A MODEL FOR THE FOREST SECTOR

M. Kallio
A. Propoi
R. Seppälä

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INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

Professor Risto Seppälä is Head of the Mathematical Department
of The Finnish Forest Research Institute, Helsinki.

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ABSTRACT

This paper describes a dynamic linear programming model for studying long-range development alternatives of forestry and forest based industries at a national and regional level. The Finnish forest sector is used as an object of implementation and for numerical examples. Our model is comprised of two subsystems, the forestry and the industrial subsystem, which are linked to each other through the wood supply. The forestry submodel describes the development of the volume and age distribution of different tree species within the nation or its subregions. In the industrial submodel we consider various production activities, such as saw mill industry, panel industry, pulp and paper industry, as well as further processing of primary products. For a single product, alternative technologies may be employed. Thus, the production process is described by a small Leontief model with substitution. Besides supply of wood and demand of wood products, production is restricted through labor availability, production capacity, and financial resources. The production activities are grouped into financial units and the investments are made within the financial resources of such units. Objective functions related to GNP, balance of payments, employment, wage income, stumpage earnings, and industrial profit have been formulated. Terminal conditions have been proposed to be determined through an optimal solution of a stationary model for the whole forest sector.

The structure of the integrated forestry-forest industry model is given in the canonical form of dynamic linear programs for which special solution techniques may be employed. Two versions of the Finnish forest sector models have been implemented for the interactive mathematical programming system called SESAME, and a few numerical runs have been presented to illustrate possible use of the model.

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1. INTRODUCTION

As is the case with several natural resources, many regions of the world are now at the transition period from ample to scarce wood resources. Because the forest sector plays an important role in the economy of some countries, long-term policy analysis of the forest sector, i.e., forestry and forest industries, is becoming an important issue for these countries.

We may single out two basic approaches for analyzing long-range development of the forest sector: simulation and optimization. Simulation techniques (e.g., system dynamics) allow us to understand and to quantify basic relationships influencing the development of the forest sector (see Jegr et al. 1978, Randers 1976, Seppälä et al. forthcoming). Hence, using a simulation technique we can evaluate the consequences of a specific policy. However, using only simulation it is difficult to find a "proper" (or in some sense optimal) policy. The reason for this is that the forest sector is in fact a large-scale dynamic system and, on the basis of simulation alone, it is difficult to select an appropriate policy which should satisfy a large number of conditions and requirements. For this we need an optimization technique. Because of the complexity of the system in question, linear programming (Dantzig 1963) may be considered as the most appropriate technique for this case. It is worthwhile to note that the optimization technique itself should be used on some simulation basis; i.e., different numerical runs based on different assumptions and objective functions should be carried out to aid the selection of an appropriate policy. Specific applications of such an approach for planning an integrated system of forestry and forest industries have been presented, for instance, by Jackson (1974) and Barros and Weintraub (1979).

Already because of the nature of growth of the forests, the model should necessarily be dynamic. Therefore, in this paper we consider a dynamic linear programming (DLP) model for the forest sector. In this approach the planning horizon (e.g., a 50-year period) is partitioned into a (finite) number of time periods (e.g., 5-year periods) and for each of these shorter periods we consider a static linear programming model. A dynamic LP is then just a linear program comprising of such static models which are interlinked via various state variables (i.e., different types of "inventories", such as wood in the forests, production capacity, assets, liabilities, etc., at the end of a given period are equal to those at the beginning of the following period). In our forest sector model, each such static model comprises two basic submodels: a forestry submodel, and an industrial model of production, marketing and financing. The forestry submodel describes also ecological and land availability constraints for the forest, as well as labor and machinery constraints for harvesting and planting activities.

The industrial submodel is described by a small input-output model with both mechanical (e.g., sawmill and plywood) and chemical (e.g., pulp and paper) production activities. Also secondary processing of the primary products will be included in the model, in particular, because of the expected importance of such activities in the future.

The rate of production is restricted by wood supply (which is one of the major links between the submodels), by final demand for wood products, by labor force supply, by production capacity availability, and finally, by financial considerations.

The evaluation criterion in comparing alternative policies for the forest sector is highly multiobjective: while selecting a reasonable long-term policy, preferences of different interest groups (such as government, industry, labor, and forest owners) have to be taken simultaneously into account. It should also be noted that forestry and industry submodels have different transient times: a forest normally requires a growing period of at least 40 to 60 years whereas a major structural change in the industry may be carried out within a much shorter period. Because of the complexity of the system, it is sometimes desirable to consider the forestry and the industries on some independent basis, each with its own objective(s), and to analyze an integrated model thereafter (see Kallio et al. 1979).

The paper is divided into two parts. In the first part (Sections 2-4) we describe the methodological approach. In the second part (Section 5) a specific implementation for the Finnish forest sector is described and illustrated with somewhat hypothetical numerical examples.

2. THE FORESTRY SUBSYSTEM

Mathematical programming is a widely applied technique for operations management and planning in forestry (e.g., Navon 1971, Dantzig 1974, Kilkki et al. 1977, Newnham 1975, Näslund 1969, Wardle 1965, Ware and Clutter 1971, Weintraub and Navon 1976, Williams 1976). In this section we follow a traditional formulation of the forests' tree population into a dynamic linear programming system. We describe the forestry submodel, where the decision variables (control activities) are harvesting and planting activities, and where the state of the forests is represented by the volume of trees in different species and age groups. Because the model is formulated in the DLP framework, we single out the following: (i) state equations which describe the development of the system, (ii) constraints which restrict feasible trajectories of the forest development, (iii) planning horizon, and (iv) objective function(s).

2.1 State Equations

Each tree in the forest is assigned to a class of trees specifying the age and the species of the tree. A tree belongs to age group a (a = 1,..., N-1) if its age is at least $(a-1)\Delta$ but less than $a\Delta$, where Δ is a given time interval (for example, five years). In the highest age group a = N all trees are included which have an age of at least $(N-1)\Delta$. (Instead of age groups, we might alternatively assign trees to size groups specified by the trees' diameter.) We denote by $w_{sa}(t)$ the number of trees of species s, s = 1,2,3,..., (e.g., pine, spruce, birch, etc.) in age group a at the beginning of time period t, t = 0,1,..., T.

Let $\alpha^s_{aa'}(t)$ show the ratio of trees of species s and in age group a that will proceed to the age group a' during time period t. We shall consider a model formulation where the length of each time period is Δ. Therefore, we may assume that $\alpha^s_{aa'}(t)$ is independent of t and equal to zero unless a' is equal to a+1 (or a for the highest age group). We denote then $\alpha^s_{aa'}(t) = \alpha^s_a$ with $0 \leq \alpha^s_a \leq 1$. The ratio $1 - \alpha^s_a$ may then be called the attrition rate corresponding to time interval Δ and tree species s in age group a. We introduce a subvector $w_s(t) = \{w_{sa}(t)\}$, specifying the age distribution of trees (number of trees) for each tree species s at the beginning of time period t. Assuming neither harvesting nor planting, the age distribution of trees at the beginning of the next time period t+1 will then be given by $\alpha^s w_s(t)$ where α^s is the square $N \times N$ growth matrix, describing aging and death of the trees resulting from natural causes. By our definition, it has the form

$$\alpha^s = \begin{bmatrix} 0 & 0 & & 0 \\ \alpha^s_1 & 0 & & 0 \\ 0 & \alpha^s_2 & & \\ 0 & \cdots & \alpha^s_{N-1} & \alpha^s_N \end{bmatrix} .$$

Introducing a vector $w(t) = \{w_s(t)\} = \{w_{sa}(t)\}$, describing tree species and age distribution and a block-diagonal matrix α with submatrices α^s on its diagonal, the species and age distribution at the beginning of period t+1 will be given by $\alpha w(t)$.

We denote by $u^+(t)$ and $u^-(t)$ the vectors of planting and harvesting activities at time period t. The state equation describing the development of the forest will then be

$$w(t+1) = \alpha w(t) + \eta u^+(t) - \omega u^-(t) \quad , \tag{1}$$

where matrices η and ω specify planting and harvesting activities in such a way that $\eta u^+(t)$ and $-\omega u^-(t)$ are the incremental change in numbers of trees resulting from planting and harvesting activities, respectively.

A planting activity n may be specified to mean planting of one tree of species s which enters the first age group (a = 1) during period t. Thus, matrix η has one unit column vector for each tree species s. The nonzero element of such a column is on the row of the first age group for tree species s in equation (1).

A harvesting activity h is specified by variables $u_h^-(t)$ which determine the level of this activity (e.g., final harvesting, thinning, etc.). The coefficients ω_{ah}^s of matrix ω are defined so that $\omega_{ah}^s u_h^-(t)$ is the number of trees of species s from age group a harvested when activity h is applied at level $u_h^-(t)$. Thus, these coefficients show the age and species distribution of trees harvested when activity h is applied.

Sometimes the harvesting activities can be specified simply by the numbers of trees of species s and age a harvested during time period t. There is some danger in this specification, however, because the solution of the model may suggest that only one or very few age groups will be harvested at each time period t. This would of course be unrealistic in practice. Therefore, it is recommended that each harvesting activity is defined through a tree distribution corresponding to actual operations.

2.2 Constraints

Land. Let H(t) be the vector of total acreage of different types d of land available for forests at time period t. A land type d may refer, for instance, to a soil type. Let G^s_{ad} be the area of land species d required by one tree of species s and age group a. We assume that each tree species uses only one type of land d; i.e., only one of the elements G^s_{ad}, d = 1, 2, ..., is nonzero. Thus, if we consider more than one land type, then the tree species s may also refer to the soil. Defining the matrix $G = (G^s_{ad})$, we have the land availability restriction

$$Gw(t) \leq H(t) \quad . \qquad (2)$$

In this formulation we assume that the land area H(t) is exogenously given. Alternatively, we may endogenize vector H(t) by introducing activities and a state equation for changing the area of different types of land. Such a formulation is justified if changes in soil type over time is considered or if some other land intensive activities, such as agriculture, are included in the model.

