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Constrained mortality extrapolation to old age: An empirical assessment

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Approved by

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Abstract

The paper aims at improving the accuracy of the extrapolations of the death rates into old age by constraining the extrapolation model on presumed life expectancy at old age. Such a task is particularly important in cases where the data quality at old age, in particular the age exaggeration, does not allow for drawing reliable mortality estimates. Our tests are based on period data from the Human Mortality Database (HMD 2016). We show strong improvements in the extrapolation accuracy when constraining the extrapolation on either the empirical life expectancy or the Horiuchi-Coale (1982) or Mitra (1984) estimates. Unconstrained extrapolations and those constrained by conventional life table estimates of the life expectancy in the open age interval show substantial biases and should be avoided. Combining extrapolation with life expectancy estimates which are robust to effects of the age exaggeration appear to be a valuable tool for mortality estimation.

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Constrained mortality extrapolation to old age: An empirical assessment

Dalkhat M. Ediev

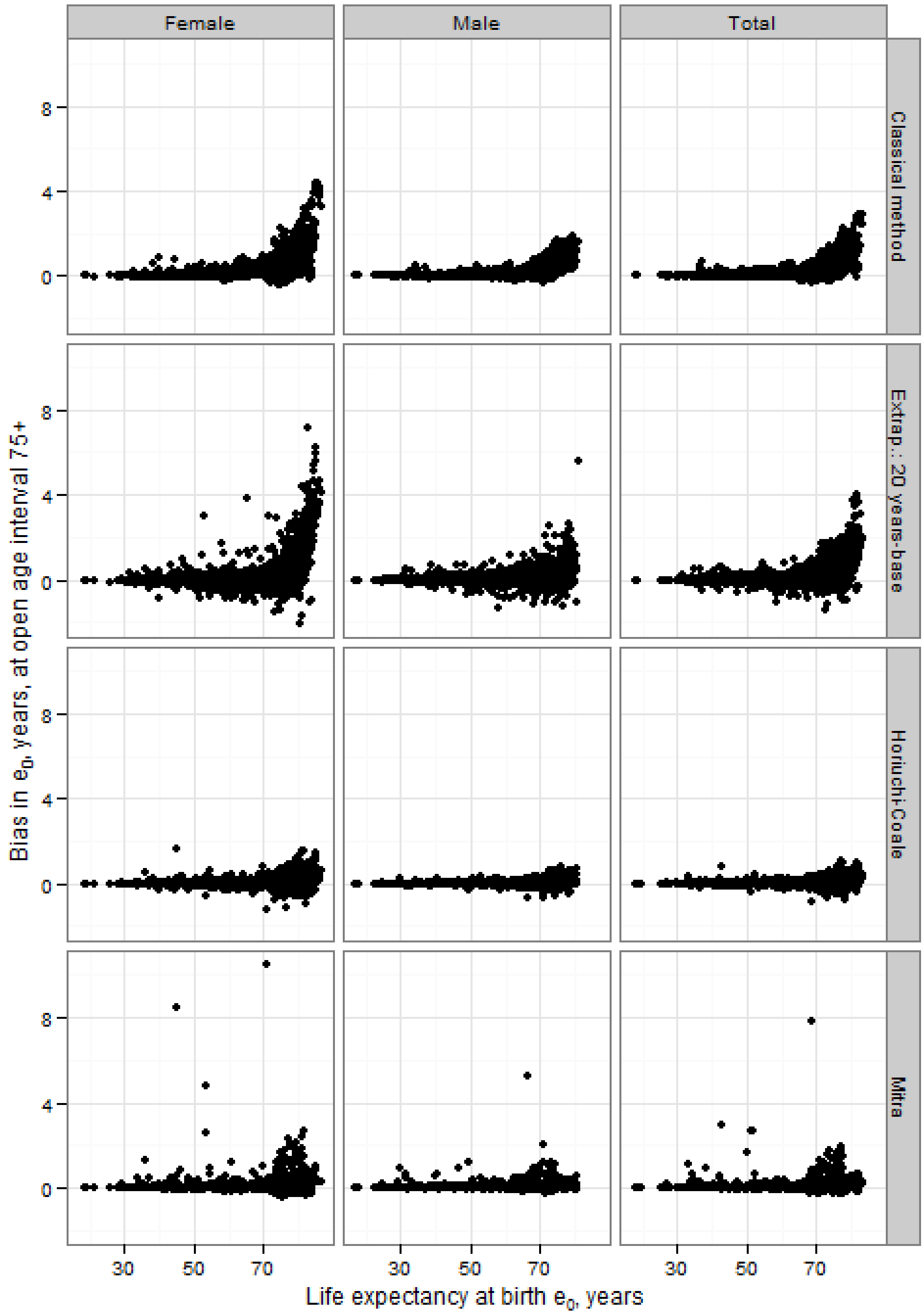
1 Introduction

Understanding mortality patterns at old age is essential for studying the processes of lifespan extension, population ageing and its consequences. The task is relatively straightforward for countries with well-established vital statistics, even though not without complications (Duthé et al. 2010; Khlat & Courbage 1996; Kibele et al. 2008; Preston et al. 1996). For populations with a lack of vital statistics, on the other hand, indirect estimates based on model life tables and other simplifications form the common way around the data limitations. Some countries are in an intermediate situation where vital statistics are available but they suffer from inaccuracies that prevent a direct estimation of old age mortality. Different groups and individuals have developed various approaches to overcome these data problems. The Statistics Centre of the Abu Dabi Emirate (SCAD), for example, uses the Coale-Guo model (Coale & Guo 1989; Coale & Kisker 1990) to extend the death rates to old age, and imputes the death rates at ages 85+ "based on proportions found in populations of other countries" (SCAD 2016).

