AN INTERACTIVE PROCEDURE FOR MULTIOBJECTIVE ANALYSIS OF WATER RESOURCES ALLOCATION

J. Kindler
P. Zielinski
L. de Maré

May 1980
WP-80-85

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

THE AUTHORS

J. Kindler is a Chairman of the Resources and Environment Area at IIASA leading also the "Regional Water Management" research task, International Institute for Applied Systems Analysis, Schloss Laxenburg, 2361 Laxenburg, Austria (on leave from the Institute of Environmental Engineering, Warsaw Polytechnic University, Warsaw, Poland).

P. Zielinski is a research scientist at the Institute of Environmental Engineering, Warsaw Polytechnic University, Warsaw, Poland.

L. de Maré is a research scientist at the Resources and Environment Area, International Institute for Applied Systems Analysis, Schloss Laxenburg, 2361 Laxenburg, Austria (on leave from the Department of Water Resources Engineering, Lund Institute of Technology, Lund, Sweden).

ACKNOWLEDGEMENT

The authors would like to express their gratitude to Prof. Dr. Andrzej Wierzbicki and Dr. Andrzej Lewandowski from IIASA (both on leave from the Institute of Automatic Control, Warsaw Polytechnic University, Warsaw, Poland) for several suggestions and comments concerning possible extensions of the proposed procedure.

This study was supported in part with funds provided by the Volkswagen Foundation.

ABSTRACT

This paper reports on part of IIASA's research concerning regional water management planning, focusing on the Western Skåne region in Southern Sweden. The IIASA's studies are concerned with four issues of particular importance to water resources management, namely, conflict resolution, criteria of choice, uncertainty, and institutional arrangements. This paper is related primarily to the first two of these issues. An interactive procedure seeking the satisfactory nondominated solution of the multiobjective water resources allocation problem is discussed. It is based on the Powell method with penalty function for the solution of scalar optimization problem and on a constraint and weighting method, or actually a reference objective method, for the solution of the multiobjective optimization problem. Application of the procedure is illustrated by an example referring to the situation in the Kävlinge River system in the Western Skåne, Sweden.

AN INTERACTIVE PROCEDURE FOR MULTIOBJECTIVE ANALYSIS OF WATER RESOURCES ALLOCATION

J. Kindler, P. Zielinski, and L. de Maré

INTRODUCTION

The past decade witnessed development of a large number of computer-aided procedures designed to assist water resources planners and managers in analysis and evaluation of multiobjective resource allocation problems. The literature referring to this topic is abundant; review and evaluation of existing procedures has been the subject of several publications such as Cohon and Marks (1975), Haimes et al. (1975), Haith and Loucks (1976), Major (1977), and Wierzbicki (1979). What is common to practically all of the multiobjective analysis procedures is that they provide mechanism for estimating the trade-offs among conflicting objectives. But it should be underlined that estimating these trade-offs is not synonymous to making the choices among conflicting objectives, especially when they are of the noncommensurable character. Thus it is inevitable that those responsible for implementation of each particular objective must become involved in the process of selecting the satisfactory nondominated solution (March and Simon, 1958). This is usually a complex process involving negotiations and bargaining among all parties concerned. The procedure presented in this paper provides an example how system analysts may contribute to this process and to the ultimate identification of a solution acceptable to all concerned. The procedure is in fact a type of reference objective method as proposed by Wierzbicki (1975, 1979).

The work discussed herewith was inspired by the situation encountered in the Kävlinge River System in the region of Western Skåne in Sweden. The Western Skåne is one of the IIASA's case studies concerned with regional water management planning. In Figure 1, general scheme of the Kävlinge River System is shown (data characterizing the system are specified in Table 1 of this paper). Water resources of the Kävlinge River are to be allocated to several different users. First, the regulated Vomb Lake serves as a source of municipal water supply for the city of Malmö. In the late 40's when the Vomb-Malmö supply scheme became operational, it was decided not only how much water Malmö may withdraw from Vomb but also certain restrictions concerning minimum flow in Kävlinge at control point A were imposed. The lake is also used for recreational purposes, and it is desired that water levels in the lake do not deviate much from a certain level considered to be optimal for this particular purpose. In the early 70's a new water-user emerged in the Kävlinge system. During a few consecutive dry years, local farmers began withdrawing considerable amount of water from the river for irrigation purposes. These additional water withdrawals aggravated not only the water quantity but also the water quality problems in the Kävlinge system; concentration of chemicals due to fertilization practices has increased substantially. It becomes clear that occasionally, especially during the low-flow periods, complete satisfaction of all water requirements in the system is impossible, both in the sense of water quantity and its quality.

