WORKING PAPER

URBANIZATION LEVEL AND URBAN CONCENTRATION: COMPARATIVE PATHS AND A PERFORMANCE INDEX

Ahmed Seifelnasr

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IIASA
International Institute for Applied Systems Analysis

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A-2361 Laxenburg, Austria

FOREWORD

Roughly 1.6 billion people, 40 percent of the world's population, live in urban areas today. At the beginning of the last century, the urban population of the world totaled only 25 million. According to recent United Nations estimates, about 3.1 billion people, twice today's urban population, will be living in urban areas by the year 2000.

Scholars and policy makers often disagree when it comes to evaluating the desirability of current rapid rates of urban growth and urbanization in many parts of the globe. Some see this trend as fostering national processes of socioeconomic development, particularly in the poorer and rapidly urbanizing countries of the Third World; whereas others believe the consequences to be largely undesirable and argue that such urban growth should be slowed down.

The controversy also extends to the way the urban population is distributed among different city size classes. In particular a condition of primacy is said to exist when an observed urban population distribution deviates from a given standard distribution, most commonly the rank-size distribution, and "overurbanization" is often diagnosed as the cause.

In this paper, Ahmed Seifelnasr, a demographer, analyzes the level, pace, and pattern of change in two urbanization dimensions: urbanization level and urban concentration. The emphasis is on regional and temporal comparisons. To facilitate the analysis a new measure called the measure of combined change is introduced and its use illustrated using the most recent data available. In addition, a graphic method that shows compactly the simultaneous change in the two dimensions is presented. Finally, the overall urbanization "performance" on these two dimensions is examined over time with the help of a summary index.

A list of the papers in the Population, Resources, and Growth Task appears at the end of this paper.

Andrei Rogers
Chairman
Human Settlements
and Services Area

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The theme of the paper, that is, the contrast between the urbanization level and urban concentration, arose mainly in discussion with Professor Andrei Rogers to whom the author is indebted.

The author is also grateful to L. Castro, U. Karlstrom, J. Ledent, and L. Mayhew for their helpful comments on an earlier draft. Special thanks are due to Maria Rogers for her excellent editing and to Susie Riley for her perfect typing.

Finally, the author would like to acknowledge his intellectual debt to all faculty and staff at the Department of Population Dynamics, the Johns Hopkins University, USA at which he was a doctoral student. Very special thanks are due in this regard to Professor Ismail Sirageldin for a rich and stimulating experience.

ABSTRACT

An attempt has been made to examine the development of the urbanization process in two basic dimensions: urbanization level and urban concentration.

To facilitate the regional and temporal comparisons, a summary measure that combines the two dimensions is introduced and its use is illustrated using data compiled in UN (1979). The historical development of the two dimensions, together with the measure, has been depicted on a graph which shows compactly the path that has been followed in the course of urbanization.

Finally, the urbanization "performance" (from the demographic point of view) is compared over time and across space with the help of a comparative index that takes values between zero and one (called the performance index). This index is found to be strongly (although tentatively) related to a summary measure of the deviations between the observed urban distribution and the empirically common rank-size distribution.

CONTENTS

URBANIZATION LEVEL AND URBAN CONCENTRATION: COMPARATIVE PATHS AND A PERFORMANCE INDEX

INTRODUCTION

In the statistical and demographic study of urbanization, the following dimensions are usually considered: level of or degree of urbanization, changes in the degree of urbanization (the tempo), distribution and concentration of urban population among city size categories, and components of urban growth. The level and the tempo of urbanization constitute the minimum and basic dimensions that must be quantified for any meaningful study of the phenomenon. The other aspects are alternatively treated as causes, effects, and dimensions of urbanization. This paper is not an attempt to review or contrast the different measures of urbanization, this can be found in Arriaga (1975). The application of different measures to world data over time and cross-sectional data can also be found in Davis (1970) and most recently in UN (1979).

In this paper, a comparative analysis over time and between regions* of two dimensions of the urbanization process is presented: namely, urbanization level and urban concentration.

*Only two groups of countries, developed and less developed are examined here. The classification of countries into these two groups is that of the UN (1979). In that study, the definition of urban areas was that adopted by each country.

In contrast to the other studies, however, the levels of the two dimensions and the change in them are examined simultaneously with the help of a new measure that combines them analytically (hereafter called the measure of combined change).

The numerical values taken by a given country or region on the two dimensions at a given period of time are considered as specifying the country's urbanization "position" relative either to the position of other countries at the same time period, or to its own position at earlier time periods. As urbanization progresses these two dimensions change concomitantly, although the magnitude and/or the direction of change may differ. As a result, the country or the region under consideration will move to a new position. Therefore, by tracing this simultaneous change over time we will be able to follow the "path" of a given country or region during the course of urbanization. These paths provide an effective tool for regional and temporal comparisons. Furthermore, since urbanization is a dynamic and cumulative process, it is considered logical to regard the amount of change that occurred during a given year in a given dimension as a quantitative measure of the demographic "performance" on that dimension of a given country or region. This led to the construction of a summary index called the performance index, with values between zero and one depending on the relative or net change in the two urbanization dimensions, which greatly facilitates the regional and temporal comparison.

All the analysis presented in this paper is based on a simple formula that combines the two dimensions analytically. The analysis is done with respect to two types of population; the urban population as defined in UN (1979) and the "city population" namely that part of the urban population that lives in cities of at least 100,000 inhabitants. As will be demonstrated later, the comparison between these two facets of urbanization is highly instructive. It also facilitates relating some part of the analysis to a given form of city size distribution, namely, the rank-size distribution, which attracts a lot of interest.

The paper consists of four sections. Section I introduces the measures used to quantify the dimensions, the measure of combined change and discusses their relationships. The urbanization paths are discussed and illustrated in Section II. In Section III the performance index is defined and its relation with the rank-size distribution is examined. Finally, the paper ends with a summary and suggestions for further applications of the ideas presented in the paper.

All the computations presented throughout the paper are based on Table 4.4 of UN (1979), which gives population, number of cities, and percentage of urban population in a given size class or above (1950-1975), by world, developed and less developed regions, major areas, and regions. It is important to note here that the data for 1975 do not include some cities that have graduated to a size class of 100,000+ between 1970 and 1975, but it was decided to include them in the analysis for this paper.

I. URBANIZATION LEVEL, URBAN CONCENTRATION, AND A MEASURE OF COMBINED CHANGE: OVERALL RELATIONSHIPS

Let n be the number of city size classes that encompass the urban population in a given area (country or region), with u_i referring to that part of the population that lives in the i^{th} size class, and u being total urban population and

$$\sum_{i}^{n} (u_i/u) = 1$$

All the observed city size distributions (either cross-nationally or longitudinally) could be looked on as lying between two extreme cases; the case of complete concentration in one size class $(u_i/u) = 1$ for some i, and $(u_j/u) = 0$ for each $j \neq i$, and the case of complete evenness $(u_i/u) = \frac{1}{n}$ for all i. Our first objective is simply to contrast the regional and the temporal differences in urban concentration by examining the movements of the corresponding city size distribution between these two extremes. Since movement towards one extreme implies movement away from the other, it is sufficient to use one case only as a "benchmark" for comparison.

To facilitate the comparison we need a summary measure (aggregate) of the extent of concentration among different size classes. The first such measure that comes to mind is the sum of absolute deviations between the observed proportions (u_i/u) and the corresponding ones for the case of complete evenness $(1/n)$. This is an easy measure to calculate and to interpret. However, its major disadvantage is that it is difficult to manipulate algebraically. An alternative measure which has been suggested by Berry (1971) in this connection, although no application was given, is the entropy of the distribution. Considering (u_i/u) as the probability that a randomly selected individual of the urban population be found in the i^{th} size class, the entropy of this distribution, according to Theil (1972) is given by

$$H = - \sum_{i}^{n} \left(\frac{u_i}{u}\right) \ln \left(\frac{u_i}{u}\right) \qquad (1)$$

where ln is the natural logarithm. Theil (1972) gives an extensive and comprehensive utilization of the entropy as a general measure of "dividedness" or concentration.