Age exaggeration is a particular obstacle in establishing empirical estimates of old age mortality. In areas where there is no tradition of documented birth registration, elderly people tend to exaggerate their age. This excludes the possibility of obtaining reliable estimates of the death rates at old ages directly from vital statistics. In Turkey, for example, where rich register data enables the calculation of detailed life tables, official estimates of old age mortality look unrealistically low (Turkish Statistical Institute 2015), possibly because of the age exaggeration. Other typical obstacles to computing the death rates at advanced old age are a small population size and the resulting erratic patterns of empirical rates at those ages (e.g. Wilmoth et al. 2007). In such cases, typically, the statistical agency would limit the analysis to death rates below the problematic age range, hence, closing the official life table at some young open age interval and limiting the usability of the table. This classical method is applied in many countries where official life tables are published with rather low ages at the beginning of the open age interval (Missov et al. 2016, Table 2).

Horiuchi and Coale (1982) have shown that life expectancy estimates based on a life table that was closed at a younger open age interval may be badly biased when the proportion of elderly population is growing, and suggested an adjustment formula to bypass this problem. Although Mitra (1984) questioned the Horiuchi-Coale correction and came up with an alternative formula , a more recent analysis (Ediev 2016) shows that the two methods are consistent with each other and provide a dramatic improvement in accuracy of life expectancy estimates as compared to the classical life table with young open age intervals.

Figure 1. Estimation errors in life expectancy at birth, obtained by selected methods for the open age interval 75+ (in years)



Notes: Methods indicated in the right-hand side of the panels: ‘Classical method’ = traditional life table with open age interval 75+ and life expectancy for the open age interval obtained as inverse to the aggregate death rate in that interval; ‘Extrap.: 20 years-base’ = detailed life table with open age interval 110+, where the death rates for ages 75-110 are obtained by extrapolation based on their rate of change in the age range 55-74 years and the Gompertz model; ‘Horiuchi-Coale’ = same as the classical method, with the life expectancy at age 75 adjusted using the Horiuchi-Coale (1982) formula; ‘Mitra’ = same as the classical method, with the life expectancy at age 75 adjusted using the Mitra (1984) formula and our modification (Eq.4). Source: (Ediev 2016) based on the data from the Human Mortality Database (2016).

Another common approach in dealing with problematic data at old age is to extrapolate the old age mortality based on the death rates at younger age in combination with some mortality model, such as the Gompertz (a popular model in earlier times) or Kannisto (the more recent favorite) models. Yet, empirical tests show that this method is not an improvement of the classical life table in terms of the accuracy of the estimated life expectancy (Ediev 2016). Selected results for the accuracy of the life expectancy at birth estimated from the various methods, here with the open age interval 75+, are presented in Figure 1. The two most common approaches, the classical life table with the open age interval 75+ and the estimate based on extrapolating the death rates into the ages 75-110, produce the worst results. In fact, the extrapolation method produces even less stable results than the classical life table. Both the Horiuchi-Coale and Mitra methods, on the other hand, substantially improve the accuracy of the estimated life expectancy.

Although inferior in accuracy to the Horiuchi-Coale and Mitra methods, the extrapolation method is appealing and in demand for being able to produce age details of mortality at old age. In this paper, we aim at developing a method that allows to keep the age details of the extrapolation method while improving its overall accuracy. To this end we use the more accurate estimates of life expectancy (Horiuchi-Coale and Mitra methods) to constrain the extrapolated rates in the open age interval. We show, that such an approach leads to estimates of the death rates which are more accurate both in general in terms of life expectancy and also for individual ages.

2 Data and methods

In our study, we use the unsmoothed single-year death rates and corresponding population exposures of the Human Mortality Database (HMD) (2016) for the most recent available calendar periods for each HMD country. Altogether, the database (data downloaded on 12.02.2016) contains 46 recent country-calendar years for each gender (males, females, total). For each of the 3x46=138 input entries, we calculate life tables by assuming alternative open age intervals (the beginning age of the open age interval spanning from a=65 to a=85) and applying various estimation methods for life expectancy in the open age interval (described in the next paragraph). Estimates of life expectancy in the open age interval will be used to improve the extrapolations of death rates into the chosen open age interval.

We consider three alternative methods for the life expectancy in the open age interval: the classical life table model, the Horiuchi-Coale adjustment, and the Mitra adjustment. In the classical life table model (Preston et al. 2001), life expectancy is inverse to the aggregated death rate:

$$e_a{}^{LT} = M_{a+}{}^{-1}, \quad (1)$$

hereinafter, a denotes the beginning age of the open age interval; e_a is life expectancy at age a; M_{a+} is the death rate in the open age interval[1]. In the Horiuchi-Coale (1982) method, estimate (1) is adjusted for the departure of the population age composition from the stationary population assumed in the classical method:

[1] The aggregated death rate for the open age interval is derived from the HMD (unsmoothed) death rates and population exposures: $M_{a+} = \frac{\sum_{x=a}^{\omega} M_x P_x}{\sum_{x=a}^{\omega} P_x}$, where M_x is the death rate and P_x is the population exposure for age x.

$$e_a{}^{HC} = M_{a+}{}^{-1} e^{-\beta_a r M_{a+}{}^{-\alpha_a}}, \quad (2)$$

here, r is the annual growth rate of the population in the open age interval (to stabilize the estimates, we average the growth rate over 10-year time periods prior to the year of estimation); α_a and β_a are the model parameters (for numerical values, see Horiuchi and Coale (1982) or the Appendix Table A1). In the Mitra (1984) method, the adjustment involves mean population age in the open age interval:

$$e_a{}^{M} = M_{a+}{}^{-1} e^{-r\left[M_{a+}{}^{-1} - \left(1 + r M_{a+}{}^{-1}\right)(\bar{x} - a)\right]}, \quad (3)$$

where $\bar{x}$ stands for the mean age of the population in the open age interval. Because the usage of the mean population age in (3) was questioned by Coale (1985) as prone to effects of age exaggeration, we replace it by the following regression based on HMD data (Ediev 2016):

$$\bar{x} = C + k_1 M_{a+}{}^{-1} + k_2 r M_{a+}{}^{-1}, \quad (4)$$

where C, k_1, k_2 are model parameters (see Appendix Table A1 for the values).

