NOT FOR QUOTATION
WITHOUT PERMISSION
OF THE AUTHOR

A DISAGGREGATED ANNUAL MODEL OF LABOR SUPPLY AND UNEMPLOYMENT, 1951-2000

Robert M. Coen and Bert G. Hickman

January 1980
WP-80-15

Working Papers are interim reports on work of the International Institute for Applied Systems Analysis and have received only limited review. Views or opinions expressed herein do not necessarily represent those of the Institute or of its National Member Organizations.

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

ABSTRACT

The quantity of manhours available as a factor input is one of the principal determinants of an economy's potential output. Although such a measure of labor supply may be strongly influenced by population developments, there are several aspects of economic behavior that intervene between growth of population and growth of manhours. These include the willingness of individuals to join the labor force, the aggregate employment rate of those in the labor force, and hours worked per employee. In the United States, for example, the working-age population was 53 percent larger in 1977 than in 1947, yet aggregate manhours worked increased by only 40 percent, and private manhours rose by only 27 percent.

This paper presents an integrated empirical analysis of the links between population and labor supply. Determinants of labor force participation by age and sex, annual hours per worker, and the unemployment rate are examined for the post-World War II period. The relations estimated for these variables

are designed for use in a new version of the Hickman-Coen annual growth model of the United States, but we take advantage here of the fact that they form a subsystem which jointly determines the supply of manhours, the high-employment unemployment rate, and the natural rate of unemployment. We show how the subsystem can be so used and give conditional estimates of these variables for the historical period 1951-77 and the future through the year 2000.

A DISAGGREGATED ANNUAL MODEL OF LABOR SUPPLY AND UNEMPLOYMENT, 1951-2000

Robert M. Coen and Bert G. Hickman[1/]

This paper presents an empirical model of labor supply that jointly determines aggregate manhours, the high-employment unemployment rate, and the natural rate of unemployment, conditional on population and real wages. The model's structural relations are a subset of equations from a new version of the Hickman-Coen annual growth model of the United States (Hickman and Coen 1976) and are estimated with annual time series for the post-World War II period. They include: (a) labor force participation equations by age and sex, capturing labor force responses to employment opportunities and real wages; (b) an equation relating hours per worker to variations in labor demand, the real wage and the sex composition of the work force; and (c) an aggregate unemployment equation in which the natural rate of unemployment depends on the degree of mismatch between job requirements and workers' skills, the size of the military, and unemployment benefits, and deviations between the actual and natural unemployment rates are accounted for by unanticipated wage changes and cyclical fluctuations in labor demand, à la search theory.

The model is solved for conditional estimates of manhours, the high-employment unemployment rate, and the natural rate of unemployment for the historical period 1951-77 and for the future through the year 2000. Special attention is given to the impacts of past and projected changes in the structure of the population on labor supply and unemployment.

LABOR FORCE PARTICIPATION EQUATIONS BY AGE AND SEX

Specification of the disaggregated participation equations parallels that of the aggregate equation developed in our previous work (Coen 1973, Hickman and Coen 1976:67-75). We define the participation rate of the i-th age sex group as the ratio of civilian labor force to noninstitutional population in that group, LC_i/NNI_i. Each group's participation rate is assumed to be influenced by the real after-tax hourly wage rate, W1/PCE[2/], and by variations in the state of the job market affecting both the probabilities of obtaining work and workers' realized incomes. The employment-population ratio, E/NNI, and average annual hours per worker, AH1, are presumed to be the principal indicators of cyclical changes in job opportunities and earnings at a given real wage. A positive relationship between participation and E/NNI and AH1 would indicate that cyclical declines in employment or hours lead some individuals to become so discouraged about their chances of finding a job, or about the incomes they will earn if employed, that they leave the labor force. Existence of these "discouraged-worker" effects, particularly among women and teenagers, has been a principal theme in studies of the short-run elasticity of labor supply[3/].

Since labor force here encompasses only civilians, we allow for influences of military recruitment on participation by including the ratio of armed forces to population, LA/NNI. There are several ways in which mobilizations might affect the size of the labor force. New recruits might be drawn in part from the current labor force; potential draftees might withdraw from work and return to school in hope of avoiding military service; and the loss of income associated with military service for young males may lead other members of their families to seek employment in the hope of maintaining family income.

Finally, we introduce a time trend, TIME5, to capture changing institutional and social factors not otherwise accounted for.

Table 1 gives estimates of the participation equations for males for the period 1950-77. The equations were routinely corrected for first-order autocorrelation in the residuals, RHO being the estimated autocorrelation coefficient. DW is the Durbin-Watson ratio for the RHO-adjusted disturbances, and SEE is the standard error of estimate.

Changes in the real wage are seen to have insignificant effects on male participation, with the possible exception of a positive relation for males age 45-54. Participation rates of 25-64 year olds are evidently insensitive to variations in employment and hours, while those of younger and older males generally decline with employment (discouraged-worker effects) but move inversely to hours (added-worker effects). The impacts of a rise in the armed forces are substantial and negative for 18-24 year olds, negative again but smaller for those 25-34,