The analytical procedure presented in this paper was developed as an aid to be used interactively by those involved in the decisions concerning allocation of water resources in the system under the resource scarcity situations. The procedure can be used in the water management planning context to develop certain rules how to allocate resources at the different levels of their scarcity. It can also be used in the operational context to make allocational decisions in the face of current water availability situation.

THE MODEL

Description of Decision Variables

The model contains 11 decision variables (see Figure 1):

x_1 - irrigation withdrawals for agricultural area 114 (m^3/ha),

x_2 - irrigated part of agricultural area 114 (ha),

x_3 - release from Vomb Lake (m^3/s),

x_4 - irrigation withdrawals for agricultural area 104 (m^3/ha),

x_5 - irrigated part of agricultural area 104 (ha),

x_6 - irrigation withdrawals for agricultural area 64 (m^3/ha),

x_7 - irrigated part of agricultural area 64 (ha),

x_8 - water intake for Malmö (m^3/s),

x_9 - application rate of fertilization for the irrigated part of agricultural area 114 (kg/ha),

x_{10} - application rate of fertilization for the irrigated part of agricultural area 104 (kg/ha),

x_{11} - application rate of fertilization for the irrigated part of agricultural area 64 (kg/ha).

Objective Functions

The multiobjective optimization problem has the following noncommensurable objective functions:

- Yield effects of irrigation and fertilization in agricultural areas 114, 104 and 64 :

$$J_1(\underline{x}) = f_1(x_1, x_2, x_9) \quad (1)$$

$$J_2(\underline{x}) = f_2(x_4, x_5, x10) \quad (2)$$

$$J_3(\underline{x}) = f_3(x_6, x_7, x_{11}) \quad (3)$$

where f_1, f_2, f_3 are nonlinear functions (see Figure 2).

- Water deficit in Malmö:

$$J_4(\underline{x}) = P - x_8 \quad (4)$$

- Deviation from the minimum flow required at the control point A:

$$J_5(\underline{x}) = |QN - (x_3 - \phi_{104}x_4x_5\frac{1}{T} - \phi_{64}x_6x_7\frac{1}{T} + q_3 + q_4)| \quad (5)$$

- Deviation of the actual water storage level in Vomb Lake from the level optimal for recreation:

$$J_6(\underline{x}) = g[S_{opt}] - g[S_0 + (q_1 + q_2 - \phi_{114}x_1x_2\frac{1}{T} + x_3 - x_8) \frac{T}{1000000}] \quad (6)$$

where g is the volume curve of Vomb Lake (see Figure 3) and the argument of function g is storage volume in (Mm^3).

- Concentration of pollutants at the control point A (deviation from maximum acceptable concentration)

$$J_7(\underline{x}) = c_{15} - c_o \quad . \quad (7)$$

In the above functions P represents water requirements of Malmö (in m^3/s), QN is the minimum flow required at the control point A (in m^3/s), T is the length of time period (in s), ϕ_{114}, ϕ_{104}, and ϕ_{64} are water loss coefficients in agricultural areas 114, 104 and 64 respectively, q_1, q_2, q_3, q_4 are inflows to the system (in m^3/s), so is the initial storage volume of Vomb Lake (in m^3/s T), S_o is the maximum acceptable concentration of pollutants at the control point A (in mg/l), c_{15} is the actual concentration of pollutants at the control point A.

The functions J_1, J_2, J_3 should be maximized, functions J_4, J_5, J_6 and J_7 should be minimized. However, in order to simplify computations, the signs of functions J_1, J_2, J_3 were changed to obtain minimization of all objectives.

The first three objective functions represent the yield effects of irrigation and fertilization in three agricultural areas and are measured in natural units (for example tons of crop). The functions J_4 and J_5 are measured in m^3/s, function J_6 in meters, and function J_7 in mg/l.

Constraint Set

The set of 25 constraints can be divided into six groups. The first group of constraints states that none of the decision variables must be negative:

$$x_i \geq 0 \quad . \qquad i = 1, 2, \ldots, 11 \tag{8}$$

The second group states that the flows in all reaches of the system must be nonnegative:

$$x_1 x_2 \frac{1}{T} \leq q_1 \quad , \tag{9a}$$

$$x_4 x_5 \frac{1}{T} \leq x_3 + q_3 - x_6 x_7 \frac{1}{T} \quad , \tag{9b}$$

where q_1, q_3 are inflows to the system, T is the length of time period.

These two constraints are sufficient for nonnegativity of flows in all reaches.

The third group describes relationships between decision variables and physical constraints of each variable:

$$x_2 \leq A_{114} \tag{10a}$$

$$X_5 \leq A_{104} \tag{10b}$$

$$x_7 \leq A_{64} \tag{10c}$$

$$x_9 \leq F_{114} \tag{10d}$$

$$x_{10} \leq F_{104} \tag{10e}$$

$$x_{11} \leq F_{64} \quad , \tag{10f}$$

where A_{114}, A_{104}, A_{64} are the maximum (potential) acreages available for irrigation in areas 114, 104 and 64 respectively, and F_{114}, F_{104}, F_{64} are the optimal application rates of fertilization in these areas.