H is zero when there is a complete concentration of urban population in one size class. The maximum value of H is ln (n), where n is the number of size classes, and will only be reached when there is a complete evenness in the distribution, i.e., when all probabilities are equal to (1/n).* Therefore, an increasingly skewed distribution is one in which H is decreasing.

The entropy H will be our measure of the overall degree of urban concentration, and will, as mentioned earlier, be contrasted in some parts of the paper with the case of uniform distribution. However, this does not imply that we regard the uniformity of urban population distribution among different size classes as the optimum distribution. Simply, we need a standard distribution (or a standard value of H) for comparative purposes, and the uniform distribution fulfills this need.

*This is only correct if all size classes are of equal length. If not, as in our case, then the probabilities have to be changed accordingly. In this case the maximum value of H (which can be reached empirically) will be less than ln (n). However, for simplicity, ln (n) will be used here as the maximum value.

Following the UN (1979), seven city size classes will be used with the first class consisting of all cities with less than 100,000 inhabitants and the last class consisting of cities with four million inhabitants or more. In this case, n = 7 and we have

$$H = - \sum_{1}^{7} \left(\frac{u_i}{u}\right) \ln \left(\frac{u_i}{u}\right) \qquad (1')$$

Let P_c be that part of urban population that lives in cities of 100,000 inhabitants or more. To distinguish this population from the total urban population, it will be referred to as the city population. Letting P_{ci} be that part of city population that lives in the i^{th} size class, the entropy of this distribution is given by

$$H_c = - \sum_{1}^{6} \left(\frac{P_{ci}}{P_c}\right) \ln \left(\frac{P_{ci}}{P_c}\right) \qquad (2)$$

where $P_c = \sum_{1}^{6} P_c$. Similarly, H_c will be used to measure the level of concentration of city population.*

Tables 1 and 2 show the individual probabilities and the corresponding entropies for both urban and city populations, for developed and less developed countries from 1950 to 1975. This table shows that entropy H for the urban population is increasing over time for both developed and less developed countries, with larger increases in the case of developed countries. This is also accompanied by increasing concentration in the last size class (four million or more). There is no contradiction here since H measures the overall change in the distribution. Although it is not presented here, the sum of absolute differences between

*One may question the use of the entropy as a comparative measure of the overall shape of a given distribution since two completely different sets of probabilities can lead to the same value of the entropy. However, this point is not relevant to our use of the entropy as a measure of the closeness to the uniform distribution, since for a given number and division of classes, the entropy will increase (decrease) if and only if, there is a movement towards (away from) the uniform distribution.

Table 1. Urban population: distribution[a] and level of concentration 1950-1975, developed and less developed countries.

City Size Class (000)	1950 D[b]	1950 LD	1955 D	1955 LD	1960 D	1960 LD	1965 D	1965 LD	1970 D	1970 LD	1975 D	1975 LD
Urban to												
100	.4218	.4608	.3946	.4561	.3702	.4618	.3541	.4452	.3432	.4299	.3350	.4185
100- 250	.1140	.1311	.1231	.1216	.1223	.1062	.1217	.1019	.1149	.0910	.1008	.0714
250- 500	.0933	.0880	.0893	.0846	.0919	.0888	.0917	.0906	.0977	.0945	.1067	.0885
500-1000	.0938	.0940	.0944	.0946	.0991	.0851	.1007	.0851	.1041	.0868	.1042	.0885
1000-2000	.0864	.0958	.1032	.1041	.0971	.0920	.0959	.0906	.1006	.0976	.1009	.0917
2000-4000	.0670	.0748	.0528	.0883	.0744	.0696	.0794	.0817	.0857	.0747	.0945	.0767
4000+	.1239	.0563	.1425	.0513	.1449	.0966	.1566	.1049	.1540	.1251	.1579	.1520
$\sum$	1	1	1	1	1	1	1	1	1	1	1	1
H^c	1.7064	1.6402	1.7305	1.6487	1.7731	1.6517	1.7901	1.6811	1.8097	1.7002	1.8204	1.7047

[a] (u_i/u) where u_i is the urban population that resides in the size class i and u is the total urban population, $i = 1,2,\ldots,7$.

[b] D = Developed countries, LD = less developed countries.

[c] $H = -\sum_i (u_i/u) \log (u_i/u)$.

Source: Computed from UN (1979:Table 4.4)

Table 2. City population: distribution[a] and level of concentration 1950-1975, developed and less developed countries.

City size class (000)	1950		1955		1960		1965		1970		1975	
	D[b]	LD	D	LD	D	LD	D	LD	D	LD	D	LD
100- 250	.1970	.2428	.2034	.2238	.1942	.1975	.1883	.1839	.1749	.1596	.1514	.1224
250- 500	.1612	.1632	.1477	.1552	.1460	.1648	.1419	.1632	.1487	.1659	.1605	.1522
500-1000	.1622	.1738	.1558	.1736	.1574	.1581	.1559	.1534	.1583	.1523	.1566	.1744
1000-2000	.1494	.1773	.1705	.1911	.1542	.1713	.1485	.1633	.1532	.1712	.1518	.1579
2000-4000	.1157	.1384	.0872	.1622	.1182	.1289	.1230	.1473	.1304	.1314	.1421	.1319
4000+	.2145	.1042	.2354	.0942	.2300	.1795	.2424	.1889	.2345	.2194	.2376	.2612
$\sum$	1	1	1	1	1	1	1	1	1	1	1	1
H_c[c]	1.7731	1.7600	1.7512	1.7618	1.7691	1.7832	1.7661	1.7872	1.7739	1.7790	1.7751	1.7573

[a] P_{ci}/P_c where P_{ci} is the city population that resides in size class i, P_c is the total city population that resides in cities of at least 100,000 people, i = 1,2,...,6.

[b] D = Developed countries, LD = Less developed countries.

[c] $H_c = - \sum_i (P_{ci}/P_c) \log (P_{ci}/P_c)$.

Source: Computed from UN (1979:Table 4.4) the individual probabilities and the uniform probabilities (1/n) declined by 24 percent in the case of developed countries and by 10 percent in the case of less developed countries during the period 1950 to 1970. There is no clear trend in the case of city population and this will be exposed clearly in the next section.

The most common measure of the degree of urbanization or urbanization level is the proportion (or percentage) of the total population that lives in urban areas. It will be used here to quantify the first dimension of the urbanization process. Thus let,

$$\alpha(t) = \left(\frac{u}{T}\right)_t = \sum_{1}^{7} \left(\frac{u_i}{T}\right) \qquad (3)$$

be the degree or level of urbanization for a given country or region in year t, where u is the urban population, and T the total population.

As urbanization proceeds both the level of urbanization (α) and urban distribution (u_i/u) change, which results in the changing of the individual components (u_i/T). These components show how the level of urbanization is distributed among different city size classes. Thus the changing pattern of (u_i/T) over time reflects the simultaneous, or the "combined", change in the level α and in the distribution (u_i/u).

To measure the overall change in the distribution of the level of urbanization, a formula equivalent to that used in defining the entropy H immediately suggests itself:

$$\hat{H} = - \sum_{1}^{7} \left(\frac{u_i}{T}\right) \ln \left(\frac{u_i}{T}\right) \qquad (4)$$

which is minus the weighted average of ln (u_i/T). By dividing both sides of equation (4) by α, the new weights (u_i/u) now sum to one and the new measure:

$$H^{*} = - \sum_{1}^{7} \left(\frac{u_i}{u}\right) \ln \left(\frac{u_i}{T}\right) \qquad (5)$$

is equal to minus the expected value of $\ln (u_i/T)$. It follows immediately that

$$\ln (\alpha) = H - H^* \tag{6}$$

and thus

$$\alpha = e^H e^{-H^*} \tag{7}$$

Since for a given number of city size classes (n):

$$0 \leq H \leq \ln (n)$$

it follows that

$$\ln (\frac{1}{\alpha}) \leq H^* \leq \ln (\frac{n}{\alpha}) \qquad \alpha > 0$$

For a given level of urbanization, H^* reaches its maximum with complete evenness of urban distribution.