Table 1

Labor Force Participation Equations, Males, 1950-77

Regression Coefficients and t-Statistics

	Variable								
Age Group	Constant	E/NNI	LA/NNI	W1/PCE	AH1	TIME5	RHO	DW	SEE
16-17	.51012 (1.11)	1.8759 (6.98)	.77596 (1.32)	-.00624 (0.18)	-.50520 (2.30)	-.00187 (0.57)	.86911 (9.30)	2.3017	.0080
18-19	-1.1939 (1.72)	.36970 (0.92)	-5.1045 (5.87)	-.06771 (1.32)	.70512 (2.15)	.01135 (2.14)	.90101 (11.0)	1.2159	.0121
20-24	1.3407 (1.36)	.96639 (1.66)	-10.062 (7.80)	-.03522 (0.47)	-.39830 (0.83)	-.00114 (0.17)	.80447 (7.17)	1.2500	.0171
25-34	1.1794 (4.32)	.10727 (0.67)	-1.3986 (3.88)	-.00161 (0.08)	-.11127 (0.84)	-.00158 (0.88)	.68540 (4.98)	1.5375	.0046
35-44	.98457 (5.79)	.02763 (0.28)	-.30821 (1.38)	.00431 (0.34)	-.00041 (0.00)	-.00151 (1.36)	.59611 (3.93)	1.5288	.0029
45-54	1.0499 (10.3)	-.07898 (1.32)	-.07436 (0.57)	.01711 (2.23)	.02580 (0.52)	-.00383 (5.33)	.85046 (8.55)	2.1141	.0017
55-64	1.9044 (5.74)	-.06626 (0.35)	.21843 (0.53)	.01990 (0.82)	-.24589 (1.58)	-.01324 (5.03)	.91353 (11.9)	1.9025	.0057
65+	1.0689 (2.84)	.56278 (2.54)	.24300 (0.50)	.01352 (0.48)	-.31880 (1.75)	-.01170 (4.49)	.82907 (7.85)	2.0114	.0065

and otherwise negligible. Significant trends appear in the equations for 18-19 year olds (upward) and for those 45 years of age and older (downward).

Female participation rates have behaved quite differently from male rates. Until the mid-1960's, rates for younger women were relatively stable, while those for older women were generally rising. Thereafter, the former rose dramatically and the latter either levelled off or increased at a much slower pace. The variables used to explain the male participation rates cannot account for these patterns. One could, of course, introduce appropriate breaks in the time trends to improve the equations (see, for example, Perry 1977), but we have chosen instead to examine an hypothesis proposed by Easterlin (1978) which accords an important role to demographic changes in determining participation rates of women.

Easterlin argues that participation of women has been much influenced by changes in the age structure of the male population, at least since immigration to the U.S. was restricted in the 1920's. Assuming that younger and older males are not substitutable in the eyes of employers, and that younger males have a more or less fixed participation rate, a rise in the ratio of younger to older males should drive down the earnings of younger males, below what they might have aspired to earn. As a result, younger women seek employment in greater numbers to supplement their husbands' earnings. Assuming that younger and older women are highly substitutable to employers, the influx of younger women tends to restrict job opportunities for older women, who leave the labor force discouraged. Thus, according to this hypothesis, we should expect participation rates of younger women to be positively related to the ratio of younger to older males, while those of older women should be negatively related to the same ratio.

The female participation equations in Table 2 test the Easterlin hypothesis by adding a new variable NNIMRAT, the the ratio of males 16-34 to those 35-64 in the noninstitutional population. (NNIMRAT was about 0.86 in 1949, declined rather steadily to 0.76 in 1962, and then rose steadily to 1.08 in 1977.) The hypothesis is strikingly confirmed. NNIMRAT is highly significant in all but one of the equations, and the sign of its coefficient changes from positive to negative as we move from the younger groups to the older.

The configuration of signs on NNIMRAT and on the time trends suggests the following interpretations of participation patterns. For females 16-44, positive trends were offset by the decline in NNIMRAT through the early 1960's; thereafter, the trends and growth of NNIMRAT reinforced one another resulting in large increases in participation. For females 45-64, positive trends and NNIMRAT interact in the opposite way, reinforcing one another to augment participation through the early 1960's, after which time the growth in NNIMRAT begins to offset the trends.

Some other characteristics of the equations for females are worth noting. The cyclical indicators of labor market conditions, E/NNI and AH1, have much stronger effects than in the case of males. Significant discouraged-worker responses to employment changes are evident for most female groups, the greatest sensitivity occurring in the younger groups. Modest added-worker responses are found, however, for changes in hours

Table 2

Labor Force Participation Equations, Females, 1950-77

Regression Coefficients and t-Statistics

Age Group	Constant	E/NNI	LA/NNI	W1/PCE	AH1	TIME5	NNIMRAT	RHO	DW	SEE
16-17	.53025	1.7193	.69599	-.09257	-.60724	.00035	.36902	.09890	1.7879	.0087
	(1.16)	(5.51)	(1.37)	(2.85)	(2.94)	(0.12)	(10.1)	(0.53)		
18-19	.61250	.84376	-.18346	-.10148	-.29691	.00405	.21775	.02463	1.7404	.0074
	(1.61)	(3.21)	(0.45)	(3.80)	(1.75)	(1.75)	(7.34)	(0.13)		
20-24	.22013	.46825	-.05560	-.01805	-.18167	.00405	.34502	.25623	1.7970	.0064
	(0.62)	(1.99)	(0.13)	(0.70)	(1.10)	(1.81)	(11.1)	(1.40)		
25-34	-.09388	.33813	.69006	-.05409	-.19154	.01269	.31781	.93681	1.3555	.0062
	(0.23)	(1.65)	(1.50)	(2.05)	(1.14)	(1.89)	(1.90)	(14.2)		
35-44	.50177	.43106	-.10541	-.01604	-.25749	.00298	.21501	.75951	1.2825	.0047
	(1.81)	(2.59)	(0.29)	(0.77)	(1.92)	(1.43)	(3.47)	(6.18)		
45-54	.87955	.53136	-.41049	-.02633	-.32209	.00734	-.27486	.38565	1.5710	.0052
	(2.91)	(2.73)	(1.10)	(1.19)	(2.25)	(3.83)	(9.30)	(2.21)		
55-64	-.10664	.14730	-.02038	.03304	.14095	.00799	-.35585	.68973	1.6256	.0065
	(0.28)	(0.63)	(0.04)	(1.15)	(0.76)	(3.01)	(5.34)	(5.04)		
65+	.32180	.27528	-.31263	.02108	-.12608	-.00222	-.10092	.13620	1.9204	.0035
	(1.71)	(2.16)	(1.48)	(1.58)	(1.48)	(1.91)	(6.59)	(0.73)		

in several groups. The real wage also appears to play a larger role in determining female participation than male participation. In particular, there is a significant inverse relation between participation rates of younger women and the real wage. The armed forces variable, on the other hand, does not contribute much to explaining female participation.

