AN IMPROVED METHODOLOGY FOR CONSTRUCTING INCREMENT-DECREMENT LIFE TABLES FROM THE TRANSITION PERSPECTIVES

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FOREWORD

Interest in human settlement systems and policies has been a central part of urban-related work at the International Institute for Applied Systems Analysis (IIASA) from the outset. From 1975 through 1978 this interest was manifested in the work of the Migration and Settlement Task, which was formally concluded in November 1978. Since then, attention has turned to dissemination of the Task's results, to the conclusion of its comparative study, and to the exploration of possible future work that might apply the newly-developed mathematical methodology to other research topics.

This paper outlines a new and improved procedure for calculating a multistate life table from transition data. A novel feature of this method is the use of transition proportions conditional on survival. The exposition draws on migration data for the United States and the United Kingdom.

Papers summarizing previous work on migration and settlement at IIASA are listed at the back of this paper.

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ABSTRACT

Satisfactory calculation methods are currently available for implementing the movement approach to the calculation of increment-decrement life tables. By contrast only heuristic calculation methods have been suggested for implementing the alternative approach; namely, the transition approach, which is relevant in the analysis of interregional migration from census information.

This paper presents an improved methodology for calculating increment-decrement life tables from the transition perspective. First, it suggests a method for estimating transition probabilities which is an interpolative variant of Rees and Wilson's averaging method tailored to the type of data commonly available. Then, it proposes an alternative to the usual linear integration method for calculating the multistate counterpart of the L-statistics of an ordinary life table.

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AN IMPROVED METHODOLOGY FOR CONSTRUCTING INCREMENT-DECREMENT LIFE TABLES FROM THE TRANSITION PERSPECTIVE

INTRODUCTION

Increment-decrement life tables--life tables that recognize entries as well as withdrawals from alternative states--are used increasingly in various fields of demography, e.g., in the analysis of marital status, labor force participation, and interregional migration.

In practice, the crux of their calculation resides in the estimation of a set of transition probabilities from which multistate life table functions--generalizing the statistics contained in the columns of the ordinary life table--can be calculated.

For the purpose of such an estimation, two alternative approaches are available: the movement and the transition approaches. Rather than being competitive, they are complementary in that the choice of either approach is dictated by the data at hand. In most cases, mobility data are available in a format consistent with the movement approach. The only notable exception relates to the analysis of interregional migration when the mobility data come from population censuses, i.e., when they relate to migrants rather than to migrations.

From a methodological point of view, the implementation of the movement approach raises relatively few problems since satisfactory calculation methods, generalizing those classically used for the construction of an ordinary life table, have been recently developed. By contrast, the few methods suggested for the implementation of the alternative (transition) approach are much less adequate, because they are more heuristic in nature.

Perhaps, the most reliable method available from the transition perspective is the one set forth by Rees and Wilson (1977) in which the age-specific transition probabilities are taken as arithmetic averages of the observed survivorship proportions for the two consecutive age groups located on each side of the exact age concerned. We show, in this paper, that it is relatively easy to improve on this method by interpolating between the observed survivorship proportions in a less crude fashion (through the use of cubic spline functions). An interesting feature of this method is that, unlike Rees and Wilson's original method, it is applicable to unequal age and time intervals, i.e., it can be used if the width of each age group is different from the length of the period over which the data are collected.

Nevertheless, like other estimation methods focusing on survivorship proportions, the new method proposed generally fails because, in most cases, the necessary migration data are not available. Thus, we also propose in this paper an amendment of these estimation methods to produce a methodology tailored to the type of data generally available (i.e., age-specific mortality rates and transition proportions conditional on survival).

Yet another element of importance in the construction of an increment-decrement life table relates to the calculation of the numbers of person-years lived in the alternative states (i.e., the multistate life table functions generalizing the L-statistics of the ordinary life table). In general, this does not raise any real problem since the assumptions underlying the estimation of the age-specific transition probabilities determine as well the formulas necessary for the calculation of such statistics. Only in the case of the Rees and Wilson method, the estimation of the age-specific transition probabilities has no stringent implications for the calculation of the generalized L-statistics.

In view of this, we propose in the latter part of this paper an alternative to the usual linear integration method, which can be used, in the case of the Rees and Wilson averaging method and the extensions we propose here, for calculating numbers of person-years living that are in closer agreement with the available data.

The present paper consists of five sections. Section One, intended as a background section, provides a short review of the generalization of life table concepts to the increment-decrement case and briefly contrasts the movement and transition approaches. Section Two critically examines the two options currently available for the estimation of age-specific transition probabilities from the transition perspective. Our interpolative variant of Rees's and Wilson's averaging method is presented in Section Three, whereas Section Four discusses the amendment of the relevant estimation methods from survivorship proportions to the type of data commonly available (i.e., mortality rates plus transition proportions conditional on survival). Finally, Section Five describes the alternative to the linear integration method suggested for the calculation of the generalized L-statistics. Our exposition is illustrated throughout by means of examples relating to interregional migration in the United States (for females only) and in the United Kingdom.

1. INCREMENT-DECREMENT LIFE TABLES: A BRIEF REVIEW

Life table models are helpful devices for following a group of people, born at the same moment, over time and age in transition between two or more states. In the simplest situation, that of the ordinary life table, there are two states, the states of being alive and dead, and the emphasis is put on the irreversible transition from the former to the latter. By contrast, the increment-decrement life table is an elaborated version which allows one to follow persons advancing through successive states and possibly reentering states formerly occupied.

Whereas the fairly simple methodology underlying the construction of the ordinary life table has long been established (see Keyfitz 1968 for an in-depth discussion of such a methodology), the methodological and empirical problems raised by the construction of increment-decrement life tables have been thoroughly and systematically discussed only very recently. The contribution of several researchers (Rogers 1973, 1975; Schoen and Nelson 1974; Schoen 1975; Rogers and Ledent 1976; Schoen and Land 1977; Krishnamoorthy 1979; Ledent 1980a) has led to the development of a formal mathematical treatment which now give increment-decrement life tables a status comparable to that of the ordinary life table.

Next, in order to facilitate the understanding of the issues discussed in this paper, we provide a concise exposition of the emerging methodology of such complex life tables.

1.1 Generalizing the Ordinary Life Table Concepts

In brief, increment-decrement life tables can be regarded as generalizations of the ordinary life table in which elements in vector or matrix format are substituted for the scalar elements of the ordinary life table. In most instances, such a generalization appears to be a relatively straightforward matter, but it is not always so (Ledent 1980a).

First, we present the generalization of the theoretical derivation of the ordinary life table to the increment-decrement case. (In the course of this presentation, Table 1, which sets out the formulas relating to the simple and complex cases, is used as a point of reference.)

Table 1. A tabular comparison of the theoretical exposition of the ordinary and increment-decrement life tables.

Source: Ledent (1980a).

Ordinary Life Table		Increment-decrement Life Table	
$\mu(y) = \lim_{dy \to 0} \frac{d(y)}{\ell(y)dy}$	(1)	${}^{i}\mu^{j}(y) = \lim_{dy \to 0} \frac{{}^{i}d^{j}(y)}{\ell^{i}(y)dy}$	(1')
$\ell(y + dy) = \ell(y) - d(y)$	(2)	$\ell^{i}(y + dy) = \ell^{i}(y) - \sum_{\substack{j=1 \\ j \neq i}}^{r+1} {}^{i}d^{j}(y) + \sum_{\substack{j=1 \\ j \neq i}}^{r} {}^{j}d^{i}(y)$	(2')
$\frac{d}{dy}\ell(y) = -\mu(y)\ell(y)$	(3)	$\frac{d}{dy}\{\underset{\sim}{\ell}(y)\} = -\underset{\sim}{\mu}(y)\{\underset{\sim}{\ell}(y)\}$	(3')
$\ell(y) = \Omega(y)\ell(0)$	(4)	$\underset{\sim}{\ell}(y) = \underset{\sim}{\Omega}(y)\underset{\sim}{\ell}(0)$	(4')
$\Omega(y) = e^{-\int_0^y \mu(t)dt}$	(5)	~~$\underset{\sim}{\Omega}(y) = e^{-\int_0^y \underset{\sim}{\mu}(t)dt}$~~	(5')
		$\underset{\sim}{\Omega}(y) = [\underset{\sim}{I} - \underset{\sim}{\mu}(\tau_n)\Delta y_n] \ldots [\underset{\sim}{I} - \underset{\sim}{\mu}(\tau_1)\Delta y_1] + (**)$	(5")
$\ell_{x+n} = p_x \ell_x$	(6)	$\underset{\sim}{\ell}_{x+n} = \underset{\sim}{p}_x \underset{\sim}{\ell}_x$	(6')
$p_x = \frac{\Omega(x+n)}{\Omega(x)}$	(7)	$\underset{\sim}{p}_x = \underset{\sim}{\Omega}(x+n)\underset{\sim}{\Omega}(x)^{-1}$	(7')