Equation (6) shows that when $\alpha = 1$, the measure of the combined change reduces to the entropy of the distribution. H^* is always larger than H, and the smaller the level of urbanization, the larger H^* is in relation to H. Two countries with the same urbanization level will have different values of H^* if they have different urban distribution (u_i/u). Two similar sets of (u_i/u) will result in different values of H^* when they are combined with two different values of α. Also two similar values of the entropy H can result from two different sets of (u_i/u), however, unless the two corresponding levels of urbanization are equal, the measure H^* will take two different values.

Equation (7) shows that the level of urbanization is equal to the product of two components: e^H and e^{-H^*}. Figure 1 shows the plot of three contour lines using equation (7). Each line represents all the different combinations of the two components (e^H and e^{-H^*}) that give the same level of urbanization, .1, .5, or 1.

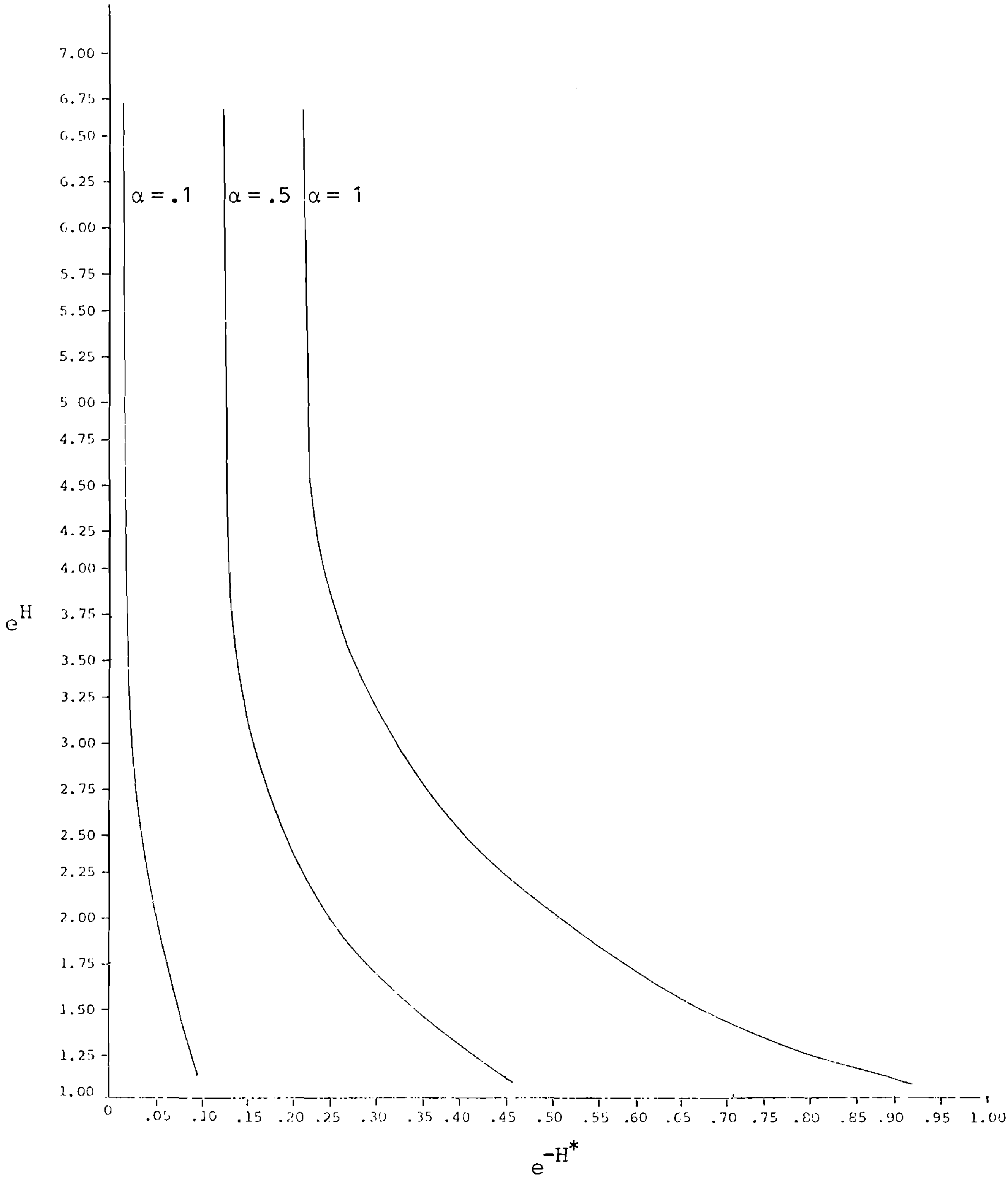


Figure 1. Contour lines of a given level of urbanization with different combinations of its two components e^H and e^{-H^*}.

With respect to city population, we have

$$c(t) = \left(\frac{P_c}{u}\right)_t = \sum_1^6 \left(\frac{P_{ci}}{u}\right)_t \tag{8}$$

$$H_c^* = - \sum_1^6 \left(\frac{P_{ci}}{P_c}\right) \ln \left(\frac{P_{ci}}{u}\right) \tag{9}$$

$$\log (c) = H_c - H_c^* \tag{10}$$

and

$$c = e^{H_c} e^{-H_c^*} \tag{11}$$

Tables 3 and 4 give the different values of H^* and H_c^* over the period 1950-1975 for developed and less developed countries. The values of H^*, decline over time (this is true for urban and city populations, except in 1960, at which time H_c^* was larger than that of 1955). The magnitude of the decline depends (as we shall see) on the relative change of α (or c) and H (or H_c).

The two measures of the levels α and c are given at the bottom of Tables 3 and 4, respectively, each as the sum of its individual components; (u_i/T) in the case of α and (P_{ci}/u) in the case of c.

The equations that define the measure of combined change (H^*) lend themselves easily to a different kind of comparison, namely, standardized comparisons, which, to the author's knowledge, has not been done in this context before. However, a prerequisite to this is the interpretation of the regional and temporal variations of the values of H^* as shown by Tables 3 and 4. This will be done in this paper, while deferring the standardized analysis to a future work.

Table 3. Urbanization level: distribution[a] and parameters 1950-1975, developed and less developed countries.

City size class (000)	1950		1955		1960		1965		1970		1975	
	D[b]	LD	D	LD	D	LD	D	LD	D	LD	D	LD
Urban to												
100	.2215	.0770	.2196	.0863	.2174	.1009	.2193	.1057	.2220	.1110	.2261	.1173
100- 250	.0599	.0219	.0685	.0230	.0718	.0232	.0754	.0242	.0743	.0235	.0680	.0200
250- 500	.0490	.0147	.0497	.0160	.0540	.0194	.0568	.0215	.0632	.0244	.0720	.0248
500-1000	.0493	.0157	.0525	.0179	.0582	.0186	.0624	.0202	.0673	.0224	.0703	.0284
1000-2000	.0454	.0160	.0574	.0197	.0570	.0201	.0594	.0215	.0651	.0252	.0681	.0257
2000-4000	.0352	.0125	.0294	.0167	.0437	.0152	.0492	.0194	.0554	.0193	.0638	.0215
4000+	.0651	.0094	.0793	.0097	.0851	.0211	.0970	.0249	.0996	.0323	.1066	.0426
$\sum = \alpha(t)$	.5254	.1671	.5563	.1892	.5873	.2185	.6194	.2374	.6468	.2582	.6749	.2803
H^{*}[c]	2.3500	3.4273	2.3172	3.3119	2.3057	3.1709	2.2698	3.1173	2.2464	3.0523	2.2146	2.9743

[a] (u_i/T) where u_i is the urban population that resides in size class i, T is the total population, i = 1,2,...,7.