Responses of the total labor force to changes in aggregate employment, etc., are not readily discernible from Tables 1 and 2, since these responses depend on the age-sex composition of the population. The estimated coefficients imply that for the years 1950-62, a one percent increase in employment would have produced about a 0.34 percent increase in the total labor force. Thereafter, the employment elasticity gradually increases to 0.37 by the mid-1960's as a result of the rise in the proportion of youth in the population. Throughout the sample period, roughly 60 percent of the increase in labor force is accounted for by women. The real wage elasticity rises from -0.04 in the early postwar years to -0.07 in the 1970's, indicating growing labor force sensitivity to the real wage as well as to employment. The wage elasticity for males as a group is virtually zero; the negative wage elasticities reflect primarily the net exodus of young females from the labor force as the wage increases.

THE AVERAGE HOURS EQUATION

Our equation for annual hours per worker in the private sector is a hybrid relation incorporating both supply and demand factors. Workers' desired hours are assumed to depend on the real after-tax wage in the private sector, W3/PCE. Actual hours may diverge from desired hours, however, due to cyclical variations in the demand for labor. Our proxy for these cyclical effects is the conventional unemployment rate, U1.

Although we do not disaggregate hours by age and sex, we allow for the possible effects of changes in the composition of the labor force on average hours. The hypothesis that naturally suggests itself is that average hours will tend to decline as the proportion of teenagers and women in the labor force rises, since these groups are more inclined to be engaged in part-time work. We test three compositional variables: the ratio of teenagers to total labor force, LCT; the ratio of women age 20 and over to total labor force, LCW; and the sum of these two ratios, LCTW.

Table 3 presents alternative estimates of the hours equation. The first equation, which resembles the one appearing in the published version of the Hickman-Coen model (1976:67-75), indicates significant negative effects of the real wage and unemployment on hours. Inclusion of the labor-force composition variables has little effect on the estimated unemployment elasticity, but it substantially reduces the real-wage elasticity. Of the three compositional variables, LCW yields the best results, and we therefore utilize the third equation in subsequent analyses.

THE HIGH-EMPLOYMENT UNEMPLOYMENT RATE AND LABOR SUPPLY

The estimated labor force and hours equations, together with several identities, constitute an interdependent system which for a given unemployment rate (and other right-hand variables exogenous to the system) jointly determines the labor

Table 3

Average Hours Equations, 1950-77[a]

Regression Coefficients and t-Statistics

Equation	Constant	W3/PCE	U1	LCT	LCW	LCTW	RHO	DW	SEE
	Variable								
1	.93234 (43.0)	-.18294 (9.36)	-.01744 (4.06)				.74125 (5.84)	1.4260	.0047
2	.81409 (6.20)	-.15242 (3.85)	-.01940 (4.11)	-.03443 (0.94)			.69233 (5.08)	1.3771	.0047
3	.44018 (4.18)	-.04848 (1.59)	-.01835 (5.50)		-.30525 (4.79)		.50808 (3.12)	1.6308	.0036
4	.55046 (5.68)	-.04574 (1.26)	-.02140 (5.91)			-.25961 (4.07)	.52615 (3.27)	1.6071	.0038

a. All variables are in logarithms.

force by age and sex, average hours, and aggregate employment. Setting the unemployment rate equal to an assumed high-employment value, manhours at high-employment could thus be estimated. Until recent years, a conventional assumption in studies of potential labor supply was that an aggregate unemployment rate of 4 percent characterized high employment; but as the composition of the labor force has shifted toward women and teenagers, groups with typically higher unemployment rates than prime-age males, the high-employment unemployment rate has generally been presumed to be rising. To allow for this possibility, we shall include the high-employment unemployment rate itself as an additional endogenous variable, in the manner described below.

Let us define U7 as the high-employment unemployment rate and use the suffix "7" to denote high-employment levels of the variables to which it is attached. For example, LC7, E7, and AH7 are high-employment levels of civilian labor force, aggregate employment, and private average hours. Average hours at high employment are then calculated as

$$AH7 = exp\,[c_{10} + c_{11}\,\ell n(W3/PCE) + c_{12}\,\ell n(U7) + c_{13}\,\ell n(LCW7)], \tag{1}$$

where the c's are estimated coefficients from the average hours regression. High-employment employment is defined by

$$E7 = (1 - (U7/100))(LC7)\ . \tag{2}$$

E7 is allocated between private employment, EP7, and public employment, EG7, according to the "normal" ratio of private to total employment, WHT1; that is,

$$EP7 = (WHT1)(E7) \tag{3}$$

$$EG7 = (1 - WHT1)(E7) \ . \tag{4}$$

The product

$$MH7 = (AH7)(EP7) \tag{5}$$

gives private manhours, our basic measure of labor supply at high employment.

