Abstract

We propose a model to investigate the dynamics of fashion traits purely driven by social interactions. We assume that people adapt their style to maximize social success and we describe the interaction as a repeated group game in which the payoffs reflect the social norms dictated by fashion. On one hand, the tendency to imitate the trendy stereotypes opposed to the tendency to diverge from them to proclaim identity; on the other hand, the exploitation of sex appeal for dating success opposed to the moral principles of the society. These opposing forces promote diversity in fashion traits, as predicted by the modeling framework of Adaptive Dynamics. Our results link the so-called horizontal dynamics—the primary driver of fashion evolution, compared with the vertical dynamics accounting for inter-class and economic drivers—to style variety.

Keywords: Adaptive Dynamics, diversity, game theory, evolutionary branching, fashion, replicator equation, social interactions.

1 Introduction

The dynamics of fashion traits has attracted much attention in the last century [Blumer, 1969, Simmel, 1904, Sproles, 1979, 1985, Veblen, 1894]. The several driving forces of fashion are best described by Sproles [1985]: "*Psychologists speak of fashion as the seeking of individuality; sociologists see class competition and social conformity to norms of dress; economists see a pursuit of the scarce; aestheticians view the artistic components and ideals of beauty; historians offer evolutionary explanations for changes in design. Literally hundreds of viewpoints unfold, from a literature more immense than for any phenomenon of consumer behavior.*" Changes in fashion traits have been documented since the XVIII century [Lowe and Lowe, 1990, Richardson and Kroeber, 1940, Robinson, 1976, Sproles, 1981, Weeden, 1977, Young, 1937] and many studies tried to formally interpret and model fashion dynamics, from both the empirical [Lowe, 1993, Lowe and Lowe, 1982, 1983] and conceptual points of view [Caulkins et al., 2007, Miller et al., 1993, Pesendorfer, 1995]. But "*The current state of fashion theory includes a loosely organized array of descriptive principles and propositions but is not formalized in that it does not specify a detailed structure of concepts, variables, and relations*" [Sproles, 1981].

We focus on the evolution of fashion traits that emerges from pure personal choice driven by social interactions, the so-called *horizontal dynamics* in the *trickle-across* [Field, 1970, Robinson, 1976, Simmel, 1904] and *trickle-up* theories—"*It now appears that some fashions, as well as some analogous nonfashion phenomena, climb the status pyramid from below, trickling up, as it were*" [Field, 1970]. We intentionally do not consider all *vertical* drivers of fashion, related to social class differences among consumers and the tendency to emulate stereotypes from higher classes (investigated in the *trickle-down* theory [Simmel, 1904, Veblen, 1894]) and to economic aspects from both the production (business, marketing, design, and manufacturing) and the consumer (budget) sides. This is in line with the view of Blumer [1969], who considers intra-class social interactions (the horizontal dynamics) dominant with respect to (vertical) inter-class and economic drivers. "*The fashion mechanism appears not in response to a need of class differentiation and class emulation but in response to a wish to be in fashion, to be abreast of what has good standing, to express new tastes which are emerging in a changing world*" [Blumer, 1969]. Change in fashion traits is for Blumer the result of "*the gradual formation and refinement of collective tastes, which occur through social interaction among people with similar interests and social experience, with the result that many people develop tastes in common.*"

Although Blumer's view is debatable—the dominance between horizontal and vertical dynamics certainly depends on the particular fashion sector considered—inter-class and economic drivers are important aspects of fashion and, as such, they have been incorporated in conceptual models [Caulkins et al., 2007, Pesendorfer, 1995], lacking however detail on the horizontal aspects. Here we do not consider the vertical drivers because our aim is to model and study the horizontal dynamics of fashion in isolation. Thus, our model directly applies to situations in which individuals are free to adapt and innovate their own style of dress, appearance, and/or behavior, and they do so in response to social interactions, rather than to strategic marketing campaigns. The closest analog in the literature to our investigation is Miller's et al. [1993] innovative paper, in which individuals adapt their style to maximize an utility based on a value linked to the style and on the reputation and style of the people in the individual's social network.

Specifically, we want to assess whether the social interaction between common-class people can be responsible of the emergence of variety in fashion traits. To this endeavour, we see Evolutionary Game Theory (EGT) [Hofbauer and Sigmund, 1998, Maynard Smith, 1982] and Adaptive Dynamics (AD) [Dercole and Rinaldi, 2008, Geritz et al., 1997, 1998, Metz et al., 1996]—two mathematical approaches borrowed from evolutionary biology—as the promising frameworks to model the evolution of social traits, fashion traits in particular.

We use the flexibility of game theory to model a dating game (as in Pesendorfer [1995]). This is the step where most modeling assumptions are made. A one-dimensional continuous trait is used (as in Miller et al. [1993] and Caulkins et al. [2007]) to abstractly describe the key stylistic attributes of consumer goods, and the strategy of an individual is identified by the adopted style. No business, marketing, and production aspects are considered, as well as no individual class differentiation. The social dynamics resulting from repeated rounds of the dating games are modeled with the standard replicator equation [Schuster and Sigmund, 1983, Taylor and Jonker, 1978] of EGT, a set of differential equations giving the change in time of the shares of a given set of coexisting styles. Social success is therefore measured by dating success [Barber, 1999] and the styles conferring the highest success will grow in share. EGT also provides the concepts and tools for discussing the robustness of a set of mainstream styles against invasion by an innovative variants. Sequences of successful small innovations can then accumulate and produce the gradual evolution of fashion envisaged by Blumer [1969]. AD exactly provides the modeling framework for this long-term evolution, including the endogenous proliferation of styles from common "classics"—evolutionary branching—and the endogenous elimination of obsolete styles—evolutionary extinction. Although AD has been primarily developed for application in evolutionary biology, innovation and competition play in social sciences the analogous role of genetic mutations and natural selection [Ziman, 2000].

The rest of the paper is organized as follows. Section 2 presents the analytical methods, the rules of social (dating) game (Sect. 2.1), the social replicator dynamics (Sect. 2.2), and the AD framework to describe fashion evolution (Sect. 2.3). Sect. 2.3 presents the model analysis, starting from a stylistic uniform society, and discusses the conditions for the diversification of fashion. Further discussion and comparison with the fashion literature and models closes the paper in Sect. 4.

2 Methods

We study a (technically infinite and well-mixed) society in which each individual has his/her own style, that we represent with a one-dimensional continuous trait or *strategy* x assumed to be positively related with the sex appeal of the adopted style. This is supported by many authors [Laver, 1937, Lurie, 1981, Steele, 1985, Veblen, 1894]. E.g., Steele [1985] argues: "*Because clothing is so intimately connected to the physical self, it automatically carries an erotic charge*," while Lurie [1981] applies psychoanalytic theory in the description of what is communicated by one's hats and umbrellas, women's handbags, men's walking sticks and ties. For example, x could measure the sizes of clothes (as in Lowe and Lowe [1990], who report fluctuations and variety in the skirt length of women's evening dresses over two centuries). For simplicity, x is assumed unbounded and can be interpreted as a physical measure through a suitable scaling.

Individuals with same style are grouped into sub-populations, which we assume to occur in finite number. Let S be the number of different styles in the society, $x_1 < x_2 < \cdots < x_S$ the traits representing the styles from the most austere to the most sexy, and $n_1, n_2, \ldots, n_S$ the corresponding fractions (or *frequencies*) of people adopting styles $x_1, x_2, \ldots, x_S$ $(n_1 + n_2 + \cdots + n_S = 1)$.

Individuals with different styles compete in their everyday life for their social success [Lowe and Lowe, 1983], here mainly focused on dating success [Barber, 1999]. We imagine that people repeatedly meet at social events involving a finite group of N randomly selected individuals and we indicate with $N_1, N_2, \ldots, N_S$ $(N_1 + N_2 + \cdots + N_S = N)$ the numbers of x_i-strategists in the selected group $i = 1, \ldots, S$. We evaluate their dating success as the expected payoff of an underlying N-player game, indicating with P_i the expected payoff for style i.

2.1 The social game

We assume the social payoff for strategy x_i to be the sum of four contributions.

- The payoff for being trendy. It is the advantage to conform to one of the mainstream styles established in the society [Efferson et al., 2008b]. We quantify it with the fraction of people in the society adopting a style similar to x_i, and we scale it through the *trendy payoff* τ. In formulas, we write

$$P_\tau(x_i) := \tau \sum_{j=1}^{S} n_j \exp(-\alpha(x_i - x_j)^2), \tag{1a}$$

  where the width of the Gaussian bell $\exp(-\alpha(x_i - x_j)^2)$, regulated by the (positive) parameter α, defines similarity. The similarity exponent α measures the *style sensitivity* of the society, i.e., how sensitive people are to differences in style. Although the sensitivity to style is rather personal and could be relative to the austere-sexy character of the style, we consider a sort of average value across the society. If α is sufficiently large, the Gaussian bell is narrow and the trendy payoff for strategy x_i is essentially scaled by the fraction of people adopting x_i. Independently of α, if everyone is adopting the same style x_1, then $P_\tau(x_1) = \tau$.

- The payoff for originality. It is the advantage to be distinct from the styles present at a social event, i.e., a gain/loss to be minority/majority [Esposito, 2011]. We quantify it by removing from $\frac{1}{2}$ the fraction of people in the social event adopting a style similar to x_i, and we scale it through the *originality payoff* ε. In formulas, we write

$$P_\varepsilon(x_i) := \varepsilon\Big(\tfrac{1}{2} - \sum_{j=1}^{S} \frac{N_j}{N} \exp(-\alpha(x_i - x_j)^2)\Big), \tag{1b}$$

where the Gaussian bell defining similarity is the same as in (1a). If α is sufficiently large, the originality payoff for strategy x_i is essentially scaled by $\frac{1}{2}$ minus the fraction of people adopting x_i. Independently of α, if style i is highly innovative (x_i far from all x_j, $j \neq i$) and adopted by one or a few individuals at a big event (large N), then $P_\varepsilon(x_i)$ maximizes at $\frac{1}{2}\,\varepsilon$. On the other hand, if everyone is adopting the same style the payoff drops to $-\frac{1}{2}\,\varepsilon$.

