THE APPLICATION OF GAME THEORY AND GAMING TO CONFLICT RESOLUTION IN REGIONAL PLANNING

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PREFACE

This working paper contains firstly, a revised version of a paper presented at the joint IIASA - IASI/CNR - IRPET Conference in Florence in April 1980. It also contains a report on a gaming experiment, related to regional planning carried out at this conference.

A very similar gaming experiment was carried out in Lund, in Southern Sweden with Swedish water planners in November, 1979. This game is described in IIASA's Working Paper WP-80-38. Both gaming experiments in turn refer to a study on water resources planning in Western Skane, Sweden. The actual planning situation as well as certain methodological problems involved in allocating costs in joint water projects are discussed in another IIASA Working Paper, WP-79-77.

The gaming experiment described in this paper is concerned not only with developing methods for regional planning, but also more generally, with developing gaming as a systems analysis tool. The playing of virtually identical games in different countries, is one of the focal points of the present IIASA research project on gaming, presented in IIASA's Working Paper WP-79-30.

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ABSTRACT

Firstly, the concepts of game situations, game, game theory, theory of game playing and gaming are introduced. Game theory and gaming are presented as system analysis tools, complementing each other, for the analysis of decisions in game situations.

Next, the great amount of game situations existing in regional planning are discussed. An attempt at a taxonomy is given, to indicate what kind of models can be brought in from other areas. Some examples of different situations are given.

Finally, the application of game theory and gaming to a specific problem, namely cost allocation in regional water resources management, is discussed and a small game is presented. The playing of this game in Italy by Tuscan regional planners is presented, and the outcome of this gaming experiment is compared with the solutions suggested by some game theoretical methods.

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THE APPLICATION OF GAME THEORY AND GAMING TO CONFLICT RESOLUTION IN REGIONAL PLANNING

I. Ståhl

INTRODUCTION

In regional planning there exist, just as in many other areas of economic life, a great many decision situations in which a particular feature is, that some decision makers are strategically dependent on other decision makers. This feature makes it suitable to sort out these decision situations separately, calling them game situations. The decision maker can in these game situations, by allowing the game situation to be modeled as a game, be aided by methods such as game theory, theory of game playing and gaming.

Some new concepts have been introduced here. It is, therefore, important to define these concepts. It should be stressed from the outcome that there is great confusion as to terminology in this area, and that the system of definition presented below is not in general acceptance, but neither is any other terminology in this area.*

Let us first define a "game situation". This is a decision situation in which the actions of the studied decision maker will noticeably influence the pay-off of some other decision maker, whose decision in turn will noticeably influence the pay-off of the first decision maker. If this influence to some extent is negative, i.e., when one party's pay-off goes up, the other one's will go down, we have a "conflict situation".

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*For a discussion of terminology in this area, see Ståhl (1979).

This is no doubt, the most important type of game situation. There is, however, also another group of game situations, namely "coordination situations". Take, for instance, two persons on each side of a door, moveable in both directions. If both persons push or both pull, both are worse off; if one pulls and the other one pushes, both are better off.

The theoretical problem with game situations is that one person cannot simply optimize, since there is at least one other person trying to optimize at the same time, and then will influence the results of the first one. Hence, the decision problem in game situations must be studied more carefully by methods such as game theory, theory of game playing and gaming.

All these three methods, to which we shall return with definitions in a moment, commonly deal with a model of the gaming situation, i.e., an abstract description of the main features of the gaming situation. Hence, the single word "game" is used in the special sense of being the more precise description, including the "rules of the game", etc., of the game situation that one might want to study.

Hence, the game chess can, e.g., be seen as a model, although crude, of the game situation "warfare in Persia 1000 B.C." and the game Monopoly as a model, just as crude, of the game situation,"real estate dealings in New York in the thirties".

Now, game theory is one special theory for studying games, namely that theory which presupposes some kind of rational behavior on the part of the game participants, i.e., the actors assumed in the game. This rationality includes, first of all, the assumptions of rational behavior in a non-conflict or non-game situation, i.e., a rationality which implies optimization. To these basic assumptions are added assumptions that specifically refer to the game situation, involving assumptions of correct expectations. In particular, we assume that every party realizes that all other parties are rational optimizers.

There are in some situations, however, reasons to be sceptical about the relevance of the game theoretic concepts not only as regards the use of the theory for the prediction of what will happen, but also for using the theory in a normative sense, for advice on what one shall do. This scepticism is partly due to gaming, i.e., to studies of how people actually have behaved when playing games. We will soon return to the concept of gaming, but it should be mentioned here, that in some games, intelligent decision makers, having ample time for their decision and high motivation to play well, have played consistently contrary to what game theory suggests. Hence, parallel to game theory, there is slowly evolving what I would like to call here "a theory of game playing".

To this one assigns all theory about game playing, both game theory and a theory regarding parties who are not fully "rational" in the way we have defined game theoretic rationality above. In particular, the assumptions regarding correct expectations are violated. It is clear that the latter theory would be far less built on axioms or on a small set of specific behavioral assumptions than game theory just discussed. The new theory would rely more on "heuristics" i.e., simple rules of thumb rather than the traditional game theory.

The difference between "game theory" and the more general "theory of game playing" can e.g., be exemplified by chess. Here game theory can, using one of the oldest theorems in game theory, (that of Zermelo, 1911), only say that there exists a solution in the form of an equilibrium point in so-called "pure strategies". This implies that there exists an optimal chess playing program which does not use any random devise in choosing the moves. The optimal program would thus be completely deterministic.

If we look at the "theory of chess playing", for example, developed for the computer chess programs, we have that successful computer chess programs, winning championship tournaments against other computer chess programs, have to involve random elements. For instance, a random number might determine whether the computer in a particular situation will move the queen two steps or three steps. Hence, the chess strategy chosen will not be a so-called pure strategy.