Besides land availability constraints, requirements for allocating land for certain purposes (such as preserving the forest as a water shed or as a recreational area) may be stated in the form of inequality (2). In such a case (the negative of) a component of H(t) would define a lower bound on such an allocation, while the left hand side would yield the (negative of) land allocated in a solution of the model.

Sometimes constraints on land availability may be given in the form of equalities which require that all land which is made available through harvesting at a time period should be used in the same time period for planting new trees of the type appropriate for the soil. Forest laws in many countries even require following this type of pattern.

Labor and other resources. Harvesting and planting activities require resources such as machinery and labor. Let $R^+_{gn}(t)$ and $R^-_{gh}(t)$ be the usage of resource g at the unit level of planting activity n and harvesting activity h, respectively. Defining the matrices $R^+(t) = \{R^+_{gn}(t)\}$ and $R^-(t) = \{R^-_{gh}(t)\}$, and vector $R(t) = \{R_g(t)\}$ of available resources during period t, we may write the resource availability constraint as follows:

$$R^+(t)u^+(t) + R^-(t)u^-(t) \leq R(t) \quad . \tag{3}$$

Wood supply. The requirements for wood supply from forestry to industries can be given in the form:

$$S(t)u^-(t) = y(t) \quad , \tag{4}$$

where vector $y(t) = \{y_k(t)\}$ specifies the requirements for different timber assortments k (e.g., pine log, spruce pulpwood, etc.), and matrix S(t) transforms quantities of harvested trees of different species and age into the volume of different timber assortments. Note that the volume of any given tree being harvested is assigned in (4) to log and pulpwood in a ratio which depends on the species and age group of the tree.

2.3 Planning Horizon

The forest as a system has a very long transient time: one rotation of the forest may in extreme conditions require more than one hundred years. Naturally, various uncertainties make it difficult to plan for such a long time horizon. On the other hand, if the planning horizon is too short we cannot take into account all the consequences of activities implemented at the beginning of the planning horizon. As a compromise we may think of a planning horizon of 50 to 80 years. Thus, if one period represents an interval of five years, the model will constitute 10 to 16 stages. It should be noted that such a planning horizon is unnecessarily long for the industrial subsystem and too short for the forestry subsystem. In order to eliminate the latter difficulty, it is desirable to analyze a stationary regime for the forests. In this case we set $w(t+1) = w(t) = w$, for all t. Similarly planting and harvesting activities are taken independent of time; i.e., $u^+(t) = u^+$ and $u^-(t) = u^-$, for all t. The state equation (1) can then be restated as

$$w = \alpha w + \eta u^{+} - \omega u^{-} \quad . \tag{1a}$$

Imposing constraints (2) through (4) on variables w, u^{+}, and u^{-}, we can solve the static linear programming problem and find an optimal stationary state w^{*} of the forest (and corresponding harvesting and planting activities). This approach has been used, for instance, by Rorres (1978) for finding the stationary maximum yield of a harvest. The solution of a dynamic linear program with terminal constraints

$$w(T) = w^{*}$$

yields the optimal transition to this stationary state.

Another way of introducing a stationary state is to consider an infinite period formulation and to impose constraints $w(t) = w(t+1)$, $u^{-}(t) = u^{-}(t+1)$ and $u^{+}(t) = u^{+}(t+1)$, for all $t \geq T$. If the model parameters for period t are assumed independent of time for all $t \geq T$, then the dynamic infinite horizon linear programming model may be formulated as a T+1 period problem where the last period represents a stationary solution for periods $t \geq T$, and the first T periods represent the transition from the initial state to the stationary solution.

There is a certain difference in these two approaches of handling the stationary state. In the first approach, when (5) is applied, we first find the optimal stationary solution independently of the transition period, and thereafter we determine the optimal transition to this stationary state. In the latter approach we link the transition period with the period corresponding to the stationary solution. The linkage takes place in the stationary state variables which are determined in an optimal way taking into account both time periods simultaneously.

2.4 Objective Functions

The forest management described above, has a very multiobjective nature. For example, the following objectives have been mentioned (Dantzig 1974, Steuer and Schuler 1978):

1) obtaining higher yields of round wood; 2) preserving the watershed; 3) preserving the forest as a recreational area; 4) making the forest resilient to diseases, fire, droughts, etc. Some of these objectives may be included in objective function(s), while others can be given as constraints. In Section 2.2 we considered some of these types of objectives as constraints.

A common objective which is also used as an objective function is the discounted sum of net income in forestry. This profit may be expressed as a linear combination of the decision variables:

$$\sum_{t=0}^{T-1} \beta(t)[J^{-}(t)u^{-}(t) - J^{+}(t)u^{+}(t)] \quad . \tag{6}$$

Here $J^{-}(t)$ accounts for the mill price of the wood less transportation and harvesting costs at unit level. Vector $J^{+}(t)$ refers to planting costs at unit level and $\beta(t)$ is a discounting factor. For illustrative purposes we shall use this objective function for forestry.

2.5 Forestry Model

In summary, our forestry model may now be stated as follows. Given state equation (1), an initial state $w(0) = w^0$ and a terminal state $w(T) = w^*$, find such nonnegative controls $\{u^{-}(t)\}$ and $\{u^{+}(t)\}$ $(t = 0,1,\ldots, T-1)$, which satisfy constraints (2) through (4), yield nonnegative state vectors $w(t)$ and maximize the aggregated profit defined in (6).

In this problem the vector $y(t)$ of wood supply, the (vector of) available land $H(t)$, and the availability of labor and other resources $R(t)$ are given exogenously. Therefore, policy analysis for forestry on the basis of only this submodel is very limited in its possibilities. We shall link below this submodel with an industrial submodel describing transformation of wood raw material into products.

Note that our formulation may also be considered as a regionalized forestry model. In this case we only have to extend the meaning of various indices (tree species s, planting activity n, harvesting activity h, land type d, resource g, and timber assortment k) to refer, in addition to the above, also to various subregions within the nation.

3. THE INDUSTRIAL SUBSYSTEM

We will now consider the industrial subsystem of the forest sector. Again the formulation is a dynamic linear programming model. We discuss first the section related to production and final demend of wood products, then the financial considerations and the complete industrial submodel thereafter.

3.1 Production and Demand

Let $x(t)$ be the vector (levels of) of production activities for period t, for $t = 0, 1, \ldots, T-1$. Such an activity i may include production of sawn wood, panels, pulp, paper, converted products, etc. For each single product j, there may exist several alternative production activities i which are specified through alternative uses of raw material, technology, etc. Let U be the matrix of wood usage per unit of production activity so that the wood processed by industries during period t is given by vector $Ux(t)$. Note that matrix U has one row corresponding to each timber assortment k (corresponding to the components of supply vector $y(t)$ in the forestry model). Some of the elements in U may be negative. For instance, saw milling consumes logs but produces raw material (industrial residuals) for pulp mills. This byproduct appears as a negative component in matrix U. We denote by $r(t) = \{r_k(t)\}$ the vector of wood raw material inventories at the beginning of period t (i.e., wood harvested but not processed by the industry). As above, let $y(t)$ be the amount of wood harvested in different timber assortments, and $z^+(t)$ and $z^-(t)$ the (vectors of) import and export of different assortments of wood, respectively during period t. Then we have the following state equation for the wood raw material inventory:

$$r(t+1) = r(t) + y(t) - Ux(t) + z^{+}(t) - z^{-}(t) \quad . \tag{7}$$

In other words, the wood inventory at the end of period t is the inventory at the beginning of that period plus wood harvested and imported less wood consumed and exported (during that period). Note that if there is no storage (change), and no import nor export of wood, then (7) reduces to $y(t) = Ux(t)$; i.e., wood harvested equals the consumption of wood. For wood import and export we assume upper limits $Z^{+}(t)$ and $Z^{-}(t)$, respectively:

$$z^{+}(t) \le Z^{+}(t) \quad \text{and} \quad z^{-}(t) \le Z^{-}(t) \quad . \tag{8}$$

The production process may be described by a simple input-output model with substitution. Let $A(t)$ be an input-output matrix having one row for each product j and one column for each production activity i so that $A(t)x(t)$ is the (vector of) net production when production activity levels are given by $x(t)$. Let $m(t) = \{m_j(t)\}$ and $e(t) = \{e_j(t)\}$ be the vectors of import from and export to the forest sector, respectively, for products j. Then, excluding from consideration a possible change in the product inventory, we have

$$A(t)x(t) + m(t) - e(t) = 0 \quad . \tag{9}$$

Both for export and for import we assume externally given bounds $E(t)$ and $M(t)$, respectively:

$$e(t) \le E(t) \quad , \tag{10}$$

$$m(t) \le M(t) \quad . \tag{11}$$

Production activities are further restricted through labor and mill capacities. Let $L(t)$ be the vector of different types of labor available for the forest industries. Labor may be classified in different ways taking into account, for instance, type of production, and the type of responsibilities in the production process (e.g., work force, management, etc.). Let $\rho(t)$

be a coefficient matrix so that $\rho(t)x(t)$ is the (vector of) demand for different types of labor given production activity levels $x(t)$. Thus we have

$$\rho(t)x(t) \leq L(t) \quad . \tag{12}$$

We will consider the production (mill) capacity as an endogenous state variable. Let $q(t)$ be the vector of the amount of different types of such capacity at the beginning of period t. Such types may be distinguished by region (where the capacity is located), by type of product for which it is used and by different technologies to produce a given product. Let $Q(t)$ be a coefficient matrix so that $Q(t)x(t)$ is the demand (vector) for these types of capacity. Such a matrix has nonzero elements only when the region-product-technology combination of a production activity matches with that of the type of capacity. The production capacity restriction is then given as

$$Q(t)x(t) \leq q(t) \quad . \tag{13}$$

The development of the capacity is given by a state equation

$$q(t+1) = (I-\delta)q(t) + v(t) \quad , \tag{14}$$

where δ is a diagonal matrix accounting for (physical) depreciation and $v(t)$ is a vector of investments (in physical units). Capacity expansions are restricted through financial resources. We do not consider possible constraints of other sectors, such as heavy machinery or building industry, whose capacity may be employed in investments of the forest sector.