- The payoff for being sexy. It is a relative advantage for sexy vs. austere styles when competing for dating at a social event [Laver, 1937, Lurie, 1981, Steele, 1985, Veblen, 1894]. We quantify it by weighting the differences in strategy w.r.t. the competitors present at the social event, by the probabilities of interacting with such competitors, and we scale it through the *sexy payoff* σ. In formulas, we write

$$P_\sigma(x_i) := \sigma \sum_{j=1}^{S} (x_i - x_j) \frac{N_j}{N-1}. \tag{1c}$$

Note that the sexy payoff σ is defined per unit of style difference w.r.t. the competitor and that the probability of interaction with a same-style competitor is actually $(N_i-1)/(N-1)$, but this is irrelevant in (1c).

- The payoff for respecting morality. It is an absolute judgment on style, given by the morality codes uniformly accepted in the society [Lowe and Lowe, 1983]. It solely depends on the style adopted by an individual, and it is expressed as

$$P_\mu(x_i) := \mu\big(1 - \exp(\beta(x_i - x_0))\big), \tag{1d}$$

where μ is the *morality payoff* obtained by an extremely austere style ($x_i \to -\infty$) and x_0 represents a *morality threshold* somehow separating austere from sexy (immoral) styles. The payoff is positive for austere styles ($x_i < x_0$) and negative for sexy styles ($x_i > x_0$), with and exponential punishment towards highly immoral display. The morality exponent β is a sort of average *moral sensitivity* of the society.

The expected payoff P_i for a focal x_i-strategist attending a social event is then obtained by weighting the sum of the four above contributions over all possible compositions of the other $N-1$ attendants.

For example, for $S = 1$, there is a uniform style x_1 in the society ($n_1 = 1$) and each social event is composed by x_1-strategists. Their payoff is trivially given by

$$P_1 = \tau - \tfrac{1}{2}\,\varepsilon + \mu(1 - \exp(\beta(x_1 - x_0))), \tag{2}$$

where the payoff for being trendy is maximal (τ) and that for originality minimal ($-\frac{1}{2}\,\varepsilon$), whereas there is no advantage to be sexy all individuals being equal.

For $S = 2$, two-style society, the probability that a focal x_1-strategist attends a social event with N_2 x_2-strategists (and $N_1 - 1$ other x_1-strategists) is given by the binomial distribution $\binom{N-1}{N_2} n_1^{N-1-N_2} n_2^{N_2}$.

Similarly, a focal x_2-strategist attends a social event with N_1 x_1-strategists (and N_2-1 other x_2-strategists) with probability $\binom{N-1}{N_1} n_1^{N_1} n_2^{N-1-N_1}$. The expected payoff P_1 and $P2$ are therefore given by

$$P_1 = \sum_{N_2=0}^{N-1} \big(P_\tau(x_1) + P_\varepsilon(x_1) + P_\sigma(x_1) + P_\mu(x_1)\big) \binom{N-1}{N_2} n_1^{N-1-N_2} n_2^{N_2}, \tag{3a}$$

$$P_2 = \sum_{N_1=0}^{N-1} \big(P_\tau(x_2) + P_\varepsilon(x_2) + P_\sigma(x_2) + P_\mu(x_2)\big) \binom{N-1}{N_1} n_1^{N_1} n_2^{N-1-N_1}. \tag{3b}$$

Then, substituting the expressions (1a–d) into (3) and computing the resulting sums (see Appendix), we obtain the following expressions:

$$\begin{aligned} P_1 = {} & \tau\big(n_1 + n_2 \exp(-\alpha(x_1-x_2)^2)\big) + \varepsilon\big(\tfrac{1}{2} - \tfrac{1}{N} - \tfrac{N-1}{N} n_1 - \tfrac{N-1}{N} n_2 \exp(-\alpha(x_1-x_2)^2)\big) \\ & + \sigma n_2 (x_1 - x_2) + \mu\big(1 - \exp(\beta(x_1 - x_0))\big), \end{aligned} \tag{4a}$$

$$\begin{aligned} P_2 = {} & \tau\big(n_1 \exp(-\alpha(x_2-x_1)^2) + n_2\big) + \varepsilon\big(\tfrac{1}{2} - \tfrac{1}{N} - \tfrac{N-1}{N} n_1 \exp(-\alpha(x_2-x_1)^2) - \tfrac{N-1}{N} n_2\big) \\ & + \sigma n_1 (x_2 - x_1) + \mu\big(1 - \exp(\beta(x_2 - x_0))\big). \end{aligned} \tag{4b}$$

Note the originality terms in (4a), where the three contributions removed from $\frac{1}{2}$ are related to the focal individual ($1/N$), to the other same-style individuals, and to the other competing style.

The computation of the expected payoff P_i, $i = 1, \ldots, S$, for the general case of a S-style society ($S > 2$) is more involved and reported in Appendix. It essentially makes use of the multinomial probability distribution for the composition of the $N-1$ attendants of the social event met by the focal x_i-strategist. The result is

$$P_i = \tau \sum_{j=1}^{S} n_j \exp(-\alpha(x_i - x_j)^2) + \varepsilon\Big(\tfrac{1}{2} - \tfrac{1}{N} - \tfrac{N-1}{N} \sum_{j=1}^{S} n_j \exp(-\alpha(x_i - x_j)^2)\Big) + \sigma \sum_{j=1}^{S} n_j (x_i - x_j) + \mu\big(1 - \exp(\beta(x_i - x_0)\big), \tag{5}$$

which naturally generalizes P_1 and P_2 in eqs. (4a,b).

2.2 The social dynamics

We model the competition between the different styles present in the society with the standard replicator equation [Schuster and Sigmund, 1983, Taylor and Jonker, 1978]. It deterministically describes the change in time of the styles' frequencies resulting from repeated rounds of the game defined in Sect. 2.1, assuming that the rounds take place on a faster time scale. Being randomly selected for repeated social events, each individual with style x_i realizes, on average, a dating success quantified by the expected payoff P_i, and decides whether to change from style x_i to x_j with a probability proportional to the payoff difference $P_j - P_i$, $j = 1, \ldots, S$. Separating the *game time scale*—the time scale of everyday life—from the *social time scale* on which shifts from one style to another can be observed—typically from one season to the next—the replicator equation,

$$\tfrac{\mathrm{d}}{\mathrm{d}t} n_i(t) = n_i(t)(P_i - \bar{P}), \quad \bar{P} := \sum_{j=1}^{S} n_j(t) P_j, \quad i = 1, \ldots, S, \tag{6}$$

says that style i gains share at time t (i.e., $\dot{n}_i(t) > 0$) if the expected payoff P_i of the x_i-strategist is higher than the average payoff $\bar{P}$ in the society, where t here spans the social time scale.

Note that

$$\sum_{j=1}^{S} \tfrac{\mathrm{d}}{\mathrm{d}t} n_j(t) = \sum_{j=1}^{S} n_j(t) P_j - \bar{P} \sum_{j=1}^{S} n_j(t) = 0 \tag{7}$$

by definition of $\bar{P}$, so that the sum of the frequencies remains constant at 1.

The styles' frequencies converge, in the long run, to one of the attractors of the replicator eq. (6). Although the nonlinearity of the equation allows for nonstationary (periodic or chaotic) attractors, we focus on stable equilibria, at which $\mathrm{d}n_i/\mathrm{d}t = 0$ for all i. Only positive equilibria (i.e., $\bar{n}_i > 0$ for all i) are of interest, since negative frequencies make no sense and a reduced-order replicator eq. should be used if some of the styles are absent at the equilibrium. All styles gain the same average payoff $\bar{P}$ at a positive equilibrium.

Taking the constraint (7) into account, and denoting the equilibrium frequencies with an overbar, equilibria can be computed by solving

$$\left(P_i - P_{i+1}\right)\Big|_{\substack{n_j=\bar{n}_j, j=1,\dots,S \\ \bar{n}_S=1-\sum_{j=1}^{S-1}\bar{n}_j}} = 0, \quad i = 1,\dots,S-1, \tag{8}$$

for the unknowns $\bar{n}_1,\dots,\bar{n}_{S-1}$ and then setting $\bar{n}_S = 1-\sum_{j=1}^{S-1}\bar{n}_j$. The stability of equilibria can be checked by looking at the eigenvalues of the linearized dynamics in their vicinity (negative real parts of all but one eigenvalues implying stability—one eigenvalue is always null by the constraint (7)).

Finally, recall that the equilibrium frequencies are functions of the corresponding styles, that play in (6) the role of model parameters. Packing styles and frequencies in S-dimensional vectors, we compactly write

$$\mathrm{n} := (n_1,\dots,n_S) = \bar{\mathrm{n}}(\mathrm{x}) := (\bar{n}_1(\mathrm{x}),\dots,\bar{n}_S(\mathrm{x})), \quad \mathrm{x} := (x_1,\dots,x_S). \tag{9}$$

2.3 The fashion dynamics

Suppose S different mainstream styles steadily coexist in the society at a stable and positive *social equilibrium* (9) of the replicator eq. (6). If an innovative style x' is introduced by one or a few individuals in the society, i.e., with infinitesimal frequency n', the success or flop of the new style can be discussed by extending model (6), locally to the equilibrium (9), with the equation

$$\tfrac{\mathrm{d}}{\mathrm{d}t} n'(t) = n'(t)(P' - \bar{P}), \tag{10}$$

where P' is the expected payoff of the new style x' just after its introduction. Note that $\mathrm{n} = \bar{\mathrm{n}}(\mathrm{x})$ and $n' = 0$ is an equilibrium of the (locally) extended replicator equation (6, 10) and its instability/stability corresponds to the initial success/flop of x'.

