COST BENEFIT ANALYSIS OF LABOR ALLOCATION AND TRAINING SCHEMES

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PREFACE

Discrepancies in local labor markets occur as unsatisfactory matching of skill within the same region as well as redundant supply and insatiated demand among regions. Some of this discrepancy could--in principle--be removed by letting supply in one region meet demand in another. A reallocation policy of this kind poses a few questions of prominent concern:

1. Can economic disvalue arising from imperfection of labor markets at a regional level be mathematically assessed?
2. Is it possible to define a regional measure of inefficiency on both sides--demand and supply--of the labor market?
3. How should vacancies be distributed over skill and space to alleviate inefficiency?

These questions are investigated in this paper and a short-run solution is obtained via the primal-dual linear programming formulation of the problem.

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1. INTRODUCTION

Much of the current debate over efficient allocation of scarce resources involves a revised discussion of social sources of profit [1,2,3]. Inefficient allocations often result in so-called external diseconomies such as ecological damage, transportation congestion, communication costs and the like. If these factors are bound to play a dominant role in profit formation and growth, it appears appropriate that their effects should be explicitly introduced into economic analysis.

In this paper we address ourselves to inefficient allocations of jobs to vacancies occurring in a regional labor market.

As it has been observed by several investigators [4,5,6] there exist structural discrepancies between labor demand and supply at a local level, even in presence of an excess labor supply at a national level. This feature has led to the introduction of several theories, sometimes re-enforcing each other, in some case overlapping, often in sharp antagonism.

Neoclassical Theory postulates an automatic saturation of labor market sustained by wage rate adjustment to the value of marginal labor product. Thus, excess supply is met by lower, excess demand by higher wages. However a closer look into modern industrial infrastructure soon reveals the existance of jobs where qualification requirements come prior to wage levels, i.e. lack of skill is not always off-tradeable by lower wages on the management side. A similar rigidity is observable on the supply side. Relatively high wages but unsatisfactory working conditions (instability, environment, shifts ets.) may curb down labor supply or fail to attract labor at all.

Recognizing this feature, some authors introduced the conceptual device of a "dual labor market". This comprises a primary market linked to large, capital-intensive industries providing high wages, job stability and excellent working conditions: a secondary market with opposite features. Within each sector, the laws of marginal theory still hold but communication and information between sectors--if present--is one-directional, i.e. from primary to secondary [4, 5, 6 and 7]. In order for this theory to be workable at a regional level, it is necessary to assume spatial homogeneity within each market. While this may be true for certain advanced industrial cities, where large high-wage industries moved out of the city centres and small labor-intensive industries remained in the center, it appears questionable to postulate homogeneity in an integrated interregional analysis. Different regions may have very different development patterns and the distinction between primary and secondary market may lose much of its explanatory value when projected on a spatial dimension.

An extension of the dual labor market concept was recently introduced by [8, 9 and 10] in the form of a multi-segmented labor market. Such a finely articulated description may well be warranted on a micro-scale for sociological assessment of behavioral complexity. From the standpoint of economic analysis, however, we see no reason to further pursue this methodological trend outside the attempt to recover within each segment of the labor market the validity of marginal theory. Unwilling to enter the dispute here by pledging allegiance to a new theory or a new definition on this sociologically inflated subject, we will rather assume, along with [11], a Keynesian framework of downward sticking wages and existence of equilibrium under partial employment. Our purpose in this paper is to propose a method to assess the economic disvalue arising from imperfection of labor market mechanisms in combining supply and demand at a regional level.

Discrepancies in local labor market occur as unsatisfactory matching of skills within the same region as well as redundant supply and insatiated demand among regions. It is assumed that each of those discrepancies can be assigned a social disvalue in terms of re-training and/or commuting-migrating costs. This assumption permits to formulate the problem of allocating jobs to vacancies in the short-run as the minimization of economic disvalue of market imperfection. This minimization problem will be translated into the standard format of a Linear Programming Problem. The economic interpretation of the associated dual problem permits to evaluate the social cost-benefit of creating a job in a given region and skill, as well as the marginal value-disvalue of a new unemployee entering the labor market. It is hoped that the results of this kind of analysis will offer valuable information for an active regional policy both on the supply and the demand side.