Suppose we have a system of r+1 states (r intercommunicating states plus the state of death) in which the initial cohort is allocated among s states ($1 \leq s \leq r$)* and let $l^i(0)$ denote the "radix" of state i. The main problem here is to estimate the state-specific curves of survivors $l^i(y)$ at each age y. Such an estimation starts with the definition of the instantaneous mobility rates ${}^i\mu^j(y)$ [equation (1')] generalizing that of the instantaneous mortality rate $\mu(y)$ [equation (1)] of the ordinary life table. Substituting equation (1') in the definitional equation (2') showing the increments and decrements to each group $l^i(y)$ leads to the differential equation (3') which appears as a straightforward vector extension of the basic differential equation (3) of the ordinary life table.

Equation (3) admits r linearly independent solutions which can be expressed as equation (4), a straightforward matrix extension of the ordinary life table solution (4). These r independent solutions of (3') are the r multistate stationary populations that are generated by a unit (or arbitrary) radix in each of the r states (regàrdless of whether some of the states are initially empty or not).

Note that $\underset{\sim}{\Omega}(y)$--the matrix showing the state-specific survival probabilities at age y of the members of each radix--cannot be simply expressed as a function of the instantaneous mobility rates. The straightforward generalization of (5) into (5') does not hold and $\underset{\sim}{\Omega}(y)$--as shown in equation (5") --has to be determined by the infinitesimal calculus of Volterra (Schoen and Land 1977). $\underset{\sim}{\Omega}(y)$ is a proper transition probability matrix which allows one to derive the number of survivors $\underset{\sim}{l}_x$ at fixed ages x = 0,n,2n,..., by applying--as shown in equation (6')--a set of age-specific transition probability matrices $\underset{\sim}{p}_x$ defined as in equation (7'): again, this appears as a straightforward matrix extension of the ordinary life table case.

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*In most applications, (analysis of marital status, labor force participation, or birth parity), there is a unique "radix" (s = 1). But, in the case of interregional migration, the initial cohort is allocated among all intercommunicating states (s = r).

We now turn to the generalization of the life table functions to the increment-decrement case (see Table 2 for an illuminating tabular exposition of this generalization). Corresponding to the L_x-function of the ordinary life table [see equation (8)], one can define the multistate life table function $\underset{\sim}{L}_x$ [see equation (8')] whose (i,j)-th element has a two-fold interpretation as in the ordinary life table. First, it may represent the number of people born in state j and alive in state i of the life table between ages x and x+n. Alternatively, it may be considered as the number of person-years lived in state i between those ages by the members of the j-th radix.

From there, it is then possible to define a matrix of total number of person-years lived in prospect by the survivors at any age x of the initial radices [equation (9')] as well as a matrix of expectations of life by place of residence at age x [equation (10')].

Another generalization of interest here is that of the age-specific death rates m_x and survivorship proportions s_x: their multistate counterparts constitute, in effect, the basis for the implementation of the movement and transition approaches to the estimation of the age-specific transition probabilities $\underset{\sim}{p}_x$ from which all the multistate life table functions can be calculated.

It turns out, as shown by Ledent (1980a), that the existence of a predetermined mobility pattern as defined in continuous terms by (1') does not lead to the constancy of age-specific mobility rates $\underset{\sim}{m}_x$ or survivorship proportions $\underset{\sim}{s}_x$ but to the constancy of such proportions further indexed by place of birth. This, of course, only applies to the multiradix case (the case of interregional migration) for there is no ambiguity in the uniradix case since there exists a unique state of birth.

In analytic terms, the above means that the matrix extensions of (11) and (12), such as (11') and (12'), do not hold in the multiradix case because the values of $\underset{\sim}{m}_x$ and $\underset{\sim}{s}_x$ are affected by the state allocation of the initial cohort. Nevertheless,

Table 2. The vector generalization of the ordinary life table functions.

Source: Ledent (1980a).

Ordinary Life Table		Increment-decrement Life Table	
$L_x = \int_0^n \ell(x+t)dt$	(8)	$\underset{\sim}{L}_x = \int_0^n \underset{\sim}{\ell}(x+t)\, dt$	(8')
$T_x = \int_0^\infty \ell(x+t)dt$	(9)	$\underset{\sim}{T}_x = \int_0^\infty \underset{\sim}{\ell}(x+t)\, dt$	(9')
$e_x = \frac{T_x}{\ell_x}$	(10)	$\underset{\sim}{e}_x = \underset{\sim}{T}_x \underset{\sim}{\ell}_x^{-1}$	(10')
$m_x = \frac{\ell_x - \ell_{x+n}}{L_x}$	(11)	$\underset{\sim}{m}_x = (\underset{\sim}{\ell}_x - \underset{\sim}{\ell}_{x+n}) \underset{\sim}{L}_x^{-1}$	(11')
$s_x = \frac{L_{x+n}}{L_x}$	(12)	$\underset{\sim}{s}_x = \underset{\sim}{L}_{x+n} \underset{\sim}{L}_x^{-1}$	(12')

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in practice, these relationships are assumed to hold for they greatly facilitate the applied construction of increment-decrement life tables: as shown by Ledent (1978), the approximation introduced in the process by such an assumption is adequate.

1.2 Contrasting the Movement and Transition Approaches

The above theoretical exposition of increment-decrement life tables shows that the applied calculation of such tables immediately follows from the knowledge of the set of age-specific transition probabilities $\tilde{p}_x$. In effect, the availability of these probabilities immediately allows for the calculation of the survivors $\tilde{l}_x$ [using equation (6)] and that of the number of person-years lived $\tilde{L}_x$ [once a method for integrating $\tilde{l}(y)$ over each interval is chosen]. Then each value of $\tilde{T}_x$ is obtained by summing the L-statistics over all ages greater than or equal to x. Finally, the application of (10') through (12') provides the values of the remaining multistate life table statistics.

Thus, the crucial problem to be examined is the estimation of the age-specific transition probabilities $\tilde{p}_x$ from the observed data. For the purpose of this estimation, two alternative approaches--which appear to be complementary rather than competitive--have emerged: the movement and the transition approaches.

The former approach devised by Schoen (1975) views "passage" from one state to another as an instantaneous event in much the same ways as a death. Consistent with the approach commonly taken in the ordinary life table, it requires input data in the form of exposure/occurrence rates, estimated from primary data on interstate moves (i.e., transfers observed between each pair of sending and receiving states regardless of the states in which the individuals concerned are present at the beginning and end of the observation period). The movement approach is thus relevant to the analysis of marital status, labor force participation, as well as birth parity, since the data relating to the events involved come in the form of such interstate moves.

By contrast, the transition approach developed by Rogers (1973, 1975) conceives interstate "passage" as the result of a change in an individual's state of presence between two points in time. It is thus widely used in the analysis of interregional migration since population censuses typically give information concerning the number of persons in given age categories who were in another region one or five years earlier.

Note, however, that the analysis of interregional migration can also be approached from the movement perspective for nations which, such as Sweden or the Netherlands, maintain population registers: in effect, the knowledge from these registers of individuals' places of residence at all times allows one to construct migration statistics in the form of moves rather than of transitions.

To date, the implementation of the movement approach appears to have been successfully developed: the methodology underlying the estimation of the set of the transition probability matrices $\underset{\sim}{p}_x$ from the movement perspective is, in all cases, an elegant one, based on rigorous and transparent assumptions. For example, the extension by Rogers and Ledent (1976) of the work of Schoen and Nelson (1974) and Schoen (1975) has led to a simple matrix formula expressing the transition probabilities in terms of occurrence/exposure rates which are easily measurable from the primary data consistent with this approach.*

By contrast, the implementation of the transition approach has not been carried to the same degree of development. The fact is that the few methods proposed (Rogers 1973, 1975; Rees and

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*Based on a linear integration method for the calculation of $\underset{\sim}{L}_x$, this formula appears as a straightforward matrix extension of the formula relevant to the estimation of the age-specific survival probabilities in the ordinary life table.