To improve the extrapolation performance, we constrain the parameters of extrapolation models to either the empirical e_a or to one of the estimates (1)-(3). We consider two popular mortality models that represent typical assumptions about mortality change at old age, the Gompertz and the Kannisto models. Both models contain only two parameters[2], one of which may be determined by fixing the model death rate at the age below the open age interval, M_{a-1}, to its empirical value. In the Gompertz (1825) model:

$$M_x = C e^{b(x-a+1)}, \quad (5)$$

M_x being the central death rate at age x; we set $C = M_{a-1}$. In the Kannisto model (Doray 2008; Thatcher et al. 1998):

$$M_x = \frac{C e^{b(x-a+1)}}{1 + C e^{b(x-a+1)}}, \quad (6)$$

we set $C = \frac{M_{a-1}}{1 - M_{a-1}}$. The second parameter, b, in either model can be fit to the life expectancy in the open age interval e_a (either the actual one or one of the estimates (1)-(3)). We use the standard one-dimensional optimizer of the R package (R Core Team 2016) in finding the parameter b best-fit to the assumed e_a.

3 Results

The potential to improve the extrapolation model by constraining its parameters is demonstrated in Figure 2. It features extrapolations (conventional and constrained by $e_a{}^{HC}$ (2)) produced by applying the Gompertz and Kannisto models to the death rates in Japan in 2012, at three selected open age intervals (a=65, 75, or 85 years). In all cases, the constrained extrapolations fit the empirical rates better than the unconstrained ones, although the improvement was small in the case of males, open age interval 65+. In most cases, the conventional extrapolations mislead the production of death rates several times

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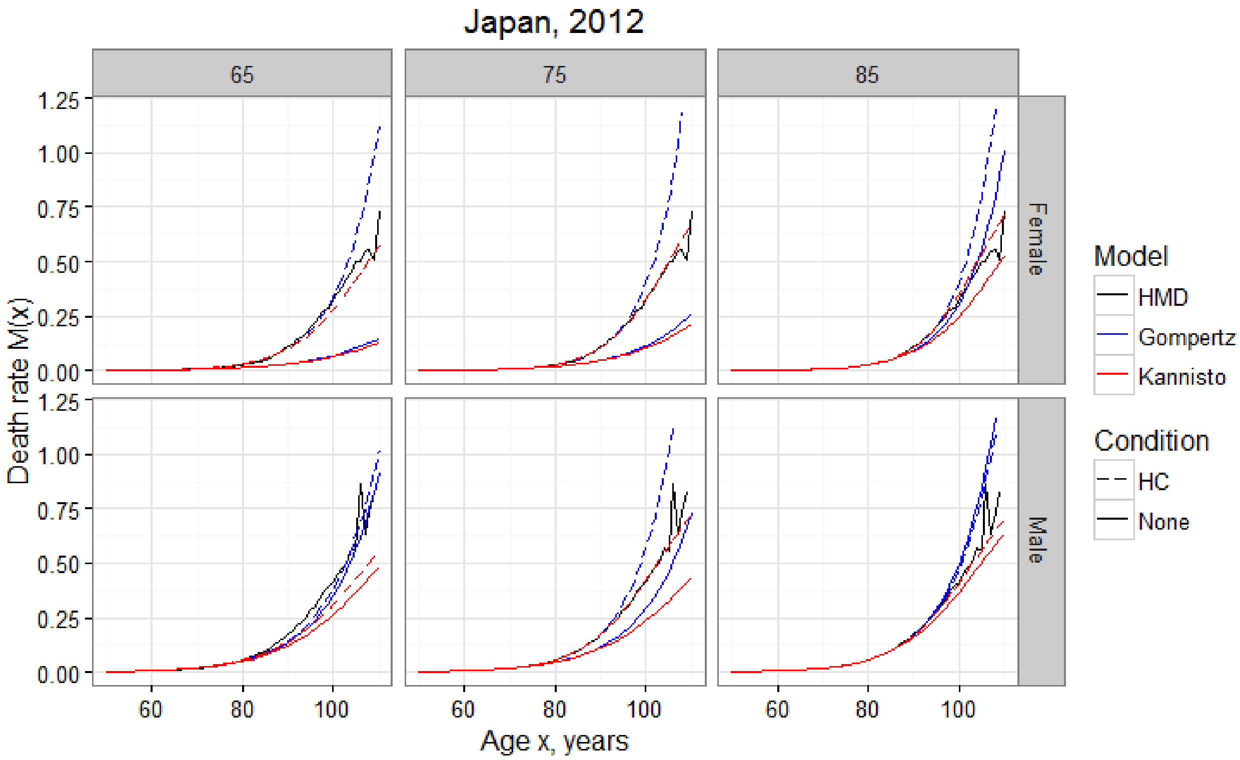
[2] The two models are also known in their three-parameter variants (the Makeham (1860) model and the full Kannisto model (Thatcher et al. 1998) with the background mortality term). Those are not presented here, because inclusion of the background mortality did not change our results substantially.

lower than the actual rates at old age while the constrained extrapolations (more so the Kannisto model) stay close to the empirical curve.

Figure 2. Constrained and unconstrained extrapolations of the death rates for Japan, 2012, men and women, at various ages at the start of the extrapolation (*a*=65, 75, or 85 years as shown in the upper parts of the panels)



Notes: 'HC' = extrapolation constrained by life expectancy e_a from the Horiuchi-Coale method; 'None' = no constraints imposed (model parameters are estimated on the detah rates for a 20-years age frame below age a); 'HMD' = original death rates from the Human Mortality Database (HMD 2016); 'Gompertz' = death rates extrapolated using the Gompertz model at ages a+; 'Kannisto' = death rates extrapolated using the Kannisto model at ages a+.