3.2 Finance

We will now turn our discussion to the financial aspects. We partition the set of production activities i into financial units (so that each activity belongs uniquely to one financial unit). Furthermore, we assume that each production capacity is assigned to a financial unit so that each production activity employs only capacities assigned to the same financial unit as the activity itself.

Production capacity in (14) is given in physical units. For financial calculations (such as determining taxation) we define a vector $\bar{q}(t)$ of fixed assets. Each component of this vector determines fixed assets (in monetary units) for a financial unit related to the capacity assigned to that unit. Thus, fixed assets are aggregated according to the grouping of production activities into financial units, for instance, by region, by industry, or by groups of industries.

Financial and physical depreciation may differ from each other; for instance, when the former is defined by law. We define a diagonal matrix $(I-\bar{\delta}(t))$ so that $(I-\bar{\delta}(t))\bar{q}(t)$ is the vector of fixed assets left at the end of period t when investments are not taken into account. Let $K(t)$ be a matrix where each component determines the increase in fixed assets (of a certain financial unit) per (physical) unit of an investment activity. Thus the components of vector $K(t)v(t)$ determine the increase in fixed assets (in monetary units) for the financial units when investment activities are applied (in physical units) at a level determined by vector $v(t)$. Then we have the following state equation for fixed assets:

$$\bar{q}(t+1) = (I-\bar{\delta}(t))\bar{q}(t) + K(t)v(t) \quad . \tag{15}$$

For each financial unit we consider external financing (long-term debt) as an endogenous state variable. Let $\ell(t)$ be the (vector of) beginning balance of external financing for different financial units in period t. Similarly, let $\ell^+(t)$ and $\ell^-(t)$ be the (vectors of) drawings of debt and the repayments made during period t. In this notation, the state equation for long-term debt is as follows:

$$\ell(t+1) = \ell(t) + \ell^+(t) - \ell^-(t) \quad . \tag{16}$$

We will restrict the total amount for long-term debt through a measure which may be considered as a realization value of a financial unit. This measure is a given percentage of the total assets less short-term liabilities. Let $\mu(t)$ be a diagonal matrix of such percentages, let $b(t)$ be the (endogenous vector of) total stockholders equity (including cumulative profit and stock). Then the upper limit on loans is given as

$$[I-\mu(t)]\ell(t) \leq \mu(t)b(t) \quad . \tag{17}$$

Alternatively, external financing may be limited, for instance, to a percentage of a theoretical annual revenue (based on available production capacity and on assumed prices of products). Note that no repayment schedule has been introduced in our formulation, because an increase in repayment can always be compensated by an increase of drawings in the state equation (16).

Next we will consider the profit (or loss) from period t. Let $p^+(t)$ and $p^-(t)$ be vectors whose components indicate profits and losses, respectively, for the financial units. By definition, both profit and loss cannot be simultaneously nonzero for any financial unit. For a solution of the model, this fact usually results from the choice of an objective function.

Let $P(t)$ be a matrix of prices for products (having one column for each product and one row for each financial unit) so that the vector of revenue (for different financial units) from sales $e(t)$ outside the forest industry is given by $P(t)e(t)$. Let $C(t)$ be a matrix of direct unit production costs, including, for instance, wood, energy, and direct labor costs. Each row of $C(t)$ refers to a financial unit and each column to a production activity. The (vector) of direct production costs for financial units is then given by $C(t)x(t)$.

The fixed production costs may be assumed proportional to the (physical) production capacity. We define a matrix $F(x)$ so that the vector $F(t)q(t)$ yields the fixed costs of period t for the financial units. According to our notation above, (financial) depreciation is given by the vector $\overline{\delta}(t)\overline{q}(t)$.

We assume that interest is paid on the beginning balance of debt. Thus, if $\varepsilon(t)$ is the diagonal matrix of interest rates, then the vector of interest paid (by the financial units) is given by $\varepsilon(t)\ell(t)$. Finally, let D(t) be (a vector of) exogeneously given cash expenditure covering all other costs. Then the profit before tax (loss) is given as follows:

$$p^{+}(t) - p^{-}(t) = P(t)e(t) - C(t)x(t) - F(t)q(t) - \overline{\delta}(t)\overline{q}(t) - \varepsilon(t)\ell(t) - D(t) \quad . \tag{18}$$

The stockholder equity b(t), which we already employed above, satisfies now the following state equation:

$$b(t+1) = b(t) + [I-\tau(t)]p^{+}(t) - p^{-}(t) + B(t) \quad , \tag{19}$$

where $\tau(t)$ is a diagonal matrix for taxation and B(t) is the (exogenously given) amount of stock issued during period t.

Finally, we consider cash (and receivables) for each financial unit. Let c(t) be the vector of cash at the beginning of period t. The change of cash during period t is due to the profit after tax (or loss), depreciation (i.e., noncash expenditure), drawing of debt, repayment, and investments. Thus we assume that the possible change in cash due to changes in accounts receivable, in inventories (wood, end products, etc.) and in accounts payable cancel each other (or that these quantities remain unchanged during the period). Alternatively, such changes could be taken into account assuming, for instance, that the accounts payable and receivable, and the inventories are proportional to annual sales of each financial unit.

Using our earlier notation, the state equation for cash is now

$$c(t+1) = c(t) + [I-\tau(t)]p^{+}(t) - p^{-}(t) + \overline{\delta}(t)\overline{q}(t) + \ell^{+}(t) - \ell^{-}(t) - K(t)v(t) + B(t) \quad . \tag{20}$$

3.3 Initial State and Terminal Conditions

In our industrial model, we now have the following state vectors: wood raw material inventory $r(t)$, (physical) production capacity $q(t)$, fixed assets $\bar{q}(t)$, long-term debt $\ell(t)$, cash $c(t)$, and total stockholders equity $b(t)$. For all of them we have an initial value and possibly a limit on the terminal value. We shall refer to the initial and terminal values by superscripts 0 and *, respectively; i.e., we have the initial state given as

$$\begin{aligned} r(0) &= r^0 \quad , \quad q(0) = q^0 \quad , \quad \bar{q}(0) = \bar{q}^0 \quad , \\ \ell(0) &= \ell^0 \quad , \quad c(0) = c^0 \quad , \quad b(0) = b^0 \quad , \end{aligned} \tag{21}$$

and a terminal state restricted, for instance, as follows:

$$\begin{aligned} r(T) &\geq r^* \quad , \quad q(T) \geq q^* \quad , \quad \bar{q}(T) \geq \bar{q}^* \quad , \\ \ell(T) &\leq \ell^* \quad , \quad c(T) \geq c^* \quad . \end{aligned} \tag{22}$$

The initial state is determined by the state of the forest industries at the beginning of the planning horizon. The terminal state may be determined as a stationary solution similarly as we described for the forestry model above.

If we consider the wood supply $y(t)$ being exogenous, we now have an industrial submodel which may be analyzed independently from the forestry submodel. A more complete duscussion on objectives will be given in the next section, but for illustrative purposes, we may choose now the discounted sum of industrial profits (after tax) as an objective function:

$$\sum_{t=0}^{T-1} \beta(t)\left[(I-\tau(t))p^+(t)-p^-(t)\right] \quad . \tag{23}$$

Here $\beta(t)$ is a (row) vector where components are the discounting factors for different financial units (for period t).

3.4 Industrial Model

We may now summarize the industrial model. Given initial state (21), find nonnegative control vectors x(t), $z^+(t)$, $z^-(t)$, m(t), e(t), v(t), $\ell^+(t)$, $\ell^-(t)$ $p^+(t)$, and $p^-(t)$, and nonnegative state vectors r(t), q(t), $\bar{q}(t)$, $\ell(t)$, c(t), and b(t), for all t which satisfy constraints and state equations (7) - (20), the terminal requirements (22), and maximize the linear functional given in (23).

As was the case with the forestry model, our industrial model may also be considered being regionalized. Again various indices (such as production activities, production capacities, etc.) should also refer to subregions within the country. Various transportation costs will then be included in direct production costs. For instance for a given product being produced within a given region there may be alternative production activities which differ from each other only in the source region of raw material.

4. THE INTEGRATED SYSTEM

We will now consider the integrated forestry--forest industries model. First we have a general discussion on possible formulations of various objective functions for such a model. Thereafter, we summarize the model in the canonical form of dynamic linear programming. A tableau representation of the structure of the integrated model will also be given.

4.1 Objectives

The forest sector may be viewed as a system controlled by several interest groups or parties. Any given party may have several objectives which are in conflict with each other. Obviously, the objectives of one party may be in conflict with those of another party. For instance, the following parties may be taken into account: representatives of industry, government, labor, and forest owners. Objectives for industry may be the development of profit of different financial units. Government may be interested in the increment of the forest sector to the gross national product, to the balance of payments, and to employment. The labor unions are interested in employment and total wages earned in forestry and different industries within the sector. Objectives for forest owners may be the income earned from selling and harvesting wood. Such objectives refer to different time periods t (of the planning horizon) and possibly also to different product lines. We will now give simple examples of formulating such objectives into linear objective functions.

Industrial profit. The vector of profits for the industrial financial units was defined above as $[I-\tau(t)]p^{+}(t) - p^{-}(t)$ for each period t. If one wants to distinguish between different financial units, then actually each component of such a vector may be considered as an objective function. However, often we aggregate such objectives for practical purposes, for instance, summing up discounted profits over all time periods, summing over financial units, or as in (23), summing over both time periods and financial units.