The fourth group of constraints is that water inflow to each agricultural area cannot exceed the optimal irrigation rate:

$$x_1 \leq OR_{114} - p \tag{11a}$$

$$x_4 \leq OR_{104} - p \tag{11b}$$

$$x_6 \leq OR_{64} - p \quad , \tag{11c}$$

where OR_{114}, OR_{104}, OR_{64} are the optimal application rates of irrigation water, and p is precipitation (in m^3/ha).

The fifth group of constraints is related to the regulated Vomb Lake (storage reservoir):

$$x_3 + x_8 \leq S_0 + q_1 + q_2 + \phi_{114} x_1 x_2 \frac{1}{T} \quad , \tag{12a}$$

where S_0 is the initial storage volume of Vomb. This constraint means that the volume of water released from the reservoir cannot exceed the contents of the reservoir at the beginning of the period plus inflow into the reservoir during that period.

$$S_0 + q_1 + q_2 + \phi_{114} x_1 x_2 \frac{1}{T} - x_3 - x_8 \leq S \quad , \tag{12b}$$

where S is the total capacity of Vomb Lake in Mm^3. This constraint states that the contents of the reservoir at the end of the period considered cannot exceed the capacity of the reservoir.

The sixth group contains only one constraint which states that water intake for Malmö cannot exceed water requirements of this city:

$$x_8 \leq P \quad . \tag{13}$$

SOLUTION PROCEDURE

Mathematically, the above optimization problem may be written as follows:

$$\min \underline{J}(\underline{x}) = [-J_1(\underline{x}), -J_2(\underline{x}), -J_3(\underline{x}), J_4(\underline{x}), J_5(\underline{x}), J_6(\underline{x}), J_7(\underline{x})] \tag{14}$$

subject to:

$$g_i(\underline{x}) \leq 0 \qquad i = 1, 2, \ldots, 25$$

where the functions $J_j(x)$ and some $g_i(x)$ defined above are nonlinear.

The interactive procedure seeking the satisfactory nondominated solution of problem (14) is based on the Powell method (Powell, 1969) with penalty functions for the scalar optimization problem and on the constraint and weighting method (Cohon and Marks, 1975), which is in fact a type of reference objective method (Wierzbicki, 1975, 1979), for the multiobjective optimization problem.

The solution procedure can be described in four steps:

(1) A large number of feasible solution is generated. The solution $\hat{\underline{x}}$ for which the scalar objective function

$$F(\underline{x}) = \sum_{i=1}^{7} J_i(\underline{x}) \qquad (15)$$

has the best value, is taken as the initial point for the nonlinear optimization.

(2) Carry out the minimization of the nonlinear scalar problem

$$\min F(\underline{x}) = \sum_{i=1}^{7} J_i(\underline{x}) \qquad (16)$$

s.t.

$g_i(\underline{x}) \leq 0 \ , \quad i = 1, 2, \ldots, 25 \ ,$

applying the Powell method with penalty functions and taking the point $\hat{\underline{x}}$ as a starting point for the optimization. Let the result of optimization be $\underline{w}$.

(3) The result of optimization $\underline{w}$ is one of the set of noninferior solutions. The levels of objective functions attained for solution $\underline{w}$ are now known. This solution and levels of objective functions for $\underline{w}$ are presented to the decision-maker (DM). Moreover, all other pertinent information is also presented - for example, the maximal and minimal attainable levels of objective functions which are easy to estimate in the considered example. The DM has to answer two following questions:

- o The satisfaction of which objective should be improved?
- o How much the satisfaction of other objectives can be changed?

To answer these questions, the DM makes use of the computed and the extreme levels of objective functions $J_i(\underline{x})$. If no improvement is desired the multiobjective problem has been solved and vector $\underline{w}$ with $J_i(\underline{w})$

represents the ultimate solution. Otherwise, the DM indicates the objective function $J_n(\underline{x})$ which should be improved and specifies the values of constraints B_i (otherwise called reference objective levels) for other objective functions $J_1(\underline{x}), J_2(\underline{x}), \ldots, J_{n-1}(\underline{x}), \ldots, J_7(\underline{x})$.