The expected payoff P' can be computed using formula (5) as follows. Renaming x' and n' with x_{S+1} and n_{S+1}, we can in fact use (5) with $S + 1$ styles and $i = S + 1$ and note that the infinitesimal frequency n_{S+1} annihilates all last terms in the sums. That is

$$P' = \tau\sum_{j=1}^{S} n_j \exp(-\alpha(x'-x_j)^2) + \varepsilon\Big(\tfrac{1}{2} - \tfrac{1}{N} - \tfrac{N-1}{N}\sum_{j=1}^{S} n_j \exp(-\alpha(x'-x_j)^2)\Big) + \sigma\sum_{j=1}^{S} n_j(x'-x_j) + \mu\big(1-\exp(\beta(x'-x_0)\big). \tag{11}$$

For $S = 1$ (innovation in a uniform society) we have

$$P' = \tau\exp(-\alpha(x'-x_1)^2) + \varepsilon\big(\tfrac{1}{2} - \tfrac{1}{N} - \tfrac{N-1}{N}\exp(-\alpha(x'-x_1)^2)\big) + \sigma(x'-x_1) + \mu(1-\exp(\beta(x'-x_0))), \tag{12}$$

whereas for $S = 2$ (innovation in a two-style society) we have

$$\begin{aligned} P' &= \tau\big(n_1\exp(-\alpha(x'-x_1)^2) + n_2\exp(-\alpha(x'-x_2)^2)\big) \\ &+ \varepsilon\big(\tfrac{1}{2} - \tfrac{1}{N} - \tfrac{N-1}{N} n_1\exp(-\alpha(x'-x_1)^2) - \tfrac{N-1}{N} n_2\exp(-\alpha(x'-x_2)^2)\big) \\ &+ \sigma\big((x'-x_1)n_1 + (x'-x_2)n_2\big) + \mu\big(1-\exp(\beta(x'-x_0))\big). \end{aligned} \tag{13}$$

Once P' is computed, we can use eq. (10) to evaluate the initial (relative) growth rate $\mathrm{d}n'/\mathrm{d}t/n'$ of the innovation, locally to the social equilibrium (9). In biological terms, it gives the *invasion fitness* of the innovative style, that we indicate with

$$\lambda(\mathrm{x}, x') = \left(P' - \bar{P}\right)\big|_{\mathrm{n}=\bar{\mathrm{n}}(\mathrm{x})}. \tag{14}$$

Note that $\lambda(\mathrm{x}, x')$ depends only on the mainstream styles in vector x and on the innovative style x'. Technically, it is the eigenvalue associated to the equilibrium (9) (extended with $n' = 0$) along the direction of invasion. Positive/negative fitness implies the success/flop of the style, according to model (6, 10, 11).

Adaptive Dynamics [AD; Dercole and Rinaldi, 2008, Geritz et al., 1997, 1998, Metz et al., 1996] — originally developed for modeling the evolution of phenotypic traits in biology—is here used to describes the evolution of fashion, resulting from small stylistic innovations that replace the former mainstream styles. Assuming that innovations are small ensures a gradual (mathematically continuous) evolution of the mainstream traits, and this is envisaged in biology as well as in the context of fashion. For example, Blumer [1969] supports the historical continuity of fashion change, where new styles evolve from those previously established in the society. And Lowe and Lowe [1983] assume that fashion change is ruled by inertia (e.g., if skirts have been progressively rising for the last few years, they will continue to rise up to an extreme) and resistance to that motion (large year-to-year jumps in one direction create force back the other way).

AD further assumes that innovations are sufficiently rare on the social time scale. Although this is not strictly necessary [Meszéna et al., 2005], it keeps the AD-picture simple. It guaranties that the frequencies of the mainstream styles are close to the equilibrium (9) whenever an innovation occurs, and that the (globally) extended replicator dynamics (i.e., eq. (6) with S increased by one, $x_{S+1} := x'$, and $n_{S+1} := n'$) has time to converge to a new social equilibrium before the next innovation.

With respect to the locally extended replicator equation (6, 10, 11), the globally extended one describes the mainstream-innovative competition far from the equilibrium (9). It is the so-called *resident-invader* model of AD. Interestingly, it does not need to be analyzed, as invasion of an x_i-innovation under a nonvanishing *selection gradient*

$$s_i(\mathrm{x}) := \tfrac{\partial}{\partial x'} \lambda(\mathrm{x}, x')\big|_{x'=x_i} \tag{15}$$

has been shown to imply the substitution of the former mainstream style x_i by x' [Dercole and Rinaldi, 2008, Geritz, 2005, Meszéna et al., 2005]. That is, if the fitness (14) of the innovation—at first-order given by $s_i(\mathrm{x})\,(x' - x_i)$—is positive, then the resident-invader trajectory starting sufficiently close to equilibrium (9) and with arbitrarily small n_{S+1} converges to the new equilibrium at which $n_i = 0$, $n_{S+1} = \bar{n}_i(x_1, \dots, x_{i-1}, x', x_{i+1}, \dots, x_S)$, and $n_j = \bar{n}_j(x_1, \dots, x_{i-1}, x', x_{i+1}, \dots, x_S)$, $j \neq i$. Then, renaming x' with x_i and n_{S+1} with n_i, the society is back characterized by S mainstream styles $x_1, \dots, x_S$, with style i slightly changed.

Thus, as long as the selection gradients do not all vanish, fashion evolution proceeds by sequences of small successful innovations in the directions dictated by (15), $i = 1, \dots, S$. Unsuccessful innovations (i.e., those for which $s_i(\mathrm{x})\,(x' - x_i) < 0$) are obviously lost. In the limit of infinitesimal innovations, evolution become smooth and described by the so-called AD *canonical equation* [Champagnat et al., 2006, Dieckmann and Law, 1996]

$$\dot{x}_i = \tfrac{1}{2} \rho\, \bar{n}_i(\mathrm{x})\, s_i(\mathrm{x}), \tag{16}$$

where the dot-notation represents the time derivative on a slower *fashion time scale* (decades, centuries), the factor $\frac{1}{2}$ takes into account that half of the innovations are on average unsuccessful, and the constant ρ is proportional to the frequency and average breadth (the variance) of innovations. The AD canonical equation give the expected evolutionary path, averaging among all the possible innovations (see Dercole and Rinaldi [2008], Chap. 3, for a simplified comprehensive derivation).

The most interesting aspect of AD is to account for the evolution of diversity in the system. The number of coexisting styles increases through *evolutionary branching*—the diversification of two initially similar styles after the coexistence of an innovation with its mainstream generator—and is pruned by *evolutionary extinction*—the evolution of x toward a boundary in trait space at which some of the components of the social equilibrium $\bar{\mathrm{n}}(\mathrm{x})$ vanish, or the equilibrium itself stops to exist.

Mainstream-innovative coexistence can only occur in the vicinity of a *fashion equilibrium* $\bar{\mathrm{x}}$ at which all selection gradients (15), $i = 1, \dots, S$, vanish. It is possible, for an x_i-innovation under the condition

$$C_i := \tfrac{\partial^2}{\partial x \partial x'} \lambda(\mathrm{x}, x')\Big|_{x'=x_i, \mathrm{x}=\bar{\mathrm{x}}} < 0 \tag{17}$$

[Dercole and Geritz, 2016, Geritz, 2005, Meszéna et al., 2005], whereas the diversification of the two initially similar coexisting styles—and hence their establishment as distinct mainstream styles—occurs under the condition

$$B_i := \tfrac{\partial^2}{\partial x'^2} \lambda(\mathrm{x}, x')\Big|_{x'=x_i, \mathrm{x}=\bar{\mathrm{x}}} > 0 \tag{18}$$

[Geritz et al., 1997, 1998]. Specifically, under $B_i \neq 0$, the two similar coexixting styles are under opposite selection gradients. If $B_i > 0$, the fitness landscape is an upward parabola w.r.t. x' locally to $\mathrm{x} = \bar{\mathrm{x}}$—a fitness minimum—so that close to $\bar{\mathrm{x}}$ both more and less sexy innovations are favored in style i. After coexistence, assuming (without loss of generality) $x' > x_i$, $B_i > 0$ implies that more sexy innovations are favored in styles x', whereas the opposite occurs in style x_i. In the AD jargon, selection is said to be *disruptive*. Otherwise, $B_i < 0$, the fitness landscape is a downward x'-parabola locally to $\mathrm{x} = \bar{\mathrm{x}}$—a fitness maximum—so close to $\bar{\mathrm{x}}$ most innovations are rejected. If $C_i < 0$, coexistence is possible, however, innovative styles in between x_i and x' perform better, so that style i does not branch. Selection is in this case *stabilizing*, i.e., acting against diversification.

If branching is possible for several i, generically it develops only in the style with largest B_i (largest rate of initial divergence), the other incipient branchings being "missed" [Kisdi, 1999, Landi et al., 2013]. The fastest trait divergence in fact alters the society faced by the other pairs of similar coexisting styles, and typically breaks the conditions for their coexistence. If branching is not possible for any i, then the fashion equilibrium $\bar{\mathrm{x}}$ is a terminal point of fashion evolution.

Note that the coexistence and branching conditions (17) and (18) are only sufficient. In case C_i and/or B_i do vanish, the higher-order terms in the fitness expansion play a role. These degenerate situations can be studied in the spirit of bifurcation analysis [Della Rossa et al., 2015, Dercole and Geritz, 2016], but this is not discussed. Further note that condition (17) involves, by the chain rule, the x-derivative of the social equilibrium $\bar{\mathrm{n}}(\mathrm{x})$.

After evolutionary branching, the dynamics of fashion is described by an extended canonical equation, that can lead the new mainstream styles to a new fashion equilibrium and further branch. Similarly, after evolutionary extinction, the dynamics of fashion is described by a new canonical equation, this time with a reduced number of survived coevolving styles.

3 Results

Let's start the analysis of the model from a stylistic uniform society, with style x_1 and social equilibrium $\bar{n}_1(x_1) = 1$. The invasion fitness of an innovative style x' is obtained from (14), where P' and $\bar{P} = P_1$ are given by (12) and (2), respectively, i.e.,

$$\begin{aligned}\lambda(x_1, x') = \big(P' - P_1\big)\big|_{n_1=1} = {} & \tau\big(\exp(-\alpha(x'-x_1)^2) - 1\big) + \varepsilon\big(1 - \tfrac{1}{N} - \tfrac{N-1}{N}\exp(-\alpha(x'-x_1)^2)\big) \\ & + \sigma(x'-x_1) + \mu\big(\exp(\beta(x_1-x_0)) - \exp(\beta(x'-x_0))\big). \qquad (19)\end{aligned}$$

The style x_1 evolves according to eq. (16), that (setting $\rho = 2$) takes the following specific form:

$$\dot{x}_1 = s_1(x_1) := \sigma - \mu\beta\exp(\beta(x_1 - x_0)). \qquad (20)$$

The two driving forces regulating fashion dynamics in eq. (20) are sex appeal (σ) and morality ($\mu\beta$), respectively pushing for sexy and austere styles. They balance at the fashion equilibrium

$$\bar{x}_1 = x_0 + \tfrac{1}{\beta}\ln\big(\tfrac{\sigma}{\mu\beta}\big), \qquad (21)$$

defined by $s_1(\bar{x}_1) = 0$. Note that the trendy payoff τ and the originality payoff ε play no role in eq. (20). The reason is that the advantage to be trendy (τ) and that of being original (ε) are marginal when x' moves toward x_1 (their contributions to the fitness (19) vanish quadratically), whereas the advantage/disadvantage conferred by sex appeal and morality are dominant (their contributions to the fitness are linear with the difference $x' - x_1$). As we now see, the opposed pressures to be trendy and original drive fashion diversification.