Increment to gross national product. For the purpose of defining the increment of the forest sector to the GNP we consider the sector as a "profit center" where no wage is paid to the employees within the sector, where no price is paid for raw material originating from this sector, and where no taxes exist. The increment to the GNP is then the profit for such a center. We will now make a precise statement of such a profit which may also be viewed as the valued added in the forest sector.

Let $P'(t)$ be a price vector so that $P'(t)e(t)$ is the total revenue from selling wood products outside the forest sector. Let $C'(t)$ be the vector of direct production unit costs excluding direct labor cost and cost of raw material which originates from the forest sector. Let $\hat{R}(t)$ and $\check{R}(t)$ be vectors of unit cost of planting and harvesting activities, respectively, excluding labor costs. For simplicity, we may assume that these latter two cost components include both operating and capital cost for machinery. The direct operating costs (excluding wages and wood based raw material) is then given, for period t, by

$C'(t)x(t) + \hat{R}(t)u^+(t) + \check{R}(t)u^-(t)$. Also the import and export of wood based raw material influence the GNP. Let $\hat{Z}(t)$ and $\check{Z}(t)$ be price vectors for imported and exported wood raw material, respectively, and let $M'(t)$ be the price vector of imported wood based products (to be used as raw material). Thus, the following term should be added to the GNP of period t: $\check{Z}(t)z^-(t) - \hat{Z}(t)z^+(t) - M'(t)m(t)$. The influence of the change in the wood inventory may be neglected in our model. For the fixed costs all except the labor costs will be taken into account. Let $F'(t)$ be the vector of such costs per unit of production capacity, let $\delta'(t)$ be the vector of depreciation factors, and $\varepsilon'(t)$ the vector of interest rates (for various financial units). Then the negative increment of the fixed costs, depreciation and interest to the GNP is given by $F'(t)q(t) + \delta'(t)\overline{q}(t) + \varepsilon'(t)\ell(t)$. Summing up, the increment of the forest sector to the GNP of period t is given by the following expression:

$$P'(t)e(t) - C'(t)x(t) - \hat{R}(t)u^+(t) - \check{R}(t)u^-(t) - \hat{Z}(t)z^+(t)$$

$$+ \check{Z}(t)z^-(t) - M'(t)m(t) - F'(t)q(t) - \delta'(t)\overline{q}(t) - \varepsilon'(t)\ell(t).$$

Increment to balance of payments. The increment of the forest sector to the balance of payments has a similar expression to the one above for the GNP. The changes to be made in this expression are, first, to multiply the components of the price vector $P'(t)$ by the share of exports in the total sales $e(t)$; second, to multiply the components of the cost vectors $C'(t)$, $\hat{R}(t)$, $\check{R}(t)$, and $F'(t)$ by the share of imported inputs in each cost term; third, to multiply each component of $\varepsilon'(t)$ by the share of foreign debts (among all long-term debts) of the financial unit; and finally, to replace the depreciation function $\delta'(t)\overline{q}(t)$ by investment expenditures $K'(t)v(t)$, where $K'(t)$ is a vector expressing investments in imported goods (per unit of production capacity).

Employment. Total employment (in man-years per period) for each time period t for different types of labor, in different activities and regions, has already been expressed in the left hand side expressions of inequalities (3) and (12). The expression for forestry is given by (part of the component of) the vector $R^{+}(t)u^{+}(t) + R^{-}(t)u^{-}(t)$ and for the industry by the vector $\rho(t)x(t)$.

Wage income. For each group of the work force, the wage income for period t is obtained by multiplying the expressions for employment above by the annual salary of each such group.

Stumpage earnings. Besides the wage income for forestry (which we already defined above), and an aggregate profit (as expressed in (6)), one may account for the stumpage earnings; i.e., the income related to the wood price prior to harvesting the tree. Such income is readily obtained by the timber assortments if the components of the harvesting yield vector y(t) are multiplied by the respective wood prices.

4.2 The Integrated Model

We will now summarize the integrated forestry-industry model in the canonical form of dynamic linear programming (Propoi and Krivonozhko 1978). Denote by X(t) the vector of all state variables (defined above) at the beginning of period t. Its components include the trees in the forest, different types of production capacity in the industry, wood inventories, external financing, etc. Let Y(t) be the nonnegative vector of all controls for period t, that is, the vector of all decision variables, such as levels of harvesting or production activities. An upper bound vector for Y(t) is denoted by $\hat{Y}(t)$ (some of whose components may be infinite). We assume that the objective function to be maximized is a linear function of the state vectors X(t) and the control vectors Y(t), and we denote by $\gamma(t)$ and $\lambda(t)$ the coefficient vectors for X(t) and Y(t), respectively, for such an objective function. This function may be, for instance, a linear combination of the objectives defined above. The initial state X(0) is denoted by X^0, and the terminal requirement for X(T) by X^*. Let $\Gamma(t)$ and $\Lambda(t)$ be the coefficient matrices for X(t) and Y(t), respectively, and let $\xi(t)$ be the exogenous right hand side vector in the state equation for X(t).

Let $\Phi(t)$, $\Omega(t)$, and $\psi(t)$ be the corresponding matrices and the right hand side vector for the constraints. Then the integrated model can be stated in the canonical form of DLP as follows:

find $Y(t)$, for $0 \leq t \leq T-1$, and $X(t)$, for $1 \leq t \leq T$, to

$$\text{maximize} \sum_{t=0}^{T-1} (\gamma(t)X(t)+\lambda(t)Y(t)) + \gamma(T)X(T) \quad ,$$

subject to

$$X(t+1) = \Gamma(t)X(t) + \Lambda(t)Y(t) + \xi(t) \quad , \qquad \text{for } 0 \leq t \leq T-1 \quad ,$$

$$\Phi(t)X(t) + \Omega(t)Y(t) \mathrel{\hat{=}} \psi(t) \quad , \qquad \text{for } 0 \leq t \leq T-1 \quad ,$$

$$0 \leq X(t) \quad , \quad 0 \leq Y(t) \leq \hat{Y}(t) \quad , \qquad \text{for all } t \quad ,$$

with the initial state

$$X(0) = X^0 \quad ,$$

and with terminal requirement

$$X(T) \mathrel{\hat{=}} X^* \quad .$$

The notation $\hat{=}$ for the constraints and terminal requirement refers either to =, to $\leq$ or to $\geq$, separately for each constraint. The coefficient matrix (corresponding to variables $X(t)$, $Y(t)$, and $X(t+1)$) and the right hand side vector of the integrated forestry-industry submodel of period t are given as

$$\begin{bmatrix} -\Gamma(t) & -\Lambda(t) & I \\ \Phi(t) & \Omega(t) & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \xi(t) \\ \psi(t) \end{bmatrix} \quad ,$$

respectively. Their structure has been illustrated in Figure 1 using the notation introduced in Sections 2 and 3.

w	r	q	$\bar{q}$	ℓ	b	c	u^+	u^-	y	x	z^+	z^-	m	e	v	ℓ^+	ℓ^-	p^+	p^-	
$-\alpha$							$-\eta$	ω												
	$-I$								$-I$	U	$-I$	I								$= 0$
		$\delta - I$													$-I$					$= 0$
			$\bar{\delta} - I$												$-K$					$= 0$
				$-I$												$-I$	I			$= 0$
					$-I$													$\tau - I$		$= 0$
			$-\bar{\delta}$			$-I$														$= 0$
G															K	$-I$	I	$\tau - I$	I	$= B$
							R^+	R^-												$\leq H$
								S	$-I$											$\leq R$
										A			I	$-I$						$= 0$
										ρ										$= 0$
		$-I$								Q										$\leq L$
				$\mu - I$																≤ 0
		F	$\bar{\delta}$	ε											μ					$\geq -\mu B$
										C				$-P$				$+I$	$-I$	$= -D$
Upper bounds:											Z^+	Z^-	M	E						

Figure 1. The constraint matrix $\begin{bmatrix} -\Gamma(t) & -\Lambda(t) \\ \Phi(t) & \Omega(t) \end{bmatrix}$, the right hand side vector $\begin{bmatrix} \xi(t) \\ \psi(t) \end{bmatrix}$, the state vector X(t), the control vector Y(t), and the upper bound vector $\hat{Y}(t)$ for the submodel of period t of the integrated forestry--industry model.

5. APPLICATION TO THE FINNISH FOREST SECTOR

5.1 Implementation

Two versions of the integrated model were implemented for the SESAME system (Orchard-Hays 1978) (a large interactive mathematical programming system designed for an IBM/370 and operating under VM/CMS). The model generators are written using SESAME's data management extension, called DATAMAT. An actual model is specified by the data tableaux of the generator programs.

Our two versions have been designed for the Finnish forest sector. Both of them may have at most ten time periods each of which is a five year interval. In each case, the country is considered as a single region. The main differences between our small and large version are in the number of products, financial units, and the tree species considered in the forest. Table 1 shows the dimensions of the two models.

For the small version, the seven product groups in consideration are sawn goods, panels, further processed mechanical wood products, mechanical pulp, chemical pulp, paper and board, and converted paper products. For each group we consider a separate type of production capacity and labor force. In this small version, we have aggregated all production into one financial unit. Only one type of tree represents all tree species in the forests. The trees are classified into 21 age groups. Thus, the interval being five years, the oldest group contains trees older than 100 years. Two harvesting activities were made available: thinning and final harvesting. The main timber assortments in consideration are log and pulpwood.

The larger version has the following 17 product groups: sawn goods, plywood, particle board, fiberboard, three types of further processed mechanical products, mechanical pulp, Si-pulp, Sa-pulp, newsprint, printing and writing paper, other papers, paperboard, and three types of converted paper products. Again for each such group we have a separate type of production capacity as well as labor force. The production is aggregated into seven

Table 1. Characteristic dimensions of the small and the large versions of the Finnish forest sector model.