(4) Define the new optimization problem:

$$
\begin{aligned}
&\min J_n(\underline{x}) \\
&\text{s.t.} \quad g_i(\underline{x}) \le 0 \qquad i=1, 2, \ldots, 25 \\
&\qquad J_1(\underline{x}) \le B_1 \\
&\qquad J_2(\underline{x}) \le B_2 \\
&\qquad \ldots\ldots\ldots\ldots \\
&\qquad J_{n-1}(\underline{x}) \le B_{n-1} \\
&\qquad J_{n+1}(\underline{x}) \le B_{n+1} \, . \\
&\qquad \ldots\ldots\ldots\ldots \\
&\qquad J_7(\underline{x}) \le B_7
\end{aligned}
\tag{17}
$$

where $B_k \ge J_k(\underline{w})$ for all k (18)

and at least one of inqeualities (18) must be satisfied as a strict inequality.

The result of optimization problem (17) $\underline{w}^1$ is such that $J_n(\underline{w}^1) < J_n(\underline{w})$ and satisfaction of at least one of the other objectives is worse than for the solution $\underline{w}$.

The result $\underline{w}^1$ and the levels of objective functions $\underline{J}(\underline{w}^1)$ are presented to DM (return to step (3)).

It should be noted that the solution of the new optimization problem in step (4) is clearly a noninferior solution, if the additional constraints (17) are strictly satisfied. However, the penalty function technique and Powell method are used for dealing with these constraints; it may happen that these constraints are not precisely met. On the other hand, if this technique is interpreted as a type of reference objective procedure with penalty scalarization, the noninferiority of the solution can be guaranteed even if the constraints (17) are not met. In fact, if one considers the penalty scalarizing function:

$$s(J(\underline{x})) = J_n(\underline{x}) + \rho \sum_{i \neq n} (J_i(\underline{x}) - B_i)_+^2 \quad , \tag{19}$$

where $\rho > 0$ is a penalty coefficient and $(y) = \max(0, y)$, then the function s is strictly order-preserving and its minimal points correspond to noninferior solutions even if $J_i(\underline{x}) > B_i$ (see Wierzbicki, 1979). Therefore, it is even possible to simplify the Powell method and stop its iterations earlier, when some of the new constraints (17) are not met precisely.

Moreover, after minimizing the penalty scalarizing function (19) under the constraints $g_i \leq 0$, i=1, .., 25, one can *a posteriori* determine the weighting coefficients λ_i or trade-offs between various objective functions. These coefficients are determined by:

$$\lambda_n = 1; \; \lambda_i = 2g(J_i(\hat{\underline{x}}) - \bar{B}_i)_+, \; i \neq n \quad , \tag{20}$$

where $\hat{\underline{x}}$ is the noninferior solution obtained through the minimization of $s(J(x))$ and $B_i = \bar{B}_i$, if the Powell method was not applied to the constraints (17); if the Powell method of shifting constraints was applied, $\bar{B}_i$ denotes B_i modified by the resulting shift of constraints. The *a posteriori* information on trade-offs can help the DM, since:

$$\frac{\partial J_n(\hat{\underline{x}})}{\partial J_i(\hat{\underline{x}})} = \lambda_i \tag{21}$$

and he knows, in first-order approximation, what an improvement of $J_n(\underline{x})$ he can expect if he changes the reference objective levels B_i for $J_i(\underline{x})$.

However, in the first implementation of the proposed procedure, no provision was made for presentation of the additional *a posteriori* information on trade-offs to the DM. Similarly, no other type of the penalty scalarizing function than (19) was preliminarily investigated. One could consider, for example, the following scalarizing function:

$$s(J(\underline{x})) = \sum_{i=1}^{7} J_i(\underline{x}) + \rho \sum_{i=1}^{7} (J_i(\underline{x}) - B_i)_+^2 \qquad (22)$$

and ask the DM to specify desired reference objective levels B_i for all objective functions; as shown by Wierzbicki (1979), these reference objective levels need not be attainable as long as the scalarizing function is order-preserving.

SOLUTION AND RESULTS

The general scheme of the system in question is shown in Figure 1. The system is characterized by several parameters describing water inflows, precipitation, concentration of pollutants, the reservoir, fertilization and irrigation application rates. These parameters are specified in Table 1 (because of the initial stage of the analysis some of them are of a hypothetical character).

The results of the test application of the proposed procedure are shown in Tables 2 and 3. In Table 2, computer printout of the first noninferior solution is presented. At the background of the worst and the best solution for each of the seven objective functions, their actual (computed) values are indicated together with the percentage to which each objective is satisfied. Next the values of all eleven decision variables are indicated as well as the initial point for the generation of the next noninferior solution. The DM is asked which objective is satisfied least to him. The answer is objective No. 4 which is satisfied in 0.1% only. Next the DM indicates the acceptable levels to which satisfaction of other objectives may be changed (not less than 250.0 for the first three objectives, and not more than 2.0, 1.6, 1.7 and 0.9 for the four remaining objectives).