The example presented above is characteristic of improvements to the extrapolation method from constraining it to life expectancy estimates. In Figure 3, we present results for extrapolation errors when extrapolation starts at age 85 years. It features boxplots of errors in age-specific death rates pooled over five-year age intervals for the Gompertz and Kannisto models. We pool together results for males, females, and both sexes combined for all HMD countries, because there appeared to be similar error patterns across different population subgroups. Extrapolations constrained by either the empirical life expectancy from the HMD[3] or Horiuchi-Coale and Mitra estimates are substantially more accurate, less biased and/or more stable at ages below 97.5 for the Gompertz model and ages below 107.5 for the Kannisto model. The extrapolation

[3] By 'empirical' we denote the life expectancies calculated from the HMD raw data on the death rates. Because the HMD life tables are based on smoothing the raw death rates, our 'empirical' life expectancies may somewhat differ from the values in HMD.

constrained by the empirical life expectancy, expectedly, outperforms other methods at youngest age groups, although its advantage to the Horiuchi-Coale or Mitra methods fades away by about age 95 years. Unconstrained extrapolation and extrapolation constrained by the classical estimate (1) perfom worse except at the oldest age where volatility of original data seems to overshadow differences between the methods. The Kanniso model appears to better fit the age pattern of period mortality at advanced age in terms of both the bias and the spread of the errors. Conterintuitively, the constrained extrapolations (except for the one constrained by the classical estimate (1)) outperform the unconstrained extrapolation even at the youngest age interval, although the constraints should have loosened the fit of the models to data below and around age 85. Even constraining the extrapolation on the classical (biased) estimate of the life expetancy at the open age interval (ea.LT) somewhat stabilizes the extrapolation results, except at very old and the youngest age.

Figure 3. Boxplots of errors in the age-specific death rates pooled over five-year age intervals under alternative constraints imposed over the extrapolation of the death rates into the open age interval 85+

Notes: 'ea' = the extrapolation is constrained by actual e_{85} from the HMD; 'HC' = extrapolation constrained by e_{85} from the Horiuchi-Coale method; 'M.regr' = extrapolation constrained by e_{85} from the modified Mitra method; 'ea.LT' = extrapolation constrained by e_{85} from the conventional life table with open age inerval 85+; 'None' = no constraints imposed (model parameters are estimated on the death rates for a 20-years age frame below age 85). 'Gompertz' = death rates extrapolated using the Gompertz model; 'Kannisto' = death rates extrapolated using the Kannisto model. Data: only the most recent available year for each country in the HMD, female, male and total populations pooled together.

Similar results apply to the errors in terms of the remaining life expectancy (Figure 4), although biasness and less stability of the conventional extrapolations appear strongly already at young age.

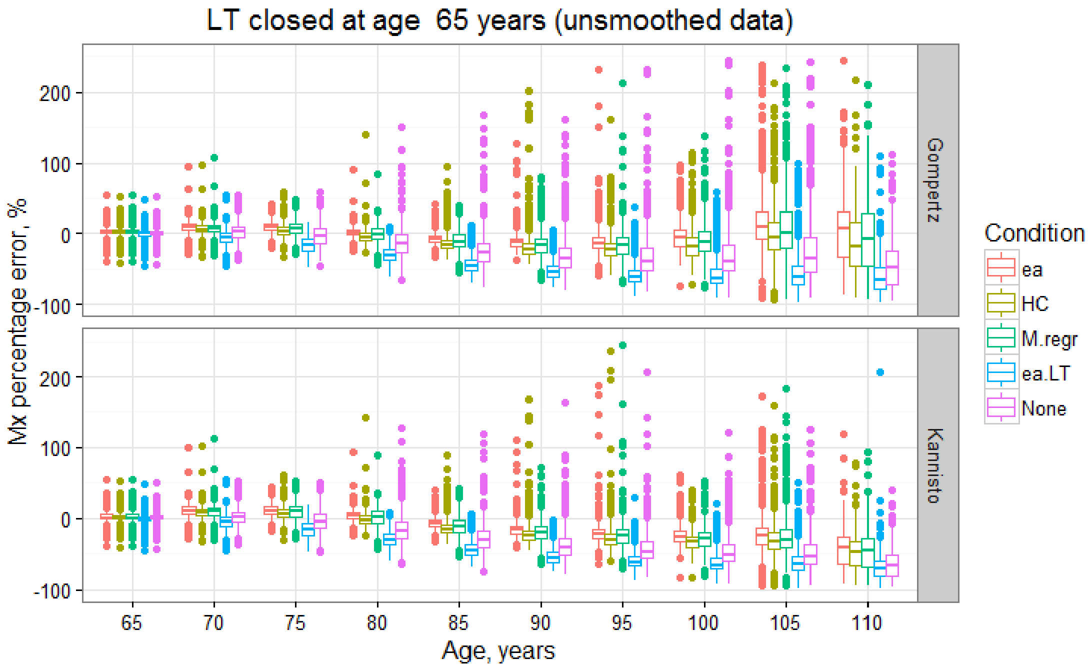
Figure 4. Boxplots of errors in remaining life expectancy by age pooled over five-year age intervals under alternative constraints imposed over the extrapolation of the death rates into the open age interval 85+

Notes: 'ea' = the extrapolation is constrained by actual e_{85} from the HMD; 'HC' = extrapolation constrained by e_{85} from the Horiuchi-Coale method; 'M.regr' = extrapolation constrained by e_{85} from the modified Mitra method; 'ea.LT' = extrapolation constrained by e_{85} from the conventional life table with open age inerval 85+; 'None' = no constraints imposed (model parameters are estimated on the death rates for 20-years age frame below age 85). 'Gompertz' = death rates extrapolated using the Gompertz model; 'Kannisto' = death rates extrapolated using the Kannisto model. Data: only the most recent available year for each country in the HMD, female, male and total populations pooled together.

Extrapolation from age 85 onwards may be a feasible option for reconstructing or graduating the death rates for countries with decent data quality below age 85 (such extrapolations used to be part of the World Health Organization's methodology, and smoothing rates at that age are part of HMD methods protocol). It is too optimistic an option, however, for countries with poorer data, particularly with strong age exaggeration. Results which are more relevant for the problematic data cases are presented in Figures 5 and 6. Here we feature estimation errors for extrapolations into open age interval 65+. All in all, results for the younger open age interval are similar to those presented above for the age interval 85+. However, the price for not or wrongly constraining the extrapolation is considerably higher. It is interesting to note that constrained extrapolations starting from age 65 yield not much higher errors by age 100 as compared to the errors of the unconstrained extrapolation starting at age 85. Also notably, the 'ideal' constraining on the actual life expectancy at age 65 provides better results, also as compared to the