	Small version	Large version
Number of time periods *	10	10
Length of one period in years *	5	5
Number of regions	1	1
Number of tree species	1	3
Number of age groups for trees*	21	21
Harvesting activities*	2	6
Soil types	1	1
Harvesting and planting resources	1	1
Timber assortments	2	6
Production activities	7	17
Types of labor in the industry	7	17
Types of production capacity	7	17
Number of financial units	1	7
Number of rows in a ten period LP	520	2320
Number of columns in a ten period LP	612	3188

*The value may be specified arbitrarily by the model data. The numbers show the actual values being used.

financial units: saw mills, panels production (plywood, particle board, and fiberboard), further processing of primary mechanical wood products, mechanical pulp mills, chemical pulp mills, paper and board mills, and production of converted paper goods.

Three species of trees appear in the larger version: pine, spruce, and birch. For each of these we apply the same 21 age groups as in the small version. The two harvesting activities (thinning and terminal harvesting) and the two main timber assortments (log and pulpwood) are now considered separately for each of the three tree species.

The data for both of the versions of the Finnish model was provided by the Finnish Forest Research Institute. It is partially based on the official forest statistics (Yearbook of Forest Statistics 1977/1978) published by the same institute. Validation runs (which eventually resulted in our current formulation) were carried out by contrasting the model solutions with the experience gained in the preceeding simulation study of the Finnish forest sector by Seppälä, Kuuluvainen and Seppälä (forthcoming).

5.2 Numerical Examples

For illustrative purposes we will now describe a few test runs: two with the small version and one with the larger one. Most of the data being used in these experiments corresponds approximately to the Finnish forest sector. This is the case, for instance, with the initial state; i.e., trees in the forests, different types of production capacity, etc. Somewhat hypothetical scenarios have been used for certain key quantities, such as final demand, and price and cost development. Thus, the results obtained do not necessarily reflect reality. They have been presented only to illustrate a few possible uses of the model.

For each test run a ten (five year) period model was constructed. Labor constraints both for indsutry and for forestry were temporarily relaxed. At this stage, no further processing activity for mechanical wood products but one activity for converted paper products was considered. Both wood import and export were excluded, and pulp import to be used for paper production was allowed only in the larger version of the model. The assumed demand of wood products is given in Table 2. At the end of the planning horizon, we require that in each age group there is at least 80 percent of the number of trees initially in those groups. For production capacity a similar terminal requirement is 50 percent. Initial production capacity is given in Table 3 and the initial age distribution of trees in Figure 8 below.

For the first run the discounted sum of industrial profits (after tax) was chosen as an objective function. Such an objective may reflect the industry's behavior given the cost structure, price development, and other parameters. The results have been illustrated in Figures 2 through 7. The mechanical processing activities are limited almost exclusively by the assumed demand of sawn goods and panels. The same is true for converted paper products. However, both mechanical and chemical pulp produced is almost entirely used in paper mills, and therefore, the potential demand for export has not been exploited. Neither have the possibilities for exporting paper been used fully. As shown in Figure 5, paper export is declining sharply from the level of 5 million ton/year, approaching zero towards the end of the planning horizon. This is due to the stongly increasing production of converted paper products. The corresponding structural change of the production capacity of the forest industry over the 30 year period from 1980 to 2010 is given in Table 3. (The sudden decrease in production of panels and converted paper products is a "planning horizon effect" which often appears in dynamic LP solutions. Usually it is due to inappropriate accounting for the future in terminal conditions. For instance, in our case only a reasonable state was required at the end of the planning horizon, while an optimal stationary state might have been more appropriate.)

Table 2. Assumed annual demand of wood products in Runs 1 - 3.

Period	Sawn wood Mm^3/y	Panels Mm^3/y	Mech. pulp Mton/y	Chem. pulp Mton/y	Paper and board Mton/y	Converted paper prod. Mton/y
1980-84	7.0	1.7	.02	1.2	4.8	0.5
1985-89	7.5	2.0	.01	1.1	5.8	0.7
1990-94	8.0	2.2	.01	1.0	7.0	0.9
1995-99	8.8	2.5	.01	0.9	8.3	1.2
2000-04	9.3	2.8	.01	0.8	9.8	1.6
2005-09	9.7	3.2	.01	0.7	11.6	2.1
2010-14	10.2	3.6	.01	0.7	13.2	2.9
2015-19	10.7	4.1	.01	0.6	15.1	3.8
2020-24	11.2	4.6	.01	0.6	17.1	5.1
2025-29	11.6	5.2	.01	0.6	19.2	6.9

Table 3. Production capacity initially and in 2010 according to Runs 1 - 3.

Product	Production capacity				Unit
	Initial	Year 2010			
		Run 1	Run 2	Run 3	
Sawn wood	7.0	10.2	10.2	10.2	M m^3/year
Panels	1.7	3.6	3.6	3.6	M m^3/year
Mechanical pulp	2.2	1.9	2.2	0.5	M ton/year
Chemical pulp	4.0	4.3	5.8	5.0	M ton/year
Paper (and board)	6.2	6.2	7.3	8.7	M ton/year
Converted paper and board products	0.5	2.9	2.9	2.9	M ton/year

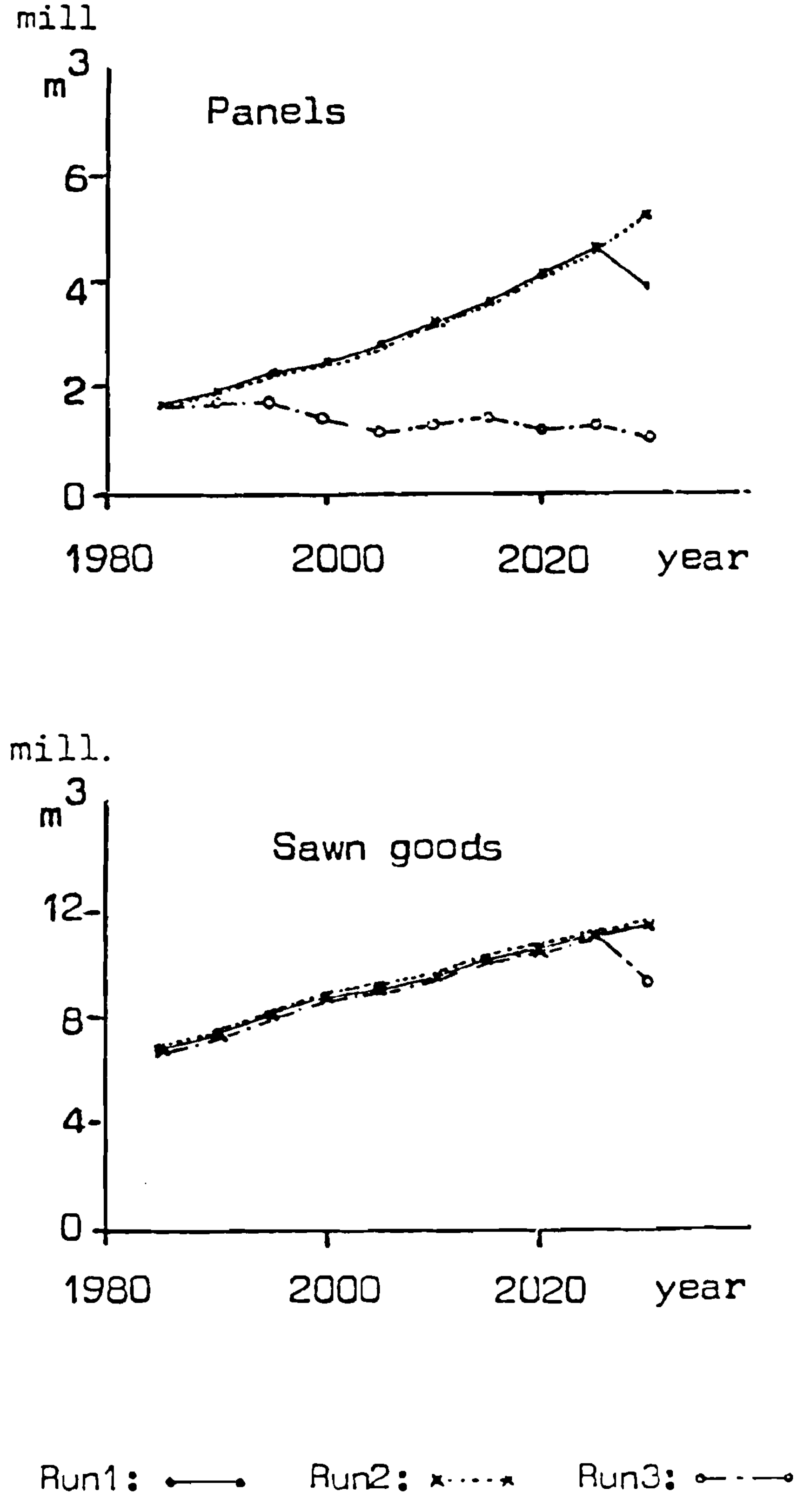

Figure 2. Annual production of sawn wood and panels (in millions of m^3 per year).