In Table 3, all eight noninferior solutions are presented. It can be seen that in the second solution the degree of satisfaction of the objective No.4 has been improved (up to 25.4%), mostly at the cost of worsening the degree of satisfaction of the first three objectives. The DM was not entirely satisfied with the improvement concerning objective No.4 and he indicated that it should be improved further. He also indicated that the levels of the first three objectives should again be not less than 250.0, and the levels of the four remaining objectives should be not higher than 1.6, 1.8, 1.9 and 0.9, respectively.

The third noninferior solution indicates further improvement of the degree of satisfaction of objective No. 4 (up to 48.1%), mostly at the cost of objective No.5. The DM still wants to improve objective No.4 relaxing somewhat his requirements concerning objective No.6 (nor more than 2.4).

In accordance with the DM's wishes the degree of satisfaction of objective No.4 is next improved up to 72.1% at a cost of objective No.6 (the level of this objective function goes up to 2.4). The DM still wants to improve objective No. 4--now he is ready to sacrifice objectives Nos. 2 and 3 (not less than 200.0).

Moving to the next solution one can see that objective No.4 is now satisfied to 87.9% what the DM finds to be satisfactory. Now he turns his attention to objective No.2 and wants to see how much it can be improved, relaxing a bit his requirements concerning objectives No.1 (not less than 150.0) and No.6 (not more than 2.5).

In can be seen that the objective No.2 can be completely satisfied indeed, mostly at a cost of objective No.1. Now the DM wants to develop better understanding how much objective No.7 can be improved if he relaxes his requirements concerning objectives No.1 (not less than 100.0), Nos. 2 and 3 (not less than 80.0), and No.4 (not more than 0.5). Requirements concerning objective No.6 are a bit more stringent than before (not more than 2.4). The required degree of satisfaction of the remaining objectives Nos. 6 and 7 is maintained at the same level as before.

In the next solution it can be seen that objective No.7 can be improved very slightly only (from 89.2% to 89.5%); this is followed by some improvement of objectives Nos. 5 and 6, however, the degree of satisfaction of all first four objectives is considerably reduced. In spite of all that, the DM is determined to check once again how much can be gained in objective No.7 by almost complete relaxation of his requirements concerning all other objectives.

The results are presented in the last column of Table 3. At this point the DM decided that he has learned enough about the trade-offs among conflicting objectives in his allocation problem and the test application of the proposed procedure was brought to an end.

The computer program implementing the procedure is written in the FORTRAN language; its applicability was tested on the PDP 11/70, CYBER 70, and UNIVAC 1108 machines and for the seven objectives problem presented herewith generation of each non-inferior solution required always less than 1 s CPU.

CONCLUSIONS

At this point a legitimate question would be how the available resources are to be finally allocated among the conflicting objectives. This paper does not provide an answer to this question, since the procedure presented above does not employ any value judgements concerning the relative priorities among various objectives. It is inescapable that these priorities must be defined by the DM himself and the only question is when and how should they be defined. The authors of this paper do believe that instead of making *a priori* decision in this respect, it is much more advisable to define the priorities in the process of learning more about the trade-offs among the conflicting objectives. The procedure presented in this paper is intended to serve this purpose and the possibilities of its further improvement will continue to be investigated along the lines indicated in the text.

REFERENCES

Cohon, J.L., and D.H. Marks (1975). A Review and Evaluation of Multiobjective Programming Techniques. Water Resources Research, Vol. II, No.2, pp. 208-220.

Haimes, Y.Y., W.A. Hall, and H.T. Freedman (1975). Multiobjective Optimization in Water Resources Systems, Elsevier Scientific Publishing Company, New York.

Haith, D.A., and D.P. Loucks (1976). Multiobjective Water-Resources Planning. In A.K. Biswas (Ed.) Systems Approach to Water Management, McGraw-Hill, New York.

Major, D.C. (1977). Multiobjective Water Resource Planning. Amer. Geophysical Union, Washington, D.C.

March, J.C., and H.A. Simon (1958). Organizations. John Wiley and Sons, New York.

Powell, M.D.J. (1969). A Method for Nonlinear Constraints in Minimization Problems. In R. Fletcher, Optimization, Academic Press.

Wierzbicki, A.P. (1975). Penalty Methods in Solving Optimization Problems with Vector Performance Criteria. Proceedings of VI-th IFAC World Congress, Cambridge, Boston.

Wierzbicki, A.P. (1979). A Methodological Guide to Multiobjective Optimization. International Institute for Applied Systems Analysis, Laxenburg, Austria, Working Paper WP-79-122.

