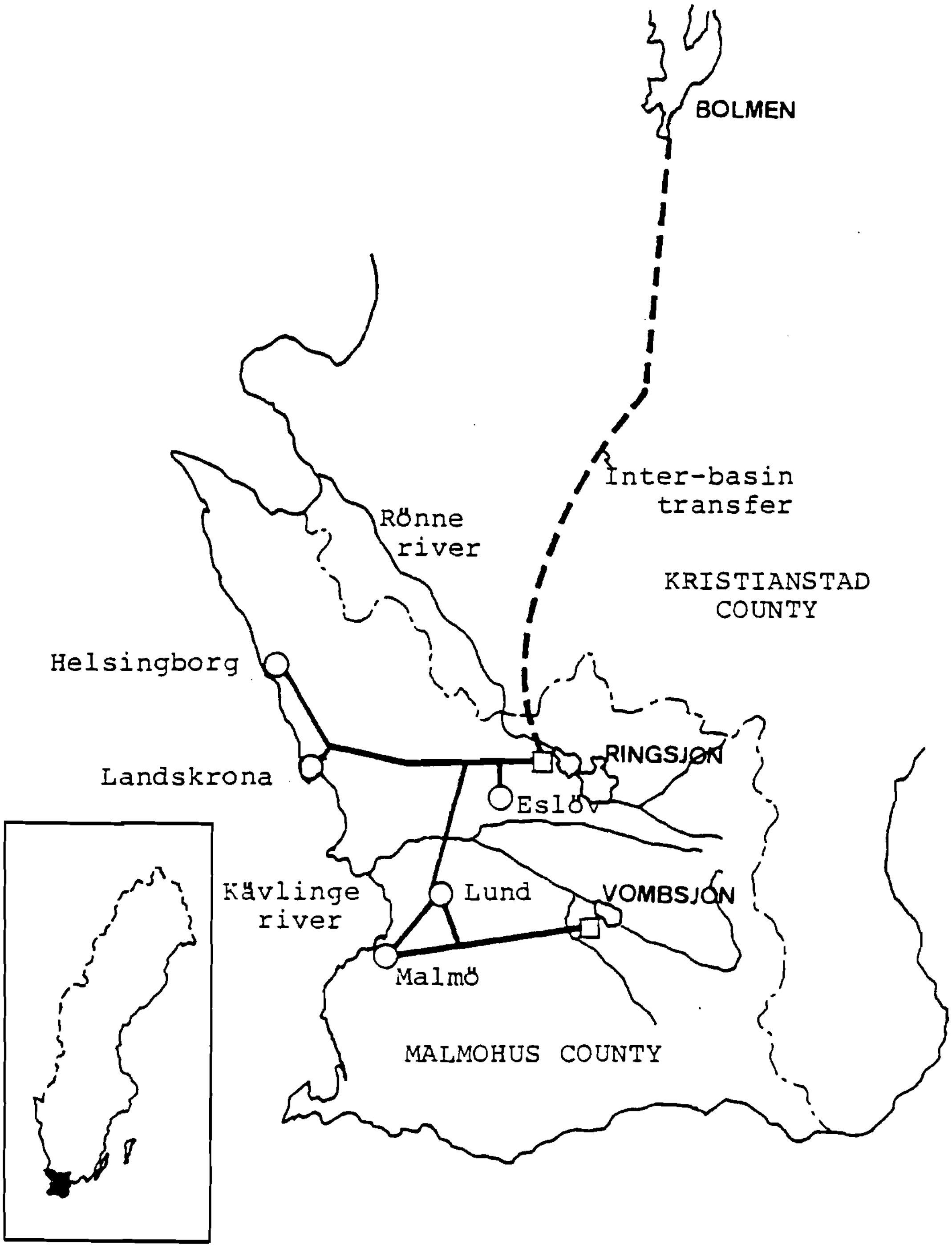


Figure 1. Study Area - Western Skåne, Sweden

sets of municipalities. A company, called Sydvatten, was formed in the late 1960's by several municipalities to plan long-term water supply, and management for the region. It now consists of 12 municipalities and is implementing the new inter-basin water transfer project described above.

The preceding discussion suggests the main characteristics of water resource management problem of the region. First, the physical framework of water resource management is rigidly set for the region, since the Bolmen system will be in operation sooner or later to provide sufficient quantity of water and no substantial investments in wastewater treatment plants are expected in the near future. Thus nothing much can be done about facilities location and scheduling related to water; more important is operational planning. Secondly, introduction of the Bolmen system will not only provide additional water for municipal use but it will also increase the possibility of reallocating local water resources to other uses, mainly recreation and agriculture. The basic question therefore is how to allocate available water to agricultural and recreational uses as well as to municipal uses; or in other words, how to operate the Bolmen, Ring and Vomb systems joingly for collective benefits of the region evaluated on a broader base to satisfy different users in all the municipalities, which act in principle independently of each other.

IV. GENERAL FORMULATION OF WATER RESOURCE ALLOCATION MODEL

Given highly decentralized institutional system of the region and three major water uses - municipal, agricultural and recreational - to be considered, the allocation problem typically takes the form of multiple decision-maker multiple-objective problem. This problem can be decomposed into more tractable subproblems as suggested before; specifically a two-level hierarchical structure is introduced. On the upper level, a multiobjective problem with objectives related to physical attributes as well as a cost objective is formulated. That is, physical possibilities of the regional water supply system and costs associated with operating the entire system to obtain a particular performance are analyzed.

Given a solution on the upper level, a set of multi-objective problems with multiple decision-makers may be solved on the lower level. Resolution of conflicts among independent water users derived from interactions of their uses through the physical attributes of the upper level problem can be aided by these multi-objective models on the lower level.

Upper Level Problem

The upper level problem may be formally written as:

[1a]
$$\max_{\underline{x}} \underline{f}(\underline{x}) \qquad (1)$$

subject to

$$\underline{g}(\underline{x}) \leq 0 \quad . \qquad (2)$$

This is to be solved following the procedure called STEM (Benayoun et al, 1971). This method is a type of reference objective method, but instead of changing reference levels at each iterative step as most methods in this class do, a decision-maker is supposed to do the following. First the decision-maker identifies satisfactory objectives, if any, based on the results of the previous step, and secondly for each satisfactory objective specify the amount of permissible reduction in this objective attainment in order to improve values of unsatisfactory objectives.

STEM uses the following penalty scalarizing function:

$$s(\underline{f} - \underline{m}) = \max_j w_j \{m_j - f_j(\underline{x})\} \longrightarrow \min_{\underline{x}} \quad , \qquad (3)$$

where m_j is the maximum attainable level of objective $f_j(\underline{x})$ over the feasible region $\underline{g}(\underline{x}) \leq 0$ in the absence of other objectives, and the weights $\underline{w}$ are determined as follows. First, a sensitivity parameter γ_j and a scaling parameter α_j are defined as:

$$\gamma_j = \begin{cases} \dfrac{m_j - n_j}{m_j} & \text{if } m_j > 0 \\[2ex] \dfrac{n_j - m_j}{n_j} & \text{if } m_j \leq 0 \quad , \end{cases} \qquad (4)$$

and

$$\alpha_j = \begin{cases} m_j \sum_\ell \frac{\gamma_\ell}{m_\ell} & \text{if } m_\ell > 0 \text{ for all } \ell \\ \{m_j + (1 - \min_\ell m_\ell)\} \sum \frac{\gamma_\ell}{m_\ell + (1 - \min_i m_i)} & \text{if any } m_\ell \leq 0 \end{cases} \quad (5)$$

where n_j is the minimum value assumed by $f_j(\underline{x})$ over the feasible region. Then the weight is given by

$$w_j = \frac{\gamma_j}{\alpha_j} \quad . \quad (6)$$

The reasoning behind this definition is the following. If the value of $f_j(\underline{x})$ does not vary much from its maximum attainable level m_j for various solution vector $\underline{x}$, this objective is not sensitive to a variation in the weighting values w_j, and thus a relatively small weight can be assigned. As the variation in $f_j(\underline{x})$ becomes larger with changes in $\underline{x}$, the weight w_j will become correspondingly large.

As noted by Cohon (1978), this rather elaborate procedure for the calculation of weights is supposed to minimize the need for value judgements. Thus this method may be more suitable when a decision-maker can not be easily identified for each objective of the upper level problem and evaluation of alternatives generated by the upper level problem therefore is left for the lower level. In this case the function of the upper level problem is to obtain a reasonably balanced solution at each iterative step. Note, however, that the relative weight calculated by this procedure has nothing to do with the relative political importance of the corresponding objective (Haith and Loucks, 1976), and the trade-off coefficients can be calculated *a posteriori* after a solution is obtained.