The coexistence and branching conditions (17) and (18) for the mono-style equilibrium (21) become

$$C_1 = -2\alpha\big(\tfrac{N-1}{N}\varepsilon - \tau\big) \quad \text{and} \quad B_1 = 2\alpha\big(\tfrac{N-1}{N}\varepsilon - \tau\big) - \sigma\beta. \qquad (22)$$

If the originality payoff ε is large compared to the trendy payoff τ, both the coexistence ($C_1 < 0$) and branching ($B_1 > 0$) conditions are satisfied. Then, two similar styles (x_1, x_2) close to $(\bar{x}_1, \bar{x}_1)$ can coexist and further innovations in the two styles lead to their diversification. Note that if N is sufficiently large—big social games, e.g. those virtually played on social networks—the coexistence condition reduces to $\varepsilon > \tau$, whereas branching requires $2\alpha(\varepsilon - \tau)$ to overcome the stabilizing force $\sigma\beta$. If the advantage of being more sexy (σ) and that of being more austere (more properly measured by the morality exponent β rather than by the payoff μ for the extremely austere style) are too large, the fashion dynamics is strongly stabilized at $\bar{x}_1$ ($-\sigma\beta$ is indeed the eigenvalue of the equilibrium (21), i.e., $\mathrm{d}\dot{x}_1/\mathrm{d}x_1|_{x_1=\bar{x}_1}$) and this stabilizing force acts against branching. That is, the dynamic stability of the mono-style equilibrium (21)—its attractiveness for the mono-style canonical equation (20)—contributes to its *evolutionary stability*—the property of being uninvadable by small innovations [Dercole and Rinaldi, 2008, Geritz et al., 1997, 1998, Metz et al., 1996].

It is thus the need to proclaim identity and individual affirmation (ε) that fosters fashion variety. Moreover, after mainstream-innovative coexistence, fashion diversification is favored/disfavored in societies where people are more/less sensitive to style (large/small α). Although these results might seem unsurprising—ε and τ measuring the advantage and disadvantage of rarity—note that while a premium to rarity obviously favors the coexistence of different styles, their divergence by sequences of further innovations is less trivial. Indeed, after mainstream-innovative coexistence, innovative styles that are more sexy or more austere of both of the two coexisting ones do exploit originality better than innovations in between. All this is confirmed by the following analysis of the model for $S \geq 2$.

After branching, the society is characterized by two mainstream styles, x_1 and x_2, $\mathrm{x} = (x_1, x_2)$, coexisting at the social equilibrium

$$\bar{n}_1(\mathrm{x}) = \tfrac{1}{2}\left(1 + \frac{\sigma(x_2 - x_1) + \mu\big(\exp(\beta(x_1 - x0)) - \exp(\beta(x_2 - x0))\big)}{\tau\big(1 - \exp(-\alpha(x_1 - x_2)^2)\big) - \varepsilon\frac{N-1}{N}\big(1 - \exp(-\alpha(x_1 - x_2)^2)\big)}\right), \quad \bar{n}_2(\mathrm{x}) = 1 - \bar{n}_1(\mathrm{x}) \qquad (23)$$

(obtained by solving eq. (8), $i = 1$). The fitness of an innovative style x' similar to x_1 or x_2 can be computed from (14), where P' is given in (13) and $\bar{P} = n_1P_1 + n_2P_2$ with P_1 and P_2 in (4a,b).

The two coexisting styles coevolve according to a two-dimensional AD canonical equation (see eqs. (15) and (16), $i = 1, 2$), that is here not shown because long and not particularly easy to be interpreted. Its simulation, starting from $x_2 > x_1$ close to the mono-style fashion equilibrium $\bar{x}_1$ is however pictured in Fig. 1 (two-style society). The two initially similar styles further and further differentiate. They coexist by exploiting different niches of the social game, the (slight minority of) austere x_1-strategists (see the gray scale indicating the x_1- and x_2-frequencies) losing sex appeal but gaining in moral reputation and originality, compared with the (slight majority of) the sexy x_2-strategists. Eventually, the two-style fashion dynamics converge to an equilibrium at which both the austere and the sexy styles can branch (see the coexistence and branching conditions in the caption of Fig. 1). As generically expected, branching actually develops only in style 1 (with faster rate of initial divergence, $B_1 > B_2$).

After the second branching, the society is characterized by three mainstream styles, x_1, x_2, and x_3, $\mathrm{x} = (x_1, x_2, x_3)$. The frequencies $\bar{n}_1(\mathrm{x})$, $\bar{n}_2(\mathrm{x})$, $\bar{n}_3(\mathrm{x}) = 1 - \bar{n}_1(\mathrm{x}) - \bar{n}_2(\mathrm{x})$ of the corresponding social equilibrium can be computed analytically, solving eq. (8) for $i = 1, 2$, but the resulting expressions are long and not shown. The fitness of an innovative style x' similar to x_1, x_2, or x_3 can be computed from (14), where P' and $\bar{P} = n_1P_1 + n_2P_2 + n_3P_3$ can be obtained form eqs. (11) and (5) for $S = 3$.

The three coexisting styles coevolve according to a three-dimensional AD canonical equation (see eqs. (15) and (16), $i = 1, 2, 3$) (not shown). Its simulation, starting from $x_2 > x_1$ close to the equilibrium style $\bar{x}_1$ of the two-style society and $x_3 = \bar{x}_2$ is pictured in Fig. 1 (three-style society). The two initially similar styles differentiate, again with a (slight) minority of more austere strategists (x_1) exploiting morality and originality, with respect to a (slight) majority of more sexy strategists (though still austere, $x_2 < x_0 = 0$). Eventually, the three-style fashion dynamics converge to a fashion equilibrium at which branching is possible in all styles. Again branching occurs in style 1 (largest B_i, see caption), but this is not shown in the figure.

Note that upon convergence to the fashion equilibrium, the time at which branching is triggered is rather arbitrary (see the color transitions in Fig. 1). In reality, it is related to the time of occurrence of the innovation leading to branching.

Also note that styles close to x_0 return in vogue during the three-style society, after a long period characterized by two opposed mainstream styles far from x_0. Thus, recurrent diversification could also

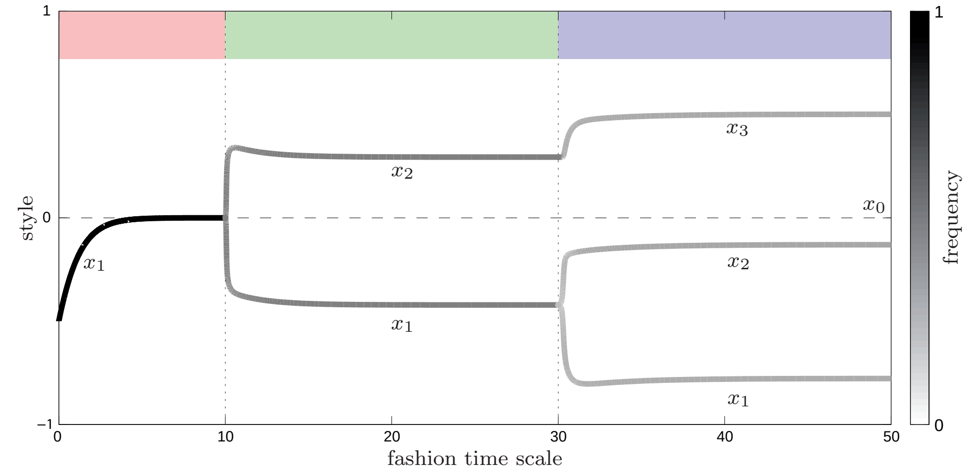

Figure 1: Simulated fashion evolution. Uniform style society (reddish): initial strategy $x_1(0) = -0.5$, equilibrium $\bar{x}_1 = 0$ (see (21)), coexistence and branching conditions $C_1 = -160$, $B_1 = 159$. Two-style society (greenish): initial strategy $(x_1(10), x_2(10)) = (\bar{x}_1 - 10^{-3}, \bar{x}_1 + 10^{-3})$, equilibrium $(\bar{x}_1, \bar{x}_2) = (-0.42, 0.29)$, coexistence and branching conditions $C_1 = -79.68$, $B_1 = 74.60$, $C_2 = -80.29$, $B_2 = 74.55$. Three-style society (bluish): initial strategy $(x_1(10), x_2(10), x_3(10)) = (\bar{x}_1 - 10^{-3}, \bar{x}_1 + 10^{-3}, \bar{x}_2)$, equilibrium $(\bar{x}_1, \bar{x}_2, \bar{x}_3) = (-0.78, -0.13, 0.50)$, coexistence and branching conditions $C_1 = -51.53$, $B_1 = 44.98$, $C_2 = -51.61$, $B_2 = 41.70$, $C_3 = -53.09$, $B_3 = 44.36$. Gray scale: styles' frequencies. Parameters: $\tau = \sigma = \mu = \beta = 1$, $\varepsilon = \alpha = N = 10$, $x_0 = 0$.

explain the revival of old-fashion styles.

Following the numerical procedure developed in [Landi et al., 2013], the conditions $C_i = 0$ and $B_i = 0$ can be continued in two-dimensional parameter spaces, producing the contour lines separating the regions in which the coexistence and branching discriminants, C_i and B_i, are respectively positive and negative. This can be done for the mono-style, as well as for the two-, three-, and S-style fashion equilibria, $S \geq 4$, and the result for different pairs of model parameters is shown in Fig. 2.