Economy-wide average hours and the economy-wide real after-tax wage, which enter the labor force participation equations, are calculated as weighted averages of their private- and public-sector values. Assuming that average hours in the public sector are constant at 2,000 per year, we have:

$$AH17 = [(AH7)(EP7) + 2.0(EG7)] / E7 \tag{6}$$

$$W17/PCE = [(W3/PCE)(MH7) + (WG/PCE)(2.0)(EG7)] / (AH17)(E7) \ , \tag{7}$$

where WG is the after-tax money wage in the public sector. High-employment labor force for the i-th age-sex group (there are 16 groups in all) is then given by

$$LC7_i = NNI_i[c_{20,i} + c_{21,i}(E7/NNI) + c_{22,i}(LA/NNI) + c_{23,i}(W17/PCE) + c_{24,i}(AH17) + c_{25,i}(TIME5) + c_{26,i}(NNIMRAT)] \ , \tag{8}$$

and the identities

$$LC7 = \sum_{i=1}^{16} LC7_i \tag{9}$$

$$LCW7 = \sum_{i=11}^{16} LC7_i/LC7 \tag{10}$$

define the total labor force and the proportion of women age 20 and over in the labor force at high employment.

Finally, we express U7 as a weighted average of unemployment rates by age and sex in a high-employment base year, the weights being the proportions of the high-employment labor force in each age-sex group. Thus,

$$U7 = \sum_{i=1}^{16} (U7_i^B)(LC7_i) / LC7 . \tag{11}$$

As the composition of the labor force changes over time, U7 will, of course, vary. A unique feature of the system (1) - (11) is that it jointly determines the weights and U7 and assures that their values are consistent with the behavioral relations (1) and (8).

We have solved this system for the sample period 1951-77 and for the post-sample years 1978-2000. We choose 1956 as the base year for the unemployment rates by age and sex, since it is widely regarded as a time of high employment, the aggregate unemployment rate being 4.1 percent. The 1956 unemployment rates are as follows:

Age Group	Males	Females
16-17	11.7	13.2
18-19	10.4	9.9
20-24	6.9	6.3
25-34	3.3	4.8
35-44	2.6	3.9
45-54	3.0	3.6
55-64	3.5	3.6
65+	3.5	2.3

The population projections, which are crucial inputs for the post-sample calculations, are derived from the Census Series II estimates of total population by age and sex (U.S. Department of Commerce 1977). The Census estimate for each group is adjusted to a noninstitutional basis by applying the corresponding 1974 ratio of noninstitutional to total population. To aid the reader in interpreting our results, Table 4 presents growth rates of noninstitutional population by age and sex for selected intervals in the sample and post-sample periods.

Real wages and the ratio of armed forces to population are set equal to their actual values for the sample period; thereafter, real wages are assumed to grow at 2 percent per year, and the armed forces ratio is held constant at its 1977 level, 0.013. The trend value of EP/E is used for WHT1 through 1974; WHT1 is then held constant at its 1974 level, 0.846.

Our initial projections of the system produced unreasonably large declines in the participation rates of males age 55 and over, groups for which large negative trends in participation are evident over the sample period. Tendencies toward earlier retirement probably account for the strength of these trends, but in view of recent legislation raising the retirement age, among other things, there is good reason to believe that they will not persist. (If they do, there will be no males age 65 and over in the labor force by the year 2000!)
Hence, we decided to suppress the trend coefficients for these two groups beginning in 1978. With these adjustments, the projections indicate gradual increases in the participation rates

Table 4

Growth Rates of Noninstitutional Population
by Age and Sex, 1951-2000

Annual Exponential Rates of Changes

Period	16-17	18-19	20-24	25-34	35-44	45-54	55-64	65+
				Males				
Sample								
1951-1955	1.61	-0.18	-1.56	0.35	1.33	1.31	0.72	2.92
1956-1960	4.42	2.83	1.11	-0.93	0.99	1.78	1.15	1.57
1961-1965	4.30	4.96	3.53	-0.47	0.35	1.04	1.61	2.06
1966-1970	1.86	2.54	4.71	2.51	-1.14	0.97	1.44	1.12
1971-1977	1.29	2.17	2.21	3.84	0.29	0.12	1.47	1.83
Projections								
1978-1980	-1.00	0.04	1.45	3.01	2.94	-0.94	1.33	1.68
1981-1985	-3.00	-3.03	-0.39	1.94	4.04	-0.20	0.60	1.71
1986-1990	-1.97	-0.35	-2.66	0.60	3.11	2.39	-0.79	1.72
1991-1995	1.37	-2.02	-1.37	-1.49	1.86	4.00	-0.17	0.98
1996-2000	3.11	3.55	0.25	-2.04	0.55	3.08	2.44	0.18
				Females				
Sample								
1951-1955	1.71	-0.18	-1.70	0.22	1.62	1.56	1.54	4.03
1956-1960	4.46	2.88	0.96	-1.18	1.12	2.24	1.79	2.76
1961-1965	4.39	5.06	3.72	-0.44	0.32	1.13	1.67	2.97
1966-1970	1.91	2.47	4.54	2.25	-1.21	1.36	1.98	2.17
1971-1977	1.16	2.28	2.26	3.84	0.40	0.00	1.41	2.58
Projections								
1978-1980	-1.10	-0.02	1.46	2.89	2.92	-1.16	1.28	2.31
1981-1985	-3.00	-3.02	-0.40	1.94	3.91	-0.22	0.41	1.90
1986-1990	-2.06	-0.46	-2.67	0.62	3.04	2.40	-1.01	1.80
1991-1995	1.30	-2.09	-1.45	-1.47	1.88	3.87	-0.19	1.06
1996-2000	3.11	3.55	0.16	-2.05	0.60	3.01	2.43	0.32

of both groups between 1977 and 2000 - from 0.74 to 0.81 for the 55-64 group, and from 0.21 to 0.29 for those 65 and over.[4/]

The computed values of U7 are shown in Table 5. The series declines very slightly from 1951-55, then rises to a peak in 1977. The modest gyrations in U7 over the sample period are primarily due to fluctuations in the armed forces, as can be seen by comparing U71, based on trend values of LA/NNI, with U7. On average, U7 exceeded the actual unemployment rate by 0.30 of a percentage point during the first half of the 1950's and by 0.48 of a percentage point in the last half of the 1960's; otherwise the actual unemployment rate was well above U7, the largest one-year gap being 3.76 percentage points in 1975.