The reason for this random choice is as follows: The computer can only look, e.g., a handful number of moves ahead, say n moves.* If the program were completely deterministic the opponent could always get the chess playing computer into a trap by thinking n + 1 or n + 2 moves ahead. Because of the random choices, the computer will not be completely predictable and can hence not as easily be fooled into a trap.

Let us next go back to our final concept, namely that of gaming. Gaming implies the actual playing of a game, i.e, that actual human beings play the game, often portraying a real game situation.

We can distinguish between game experiments and gaming, using the first word for very simple games which do not to any extent try to portray any game situation existing in reality and which generally do not involve any interactive simulation. We shall hence reserve the word "gaming" for "interactive simulation, with more than one player in a game attempting to portray the main features of some real game situation".

In this case, it shall be mentioned that gaming can mainly be divided into two separate forms, rigid rule gaming and free form gaming.

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*See, e.g., Berliner (1973)

In rigid rule gaming, the game is precisely formulated prior to the start of the gaming, i.e., the model of the game situation in the form of rules, pay-off tables, etc., is constructed, usually by a game constructor, prior to the playing of the game.

In so-called free form gaming, the game, i.e., the model of the game situation, is at least partly constructed by the participants during the actual gaming exercise. Hence the model building is partly carried out by the gaming participants. The role of the game constructor might only be to present vaguely the game situation that shall be modeled.

We shall mainly deal with rigid form games here, since they are generally of the greatest interest for the specific type of operational games that we shall work at later. As understood from the comments regarding the theory of game playing, one of the purposes of gaming is to help move towards a more realistic and relevant theory of game playing, away from game theory in those instances where this theory has failed. One method of determining when game theory is valid is, as mentioned, gaming. If one, by repeated gaming, allowing various key variables as regards the institutional set-up to vary, has been unable to get the players to play according to, or even wanting to play according to game theory, one would probably be wise to question the fundamental assumptions of this game theory rather than seeking to apply the theory directly in practice. When looking for alternative assumptions one can then be aided by studying the actual game behavior.

There are for many games, several competing game theoretic concepts leading to different solutions. Gaming can then be used as an aid in choosing between these different, competing game theoretic concepts. We shall below look more closely at how gaming can be used to compare some different game theoretic concepts in a game depicting the allocation of cost for water resources to different regions or municipalities.

Furthermore, some game theoretic concepts present a large set of equally likely solutions. Gaming can also be used for distinguishing between the different solutions presented by one specific game theoretic concept.

It should be stressed that gaming in its turn, greatly relies on game theoretic concepts as concerns the definition of the rules of the game, and the setting up of the hypotheses to be tested. It also provides a language for analysing the gaming process.

We can hence see game theory and gaming as complementing each other and together providing an important systems analysis tool for studying actual game situations.

GAME SITUATIONS IN REGIONAL PLANNING

The question next arises as to whether these methods are applicable to decision situations of interest for regional planning. The question is: Do game situations really exist in regional planning?

The answer is yes. Some examples will be used to indicate that this is very much the case. We shall present these examples in the framework of a very preliminary taxonomy of game situations in regional planning in order to increase the possibilities of generalising out of the examples.

The main purpose of this attempt at a taxonomy is to make it somewhat easier to see what kind of game theory, or other type of theory, one could bring from other areas of study, mainly the economics of market behavior.

We shall first distinguish between horizontal and vertical game situations:

In horizontal game situations, the main part of the game takes place between different regions on the same decision level, i.e., of the same status in the decision hierarchy. In this connection, it should be stressed that we shall use the word region here in the simple sense of a separately administered geographical area. A region can hence be either a county, a city or a municipality. Hence, the horizontal games concern, e.g., games between counties, or games between cities or games between municipalities.

Vertical games to which we shall return later, involve game situations, where the decisions of the studied region are affected by decision units placed on some scale either "above" or "below" this studied region.

The horizontal conflicts can be seen as arising from much the same reasons as why pure competition will not generally lead to a socially optimal solution, i.e., the main causes are so-called external diseconomies and economies of scale.*

The word external diseconomies refers to the fact that one region will gain by taking actions that will affect another region negatively. Pollution is a good example; often one will find that polluting industries will be located at the very borders to other regions. If you let a region also stand for a country and let polluting industries be exemplified by nuclear plants, a good example is provided if you look at a map of nuclear installations in Europe.

The other region, with a polluting plant on its border, will take action, e.g., by itself locating or at least threatening to locate polluting plants close to the border of the first region.

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*See further, e.g., Quirk and Saposnick (1968).

The action of one region will be dependent on what one believes the other region will do.

For the study of these problems, concerning fairly similar units, the debate in game theory regarding whether one should apply so-called cooperative or non-cooperative game theory becomes fundamental. Is it possible for the regions to enforce cooperation? Are there too many for this? Have they got possibilites to compensate, by so-called side payments--that is a kind of bribe--those regions, who otherwise would gain comparatively little by cooperation and hence by more reluctant to cooperate? *

As regards the economies of scale, we have the case when e.g., two regions can produce a certain amount of services or goods more cheaply together in a joint plant than if each region produced on its own for only its own requirements. One example is hospitals run by several regions; another example is jointly owned facilities in the public utilities field, e.g., water production. One game situation arises here in connection with the cost allocation problem.

Let us exemplify this with the water production example. Regions can in many cases get their water demand fulfilled more cheaply by building a joint facility than by building separate ones. The amount of cost savings depends on which regions join together, the greatest cost savings often resulting when adjacent regions join together. A problem of cost allocation arises from the fact that the fixed costs of the construction of the plant cannot be assigned to the regions in any unique way. While both regions have a joint interest in joining together, they have opposing interests as to how to divide these cost savings. They can influence this share, e.g., by threatening not to join, to join with other regions instead, etc. We shall later refer to a more concrete example of how to solve this.

We next turn to the vertical game situations. The vertical game situations correspond very closely to vertical models for markets.

A division of the vertical models according to market theory would be to distinguish between a) situations where the studied region has only one counterpart on the other side, and b) situations where the studied region is one of many parties on one level in the systems hierarchy and c) where the studied region is alone on its level in the systems hierarchy, but has many counterparts on the other level. **

a. The most simple vertical game is obtained where there is one region bargaining with one other unit on another level, e.g., with the central government or with a large corporation.