Using the above penalty scalarizing function with appropriate weights, the upper level problem is to minimize the upper bound d of the weighted deviation $w_j\{m_j - f_j(\underline{x})\}$:

$$[1b]\quad \left[\begin{array}{ll} \min\limits_{\underline{x}} \quad d & (7) \\ \text{subject to} & \\ d \geq w_j\ \{m_j - f_j(\underline{x})\} \qquad \forall\ j & (8) \\ \underline{g}(\underline{x}) \leq 0 & (9) \end{array}\right.$$

The solution to this problem gives values of decision variables $\underline{x}^*$ and the objective function values $\underline{f}^*$. The next step of the procedure is to show the results to the decision-maker and ask him to identify satisfactory and unsatisfactory objectives, and for each satisfactory objective f_j to specify the amount Δf_j that can be sacrificed to improve attainment levels of unsatisfactory objectives.

Depending on a particular institutional framework, there are alternative modes of operating the model at this stage. These are described in the remaining part of this section with the aid of Figure 2. If a "single" decision-maker can be identified for each objective of this upper level problem as discussed in Section II, each of them can tell whether the level of his objective attained by the previous iteration is satisfactory, and if satisfactory, even specify permissible reduction of his objective in the light of a planned or expected level for his own objective. Even in more decentralized institutional systems, satisfactory and unsatisfactory objectives and amounts of permissible reduction for the satisfactory objectives may be specified joingly by multiple decision-makers without aid of any mathematical model. For this to be possible, all the decision-makers' specifications have to be aggregated in some way. Obviously if one decision-maker finds attainment of a certain objective unsatisfactory this is an unsatisfactory objective and is so specified in the next iterative step. How the aggregation of permissible levels can be done depends on a particular problem and objectives.

Once the values $\Delta \underline{f}$ are specified for all the objectives (where $\Delta f_j = 0$ for any unsatisfactory objective f_j), the next iteration of the upper level problem is performed by solving the following modified problem:

$$
[1c] \quad \left[\begin{array}{lll}
\min\limits_{\underline{x}} & d & (10) \\
\text{subject to} & & \\
\quad d \geq w_j\{m_j - f_j(\underline{x})\}, & \forall_j & (11) \\
\quad \underline{f}(\underline{x}) \geq \underline{f}(\underline{x}^*) - \Delta \underline{f} \quad , & & (12) \\
\quad \underline{g}(\underline{x}) \leq 0 \quad , & & (13)
\end{array} \right.
$$

where $w_j = 0$ if $\Delta f_j = 0$ and the weighting $\underline{w}$ has been rescaled accordingly. The iterative procedure continues until all the upper level objectives are found satisfactory (see the upper half of Figure 2.a).

Termination of the procedure with successful identification of a compromise solution implies that all the decision-makers have agreed on basic operating rules of the entire system. The remaining question is, given these "optimal" values of decision variables $\underline{x}$ and objectives $\underline{f}$, how to allocate water in different parts of the system among concerned water-users and also how to allocate total costs of the system operation. A set of multi-objective allocation problems may be solved on the lower level to aid the resolution of conflicts among multiple decision-makers - i.e. independent water-users (see the lower half of Figure 2.a).

Specification of satisfactory and unsatisfactory objectives and permissible reductions for the satisfactory objectives by multiple decision-makers may not be so easy as described above in some cases for two major reasons: (i) Each decision-maker may not be able to tell if a certain objective on the upper level is satisfactory without interpreting it in terms of his own objective; (ii) Decision-makers may not know how their objectives conflict with each other, and aggregation of individual specification of permissible reduction for satisfactory objectives may not lead to reasonable results. In this case, some device would be necessary to aid decision-makers to determine which objectives in the upper level problem are satisfactory and also to compromise the conflicting objectives with each other. Again a set of multi-objective models on the lower level can be used for this purpose as described in the following (see also Figure 2.b).

Lower Level Problem.

The lower level problem may be formulated using the "optimal" values $\underline{f}^*$ of the upper level objectives as inputs. One of the lower level problems may be formally represented as:

[2a]

$$\max_{\underline{y}} \quad \underline{F}(\underline{y}|\underline{f}^*) \tag{14}$$

subject to

$$\underline{G}(\underline{y}|\underline{f}^*) \leq \underline{0} \quad . \tag{15}$$

To clarify the structure of the entire problem, this may be written in the following form, assuming the $\underline{y}$ terms and $\underline{f}^*$ terms are separable in each constraint.

[2b]

$$\max_{\underline{y}} \quad \underline{F}(\underline{y}) \tag{16}$$

subject to

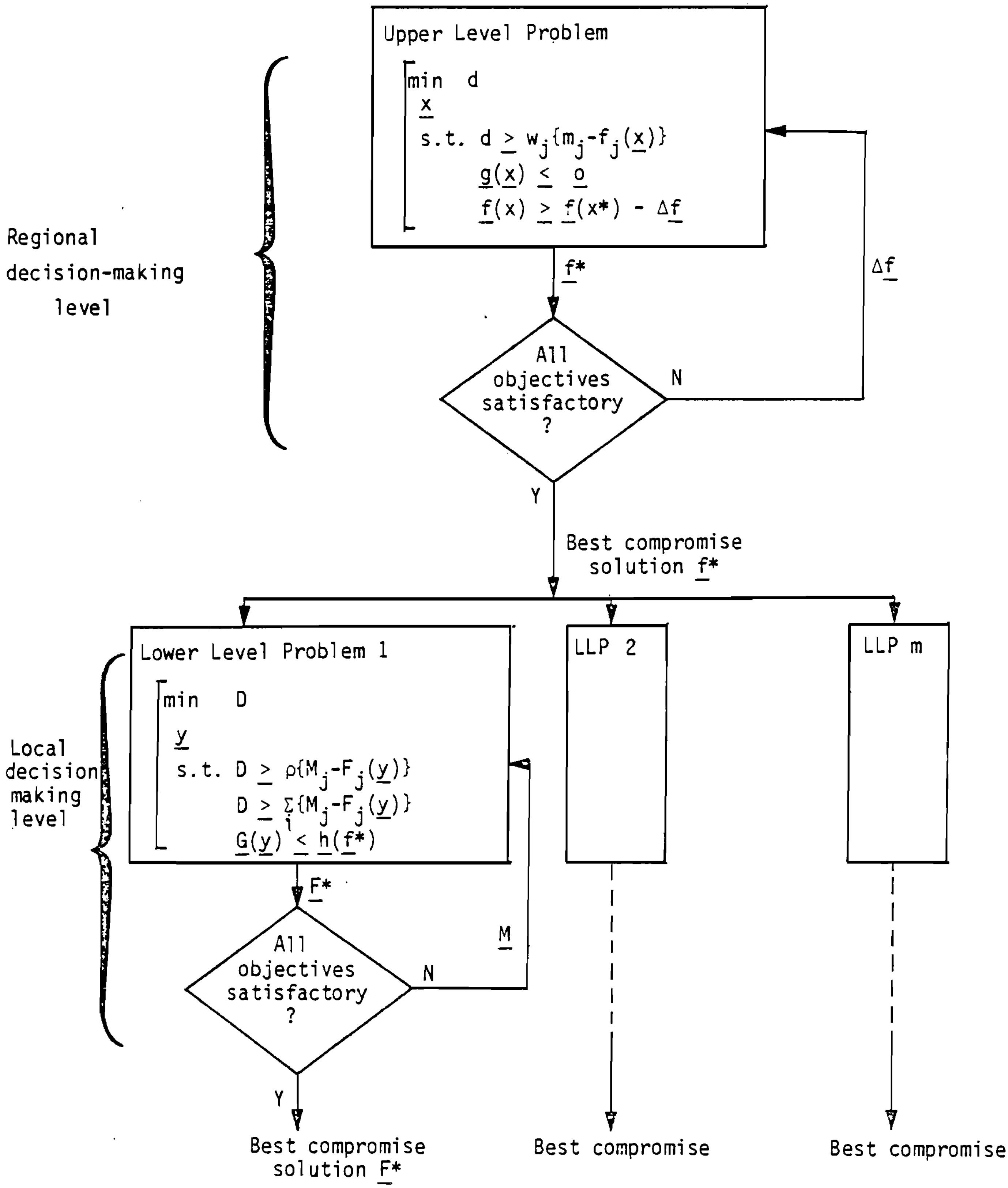
$$\underline{G}(\underline{y}) \leq \underline{h}(\underline{f}^*) \quad . \tag{17}$$

Attainable levels of the lower level objectives naturally depend on the values of $\underline{f}^*$.