Since U7 is a weighted average of the 1956 unemployment rates, and since the labor force proportions that constitute the weights depend very strongly on the composition of the non-institutional population, it would seem that trends in U7 would largely be attributable to changes in the population distribution. To examine this supposition, we computed a third variant, U72, which like U71 is based on the trend values of LA/NNI, but which holds the composition of the noninstitutional population by age and sex fixed at that observed in 1956. It should be noted that only the composition of NNI is fixed; total NNI follows its actual path over the sample period. We see that there would have been some slight upward drift in the high-employment un-employment rate even in the absence of large changes in the

Table 5

Actual and High-Employment Unemployment Rates, 1951-2000
(In Percent)

Sample Period

Year	U1	U7[a]	U71[b]	U72[c]
1951	3.31	4.20	4.19	4.18
1952	3.03	4.15	4.18	4.17
1953	2.91	4.10	4.14	4.17
1954	5.55	4.12	4.14	4.17
1955	4.39	4.11	4.13	4.16
1956	4.13	4.14	4.12	4.16
1957	4.27	4.14	4.13	4.16
1958	6.80	4.16	4.15	4.17
1959	5.47	4.18	4.18	4.18
1960	5.53	4.21	4.20	4.18
1961	6.69	4.24	4.23	4.19
1962	5.54	4.22	4.23	4.19
1963	5.67	4.26	4.28	4.20
1964	5.18	4.29	4.31	4.19
1965	4.52	4.33	4.35	4.19
1966	3.79	4.37	4.39	4.20
1967	3.85	4.38	4.42	4.20
1968	3.58	4.39	4.45	4.21
1969	3.51	4.43	4.49	4.22
1970	4.94	4.48	4.52	4.22
1971	5.94	4.53	4.56	4.22
1972	5.59	4.58	4.59	4.23
1973	4.85	4.62	4.62	4.23
1974	5.58	4.65	4.65	4.25
1975	8.45	4.69	4.67	4.26
1976	7.69	4.69	4.68	4.26
1977	7.04	4.70	4.68	4.27
Average Values				
1951-55	3.84	4.14	4.16	4.17
1956-60	5.24	4.17	4.16	4.17
1961-65	5.52	4.27	4.28	4.19
1966-70	3.93	4.41	4.45	4.21
1971-75	6.14	4.61	4.62	4.24
1976-77	7.37	4.70	4.68	4.27
1951-60	4.54	4.15	4.16	4.17
1961-70	4.73	4.34	4.37	4.20
1971-77	6.49	4.64	4.64	4.25

Projection

Year	U7
1978	4.68
1979	4.67
1980	4.66
1981	4.64
1982	4.60
1983	4.56
1984	4.52
1985	4.48
1986	4.45
1987	4.43
1988	4.40
1989	4.36
1990	4.32
1991	4.28
1992	4.26
1993	4.23
1994	4.22
1995	4.20
1996	4.19
1997	4.19
1998	4.20
1999	4.21
2000	4.21
Average Values	
1978-80	4.67
1981-85	4.56
1986-90	4.39
1991-95	4.24
1996-00	4.20
1981-90	4.48
1991-00	4.22

a. Based on actual values of LA/NNI.
b. Based on trend values of LA/NNI.
c. Based on trend values of LA/NNI and constant (1956) distribution of population by age and sex.

distribution of NNI. The reason for this is that changes in wages and armed forces and the influences of trends would still have worked in the direction of increasing the labor force proportions of groups with relatively high base-year unemployment rates. Nonetheless, the leading role of population distribution as a determinent of U7 is evident. Had it not been for changes in this distribution, U7 would have risen by only about 0.1 instead of about 0.6 of a percentage point.

Turning to the projections, we see the striking effects of the increase in the average age of the population following recent declines in the birth rate. U7 declines steadily from its 1977 peak of 4.70 percent to a low of 4.19 percent in 1996-97.

Table 6 reports growth rates of labor supply associated with the high-employment unemployment rate, U7. The growth rate of high-employment private manhours accelerated throughout the sample period despite the upward trend in U7 and declines in AH7. Increasing growth of NNI, accompanied by a rising aggregate participation rate, especially from 1966-77, accounted for this phenomenon. But the growth rate of manhours is projected to decline well into the 1990's. Although (a) U7 will be falling, (b) decreases in AH7 are forecast to be more moderate than those recently experienced, and (c) the aggregate participation rate should continue to rise (from 0.62 in 1977 to 0.67 in 2000), these factors combined are not strong enough to offset the declining growth of NNI.

Table 6

Measures of Labor Supply Growth at
High Employment, 1951-2000

Annual Exponential Rates of Change

Period	Noninstitutional Population	Civilian Labor Force	Employment[a]	Annual Hours Per Worker[a]	Manhours[a]
		Sample Period			
1951-1955	1.11	0.83	0.56	-0.34	0.22
1956-1960	1.21	1.46	1.14	-0.55	0.59
1961-1965	1.52	1.24	0.91	-0.22	0.69
1966-1970	1.63	2.13	1.78	-0.77	1.01
1971-1977	1.76	2.49	2.32	-0.38	1.94
		Projections			
1978-1980	1.50	1.98	1.99	-0.24	1.75
1981-1985	1.07	1.54	1.58	-0.25	1.33
1986-1990	0.82	1.05	1.08	-0.19	0.89
1991-1995	0.64	0.87	0.90	-0.20	0.70
1996-2000	0.84	1.24	1.24	-0.18	1.06

a. Private sector.