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*See further, e.g., Telser (1972).
**See further, e.g., Shubik (1959) and Krelle (1961).

Here we have something similar to what is called bilateral monopoly in economic theory, a situation with one seller and one buyer. We can study such a situation with pure two person bargaining theory, a theory which is well-developed (see Stahl, 1972). We can, e.g., envisage a large corporation responsible for a considerable size of employment in the region, threatening to close down a plant unless it gets a strong subsidy in some form from the regional government. The size of the subsidy will hence be subject to some hard bargaining.

b. Let us next look at the case when the studied region is on the side with many parties in the studied vertical conflict, facing one opponent on the other level of hierarchy. Theoretically, one could compare this to a monopoly-oligopsony situation (i.e., with one buyer and several sellers) with a focus on the side of the competing buyers.

If we only have a pure fight for government grants, we have a common budgetary game, widely studied in literature.* This game has mainly been studied as regards the budget "infighting" among different government departments, where means are allocated partly on the basis of appropriation requests. The game situation now is that each region has to consider how much funds the other regions will demand. There might be a general slash in appropriations affecting all regions in a similar way. The size of this slash will be dependent on how much the sum of total requests will go above available funds. From this point of view, a region will be unwise if it demands considerably more than the other regions, since it might then become subject to extra investigations, etc., possibly resulting in special budget slashes. Hence, the demands of a particular region will be made in consideration of how much one believes the other regions will demand. This in turn, of course, will depend on what they believe about one's own demands.

Another game situation involving regions and central government, but of a slightly different type, is the game situation when the government has specific employment to offer, e.g., a government agent to locate, and different regions struggle to get this agency. This occurred in Sweden a couple of years ago, when a great many government agencies in Stockholm were to be relocated into the Swedish countryside. There was a great conflict between various regions as to which agencies should go to which region. At this stage, it would be suitable for a region not only to present the region's needs, but also stress the region's capabilities and offer concrete advantages. The game will hence concern more dimensions than the simple budget game.

A game situation similar to the struggle between regions over government agencies, is the game when regions compete for some major corporation to locate a plant. In this case, the

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*See Hofstede (1972).

needs of the region no longer matter, but it becomes a simple game of outbidding your competitors by offering larger loans, better training of personnel, etc. However, a region should not offer more than just to "win" the struggle. We get a bidding game, extensively studied in the literature*. Here the ability to guess what the other regions will offer becomes crucial.

c. We next turn to those vertical game situations where we can regard the region to be the sole decision maker on the one side, with many decision makers on the opposing side. These other decision makers can, e.g., be smaller corporations, land-owners, voters, etc.

An interesting game situation in this connection is what we can call the land-holder's game. In local planning, when e.g., building a road, a municipality might require getting some land plots. Let us assume 100 already held by private land owners. One way of aquiring the land is by ordinary purchases. The prices paid will generally go up higher and higher, as there are fewer and fewer land-plots left. If, for example, 99 land-plots have been bought, the remaining 100th land owner has then some kind of a veto on the whole project. He can then obtain a very high price.

Hence, at some stage in the process, the municipality will rather resort to expropriation than buying at these very high prices. This expropriation is, however, not free of cost. Not only are there legal costs, but more importantly, also costs of bad will, risks of protests and disruptions in face of expropriation . Hence, a region might not want to start expropriating at too early a stage. On the other hand, the land-owners will generally find it economically profitable to be the last one to be purchased prior to the start of the expropriations. Therefore, we have quite a complicated game situation, with the land owners trying to guess when expropriations start and when other land-owners are going to sell out, while the regional planners must try to guess how long different land-owners are going to try to delay their sales.

Although it is a complicated game situation, it is of relevance, at least in Sweden, where we have been asked to try to make a game to study this situation parallel to a computer simulation model, involving game theoretic ideas.

In this context, it should be mentioned that many gaming activities deal with land usage planning, and similar planning such as urban zoning planning etc. Most games used in these areas are, in contrast to the games focussed on here, of the free form type. This means that there are in these cases, smaller possibilities of using game theory as a complement to gaming. These types of games are therefore, of somewhat less interest for the present purpose. Since there are several excellent books in this area,

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*See, e.g., Amihud (1976) and Ponssard (1977) chapter 3.

we shall not deal further with these kinds of games.*

Finally, regarding the game of the regional decision-maker when against the voter, we arrive at a special kind of game, if we allow the regional decision-makers to in turn be divided into several groups, i.e., political parties. The focus of the game then shifts from the vertical interaction between government and other parties towards the horizontal struggle between the parties. We then have a well-known political game much discussed in the literature in political science.** The problem for the parties, is according to this literature on which voters a party shall focus its interest and hence, what opinion a party shall have on certain issues in order to maximize its votes.

Let us limit ourselves to the study of one particular issue. Let us assume that the parties involved here, prior to declaring their opinion, have equal possibilities of attracting the voters. The question is then, where a party should "locate" its political stand along a dimension where the opinion of the voters are assumed to be fairly evenly distributed. This has in the literature been compared to the decision of where to locate your ice cream stand on a beach, where the people are well spread out and will buy from the nearest ice cream stand. Theory will then show that, if there are two parties, they should both locate right in the middle. If there are four parties, then two should locate at the 25 per cent point and the other two at the 75 per cent point. See Figure 1.