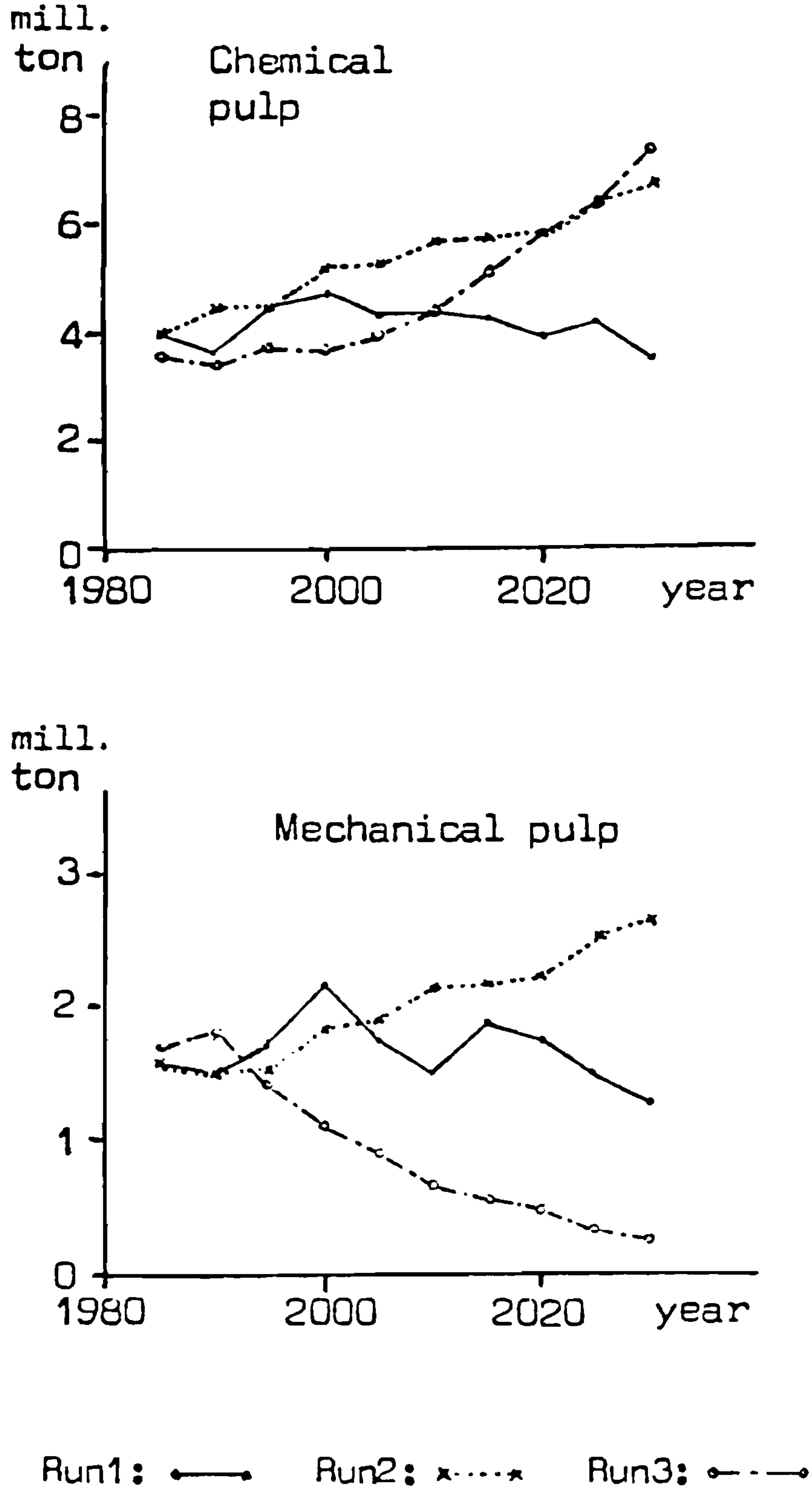

Figure 3. Annual production of pulp (in millions of ton per year).

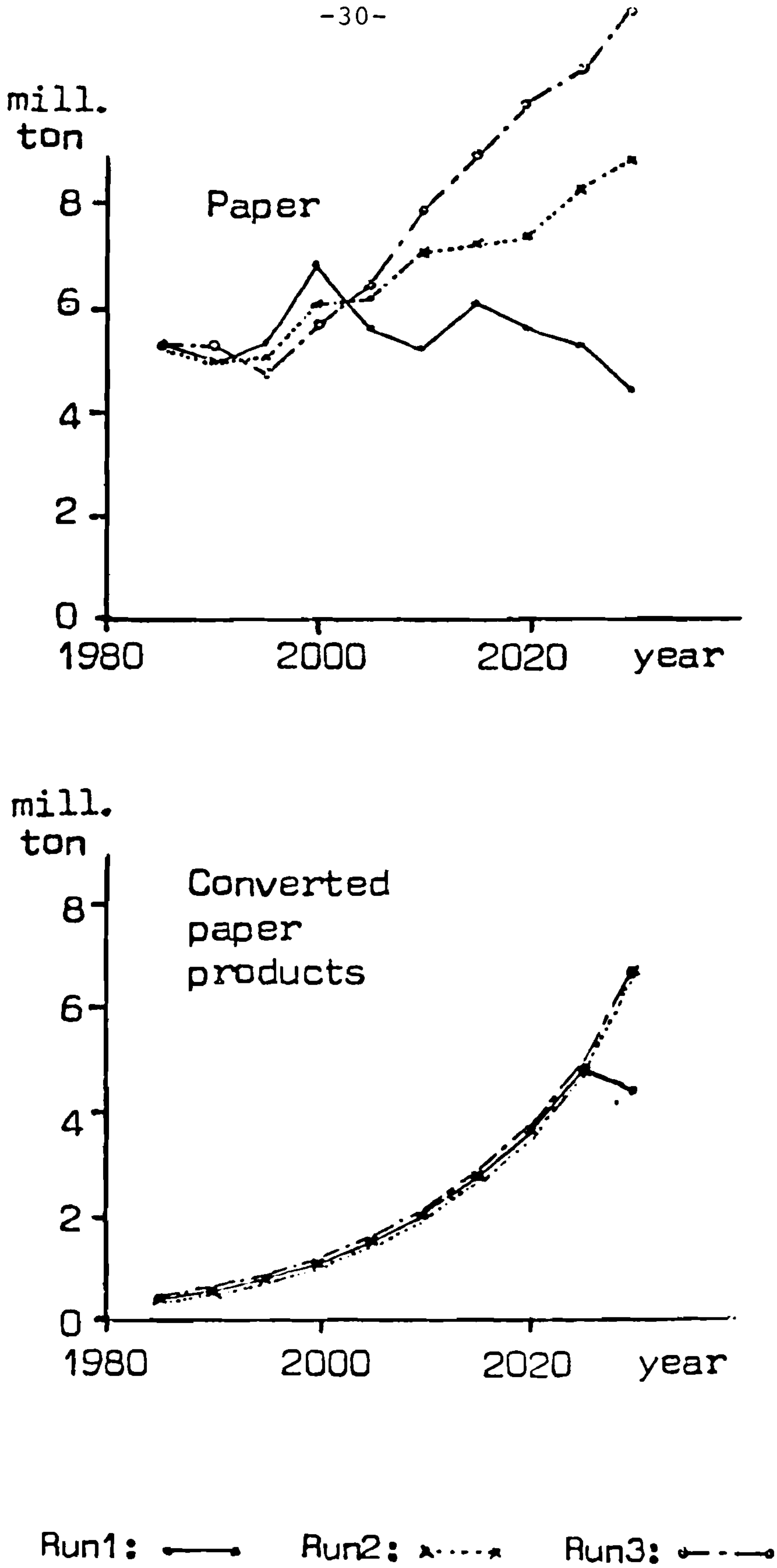

Figure 4. Annual production of paper and converted paper products (in millions of ton per year)

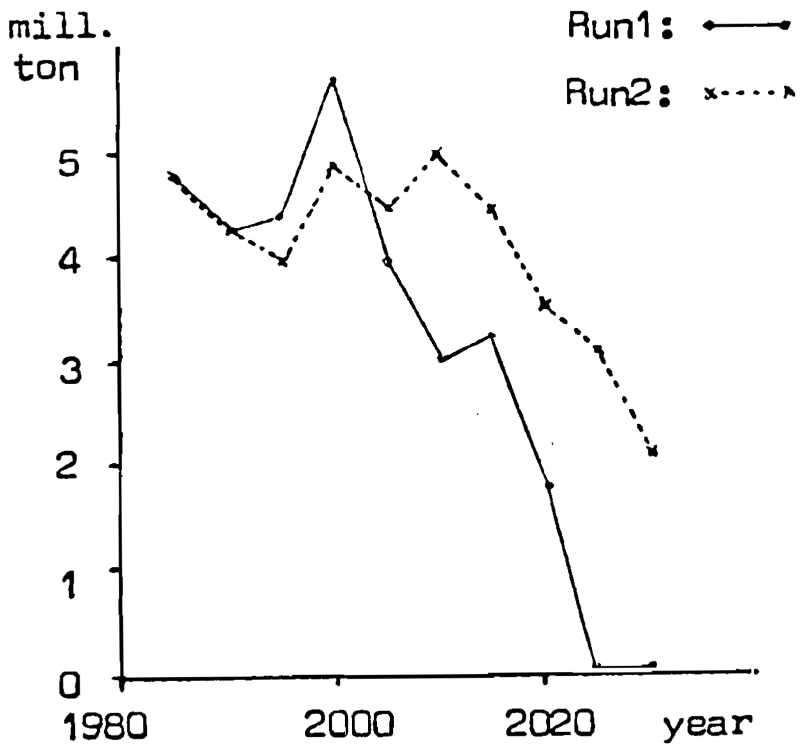

Figure 5. Paper export (in millions of ton per year)