2. A TYPOLOGY OF UNEMPLOYMENT

The main feature of our approach is to regard unemployment under two fundamental dimensions, skill and space. Within the same region labor demand and supply may not match due to skill discrepancies, although some of this discrepancy could be removed by letting supply in one region meet demand in another. This viewpoint requires, of course, a rather precise definition of a labor market region. Several definitions have been proposed in this regard, see for instance [2]. For our purposes it will be sufficient to introduce the following "closure" criterion: labor market regions are to be self-contained with respect to home-work commuting. On this basis it is possible to categorize unemployment under the five headings suggested by D. Gleave and D. Palmer [13] and freely re-phrased here:

frictional: *unemployed workers who could be employed in the same labor market (region) and occupation (skill) because there exist sufficient vacancies;*

spatial-structural: *unemployed workers who could find a job in the same occupation in a different labor market;*

occupational-structural: *unemployed workers who could gain employment withint their region if they covld be retrained and learn another skill;*

spatial-occupational-structural: *unemployed workers who would need to change both region and occupation to obtain employment;*

demand-deficient: *unemployed due to excess supply over the total number of vacancies.*

In [13] a partition algorithm is proposed to classify in the above five categories the unemployed by region and skill. That procedure, however, suffers from two major shortcomings. First, the solution obtained is dependent on the ordering according to which the single classes are defined. Secondly, the proportional criterion used in the assignment of vacancies to unemployment seems devoid of a theoretical justification.

A theoretical foundation for a classification procedure ought to be sought under the assumption of rational allocation of unemployment to vacancies. Let us assume throughout that our labor market comprises N regions and M skills. Assume that a social utility function*

$$U = U(\underline{x}, \underline{c}, \underline{l}, \underline{w}) \tag{1}$$

is assigned, where the arguments have the following meanings:

*The term "social utility function" will be freely used throughout the paper as a surrogate to "objective function", "performance index", "optimality criterion" etc. Although it will be occasionally abbreviated as "utility function" we stress that no relationship between utility and individual preferences is investigated in the present context.

$\underline{x} = \{x_{ir}^{js}\}$ vector of re-allocated unemployed. x_{ir}^{js} is the number of unemployed in skill i of region r re-allocated in skill j of region s,

$\underline{c} = \{c_{rs}\}$ cost-vector associated to labor movement (from region r to s),

$\underline{l} = \{l_{ij}\}$ cost-vector associated to retraining (from skill skill i to j),

$\underline{w} = \{w_{ir}\}$ vector of wage distribution over skills and regions,

$$i,j \in \{1,2,\ldots,M\} \quad , \qquad r,s \in \{1,2,\ldots,N\} \quad .$$

A rational reallocation of unemployment to vacancies requires--in principle--the solution of the following optimization problem:

$$\max_{\underline{x}} \quad U(\underline{x},\underline{c},\underline{l},\underline{w}) \quad , \tag{2}$$

s.t.

$$\sum_j \sum_s x_{ir}^{js} \leq D_{ir} \quad , \qquad \forall i,r \quad , \tag{3}$$

$$\sum_i \sum_r x_{ir}^{js} = V_{js} \quad , \qquad \forall j,s \quad , \tag{4}$$

$$x_{ir}^{js} \geq 0 \quad ,$$

with

D_{ir}: unemployment in sector i and region r,

V_{js}: vacancies in sector j and region s.

Remark: Constraints (3) and (4) imply that

$$\sum_i \sum_r D_{ir} \geq \sum_j \sum_s V_{js} \quad ,$$

i.e., total unemployment, over all regions exceeds total number of vacancies. Assuming existence conditions for a solution of problem (2)-(4) to be satisfied, let ${}^*x_{ir}^{js}$ be the optimal solution of problem (2)-(4). Then, the spatial distribution of unemployment according to the five categories mentioned above results in the definitions:

frictional:	$\{{}^{*}x_{ir}^{ir}\}$,	
spatial-structural:	$\{{}^{*}x_{ir}^{is}\}$,	$s \neq r$
occupational-structural:	$\{{}^{*}x_{ir}^{jr}\}$,	$j \neq i$
spatial-occupational-structural:	$\{{}^{*}x_{ir}^{js}\}$,	$j \neq i, \quad s \neq r$
demand-deficient:	$\{e_{ir}\}$,	

with

$$e_{ir} = D_{ir} - \sum_j \sum_s {}^{*}x_{ir}^{js} \quad .$$

3. OPTIMAL LABOR ASSIGNMENT AND WORKPLACE LOCATION

The classification of unemployment requires a precise definition of the utility function. We will assume that social utility is measured by the increase in GNP made possible by the reallocation of the labor force, after deduction of re-location costs. Therefore, the utility function will be assumed to be

$$U = \Delta Q - C \quad ,$$

where ΔQ is the total production increase and C the total re-location cost.