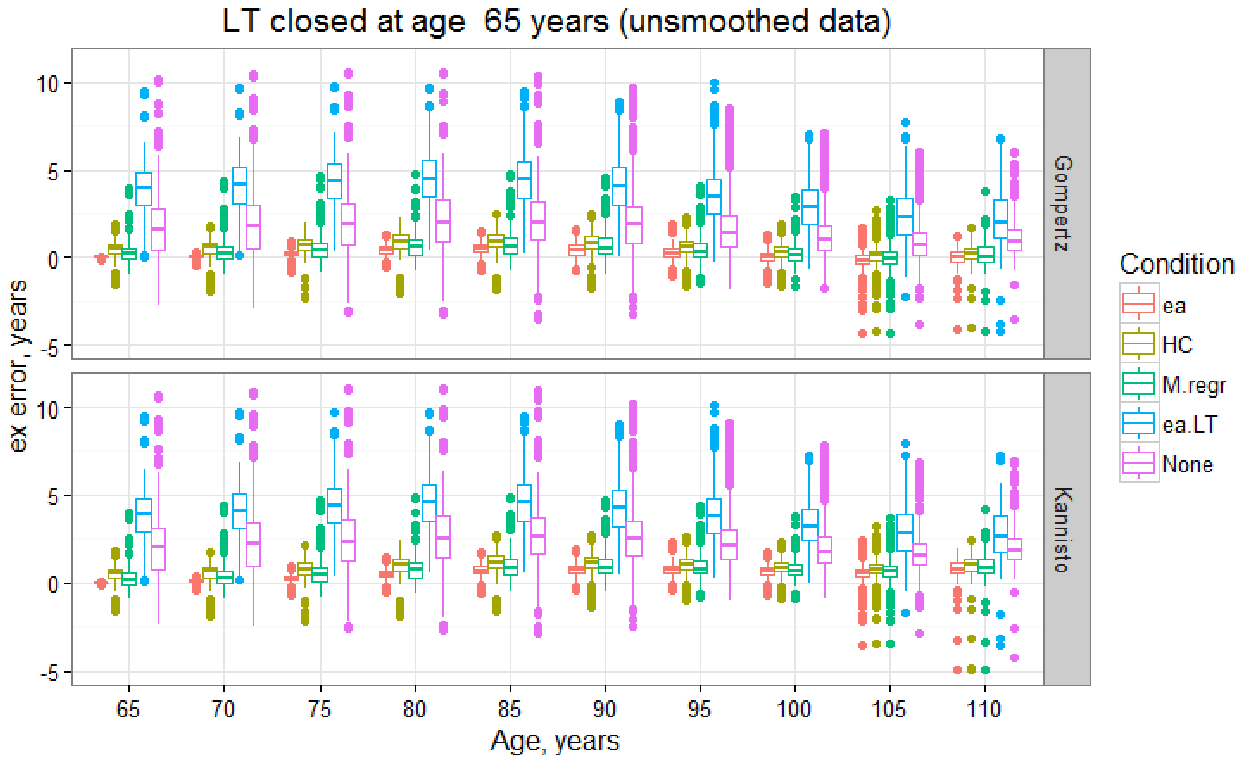
Horiuchi-Coale and Mitra methods, throughout the entire age range up to age 105. This highlights the importance of further developing the Horiuchi-Coale and Mitra methods in order to reduce their remaining estimation biases. It may also be noticed that, unlike in the case of the more advanced open age interval, the Kannisto model shows stronger systematic biases at old age when starting the extrapolation at age 65. At age 95 years and older, the bias of the (65+) Kannisto model is even stronger than that of the Gompertz model, although the wider spread of errors of the latter indicates its poorer performance. This may be taken as indication of the need for improving the mortality extrapolation models by allowing for higher flexibility of the produced mortality curve. In particular, either the three-parametric Kannisto model (Thatcher et al. 1998) or the Perks (1932) model might have offered the necessary flexibility to the mortality curve. However, our experiments with the three-parameter Kannisto and Gompertz-Makeham models including the constant background mortality term (results not shown here) did not lead to smaller biases in either model. It is also worthwhile noting that the Kannisto model shows only small biases until age 105 when tested on HMD data for the calendar year 1970 (results not shown here).

Figure 5. Boxplots of errors in age-specific death rates pooled over five-year age intervals under alternative constraints imposed over the extrapolation of the death rates into the open age interval 65+



Notes: ‘ea’ = the extrapolation is constrained by actual e_{65} from the HMD; ‘HC’ = extrapolation constrained by e_{65} from the Horiuchi-Coale method; ‘M.regr’ = extrapolation constrained by e_{65} from the modified Mitra method; ‘ea.LT’ = extrapolation constrained by e_{65} from the conventional life table with open age inerval 65+; ‘None’ = no constraints imposed (model parameters are estimated on the death rates for 20-years age frame below age 65). ‘Gompertz’ = death rates extrapolated using the Gompertz model; ‘Kannisto’ = death rates extrapolated using the Kannisto model. Data: only the most recent available year for each country in the HMD, female, male and total populations pooled together.