The top panels of the figure show the effect of the model parameters controlling the two social mechanisms responsible for branching, i.e., conformity and identity, as measured by the payoffs for being trendy and original. Panel a illustrates our main result. If the originality payoff ε is sufficiently larger than the trendy payoff τ, then style variety is fostered by the social interaction. The straight regions' boundaries, with almost 45-degree inclination, suggest that the difference between the two payoffs basically matters for branching. However, a careful inspection of the innovation fitness (14), and of the expression (11) of P' in particular, shows that $\left(\frac{N-1}{N}\varepsilon - \tau\right)$ is the quantity that actually matters, thanks to the use of the same similarity exponent α in eqs. (1a,b). For the first branching this is evident from the coexistence ($C_1 < 0$) and branching ($B_1 > 0$) conditions in (22), but the property remains true also for the further branchings and $\frac{N-1}{N}$ is actually the inclination coefficient of the regions' boundaries in panel a. This property is due to the fact that the conformity and identity mechanisms work at different scale in our model. The former at the scale of the whole society—because we assume the established mainstream styles to be globally known (by word of mouth or social and communication media)—the latter at the local scale of the social events. As a result, the effect of the originality payoff ε is scaled by $(N-1)/N$. As the scaling quickly saturates to one with N, the difference $\varepsilon - \tau$ is, in practice, the key parameter for branching when the game group size N is, say, at least 10 ($N = 10$ in panel a). This is confirmed in panel b, which shows how in small-group interactions branching requires a stronger unbalance between the conformity and identity premiums. Similarly, panel c shows that a stronger unbalance is required if people are less sensitive to differences in style (small α).

The bottom panels of Fig. 2 show the effect of the model parameters controlling the other two social mechanisms, sex appeal and morality, that are responsible of the stability (dynamic and evolutionary) of

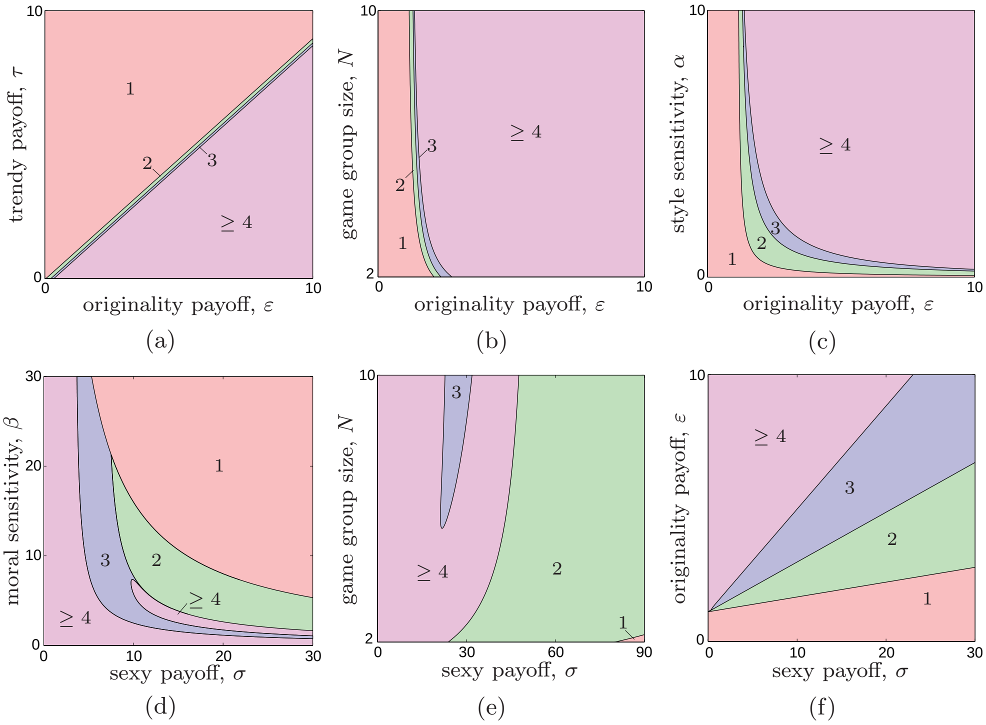

Figure 2: Regions in two-dimensional parameter spaces allowing the diversification of fashion. Starting from a stylistic uniform society, no branching is possible in region 1, whereas branching takes place and allows the coexistence of 2, 3, ≥ 4 different mainstream styles. Other parameters as in Figure 1.

the mono-style equilibrium. Panel d shows the effect of the two main parameters, the sexy payoff σ and the people's sensitivity β to morality. As suggested by the branching condition ($B_1 > 0$) in (22), the panel (with somehow hyperbolic regions' boundaries) confirms that the product $\sigma\beta$ is the relevant quantity for branching. Branching indeed develops under disruptive selection, that initially makes the two branching styles diverge, one becoming more sexy, the other more austere (see Fig. 1). Too large values of σ and β respectively oppose the development of the austere and sexy branches, and therefore prevent selection from being disruptive. Panel e shows how the group size N affects the stabilizing force of σ (a similar result holds for β). Finally, panel f considers the mixed effects of the original-trendy unbalance $\varepsilon - \tau$ and the stabilizing force $\sigma\beta$ (the effect of ε and σ at constant τ and β is actually shown). Consistently with the branching condition ($B_1 > 0$) in (22), the panel shows a proportionality between the effects of the two mechanisms in fostering and preventing branching, respectively. The disruptive force produced the seek of identity can then be counterbalanced by the relative advantages of sexy and austere styles. This is a non-trivial conclusion supported by our model.

The ranges of the parameter values used in Fig. 2 show all relevant regions (1, 2, 3, ≥ 4), see, e.g., the small region 1 (no branching) obtained in panel e for large sexy payoff σ in small-group games, indicating that the stabilizing force of σ must be very strong to prevent branching in a society with a significant premium for originality ($\varepsilon - \tau = 9$ from Fig. 1). As for the physical meaning of the numerical values, recall that payoffs measure the individual's dating success at repeated social events, so that they can be arbitrarily rescaled. In particular, recall that the sexy payoff σ is the payoff obtained per unit of style difference w.r.t. a competitor (see eq. (1c)). E.g., if x is the skirt length of women's evening dresses (as in Lowe and Lowe [1990]), $\sigma = 90$ compared with $\sigma = 1$ (see panel e) means that the obtained dating success is 90-times more sensitive to the skirt length. Another way to physically interpret the parameter values comes from the fitness definition in eq. (14). The fitness gives the initial rate of share increase of a successful innovation along the social time scale, and is obtained as the expected payoff surplus obtained by the innovative style compared with the average in the society. This shows how payoffs are numerically relative to the chosen time scale and offers an empirical way to identify payoff values from data in a specific context.

All panels in Fig. 2 suggest that a full cascade of branching is possible. We have checked this up to $S = 3$, i.e., going from region 3 to region ≥ 4 along with parameter changes, the three-style fashion equilibrium changes nature, from a terminal point of fashion dynamics to a branching point. Note that some of the regions' boundaries separate regions with non-consecutive S, e.g. regions 2-4 in panels d and e, meaning that crossing the 2-4 boundary the two- and three-style equilibria become both of branching type, or the three-style equilibrium is already so in region 2 close to the boundary, though not reachable by the fashion dynamics starting from a two-style society.

Further note that the effect of the morality payoff μ (as well as that of the morality threshold x_0) is not shown in Fig. 2, because there is no such effect. We already noted that μ does not enter the coexistence and branching conditions for the mono-style equilibrium (see (22)), and this is confirmed by the numerical analysis for the further branchings.

We close the analysis of the model with a technical note on the numerical analysis. When the expression for the social equilibrium $\bar{\mathrm{n}}(\mathrm{x})$ is not available, because the eqs. in (8) cannot be solved analytically, we need to solve numerically also for the derivatives of $\bar{n}_i(\mathrm{x})$ w.r.t. x_j, $i, j = 1, \ldots, S$, to evaluate the coexistence conditions C_i in (17) (see [Landi et al., 2013] for details).

4 Discussion

We have shown by means of a mathematical model that purely social interactions—the horizontal dynamics of fashion theory [Field, 1970, Robinson, 1976, Simmel, 1904]—promote fashion variety. We have intentionally omitted in the model all vertical drivers of fashion dynamics [Simmel, 1904, Veblen, 1894], i.e., the social aspects related to class differentiation and all economic aspects from both the production (business, marketing, design, and manufacturing) and the consumer (budget) sides.

We have considered a society of common-class people and we have linked the individuals' choice of style with their performance in a social game. The style is described by a one-dimensional continuous trait that represents, in agreement with many authors in the fashion literature [Laver, 1937, Lurie, 1981, Steele, 1985, Veblen, 1894], an abstraction of the sex appeal attributed to consumer goods. The share of different styles in the society is ruled by the replicator dynamics of EGT [Hofbauer and Sigmund, 1998, Maynard Smith, 1982], saying that the styles with higher expected payoff in the game have higher chances to be adopted, by imitation or in response to word-of-mouth diffusion. Stylistic innovations are introduced on a slower time scale by one or a few individuals and compete with the mainstream styles established in the society. AD [Dercole and Rinaldi, 2008, Geritz et al., 1997, 1998, Metz et al., 1996] deterministically describes the evolution of fashion resulting from sequences of successful small innovations, that accumulate in a macroscopic and continuous change of style. Remarkably, close to the stationary points of the fashion dynamics, mainstream and innovative styles can coexist under disruptive social selection. That is, further successful innovations of the two initially similar styles make them more and more distinct, a process called evolutionary branching that establishes the innovation as a new mainstream style in the society.

Our model is in line with the view of Blumer [1969], who consider fashion change as dominated by intra-class social interactions (the horizontal dynamics), rather than governed by (vertical) inter-class and economic drivers. Consistently with the social principles of fashion theory [Blumer, 1969, Lowe and Lowe, 1983, Sproles, 1985], we have taken the following four rewarding mechanisms into account in the social game: a payoff for being trendy, i.e., to conform to one of the established mainstream styles [Efferson et al., 2008b]; a payoff for being original, i.e., to proclaim identity being distinct from the mainstream styles [Esposito, 2011]; a payoff for being sexy in the competition for dating [Laver, 1937, Lurie, 1981, Steele, 1985, Veblen, 1894]; a payoff for respecting the morality codes uniformly accepted in the society [Lowe and Lowe, 1983]. The payoffs are measured in terms of dating success at a social event [Barber, 1999], attended by a group of randomly selected individuals.

Starting from a stylistic uniform society, we have formally shown that the tradeoff between sexy and austere styles stabilizes fashion evolution at an equilibrium style, that is as sexy/austere as large/small is the ratio of the corresponding payoffs. This is not surprising, but interestingly shows that the other cardinal social tradeoff of fashion, conformity vs. identity, plays no role in the evolution of a single style. The trendy and originality payoffs are, in contrast, the crucial parameters regulating evolutionary branching. Our main conclusion is that a relative advantage to be original vs. trendy fosters fashion variety. This result might also seem unsurprising, since a premium to minorities allows the coexistence between innovative and mainstream styles (no one can dominate the other), regardless of their relative sexiness (see the coexistence condition $C_1 < 0$ in (22)). However, that the premium is also responsible for the divergence of the two initially similar coexisting styles, as predicted by evolutionary branching (see the branching condition $B_1 > 0$ in (22)), is more remarkable. Moreover, the endogenous generation of different styles is obtained with a homogeneous model society, in which all individuals uniformly perceive, measure, and judge the influence of style in the social interaction. This simplifying assumption makes our main result even stronger.