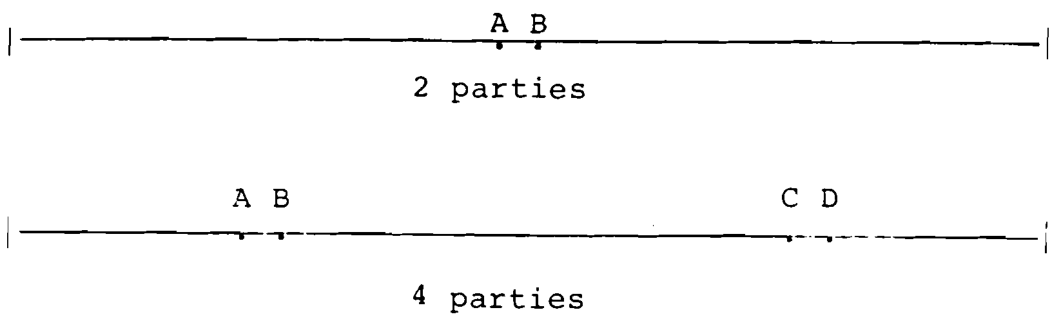

Figure 1. Location of political parties on an issue.

GAME THEORETICAL ASPECTS OF COST ALLOCATION

As indicated earlier, one actual case of application of both game theory and gaming concerns cost allocation in water resources management; in particular, the Southern Sweden Water Case studied by IIASA. The application of game theory to the actual problem was carried out by P. Young, N. Okada and T. Hashimoto (1979).

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*See e.g., Duke (1974) Feldt (1972) and in Italian Bottari (1978).

**See e.g., Brams (1975) and Ordeshook (1978).

First, their work will be reviewed briefly. The question is how costs should be allocated in a water project when different municipalities join together to develop water supplies. The cost allocation problem arises from the fact that there are, in general, economies of scale in the construction of water facilities. One problem of cost allocation in the case of such a coalition of municipalities is that the fixed costs of construction of the plant cannot be assigned to the municipalities in any unique way.

One can only propose various principles on which such allocations should depend. Suitable principles can be found, e.g., in game theory. One principle,"individual rationality", is that no municipality shall pay a higher cost than it would have to pay, if it were to fulfill its water needs completely on its own. Another principle, which we call the "full cost" principle, is that total costs should be covered, leaving no surplus and no loss to any third party. In this specific case, we study a situation in which six municipalities are involved, and where the total cost when all six cooperate, is lower than the total cost of any other combination.

Basic rationality principles would then further say that the "grand" coalition of all six municipalities should be formed, since each party can then be in a better situation than it could be under another arrangement involving higher total costs.

Furthermore, one can add an additional demand, namely "group rationality", implying that the sum of payments made by the members of every coalition which is smaller than the grand coalition, should not be larger than the cost that this coalition incurs if it is working on its own. This implies, for instance, that the coalition consisting of parties 1,2, and 3 would <u>not</u> agree to paying x_1,x_2,x_3, if the payments $x_1 + x_2 + x_3$ are higher than the total costs would be to these three parties if they only formed the three-party coalition 123.

The set of all allocations satisfying the principles stated above, (individual rationality, full cost, grand coalition, group rationality), are said to constitute "the core". In some cases, the core might not exist; in many cases, like the one studied in this concrete case, it exists, but is in no way unique.

There are several ways of obtaining a unique allocation within the core. In this case, three are discussed: the Nucleolus, the Proportional Least Core, and the Weak Least Core. Common to all of these concepts is that the solution is obtained by application of linear programming, where one seeks to minimize some kind of subsidy rate and where one deducts some transformation of this subsidy rate from the total cost of each coalition. The constraints of the LP-program, hence, imply the requirements that the sum of the payments made by the members of a specific coalition should not exceed the total costs of this coalition <u>minus</u> some transformation of this subsidy rate.

The three ways of obtaining a core solution differ with respect to the way the subsidy rate is transformed. The LP-program determines a unique value of the subsidy rate, which in turn gives a unique cost allocation. The three core concepts also differ with regard to the extent they satisfy the so-called monotonicity principle, that if total costs go up, no party should be charged more.

A fourth solution concept based on game theory, but not necessarily within the core, was also discussed: The Shapley Value. One way of representing this value is the following: The grand coalition is formed step by step; first one party joins together with another part to form a two-party coalition. Then one more party is added to form a three-party coalition, and then another party is added to form a four-party coalition, etc., until finally the grand coalition is formed. There are many (in an n-person game n!) ways or orders in which such a procedure can take place, depending on which party "signs up" first, and which party "signs up" next. For each order, a party joining a coalition is thought only to pay the incremental costs (i.e., the difference between the cost of the new coalition and the cost of the one he joins). The Shapley value for each party is then the party's average payments, computed over all n! coalition formation orders.

Finally, a fifth method was presented. This was a modified version of the Separable Cost-Remaining Benefits (SCRB). This method has been developed specifically for practical use in water resources planning. We define the marginal cost for a party as the marginal cost of being the last to join the grand coalition. Next the "remaining benefit" is defined as the difference between the cost if the municipality goes alone and its marginal costs. The payment made by a party is then computed as the marginal cost plus its share of the non-allocated costs, where the share is set in relation to the party's share of remaining benefits.

On the basis of the real situation in Southern Sweden, a cost table for various coalitions was computed. Although in reality there are 18 municipalities in this region, it was found practical and realistic to group these into six units which for this purpose can be regarded as acting as independent municipalities. It is sufficient to note here that the symbols A,H,K,L,M and T denote the main municipalities in each group, with L denoting the university town Lund and M, Malmö, the largest city in the region. The joint cost function was thus computed for each of the possible coalitions that these six municipality groups could form. The results are in table 1 below, where costs are specified in millions of Swedish Crowns.

On the basis of this data, the allocations were computed according to the five procedures discussed above, and also on the basis of population and demand. These cost allocations in millions of Swedish Crowns are given in table 2.

Table 1. Total cost of each possible coalition.