To solve the lower level problem, the following penalty scalarizing function may be used

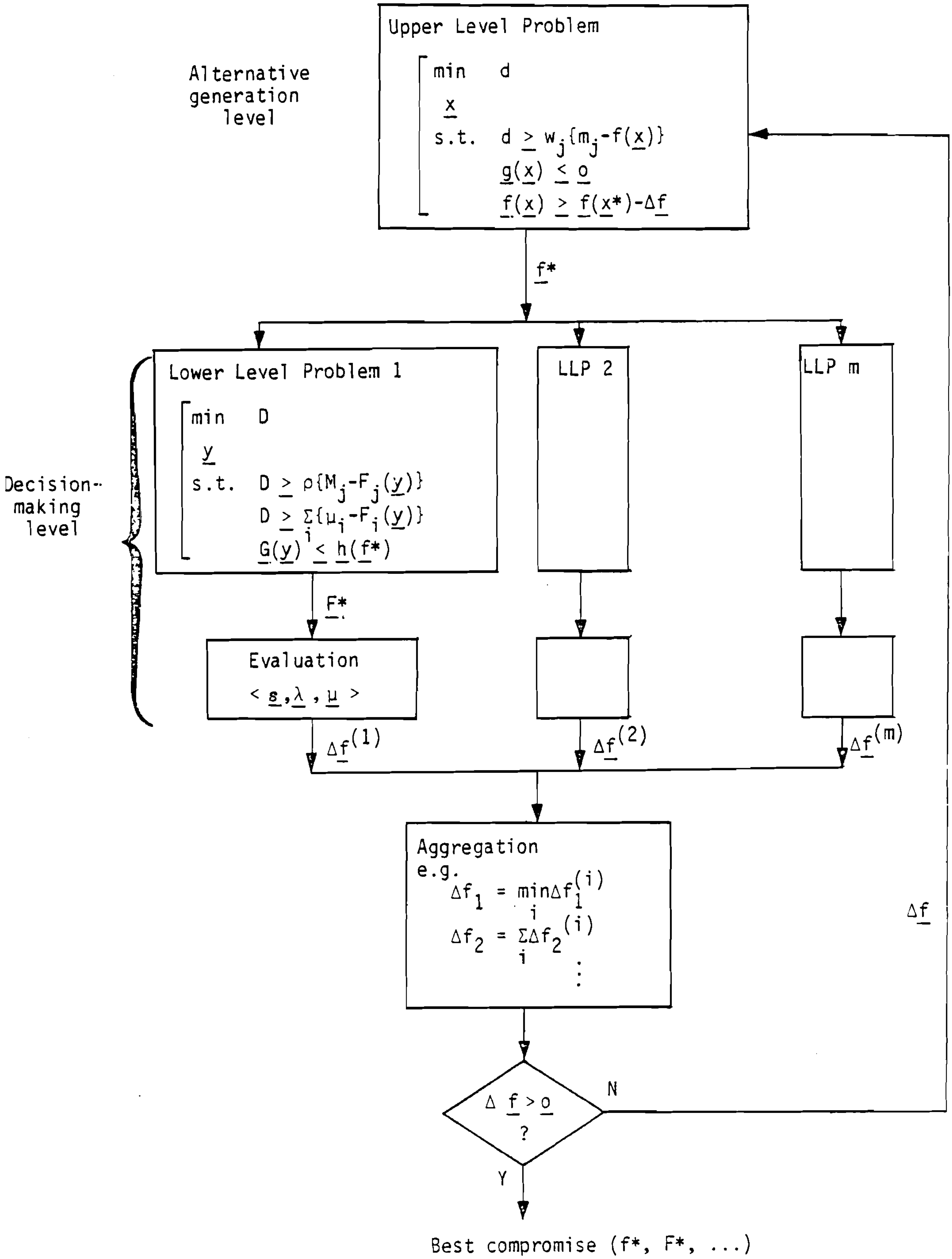
$$S(\underline{F} - \underline{M}) = \max\ [\rho \max_j\{M_j - F_j(\underline{y})\},\ \sum_i\{M_i - F_i(\underline{y})\}] + \\ + \sum_i \varepsilon_i\{M_i - F_i(\underline{y})\} \longrightarrow \min_{\underline{y}} \quad , \tag{18}$$

where M_j is a reference level of the objective $F_j(\underline{y})$, ρ is a penalty coefficient which is greater than or equal to the number of objectives and $\underline{\varepsilon}$ is a non-negative vector of parameters. Each



(a) Two-level decision-making

Figure 2. Alternative Modes of Implementing the Two-Level Multiobjective Model



(b) More decentralized decision-making

objective is assumed to be normalized so that the deviations $M_j - F_j(\underline{y})$ are comparable on the same basis. This function is found most reasonable by Kallio et al (1980). In the subsequent application, this function is used with ρ equal to the number of objectives and $\underline{\varepsilon} = \underline{0}$. Selection of a particular scalarizing function, in general, represents certain reasonableness and fairness to be incorporated in a compromise-aiding procedure, but it is not decisive of the validity of the entire planning structure formulated herein. Other forms of functions may as well be used (see Wierzbicki, 1979, for other practical forms).

By defining the bound D for $\rho\{\mu_j - F_j(\underline{y})\}$ and $\Sigma\{\mu_i - F_i(\underline{y})\}$, the lower level problem is solved in the following form:

$$\text{[2c]} \quad \left[\begin{array}{ll} \min\limits_{\underline{y}} \quad D & (19) \\ \text{subject to} & \\ \quad D \geq \rho\{M_j - F_j(\underline{y})\} \qquad \forall \; j & (20) \\ \quad D \geq \sum\limits_i \{M_i - F_i(\underline{y})\} & (21) \\ \quad \underline{G}(\underline{y}) \leq \underline{h}(\underline{f}^*) \quad . & (22) \end{array}\right.$$

As stated before, in order to specify amounts of permissible reduction $\Delta\underline{f}$ for the upper level problem, it may in some cases be necessary to relate $\Delta\underline{f}$ to objectives F_j of the lower level problem. First, trade-off coefficients $\underline{\mu}$ among objectives can be determined *a posteriori* for reference objective methods (Wierzbicki, 1980). Secondly, examination of slack variables $\underline{s}$ associated with non-binding constraints (22) of the lower level problem may tell how much the attained level $\underline{f}^*$ of the upper level objectives can be reduced without sacrificing the lower level objectives $\underline{F}$. Also the dual variables $\underline{\lambda}$ associated with binding constraints (22) can relate sacrifice ΔF_j of the lower level objectives F_j to its effects on the objectives of the upper level problem. Figure 2.b illustrates the mode of model implementation in this case.

V. REGIONAL WATER ALLOCATION MODEL

Objectives

The upper level problem concerns operation of the regional water supply system consisting of Bolmen, Ringsjön and Vombsjön as water sources (see Figure 1). The following five objectives are considered for the entire vegetation season of a certain year in this application:

(1) Minimize total costs of operating the whole system;

(2) Minimize deviation of water level in Vombsjön from the optimal level for recreational purposes;

(3) Maximize minimum release from Vombsjön for downstream uses;

(4) Minimize water level deviation from the maximum level in Ringsjön for recreational purposes;

(5) Maximize minimum release from Ringsjön for downstream uses.

As stated in Section III, water quality problem is a more serious disturbance for recreational uses of Ringsjön. However, effects of inflow water quality on lake water quality depend on the storage volume of the lake. This is why minimization of water level deviation from the maximum level in Ringsjön is taken as a proxy of water quality objective.

Constraints

A firm yield type model (a kind of implicit stochastic model) may provide a suitable structure for the upper level problem. The following mass balance equations hold for Vombsjön among the inflow q_t^V, the yield Y_t^V, the precipitation P_t^V, evaporation and seepage losses l_t^V, and the release to downstream r_t^V, all in time period t, and the storage S_t^V, S_{t+1}^V in Vombsjön in periods t and t+1:

$$S_t^V + q_t^V + P_t^V - l_t^V - Y_t^V - r_t^V = S_{t+1}^V \; . \quad \forall t \qquad (23)$$

The time periods t=1,2,...,T cover the entire vegetation season. The inflow time-series $\{q_t\}$ are either generated by synthetic hydrology or taken from historical records. Assume the lake is full at the beginning of the vegetation season:

$$S_o^V = S_{max}^V \quad . \tag{24}$$

The water level h_t^V of the lake is a function of the storage:

$$h_t^V = f^V(S_t^V) \quad . \qquad \forall t \tag{25}$$

The similar constraints apply to Ringsjön, too. That is,

$$S_t^R + q_t^R + P_t^R - l_t^R - Y_t^R - r_t^R = S_{t+1}^R \quad , \quad \forall t \tag{26}$$

$$S_o^R = S_{max}^R \quad , \tag{27}$$

$$h_t^R = f^R(S_t^R) \quad . \qquad \forall t \tag{28}$$

The following variables are defined for water level in, and release downstream from, both lakes:

$$\underset{\sim}{h}^V = \min_t h_t^V \quad , \tag{29}$$

$$\underset{\sim}{r}^V = \min_t r_t^V \quad , \tag{30}$$

$$\underset{\sim}{h}^R = \min_t h_t^R \quad , \tag{31}$$

$$\underset{\sim}{r}^R = \min_t r_t^R \quad . \tag{32}$$

Also defined are the maximum water level h_{max}^R of Ringsjön and the water level $\hat{h}^V$ of Vombsjön which is considered optimal for recreational purposes.