THE UNEMPLOYMENT EQUATION

Our unemployment equation discriminates between the short-run or disequilibrium components primarily stressed in search theory and the long-term or equilibrium components of the mismatch hypothesis[5/]. Given the decision to enter or remain in the labor force, one must still decide whether to retain one's job if employed, or if unemployed, whether to accept new offers as they become available or to continue instead to search for a better opening. It is not implied that all unemployment represents a voluntary decision to wait for a better offer, however, since workers in distressed locations or who are lacking in skills or education may not receive offers of any kind for lengthy periods despite their search. Because the composition of the labor force has increasingly shifted towards higher proportions of teenagers and women since the mid-1950's, a growing mismatch between job requirements and workers' skills has tended to raise the equilibrium or "natural" unemployment rate.

In this context the natural rate is conceptually the aggregate unemployment rate which at a given time would be consistent with general equilibrium in the labor and commodity markets, after due allowance for market imperfections, and would yield an equilibrium structure of real wage rates (Friedman 1968). In our empirical formulation it is assumed that the natural rate has been influenced by three principal factors over the postwar period: the shifts in the age-sex composition of the labor force, the size of the armed forces relative to population, and the ratio of average unemployment benefits to the private wage rate. Fluctuations of the actual unemployment rate relative to the natural rate are explained by two disequilibrium search variables: the ratio of the actual to the expected wage rate and the ratio of actual to high-employment employment.

In order to state the behavioral hypotheses concerning job search and mismatch, it is convenient to decompose the unemployment rate into two multiplicative components. The first is the annual search ratio, defined as USR = UNS/LC, where UNS is the annual flow of new job searchers and LC is the civilian labor force. The second component is the average duration of search, UADS, defined as the average number of weeks per spell of unemployment divided by the 52 weeks of the year. Since the number of persons unemployed, UN1, equals the product of UNS and UADS, it follows that:

$$U1 \equiv UN1 \;/\; LC \equiv (UNS)(UADS) \;/\; LC \equiv (USR)(UADS) \;. \qquad (12)$$

Hence the unemployment rate will vary proportionately to changes in either USR or UADS.

The Average Duration of Search

A basic hypothesis of search theory is that the average duration of search, UADS, should be inversely related to deviations of actual from expected wage rates. A worker's optimal search strategy is to select a reservation wage and to visit a sequence of potential employers until he or she is offered a wage at or above the reservation level. The reservation wage will be selected from the expected probability distribution of wage offers, and this will determine the expected probability of obtaining an offer at or above the reservation level on any particular visit. The average number of visits before an acceptable offer is received is the reciprocal of this probability, and the average number of visits times the average length of time between visits (assumed constant) equals the average duration of search.

An upward shift of the actual relative to the expected wage distribution will increase the probability of an acceptable offer on any given visit and thereby reduce the average duration of search and unemployment rate. In our formulation the expected and actual wage distributions are represented by their respective means without explicit use of higher moments. We also include, however, the relative deviation of actual employment from its high-employment level as an argument in the equation for UADS. This is because the increased job vacancies associated with a cyclical upshift of labor demand would increase the probability of an acceptable offer on a particular visit even if the realized mean wage did not increase relative to that expected.

Another factor which may affect UADS is the availability of unemployment compensation, since this reduces the cost of unemployment to the worker (Feldstein 1975). An increase of average unemployment benefits relative to average weekly earnings, for example, may lengthen the average period of search. Since unemployment benefits are nontaxable, the appropriate comparison is with average after-tax earnings from employment.

The average duration of search may vary inversely with the relative size of the armed forces. New military recruits will consist primarily of young males, an age group in which unskilled workers are especially prevalent. A mobilization will therefore tend to reduce UADS by removing disproportionately from the civilian labor force a population group in which the probability of a job offer at even the minimum legal wage is small.

Conversely, an increase in the proportions of teenagers and women in the population will increase UADS, since various wage inflexibilities inhibit firms from creating more jobs for unskilled workers even as vacancies for skilled workers go unfilled.

The Search Ratio

The ratio of new job searchers to the civilian labor force, USR, should respond to the same determinants as does UADS, and usually with the same sign. As labor demand increases cyclically, firings and layoffs decrease. This reduced flow of involuntary separations is partly offset by induced increases in quits and the number of new entrants and reentrants to the labor force; but the overall relationship between employment fluctuations and the search ratio is likely to be negative. Similarly, an increase in the armed forces reduces the search ratio. New military recruits come primarily from age groups with high unemployment rates, and when an unemployed recruit leaves the civilian labor force the numerator of the search ratio is reduced relatively more than the denominator. For the same reason, if young potential entrants or reentrants to the civilian labor force enlist or are drafted, or stay at school to avoid the draft, the search ratio is decreased.

Relative increases in unemployment compensation may raise USR by inducing additional voluntary quits in order to search for better jobs while temporarily supported by unemployment benefits. A rising proportion of women and teenagers in the labor force will tend also to increase the search ratio owing to the increased mismatch between job openings and skills.

Unlike UADS, USR may be positively related to unexpected changes in wage rates. A greater than expected wage increase may tend to raise the search ratio by inducing an increase in quits as workers reevaluate their prospects of obtaining a significantly higher wage by finding a new job. Additional entries into the labor force by formerly discouraged workers in response to the improved wage prospects may also raise the search ratio. Nevertheless, we expect a net negative correlation between the unemployment rate and the wage deviation terms, owing to the strong negative influence of the latter on the average duration of search.