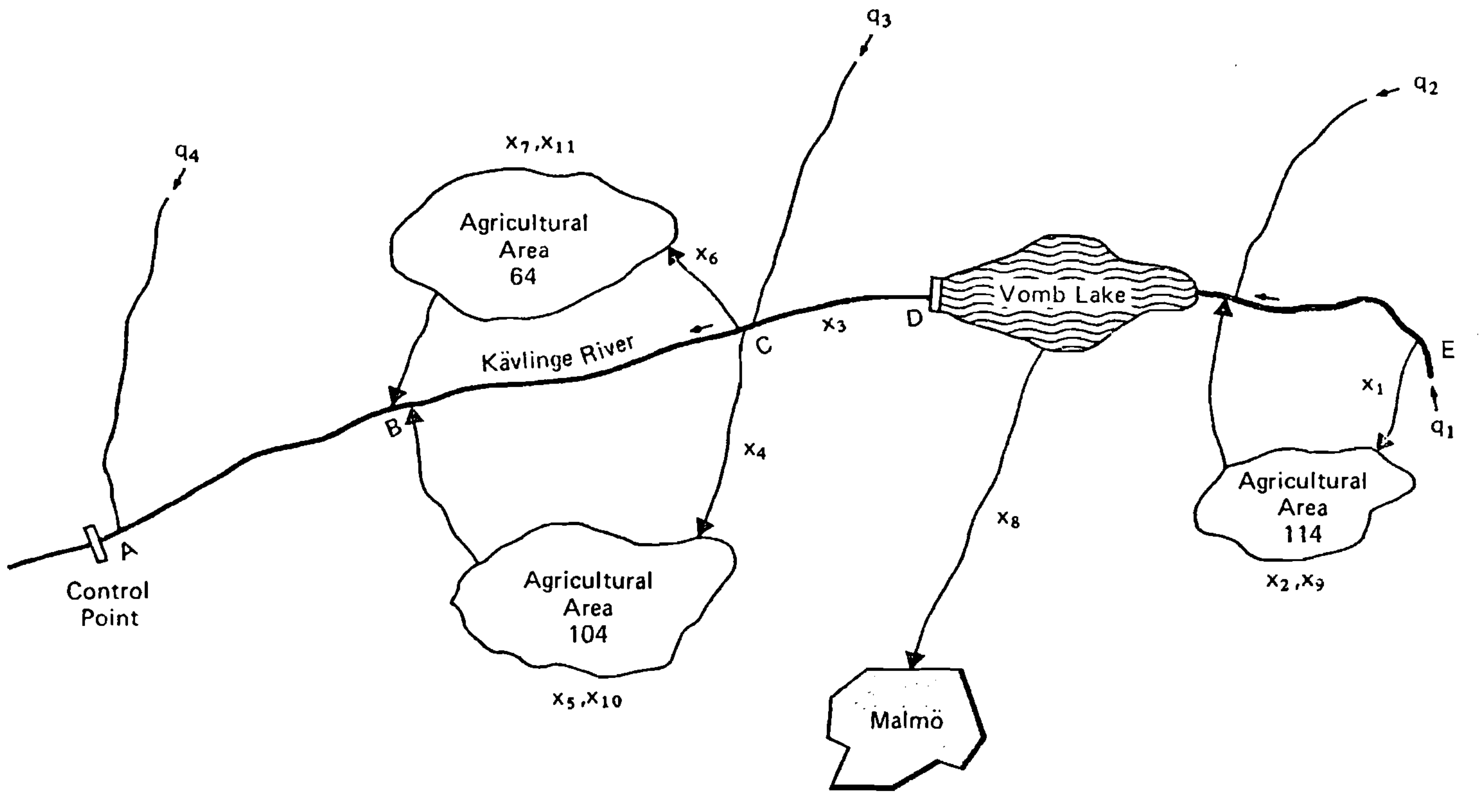

Figure 1. General Scheme of the Kävlinge River System

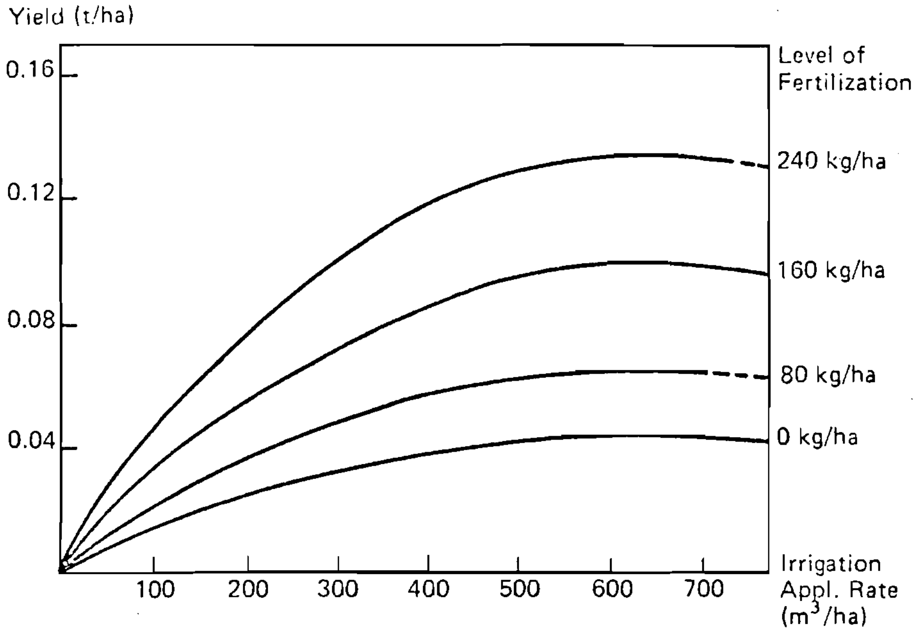

Figure 2. Yield Effects of Irrigation and Fertilization

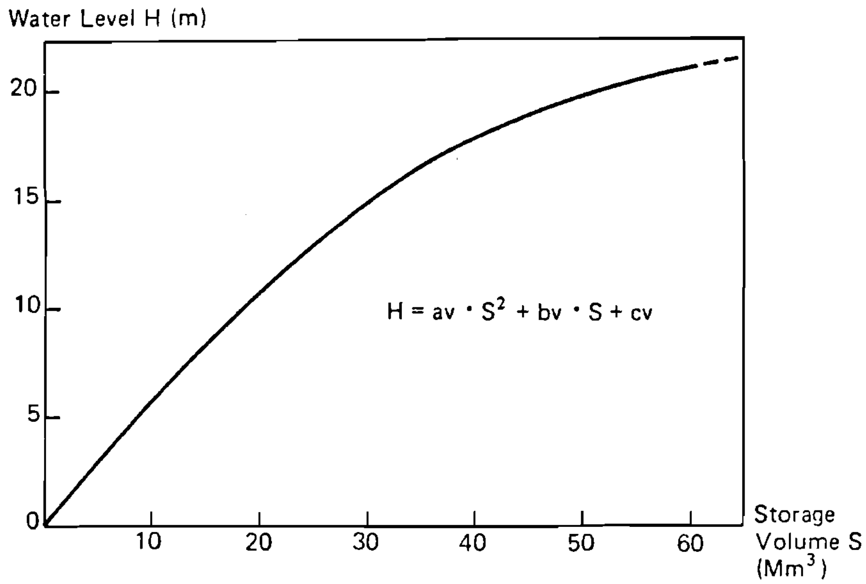

Figure 3. Volume Curve of the Vomb Lake

Table 1 System Parameters

Parameter	Symbol	Value	Unit
Inflows to the system	q_1	1.8	m^3/s
	q_2	1.5	m^3/s
	q_3	0.8	m^3/s
	q_4	0.7	m^3/s
Precipitation	p	10.0	mm
Concentration of pollutants:			
- minimum at control point A	c_o	0.05	mg/l
- maximum at control point A	c_{max}	10.0	mg/l
- initial concentration in four inflows	c_1	1.0	mg/l
	c_2	1.0	mg/l
	c_3	2.0	mg/l
	c_4	1.5	mg/l
Water requirements of Malmö	P	2.0	m^3/s
Initial storage volume of Vomb Lake	S_o	30.0	Mm^3
Capacity of Vomb Lake	S	80.0	Mm^3
Storage level of Vomb Lake optimal for recreation	S_{cpt}	29.0	Mm^3
Length of time period	T	2.59×10^6	s
Minimum required flow at the control point A	QN	6.0	m^3/s