Costs associated with each component of the water supply system are defined as follows. Costs of transporting water from Bolmen to treatment works at Ringsjön are given by

$$C^B = C^B(Y^B) \quad . \tag{33}$$

Costs of treating water from Bolmen and Ringsjön, and treatment costs for Vombsjön water are respectively

$$C^R = C^R(Y^R,\ Y^B) \quad , \tag{34}$$

$$C^V = C^V(Y^V) \quad . \tag{35}$$

Costs of local distribution of water depend on both water demand $\underline{x}$ for five major municipalities shown in Figure 1, connected to the supply system and origins of water:

$$C^L = C^L(\underline{x},\ Y^V,\ Y^R + Y^B) \quad . \tag{36}$$

The total cost is given by

$$TC = C^B + C^R + C^V + C^L \quad . \tag{37}$$

Mass balance relationships have to be satisfied between water demand for some municipalities and the yield from alternative water sources. Typically the following should always hold:

$$Y^R + Y^V + Y^B = x_H + x_{LA} + x_E + x_{LD} + x_M \quad , \tag{38}$$

where x_H, x_{LA}, x_E, x_{LD} and x_M represent the water demands of Helsingborg, Landskrona, Eslöv, Lund and Malmö, respectively. Also the water demand of each municipality has to be satisfied.

In addition, there may be other relevant constraints related to requirements on final storage or minimum water levels of Vombsjön and Ringsjön, and capacities of treatment plants or intake facilities at Vombsjön and Ringsjön.

The regional water allocation problem is formulated as the following multi-objective program:

$$\min\ TC \tag{39}$$

$$\min\ |\underset{\sim}{h}^V - \hat{h}^V| \tag{40}$$

$$\max\ \underset{\sim}{r}^V \tag{41}$$

$$\min\ h^R_{max} - \underset{\sim}{h}^R \tag{42}$$

$$\max\ \underset{\sim}{r}^R \tag{43}$$

subject to the constraints (23)~(38) and additional constraints if appropriate.

VI. KÄVLINGE RIVER BASIN MODEL

A Lower Level Problem

Based on both jurisdictional and water-shed boundaries, five aggregated agricultural users can be identified, who are assumed to act in principle independently. Also two separate recreational activities - the lake-based one and the one in the downstream of Kävlinge river - are identified. The system is schemetically represented by Figure 3. The agricultural users are identified by superscript S1, S2, Ld, Ev and KL, which stand for the municipality of Sjöbo in upper Kävlinge sub-basin and in Klingavalsån sub-basin, Lund in Klingavalsån, Eslov in mid-stream Kävlinge and the municipalities of Kävlinge and Lomma (combined) in downstream Kävlinge, respectively. The recreational activities are represented by V and K for Vombsjön and Kävlinge.

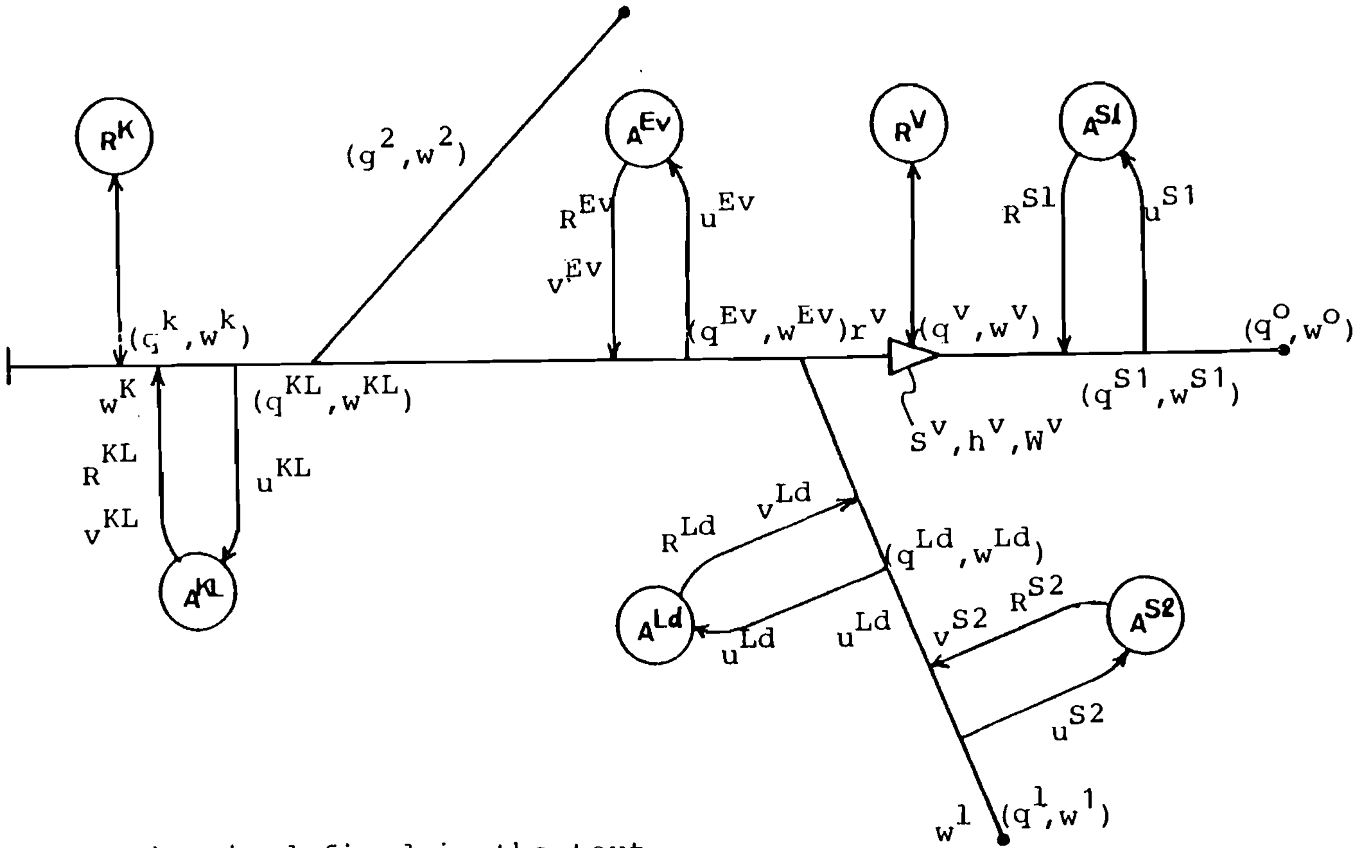
Objectives

Assume each agricultural water user tries to increase crop yield from his area by irrigation and fertilization. Alternatively net benefits from agricultural production may be used as the objective for each agricultural user, but this is not attempted here.

Water-based recreational activities depend on quality and quantity of water available at recreation sites, lake water level and its fluctuation and other factors. Since no model is available to relate these different factors to a more commensurable or composite measure in order to represent recreational activities just consider in this study optimizing water quality at appropriate points - i.e. at Vombsjön and at the downstream Kävlinge.

Constraints

For the agricultural user S1, the following constraints apply. First, the mass balance for soil moisture in agricultural land



Notation is defined in the text

Figure 3. Schematic Representation of Kävlinge River Basin with Five Agricultural Water Users A^{S1}, A^{S2}, A^{Ld}, A^{Ev}, A^{KL} and Two Recreational Activities R^{V}, R^{K}

land is given by

$$m_t^{S1} + u_t^{S1} - v_t^{S1} - E_t^{S1} + P_t^{S1} = m_{t+1}^{S1} \quad , \forall t \qquad (44)$$

where m_t^{S1} and m_{t+1}^{S1} represent soil moisture in time periods t and t+1, u_t^{S1}, v_t^{S1}, E_t^{S1} and P_t^{S1} are irrigation, surface run-off, evapotranspiration and precipitation for the agricultural land in period t. Seepage to groundwater is ignored. The difference between actual and potential evapotranspiration E_t^{S1} and E_{pt} in period t divided by the latter defines relative moisture deficit d_t^{S1}:

$$d_t^{S1} = (E_{Pt} - E_t^{S1})/E_{pt} \quad . \quad \forall t \qquad (45)$$

A yield reduction function $a_t^{S1}(d_t^{S1})$ for period t can be defined as a function of the relative moisture deficit, and the effect of fertilizer application f^{S1} on crop yield is represented by another function $b(f^{S1})$. The production function for this agricultural activity relate these functions to the total yield per unit area Y^{S1}. Assuming linearity, the production function is given by

$$Y^{S1} = Y_{max}^{S1}[1 - \sum_t \{1 - a_t^{S1}(d_t^{S1})\}] + b(f^{S1}) \quad , \qquad (46)$$

where Y_{max}^{S1} is the maximum potential yield without fertilization, if there is no moisture deficit during the entire vegetation season. The similar relationships hold for other agricultural users.