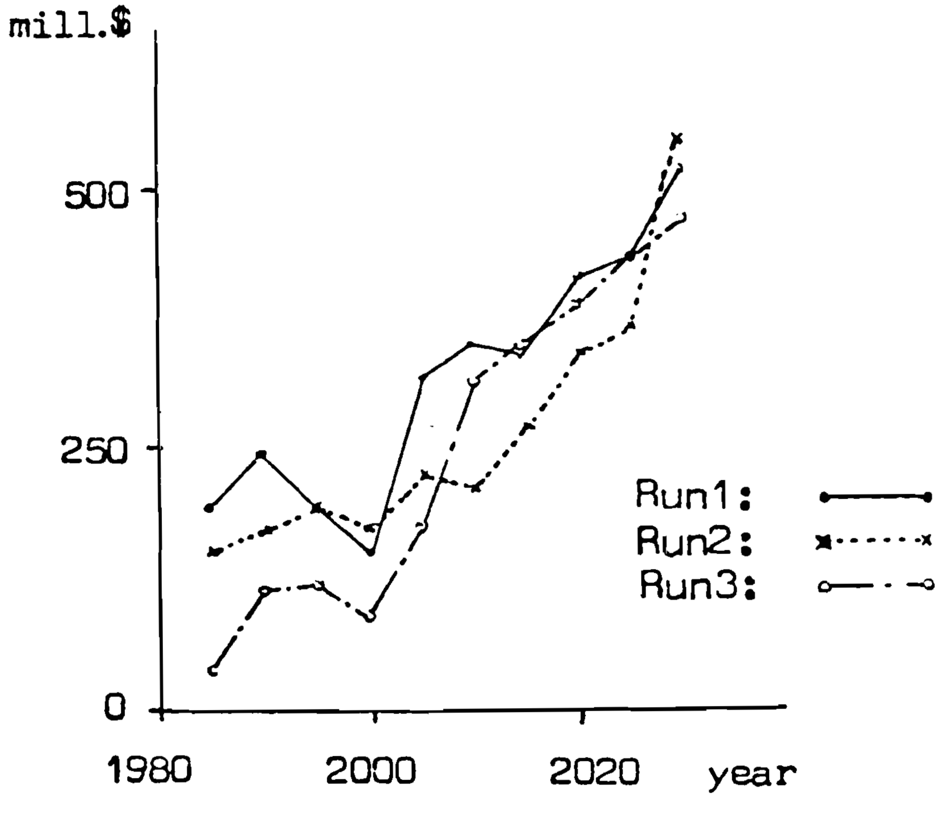

Figure 6. Industrial profit (in millions of dollars per year).

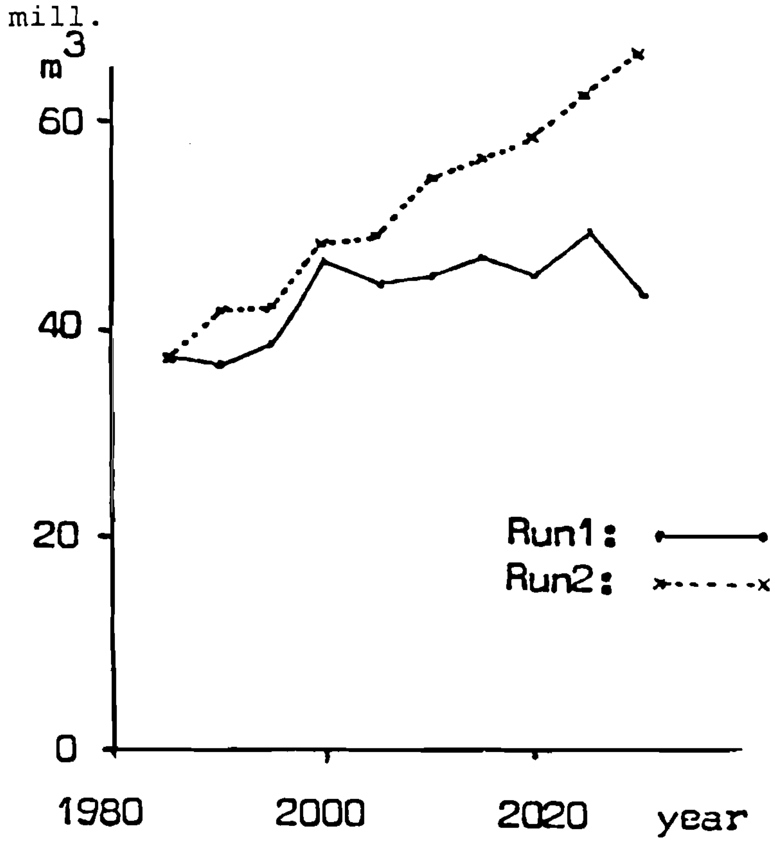

Figure 7. Industrial use of round wood (in millions of m^3 per year).

The use of wood has been shown in Figure 7. At the beginning the industrial use of wood increases from about 40 million m^3/year to the level of 45 million m^3/year and stays rather steadily there. According to Figure 6, the industrial profit increases from the annual level of .2 billion dollars towards the end of the planning horizon to around .5 billion dollars per year.

For the second run we have chosen the discounted sum of the increments of the forest sector to gross national product as an objective function. The results have been illustrated using dotted lines in the same Figures 2 through 7.

Compared with the previous case, there is no significant difference in the production of sawn goods, panels and converted paper products for which export demand again limits the production. However, there is a significant difference in pulp and paper production. Pulp (both mechanical and chemical) is now produced to satisfy fully the demand for export. Paper production is now steadily increasing from 5 million ton/year to nearly 9 million ton/year. Paper export is still declining again due to increasing use for the converting processes of paper products. Therefore, the export demand for paper is not fully exploited.

The bottleneck for paper production now is the biological capacity of the forests to supply wood. The use of round wood increases from about 40 million m^3/year to the level of 65 million m^3/year. The increase in the yield of the forests may be explained by the change in the age structure of the forests during the planning horizon. Such change over the period 1980-2010 has been illustrated in Figure 8.

We notice a significant difference in the wood use between these first two runs. We may conclude that in the first run (the profit maximization) the national wood resources are being used in an inefficient way; i.e., under the assumed price and cost structure the poor profitability of the forest industry results in an investment behavior which does not make full use of the forest resources.

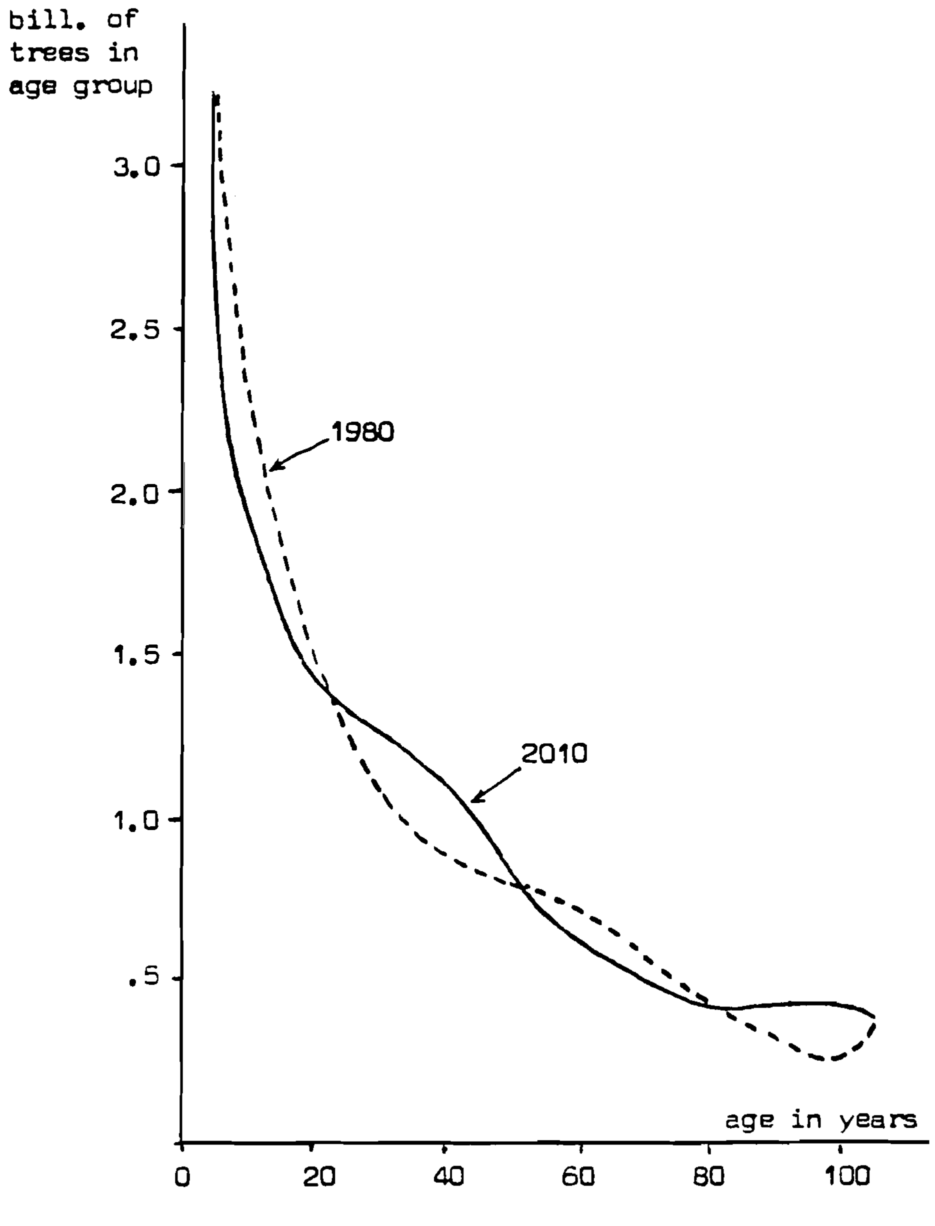

Figure 8. Age distribution of trees in 1980 and in 2010 according to Run2.

The third run is the same as the first one except that the larger version of the model was used and pulp import was allowed to be used in paper mills. The production of sawn goods and converted paper products, as described by broken lines in Figure 2, still meet the export demand. However, panel production is declining and it falls well below the level of the previous runs. The reason is that panel production is now considered as a separate financial unit which cannot afford to keep up its production capacity. Thus, an increase in panels production appears to be possible only if it is supported from other product lines. Similarly, the use of spruce for mechanical pulp appears unprofitable so that its production is declining. Production of Si-pulp (for which spruce pulpwood is used) grows steadily from 5 million ton/year to about 10 million ton/year. No spruce is used for Sa-pulp but both the use of pine and birch for Sa-pulp increase over time so that the total production of chemical pulp increases from about 3.5 million ton/year to the level of 7 million ton/year during the planning horizon. Thus chemical pulp production somewhat exceeds the amount produced in the first run.