[b] D = Developed countries, LD = Less developed countries.

[c] $H^{*} = - \sum_i (u_i/u) \log (u_i/T)$.

Source: Computed from UN (1979:Table 4.4).

Table 4. Proportion of population living in cities of at least 100,000 people: distribution[a] and parameters, 1950-1975, developed and less developed countries.

City size class (000)	1950		1955		1960		1965		1970		1975	
	D[b]	LD	D	LD	D	LD	D	LD	D	LD	D	LD
100- 250	.1139	.1309	.1231	.1217	.1223	.1063	.1216	.1020	.1149	.0910	.1007	.0712
250- 500	.0932	.0880	.0894	.0844	.0920	.0887	.0917	.0905	.0977	.0946	.1067	.0885
500-1000	.0938	.0937	.0943	.0944	.0991	.0851	.1007	.0851	.1040	.0868	.1041	.1014
1000-2000	.0864	.0956	.1032	.1039	.0971	.0922	.0959	.0906	.1006	.0976	.1009	.0918
2000-4000	.0669	.0746	.0528	.0882	.0744	.0694	.0794	.0817	.0856	.0749	.0945	.0767
4000+	.1241	.0562	.1425	.0512	.1449	.0966	.1566	.1048	.1540	.1251	.1580	.1519
$\sum = c(t)$	.5783	.5391	.6054	.5438	.6298	.5383	.6459	.5547	.6568	.5701	.6650	.5815
H_c^* [c]	2.3207	2.3779	2.2528	2.3712	2.2312	2.4031	2.2027	2.3771	2.1935	2.3414	2.1822	2.3002

[a] P_{ci}/u , see notes of the previous tables, $i = 1,2,\ldots,6$.

[b] D = Developed countries, LD = Less developed countries.

[c] $H_c^* = - \sum_i (P_{ci}/P_c) \log (P_{ci}/u)$.

Source: Computed from UN (1979:Table 4.4).

II. THE URBANIZATION PATHS

As urbanization progresses or evolves, there exists a simultaneous change in the level of urbanization and the degree of urban concentration although the magnitude and the direction of change may differ. Equation (8) and similarly equation (11) which show that the level of urbanization can be factored to be a product of two components: a concentration component (e^{H}) and another component which combines the change in the level and in concentration (e^{-H^*}), can be used to trace graphically those simultaneous changes. Figures 2 and 3 show the observed urbanization paths for urban population and for city population. In both figures the horizontal scale shows the values of (e^{-H^*}) while values of the (e^{H}) are plotted along the vertical scale. The corresponding values of the urbanization level (α or c) can be read by relating the point which represents a given combination of (e^{H}) and (e^{-H^*}) to one of the three contour lines shown in the same graph.† Each line represents the collection of all possible combinations of (e^{H}) and (e^{-H^*}) which result in the same level of .2, .4, or .6 in the case of urban population, and .5, .6, or .7 in the case of city population. The paths are shown as lines connecting the different points in the direction of the arrows, with the first point referring to 1950 and the last to 1975.

To facilitate the comparison, two hypothetical paths (a) and (b), having the same initial position as the observed path, are drawn in Figure 2 for both developed and less developed countries. In path (a) we assumed that beginning from the second period (1955), the amount of change (increase) in the level of urbanization will equal the observed change in the level of concentration. Path (a) then, is the path of equal change. Accordingly, a country having the same initial position but following path (a) would have different levels of urbanization in each time period (from either developed or less developed countries, as is

†The approach of drawing the contour lines on the same graph that shows the values of the two components is similar to that used by Coale et al. (1979) to analyze the changes in fertility in Russia.

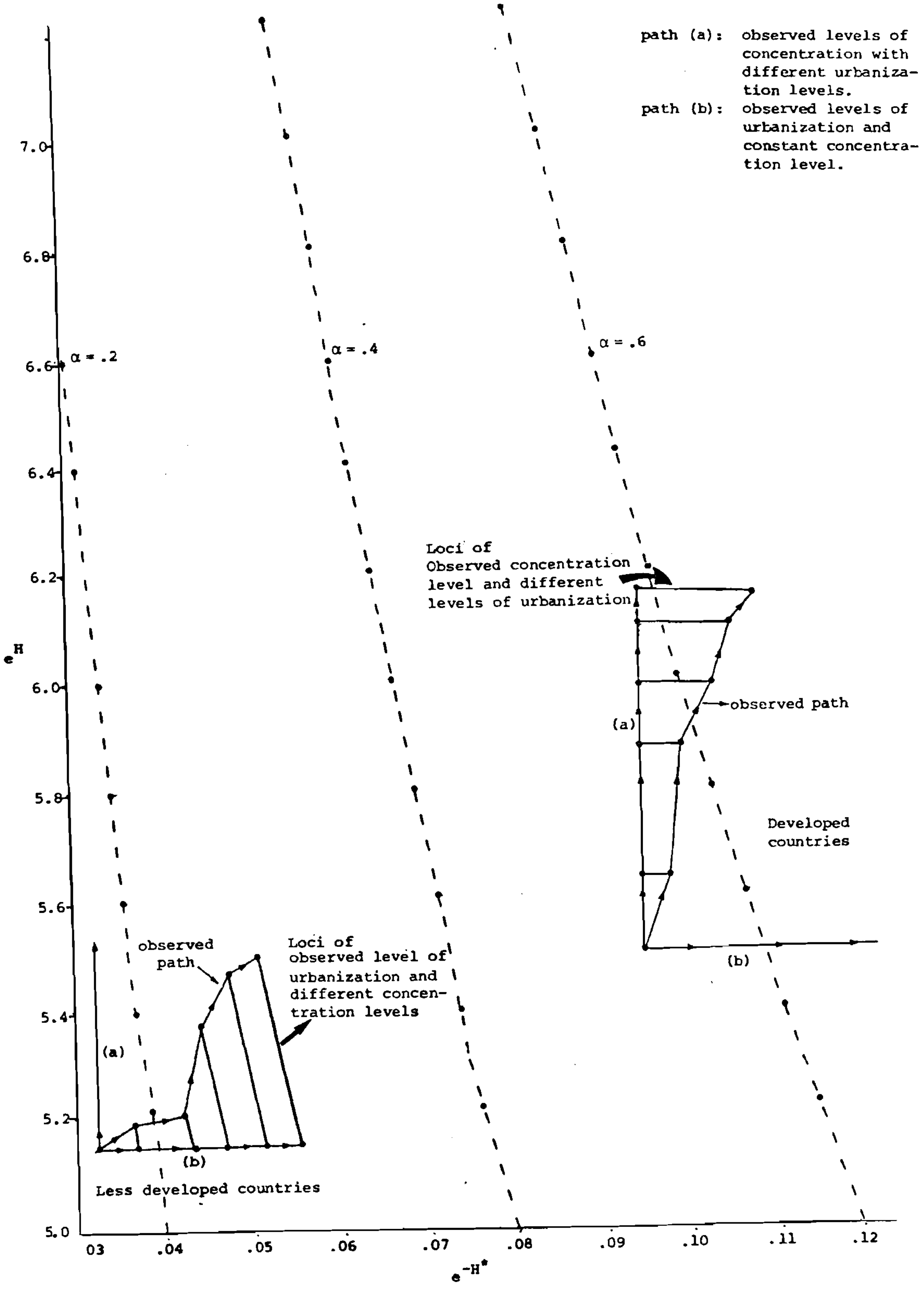


Figure 2. The urbanization path (urban population): changes in urbanization level (α) and urban concentration, observed and hypothetical paths; developed and less developed countries, 1950-1975.