Potential acreage available for irrigation	A_{114}	3000.0	ha
	A_{104}	2500.0	ha
	A_{64}	2300.0	ha

Parameter	Symbol	Value	Unit
Optimal application rate of fertilization:	F_{114}	150.0	kg/ha
	F_{104}	180.0	kg/ha
	F_{64}	180.0	kg/ha
Optimal application rate of irrigation:	OR_{114}	650.0	m^3/ha
	OR_{104}	650.0	m^3/ha
	OR_{64}	650.0	m^3/ha
Coefficients of relationship describing yield effects of irrigation and fertilization in all agricultural areas	a_1	-0.111	-
	b_1	1.42	-
	c_1	0.0	-
	a_2	-0.152	-
	b_2	1.91	-
	c_2	0.5	-
	a_3	-0.200	-
	b_3	2.70	-
	c_3	1.0	-
	a_4	-0.280	-
	b_4	3.68	-
	c_4	1.5	-
Levels of fertilization	l_1	0.0	kg/ha
	l_2	80.0	kg/ha
	l_3	160.0	kg/ha
	l_4	240.0	kg/ha
Coefficients of volume curve of the Vomb Lake	av	-0.0049	-
	bv	0.6464	-
	cv	0.0	-
Water loss coefficients in agricultural areas	Φ_{114}	0.8	-
	Φ_{104}	0.9	-
	Φ_{64}	0.8	-
Coefficient of the pollution's reduction in the Vomb Lake		0.9	