A	21.95	AHK	40.74	AHKL	48.95
H	17.08	AHL	43.22	AHKM	60.25
K	10.91	AHM	55.50	AHKT	62.72
L	15.88	AHT	56.67	AHLM	64.03
M	20.81	AKL	48.74	AHLT	65.20
T	21.98	AKM	53.40	AHMT	74.10
		AKT	54.85	AKLM	63.96
AH	34.69	ALM	53.05	AKLT	70.72
AK	32.86	ALT	59.81	ALMT	73.41
AL	37.83	AMT	61.36	HKLM	48.07
AM	42.76	HKL	27.26	HKLT	49.24
AT	43.93	HKM	42.55	HKMT	59.35
HK	22.96	HKT	44.94	HLMT	64.41
HL	25.00	HLM	45.81	KLMT	56.61
HM	37.89	HLT	46.98	AKMT	72.27
HT	39.06	HMT	56.49	AHKLM	69.76
KL	26.79	KLM	42.01	AHKMT	77.42
KM	31.45	KLT	48.77	AHLMT	83.00
KT	32.89	KMT	50.32	AHKLT	70.93
LM	31.10	LMT	51.46	AKLMT	73.97
LT	37.86			HKLMT	66.46
MT	39.41				
				AHKLMT	83.82

Table 2: Allocations in Millions of Swedish Crowns

Method	A	H	K	L	M	T
Shapley Value	20.01	10.71	6.61	10.37	16.94	19.18
Nucleolus	20.35	12.06	5.00	8.61	18.60	19.21
Proportional Least Core	19.81	12.57	4.35	9.25	19.85	17.99
Weak Least Core	20.03	12.52	3.94	9.07	20.11	18.15
S.C.R.B.	19.54	13.28	5.62	10.90	16.66	17.82
Population Proportional	10.13	21.00	3.19	8.22	34.22	7.07
Demand Proportional	13.33	16.32	7.43	7.00	29.04	10.69

Comparing these two tables, it can first of all be seen that the allocations based on population and demand violate the principle of individual rationality, for example for M, who on his own can get away with 20.81 (see table 1).

Furthermore, the SCRB and the Shapley value procedures can be criticized because none of them satisfies the principle of group rationality. Let us look at the coalition HKL. According to the SCRB procedure, HKL shall together pay 29.80 and according to the Shapley value 27.69 Should they not join the grand coalition, but remain satisfied with the three-party coalition HKL, they would only have to pay the cost of this coalition, or 27.26. Hence, neither the SCRB nor the Shapley value belong to the core.

The authors next investigate the three core solutions. They arrive at preferring the Weak Least Core, since it is only one of the three which always satisfies the monotonicity principle mentioned earlier. The Nucleolus violates this principle in this specific game; the Proportional Least Core, while satisfying the monotonicity principle in this particular game, does not fulfill it in all games.

GAME ON COST ALLOCATION

On the basis of this work, in particular on the basis of the table depicting the total cost of each possible coalition, a game has been constructed.* The details of the game is presented in the form of a game manual given as an appendix.

A first version of this game was played in Lund in Southern Sweden in November 1979. A second version was played in connection with the joint IIASA-IASI/CNR-IRPET Conference in Florence on the 9th of April 1980. The details of this game are best understood by a study of the manual presented as an appendix. This version differs only in some small details from the Lund game. The main difference is that in the Swedish game we did not present the data on population and water demand. Another rather important difference was that we were somewhat more liberal on time limits in the Italian game than in the Swedish game so that the game would not end prematurely. A less important difference was that the pay-offs were in lira, not Swedish crowns.

The reasoning behind the original gaming excercise is discussed at length in Stahl (1980). Here it suffices to mention that the main purpose was to test the relevance of the game theoretic concepts discussed above. The results of the Lund game which were partly at odds with the normative implications of some of these game theoretic concepts, made it important to play the game several more times, preferably in different countries as well. Thus, when an opportunity to play the game with

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*More details about the game is presented in Stahl (1980).

Tuscan regional planners in Florence was given, we were happy to utilize the opportunity.

Prior to the playing of the game in Lund, we made a forecast of how the game would be played in Lund, basing the forecast on how people had behaved in other games. This forecast proved to be almost as good a predictor as the game theoretic concepts which gave the best prediction, namely the Shapley Value. The Nucleolus gave almost as good a prediction, while the Least Weak Core, which as indicated above, was preferred in the more normative game theoretic analysis, was a less efficient predictor of the actual outcome in Lund.

Since the games played in Lund and Florence are virtually identical and since there did not appear to be any possibilities of saying a priori how regional planners in Italy would differ from regional planners in Sweden, our forecast was that the result would be the same as it would have been in Sweden if the same time limits had been used as in the Italian game*, namely, the following distribution: A ≐ 21.15, H = 9.70, K = 6.00, L = 9.10, M = 18.37, T = 19.50.

THE PLAYING OF THE GAME IN ITALY

The gaming experiment was carried out at Park Palace Hotel, in Florence, in the evening of April 9, 1980. The participants had been hand-picked by the IRPET management. They all had, as far as we could see, a substantial knowledge about regional planning. In fact, some wield considerable power with regard to regional planning in Tuscany.

The game took place around a small table with seats for the six players, the game leader and an interpreter. In order to minimize the difference with regard to the Swedish game, the same configuration for the players was used as in Sweden. This configuration (figure 2) is shown below.

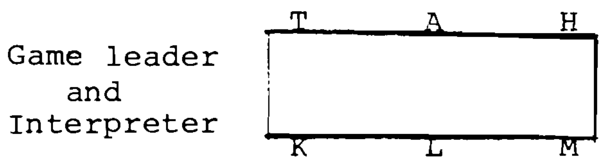

Figure 2. Seating plan

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*In Lund, the game was, due to more stringent time limits, broken up prior to the reaching of an agreement on the grand coalition. Since such an agreement was being discussed by the game participants, it was possible to determine within fairly close limits, what agreement could ultimately have been reached if there had not been this time restriction. This is the data used for the forecast here (see Stahl 1980, p.21).

The municipality roles were randomly allotted to the six players.

The game instruction had been distributed earlier to the players, both in English (i.e., the one presented in the appendix) and in Italian. This was all the information about the game that was available to the participants at the time.* The few questions that the participants had concerning the game could all be answered by referring to the manual.