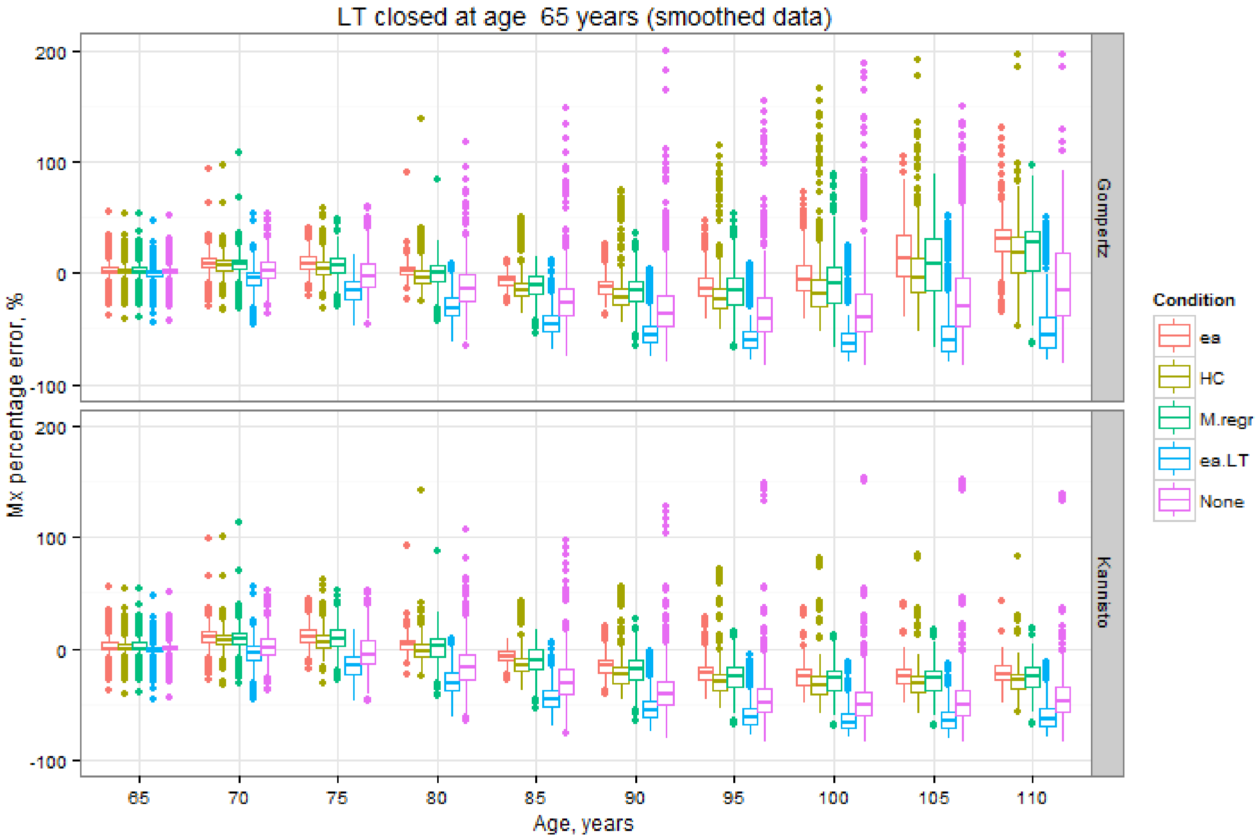
Figure 6. Boxplots of errors in remaining life expectancy by age pooled over five-year age intervals under alternative constraints imposed over the extrapolation of the death rates into the open age interval 65+



Notes: 'ea' = the extrapolation is constrained by actual e_{65} from the HMD; 'HC' = extrapolation constrained by e_{65} from the Horiuchi-Coale method; 'M.regr' = extrapolation constrained by e_{65} from the modified Mitra method; 'ea.LT' = extrapolation constrained by e_{65} from the conventional life table with open age ineral 65+; 'None' = no constraints imposed (model parameters are estimated on the death rates for 20-years age frame below age 65). 'Gompertz' = death rates extrapolated using the Gompertz model; 'Kannisto' = death rates extrapolated using the Kannisto model. Data: only the most recent available year for each country in the HMD, female, male and total populations pooled together.

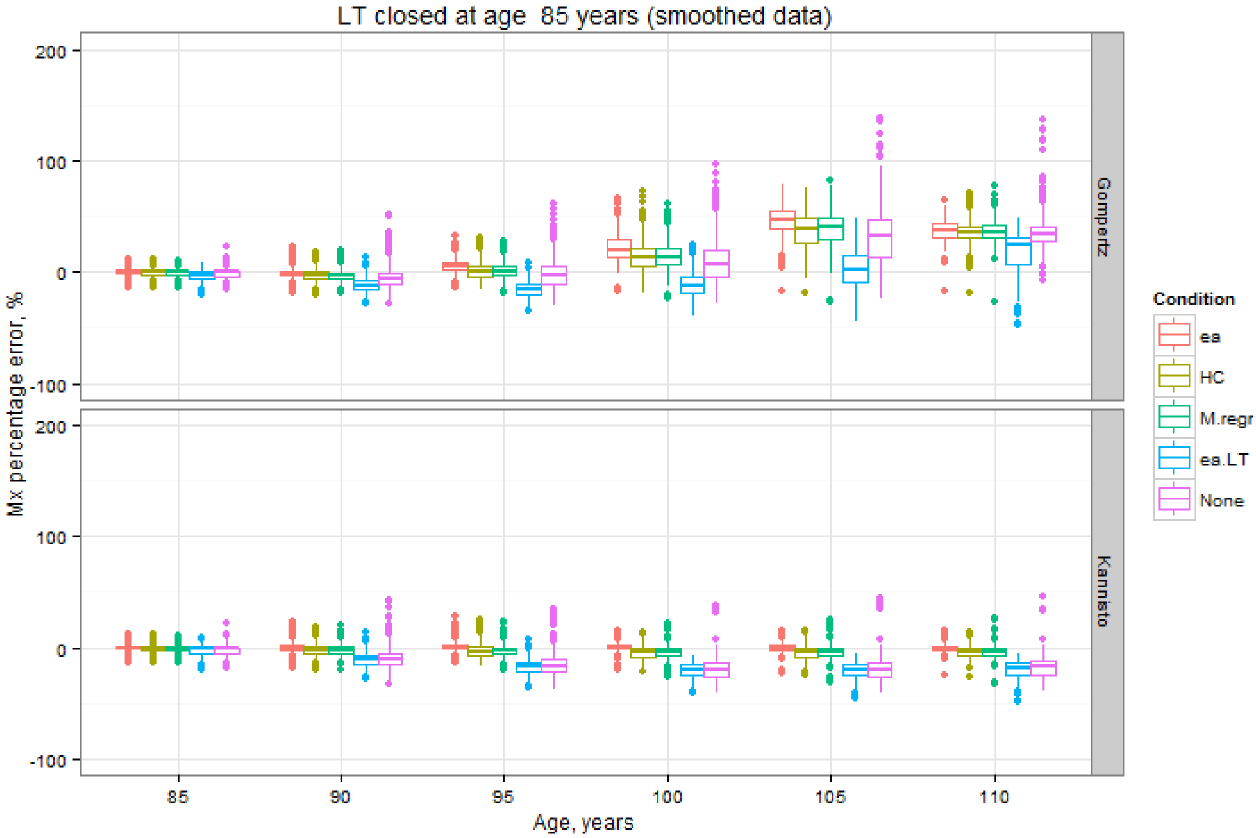
Our usage of unsmoothed raw death rates, not the smoothed life table rates, from the HMD was driven by the need to avoid possible distortions of the results by the Kannisto mortality model that are assumed when smoothing the HMD period life tables (Wilmoth et al. 2007). That same choice, however, may have increased the lack of fit of extrapolations, especially at advanced ages where the natural stochasticity of the death rates may have dominated the differences between the extrapolations. Some ideas about extrapolations on the death rates free of stochasticity may be developed when the raw death rates are replaced by the smoothed period life table death rates of the HMD (Figures 7 and 8). The advantage of the constrained extrapolations is even stronger and remains throughout the entire age span on the smoothed data. The Kannisto model clearly out-preforms the Gompertz model on the smoothed data, although this result may be a consequence of the usage of the Kannisto model itself in smoothing the HMD rates.