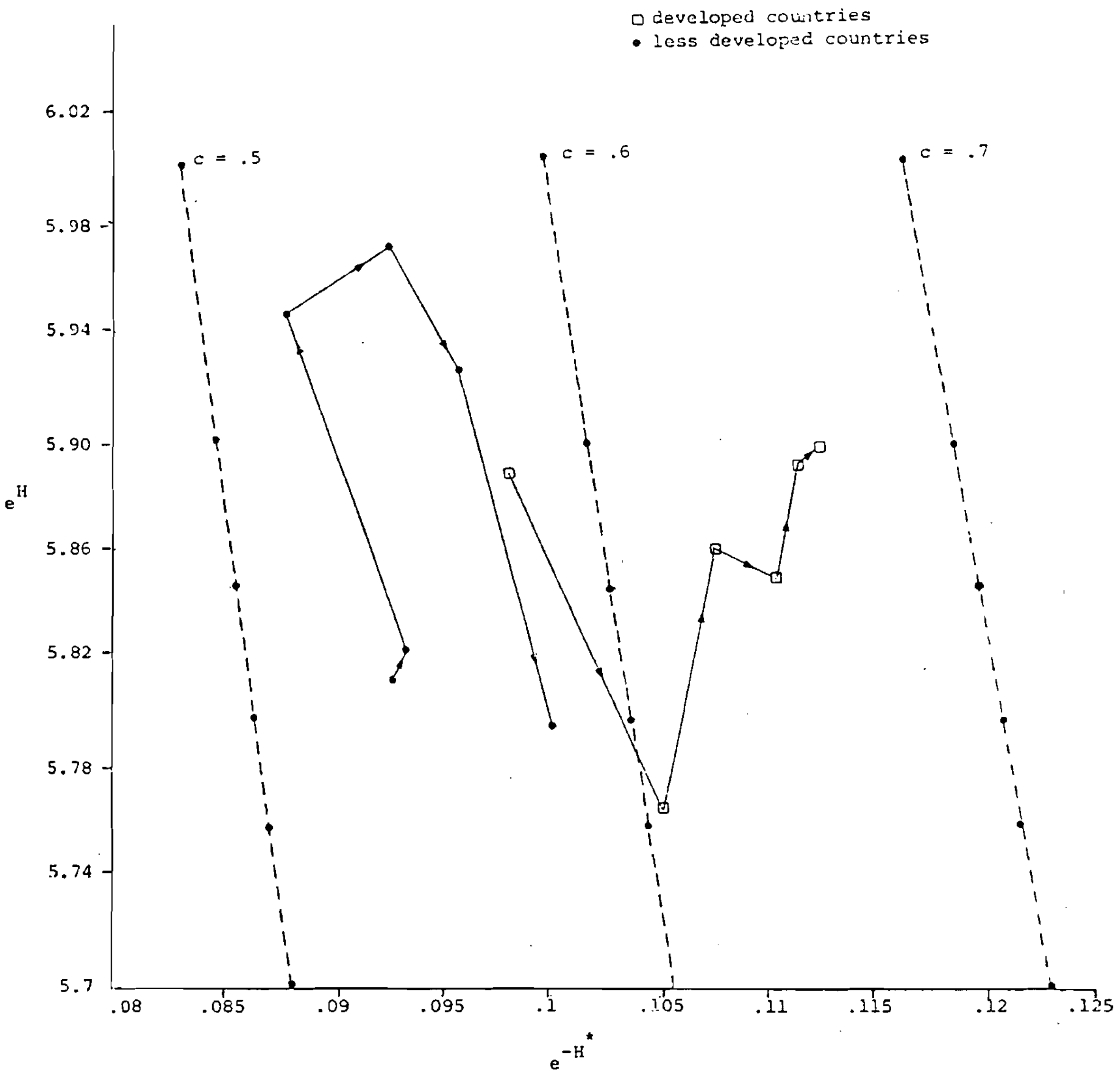


Figure 3. The urbanization path (city population): changes in the proportion or uban population living in cities of at least 100,000 (c), and the concentration level, developed and less developed countries, 1950-1975.

implied by the position of the contour lines) but the same concentration level. On the other hand, path (b) is the path that would have materialized if the initial level of concentration was kept constant for the whole period, while the urbanization level increased as observed. Path (b) then is the path of zero change in concentration. The observed path of course, lies between these two extremes.

The comparison between the observed paths can be made directly or by their relative position to the hypothetical paths (a) and/or (b). In general, the closer the observed path is to the vertical path (a) the closer the change in concentration is to the change in the urbanization level and vice versa.† Thus the relative position of the two observed paths to the path (a) indicate that the difference between the magnitude of change in the two dimensions of the urbanization process is becoming larger over time with a much larger difference in the case of less developed countries. Any path that lies between path (a) and the observed path will pass through the horizontal lines that connect the black dots on the two paths (shown for the developed countries) and thus the intersection points will have the same concentration level as the observed path but with a different urbanization level (α).

The observed path for the less developed countries is closer to the hypothetical path (b) than that of developed countries (especially in the first two time periods) and thus indicates a faster urbanization tempo and a faster decline in the measure of combined change H^*. The lines that connect the black dots on the observed path with those in the path (b) represent the same observation levels. Thus any path that lies between the observed path and the path (b) will achieve the same observed level of urbanization in each time period with different concentration levels.

†As will be made clear in the next section, the fact that the two observed paths are to the right of path (a), is due to both changes having equal (and positive) signs.

That the regional and temporal evolution of the urbanization dimensions can be shown compactly in a graphic form is illustrated again in Figure 3, which shows the observed paths for city population. This figure will not be commented upon here, except to notice that almost no change in the level of concentration was made during the 25 years for both developed and less developed countries.

III. THE PERFORMANCE INDEX

As we have seen, the two dimensions of the urbanization process, as they are considered here, change simultaneously during the course of urbanization although the magnitude and the direction of change may differ regionally and/or temporally. The last observation is very clearly demonstrated in Figure 3 which shows the urbanization path for city population. In this section an examination of the temporal pattern of the change in the two dimensions will be presented.† The amount of this change will then be used to construct an index which can be used to summarize the relative performance during the course of urbanization, as it is reflected by these two dimensions for a given country or region.

Let α_t, α_{t+5}, and H_t, H_{t+5} represent two consecutive values of levels of urbanization and the degree of urbanization concentration for a given country or region. The change in the second dimension may be represented by the difference $\Delta H = H_{t+5} - H_t$. Assuming exponential mode of growth in the level of urbanization between (t) and (t+5), the amount of change in the first dimension is conventionally measured as $\ln(\alpha_{t+5}/\alpha_t) = 5\ r_\alpha$, where r_α is the average rate of growth between (t) and (t+5). Similarly ΔH_c and $5\ r_\alpha$ may be defined with respect to city population. How are these two amounts of change related? Figure 4 summarizes all possible cases together with a plot of the actual values which are also given in Table 5. It is clear that the empirical

†Without loss of generality, and in accordance with the data set used in this study, we only consider a 5-year time period.