For branching to occur, the unbalance between the trendy and originality payoffs must overcome the stabilizing forces of sex appeal and morality. Moreover, the coexistence of different styles is promoted in highly connected societies (large N) and, after mainstream-innovative coexistence, branching is more easily triggered if people are more sensitive to style (see the effect of a large α on $B_1 > 0$ under $C_1 < 0$ in (22)). And all the above results have been confirmed numerically starting from the two-style society developing after the first branching up to a 4-style society, suggesting that a cascade of branchings can generate a rich fashion variety in societies and fashion sectors where originality is highly considered.

Our model is intentionally minimal and at the same time general. It is aimed at answering the basic question on whether the sole social principles of fashion—conformity, identity, sexiness, and morality—can endogenously generate diversity of style. As such, it cannot be tailored to fit a specific historical example or used for quantitative prediction. Moreover, without considering inter-class and economic drivers, our model directly pertains to particular kind of fashion issue, e.g. men shaving style and, to a certain extent, women hairstyle, characterized by little budget constraints and individuals free to adapt and innovate their own style.

Perhaps the best validation of our model is the qualitative interpretation of the blooming of styles observed in the western societies in the 1990s [Evans, 2007] Most likely due to the economic recession (an economic driver), people in the 1990s were no longer used to follow fashion slavishly, a sharp contrast to the highly "a la mode" 1970s and 1980s [Steele, 2000]. Fashion in the 1990s was free around a new standard of minimalism, and styles of stark simplicity became the vogue. The anti-conformist approach to fashion led to the popularisation of the casual chic look, a trend which continued into the 2000s. From the 1970s to the 1990s people shifted value from conformity to identity (reduced trendy payoff τ and increased originality payoff ε) and this, according to our model, generated a variety of contrasting styles, from the most moral to the most libertine.

A variety of mainstream styles was also observed in the 1920s [Blum, 1981], compared to the uniform and formal style of the XIX century. On the wave of optimism brought by the end of the war, social customs and morals were indeed relaxed and the mood became more informal and youthful (lower μ and β and higher σ and ε). However, the conformism to the mainstream styles vertically dictated by the fashion designers and the high class remains significant (large τ), so it is questionable whether the variety of style was vertically or horizontally generated. Similarly, the prosperity developed in the western societies after World War II generated a variety of different styles (e.g. in women skirt length, from the mini-skirt to knee- and ankle-lengths, see e.g. [Buckley and Fawcett, 2001, Steele, 2000], compared with the uniform and sober clothing before and during the war (reflecting the economic situation and political regimes). This is however again a mix of vertical and horizontal drivers.

Additionally, fashion trends throughout the 1990s recycled styles from previous decades [Evans, 2007], notably the 1950s, 1960s and 1970s [Laver et al., 2002, Mendes and de la Haye, 1999, Tortora and Eubank, 2010], and this is also shown by our model. We have found no fashion cycles, in the strict sense of periodic solutions of the AD canonical equation (16), but our simulations indeed show that recurrent diversification is also accompanied by the return in vogue of old-fashioned (e.g. vintage) styles. Although fashion cycles are often thought to be vertically dictated [Caulkins et al., 2007, Sproles, 1981]—as recurrent innovations put forward by fashion producers [Pesendorfer, 1995]—or due to a broader external, e.g. cultural, change [Frings, 1999, Lowe and Lowe, 1990], a periodic evolution of fashion could be possible under particular horizontal hypotheses (as envisaged in Young [1937] and by Miller et al. [1993]). And even a never-ending and always new chaotic evolution [Dercole and Rinaldi, 2010, Dercole et al., 2010a] could be found with different hypotheses on the social game. Or even cycles of recurrent branching and extinction [Dercole, 2003]—evolution first leads to a branching point, then drives one of the two developing style to extinction—could best describe the evolution of some fashion sectors. E.g., when the seek for originality separates a neutral style into two sexy and austere alternatives, and further innovation makes the sexy style so provocative up to the collapse under the morality judgment. But all this is left for further research.

Back to our modeling assumptions, we find the basic assumptions of AD consistent with the classical hypotheses on fashion dynamics. Specifically, the assumption of small innovations and the smooth fashion dynamics provided by the AD canonical equation are in line with the views of Blumer [1969] and Lowe and Lowe [1983], who see fashion change as a gradual and continuous process, in which new fashions evolve from those previously established in the society. In contrast, the assumption of rare innovations—formally needed to separate the social time scale of everyday life from the time scale of fashion evolution—might not well fit some fashion sectors, e.g. those related to consumer hi-tech, but can be relaxed [Meszéna et al., 2005]. And even when large innovations do occasionally occur, the AD framework can handle them. The resident-invader model (the globally extended replicator eq. (6) with $S+1$ styles, the last being the invader) can be used to establish the fate of the innovation—whether it replaces some of the mainstream styles or coexists with them—and once the new social equilibrium is found, a new phase of gradual innovation can be described by the corresponding canonical equation. Fashion evolution is hence described by phases of gradual evolution punctuated by major breakthroughs.

Alternatively, one can use an individual-based stochastic approach, as originally done in the context of horizontal fashion dynamics by Miller et al. [1993]. Miller's et al. model considers an explicit social network and divides the social peers of each individual into two classes based on (static) reputation. With synchronous discrete transitions, each individual adapts his/her own style by maximizing an utility based on a personal value given to the style and on the tendency to emulate/diversify from respected/disrespected neighbors. In contrast, our model is population based (a mean-field for the all-to-all network), class-unstructured, and explicitly describes the social mechanisms of the interaction. While Miller's et al. find both uniform and heterogeneous solutions, style variety is a priori induced by the class division according to reputation and by the locality of the interaction, whereas the emergence of style variety through evolutionary branching is endogenous in our model. Miller's et al. also find cyclical behaviors, supporting the idea that purely social interactions might sustain fashion cycles.

By a sort of continuity argument, our model can be realistically applied to describe fashion phenomena in which the vertical inter-class and economic drivers are sub-dominant with respect to the social interactions [Blumer, 1969] (e.g. the above interpretation of style branching in the 1920s and after World War II up to the 1980s) [Buckley and Fawcett, 2001, Laver et al., 2002, Mendes and de la Haye, 1999, Tortora and Eubank, 2010]. On the other hand, our results have a speculative value in situations in which the vertical drivers cannot be neglected, in the sense of showing what would be the scenario were the vertical drivers absent, or the significance of the vertical drivers were the predicted horizontal scenario different from the one observed. E.g., a monopolistic producer could decide to inhibit fashion variety in a situation in which our model suggests branching.

Whether the vertical inter-class and economic drivers significantly affect the dynamics of fashion or not is debatable [Blumer, 1969, Lowe and Lowe, 1990, Miller et al., 1993, Simmel, 1904, Sproles, 1985]. In any case, our model provides a mechanistic way of describing the social interactions that are relevant for fashion, a low level block on the top of which the class and the economic levels could be vertically added. In a monopoly [Pesendorfer, 1995], in a duopoly [Caulkins et al., 2007], or in a market with several producers, a fashion maker could introduce (costly) innovations to steer the population in a direction that is good for his/her own business. And the competition between different producers could be modeled as a strategic business game, where the producers' strategies could evolve, in response to strategic innovations, and perhaps lead to branching and/or extinctions of fashion makers.

Another natural extension for our social level is that of considering the structure of the social network. All-to-all connectivity is a simplification that gives compact, mean-field, population models. Although the large and increasing connectivity of the hi-tech age seems to shift the interaction model to all-to-all and large group games, there is, at the same time, increasing recognition that the population structure can critically affect functioning in EGT (see e.g. [Débarre et al., 2014, Tan et al., 2014]). With an explicit network structure, the model becomes individual based, with rounds of the social game played within groups of neighbors extracted in the network graph. The challenge is to derive compact mean-field models, with particular attention to the model parameters describing the network structure. And the exogenous and/or the endogenous dynamics of the network is another important aspect to be taken into account. This will be the hot topic of research in EGT for the next years. Specifically for the social aspects of fashion, interesting positive (as well as negative) network externalities, based on the number of neighbors who are consuming a particular brand or product, could be considered, in the spirit of Miller's et al. [1993] reputation mechanism.

Finally, our work enlarges the number of studies that make use of the evolutionary paradigm outside biology, to study, just to mention a few, social [Axelrod and Hamilton, 1981, Mesoudi, 2009, Shennan, 2001], economic [Henrich and Boyd, 2008], technological [Dercole et al., 2008, 2010b], cultural [Boyd and Richerson, 1988, Efferson et al., 2008a, Foley and Mirazón Lahr, 2011, Henrich and McElreath, 2003], linguistic [Nowak and Krakauer, 1999], and religious [Atran and Henrich, 2010, Doebeli and Ispolatov, 2010] evolution and diversification through human history [Mesoudi, 2011, Richerson and Christiansen, 2013], with mutual benefits of the various social science disciplines.