Figure 7. Boxplots of errors in age-specific death rates pooled over five-year age intervals under alternative constraints imposed over the extrapolation of the death rates into the open age interval 65+



Notes: 'ea' = the extrapolation is constrained by actual e_{65} from the HMD (smoothed life table death rates); 'HC' = extrapolation constrained by e_{65} from the Horiuchi-Coale method; 'M.regr' = extrapolation constrained by e_{65} from the modified Mitra method; 'ea.LT' = extrapolation constrained by e_{65} from the conventional life table with open age inerval 65+; 'None' = no constraints imposed (model parameters are estimated on the death rates for 20-years age frame below age 65). 'Gompertz' = death rates extrapolated using the Gompertz model; 'Kannisto' = death rates extrapolated using the Kannisto model. Data: only the most recent available year for each country in the HMD (smoothed life table death rates), female, male and total populations pooled together.

Figure 8. Boxplots of errors in age-specific death rates pooled over five-year age intervals under alternative constraints imposed over the extrapolation of the death rates into the open age interval 85+



Notes: 'ea' = the extrapolation is constrained by actual e_{85} from the HMD (smoothed life table death rates); 'HC' = extrapolation constrained by e_{85} from the Horiuchi-Coale method; 'M.regr' = extrapolation constrained by e_{85} from the modified Mitra method; 'ea.LT' = extrapolation constrained by e_{85} from the conventional life table with open age inerval 85+; 'None' = no constraints imposed (model parameters are estimated on the death rates for 20-years age frame below age 85). 'Gompertz' = death rates extrapolated using the Gompertz model; 'Kannisto' = death rates extrapolated using the Kannisto model. Data: only the most recent available year for each country in the HMD (smoothed life table death rates), female, male and total populations pooled together.

4 Conclusion

The presented results confirm that the conventional extrapolations of the death rates into old age bear strong biases in the death rates and remaining life expectancies. These biases may be efficiently reduced when constraining the extrapolations on life expectancy in the open age interval. Using, for that purpose, the life expectancy estimated from the Horiuchi-Coale or Mitra methods provides substantial improvements in the extrapolations. Combining improved estimates of expectation of life at old age with detailed extrapolations of the age-specific death rates provides a practical tool that may be recommended in all cases where direct usage of mortality data is limited by data quality issues at advanced age. Notably, the best constrained extrapolations starting at age 65 yielded, by advanced old age, errors not principally larger than the conventional unconstrained extrapolations starting at age 85. This opens new possibilities in correcting data that is corrupt by age exaggeration, and in smoothly extending life tables to advanced old age when empirical rates show erratic patterns.

Our finding of considerably better fit of extrapolations constrained by the empirical life expectancy at old age as compared to the extrapolations constrained by Horiuchi-Coale or Mitra estimates suggest the importance of further developing the methods of estimating life expectancy at old age. One strategy in that direction may be a recursive combination of adjustments to life expectancy and of extrapolations. While the Horiuchi-Coale and Mitra methods rely on assuming a stable population age composition, one may construct a better model of age composition by using the extrapolated death rates in the open age interval to predict unknown population exposures. Such a model may improve the accuracy of estimated life expectancy for the open age interval that, in turn, may be used to improve the extrapolation model itself.

Another practical way of improving the performance of life expectancy estimates and extrapolations may be based on carrying out an analysis on country-basis, because age patterns of death rates and population age compositions typically bear substantial country-specific regularities.

Our results indicate more stability, at old age, of the logistic model as compared to the Gompertz curve. Yet, the substantial systematic biases of the Kannisto model at extrapolating the death rates at ages 65+ suggest that a more flexible logistic curve may provide better results for contemporary period mortality.