Let quantity and quality of inflow in period t into the system on three tributaries be (q_t^o, w_t^o), (q_t^1, w_t^1) and (q_t^2, w_t^2) as shown in Figure 2 (where the subscript t is omitted for clarity). Let q_t^{S1}, q_t^{S2}, q_t^{Ld}, q_t^{Ev} and q_t^{KL}

denote the quantity of water available for respective agricultural users. These are given by the following:

$$q_t^{S1} = q_t^{o} \quad . \tag{47}$$

$$q_t^{S2} = q_t^{1} \quad . \tag{48}$$

$$q_t^{Ld} = q_t^{S2} - (u_t^{S2} - v_t^{S2})A^{S2} + I_t^{S2Ld} \quad . \tag{49}$$

$$q_t^{Ev} = r_t^{vo} - (q_t^{vo} - q_t^{v}) + q_t^{Ld} - - (u_t^{Ld} - v_t^{Ld})A^{Ld} + I_t^{LdEv} \tag{50}$$

$$q_t^{KL} = q_t^{Ev} - (u_t^{KL} - v_t^{KL})A^{KL} + I_t^{EvKL} + q_t^{2} \quad . \tag{51}$$

Here A^{S2}, A^{Ld} and A^{KL} represent respective agricultural area, and I_t^{S2Ld}, I_t^{LdEv} and I_t^{EvKL} are interflows between agricultural sites S2 and Ld, Ld and Ev, and Ev and KL, respectively. Note in the equation for q_t^{Ev}, the release from Vombsjön r_t^{vo} given as an output from the upper level problem is adjusted by difference between original and actual inflow q_t^{vo} and q_t^{v} into Vombsjön; the latter is given by

$$q_t^{v} = q_t^{S1} - (u_t^{S1} - v_t^{S1})A^{S1} + I_t^{S1V} \quad , \tag{52}$$

where I_t^{S1V} is interflow between agricultural site S1 and Vombsjön. Finally, flow q_t^{K} in period t at control point of downstream Kävlinge river is given by

$$q_t^{K} = q_t^{KL} \quad (u_t^{KL} - v_t^{KL})A^{KL} \quad . \tag{53}$$

Irrigation rate at each agricultural site is constrained by flow available at the site:

$$u_t^{AG} \leq q_t^{AG} \quad \forall t, \tag{54}$$

where AG represents any agricultural user.

Next, water quality at each agricultural site and at Vombsjön and the downstream Kävlinge is also computed based on mass balance for a conservative substance (e.g. total phosphorus, a major concern in agricultural run-offs). For time period t, these are given as follows:

$$w_t^{S1} = w_t^{o} \quad , \tag{55}$$

$$w_t^{S2} = w_t^{1} \quad , \tag{56}$$

$$w_t^{Ld} = \{(q_t^{S2} - u_t^{S2}A^{S2})w_t^{S2} + v_t^{S2}A^{S2}R_t^{S2} + \\ + I_t^{S2Ld}L_t^{S2Ld}\}/q_t^{Ld} \quad , \tag{57}$$

$$w_t^{Ev} = [\{r_t^{vo} - (q_t^{vo} - q_t^{v})\}w_t^{v} + (q_t^{Ld} - u_t^{Ld}A^{Ld})w_t^{Ld} \\ + v_t^{Ld}A^{Ld}R_t^{Ld} + I_t^{LdEv}L_t^{LdEv}]/q_t^{Ev} \quad , \tag{58}$$

$$w_t^{KL} = \{(q_t^{Ev} - u_t^{Ev}A^{Ev})w_t^{Ev} + v_t^{Ev}A^{Ev}R_t^{Ev} + \\ + I_t^{EvKL}L_t^{EvKL} + q_t^{2}w_t^{2}\}/q_t^{KL} \quad , \tag{59}$$

$$w_t^{v} = \{(q_t^{S1} - u_t^{S1}A^{S1})w_t^{S1} + v_t^{S1}A^{S1}R_t^{S1} + \\ + I_t^{S1V}L_t^{S1V}\}/q_t^{v} \quad , \tag{60}$$

$$w_t^{K} = \{(q_t^{KL} - u_t^{KL}A^{KL})w_t^{KL} + v_t^{KL}A^{KL}R_t^{KL}\}/q_t^{K} \quad , \tag{61}$$

where R_t^{S2}, R_t^{Ld}, R_t^{Ev}, R_t^{S1} and R_t^{KL} are quality of surface run-offs from respective agricultural site, and L_t^{S2Ld}, L_t^{LdEv}, L_t^{EvKL} and L_t^{S1V} represent quality of interflow. Since the operating rule for Vombsjön has been specified by the upper level problem, quality W_t^{V} of lake water in Vombsjön can also be given by simple mass balance for the conservative substance:

$$W_t^{v} = (q_t^{v}w_t^{v} + S_t^{vo}W_{t-1}^{v})/(S_t^{vo} + q_t^{v}) \quad , \quad \forall t \tag{62}$$

where S_t^{vo} is the storage in period t specified by the upper level problem. The quality of agricultural run-offs in general is a function of irrigation rate and fertilizer application:

$$R_t^{AG} = R_t^{AG}(u_t^{AG}, f^{AG}) \quad , \tag{63}$$

where AG represents any agricultural user.

Finally some additional constraints are necessary to complete the list. Define bounds for water quality in Vombsjön and downstream Kävlinge:

$$W_t^V \leq \bar{W}^V \quad , \tag{64}$$

$$w_t^K \leq \bar{w}^K \quad . \qquad \forall t \tag{65}$$

Set water quality standard and minimum flow requirements at the control point of downstream Kävlinge.

$$w^K \leq w_{max}^K \quad , \tag{66}$$

$$q_t^K \geq q_{min}^K \quad . \tag{67}$$

The Kävlinge river basin problem is formulated as the following multi-objective program:

$$\max \quad Y^{S1} \tag{68}$$

$$\max \quad Y^{S2} \tag{69}$$

$$\max \quad Y^{Ld} \tag{70}$$

$$\max \quad Y^{Ev} \tag{71}$$

$$\max \quad Y^{KL} \tag{72}$$

$$\min \quad \bar{W}^V \tag{73}$$

$$\min \quad \bar{w}^K \tag{74}$$

subject to the constraints (44)~(67) and others suggested in the text above.

VII. NUMERICAL EXAMPLE

The two-level multi-objective program for water resource allocation problem in Western Skåne is solved using as much as possible data obtained for the region. The model implementation scheme given in Figure 2.a is followed for this example. Non-linear relationships are approximated by either linear or piece-wise linear functions. A month is taken as a time period t for the upper level problem, and t=1,2,3,4 and 5 cover the vegetation season - May through September - and lower level problems deal with the same season.

Upper Level Problem - Regional Water Allocation Model

Data

Water demand for five municipalities connected to the regional water supply system is based on forecasts made for the year 2000 and given in Table 1. Data given in Table 2 on inflow into Vombsjön and Ringsjön, precipitation and evaporation for these lakes have been taken from records for year 1976, which is a recent dry year. The storage functions for Vombsjön and Ringsjön are approximated by linear functions for ranges of concern:

$$h_t^V = 0.087S_t^V + 13.1 \text{ (m)}$$
$$45 \leq S_t^V \leq 90 \text{ (Mm}^3\text{)}$$
$$h_t = 0.026S_t^R + 49.2 \text{ (m)}$$
$$109 \leq S_t^R \leq 214 \text{ (Mm}^3\text{)}$$

Assume water level of Vombsjön that is considered optimal for recreational activities be 20.9 m. Unit cost of treating and transporting water is given on each arc of the network in Figure 4. Additionally minimum requirements are set for release from Vombsjön and Ringsjön as $\underline{r}^V \geq 0.78$ Mm^3/month and $\underline{r}^R \geq 1.30$ Mm^3/month based on current operating rules for these lakes.

Solution and Results

A preliminary step of STEM is to construct a pay-off matrix. Each row of the pay-off matrix given in Table 3 corresponds to the solution vector obtained by solving an auxiliary problem which is to optimize one objective subject to the same set of

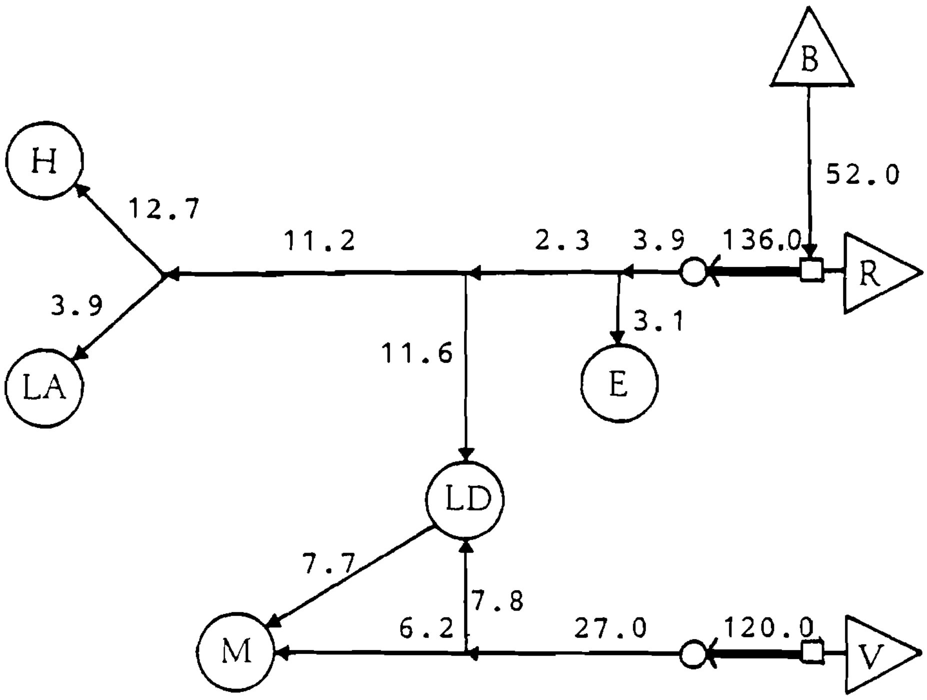
Table 1. Forecast Water Demand for Five Municipalities