Assuming for Q a differentiable dependence on capital K labor l, human capital H, we will have

$$Q = Q(K,\underline{l},H) \quad ,$$

where labor is regarded as a vector indexed by skill and region.

In a short-run perspective we may assume constant capital. Therefore we get, up to first order approximation

$$\Delta Q = \sum_{js} \frac{\partial Q}{\partial l_{js}} \Delta l_{js} + \frac{\partial Q}{\partial H} \Delta H \quad . \tag{5}$$

Letting

$$\frac{\partial Q}{\partial l_{js}} \triangleq \beta w_{js}$$

be the prevailing wage for skill j in region s, we have also

$$\Delta l_{js} = \sum_{ir} x_{ir}^{js} \quad .$$

The increase in human capital can be measured by the increase in potential wage rates, i.e.

$$\Delta H = \sum_{ijrs} (w_{js} - w_{ir}) \; x_{ir}^{js} \quad .$$

Assuming constant marginal productivity of human capital

$$\frac{\partial Q}{\partial H} = \alpha$$

we obtain

$$\Delta Q = \sum_{ijrs} [(\beta+\alpha) w_{js} - \alpha w_{ir}] x_{ir}^{js} \quad .$$

The cost term C in the utility function is the sum of movement and retraining costs

$$C = \sum_{ijrs} (c_{rs} + t_{ij}) \; x_{ir}^{js} = \sum_{ijrs} [\beta w_{js} + \alpha (w_{js} - w_{ir})] \; x_{ir}^{js} \quad .$$

where c_{rs} and t_{ij} are the prevailing unit costs to move from r to s and to retrain from i to j.

In compact vector notation, the utility function takes on the linear form

$$U = \underline{\gamma}^T \underline{x} \quad ,$$

where all cost components have been arranged in the vector

$$\underline{\gamma} = \{\gamma_{ir}^{js}\} \quad ; \quad \gamma_{ir}^{js} \triangleq (\beta+\alpha)w_{js} - \alpha w_{ir} - c_{rs} - t_{ij} \qquad (6)$$

Remark 1.

This notation requires the components of vector $\underline{\gamma}$ and $\underline{x}$ to be arranged in the same order. To avoid ambiguity we will agree that component x_{ir}^{js} occupies position*

$$k = 1 + (s-1) + (j-1)N + (r-1)NM + (i-1)NM^2 \quad , \qquad (7)$$

in vector $\underline{x}$ (and similarly for $\underline{\gamma}$).

On these premises it is possible to formulate the optimization problem

$$\max \underline{\gamma}^T \underline{x} \quad , \qquad (8)$$

$$A\underline{x} \leq \underline{D} \quad , \qquad (9)$$

$$\left.\begin{aligned} B\underline{x} &= \underline{V} \quad , \\ \underline{x} &\geq \underline{0} \quad , \end{aligned}\right. \qquad (10)$$

where A is an $NMx(NM)^2$ matrix with the following structure [induced by (7)]

$$A \triangleq \begin{bmatrix} \underline{1}^T \underline{0}^T \ldots \underline{0}^T \\ \underline{0}^T \underline{1}^T \ldots \underline{0}^T \\ \ldots \quad \ldots \\ \underline{0}^T \underline{0}^T \ldots \underline{1}^T \end{bmatrix} \quad ; \qquad \begin{aligned} \underline{1}^T &= \{111\ldots1\} \sim 1 \text{ x } NM \\ \underline{0}^T &= \{000\ldots0\} \sim 1 \text{ x } NM \end{aligned}$$

and B an NM x $(NM)^2$ matrix

$$B \triangleq [I|I|\ldots|I] \quad ; \qquad I \sim (NM)x(NM) \text{ identity matrix,}$$

$$D = \{D_{ir}\} \quad ; \qquad \underline{V} = \{V_{js}\} \quad ,$$

*This is a standard way to store multidimensional arrays into a one dimensional vector.

and component D_{ir} occupies position

$$k = 1 + (r-1) + (i-1)N = r + (i-1)N$$

in vector $\underline{D}$ (and likewise for $\underline{V}$ with r and i replaced by s and j).