Wilson 1977) are much more heuristic in nature: they are either based on simplistic assumptions or on a succession of approximations. The essence of these methods is discussed next.

But, before turning to the next section, it appears worthwhile to contrast further the movement and transition approaches from a more practical point of view. In effect, the above judgement that the movement approach is more satisfactory than the transition approach is only valid from a theoretical viewpoint. Ledent (1980a) presumes that the existing calculation methods--regardless of their sophistication--lead to less reliable and accurate results with the movement approach than with the transition approach. The argument underpinning this presumption is that the Markovian formulation of the underlying methodologies creates a certain mobility pattern which does not reflect the real world and that this affects the transition approach much less than the movement approach.

2. ESTIMATING THE TRANSITION PROBABILITIES: EXISTING METHODS FROM THE TRANSITION PERSPECTIVE

As for the estimation of the set of age-specific transition probabilities needed to initiate the calculation of increment-decrement life tables from the transition perspective, there are currently two main classes of calculation methods:

(a) the first class of methods relies on computing formulae expressing the various survival probabilities in terms of adequately measured migration rates*

(b) the second class of methods is based on alternative formulas involving the use of survivorship proportions as input variables.

These two categories are critically examined below.

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*These migration rates which refer to transitions rather than moves are different in nature from the mobility rates entering the age-specific matrices $\underset{\sim}{m}_x$.

2.1 Estimation From Mortality and Migration Rates

Rogers (1973, 1975) proposed the first computing formulae for estimating age-specific transition probabilities by applying a method that seeks the equality of the mortality and migration rates in the observed and life table populations.

According to him, the probability for an individual present at age x in region i* to survive n years later in region j is given by

$$ {}^{i}p_{x}^{j} = \frac{n \; {}^{i}M_{x}^{j}}{1 + \frac{n}{2}\left({}^{i}M_{x}^{\delta} + \sum_{k \neq i} {}^{i}M_{x}^{k}\right)} \tag{13} $$

while the probability for this same individual to survive in the same region is equal to:

$$ {}^{i}p_{x}^{i} = \frac{1 - \frac{n}{2}\left({}^{i}M_{x}^{\delta} + \sum_{k \neq i} {}^{i}M_{x}^{k}\right)}{1 + \frac{n}{2}\left({}^{i}M_{x}^{\delta} + \sum_{k \neq i} {}^{i}M_{x}^{k}\right)} \tag{14} $$

In both (13) and (14), ${}^{i}M_{x}^{\delta}$ and ${}^{i}M_{x}^{k}$ $(k = 1,\ldots,r;\ k \neq i)$ represent adequate death and destination-specific migration rates characteristic of the observed population of region i aged x to x+n.

To arrive at these formulae, Rogers used a methodology denoted as "Option 1" which focuses on the evolution over the next n years of the group l_{x}^{i} of people exactly aged x in region i. (Note the difference with the methodology used in the movement approach in which the focus is on the evolution

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*Because the transition perspective mainly applies to the case of interregional migration, the word region is substituted for state in the remainder of this paper.

of the size of the group of people living in region i over a n-year period regardless of the region of residence at age x.) Its main assumption is that it does not allow for multiple moves within each age interval: in particular, an individual who moves to another region cannot die before the end of the interval. (Note that this assumption enables Rogers to assimilate the cohort death rates he considers with the regional death rates.)

From a practical point of view, the only difficulty in applying the "Option 1" methodology relates to the possibility of correctly evaluating the destination-specific migration rates to be entered in formulas (13) and (14). Rogers (1975) suggests that, from migration data in the form of transitions (migrants) coming from a population census, one can simply measure ${}^{i}M_{x}^{j}$ $(j \neq i)$ as follows:

$$ {}^{i}M_{x}^{j} = \frac{{}^{i}K_{x}^{j}}{T\ \hat{P}_{x}^{i}} \qquad (15) $$

where ${}^{i}K_{x}^{j}$ is the number of those present in region i at age x to x+n who reside n years in region j, and T is the length of the observation period.

$\hat{P}_{x}^{i}$ is the average population aged x to x+n living in region i during the observation period.

An illustration of the above method is shown in Table 3 which sets out the age-specific transition probabilities out of the Northeast with reference to the 1965-70 four-region system of the US (for females).* It turns out that, for

*The four regions considered in this application are the four geographical units which the US Bureau of the Census refer to as Regions. The input data set has been prepared by Luis Castro from vital statistics published by the US Department of Health, Education, and Welfare (selected years) and migration data published by the US Bureau of the Census (1972).

example, a twenty-year old woman living in the Northeast has a 0.9326 probability to survive five years. Her probabilities to be in another Region are respectively 0.0164 (North Central), 0.0293 (South), and 0.0187 (West). Another illustration for the three-region system of the UK studied by Rees (1978) is provided in Table 4.

To be sure, the "Option 1" presents severe drawbacks which we discuss next. First of all, the assumption of no multiple movements within each age interval reduces somewhat the occurrence of deaths so that the total survival probabilities (${}^{i}p_x^{\delta} = 1 - \sum_k {}^{i}p_x^{k}$) and the various life expectancies are over-estimated (for a numerical evidence of this, see Ledent and Rees 1980). To circumvent this problem, one can use, as proposed in Willekens and Rogers (1978), the corresponding computing formulas of the movement approach as derived by Rogers and Ledent (1976), i.e.,

$$\underset{\sim}{p}_x = [\underset{\sim}{I} + \frac{n}{2}\underset{\sim}{M}_x]^{-1}[\underset{\sim}{I} - \frac{n}{2}\underset{\sim}{M}_x] \qquad (16)$$

where $\underset{\sim}{M}_x$ is a matrix grouping the various mortality and destination-specific migration rates. Numerically, this method--referred to as the "Option 3" method--provides better estimates of the various death probabilities (and thus of the total survival probabilities) without affecting the migration probabilities in a sensitive way (see Ledent and Rees 1980).

Secondly, the estimation of $\underset{\sim}{p}_x$ underlying the Rogers methodology assumes a linear integration method for the calculation of $\underset{\sim}{L}_x$. As shown in Ledent (1978), this is equivalent to supposing that interregional transfers (or moves) are evenly distributed within each age interval, an hypothesis which, in the case of interregional migration, does not appear to adequately reflect the age pattern of migratory moves.

Table 3. 1965-70 four-region system for the US (females): transition probabilities out of the Northeast obtained by application of the Rogers methodology.

TRANSITION FROM NORTHEAST TO

Age	Death	Northeast	North Central	South	West
0	0.020671	0.928738	0.013011	0.025022	0.012558
5	0.001692	0.961348	0.009073	0.018807	0.009080
10	0.001341	0.961213	0.010249	0.019616	0.007580
15	0.002342	0.921864	0.020219	0.037610	0.017965
20	0.002937	0.932559	0.016428	0.029339	0.018737
25	0.003729	0.949320	0.011562	0.022444	0.012965
30	0.005622	0.952279	0.010317	0.020672	0.011110
35	0.009501	0.956408	0.007897	0.018020	0.008174
40	0.014038	0.960515	0.005072	0.014212	0.006163
45	0.021320	0.959159	0.003285	0.011622	0.004614
50	0.031766	0.948197	0.002396	0.013900	0.003742
55	0.046125	0.926862	0.001923	0.020513	0.004577
60	0.067928	0.902500	0.002430	0.022556	0.004585
65	0.108212	0.869539	0.002113	0.016138	0.003998
70	0.169733	0.618792	0.001632	0.007335	0.002309
75	0.268131	0.721010	0.001733	0.006941	0.002185
80	0.384262	0.605607	0.001617	0.006475	0.002038
85	1.000000	0.000000	0.000000	0.000000	0.000000

Table 4. 1966-71 three-region system for the UK: transition probabilities out of East Anglia obtained by application of the Rogers methodology.