Paper production in this third run exceeds the level obtained in both previous runs. The reason is that imported pulp is now allowed to be used in paper mills. (Note that in the second run, the raw wood supply was the limiting factor for paper production.) As a consequence, total paper production increased from 5 million ton/year to above 11 million ton/year. The share of newsprint is about one fifth and the share of printing paper one quarter. Only paperboard production appears to decline.

From the production curves of the primary uses of wood, i.e., sawn goods, panels and pulp, we may conclude (comparing with the second run) that wood resources are again being used inefficiently. It appears that, under the assumed price and cost structure, fiber (pulp in particular) import to be used as raw material in paper mills is more profitable than the use of domestic wood raw material.

6. SUMMARY AND POSSIBLE FURTHER RESEARCH

We have formulated a dynamic linear programming model of a forest sector. Such a model may be used for studying long-range development alternatives of forestry and forest based industries at a national and regional level. Our model comprises of two subsystems, the forestry and industrial subsystem, which are linked to each other through the raw wood supply from forestry to the industries. We may also single out static temporal submodels of forestry and industries for each interval (e.g., for each five year period) considered for the planning horizon. The dynamic model then comprises of these static submodels which are coupled with each other through inventory-type of variables; i.e., through state variables.

The forestry submodel describes the development of the volume and the age distribution of different tree species within the nation or its subregions. Among others, we account for the land available for timber production and the labor available for harvesting and planting activities. Also ecological constraints, such as preserving land as a watershed may be taken into account.

In the industrial submodel we consider various production activities, such as saw milling, panel production, pulp and paper milling, as well as further processing of primary products. For a single product, alternative production activities employing, for instance, different technologies, may be included. Thus, the production process is described by a small Leontief model with substitution. For the end product demand an exogenously given upper limit is assumed. Some products, such as pulp, may also be imported into the forest sector for further processing. Besides biological supply of wood and demand for wood based products, production is restricted through labor availability, production capacity, and financial resources. Availability of different types of labor (by region) is assumed to be given. The development of different types of production capacity depends on the initial situation in the country and on the investments which are endogeneous decisions in the model. The production activities are grouped into financial units to which the respective production capacities belong. The investments are made within the financial resources of such units. External financing is made available to each unit up to a limit which is determined by the realization value of that unit. Income tax is assumed proportional to the net income of each financial unit.

The structure of the integrated forestry-forest industry model is given in the canonical form of dynamic linear programs for which special solution techniques may be employed. (See, for instance, Kallio and Orchard-Hays 1979, Propoi and Krivonozhko 1978). Objectives related to gross national product, employment and profit for industry as well as for forestry have been formulated. Terminal conditions (i.e., values for the state variables at the end of the planning horizon) have been proposed to be determined through an optimal solution of a stationary model for the forest sector.

Two verisons of the Finnish forest sector model have been implemented for the interactive mathematical programming system called SESAME (Orchard-Hays 1978). Both versions are ten period models with each period five years in length. In neither case has the country been divided into subregions. The main difference between these versions are in the number of production activities and in the number of financial units. No distinction has been made between the tree species in the smaller version whereas pine, spruce, and birch are considered explicitly in the larger one. The complete model amounts to 520 rows and 612 columns in the smaller case, and to 2320 rows and 3188 columns for the larger model.

A few numerical runs have been presented to illustrate possible use of the model. Both the discounted industrial profit and the discounted increment to the GNP were used as objective functions. The results obtained illustrate a case where the internal wood price and wage structure results in a rather poor profitability for the forest industries. This in turn amounts to an investment behavior which provides insufficient capacity for making full use of the wood resources.

However, because of somewhat hypothetical data used for some key parameters, no conclusions based on these runs should be made on the Finnish case.

The purpose of this work has been the formulation, implementation and validation of the Finnish forest sector model. Natural continuation of this research is to use the model for studying some important aspects in the forest sector. For instance, the influence of alternative scenarios of the energy price and the world market prices for wood products would be of interest. Furthermore, the studies could concentrate on employment and wage rate questions, on labor availability restrictions and productivity, on new technology for harvesting and wood processing, on the influence of inflation and alternative taxation schemes, on land use between forestry and agriculture, on site improvement, on ecological constraints, on the use of wood as a source of energy, etc. Given the required data, such studies can be carried out relatively easily.

Further research requiring a larger modeling effort may concentrate on regional economic aspects, on linking the forest sector model for consistency to the national economic model, and on studying the inherent group decision problem for controlling the development of the forest sector. The first of these three topics requires a complete revision of our model generating program and, of course, the regionalized data. The second task may be carried out either by building in the model a simple input-output model for the whole economy where the non-forest sectors are aggregated up to ten sectors. Alternatively, our current model may be linked for consistency to an existing national economic model. The group decision problem has been proposed to be analyzed, for instance, using a multicriteria optimization approach (Kallio, Lewandowski, and Orchard-Hays forthcoming) which is based on the use of reference point optimization (Wierzbicki 1979).

APPENDIX: NOTATION

Indices

a, a'	age group of trees (range 1,..., N)
d	type of forest land
g	type of resource for forestry activities
h	harvesting activity
i	production activity (of the forest industries)
j	industrial product
k	timber assortment
n	planting activity
s	tree species
t	time period (range 1,..., T)

State and control variables

$b(t)$	stockholders equity at the beginning of period t
$b^0 = b(0)$	initial level of stockholders equity
$c(t)$	cash (and receivables) at the beginning of period t
$c^0 = c(0)$	initial amount of cash
c^*	terminal requirement for cash
$e(t) = \{e_j(t)\}$	export (and sales outside the forest sector) of forest products during period t

$\ell(t)$	beginning balance of external financing for period t
$\ell^0 = \ell(0)$	initial balance of external financing
ℓ^*	terminal requirement for external financing
$\ell^+(t)$	drawings of debt during period t
$\ell^-(t)$	repayments made during period t
$m(t) = \{m_j(t)\}$	import of forest products during period t
$p^+(t)$	profits of period t
$p^-(t)$	(financial) losses of period t
$q(t)$	production capacity at the beginning of period t
$q^0 = q(0)$	initial level of production capacity
q^*	terminal requirement for production capacity
$\bar{q}(t)$	fixed assets at the beginning of period t
$\bar{q}^0 = \bar{q}(0)$	initial value of fixed assets
$\bar{q}^*$	terminal requirement for fixed assets
$r(t) = \{r_k(t)\}$	timber assortments inventory at the beginning of period t
$r^0 = r(0)$	initial level of timber assortments inventory
r^*	terminal requirement for timber assortments inventory
$u^-(t) = \{u_h^-(t)\}$	level of harvesting activities during period t
u^-	level of harvesting in a stationary solution
$u^+(t) = \{u_n^+(t)\}$	level of planting activities during period t
u^+	level of planting in a stationary solution
$v(t)$	level of investments (in physical units) during t
$w(t) = \{w_s(t)\} = \{w_{sa}(t)\}$	number of trees at the beginning of of period t
$w^0 = w(0)$	initial number of trees
w^*	terminal requirement for the number of trees
w	number of trees in a stationary solution

$x(t)$	level of production activities during period t
$X(t)$	state vector at the beginning of period t
$X^0 = X(0)$	initial state
X^*	requirement for terminal state
$y(t) = \{y_k(t)\}$	supply of timber assortments during period t
$Y(t)$	level of control activities during period t
$z^+(t)$	import of timber assortments during period t
$z^-(t)$	export of timber assortments during period t

Parameters

$\alpha^s_{aa'}(t)$	ratio of trees of species s and in age group a that will proceed to age group a' during period t
α, α^s	matrices of coefficients $\alpha^s_{aa'}(t)$
$\beta(t)$	discounting factor
$\gamma(t)$	objective function coefficients for the state vector X(t)
$\Gamma(t)$	coefficient matrix for the state vector X(t) in the state equation
δ	physical depreciation rates
$\overline{\delta}(t)$	financial depreciation rates
Δ	age interval in an age group of trees (e.g., five years)
$\varepsilon(t)$	interest rates for external financing
$\psi(t)$	right hand side vector of constraints for period t
$\Phi(t)$	coefficient matrix for the state vector X(t) in constraints for period t
η	matrix relating planting activities to the increase in the number of trees
$\lambda(t)$	objective function coefficients for the control vector Y(t)
$\Lambda(t)$	coefficient matrix for the control vector Y(t) in the state equation
ω	matrix relating harvesting activities to the decrease in the number of trees
$\Omega(t)$	coefficient matrix for the control vector Y(t) in constraints for period t
$\rho(t)$	labor requirement for different production activities
$\tau(t)$	tax factors for the industries during period t

$\mu(t)$	upper limit to external financing as a percentage of total assets less short term liabilities
$\xi(t)$	right hand side vector for the state equation of period t
$A(t)$	input-output matrix for the forest industries
$B(t)$	stock issued during period t
$C(t)$	direct unit production costs
$D(t)$	exogeneously given costs
$E(t)$	upper bound on demand of forest products
$F(t)$	fixed costs (per unit of production capacity)
$G = (G^s_{ad})$	land requirement of the species in various age groups
$H(t)$	land available for forests
I	identity matrix
$J^-(t)$	objective function coefficients for harvesting activities (an example)
$J^+(t)$	objective function coefficients for planting activities (an example)
$K(t)$	investment costs per capacity unit
$L(t)$	labor available for forest industries
$M(t)$	upper limit on import of forest products
N	number of age groups for trees
$P(t)$	prices of forest products
$Q(t)$	matrix of capacity requirements for production activities
$R(t) = \{R_g(t)\}$	resources available for forestry activities
$R^+(t) = \{R^+_{gn}(t)\}$	resource usage of planting activities
$R^-(t) = \{R^-_{gh}(t)\}$	resource usage of harvesting activities
$S(t)$	matrix transforming the trees harvested into volumes of timber assortments
T	number of time periods
U	usage of timber assortments by various production activities

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