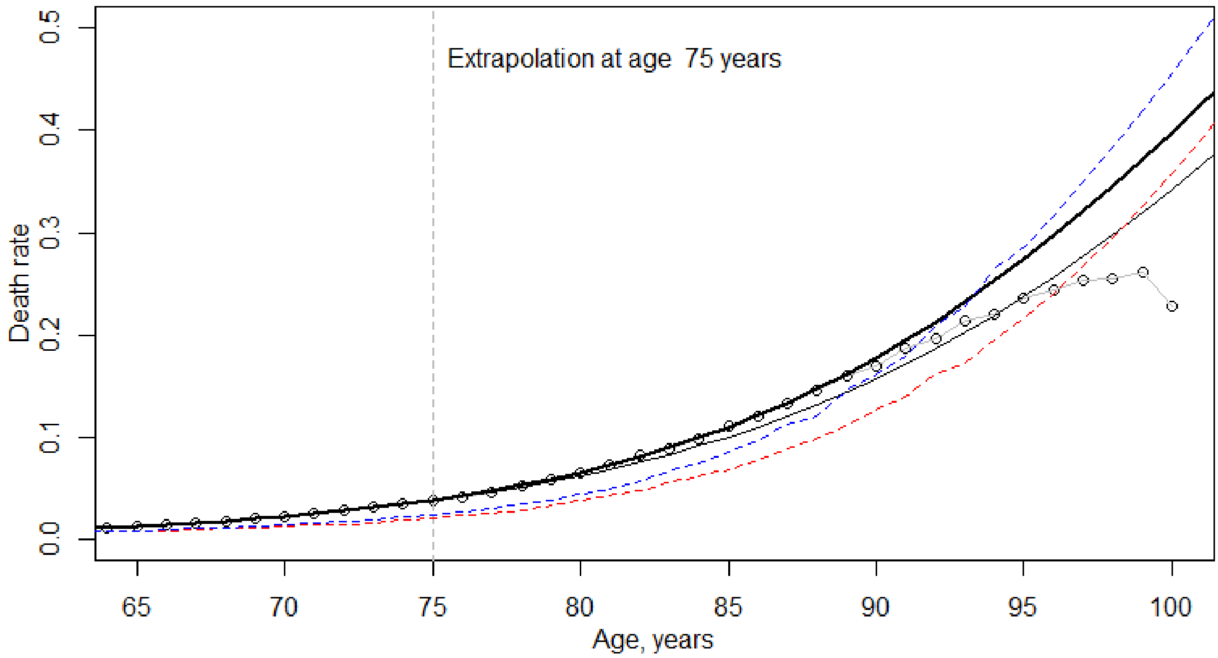
As mentioned in the introduction, mortality estimates for countries that lack vital statistics are usually based on indirect models, such as model life tables. These models, however, are themselves based on imputing the death rates at old age. Therefore, the model tables and old age mortality models for developing countries may need to be revised by improving the accuracy of the underlying empirical inputs that are used in constructing those models.

Cohort mortality is an area of useful application of extrapolations, but is missing in our study. We could not replicate the study on the cohort data, because the Horiuchi-Coale and Mitra methods are not suited for that case. Yet, our results suggest that constrained extrapolation might provide a substantial improvement in accuracy for the cohort mortality too. Even though the Horiuchi-Coale and Mitra models are not applicable to cohorts, the usage of our method in the cohort case may be facilitated by the fact that the classical estimate of life expectancy (1) is accurate when cohort age structure at old age is not affected by migration and follows closely the stationary population model (Horiuchi & Coale 1982; Mitra 1984; Ediev 2016).

We conclude with a case study that illustrates just how substantial the necessary adjustment might be to the death rates at old age when the data are affected by age exaggeration. In Figure 9, we present extrapolation results, from age 75 onwards, for the death rates in Turkey in 2013/14, both sexes combined. Official death rates (Turkish Statistical Institute 2015) (circles connected by grey line in the figure) level off at an unrealistically low level at old age (compared to the recent Japanese and Swedish death rates shown in the same figure). Quite likely, the unrealistically low official mortality rates at old age are caused by substantial age exaggeration among elderly in Turkey. When aggregating the death data in the open age interval 75+ and applying the Horiuchi-Coale method (population data comes from the World Population Prospects (United

Nations 2015)), we get remaining life expectancy e_{75} equal to 10.8 years and life expectancy at birth at 77.9 years, both slightly below the official estimates of 11.0 and 78.0 years, respectively. Kannisto-model extrapolations constrained to the Horiuchi-Coale estimates (even though largely consistent with the official rates in terms of life expectancy at birth and age 75) are substantially higher at old age as compared to the official data. Taking into account the rates in Japan and Sweden, the two long-run world leaders in life expectancy, it becomes clear that the official estimates of old age mortality in Turkey must have been strongly underestimated while the extrapolated rates look more plausible. Even the extrapolated rates may be too low at old age, as compared to the rates in Japan. Unconstrained conventional extrapolations (thinner solid line in the figure) also look unrealistically low both at advanced old age (below both Japanese and Swedish rates at ages around 100) but also at younger ages where they fall even below the official estimates.

Figure 9. Official death rates (circles connected by grey line) and extrapolations (black lines) starting at age 75 years as compared to the Japanees (2012, broken red line) and Swedish (2012, broken blue line) death rates, both sexes combined. Extrapolations are based on the Kannisto model either unconstrained (thinner balck lines) or constrained (thicker lines) to the Horiuchi-Coale estimate of the remaining life expectancy at age 75



Data: own estimates based on mortality data by the Turkish Statistical Institute (TurkStat 2015) and population data from the World Population Prospects (United Nations 2015).

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6 Appendix

Table A1. Original parameters of the Horiuchi-Coale model (2) (Alfa, Beta), the Beta-parameter of the model re-estimated on the Human Mortality Database data (Beta.hmd), and the coefficients of the mean population age model (4) estimated on HMD data (C, k_1, k_2).

Sex	a	Alfa	Beta	Beta.hmd	C	k_1	k_2
Female	40	1.0	0.283	0.321	50.045	0.241	-4.918
Female	55	1.1	0.207	0.241	61.025	0.303	-4.503
Female	65	1.4	0.095	0.100	69.200	0.335	-3.670
Female	75	1.4	0.095	0.109	77.701	0.380	-2.676
Female	85	1.4	0.095	0.104	86.460	0.470	-1.883
Female	95	1.4	0.095	0.062	95.591	0.626	-0.867
Male	40	1.0	0.283	0.330	50.924	0.196	-3.919
Male	55	1.1	0.207	0.236	61.406	0.269	-3.722
Male	65	1.4	0.095	0.102	69.229	0.318	-3.180
Male	75	1.4	0.095	0.108	77.563	0.379	-2.398
Male	85	1.4	0.095	0.102	86.355	0.482	-1.863
Male	95	1.4	0.095	0.058	95.633	0.609	-0.914
Total	40	1.0	0.283	0.308	50.839	0.206	-3.849
Total	55	1.1	0.207	0.234	61.115	0.293	-4.030
Total	65	1.4	0.095	0.099	69.117	0.335	-3.324
Total	75	1.4	0.095	0.108	77.583	0.387	-2.481
Total	85	1.4	0.095	0.102	86.405	0.477	-1.803
Total	95	1.4	0.095	0.061	95.518	0.658	-0.929

Notes: a=starting age of the open age interval. Source: (Ediev 2016).