The game then started. After a cautious start, H and L approached each other, but they did not form a coalition, seeming to prefer to form a larger coalition at once. Instead after 11 minutes, M and T formed a coalition with the distribution 19.17 to M and 20.24 to T.** This division is reached when the total costs of the two party coalition MT, 39.41, is split in proportion to the costs obtained, if each one had been on his own, i.e., M pays:

$$\frac{20.81}{20.81 + 21.98} \; 39.41 = 19.17$$

The game then focussed on a more general discussion of principles of division. For example, there was a discussion of whether the division should only be focussed on the costs of various coalitions or whether one should also take demand and population figures into account. Concerning the latter there was a problem as to whether demand or population should be considered, or if one should use some combination of these two factors.

The discussion thus went on, and at first, there was not even a consensus on whether a grand coalition should be formed or not. After 46 minutes from the beginning, there was, however, a general consensus among the six players, that a grand coalition should be formed. The arguments concerning which division scheme to use lasted 30 minutes.*** Then an agreement

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*The original version of this paper containing everything up to page 13, was distributed to the conference participants on the day after the game. Furthermore, none of the game participants had seen IIASA WP-80-38 or WP-79-77.

**All pay-offs are expressed in hundreds of lire to make comparisons with the tables presented above easier.

***In the meantime, in order for M and T not to drop out of the game, they formed a new coalition every 15 minutes, with a slightly different pay-off division each time.

was reached on the following division: A = 20.81, H = 9.55, K = 6.10, L = 8.88, M = 18.72, T = 19.77.

The reasoning behind this division, worked out mainly by players H and M, but accepted after some arguments by the others, was a three-phase division scheme involving hypothetical coalitions.

In phase 1, one assumed a coalition HKL to be formed. This is the three party coalition which gets the largest cost savings compared to what the parties can make on their own, i.e., compared to the "one-party" coalition pay-offs. These savings are then allocated to the parties in proportion to these "one-party" coalition costs. The division scheme is illustrated by table 3 (on p. 17).

In phase 2, the hypothetical coalition HKLMT was formed with M and T joining as separate units the already formed coalition HKL.* The "in-coming" costs in phase 2 of H, K and L were the "outgoing" costs of these members in phase 1, while the "in-coming" costs of M and T were their "one-coalition costs." The costs savings c(HKL) + c(M) + c(T) - c(HKLMT) were then distributed according to these "in-coming" costs and then deducted from the "in-coming" costs" to form "out-going" costs.**

Finally, in the third and final phase, party A was brought in and the grand coalition was formed. The five other parties went in with their "outgoing" costs from phase 2 while A went in with his "one-party coalition" cost. The cost savings, c(A) + c(HKLMT) - c(AHKLMT) were distributed to the parties in accordance with these "in-coming" costs. The final costs were then obtained by deducting the cost savings from these "in-coming" costs.

One can formulate this procedure in a general, recursive manner. In a phase j + 1, the "old" members, i.e., those participating in the coalition already in phase j, come in with the "out-going" costs of phase j. The "in-coming" costs of the new members are their "one-party coalition" costs. The cost savings, i.e., the sum of the "in-coming" costs minus the costs of the coalition formed, are then distributed to the members in proportion to their "in-coming" costs. The "outgoing" costs of

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*It is somewhat surprising that an assumption was made to the joining of M and T as separate units considering that the coalition MT had already been formed. It would have been more consistent with actual behavior if M and T had gone into phase 2 with 19.17 and 20.24 (see page 15) as "in-coming" costs rather than 20.81 and 21.98 (see table 1). If this had been the case, M and T would in the final agreement have paid 37.24 together instead of 38.49. This difference is, however, not big enough to have changed the general conclusions with regard to the game.

**c(HKL) denotes the costs of the coalition HKL, c(M) the costs of M going alone etc.

of the members in phase j + 1 are then obtained by deducting the costs savings from the "in-coming" costs.

Table 3: Allocation procedure in phase 1.

	Column 1	Column 2	Column 3	Column 4
	Incoming costs = one party coalition costs	Percentage of 43.87	Savings distributed according to column 2.	Outgoing cost = Incoming costs - savings (column 1 - column 3).
H	17.08	38.9%	6.46	10.62
K	10.91	24.9%	4.13	6.78
L	15.88	36.2%	6.01	9.86
Sum	43.87		16.61	27.26
- C(HKL)	-27.26			
Savings	16.61			

With the procedure starting in phase 1 with all "in-coming" costs as "one-party coalition" costs, the procedure is uniquely defined, provided one defines which of the parties "sign up" in the different phases. This signing up procedure seemed to follow the principle of which additions gave the largest marginal increase.

A procedure leading more directly in each phase to the same result, but involving a slightly different reasoning, is to directly allocate the costs of the new coalition to the members of the coalition in proportion to the "in-coming" costs. This method is in the literature known as the "Justifiable expenditure method". (See Eckstein 1958, and James and Lee 1971). It can easily be seen that this procedure leads to the same result, if we apply both procedures to e.g., a two party coalition case, with the parties called A and B. The costs savings are: $c(A) + c(B) - c(AB)$ and A's share of "in-coming" costs are:

$$\frac{c(A)}{c(A) + c(B)}$$

Hence:

$$x_A = c(A) - \frac{c(A)}{c(A) + c(B)} (c(A) + c(B) - c(AB)) =$$

$$\frac{c(A)(c(A) + c(B)) - c(A)(c(A) + c(B) - c(AB))}{c(A) + c(B)} =$$

$$\frac{c(A) \cdot c(AB)}{c(A) + c(B)} ,$$

which is the formula used in the direct allocation.

COMPARISON OF METHODS AND EXPERIMENTAL RESULTS

Having explained how the final agreement was reached, we can now compare the agreement with the solution proposed by the different theoretical concepts presented above, and with the solution from the Swedish game. In order to facilitate comparisons we sum up all values in table 4.

Table 4. Comparisons of different methods with outcome.