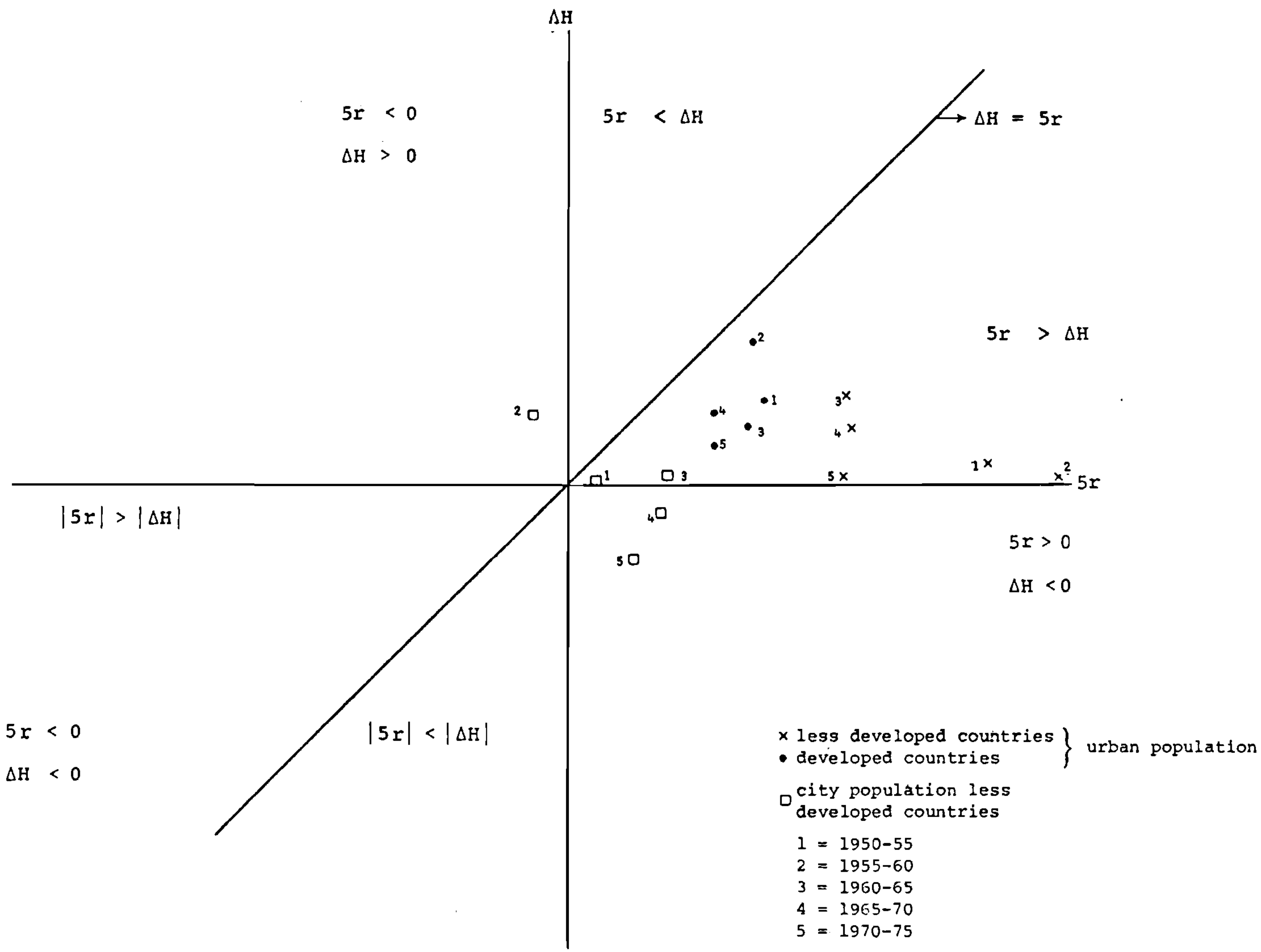


Figure 4. Relationship between change in the level of urbanization (5r) and change in the degree of concentration (ΔH): possible casses and actual values, developed and less developed countries, 1950-1975.

Table 5. Rates and amount of change in some urbanization parameters.

Parameters[a]	1950-55		1955-60		1960-65		1965-70		1970-75	
	D	LD	D	LD	D	LD	D	LD	D	LD
r_{α}	.0114	.0248	.0108	.0288	.0106	.0166	.0086	.0168	.0085	.0165
$5r_{\alpha}$	.0570	.1240	.0540	.1440	.0530	.0830	.0430	.0840	.0425	.0825
ΔH	.0242	.0086	.0426	.0030	.0170	.0294	.0196	.0191	.0107	.0045
ΔH^{*}	-.0328	-.1154	-.0114	-.1410	-.0360	-.0536	-.0234	-.0649	-.0318	-.0780
r_c	.0092	.0017	.0079	-.0021	.0051	.0060	.0034	.0055	.0025	.0039
$5r_c$	.0460	.0085	.0395	-.0105	.0255	.0300	.0170	.0275	.0125	.0195
ΔH_c	-.0219	.0018	.0179	.0214	.0031	.0040	.0078	-.0081	.0013	-.0217
ΔH_c^{*}	-.0679	-.0068	-.0216	.0319	-.0286	-.0260	-.0092	-.0356	-.0112	-.0412
$\Delta\sum\lvert 1 - \frac{O}{E}\rvert$	.3708	-.1302	-.4620	-.2988	.0420	-.2682	-.2250	.0270	-.2160	.2658

[a] See text for definitions.

Source: Computed from Tables 1 - 4.

range of variations is very narrow compared to the possible one. In addition, all the points (except one) lie below the 45^{o} line which indicates (as expected) that the magnitude of change in the level of urbanization is larger than that of the degree of concentration. Finally, it seems that the last 10-15 years show an almost constant change in the urbanization level, together with a tendency to increase in concentration for both developed and less developed countries.

This kind of comparison is greatly facilitated using the measure of combined change H^*. Recall equation (6):

$$H^* = H - \ln (\alpha) \tag{6}$$

It is easy to see that given H^*_t, H^*_{t+5} we have:

$$H^*_{t+5} - H^*_t = \Delta H^* = \Delta H - 5\ r_c \tag{12}$$

and similarly

$$H^*_c = \Delta H_c - 5\ r_c \tag{13}$$

Thus the change in the values of the measure of the combined change is the "net" change in the two dimensions between two points in time. This explains the empirical trend in the values of H^* which were given in Tables 3 and 4, namely, its decline over time with a faster decline in the case of less developed countries. A quick comparison of the relative magnitude of change in the two urbanization dimensions can thus be made by just examining how H^* changes over time.

Regarding the magnitude of change in a given dimension in a given period as a quantitative measure of the performance on that dimension in that period, the amount of change in the two dimensions may then be considered as a measure of the overall performance during the same period. Among the different ways which can be used to relate the change in the two dimensions to each other, we have chosen the form given by equations (12) and (13). Apart from its additivity, which makes it simple to interpret, it will be shown in a subsequent paper that it can be disaggregated quite easily by individual city size classes and thus can be used to evaluate the relative contribution of each size class to the change in the urbanization level and/or in the overall shape of the distribution of urban population.

Since empirically the left hand side of equation (12) or (13) is negative in most cases, the performance index is defined as follows:

$$I_{\Delta} = \frac{\Delta H^{*}(\text{obs.}) - \Delta H^{*}(\text{min.})}{\Delta H^{*}(\text{max.}) - \Delta H^{*}(\text{min.})} \qquad 0 \le I_{\Delta} \le 1 \qquad (14)$$

where ΔH^{*}(obs.) is the observed value in a given year, ΔH^{*}(min.) is the minimum possible value, which can be either theoretically defined or empirically observed and ΔH^{*}(max.) is the maximum possible value. For the purpose of this paper, the minimum and maximum value for each of the two components of the right hand side of equation (13) are defined as follows:

$$0 \le 5\ r_{\alpha} \le .5 \qquad\qquad 0 \le 5\ r_{c} \le .5$$

$$0 \le (\Delta H) \le \ln\ (n)$$

where n = 7 for urban population and n = 6 for city population. Thus we have:

$$I_{\Delta}(t) = \frac{\Delta H^{*}(t) + .5}{2.4459} \qquad \text{for urban population}$$