TRANSITION FROM EAST ANGLIA TO

Age	Death	East Anglia	Southeast	Rest of Britain
0	0.018066	0.889184	0.042182	0.050568
5	0.001737	0.931670	0.030937	0.035657
10	0.001505	0.934162	0.033102	0.031231
15	0.003374	0.863231	0.075133	0.058261
20	0.003792	0.869670	0.061003	0.065535
25	0.003174	0.898582	0.046241	0.052003
30	0.004119	0.913069	0.037602	0.045210
35	0.005909	0.931987	0.029127	0.032977
40	0.010278	0.940231	0.025842	0.023649
45	0.018140	0.945016	0.019415	0.017428
50	0.027728	0.948997	0.012864	0.010411
55	0.047964	0.932673	0.009858	0.009504
60	0.078064	0.900667	0.008127	0.013142
65	0.127602	0.852010	0.007566	0.012823
70	0.189381	0.790779	0.007136	0.012705
75	1.000000	0.000000	0.000000	0.000000

Thirdly, and more importantly, an issue raised by the estimation of the transition probabilities from the "Option 1" as well as from the "Option 3" methods is the adequacy of the migration rate measures used as input data. Clearly, the element appearing at the numerator in the definition (15) of ${}^{i}M_{x}^{j}$ is erroneous (Rees 1977; Ledent 1978): it relates to people who, on the average, are aged $x + \frac{n}{2}$ to $x + \frac{3n}{2}$ (and not x to x+n) over the observation period. More correctly, as pointed out by Rees (1978), it should be taken as a weighted average of the number of migrants observed over two consecutive intervals, i.e.,

$$ {}^{i}M_{x}^{j} = \frac{\frac{T}{2n}\ {}^{i}K_{x-n}^{j} + (1 - \frac{T}{2n})\ {}^{i}K_{x}^{j}}{T\ \hat{P}_{x}^{i}} \tag{17} $$

A numerical evaluation of the impact on the transition probabilities of the substitution of (17) for (15) is presented in Ledent and Rees (1980) with regard to one-year as well as five-year migration data.

Not only the numerator but also the denominator involved in the definition of the observed destination-specific migration rates is incorrectly evaluated. Let us recall that the methodology developed by Rogers (1973, 1975) focuses on cohorts rather than on regions. Consequently, the term $\hat{P}_{x}^{i}$ somewhat overestimates the exact value of the denominator of ${}^{i}M_{x}^{j}$ which is in fact does not appear to be easily measurable. Clearly the possibility of adequately defining and measuring the migration rates needed as inputs to the Rogers methodology casts some reservations about its reliability.

2.2 Estimation From Survivorship Proportions

The second type of estimation method from the transition perspective reflects the choice not to consider migration rates for which there exists no satisfactory definition, but instead to estimate the age-specific transition probabilities by equating observed and life table survivorship proportions.

A first method--denoted as the "Option 2" method--was proposed by Rogers (1975), one which generalizes the method sometimes used by demographers to calculate ordinary life tables and which relies on census information alone.

This generalization revolves around the following formula, linking the survivorship proportion matrices $\underset{\sim}{s}_x$ to the survival probability matrices $\underset{\sim}{p}_x$ (Rogers 1975)

$$\underset{\sim}{s}_x = (\underset{\sim}{I} + \underset{\sim}{p}_{x+n})\ \underset{\sim}{p}_x\ (\underset{\sim}{I} + \underset{\sim}{p}_x)^{-1} \tag{18*}$$

This relationship indicates that $\underset{\sim}{p}_{x+n}$ can be derived if $\underset{\sim}{s}_x$ and $\underset{\sim}{p}_x$ are known and suggests that, if $\underset{\sim}{p}_0$ is available, the series of matrices $\underset{\sim}{p}_x$ (for x = n,2n,z-n) can be obtained from the knowledge of the survivorship matrices for x = 0,n,..., z-2n.**Thus, the possibility of estimating the set of transition probabilities rests on the availability of an estimate for $\underset{\sim}{p}_0$. In effect, this does not raise any problem since the survivorship proportion matrix $\underset{\sim}{s}_{-n}$ relating to those born during the time interval considered is linked to $\underset{\sim}{p}_0$ through

$$\underset{\sim}{s}_{-n} = \frac{1}{2}\ [\underset{\sim}{I} + \underset{\sim}{p}_0] \quad , \tag{19}$$

which leads to the required estimate of $\underset{\sim}{p}_0$ by setting $\underset{\sim}{s}_{-n}$ equal to the observed $\underset{\sim}{S}_{-n}$.

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*This formula follows from assuming a linear integration formula for the calculation of the number of person-years lived $\underset{\sim}{L}_x$.

**This procedure is slightly different from Rogers's original procedure which, going backwards, relies on the availability of an estimate from $\underset{\sim}{p}_{z-n}$ which has to be obtained from other sources.

In practice, as pointed out by Ledent (1978), this procedure is much less effective here than in the case of the ordinary life table: undesirable estimates, i.e., falling outside the [0,1] range, are generally obtained even if the observed survivorship proportion matrices are not affected by any measurement error. There are at least three reasons accounting for such an unfortunate result:

(a) the observed survivorship proportions reflect the consolidation of migrating moves whose pattern may have varied over the observation period; thus, the survival probabilities observed by such a method are average values which are likely to be highly inaccurate owing to the particular averaging method implied by (18)

(b) formula (18) underlying this "Option 2" method relies on an assumption of identical migration patterns for each radix of the initial cohort, an assumption which is far from being realized (see Ledent 1980b)

(c) finally, because the "Option 2" method relies on a formula linking statistics of two consecutive age groups, estimation errors made for a given age group are passed on to the next; the "noise" thus introduced is likely to increase as one carries the estimation procedure over the whole set of age groups.

As an alternative to the "Option 2" method, Rees and Wilson (1977) simply propose to estimate the set of survival probabilities using the following approximation:

$$\underset{\sim}{p}_x = \frac{1}{2} [\underset{\sim}{S}_{x-n} + \underset{\sim}{S}_x] \qquad (20)^*$$

*This formula has to be amended in the case of the first age group for which

$$\underset{\sim}{p}_0 = \frac{1}{2} [\underset{\sim}{S}^2_{-n} + \underset{\sim}{S}_0] \qquad (21)$$

[Note that, alternatively, one could also think of estimating $\underset{\sim}{p}_x$ as the geometric (rather than arithmetic) average of the two consecutive survivorship proportion matrices, $\underset{\sim}{S}_{x-n}$ and $\underset{\sim}{S}_x$.]

An illustration of Rees and Wilson's averaging method is shown in Table 5 which sets out the transition probabilities outside East Anglia in the case of the three-region system of the UK: the observed survivorship proportions used for this application are those which Rees (1978) estimated through an adequate spatial demographic accounting method. It turns out that no matter how crude the method defined by (20) may appear, it provides quite acceptable results which, in any case, do not present the inadequacies obtained with the "Option 2" method. (Compare the figures of Table 5 with the corresponding ones in Table 4. In particular, observe the relatively large discrepancies in the case of the non-survival probabilities due to the different origin of the mortality information: vital statistics in the case of the Rogers methodology, estimated survivorship proportions in the case of the Rees and Wilson methodology.)

Table 5. 1966-71 three-region system for the UK: transition probabilities out of East Anglia obtained by application of the Rees and Wilson methodology.

TRANSITION FROM EAST ANGLIA TO

Age	Death	East Anglia	Southeast	Rest of Britain
0	0.018145	0.868918	0.055926	0.057012
5	0.002896	0.905619	0.041899	0.049386
10	0.002076	0.922461	0.036868	0.038596
15	0.003237	0.888786	0.059053	0.048924
20	0.003874	0.842247	0.080006	0.073872
25	0.003830	0.853691	0.067999	0.074480
30	0.004529	0.888243	0.049691	0.057537
35	0.006815	0.913213	0.036807	0.043165
40	0.011259	0.929925	0.028951	0.029864
45	0.018634	0.935257	0.024174	0.021935
50	0.030515	0.937086	0.017382	0.015017
55	0.050006	0.927741	0.011853	0.010400
60	0.082260	0.895768	0.009697	0.012275
65	0.130264	0.846185	0.008869	0.014682
70	0.203490	0.772598	0.008735	0.015176
75	1.000000	0.000000	0.000000	0.000000

3. AN INTERPOLATIVE VARIANT OF THE REES AND WILSON AVERAGING METHOD

Actually, the survivorship proportions $\underset{\sim}{s}_x$, central to the second class of estimation methods reviewed in Section Two, are a weighted average of the survival probabilities $\underset{\sim}{p}_{x+kn}$ where k takes on all values between zero and one. This fact suggests that an adequate estimation of $\underset{\sim}{p}_x$ can be obtained by interpolating between the observed survivorship proportions $\underset{\sim}{S}_x$, but in a less crude fashion than implied by Rees and Wilson's averaging method.