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A Computation of the expected payoffs

The expressions (4a,b) of the expected payoffs P_1 and P_2 in a two-style society are obtained from (3a,b) by exploiting the properties of the binomial distribution. Specifically, for a focal x_1-strategist, the probabilities to meet with $N_2 = 0, 1, \ldots, N-1$ x_2-strategists (and $N_1 - 1 = N - 1 - N_2$ other x_1-strategists) sum to one, i.e.

$$\sum_{N_2=0}^{N-1} \tbinom{N-1}{N_2} n_1^{N-1-N_2} n_2^{N_2} = (n_1 + n_2)^{N-1} = 1, \tag{A.1}$$

and the average numbers of x_2- and x_1-strategists in the group of N players are given by

$$\sum_{N_2=0}^{N-1} N_2 \tbinom{N-1}{N_2} n_1^{N-1-N_2} n_2^{N_2} = (N-1) n_2 \underbrace{\sum_{N_2=1}^{N-1} \tbinom{N-2}{N_2-1} n_1^{N-2-(N_2-1)} n_2^{N_2-1}}_{(n_1+n_2)^{N-2}} = (N-1) n_2 \tag{A.2a}$$

and

$$\sum_{N_2=0}^{N-1} \underbrace{N_1}_{N-N_2} \tbinom{N-1}{N_2} n_1^{N-1-N_2} n_2^{N_2} = N - (N-1) n_2 = 1 + (N-1)(1-n_2) = 1 + (N-1) n_1. \tag{A.2b}$$

Then, substituting the expressions of $P_\tau(x_1)$, $P_\varepsilon(x_1)$, $P_\sigma(x_1)$, $P_\mu(x_1)$ from (1a–d) with $S = 2$ into (3a) and separately collecting the trendy and morality payoffs—independent of N_1 and N_2—from the originality and sexy payoffs—which are linear in N_1 and N_2—we get

$$\begin{aligned} P_1 \;=\; & \big(P_\tau(x_1) + P_\mu(x_1)\big) \sum_{N_2=0}^{N-1} \tbinom{N-1}{N_2} n_1^{N-1-N_2} n_2^{N_2} \\ & + \sum_{N_2=0}^{N-1} \Big(\varepsilon\big(\tfrac{1}{2} - \tfrac{N_1}{N} - \tfrac{N_2}{N} \exp(-\alpha(x_1 - x_2)^2)\big) + \sigma(x_1 - x_2)\tfrac{N_2}{N-1}\Big) \tbinom{N-1}{N_2} n_1^{N-1-N_2} n_2^{N_2} \\ = \;& \tau\big(n_1 + n_2 \exp(-\alpha(x_1 - x_2)^2)\big) + \mu(1 - \exp(\beta(x_1 - x_0))) \\ & + \varepsilon\big(\tfrac{1}{2} - \tfrac{1}{N} - \tfrac{N-1}{N} n_1 - \tfrac{N-1}{N} n_2 \exp(-\alpha(x_1 - x_2)^2)\big) + \sigma n_2 (x_1 - x_2), \end{aligned} \tag{A.3}$$

which coincides with (4a).

Similarly for a focal x_2-strategist, we have the binomial properties

$$\sum_{N_1=0}^{N-1} \tbinom{N-1}{N_1} n_1^{N_1} n_2^{N-1-N_1} = (n_1 + n_2)^{N-1} = 1, \tag{A.4}$$

$$\sum_{N_1=0}^{N-1} N_1 \tbinom{N-1}{N_1} n_1^{N_1} n_2^{N-1-N_1} = (N-1) n_1 \underbrace{\sum_{N_1=1}^{N-1} \tbinom{N-2}{N_1-1} n_1^{N_1-1} n_2^{N-2-(N_1-1)}}_{(n_1+n_2)^{N-2}} = (N-1) n_1, \tag{A.5a}$$

and

$$\sum_{N_1=0}^{N-1} \underbrace{N_2}_{N-N_1} \tbinom{N-1}{N_1} n_1^{N_1} n_2^{N-1-N_1} = N - (N-1) n_1 = 1 + (N-1)(1-n_1) = 1 + (N-1) n_2, \tag{A.5b}$$

and the expected payoff

$$\begin{aligned} P_2 \;=\; & \big(P_\tau(x_2) + P_\mu(x_2)\big) \sum_{N_1=0}^{N-1} \tbinom{N-1}{N_1} n_1^{N_1} n_2^{N-1-N_1} \\ & + \sum_{N_1=0}^{N-1} \Big(\varepsilon\big(\tfrac{1}{2} - \tfrac{N_1}{N} \exp(-\alpha(x_2 - x_1)^2) - \tfrac{N_2}{N}\big) + \sigma(x_2 - x_1)\tfrac{N_1}{N-1}\Big) \tbinom{N-1}{N_1} n_1^{N_1} n_2^{N-1-N_1} \\ = \;& \tau\big(n_1 \exp(-\alpha(x_2 - x_1)^2) + n_2\big) + \mu(1 - \exp(\beta(x_2 - x_0))) \\ & + \varepsilon\big(\tfrac{1}{2} - \tfrac{N-1}{N} n_1 \exp(-\alpha(x_2 - x_1)^2) - \tfrac{1}{N} - \tfrac{N-1}{N} n_2\big) + \sigma n_1 (x_2 - x_1), \end{aligned} \tag{A.6}$$

which coincides with (4b).

The proof of the general formula (5) for P_i in a S-style society is fully analogous. The expected payoff P_i of a focal x_i-strategist is given by

$$P_i = \sum_{\substack{N_1+\cdots+N_S=N \\ N_i\geq 1}} \big(P_\tau(x_i) + P_\varepsilon(x_i) + P_\sigma(x_i) + P_\mu(x_i)\big)\,\mathrm{Prob}[N_1,\ldots,N_i-1,\ldots,N_S], \tag{A.7a}$$

where N_j counts the number of x_j-strategists in the group of N players (including the focal one for $j=i$) and

$$\mathrm{Prob}[N_1,\ldots,N_i-1,\ldots,N_S] := \frac{(N-1)!}{N_1!\cdots(N_i-1)!\cdots N_S!}\, n_1^{N_1}\cdots n_i^{N_i-1}\cdots n_S^{N_S} \tag{A.7b}$$

is the multinomial probability of extracting the $(N_1,\ldots,N_i-1,\ldots,N_S)$-composition for the $N-1$ non-focal players.

According to the properties of the multinomial distribution, we have

$$\sum_{\substack{N_1+\cdots+N_S=N \\ N_i\geq 1}} \tfrac{(N-1)!}{N_1!\cdots(N_i-1)!\cdots N_S!}\, n_1^{N_1}\cdots n_i^{N_i-1}\cdots n_S^{N_S} = (n_1+\cdots+n_S)^{N-1} = 1, \tag{A.8}$$

$$\begin{aligned}
&\sum_{\substack{N_1+\cdots+N_S=N \\ N_i\geq 1}} N_j\, \tfrac{(N-1)!}{N_1!\cdots(N_i-1)!\cdots N_S!}\, n_1^{N_1}\cdots n_i^{N_i-1}\cdots n_S^{N_S} \\
&= (N-1)n_j \underbrace{\sum_{\substack{N_1+\cdots+N_S=N \\ N_i,N_j\geq 1}} \tfrac{(N-2)!}{N_1!\cdots(N_i-1)!\cdots(N_j-1)!\cdots N_S!}\, n_1^{N_1}\cdots n_i^{N_i-1}\cdots n_j^{N_j-1}\cdots n_S^{N_S}}_{(n_1+\cdots+n_S)^{N-2}} = (N-1)n_j,\ j\neq i,
\end{aligned} \tag{A.9a}$$

and

$$\begin{aligned}
&\sum_{\substack{N_1+\cdots+N_S=N \\ N_i\geq 1}} \underbrace{N_i}_{N-\sum\limits_{\substack{j=1 \\ j\neq i}}^{S} N_j} \tfrac{(N-1)!}{N_1!\cdots(N_i-1)!\cdots N_S!}\, n_1^{N_1}\cdots n_i^{N_i-1}\cdots n_S^{N_S} \\
&= N - \sum_{\substack{j=1 \\ j\neq i}}^{S}(N-1)n_j = 1 + (N-1)\Big(1 - \sum_{\substack{j=1 \\ j\neq i}}^{S} n_j\Big) = 1 + (N-1)n_i.
\end{aligned} \tag{A.9b}$$

Then, substituting the expressions of $P_\tau(x_1)$, $P_\varepsilon(x_1)$, $P_\sigma(x_1)$, $P_\mu(x_1)$ from (1a–d) into (A.7a) and separately collecting the trendy and morality payoffs from the originality and sexy ones, we get

$$\begin{aligned}
P_i &= \big(P_\tau(x_i) + P_\mu(x_i)\big) \sum_{\substack{N_1+\cdots+N_S=N \\ N_i\geq 1}} \tfrac{(N-1)!}{N_1!\cdots(N_i-1)!\cdots N_S!}\, n_1^{N_1}\cdots n_i^{N_i-1}\cdots n_S^{N_S} \\
&\quad + \sum_{j=1}^{S} \Big(\varepsilon\big(\tfrac{1}{2} - \tfrac{N_j}{N}\exp(-\alpha(x_i-x_j)^2)\big) + \sigma(x_i-x_j)\tfrac{N_j}{N-1}\Big) \sum_{\substack{N_1+\cdots+N_S=N \\ N_i\geq 1}} \tfrac{(N-1)!}{N_1!\cdots(N_i-1)!\cdots N_S!}\, n_1^{N_1}\cdots n_i^{N_i-1}\cdots n_S^{N_S} \\
&= P_\tau(x_i) + P_\mu(x_i) + \varepsilon\Big(\tfrac{1}{2} - \tfrac{1}{N} - \tfrac{N-1}{N}\sum_{j=1}^{S} n_j \exp(-\alpha(x_i-x_j)^2)\Big) + \sigma\sum_{j=1}^{S} n_j(x_i-x_j),
\end{aligned} \tag{A.10}$$

which coincides with (5).

Finally, note that the above multi-index sums can be organized as

$$\sum_{\substack{N_1+\cdots+N_S=N\\ N_i\geq 1}} \equiv \sum_{N_1=0}^{N-1}\cdots\sum_{N_{i-1}=0}^{N-1-\sum_{k=1}^{i-2}N_k}\ \sum_{N_{i+1}=0}^{N-1-\sum_{k=1}^{i-1}N_k}\cdots\sum_{N_S=0}^{N-1-\sum_{k=1,k\neq i}^{S-1}N_k}, \tag{A.11a}$$

$$\sum_{\substack{N_1+\cdots+N_S=N\\ N_i,N_j\geq 1}} \equiv \sum_{N_1=0}^{N-2}\cdots\sum_{N_{i-1}=0}^{N-2-\sum_{k=1}^{i-2}N_k}\ \sum_{N_{i+1}=0}^{N-2-\sum_{k=1}^{i-1}N_k}\cdots\sum_{N_j=1}^{N-1-\sum_{k=1,k\neq i}^{j-1}N_k}\cdots\sum_{N_S=0}^{N-2-\sum_{k=1,k\neq i}^{S-1}N_k}, \tag{A.11b}$$

where $N_i = N - \sum_{k=1,k\neq i}^{S} N_k$ and $1 < i < S$ and $j > i+1$ are assumed (with little loss of generality).