Municipality	Demand (Mm^3/month)
Helsingborg	2.58
Landskrona	0.79
Lund	1.63
Malmö	6.14
Eslöv	0.55
Total	11.69

Source: Sydvatten, Prognos 73

Table 2. Hydrologic and Related Data for Upper Level Problem

	Time period 1 May	2 June	3 July	4 Aug.	5 Sept.
	Mm^3/month				
Vombsjön					
Inflow q_t^V	3.88	5.24	0.32	0.48	1.32
Precipitation p_t^V	0.81	0.59	0.78	0.20	0.59
Evaporation loss l_t^V	1.08	1.60	1.84	1.60	0.97
Ringsjön					
Inflow q_t^R	5.73	6.01	4.07	1.34	2.13
Precipitation p_t^R	1.87	1.71	1.52	1.21	1.42
Evaporation loss l_t^R	3.44	5.10	5.88	5.10	3.10



Legend: Treatment arc

Transmission Arc

The unit costs are given in 10^3 Skr/month per unit water flow expressed in Mm^3/month

Figure 4. Unit Cost of Treatment and Transmission

constraints that constitutes the upper level problem. The diagonal element of the ith row is the optimal value of the ith objective of the upper level problem, and off-diagonal elements are values attained by other objectives when the ith objective is optimized. It is interesting to note that the release from Vombsjön and Ringsjön is at respective minimum level when other objectives are optimized. That is, these objectives tend to be sacrificed in favor of others unless they are explicitly considered in multi-objective programs.

Noting three out of five objectives are to be minimized (i.e. negative of these objectives is maximized), weighting vector $\underline{w}^1$ in the upper level problem [1b] can be computed according to equation (6) as $\underline{w}^1 = \{0.358, 0.225, 0.113, 0.229, 0.075\}$. The first iteration of STEM using this weighting vector $\underline{w}^1$ yields the solution given in the first row of the matrix in Table 4. Next a question is asked to the decision-maker, to identify objectives for which the levels of attainment are satisfactory to him. Suppose he is satisfied with the release from Vombsjön and Ringsjön. Then the decision-maker is asked to specify the amounts of permissible reduction for these satisfactory objectives in order to improve attainment of three other objectives - total cost of system operation, water-level deviation in Vombsjön and Ringsjön. The decision-maker is willing to accept levels of the satisfactory objectives $\underline{r}^V$ and $\underline{r}^R$ as low as 4.79 Mm^3/month and 8.42 Mm^3/month, respectively, as indicated in Table 4.

The second solution in Table 4 is obtained by solving the modified upper level problem [1c] with the specified $\Delta f = \{0, 0, 2.0, 0. 8.0\}$ and weighting vector $\underline{w}^2 = \{0.441, 0.277, 0, 0.282, 0\}$, which is calculated in the same way as before. The weights for satisfactory objectives are set to zero. Observing the second solution, the decision-maker finds the water level deviation in Ringsjön to be satisfactorily small, and decides that it can be increased to 1.30 m at most. He also decides to accept somewhat lower attainment for two objectives that have already been found satisfactory - i.e. $\underline{r}^V \geq 4.29$ and $\underline{r}^R \geq 7.42$ - in an

Table 3. Payoff Matrix of Five Objectives of Upper Level Problem

Objectives	TC MSkr/ season	$\hat{h}^V - \underset{\sim}{h}^V$ m	$\underset{\sim}{r}^V$ Mm^3/ month	$h^R_{max} - \underset{\sim}{h}^R$ m	$\underset{\sim}{r}^R$ Mm^3/month
TC → min	9.91	3.52	0.78	0.39	1.30
$\hat{h}^V - \underset{\sim}{h}^V \rightarrow \min$	10.76	0.28	0.78	0.39	1.30
$\underset{\sim}{r}^V \rightarrow \max$	11.50	3.09	10.42	2.60	1.30
$h^R_{max} - \underset{\sim}{h}^R \rightarrow \min$	10.44	3.52	0.78	0.13	1.30
$\underset{\sim}{r}^R \rightarrow \max$	10.84	3.52	0.78	2.16	21.88

Table 4. Implementation of Upper Level Problem with Five Non-inferior Solutions

Solution No.	TC MSkr/ season	$\hat{h}^V - \underset{\sim}{h}^V$ m	$\underset{\sim}{r}^V$ Mm^3/ month	$h_{max} - \underset{\sim}{h}^R$ m	$\underset{\sim}{r}^R$ Mm^3/month
1	10.77	1.66	6.79 ↓ 4.79	1.92	16.42 ↓ 8.42
2	10.77	1.66	4.79 ↓ 4.29	1.13 ↓ 1.30	8.42 ↓ 7.42
3	10.72	1.56	4.29 ↓ 4.00	1.30	10.07 ↓ 7.00
4	10.72	1.45	4.00	1.30	10.07
5	10.64	1.56	4.00	1.30	10.07

Specification of permissible reduction for satisfactory objectives at each iterative step is indicated by arrows.

attempt to improve values of two other objectives that are still unsatisfactory to him.

For the third iteration, weighting vector is calculated to be $\underline{w}^3$ = {0.613, 0.387, 0, 0, 0}. Observing a modest improvement of total costs in the third solution, the decision-maker decides to satisfy himself with this level of attainment and wants to improve the water level deviation in Vombsjön by further sacrificing the release objectives $\underline{r}^V$ and $\underline{r}^R$ as indicated in Table 4, and the fourth solution obtained. The decision-maker may also want to find out effects of minimizing the total costs under the same conditions specified after the third iteration. The results are given as the fifth solution in Table 4.

It may happen in the course of iterative procedure that the decision-maker finds it impossible to improve some objectives to satisfactory levels without driving other objectives which have been found satisfactory in earlier steps to unsatisfactory levels; that is, STEM may fail to find a compromise solution. In this case, decision-maker's preferences have to be changed or the upper level problem has to be reformulated with modificatons such as reduced demand for municipal water uses. The previous round of iterative procedure may still serve for an educational purpose, and lead the decision-maker to change his requirements in a more realistic way. Then another round of iteration may be initiated with modified pay-off matrix.

The more details of the fifth solution is summarized in Table 5, where the storage and the water level in, and the release from, two lakes, and the water yield from three alternative sources are given for each month. Note the yield from Vombsjön and Bolmen vary among months, and Ringsjön is utilized to its maximum intake capacity for all the months. Lower level problems may be solved using this solution as the input.

Lower Level Problem - Kävlinge River Basin Model

Data

The Kävlinge river basin model - one of the lower level problems - is solved in this subsection using the conditions specified by the fifth solution (See Tables 4 and 5) of the

Table 5. Details of a Compromise Solution (the fifth solution in Table 4)

	Time period 1	2	3	4	5
Vombsjön					
Storage Mm^3	90	85	82	77	72
Water level m	20.9	20.5	20.2	19.8	19.4
Release Mm^3/month	4.00	4.00	4.00	4.00	23.28
Yield Mm^3/month	4.73	3.38	0	0	4.73
Ringsjön					
Storage Mm^3	214	206	194	180	163
Water level m	54.8	54.5	54.2	53.9	53.5
Release Mm^3/month	10.07	10.07	10.07	10.07	10.07
Yield Mm^3/month	3.55	3.55	3.55	3.55	3.55
Bolmen					
Yield Mm^3/month	3.42	4.77	8.15	8.15	3.42

upper level problem as inputs. Analysis on the lower level problem may be based on the most critical month identified by the upper level problem. However, considering different effects of irrigation in particular time of the season, the entire vegetation season is divided into three periods - May/June, July, and August/September.

Hydrologic data including water quality expressed in concentration of total phosphorus are given in Table 6 for the three periods. Also given is potential evapotranspiration for these periods.

Components of production function for agricultural water users are estimated based on the University of Uppsala study concerning effects of irrigation and fertilization on potatoes (Johansson and Linner 1977, Linner, 1979). First the yield reduction function for each period is approximated by a piecewise

Table 6. Hydrologic and Related Data for Kävlinge River Models (a lower level problem)

	Time period		
	1	2	3
Inflow q_t^o [m^3/sec]	1.17	0.12	0.09
q_t^1	0.35	0.15	0.12
q_t^2	0.47	0.19	0.24
Interflow I^{S1V} [m^3/sec]	0.59	0	0.60
I^{S2Ld}	0.35	0.14	0.12
I^{LdEv}	0.19	0.08	0.07
I^{EvKL}	3.56	2.01	1.31
Water quality [μg/ℓ total phosphorus] w_t^o, L^{S1V}	148	210	226
w_t^1, L^{S2Ld}, L^{LdEv}	92	68	69
w_t^2	830	3510	2800
L^{EvKL}	142	160	140
Potential evapotranspiration [mm/period]	195	103	119

linear function as shown in Figure 5. Effects of fertilization are expressed as incremental yield obtained by applying a certain kind of fertilizer. Piecewise linearization is used for this component, too, as given in Figure 6. The maximum yield Y_{max} without fertilization is estimated to be 27.2 ton/ha and assumed to be the same for all the agricultural sites. Field capacity of soil is 57 mm in root zone.