Suppose that, for a given system, we know the observed survivorship proportion matrices $\underset{\sim}{S}_x$ for x = 0,n,2n,... . Then for each pair of states i and j, one can interpolate between the observed survivorship proportions ${}^{i}S_x^{j}$ by using cubic spline functions which are increasingly used in the field of demography (McNeil, Trussell, and Turner 1977). The ordinate of the continuous curve thus obtained represents the probability for an individual present at age y in region i to be present in region j n years later. The required probabilities then may be found as the values of these curves at ages x = 0,n,2n,..., etc.

A difficulty arises here from the fact that, in case of low mortality levels--i.e., for the younger age groups, the application of the above procedure may yield estimates of the survivorship proportions such that $\sum {}^{i}S_x^{j} > 1$. Consequently, instead of interpolating between the retention proportions ${}^{i}S_x^{i}$, one will interpolate between the non-survival proportions ${}^{i}S_x^{\delta} = 1 - \sum_j {}^{i}S_x^{j}$ and then obtain an estimate of the retention probabilities by application of ${}^{i}S_x^{i} = 1 - \sum_{j \neq i} {}^{i}S_x^{j} - {}^{i}S_x^{\delta}$.

Once again the three-region system of the UK has been used to illustrate the above procedure: Table 6 displays the transition probabilities outside the East Anglia. A comparison of these figures with those of Table 5 obtained by application of the original method devised by Rees and Wilson reveals slightly different values. Aside from the first age group*, the major discrepancies occur--as one could expect--in the case of the age group with the highest migration propensity: the retention probability increases from .8276 to .8422 while the migration probabilities decrease significantly. Also, note that there is no significant pattern for the discrepancies observed as the sign of these discrepancies for a given transition probability varies from one age group to the next: for example, the adoption of the interpolative method increases the retention probability out of East Anglia for age groups 10-15, 15-20, 30-35, 35-40 but leads to a decrease for age groups 5-10, 20-25, 25-30. Clearly, this is an indication of the fact that our interpolative method accounts more accurately than Rees and Wilson's averaging method for the large variations and sharp overturns in migration propensities which, in most instances, characterize age-specific migration schedules. In this sense, it is felt that this variant of Rees and Wilson's method is more reliable and accurate.

Besides, our interpolative method presents definite advantages with respect to the "Option 2" method and Rees and Wilson method:

(a) it does not require the knowledge of the infant survivorship proportions $\underset{\sim}{S}_{-n}$ (relating to those born in the observation interval) which, in general, cannot be measured due to a lack of the necessary raw data

(b) it does not necessarily require observed survivorship proportions for a period T equal to the length n of the typical age group. If $T \neq n$, the above interpolative procedure allows one to estimate a set of $\underset{\sim}{p}_x$ but for $x = 0, T, 2T, \ldots$. Then if $\frac{n}{T}$ is an integer, it is readily possible to consolidate this set of transition probabilities into another one for $x = 0, n, 2n, \ldots$

*As regards the first age group, we first note the relatively low value of the non-survival probability stemming from our particular cubic spline interpolation. This could be amended by introducing additional constraints when performing such an interpolation.

Table 6. 1966-71 three-region system for the UK: transition probabilities out of East Anglia obtained by application of the interpolative variant of the Rees and Wilson methodology.

TRANSITION FROM EAST ANGLIA TO

Age	Death	East Anglia	Southeast	Rest of Britain
0	0.004847	0.885892	0.049473	0.059788
5	0.002741	0.903636	0.043019	0.050605
10	0.001570	0.934504	0.029850	0.034077
15	0.003455	0.892611	0.059059	0.044874
20	0.004027	0.827639	0.088218	0.080116
25	0.003673	0.850897	0.067778	0.077652
30	0.004264	0.891154	0.048599	0.055983
35	0.006460	0.913851	0.036025	0.043664
40	0.010805	0.932636	0.028193	0.028366
45	0.017901	0.935042	0.024943	0.022114
50	0.029437	0.938704	0.017059	0.014800
55	0.047517	0.932546	0.011127	0.008810
60	0.080658	0.897332	0.009587	0.012423
65	0.123571	0.852562	0.008718	0.015149
70	0.206893	0.769295	0.008757	0.015054
75	1.000000	0.000000	0.000000	0.000000

4. ESTIMATING THE SURVIVAL PROBABILITIES FROM MORTALITY RATES AND TRANSITION PROPORTIONS CONDITIONAL ON SURVIVAL

One difficulty common to the application of the "Option 2" method and to that of the Rees and Wilson averaging method and its interpolative variant relates to the prior availability of the set of survivorship proportions $\underset{\sim}{S}_x$ required as input.

In theory, the measurement of those survivorship proportions is performed from the following

$$ {}_{i}S_{x}^{j} = \frac{{}_{i}K_{x}^{j}(t-T,t)}{P_{x}^{i}(t-T)} \qquad (22) $$

where T is the length of the period over which changes of residence are observed

${}_{i}K_{x}^{j}(t-T,t)$ is the number of those aged x to x+n living in region j at the time of the census who were living in region i T years later

$P_x^i(t-T)$ is the number of those aged x to x+n who lived in region i T years before the census period.

In practice, however, the application of (22) is not possible because, T being generally different from the length of the intercensal period, the exact value of the denominator cannot be observed.

To circumvent this problem, Rees and Wilson (1977) have devised spatial demographic accounting methods which allow one to estimate the observed values of $\underset{\sim}{S}_x$ from the commonly available vital statistics and census migration flows. As already mentioned earlier, the observed survivorship proportions used in our UK example were obtained by Rees (1978) from one such method. The difficulty here is that Rees and Wilson's spatial demographic accounting methods for estimating the required data are based on lengthy iterative solutions whose operationalization can often be very time-consuming.

In view of this, it appears necessary to look for an alternative procedure or, more exactly, to amend the above estimation methods from survivorship proportions to make them applicable to the type of data commonly available, i.e., age-specific mortality rates from vital statistics and transition proportions conditional on survival from census information.

The general idea here is to first estimate a set of transition probabilities $\bar{\underset{\sim}{p}}_x$ conditional on survival from census migration data and then to transform them into the required set of transition probabilities by introducing independent mortality information. In analytic terms, such a procedure means that $\underset{\sim}{p}_x$ is obtained from

$$\underset{\sim}{p}_x = \bar{\underset{\sim}{p}}_x \, \underset{\sim}{p}_x^{\delta} \tag{23}$$

where $\underset{\sim}{p}_x^{\delta}$ is a diagonal matrix of survival probabilities whose elements are similar to the survival probabilities of the ordinary life table.

In practice, the implementation of this method appears as follows. First of all, one measures transition proportions ${}^{i}\bar{s}_x^{j}$ conditional on survival from

$$ {}^{i}\bar{s}_x^{j} = \frac{{}^{i}K_x^{j}(t-T,t)}{\sum_k {}^{i}K_x^{k}(t-T,t)} \tag{24}$$

Then one obtains a set of transition probabilities conditional on survival by applying Rees and Wilson's averaging formula

$$ \bar{\underset{\sim}{p}}_x = \frac{1}{2}\,[\bar{\underset{\sim}{s}}_{x-n} + \bar{\underset{\sim}{s}}_x] \quad , \tag{25}$$

or, even better, one uses the cubic spline interpolative procedure described in Section 3, to calculate the various migration probabilities ${}^{i}\bar{p}_x^{j}$ $(j \neq i)$ conditional on survival (in this latter case, the retention probabilities conditional on survival follow immediately from ${}^{i}\bar{p}_x^{i} = 1 - \sum_{j\neq i} {}^{i}\bar{p}_x^{j}$).