and

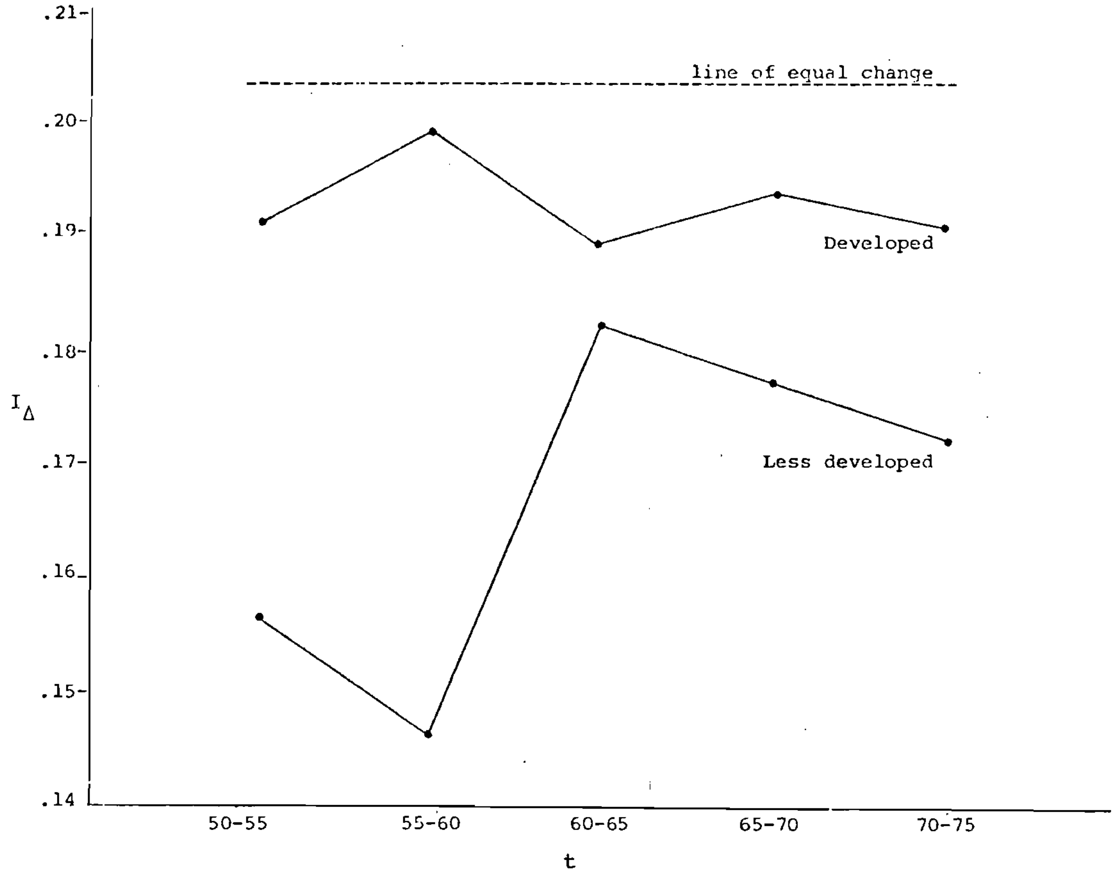
$$I_{\Delta}(t) = \frac{\Delta H^{*}(t) + .5}{2.2918} \qquad \text{for city population}$$

In the case of equal change ($\Delta H^* = 0$), I_Δ takes the values .2044 and .2182 for urban and city population respectively. Figure 5 shows the plots of the values of the index taken by developed and less developed countries together with the line of equal change. Except for the period 1955-1960, for city population, all the values are below the line of equal change. This is to be expected because of the high negative contribution of the tempo of urbanization to the index value (except for the period 1955-1960 in which the proportion of urban population living in cities of at least 100,000 inhabitants declined from its previous value in less developed countries). The developed countries in general score higher values on this index than the less developed countries.

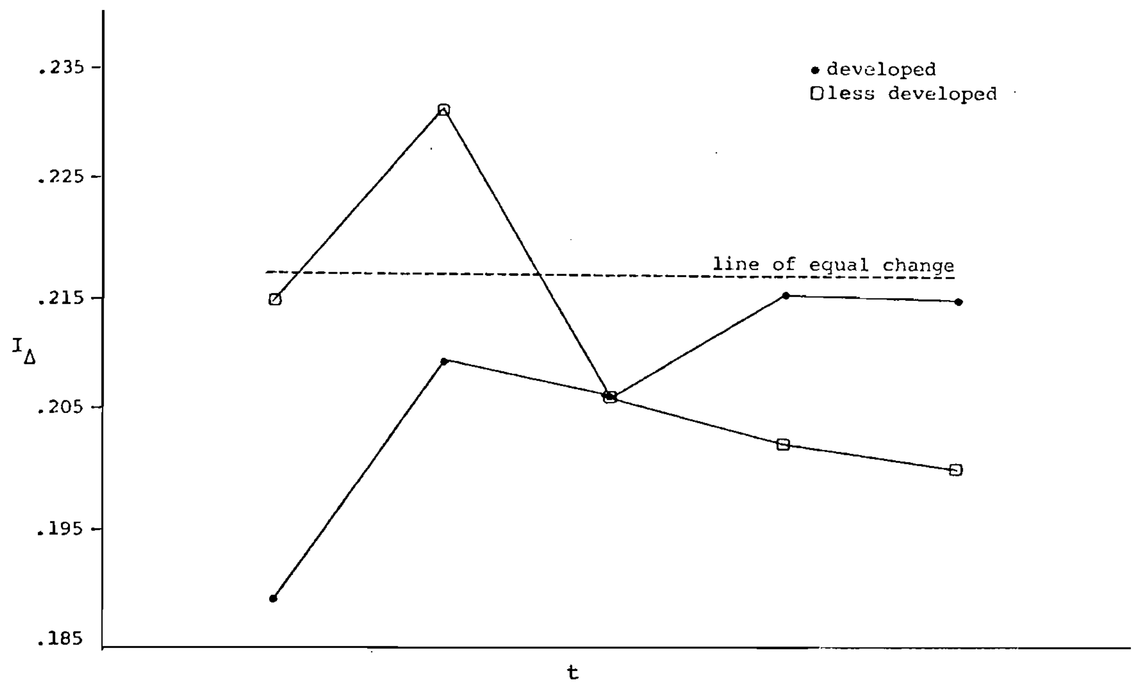
It is clear from the way this index is constructed, that for a given change in the urbanization level, a higher than otherwise score is possible only with a larger (and positive) change in ΔH, i.e., with a larger decrease in the measure of urban concentration. This explains the relatively higher scores for the developed countries.

Finally another potentially useful and interesting utilization of the index has been suggested by the data. It has been noted that the distribution of population among cities of various sizes very often closely follows a quite simple statistical rule, usually termed the "rank-size" distribution. According to one version of this statistical generalization when size of place categories are related so that the upper and lower limits of each size class represent a doubling of those in the next lower size class, the amount of population in each size group will tend to be constant.† In UN (1979) a graphical comparison of the size distribution of cities in the major regions of the world for 1970 was presented and it was evident that (in this year) the rank-size distribution was closely followed. Here we follow a different approach. For each time period, 1950, 1955, to 1975, we fitted a rank-size distribution for developed and less developed countries. The sum of the absolute proportional differences

†For a substantive and theoretical discussion of this distribution, see Berry (1971).



(a) Urban population, developed and less developed countries, 1950-1975



(b) City population, developed and less developed countries, 1950-1975

Figure 5. Values of the performance index.

between the observed (O) and the expected (E) numbers were computed for each period, i.e.,

$$\sum_{i}^{6} \left| 1 - \frac{O}{E} \right|$$

The change in this total between two time periods was also computed, i.e., the amount

$$\Delta \sum_{i}^{6} \left| 1 - \frac{O}{E} \right| = \sum_{i}^{6} \left| 1 - \frac{O}{E} \right|_{t+5} - \sum_{i}^{6} \left| 1 - \frac{O}{E} \right|_{t} \qquad (16)$$

which is shown at the bottom of Table 5. This amount was contrasted with the value of the performance index I_{Δ} in the form of a scatter diagram shown in Figure 6. The strong negative relationship between the two indices is readily apparent. Note in particular that the largest reduction in the absolute deviation in the case of less developed countries is associated with the largest value of I_{Δ}. Again the largest increase in the absolute deviation for developed countries is associated with the smallest value of the index I_{Δ}. The emphasis here on the temporal change in the variables (not on the levels themselves) is important since urbanization is a dynamic and not a static process.