The Estimating Equation

The estimating equation is

$$\begin{aligned} \ell n\, U1 = {} & a_0 + a_1\ \ell n (W4/WE) + a_2\ \ell n\ (W4/WE)_{-1} \\ & + a_3\ \ell n\ (EP/EP7) + a_4\ \ell n\ (LA/NNI) \\ & + a_5\ \ell n\ (ABU) + \ell n\ (U7), \end{aligned} \tag{13}$$

where W4 is the observed wage rate (average hourly earnings net of employers' contributions to social security programs), WE is the expected level of W4, ABU is the ratio of average weekly unemployment benefits to average weekly earnings after taxes, and the other variables are as previously defined.

The first three terms of equation (13) relate to the disequilibrium elements of unemployment, whereas the remaining terms are the determinants of the natural rate. Movements in the natural rate owing to the changes in the age-sex composition of the labor force are incorporated in U7 by construction, and to this we add the armed forces and unemployment benefits ratios with elasticities to be estimated by the regression.

Wage Expectations

An expression for the expected wage, WE, must be specified to implement the wage surprise term in (13). Autoregressive equations in W4 and in W4/PCE were tested, but the best explanations of unemployment were obtained when wage expectations were modelled from the following equation estimated over 1950-77:

$$\ell n\ [W4/(W4)_{-1}] = \underset{(7.73)}{.0451} + \underset{(2.91)}{.4416}\ \ell n\ [(PCE)_{-1}\ /(PCE)_{-2}]$$

$$\bar{R}^2 = .9988 \qquad SEE = .0199 \qquad DW = 2.049 \qquad (14)$$

On the assumption that workers ignore other systematic information in forming wage expectations, the rational expectations prediction of the expected wage is given by

$$\ell n\ (WE) = .0451 + .4416\ \ell n\ [(PCE)_{-1}\ /\ (PCE)_{-2}] + \ell n\ (W4)_{-1} \qquad (15)$$

Thus nominal wages are expected to rise by 4.5 percent per year plus a variable amount related to the rate of consumer price inflation in the preceding year. Apart from its superiority as an explanatory variable in the unemployment equation, this formulation has the advantage of incorporating a feedback from previous consumer price inflation, including that due to exogenous foreign price disturbances, into current wage expectations.

Parameter Estimates

Equation (13) was estimated by ordinary least squares over the period 1951-1977 and is shown as the first regression in Table 7. The search-theory hypotheses concerning disequilibrium unemployment are rather strongly confirmed. The coefficients of the current and lagged wage surprise terms are significant and correctly signed, indicating that unexpectedly high wage offers substantially reduce the average duration of search and the unemployment rate. A rise of private employment relative to its high-employment level has an even larger effect on unemployment as it simultaneously reduces the search ratio and the average duration of search.

The evidence about the influence of military employment and unemployment benefits on the natural rate of unemployment is weaker. Both LA/NNI and ABU have the correct signs, but their t-ratios are below the 5 percent level of significance. An alternative formulation in which ABU was corrected for broadening coverage of unemployment insurance programs, by multiplying the benefits/earnings ratio by the ratio of covered employees to the civilian labor force, resulted in a wrong sign for the

Table 7

Unemployment Equations, 1951-77 [a]

Regression Coefficients and t-Statistics

	Variable									
Equation	Constant	W4/WE	$(W4/WE)_{-1}$	EP/EP7	LA/NNI	ABU	U7	DW	$\bar{R}^2$	SEE
1	.18944 (0.32)	-2.1372 (2.38)	-2.3586 (3.54)	-9.3684 (9.30)	-.11338 (1.49)	.69536 (1.26)	1.0000 [b]	1.9509	.9461	.0609
2	-1.1687 (1.63)	-2.8262 (3.30)	-2.2380 (3.27)	-9.2709 (8.74)	-.23137 (1.83)	-.23113 [c] (1.06)	1.0000 [b]	1.8069	.9450	.0615

a. All variables are in logarithms

b. Imposed coefficient.

c. Corrected benefits-earnings ratio as described in text.

modified ABU variable, as may be seen from the second equation in Table 7. We therefore accept the first version of the equation and regard the correct signs on LA/NNI and ABU as weak confirmation of the expected contributions of these variables to variations in the natural unemployment rate over the postwar years.

The reader will have noted that our unemployment equation resembles the "Lucas labor supply fuction" as utilized in Lucas (1973) and Sargent (1976). It differs from these earlier formulations in significant ways, however. First, the fundamental labor supply function in our model is the disaggregated model of labor force participation, which determines the size of the labor force as a function of the real wage rate and population. Second, and most important, the Lucas and Sargent formulations embody the hypothesis that variations in unemployment not attributable to the unexpected component of wage changes are due to variations in the natural rate of unemployment. In contrast, the ratio of actual employment to its high-employment level is an important explanatory variable in our equation, so that cyclical fluctuations in the demand for labor have direct effects on unemployment in addition to their indirect effects operating through unexpected wage changes. The indirect wage effects are channeled through the average duration of search, whereas the direct employment effects impinge on the search ratio as well. Because of these direct employment effects, the unemployment rate would fluctuate cyclically in our model even if all wage changes were fully anticipated.

THE NATURAL RATE OF UNEMPLOYMENT

A time series estimate of the natural unemployment rate, denoted as U17, may be obtained by purging the unemployment equation (13) of its disequilibrium components. Assuming equality between the actual and expected wage and high employment of the private labor force, the equation to determine the natural rate is:

$$U17 = exp\,[\ell n\,U7 - 0.1134\ \ell n\,(LA/NNI) + 0.6954\,\ell n\,ABU + 0.1894] \quad (16)$$

Calculated values of U17, based on historical values of the armed forces and benefits/earnings ratios, are compared with the actual unemployment rate during 1951-77 in Table 8.

The natural rate is seen to rise from an average of 4.07 percent during 1951-60 to 4.35 in 1961-70 and 4.88 in 1971-77. The 1976-77 average is 4.96. The actual unemployment rate was 0.46 of a percentage point higher than the natural rate in 1951-60, 0.36 higher in 1961-70, 1.58 higher in 1971-77, and 2.38 higher in 1976-77. The actual rate fell below the natural rate during the Korean and Vietnam wars but was otherwise generally above it.