References

S. Atran and J. Henrich. The evolution of religion: How cognitive by-products, adaptive learning heuristics, ritual displays, and group competition generate deep commitments to prosocial religions. *Biological Theory*, 5:1–13, 2010.

R. Axelrod and W. D. Hamilton. The evolution of cooperation. 211:1390–1396, 1981.

N. Barber. Women's dress fashions as a function of reproductive strategy. *Sex Roles*, 40:459–471, 1999.

S. Blum. *Everyday Fashions of the Twenties.* Dover Publications, 1981.

H. Blumer. Fashion: from class differentiation to collective selection. *The Sociological Quarterly*, 10:275–291, 1969.

R. Boyd and P. J. Richerson. *Culture and the Evolutionary Process.* University of Chicago Press, 1988.

C. Buckley and H. Fawcett. *Fashioning the Feminine: Representation and Women's Fashion from the* Fin De Siecle *to the Present.* I. B. Tauris, 2001.

J. P. Caulkins, R. F. Hart, P. M. Kort, and G. Feichtinger. Explaining fashion cycles: imitators chasing innovators in product space. *Journal of Economic Dynamics and Control*, 31:1535–1556, 2007.

N. Champagnat, R. Ferrière, and S. Méléard. Unifying evolutionary dynamics: From individual stochastic processes to macroscopic models. *Theoretical Population Biology*, 69:297–321, 2006.

F. Débarre, C. Hauert, and M. Doebeli. Social evolution in structured populations. *Nature Communications*, 5:3409, 2014.

F. Della Rossa, F. Dercole, and P. Landi. The branching bifurcation of adaptive dynamics. *International Journal of Bifurcation and Chaos*, 25:1540001, 2015.

F. Dercole. Remarks on branching-extinction evolutionary cycles. *Journal of Mathematical Biology*, 47: 569–580, 2003.

F. Dercole and S. Geritz. Unfolding the resident-invader dynamics of similar strategies. *Journal of Theoretical Biology*, 394:231–254, 2016.

F. Dercole and S. Rinaldi. *Analysis of Evolutionary Processes: The Adaptive Dynamics Approach and Its Applications.* Princeton University Press, 2008.

F. Dercole and S. Rinaldi. Evolutionary dynamics can be chaotic: a first example. *International Journal of Bifurcation and Chaos*, 11:3473–3485, 2010.

F. Dercole, U. Dieckmann, M. Obersteiner, and S. Rinaldi. Adaptive dynamics and technological change. *Technovation*, 28:335–348, 2008.

F. Dercole, R. Ferriére, and S. Rinaldi. Chaotic Red Queen coevolution in three-species food chains. *Proceedings of the Royal Society of London B*, 277:2321–2330, 2010a.

F. Dercole, C. Prieu, and S. Rinaldi. Technological change and fisheries sustanibility: the point of view of adaptive dynamics. *Ecological Modelling*, 221:379–387, 2010b.

U. Dieckmann and R. Law. The dynamical theory of coevolution: a derivation from stochastic ecological processes. *Journal of Mathematical Biology*, 34:579–612, 1996.

M. Doebeli and I. Ispolatov. A model for the evolutionary diversification of religions. *Journal of Theoretical Biology*, 267:676–684, 2010.

C. Efferson, R. Lalive, and E. Fehr. The coevolution of cultural groups and ingroup favoritism. *Science*, 321: 1844–1849, 2008a.

C. Efferson, R. Lalive, P. J. Richerson, R. McElreath, and M. Lubell. Conformists and mavericks: the empirics of frequency-dependent cultural transmission. *Evolution and Human Behavior*, 29:56–64, 2008b.

E. Esposito. Originality through imitation: The rationality of fashion. *Organization Studies*, 32:603–613, 2011.

C. Evans. *Fashion at the edge: spectacle, modernity and deathliness*. Yale University Press, 2007.

G. A. Field. The status float phenomenon. *Business Horizons*, 1:45–52, 1970.

R. A. Foley and M. Mirazón Lahr. The evolution of the diversity of cultures. *Philosophical Transactions of the Royal Society B*, 366:1080–1089, 2011.

G. S. Frings. *Fashion: from concept to consumer*. Prentice Hall, 1999.

S. A. H. Geritz. Resident-invader dynamics and the coexistence of similar strategies. *Journal of Mathematical Biology*, 50:67–82, 2005.

S. A. H. Geritz, J. A. J. Metz, E. Kisdi, and G. Meszéna. The dynamics of adaptation and evolutionary branching. *Physical Review Letters*, 78:2024–2027, 1997.

S. A. H. Geritz, E. Kisdi, G. Meszéna, and J. A. J. Metz. Evolutionarily singular strategies and the adaptive growth and branching of the evolutionary tree. *Evolutionary Ecology*, 12:35–57, 1998.

J. Henrich and R. Boyd. Division of labor, economic specialization, and the evolution of social stratification. *Current Anthropology*, 49:715–724, 2008.

J. Henrich and R. McElreath. The evolution of cultural evolution. *Evolutionary Anthropology*, 12:123–135, 2003.

J. Hofbauer and K. Sigmund. *Evolutionary Games and Population Dynamics*. Cambridge University Press, Cambridge, UK, 1998.

E. Kisdi. Evolutionary branching under asymmetric competition. *Journal of Theoretical Biology*, 197:149–162, 1999.

P. Landi and F. Dercole. The evolution of fashion traits: pure social interactions promote diversity. In *Proceedings of the 8th European Nonlinear Dynamics Conference ENOC 2014*, Vienna, 2014.

P. Landi, F. Dercole, and S. Rinaldi. Branching scenarios in eco-evolutionary prey-predator models. *SIAM Journal on Applied Mathematics*, 73:1634–1658, 2013.

J. Laver. *Taste and Fashion: From the French Revolution to the Present Day*. Harrap, 1937.

J. Laver, A. de la Haye, and A. Tucker. *Costume and Fashion: A Concise History*. Thames & Hudson, 2002. 4th edition.

E. D. Lowe. Quantitative analysis of fashion change: a critical review. *Home Economics Research Journal*, 21:280–306, 1993.

E. D. Lowe and J. W. G. Lowe. Velocity of the fashion process in women's formal evening dress, 1789-1980. *Clothing & Textiles Research Journal*, 9:50–58, 1990.

J. W. G. Lowe and E. D. Lowe. Cultural pattern and process: a study of stylistic change in women's dress. *American Anthropologist*, 54:521–544, 1982.

J. W. G. Lowe and E. D. Lowe. Model of fashion change. *Advances in Consumer Research*, 11:731–734, 1983.

A. Lurie. *The Language of Clothes*. Random House, 1981.

J. Maynard Smith. *Evolution and the Theory of Games*. Cambridge University Press, Cambridge, UK, 1982.

V. Mendes and A. de la Haye. *20th Century Fashion*. Thames & Hudson, 1999.

A. Mesoudi. How cultural evolutionary theory can inform social psychology and vice versa. *Psychology Review*, 116:929–952, 2009.

A. Mesoudi. *Cultural Evolution: How Darwinian Theory Can Explain Human Culture and Synthesize the Social Sciences*. University of Chicago Press, 2011.

G. Meszéna, M. Gyllenberg, F. J. A. Jacobs, and J. A. J. Metz. Link between population dynamics and dynamics of Darwinian evolution. *Physical Review Letters*, 95:078195, 2005.

J. A. J. Metz, S. A. H. Geritz, G. Meszéna, F. J. A. Jacobs, and J. S. van Heerwaarden. Adaptive dynamics: a geometrical study of the consequences of nearly faithful reproduction. In S. J. van Strien and S. M. Verduyn Lunel, editors, *Stochastic and Spatial Structures of Dynamical Systems*, pages 183–231. Elsevier Science, 1996.

C. M. Miller, S. H. McIntyre, and M. K. Mantrala. Toward formalizing fashion theory. *Journal of Marketing Research*, 30:142–157, 1993.

M. A. Nowak and D. C. Krakauer. The evolution of language. *Proceedings of the National Academy of Sciences USA*, 96:8028–8033, 1999.

W. Pesendorfer. Design innovation and fashion cycles. *The American Economic Review*, 85:771–792, 1995.

J. Richardson and A. L. Kroeber. Three centuries of women's dress fashions: a quantitative analysis. *Anthropological Records*, 5:111–153, 1940.

P. J. Richerson and M. H. Christiansen, editors. *Cultural evolution: Society, Technology, Language, and Religion*. MIT Press, 2013.

D. E. Robinson. Fashions in shaving and trimming of the beard: the men of the illustrated London News, 1842-1972. *American Journal of Sociology*, 81:1133–1141, 1976.

P. Schuster and K. Sigmund. Replicator dynamics. *Journal of Theoretical Biology*, 100:533–538, 1983.

S. Shennan. Demography and cultural innovation: a model and its implications for the emergence of modern human culture. *Cambridge Archaeological Journal*, 11:5–16, 2001.

G. Simmel. Fashion. *International Quarterly*, 10:130–155, 1904.

G. B. Sproles. *Fashion: Consumer Behavior Toward Dress*. Burgess, 1979.

G. B. Sproles. Analyzing fashion life cycles: principles and perspectives. *The Journal of Marketing*, 45: 116–124, 1981.

G. B. Sproles. Behavioral science theories of fashion. In M. R. Solomon, editor, *The Psychology of Fashion*, pages 55–70. Lexington Books, 1985.

V. Steele. *Fashion and Eroticism*. Oxford University Press, 1985.

V. Steele. *Fifty Years of Fashion: new look to now*. Yale University Press, 2000.

S. Tan, J. Lu, G. Chen, and D. J. Hill. When structure meets function in evolutionary dynamics on complex networks. *IEEE Circuits and Systems Magazine*, 14:36–50, 2014.

P. J. Taylor and L. Jonker. Evolutionarily stable strategies and game dynamics. *Mathematical Biosciences*, 40:145–156, 1978.

P. G. Tortora and K. Eubank. *Survey of Historic Costume*. Fairchild Publications, 2010. 5th edition.

T. Veblen. The economic theory of woman's dress. *The Popular Science Monthly*, 46:198–205, 1894.

P. Weeden. Study patterned on Kroeber's investigation of style. *Dress*, 3:8–19, 1977.

A. B. Young. *Recurring Cycles of Fashion*. Harper and brothers, 1937.

J. Ziman, editor. *Technological Innovation as an Evolutionary Process*. Cambridge University Press, 2000.