The next step is to estimate the set of survival probabilities $\underset{\sim}{p}_x^{\delta}$ assuming the availability of conventional age-specific mortality rates. Actually, this task is not at all straightforward for there is a price to pay for the simplicity of the amendment underlying (23): the mortality pattern is not a characteristic of the place of occurrence but one of the place of residence at the exact age $x = 0, n, 2n, \ldots$ immediately before the age at which death occurs.

For the purpose of this task, note that if the Markovian assumption holds, we have the following relationship between the basic inputs of the movement and transition approaches (assuming further the use of a linear integration method for calculating $\underset{\sim}{L}_x$):

$$ \underset{\sim}{p}_x = (\underset{\sim}{I} + \frac{n}{2}\underset{\sim}{M}_x)^{-1}(\underset{\sim}{I} - \frac{n}{2}\underset{\sim}{M}_x) = \bar{\underset{\sim}{p}}_x\,\underset{\sim}{p}_x^{\delta} \tag{26}$$

Premultiplying the two sides of this equality by $(\underset{\sim}{I} + \frac{n}{2} \underset{\sim}{M}_x)$ and rearranging the various terms leads to

$$\underset{\sim}{M}_x = \frac{2}{n} [\underset{\sim}{I} - \bar{\underset{\sim}{p}}_x \underset{\sim}{p}_x^{\delta}][\underset{\sim}{I} + \bar{\underset{\sim}{p}}_x \underset{\sim}{p}_x^{\delta}]^{-1} \tag{27}$$

Next, we premultiply this equality by a row vector of ones $\{i\}'$. Observing that $\{i\}'\underset{\sim}{M}_x$ is a row vector of conventional mortality rates and that $\{i\}'\bar{\underset{\sim}{p}}_x \underset{\sim}{p}_x^{\delta}$ is a row vector whose typical element is ${}^{i}p_x^{\delta}$ (i.e., the probability for someone living in region i at exact age x to die wihin n years) yields

$$\{M_x^{\delta}\}' = \frac{2}{n} [\{i\}' - \{p_x^{\delta}\}'][\underset{\sim}{I} + \bar{\underset{\sim}{p}}_x \underset{\sim}{p}_x^{\delta}]^{-1} \tag{28}$$

or, after transposing,

$$\{M_x^{\delta}\} = \frac{2}{n} [\underset{\sim}{I} + \underset{\sim}{p}_x^{\delta} \bar{\underset{\sim}{p}}_x']^{-1} [\{i\} - \{p_x^{\delta}\}] \tag{29}$$

where $\{M_x^{\delta}\}$ is a column vector of conventional mortality rates

$\{p_x^{\delta}\}$ is a vector identical to the diagonal of p_x

$\{i\}$ is a column vector of ones

$\bar{\underset{\sim}{p}}_x'$ is the transpose matrix of $\bar{\underset{\sim}{p}}_x$.

Further arrangement of the terms of (29) leads to

$$\{\underset{\sim}{p}_x^{\delta}\} = \{i\} - \frac{n}{2} [\underset{\sim}{I} + \underset{\sim}{p}_x^{\delta} \underset{\sim}{p}_x'] \{M_x^{\delta}\} \tag{30}$$

Such a relationship suggests that one can calculate $\{p_x^{\delta}\}$ (and thus $\underset{\sim}{p}_x^{\delta}$) using an iterative procedure. Owing to the assumptions above, we also have that

$$\underset{\sim}{p}_x^{\delta} = (\underset{\sim}{I} + \frac{n}{2} \hat{\underset{\sim}{M}}_x^{\delta})^{-1} (\underset{\sim}{I} - \frac{n}{2} \hat{\underset{\sim}{M}}_x^{\delta}) \tag{31}$$

where $\underset{\sim}{M}_x$ is a matrix of mortality rates dependent on the place of residence at age x rather than on the place of death occurrence. Thus, a first estimate of $\underset{\sim}{p}_x^{\delta}$ can be obtained from (31) by substituting the observed estimates of the conventional mortality rates for the non-conventional ones. An improved estimate is then arrived at by using this initial estimate on the right-hand side of (30). The procedure is continued until convergence.

The amended method proposed in this section was applied to the 1965-70 US four-region system (for females only) which we considered earlier in Section Two. Tables 7 and 8 set out the transition probabilities out of the Northeast region which were obtained by application of the Rees and Wilson averaging method and its interpolative variant to the transition proportions conditional on survival estimated from the 1970 census migration data. (Note that the transition probabilities for ages 70, 75 and 80 are identical for both methods; this follows from the fact that, owing to the lack of detailed migration data for older age groups, the same transition proportions conditional on survival are assumed to apply to the last four age groups, which leads one to suppose that $\bar{\underset{\sim}{p}}_x = \bar{\underset{\sim}{S}}_x$ for all $x \geq 70$.)

5. CALCULATING THE NUMBER OF PERSON-YEARS LIVED: AN ALTERNATIVE TO THE LINEAR APPROACH

In the methodology developed by Rogers (1973, 1975), the calculation of the number of person-years lived $\underset{\sim}{L}_x$, as defined by (8'), immediately follows from the basic assumption made to estimate $\underset{\sim}{p}_x$: $\underset{\sim}{L}_x$ is estimated linearly from

$$\underset{\sim}{L}_x = \frac{n}{2} [\underset{\sim}{l}_x + \underset{\sim}{l}_{x+n}] \tag{32}$$

Table 7. 1965-70 US four-region system (for females): transition probabilities out of the Northeast obtained by application of the amended Rees and Wilson methodology.

TRANSITION FROM NORTHEAST TO

Age	Death	Northeast	North Central	South	West
0	0.021337	0.922125	0.014246	0.029578	0.012714
5	0.001730	0.958456	0.010033	0.019944	0.009838
10	0.001371	0.960990	0.009803	0.019452	0.008383
15	0.002455	0.936379	0.016455	0.030903	0.013808
20	0.003059	0.916990	0.020860	0.038069	0.021023
25	0.003815	0.929887	0.016650	0.030788	0.018860
30	0.005753	0.946088	0.011832	0.023284	0.013043
35	0.009671	0.954921	0.008478	0.017944	0.008986
40	0.014232	0.958984	0.005816	0.014516	0.006451
45	0.021535	0.956521	0.004063	0.012625	0.005256
50	0.032077	0.947334	0.002945	0.013307	0.004337
55	0.046738	0.928145	0.002296	0.018383	0.004438
60	0.068860	0.900382	0.002371	0.023413	0.004974
65	0.109253	0.862737	0.002459	0.020907	0.004644
70	0.170487	0.812326	0.002743	0.010985	0.003458
75	0.269285	0.715575	0.002417	0.009677	0.003046
80	0.385941	0.601536	0.002031	0.008132	0.002560
85	1.000000	0.000000	0.000000	0.000000	0.000000

Table 8. 1965-70 US four-region system (for females): transition probabilities out of the Northeast obtained by application of the amended interpolative variant of the Rees and Wilson methodology.

TRANSITION FROM NORTHEAST TO

Age	Death	Northeast	North Central	South	West
0	0.021309	0.933405	0.011822	0.022313	0.011150
5	0.001730	0.957807	0.010046	0.020187	0.010230
10	0.001370	0.967751	0.007968	0.016176	0.006734
15	0.002455	0.935512	0.016948	0.031901	0.013184
20	0.003061	0.909021	0.022944	0.041585	0.023389
25	0.003815	0.930517	0.016346	0.030026	0.019296
30	0.005753	0.947124	0.011558	0.023024	0.012541
35	0.009671	0.955492	0.008426	0.017594	0.008816
40	0.014232	0.959685	0.005631	0.014308	0.006144
45	0.021535	0.956897	0.003966	0.012277	0.005326
50	0.032077	0.948594	0.002898	0.012337	0.004094
55	0.046738	0.928697	0.002127	0.018115	0.004322
60	0.068855	0.898498	0.002365	0.025095	0.005187
65	0.109250	0.862159	0.002571	0.021362	0.004658
70	0.170487	0.812326	0.002743	0.010985	0.003458
75	0.269285	0.715575	0.002417	0.009677	0.003046
80	0.385941	0.601336	0.002031	0.008132	0.002560
85	1.000000	0.000000	0.000000	0.000000	0.000000

This formula was applied to our US and UK examples on the basis of the transition probabilities derived with the original Rogers methodology and the resulting numbers of person-years lived were then converted into survivorship proportions using formula (12'). Table 9 shows the survivorship proportions out of the Northeast in the case of the US example, whereas Table 10 displays similar figures relating to the East Anglia region in relation to the UK example.