	A	H	K	L	M	T
Demand proportional	13.33	16.32	7.43	7.00	29.04	10.69
Population prop.	10.13	21.00	3.19	8.22	34.22	7.07
SCRB	19.54	13.28	5.62	10.90	16.66	17.82
Shapley Value	20.01	10.71	6.61	10.37	16.94	19.18
Nucleolus	20.35	12.06	5.00	8.61	18.60	19.21
Prop Least Core	19.81	12.57	4.35	9.25	19.85	17.99
Weak Least Core	20.03	12.52	3.94	9.07	20.11	18.15
Swedish Game	21.15	9.70	6.00	9.10	18.37	19.50
Italian Game	20.81	9.55	6.10	8.88	18.72	19.77

Already a brief glance reveals how close the results of the Italian game are to those of the Swedish game. Furthermore, it can be seen that the population and demand figures have little significance to the final distribution.

In order to make the comparisons easier, however, we shall utilize three different measures of difference:

1. The sum of absolute differences. With T_i as the theoretical value and E_i as the experimental value for party i, the measure is:

$$\sum_{i=1}^{6} | T_i - E_i |$$

2. The sum of the squared differences:

$$\sum_{i=1}^{6} (T_i - E_i)^2$$

Compared to measure 1, this gives a higher relative weight to large discrepancies.

3. The sum of the relative squared differences, i.e., of the squared differences after dividing each difference by the theoretical value:

$$\sum_{i=1}^{6} (T_i - E_i)^2 / T_i \quad .$$

The idea behind this measure is that a difference is more important if it is relatively large in comparison with the "expected" value.

Applying these measures to the data in the table above, we obtain, listing the methods in order of size (pair-wise of at least two measures), the following table (Table 5):

Table 5. Measured differences between outcome of Italian game and different methods.

	Difference Measures		
	$\Sigma\|T-E\|$	$\Sigma(T-E)^2$	$\Sigma\frac{(T-E)^2}{T}$
Swedish Game	1.44	0.39	0.07
Shapley Value	6.34	8.00	0.62
Nucleolus	5.02	8.14	0.80
Proport. Least Core	9.06	17.80	1.74
Weak Least Core	9.12	18.72	2.16
SCRB	11.52	27.93	2.02
Demand Proportional	36.87	296.17	19.14
Population Proportional	53.90	655.82	50.06

From this table, we really appreciate the superiority of the forecast that the outcome of the Italian game would be the same as in Sweden. We also see how very poor the forecasts are, that the division would be proportional to demand and population figures.

Furthermore, we can study the differences between the more theoretical methods. The ranking order here is the same as in the Swedish case. It is of interest to note that the Shapley Value is on top, slightly ahead of the Nucleolus, while the Weak Least Core, which from the normative point of view had the most desirable properties, comes at the bottom among the game theoretic methods. The SCRB is, however, on two measures of difference, worse than the Weak Least Core.

It should, however, be stressed that the game theoretic methods are all fairly close together, as can be seen by the large jump to the allocations based on demand.

Finally, one can test whether the outcome lies within the core. Since the grand coalition has been formed and the distributions add up to the grand total*, it remains to test the principle of group rationality, i.e., to see if some set of players could have gotten more by forming a coalition on their own. One then finds that the group rationality is indeed violated. A,K,L,M and T together have to pay 74.28, but by forming the coalition AKLMT, they could have gotten away with 73.97, as seen from Table 1. This is, however, the only violation of the core concept.

It should be mentioned that in the Swedish game, the core concept would have also been violated only once, and also with regard to AKLMT, since these parties in the Swedish game would have had to pay a total of 74.12.**

The reason for these violations of the core concept, is the strong position that H seems to have in the early stage of the game.*** Since H therefore gets away with a lower cost, the remaining parties have to pay more. At a later stage of the game when the grand coalition is formed, the idea of excluding H does not appear to enter into anybody's mind.

COMMENTS ON THE OUTCOME OF THE GAME

The most striking result of the game was the closeness of the outcome to the game in Sweden. There are two main reasons

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*Since the actual distribution in fact concerned one more decimal than given here, the total adds up to 83.83 instead of 83.82 due to rounding off errors.

**If the final coalition had been formed, see footnote on page 14.

***This will be commented on below (page 21).

for this closeness. The first one is that the division of the costs of the grand coalition were in both cases based on a procedure involving (the real or hypothetical) formation of successively larger coalitions, with HKL first being formed, then HKLMT, and finally the grand coalition. The second reason is the following: The important division problem concerns the HKL coalition since it is here that the great cost savings are obtained. In both the Swedish and the Italian games, the same division was obtained; in Sweden by directly using the "justifiable expenditure method"; in Italy by using a method leading to identical results.

The second important result was that the final allocations were not based on demand or population. This was surprising to the extent that there was a long discussion between the gaming participants as to whether or not these figures should matter. The position of those who were arguing for the importance of demand and population was, however, weakened by their uncertainty as to whether demand or population or some combination of these figures should be used as an allocation criterion. One wonders whether their bargaining position would have been different if we had only supplied, e.g., the demand figures.*

Finally, we noticed that as in the Swedish game, the Shapley procedure fared best among the game theoretic models, closely followed by the Nucleolus, while the Weak Least Core had the poorest results. In particular, the Weak Least Core gives too low a value to K, but too high a value for H and M. In fact, all methods overestimate the cost to be paid by H, but for the Shapley value this error is not so large.

The problem of H is of special interest, since one of the players after the game, pointed out the strong strategic position of H, especially as regards the formation of the strategic coalition HKL. Since the coalition KL does not result in any savings, while the coalitions HK and HL result in considerable savings, H has a strong position as regards the division of the cost savings resulting from the formation of HKL. This fact plays, however, a very small role for the three solution methods based on the core**, since all coalition pay-offs are used as constraints on an equal basis, and the core does not take the successive coalition forming process into account. The procedure used to explain the Shapley value comprises, on the other hand, a scheme for the successive formation of coalitions, and one can wonder to what extent this accounts for the somewhat better predictive power of the Shapley Value.

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*It should be mentioned that in the Swedish game, we did not supply any figures on population and demand.