Remark 2.

This is a Linear Programming Problem. As matrix $C = [\frac{A}{B}]$ is unimodular and $\underline{V}$, $\underline{D}$ are integer-valued, any basic solution is integer.

The solution $\underline{x}^*$ of problem (8)-(10) yields the optimal assignment of unemployed workers to vacancies.

Remark 3.

Once the solution has been obtained in terms of the vector $\underline{x}^*$, it is possible to recover the individual components x_{ir}^{js} by inverting mapping (7) according to :

$$s = R[\frac{K}{N}] + N\delta(R[\frac{K}{N}]) \quad ,$$

$$j = R[\frac{K_1}{M}] + M\delta(R[\frac{K_1}{M}]) \text{ with } K_1 = Q[\frac{K}{N}] + 1 - \delta(R[\frac{K}{N}]) \quad ,$$

$$r = R[\frac{K_2}{N}] + N\delta(R[\frac{K_2}{N}]) \text{ with } K_2 = Q[\frac{K_1}{M}] + 1 - \delta(R[\frac{K_1}{M}]) \quad ,$$

$$i = R[\frac{K_3}{M}] + M\delta(R[\frac{K_3}{M}]) \text{ with } K_3 = Q[\frac{K_2}{N}] + 1 - \delta(R[\frac{K_2}{N}]) \quad ,$$

where

$Q[\cdot]$ denotes integral part,
$R[\cdot]$ denotes remainder,
$\delta(\cdot)$ is the Kronecker function.

4. SHADOW PRICES OF VACANCIES AND UNEMPLOYMENT

In this section we will attempt to assign a "market value" to inefficient allocation of labor.

It is clear from the foregoing discussion that a different distribution of work places and unemployed people results in a different value of the utility function.

Inefficient allocations in the labor market yield a suboptimal value of the utility function. Therefore it is reasonable to define "inefficiency" of the labor market as the difference between the current and the optimal value of the utility function. However we have been unable so far to disaggregate inefficiency by its most significative components. We will distinguish two major factors contributing to it:

1) unproductiveness of a potential employee who has not been able to gain employment, on the supply side;
2) for a given production level, higher (lower) relocation-retraining cost resulting from inappropriate (appropriate) vacancy assignment, on the demand side.

A formal definition of these concepts is now required. Let $y_{ir}^{(1)}$ be the contribution to inefficiency due to one unemployed in region r and skill i; and $y_{js}^{(2)}$ the contribution due to one vacancy in region s and skill j. While it is reasonable to assume $y_{ir}^{(1)} \geq 0$ no restriction will be posed on the sign of $y_{js}^{(2)}$. In this framework it is meaningless to consider the case in which a component of the re-allocation cost $\underline{\gamma}$ exceeds the value of the corresponding component of the labor market inefficiency. Therefore the following constraint results:

$$y_{ir}^{(1)} + y_{js}^{(2)} \geq \gamma_{ir}^{js} \quad \forall\ i,j,r,s \quad ,$$

or, in matrix notation

$$A^T \underline{y}^{(1)} + B^T \underline{y}^{(2)} \geq \underline{\gamma} \quad . \tag{11}$$

Inefficiency will be minimal when the total

$$L = \underline{D}^T \underline{y}^{(1)} + \underline{V}^T \underline{y}^{(2)} \tag{12}$$

is minimized on the set (11).

As it might have been expected, problem (11), (12) is the dual formulation of problem (8)-(10). Since the solution of the primal problem is bounded, the solution of this problem will be bounded. The optimal solution $\underline{y}^*$ of problem (11)-(12) yields the shadow prices of unemployment and vacancies:

$^*y_{ir}^{(1)}$ can be interpreted as the shadow cost of one unemployed in region r and skill i;

$^*y_{js}^{(2)}$ can be interpreted as the shadow price of one vacancy in region s and skill j.