Table 2 Computer Printout of the First Noninferior Solution

NUMBER OF FUNCTION	THE WORST VALUE	THE COMPUTE VALUE	THE BEST VALUE	PERCENT
1	39.270	239.365	239.365	100.0
2	32.725	274.531	274.531	100.0
3	30.107	252.569	252.569	100.0
4	2.000	1.998	.000	.1
5	6.000	1.584	.000	79.8
6	14.625	1.647	.000	92.7
7	8.000	.819	.000	89.8

DECISION VARIABLES

INFLOW TO AGRICULTURE AREA 114 X1 = .66 M3/S
THE SURFACE OF IRRIGATED AREA 114 X2 =1003. HA
RELEASE FROM VOMBSJOEN X3 = 3.81 M3/S
INFLOW TO AGRICULTURE AREA 104 X4 = .55 M3/S
THE SURFACE OF IRRIGATED AREA 104 X5 =2500. HA
INFLOW TO AGRICULTURE AREA 64 X6 = .50 M3/S
THE SURFACE OF IRRIGATED AREA 64 X7 =2300. HA
WATER INFLOW TO USER MALMOE X8 = .00 M3/S
APPL. FERT. RATE FOR AREA 114 X9 = 150. KG/HA
APPL. FERT. RATE FOR AREA 104 X10 = 180. KG/HA
APPL. FERT. RATE FOR AREA 104 X11 = 180. KG/HA

INITIAL POINT FOR THE NEXT OPTIMIZATION PROBLEM

5.704	30.028	3.813	5.679	25.002	5.684
23.001	.002	15.000	18.000	18.000	

GIVE ME YOUR OPINION
PRINT NUMBER OF THE WORST FUNCTION
>4

THE WORST OBJECTIVE FUNCTION IN YOUR OPINION
IS FUNCTION NUMBER 4
PLEASE, GIVE CONSTRAINTS FOR OBJECTIVE FUNCTIONS
>250.0 250.0 250.0 2.0 1.6 1.7 0.9

CONSTRAINTS FOR OBJECTIVE FUNCTIONS
250.000 250.000 250.000 2.000 1.600 1.700 .900

Table 3 Test Application of the Procedure
(generation of 8 noninferior solutions)

Solution No.	Computed levels of objective function No:							Interaction with DM							
	1	2	3	4	5	6	7	No. of objective to be improved	Reference objective levels (constraints) 1	2	3	4	5	6	7
1	289.865 (100%)	274.531 (100%)	252.569 (100%)	1.998 (0.1%)	1.584 (79.8%)	1.647 (92.7%)	0.819 (89.8%)	4	250.0	250.0	250.0	2.0	1.6	1.7	0.9
2	250.001 (84.1)	250.378 (90.0)	250.491 (99.1)	1.567 (25.4)	1.601 (79.5)	1.700 (92.4)	0.838 (89.5)	4	250.0	250.0	250.0	1.6	1.8	1.9	0.9
3	249.998 (84.1)	250.000 (89.9)	249.998 (98.8)	1.157 (48.1)	1.802 (76.4)	1.901 (91.4)	0.855 (81.3)	4	250.0	250.0	250.0	1.2	1.8	2.4	0.9
4	250.017 (84.1)	250.011 (89.9)	249.999 (98.8)	0.690 (72.1)	1.806 (76.3)	2.401 (88.6)	0.855 (89.3)	4	250.0	200.0	200.0	0.7	1.8	2.4	0.9
5	249.999 (84.1)	200.003 (69.2)	200.000 (76.4)	0.344 (87.9)	1.835 (75.9)	2.397 (88.6)	0.880 (89.0)	2	150.0	200.0	200.0	0.4	1.8	2.5	0.9
6	152.233 (45.1)	274.555 (100.0)	199.999 (76.4)	0.345 (87.9)	1.773 (76.8)	2.476 (88.1)	0.867 (89.2)	7	100.0	80.0	80.0	0.5	1.8	2.4	0.9
7	122.068 (33.0)	80.018 (19.6)	80.001 (22.4)	0.506 (80.8)	1.124 (86.6)	2.395 (88.6)	0.841 (89.5)	7	50.0	50.0	50.0	2.0	6.0	14.0	0.8
8	97.634 (23.3)	50.068 (7.2)	74.880 (20.1)	1.068 (52.9)	6.000 (0.0)	12.978 (13.3)	0.570 (92.9)	-	-	-	-	-	-	-	-