**The Nucleolus, the Proportional Least Core and the Weak Least Core.

IDEAS FOR FUTURE RESEARCH

As stressed already in Stahl (1980), a great many games would be required to draw any more general conclusions. Two game runs are obviously far from enough. The fact that the results of the Swedish and Italian games were so close, gives, however, additional stimulus to renewed playing of the game.

The present game gives some directions for the set up of further gaming experiments. As discussed above, the divisions to no extent reflected demand or population figures, although some parties agreed this should be the case. They were, however, uncertain as to whether demand or population figures, or a combination of these, should matter. Hence, it would be interesting to see if there would be any difference, if, e.g., only data on demand was given.

After the playing of the Swedish game, we also suggested some ideas for further theoretical work.* One concerned a modification of the Shapley value; another concerned work in the extensive form. The division scheme used in the Italian game also brings out the question of the relevancy of using the core concept only for the grand coalition. An alternative procedure would be to use some core concept in a first phase for the division of the costs of the coalition HKL. The same kind of division concept would then apply to MT. One would then re-apply this concept to the coalition HKLMT, seeing it as a two-party coalition consisting of the new parties HKL and MT. This would, e.g., determine the costs of the two groups, HKL and MT. The same procedure would then finally be used when bringing in A to form the grand coalition AHKLMT. The costs thus assigned to e.g., HKLMT as a group would then be allocated to HKL and MT, in proportion to how the costs of HKLMT were divided in the preceeding phase. Finally, the costs assigned to e.g., HKL as a group, would be allocated to the individual members H, K, and L in proportion to how the original costs of HKL were allocated to its members in the first phase.

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*See Stahl (1980) page 26

APPENDIX: GAME INSTRUCTIONS

You have been invited to participate in a simple game.

The game concerns the allocation of costs in a water project. This project aims at bringing stimulating liquid to six municipalities. You will represent one of these. On this occasion, as the sole representative of this municipality, you will represent both the producer and the consumer side.

You will participate in this project either completely on your own, or in cooperation with one or several of the other participants in the game, who are acting as representatives for other municipalities.

All in all, representatives of six municipalities, called A, H, K, L, M, and T, participate in the game. All participants (= municipalities) must in some way take part in the water project, but their costs will depend on how they form coalitions with other participants.

Should a municipality not enter into coalition with any other municipality, it will pay that sum in the allocated table which represents what each municipality would be obligated to pay if acting alone. Prior to the start of the game, each player, i.e. each representative of a municipality, will receive this sum in cash from the game leader.

Each player can, however, by acting skillfully both during the formation of coalitions and during the allocation of the total costs within the coalition, get away with a lower payment, in some cases, a considerably lower one.

The player may keep this surplus for himself (or if he wishes to do so, may donate it at the end of the game to a charitable purpose).

The details of the game are as follows:

By lottery, each player is assigned the role of the representative of one of the six municipalities. Next each player obtains the aforementioned sum of money corresponding to the maximum amount that he might have to pay, should he participate in the water project completely on his own. After this, the players sit down around the table and the coalition-formation negotiations can begin.

The players then must try to form coalitions and reach agreement on how much each of the participants in the formed coalition shall pay of the total cost to the whole coalition. This total cost for each possible coalition is seen in the attached table.

As soon as the first coalition has been formed and agreement has been reached as to the allocation of the total costs of this coalition among its members, they register the coalition with the game director. He will then record the names of the coalition participants, as well as the payment each of them would make toward the total costs of the coalition. Once a coalition has been registered, its content, i.e. the participants and the cost allocation, is announced to all participants of the game.

A coalition does not come into force, however, until 15 minutes have elapsed since its registration, and then only provided that none of its members has been registered in another coalition during this period. Hence a player can leave one coalition and join another in order to decrease the amount of his payment. Furthermore, a coalition dissolves by registering a new coalition with additional members. For new coalitions, the rule still applies that it does not come into force until it has been registered unchanged for 15 minutes.

Once a coalition has come into force, each of its members pays the game leader the amount agreed upon at the time of the registration. These participants then cease to take an active part in the game, but may remain at the table if they wish to do so.

The game continues in this way until all participants are members of a coalition which has come into force (with the possible exception of a single "leftover" participant). Should the game continue more than 90 minutes from the time of its start, it will be brought to an end and those coalitions registered (but not broken) at the time will come into force.

Finally it should be stressed that the game aims at bringing out some aspects of one of the papers presented at the conference. Hence it is important that you try as much as possible to act as one could expect a representative for a municipality to act during such negotiations, where the economic interest of the municipality are at stake.

TOTAL COST OF EACH POSSIBLE COALITION (IN LIRE)

A	2195	AHK	4074	AHKL	4895
H	1708	AHL	4322	AHKM	6025
K	1091	AHM	5550	AHKT	6272
L	1588	AHT	5667	AHLM	6403
M	2081	AKL	4874	AHLT	6520
T	2198	AKM	5340	AHMT	7410
		AKT	5485	AKLM	6396
AH	3469	ALM	5305	AKLT	7072
AK	3286	ALT	5981	ALMT	7341
AL	3783	AMT	6136	HKLM	4807
AM	4276	HKL	2726	HKLT	4924
AT	4393	HKM	4255	HKMT	5935
HK	2296	HKT	4494	HLMT	6441
HL	2500	HLM	4581	KLMT	5661
HM	3789	HLT	4698	AKMT	7227
HT	3906	HMT	5649	AHKLM	6976
KL	2679	KLM	4201	AHKMT	7742
KM	3145	KLT	4877	AHLMT	8300
KT	3289	KMT	5032	AHKLT	7093
LM	3110	LMT	5146	AKLMT	7397
LT	3786			HKLMT	6646
MT	3941			AHKLMT	8382

DATA ON POPULATION AND DEMAND

	A	H	K	L	M	T
Population (10^3)	85.0	176.3	26.8	69.0	287.3	59.5
Demand: (Mm^3/yr)	6.72	8.23	3.75	3.53	14.64	5.39

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