The solution of the dual problem permits to disaggregate the cost vector $\underline{\gamma}$ into a part $\{y_{ir}^{(1)}\}$ associated to unemployment (supply side) and a part $\{y_{js}^{(2)}\}$ associated to vacancies (demand side).

Furthermore, this allows to establish a four-fold categorization of labor market inefficiency by means of the comparative indices:

$$\bar{y}_r^{(1)} = \frac{\sum_i {}^*y_{ir}^{(1)} D_{ir}}{\sum_i D_{ir}} \quad ,$$

average inefficiency of labor market in region r, measured on the supply side;

$$\bar{y}_i^{(1)} = \frac{\sum_r {}^*y_{ir}^{(1)} D_{ir}}{\sum_r D_{ir}} \quad ,$$

average inefficiency of labor market with respect to skill i, measured on the supply side;

$$\bar{y}_r^{(2)} = \frac{\sum_i {}^*y_{ir}^{(2)} V_{ir}}{\sum_i V_{ir}} \quad ,$$

average inefficiency of labor market in region r, measured on the demand side;

$$\bar{y}_i^{(2)} = \frac{\sum_r {}^*y_{ir}^{(2)} V_{ir}}{\sum_r V_{ir}} \quad ,$$

average inefficiency of labor market with respect to skill i, measured on the demand side.

5. OPTIMAL WORK-PLACE LOCATION POLICY AND TRAINING SCHEMES

Assume that the reallocation of labor force has been completed according to scheme (8)-(10). It is obvious that, in each region, there will remain a number of unemployed

$$\sum_i e_{ir} = \sum_i (D_{ir} - \sum_j \sum_s {}^*x_{ir}^{js}) \quad , \qquad r = 1,2,\ldots,N \quad .$$

If a number W of new work-places were made available at some subsequent time how should a policy maker distribute them over skill and space to minimize inefficiency? Retaining the assumption of excess labor supply, we will take

$$W \leq \sum_i \sum_r e_{ir} \quad .$$

The unknown work-place distribution W_{ir} will have to satisfy the constraints

$$\sum_i W_{ir} \leq \sum_i e_{ir} \quad , \qquad r = 1,2,\ldots,N \quad ,$$

$$\sum_{ir} W_{ir} = W \quad ,$$

$$W_{ir} \geq 0 \quad , \qquad \forall i,r \quad .$$

The optimality criterion we suggest will again be based on efficiency. Using the inefficiency measure introduced in section 4, we define an objective function

$$I = \sum_{ir} y_{ir}^{(2)} W_{ir} \quad .$$

Minimizing this index is equivalent to using the newly created work-places W_{ir} to reduce as far as possible labor market inefficiency.

On the supply side an optimal training scheme under excess labor supply can do no better than promoting educational outputs in accordance with the skill ranking induced by the ${}^{*}y_{ir}^{(1)}$ coefficients, i.e. in each region r, efforts should be directed to re-convert training in skill i to training in skill j whenever ${}^{*}y_{ir}^{(1)} > {}^{*}y_{jr}^{(1)}$.

6. CONCLUSIONS AND SUGGESTED EXTENSIONS

The analysis we carried out has been based on an essentially static approach. Our main concern was to provide a computational tool to interpret data on regional unemployment and assist the decision maker in devising regional policies. This justifies the assumptions of exogenous costs and wage levels. Within the framework of our definitions and basic assumptions, three conclusions can be drawn:

1. the notion of labor market inefficiency can be given a quantiative basis;
2. inefficiency can be decomposed into a supply and a demand component;
3. regional and occupational indices can be defined to draw the inefficiency map of a regional market system.

In a paper that will shortly appear, the method will be applied to analyze data of an Italian region. In a long-run perspective, a more comprehensive analysis should be warranted. As some authors (see for instance [13]) have pointed out, it is unlikely that meaningful results could be obtained outside a general equilibrium framework. To our knowledge such an approach has never been attempted on a regional scale. Aside from the problem of a sufficient data-set to test a dynamical theory of unemployment, some of the assumptions contained in [13] (see for instance the market compatibility assumption) would have to be discussed in a regional context. The introduction of space into existing theories of unemployment may not be a trivial extension of those theories.

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