Of course, the linear integration method underlying (32) is not an exclusive feature of the Rogers methodology and can be applied anytime one has available a set of transition probabilities, regardless of the method of estimation used. In general, the accuracy of the generalized L-statistics thus obtained depends on the accuracy of the input data, but it also reflects:

(a) the adequacy of the method used for estimating $\underset{\sim}{p}_x$ and
(b) the validity of the linear integration assumption.

With regard to the latter, let us recall that the linear integration assumption is equivalent to an assumption that deaths and migratory moves are evenly distributed over time (Ledent 1978). Presumably, such an assumption which is acceptable for small intervals (up to n = 1 year) becomes less adequate as n increases. Thus, since the migration data commonly available relate to five-year age groups, there is a clear need to find a substitute to the linear integration method in order to strive for more realistic values of $\underset{\sim}{L}_x$. The method we propose here takes advantage of the definition of the survivorship proportions

$$\underset{\sim}{s}_x = \underset{\sim}{L}_{x+n} \underset{\sim}{L}_x^{-1} \qquad x = 0,n,\ldots,z-n \qquad (12')$$

as well as the definition of similar survivorship proportions for infants

Table 9. 1965-70 US four-region system (for females): survivorship proportions out of the Northeast obtained by application of the Rogers methodology.

TRANSITION FROM NORTHEAST TO

Age	Total	Northeast	North Central	South	West
-5	0.989665	0.964369	0.006505	0.012511	0.006279
0	0.988771	0.944382	0.011166	0.022254	0.010970
5	0.998483	0.961272	0.009649	0.019205	0.008358
10	0.998154	0.942010	0.015125	0.028307	0.012712
15	0.997362	0.927005	0.018406	0.033682	0.018270
20	0.996679	0.940564	0.014067	0.026114	0.015934
25	0.995337	0.950736	0.010937	0.021598	0.012066
30	0.992445	0.954268	0.009117	0.019379	0.009681
35	0.988236	0.958394	0.006500	0.016160	0.007182
40	0.982351	0.959840	0.004181	0.012933	0.005397
45	0.973521	0.953788	0.002839	0.012717	0.004177
50	0.961173	0.937811	0.002152	0.017078	0.004132
55	0.943259	0.915145	0.002154	0.021399	0.004561
60	0.912689	0.886865	0.002262	0.019293	0.004270
65	0.862869	0.845934	0.001950	0.011825	0.003160
70	0.785675	0.774772	0.001738	0.006970	0.002196
75	0.682801	0.672662	0.001618	0.006479	0.002041
80	0.923153	0.912899	0.001534	0.006567	0.002153

Table 10. 1966-71 three-region system for the UK: survivorship proportions out of East Anglia obtained by application of the Rogers methodology.

TRANSITION FROM EAST ANGLIA TO

Age	Total	East Anglia	Southeast	Rest of Britain
-5	0.990967	0.944592	0.021091	0.025284
0	0.990102	0.909141	0.037206	0.043755
5	0.998379	0.932866	0.032055	0.033458
10	0.997566	0.899966	0.053302	0.044298
15	0.996422	0.866227	0.068334	0.061861
20	0.996492	0.883066	0.054169	0.059257
25	0.996350	0.905407	0.042215	0.048728
30	0.994972	0.922082	0.033586	0.039305
35	0.991897	0.935961	0.027533	0.028404
40	0.985796	0.942544	0.022685	0.020568
45	0.977084	0.946949	0.016166	0.013969
50	0.962281	0.941050	0.011318	0.009913
55	0.937335	0.917226	0.008933	0.011177
60	0.898157	0.877609	0.007773	0.012775
65	0.843566	0.823839	0.007235	0.012493
70	1.702820	1.680883	0.008248	0.013690

$$\underset{\sim}{s}_{-n} = \frac{1}{n} \underset{\sim}{L}_0 \underset{\sim}{l}_0^{-1} \qquad (33)$$

Clearly, if the data available can be arranged in the form of survivorship proportions $\underset{\sim}{S}_x$ for $x = -n,0,n,\ldots,z-n$, it is then possible to calculate the set of $\underset{\sim}{L}_x$ by the repeated application of (12') rewritten as

$$\underset{\sim}{L}_{x+n} = \underset{\sim}{S}_x \underset{\sim}{L}_x \qquad x = 0,n,\ldots,z-n \qquad (34)$$

($\underset{\sim}{S}_x$ has been substituted for $\underset{\sim}{s}_x$) starting with

$$\underset{\sim}{L}_0 = n \underset{\sim}{S}_{-n} \underset{\sim}{l}_0 \qquad (35)$$

In other words, the method suggested here leads to numbers of person-years lived such that observed and life table survivorship proportions are equal*. Thus, if survivorship proportions can be observed, one can construct a certain number of multistate statistics ($\underset{\sim}{L}_x$ and the expectations of life at birth--which only depend on the generalized L-statistics) without first estimating the transition probabilities. The underlying methodology is identical to that of the "Option 2" method (see 2.1) but bypasses the first part (the estimation of $\underset{\sim}{p}_x$) which in any case has been shown inadequate.

The accuracy of the multistate statistics thus obtained appears to depend solely on the accuracy of the survivorship proportions as inputs: in contrast to the linear integration approach, it is not affected by methodological assumptions. Thus, if the estimated survivorship proportions for the UK

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*Clearly, the calculation of the life table survivorship proportions from the number of person-years lived derived as above gives back the observed survivorship proportions.

(Table 11 sets out those out of East Anglia) were truly observed, the comparison of the figures in Tables 10 and 11 would reveal the performance of the Rogers methodology. Unfortunately, this is not the case because the estimated survivorship proportions are affected by some estimation errors. In effect, the straightforward application of the above methodology yields too high values of the total expectations of life at birth in each region. The reason for this is the same as for the implementation of the Rees and Wilson methodology for estimating $\underset{\sim}{p}_x$: the mortality information does not come directly from vital statistics.

Table 11. 1966-71 three-region system for the UK: "observed" survivorship proportions out of East Anglia.

TRANSITION FROM EAST ANGLIA TO

Age	Total	East Anglia	Southeast	Rest of Britain
-5	0.983876	0.919681	0.034315	0.029880
0	0.995851	0.891759	0.047340	0.056752
5	0.998357	0.919878	0.036458	0.042021
10	0.997491	0.925044	0.037277	0.035171
15	0.996034	0.852528	0.080829	0.062677
20	0.996217	0.831966	0.079184	0.085067
25	0.996123	0.875416	0.056814	0.063893
30	0.994819	0.901070	0.042568	0.051181
35	0.991552	0.925356	0.031045	0.035150
40	0.985930	0.934494	0.026857	0.024579
45	0.976802	0.936020	0.021491	0.019291
50	0.962168	0.938152	0.013274	0.010742
55	0.937820	0.917330	0.010433	0.010056
60	0.897659	0.874207	0.008961	0.014491
65	0.841813	0.818163	0.008776	0.014874
70	1.526001	1.478805	0.018395	0.028802

In view of this unfortunate result as well as the difficulty of measuring or estimating actual survivorship proportions, it appears necessary to amend the above method of calculation so that it can be separately applied to mortality and migration information.

Of course, if one applies the above method to the available transition proportions conditional on survival, we are left with a set of number of person-years lived $\bar{L}_x$ unaffected by mortality. The problem then is one of transforming this set into yet another set accounting for mortality. This can be handled as follows:

(a) First, apply to the initial cohort l_0 the set of transition probabilities $\bar{p}_x$ conditional on survival to obtain a set of matrices $\bar{l}_x$ describing the evolution of the initial cohort in case of zero mortality. Then, calculate the matrices $\bar{L}_{(x)} = \bar{L}_x \bar{l}_x^{-1}$ to obtain "normalized" estimates of the numbers of person-years lived in case of zero mortality

(b) The next step consists of transforming the "normalized" matrices $\bar{L}_{(x)}$ in case of zero mortality into "normalized" matrices $L_{(x)}$ affected by mortality. In accordance with the method proposed in Section Four for transforming the set of conditional probabilities $\bar{p}_x$ into a set of unconditional probabilities p_x, this can be done simply by postmultiplying $\bar{L}_{(x)}$ by the following diagonal matrix: $\frac{I + P_x^{\delta}}{2}$ or equivalently $(I + \frac{n}{2} \hat{M}_x^{\delta})^{-1}$

(In the last group, the relevant matrix is $\hat{M}_z^{\delta -1}$.)