As mentioned above, the two terms on the right hand side of equation (13) can be disaggregated by city size class. With the help of a larger set of data, this fact will be used to examine the aforementioned relationship in greater detail.

IV. SUMMARY AND SUGGESTIONS FOR FURTHER APPLICATIONS

An attempt has been made to examine the development of the urbanization process in two basic dimensions: urbanization level and urban concentration.

To facilitate the regional and temporal comparisons, a summary measure that combines the two dimensions analytically is introduced and its use is illustrated with data compiled in UN

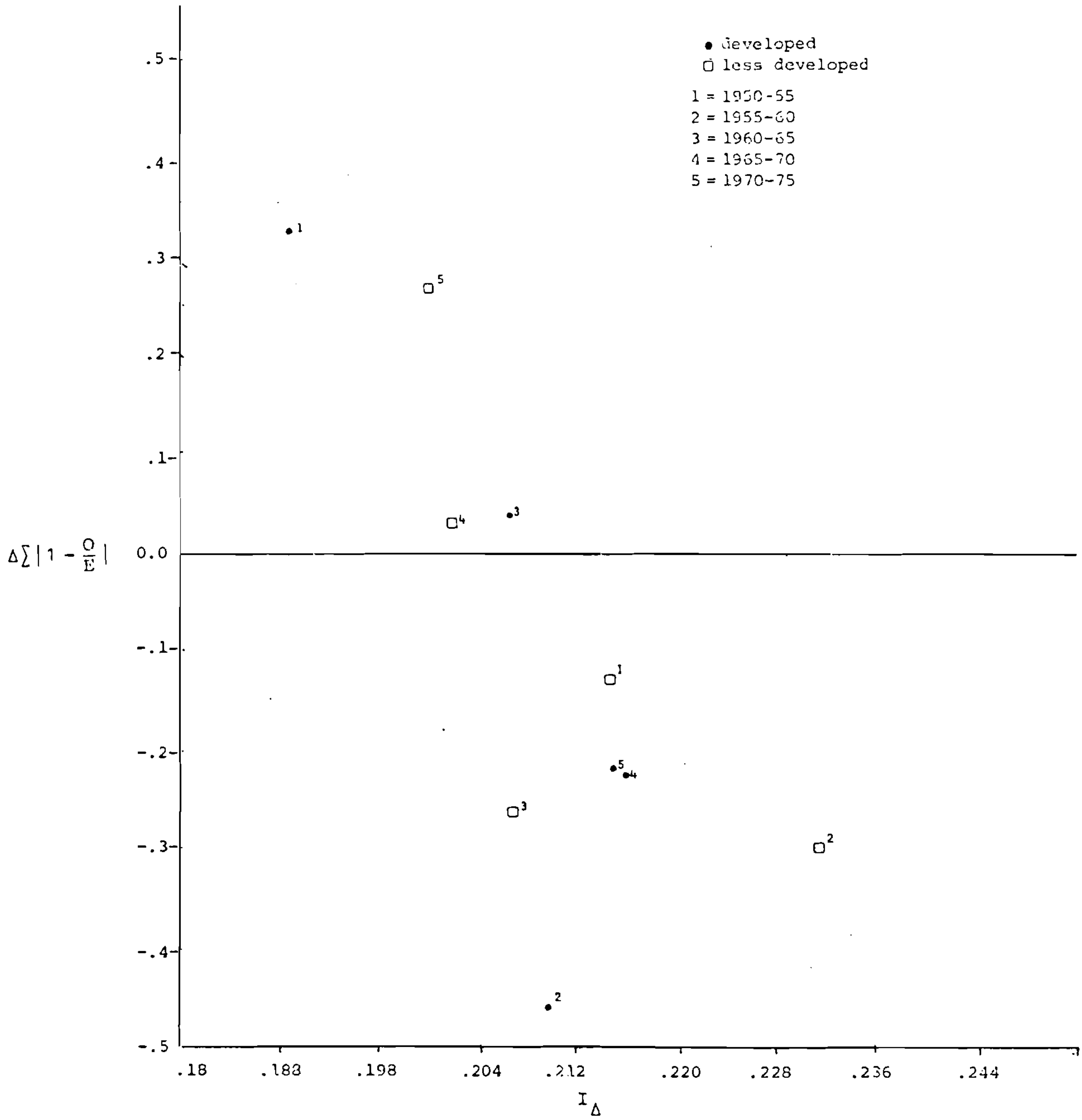


Figure 6. Values of the performance index (city population) versus a summary measure of the deviation from the rank-size distribution, developed and less developed countries, 1950-1975.

(1979). The historical development of the two dimensions, together with the new measure, has been depicted on a graph which shows compactly the path that has been followed in the course of urbanization.

Finally, a comparative index with values between zero and one, called the performance index, was defined and was found to be strongly (although tentatively) related to a summary measure of the deviations between the observed urban distribution and the empirically common rank-size distribution. It will be shown in a subsequent paper that this index (when disaggregated by city size classes) can be explicitly related to the growth rate of urban population, the growth rates of the individual size classes, and a weighted function of the size of the population in each class.

One result of this decomposition is that it now becomes possible to apply the different techniques of standardization (familiar in fertility and mortality comparisons) to the analysis of the demographic aspects of urbanization. In the meantime, however, we would like to suggest the possible uses of the ideas presented in this paper in a more general way.

Given a total that is divided into components, it is always interesting to compare the change in that total (in terms of both the magnitude and direction of change) with the change in the overall importance of individual components, namely the change in their proportional contribution to the total.

An immediate field of application is fertility, where the total can be represented by total fertility rate (TFR) and the individual components are the age-specific fertility rates. Thus we immediately have

$$H_F = - \sum f(i) \log \left(f(i)\right)$$

$$H_F^* = - \sum f(i) \log \left(F(i)\right)$$

and

$$H_F - H_F^* = \log\ (TFR)$$

where F(i) is the age-specific fertility rates and $TFR = \sum_i F(i)$ is the total fertility rate and $f(i) = F(i)/TFR$ with $\sum f(i) = 1$. H_F^* will give the combined effect of the change in the timing of fertility represented by the change in shape of the proportional contribution of the reproduction ages to TFR f(i) and of the change in average family size represented by TFR. We may also define a fertility path similar to the urbanization path defined earlier. This can be used effectively in comparative analysis over time and across space of the change in these two dimensions of fertility.

The same approach can be applied to the migration by replacing TFR with GMR (Gross Migraproduction Rate) and F(i) by M(i), the age-specific migration rate; Castro and Rogers (1979).

With respect to mortality, Keyfitz's (1977) analysis of the effect of a uniform change in death rates on the expectation of life is in the same line. His quantity H derived as the proportional change in the expectation of life has the same form as our measure H^* with l(a) playing the role of (u_i/T).

Finally it is interesting to note that our measure H^* may be made (after a suitable change in scale) to have the same form as the quantity ϕ, defined as the reproductive potential and derived using some concepts from statistical mechanics and assuming stable age distribution, in Demetrius (1980).

This function has the form

$$\phi = - \frac{\sum p_j \log\ (l_j\ m_j)}{\sum\ _j p_j}$$

where l_j is the probability of surviving to age j, m_j is the fecundity at age j and $p_j = (l_j\ m_j/\lambda^j)$, λ being the growth ratio. No application was given, however. The utilization and the interpretation given to the measure H^* in the course of this paper may be carried over to ϕ and some possible use of it within the context of stable population theory suggests itself.

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