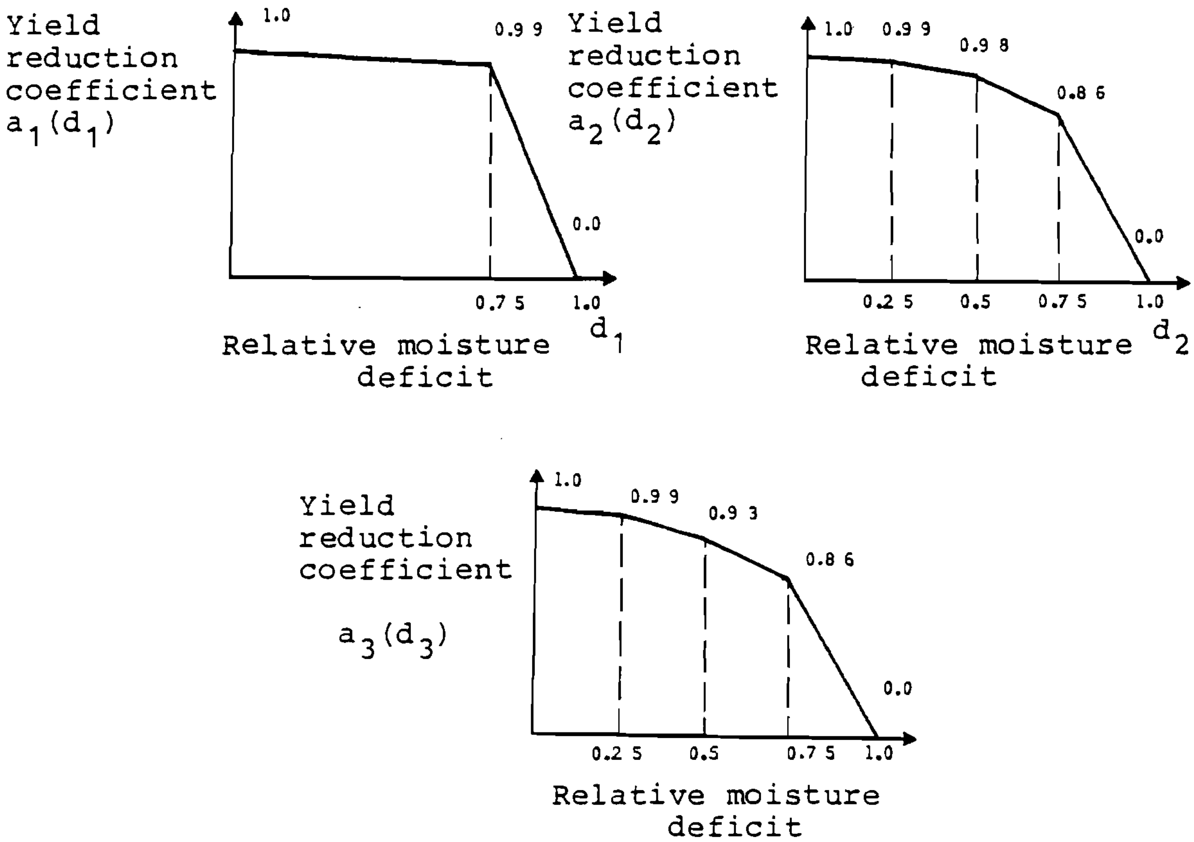


Figure 5. Piecewise Linear Approximation of Yield Reduction Functions for Three Periods

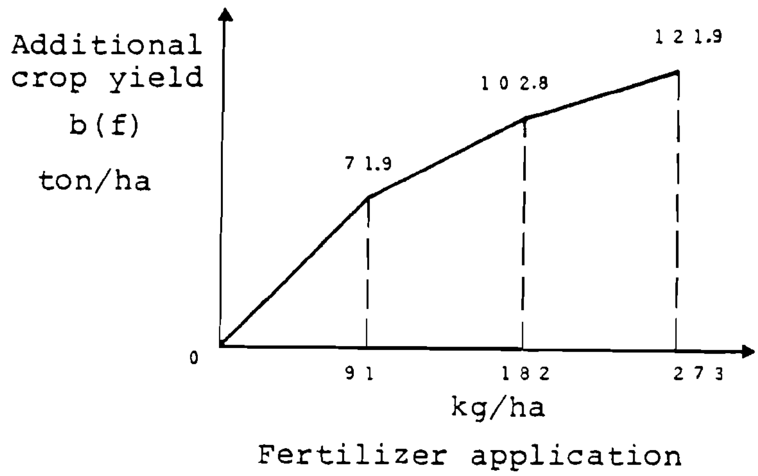


Figure 6. Piecewise Linear Approximation of Fertilization Effects

Irrigation area for each agricultural user is given in Table 7. Surface run-offs from agricultural sites depend on soil moisture, irrigation practice and other factors, but in this example, simple water loss coefficients as given in Table 7 are used as rough estimate of a part of irrigation water not returned to the stream system (i.e. consumptive uses).

Solutions and Results

To solve the lower level problem, the penalty scalaring function (18) is used with $\rho = 7$ (number of objectives) and $\underline{\varepsilon} = \underline{0}$, after all the objectives are normalized so that deviation from reference objective levels are comparable to each other. To normalize the objectives, first maximum attainable level in the absence of other objectives was found out for each objective, by optimizing each objective subject to the same constraints that constitute the lower level problem. Table 8 gives the maximum values for seven objectives, where water quality at Vombsjön and downstream Kävlinge is expressed in terms of improvement from the worst - 506 μg/l and 800 μg/l of total phosphorus, respectively. All the objectives are divided by the respective maximum values (minimum value for each objective is zero).

A reference point at first is taken to be the maximum values of all the objectives. This point naturally cannot be attained. The reference point optimization with the penalty scalarizing function defined above yields the results given in the first two rows of Table 9. Attainment ratio AR for each objective is calculated according to

$$AR = \frac{\text{Attained value}}{\text{Best value}} \quad , \qquad (75)$$

and given also in Table 9. The attainment ratios are well balanced among seven objectives in this problem. The decision-makers may accept this solution, but if not, the reference point is changed and the same procedure will be followed. A simple way suggested by Kallio et al (1980) to change the reference point is to ask each decision-maker to move his reference

Table 7. Irrigation Areas and Water Loss Coefficients for Five Agricultural Sites

Agricultural site	S1	S2	Ld	Ev	KL
Irrigation area in ha	974	440	309	820	897
Water loss coefficient	0.9	0.9	0.9	0.8	0.8

Table 8. Maximum Attainable Values for Seven Objectives of Lower Level Problem

Objective	Y^{S1}	Y^{S2}	Y^{Ld}	Y^{Ev}	Y^{KL}	$\bar{W}^{v'}$	$\bar{w}^{k'}$
Maximum Value	39.39	39.21	38.55	34.00	32.81	403.0	521.4
	ton/ha					µg/l total phosphorus*	

*Water quality objectives $\bar{W}^v$ and $\bar{w}^k$ are expressed in terms of improvement from the worst: i.e. $\bar{W}^{v'} = 505.8 - \bar{W}^v$ and $\bar{w}^{k'} = 800 - \bar{w}^k$.

Table 9. Two Non-inferior Solutions of Lower Level Problem

Solution		Objectives Y^{S1} (ton/ha)	Y^{S2}	Y^{Ld}	Y^{Ev}	Y^{KL}	$\bar{W}^{v'}$ (µg/l total phosphorus*)	$\bar{w}^{k'}$
1	Value of objective	29.83	29.66	29.17	25.77	24.93	353.4	394.7
	Attainment ratio	0.757	0.756	0.757	0.758	0.760	0.877	0.757
2	Value of objective	29.98	28.81	28.34	26.70	25.83	369.3	396.1
	Attainment ratio	0.736	0.735	0.735	0.785	0.787	0.916	0.760

* Water quality objectives $\bar{W}^v$ and $\bar{w}^k$ are expressed in terms of improvement from the worst: i.e. $\bar{W}^{v'} = 505.8 - \bar{W}^{v'}$ and $\bar{w}^{k'} = 800 - \bar{w}^k$.

objective level towards the solution just obtained, at least some fraction β ($o < \beta \leq 1$) of the entire distance between the solution and the original reference level.

It may be useful for the decision-makers to see more details of the solution obtained in order to determine if the solution is acceptable, or if not, how the reference levels should be changed. In Table 10, soil moisture, evapotranspiration, moisture deficit and irrigation rate in each period, rate of fertilizer application and the yield per hectare are given for each agricultural user, and water quality at Vombsjön and the downstream Kävlinge in each period is also given. The results show, for instance, that agricultural user S1 applies 117 mm of water for irrigation in period 1 to obtain maximum soil moisture in period 2, which is the most crucial period for crop yield, and 32 mm and 8 mm in the subsequent periods; 33.2 kg/ha of total phosphorus is also applied during the season to obtain eventually the crop yield 29.8 ton/ha.