(c) Finally, postmultiplying each "normalized" matrix $L_{(x)}$ by l_x (obtained by application to the initial cohort l_0 the set of transition probabilities p_x) leads to the number of person-years lived L_x sought.

This amended methodology was applied to our US example using the conditional and unconditional transition probabilities obtained by application of the amended interpolative variant of Rees and Wilson's averaging method. The resulting numbers were then converted into survivorship proportions using formula (12'): Table 12 shows the survivorship proportions out of the Northeast. (Compare these with the corresponding survivorship proportions obtained with the Rogers methodology shown in Table 8.)

Table 12. 1965-70 US four-region system (for females): survivorship proportions out of the Northeast obtained by application of the alternative to the linear integration method.

		TRANSITION FROM NORTHEAST TO			
Age	Total	Northeast	North Central	South	West
-5	0.989346	0.953254	0.008830	0.019707	0.007555
0	0.968462	0.944519	0.011313	0.021741	0.010888
5	0.998454	0.963111	0.008676	0.017986	0.008682
10	0.998091	0.958176	0.010924	0.020911	0.008079
15	0.997249	0.914825	0.021987	0.040900	0.019537
20	0.996575	0.919082	0.019736	0.035248	0.022509
25	0.995230	0.940129	0.013562	0.026330	0.015208
30	0.992305	0.951093	0.010096	0.020241	0.010874
35	0.988067	0.958431	0.006861	0.015675	0.007100
40	0.982161	0.958225	0.004767	0.013376	0.005792
45	0.973258	0.953298	0.003355	0.011894	0.004710
50	0.960695	0.939537	0.002526	0.014689	0.003943
55	0.942443	0.913575	0.002048	0.021939	0.004881
60	0.911594	0.879386	0.002640	0.024591	0.004977
65	0.861705	0.838213	0.002224	0.017074	0.004194
70	0.784250	0.763148	0.002019	0.015318	0.003765
75	0.680549	0.662726	0.001745	0.012903	0.003175
80	0.917519	0.895993	0.002372	0.015311	0.003845

CONCLUSION

Focusing on the construction of increment-decrement life tables from the transition approach, this paper has attempted to provide an alternative to the Rogers (1973, 1975) methodology that is hampered by the difficulty of defining and measuring the migration rates required as inputs.

Initially the methodology sought was centered around the use of more familiar input data, namely survivorship proportions. An interpolative variant to the Rees and Wilson (1977) methodology for estimating transition probabilities was first provided. But the realization that errors in the measurement or estimation of the observed survivorship proportions (see the UK case) plus the impossibility, in most cases, of measuring or estimating such proportions led us to redirect the methodology so as to account separately for mortality and migration.

The final methodology that we propose can be summarized as follows:

(a) The estimation of the transition probabilities starts with the estimation of a set of transition probabilities conditional on survival from the migration data usually found in census results. The method used is an interpolative variant of Rees and Wilson's averaging method. Then, this set of conditional transitional probabilities is transformed into the set of required probabilities by application of a diagonal matrix of survival probabilities estimated from conventional vital statistics, as shown in Section Four.

(b) As for the calculation of the number of person-years lived, it is again performed in two steps. First, the transition proportions conditional on survival easily derived from the census migration data are used to estimate numbers of person-years lived unaffected by mortality. These numbers are then transformed as shown in Section Five to account for the effect of mortality.

The application of this methodology to the UK case yields results, some of which are shown in Tables 13 and 14. (Compare these with the figures shown in Tables 6 and 12 for an assessment of the inaccuracies to which the use of unconditional survivorship proportions leads.)

How does this new methodology compare with the Rogers methodology? Perhaps, the simplest way to evaluate their numerical differences is to contrast the expectations-of-life statistics to which they lead for both our US and UK examples. The figures set out in Table 15 indicate that:

(a) The new methodology yields slightly smaller total expectations of life (because, unlike the Rogers methodology, it does not include any constraint on the possibility of multiple moves).

Table 13. 1966-71 three-region system for the UK: transition probabilities out of East Anglia obtained by application of the generalized interpolative variant of the Rees and Wilson methodology.

TRANSITION FROM EAST ANGLIA TO

Age	Death	East Anglia	Southeast	Rest of Britain
0	0.019087	0.873235	0.048756	0.058921
5	0.001796	0.904472	0.043069	0.050663
10	0.001555	0.934548	0.029833	0.034064
15	0.003594	0.892466	0.059063	0.044877
20	0.004002	0.827675	0.088213	0.080110
25	0.003359	0.851161	0.067801	0.077679
30	0.004321	0.891097	0.048599	0.055983
35	0.006146	0.914139	0.036036	0.043679
40	0.010597	0.932827	0.028202	0.028374
45	0.018555	0.934398	0.024935	0.022111
50	0.028150	0.939949	0.017080	0.014821
55	0.048518	0.931563	0.011131	0.008788
60	0.079058	0.898869	0.009601	0.012473
65	0.129125	0.847237	0.008655	0.014983
70	0.191598	0.784056	0.008938	0.015408
75	1.000000	0.000000	0.000000	0.000000

Table 14. 1966-71 three-region system for the UK: survivorship proportions out of East Anglia obtained by application of the alternative to the linear integration method.

TRANSITION FROM EAST ANGLIA TO

Age	Total	East Anglia	Southeast	Rest of Britain
-5	0.990456	0.925833	0.034544	0.030079
0	0.989559	0.886051	0.047071	0.056438
5	0.998323	0.919848	0.036456	0.042019
10	0.997430	0.924983	0.037274	0.035172
15	0.996204	0.852672	0.080845	0.062687
20	0.996279	0.832050	0.079169	0.085060
25	0.996159	0.875449	0.056816	0.063894
30	0.994754	0.901024	0.042567	0.051163
35	0.991628	0.925435	0.031053	0.035141
40	0.985439	0.934034	0.026855	0.024549
45	0.976679	0.935915	0.021501	0.019263
50	0.961834	0.937805	0.013312	0.010717
55	0.936605	0.916128	0.010468	0.010009
60	0.896946	0.873506	0.009011	0.014429
65	0.841795	0.818145	0.008830	0.014820
70	1.698976	1.645539	0.019980	0.033456

Table 15. Contrasting the expectations of life at birth and their regional shares obtained by application of (a) the Rogers methodology and (b) the improved methodology proposed in this paper.

(a) ROGERS METHODOLOGY						(b) PROPOSED METHODOLOGY				
					A. United States (females)					
Total	1	2	3	4		Total	1	2	3	4
74.55	54.13	5.07	10.10	5.24	1. Northeast	74.41	52.67	5.49	10.74	5.52
74.43	3.76	52.16	10.48	8.04	2. North Central	74.26	4.24	50.02	11.57	8.43
74.40	5.06	7.88	54.54	6.93	3. South	74.25	5.77	8.72	52.10	7.65
75.57	3.89	7.94	11.32	52.42	4. West	75.32	4.60	9.12	12.62	48.99

B. United Kingdom

Total	1	2	3		Total	1	2	3
73.02	42.19	13.97	16.85	1. East Anglia	72.73	38.00	15.60	19.12
72.66	2.59	54.06	16.01	2. Southeast	72.48	2.71	52.17	17.60
71.76	1.26	8.27	62.23	3. Rest of Britain	71.71	1.36	8.66	61.69

(b) The new methodology leads to expected numbers of years spent in (outside) the region of birth which are smaller (greater) than the figures obtained with the Rogers methodology (this is consistent with our earlier findings on migration probabilities). For example, on the basis of the methodology proposed here, the number of years that a female born in the West of the United States can expect to spend in that region is 48.99 years (out of a total of 75.32 years) versus 52.42 years as obtained with the alternative methodology.

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