The downstream agricultural users Ev and KL cannot apply fertilizer, and Ev cannot even irrigate, since combined effects of fertilization and irrigation on water quality at control point downstream of Kävlinge river are much more significant for these agricultural users than the upstream users. The agricultural users Ev and KL may find such a solution unfair. In the next step therefore, Ev and KL move their reference objective levels only 40% towards this solution, while the upstream agricultural users S1, S2 and Ld are required to move 60% of the entire distance between the solution and the original reference point; recreationalists V and K move their respective objective levels by 50% of the entire distance.

Values of seven objectives and attainment ratios for the second step solution are given in the last two rows of Table 9. Details summarized in Table 11 show that the downstream agricultural users Ev and KL can now apply both irrigation and fertilization in a modest degree to increase their crop yields as a result of reduced fertilization by upstream agricultural users. They can further improve their production, if they succeed in manipulating next moves of the reference objective levels in their favor.

Table 10. Details of Solution 1 (in Table 9) of Lower Level Problem

		Soil moisture mm			Evapotranspiration mm			Moisture deficit %			Irrigation rate mm			Fertilization kg/ha	Crop yield ton/ha
		1	2	3	1	2	3	1	2	3	1	2	3		
Agricultural water user	S1	40	57	56	195	103	119	0	0	0	117	32	8	33.2	29.83
	S2	40	57	40	132	103	112	33	0	6	93	40	32	33.5	29.66
	Ld	40	57	45	97	75	89	51	27	25	1	0	0	34.7	29.17
	Ev	40	57	31	61	79	79	69	23	34	0	0	0	0	25.77
	KL	40	49	53	49	52	75	75	50	37	6	4	3	0	24.93

Water quality [μg/l total phosphorus]	Time period 1	2	3
$\bar{w}^v$	123.3	136.3	152.4
$\bar{w}^k$	183.3	405.3	235.0

Table 11. Details of Solution 2 (in Table 9) of Lower Level Problem

		Soil moisture mm			Evapotranspiration mm			Moisture deficit %			Irrigation rate mm			Fertilization kg/ha	Crop yield ton/ha
		1	2	3	1	2	3	1	2	3	1	2	3		
Agricultural water user	S1	40	57	57	195	103	119	0	0	0	117	32	8	22.5	28.98
	S2	40	57	40	132	103	112	33	0	6	93	40	32	22.8	28.82
	Ld	40	57	45	97	75	89	51	27	25	1	0	0	24.2	28.34
	Ev	40	57	31	61	83	89	69	23	25	2	2	11	3.2	26.70
	KL	40	49	53	49	52	84	75	50	29	6	4	12	3.8	25.83

Water quality [μg/l total phosphorus]	Time period 1	2	3
$\bar{W}^v$	116.0	124.8	136.5
$\bar{w}^k$	182.0	403.9	222.5

Summary and Conclusions

The water resource allocation problem in Western Skåne, Sweden, formulated in Sections V and VI as a two-level multi-objective program, was solved in this section. The example illustrates how this model can be solved using reference objective methods. The upper level problem dealing with the region as a whole was solved using the procedure called STEM, which identifies a compromise solution in a relatively small number of iterations. As an example of the lower level problems, the Kävlinge river basin model was solved using a solution from the upper level as inputs. A particular penalty scalarizing function was used with reasonable results.

The example studied here also indicates the feasibility of implementing the model by interactions with multiple decision-makers. With this respect, an obvious advantage of a reference objective method is in its simplicity. First, dialogues between decision-makers and a planner to reveal their "preferences" are rather simple; the decision-makers can think in terms of real values of objectives and specify a desired level for each objective of concern. When multiple decision-makers are involved, their preferences may be aggregated in a simpler and more natural way for reference objective methods than other kinds of multi-objective methods.

Another advantage of using a reference objective method is flexibility that it allows in implementing multi-objective models. Since decision-makers do not know in advance the range of feasible solutions or even their own preferences, implementation of a multi-objective model has to proceed in a stepwise manner, and at each step the model has to be modified taking account of information obtained from the decision-makers. Multi-objective methods should also allow for changes in decision-makers preferences during the iterative procedure that may occur as a result of learning and interaction among themselves.

A few more points specific to the example studied here are noted. First, it was shown by the upper level problem that amounts of water drawn from three alternative sources - Vombsjön, Ringsjön and Bolmen - vary depending on relative weight given to each objective of upper level problem. If more emphasis is placed on regulation of water level in Vombsjön and Ringsjön for recreational purposes or on increasing release from these lakes for downstream users (mostly agricultural), more water is introduced from Bolmen, although this source is inferior from the viewpoint of the cost objective.

Secondly, a classical problem of upstream/downstream conflicts was clearly illustrated by the lower level problem. A possible way suggested by this example to resolve such conflicts is to adjust moves of reference objective levels at each iteration.

The third point illustrated by the example is that although dialogues between decision-makers and a planner take place in objective space, more detailed information on each solution obtained should also be presented to the decision-makers in order to enable them to make unerring decisions concerning acceptability of the solution. For instance, the first solution of the lower level problem appears to be well balanced with respect to attainment ratios of objectives, but some decision-makers may claim it unfair by looking at what that solution implies in terms of their own activities.

VIII. USES OF THE MODEL AND POSSIBLE EXTENSIONS

A typical use of the two level multi-objective water allocation model studied in this paper may be visualized as follows. At the beginning of a dry season, all the independent water users sit together to determine basic rules for operating Vomb, Ring and Bolmen systems jointly. The upper level model will be mobilized using appropriate data to generate information on physical possibilities of the system, which vary depending on meteorological and other conditions in a particular year, and the total cost of system operation is also computed. The water users will evaluate alternative operating policies in the light of their own objectives. Their conflicting interests can be adjusted with the aid of lower level models like the one studied here for Kävlinge river basin.

The model can also be used for longer-range planning if some normative mechanism which accords with existing or proposed institutional arrangements is incorporated into the lower level models. For instance, it may provide guidelines for permitting new water rights to agricultural users in an equitable way and also in a way compatible with recreational activities. Such a specification of equity or reasonableness is partly reflected in selection of a particular penalty scalaring function to be used in reference objective approach to multi-objective problems.

Naturally usefulness of the model as a practical planning tool depends heavily on how to specify each component of the model, which in turn is dependent on availability of data for a particular region of concern. With this respect, there exist many limitations in the way the model was solved in this paper for Western Skåne region, which motivate further extensions. Especially improvements are required in many aspects describing agricultural activities.

First, more than one crop need to be considered in combination with different soil types. Irrigation and fertilization practice may be different depending on each combination. Naturally a production function has to be estimated for each crop taking account of soil type, too.

Effects of irrigation and fertilization on receiving water bodies have to be treated more carefully. Surface run-offs from agricultural areas are function of soil moisture, irrigation practice and other factors. Nitrogen leaching from agricultural areas and its effects on groundwater may also be considered. More than a single water quality parameter, non-conservative as well as conservative, may have to be considered to evaluate the effects of agricultural activities on receiving water. This calls for more sophisticated modeling for surface water and groundwater response.

A better objective for each agricultural water user is net benefits rather than crop yield itself, since irrigation and fertilization involve costs and prices for crops may change in future.

Water-based recreational activities depend not only on water quality but also on many other factors. A model to relate different factors to a more commensurable or composite measure to represent the recreational activities may be desirable. Consideration of effects of recreational activities on receiving water may also be necessary.

The entire model hinges on quality of hydrologic data. Characterization of streamflow and interflow by rainfall-runoff analyses and synthetic hydrology may help in this respect. The upper level problem can be solved repeatedly with various combinations of inflow time-series for Vombsjön and Ringsjön. This allows to incorporate different levels of hydrologic uncertainty into the model. In this connection, more explicit introduction of risk-related objectives (e.g. reliability, resilience and vulnerability) may be appropriate.

Other lower level problems have to be formulated and solved in appropriate ways to complete the analysis of the regional water resource allocation. Rönne river basin problem may be formulated in the similar way as the Kävlinge river basin problem studied in this paper.

The municipal sector may be treated separately on the lower level from agricultural and recreational sectors. This is justified in the case of Western Skåne, since municipal water uses do not interact much with other uses except through water quantity in Vombsjön and Ringsjön, which is dealt with by the upper level problem. A major allocation problem concerning the municipal water users is how to allocate total costs of system operation. Other relevant studies are available for this general problem (see, for example, Young et al 1980).

However, if for instance, it turns out during the implementation of the upper level problem that attainment of the cost objective cannot be brought down to a satisfactory level, or if other objectives cannot be satisfied fully without driving the cost objective to an unsatisfactory level, the upper level problem itself has to be modified by relaxing certain requirements that constitute the constraint set of the upper level problem. One way is to suppress water demand for municipal uses by using appropriate measures (Kindler, Maidment and Gouevsky, 1980). If this option can be defined in specific terms, it could as well be incorporated formally in the upper level problem.

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