Successive mobilizations and demobilizations during and after these wars, as well as some slight fluctuations in ABU, induced corresponding swings in the natural rate. This may be seen by comparing U171, based on trend values of the armed forces and benefits/earnings ratios, with U17. Had these ratios not fluctuated about their trends, the natural rate would have remained virtually constant between 1951 and 1956 and risen steadily thereafter.

Table 8

Actual and Natural Unemployment Rates, 1951-2000
(In Percent)

Sample Period

Year	U1	U17[a/]	U171[b/]	U172[c/]
1951	3.31	3.94	4.03	4.01
1952	3.03	3.94	4.03	4.02
1953	2.91	3.85	4.00	4.03
1954	5.55	3.96	4.02	4.04
1955	4.39	3.95	4.03	4.06
1956	4.13	4.07	4.03	4.07
1957	4.27	4.08	4.05	4.09
1958	6.80	4.32	4.09	4.11
1959	5.47	4.20	4.13	4.13
1960	5.53	4.43	4.17	4.15
1961	6.69	4.45	4.21	4.17
1962	5.54	4.32	4.23	4.19
1963	5.67	4.39	4.29	4.21
1964	5.18	4.28	4.34	4.22
1965	4.52	4.25	4.39	4.24
1966	3.79	4.29	4.45	4.26
1967	3.85	4.29	4.50	4.28
1968	3.58	4.29	4.55	4.30
1969	3.51	4.38	4.61	4.32
1970	4.94	4.56	4.66	4.34
1971	5.94	4.64	4.71	4.36
1972	5.59	4.81	4.75	4.38
1973	4.85	4.81	4.80	4.40
1974	5.85	4.93	4.85	4.44
1975	8.45	5.03	4.90	4.47
1976	7.69	4.99	4.92	4.48
1977	7.04	4.92	4.94	4.51

Projection

Year	U17
1978	4.91
1979	4.90
1980	4.89
1981	4.87
1982	4.82
1983	4.78
1984	4.74
1985	4.70
1986	4.67
1987	4.64
1988	4.61
1989	4.57
1990	4.53
1991	4.49
1992	4.46
1993	4.44
1994	4.42
1995	4.40
1996	4.39
1997	4.39
1998	4.40
1999	4.41
2000	4.42

Average Values

Year	U1	U17[a/]	U171[b/]	U172[c/]
1951-55	3.84	3.93	4.02	4.03
1956-60	5.24	4.22	4.09	4.11
1961-65	5.52	4.34	4.29	4.21
1966-70	3.93	4.36	4.55	4.30
1971-75	6.14	4.84	4.80	4.41
1976-77	7.37	4.96	4.93	4.50
1951-60	4.54	4.07	4.06	4.07
1961-70	4.73	4.35	4.42	4.25
1971-77	6.49	4.88	4.84	4.43

Year	U17
1978-80	4.90
1981-85	4.78
1986-90	4.60
1991-95	4.44
1996-00	4.40
1981-90	4.69
1991-00	4.42

a. Based on U7 and actual values of LA/NNI and ABU.
b. Based on U71 and trend values of LA/NNI and ABU.
c. Based on U72 and trend values of LA/NNI and ABU.

The third variant, U172, appearing in Table 8 is again based on trend values of LA/NNI and ABU, but it illustrates how the natural rate would have behaved had the 1956 population distribution prevailed throughout the period. U171 and U172 are roughly the same through 1959. With a constant population distribution, the natural rate would have risen by 0.38 of a percentage point by 1977, whereas with changes in this distribution, the rate rises by 0.81. Thus, somewhat less than half the increase in the natural rate may be attributed to changes in LA/NNI and ABU, the remainder being due to shifts in the age-sex structure of the population.

The projections of U17 shown in the last column are based on the projected path of U7 (Table 5) and on the assumption that LA/NNI and ABU will remain at their 1977 levels. With no change in these ratios, U17 remains above U7 by a roughly constant 0.2 of a percentage point. However, were the armed forces to rise from 1.3 to about 2.0 percent of the population between 1977 and 2000, or were unemployment benefits to decline from 40.4 to 37.7 percent of private earnings, the high-employment and natural rates would be identical in 2000.

The equation system (1) - (11) can be used to determine labor supply growth conditional on the natural rate of unemployment. All that is required algebraically is to replace U7 by U17 and delete equation (11). We do not display the results of such calculations because they are very similar to those given in Table 6. Over the sample intervals, manhours associated with the natural rate grow more rapidly in 1951-55 (0.35 rather than 0.22 percent year) and in 1961-65 (0.79 compared to 0.69).

otherwise, they lag behind the growth of high-employment manhours (0.46 compared to 0.59 in 1956-60, 0.96 compared to 1.01 in 1966-70, and 1.90 compared to 1.94 in 1971-77). The projected rates of growth of manhours to 2000 are virtually identical for U7 and U17.

CONCLUSION

The principal novelty of the model developed in this paper is that it jointly determines labor supply, the high-employment unemployment rate, and the natural rate of unemployment. The model highlights in several ways the usefulness and importance of age-sex disaggregation in studying aggregate labor supply and unemployment. Differences in labor force participation behavior among age-sex groups account in part for the changing relationship between population growth and labor force growth, and the age structure of the male population appears to play a central role in shaping patterns of female participation. Much of the secular decline in average hours is explained by the growing proportion of women in the labor force. Finally, age-sex disaggregation makes possible endogenous determination of a high-employment unemployment rate whose variations over time are attributable to structural changes in the labor force and consequent mismatches between job requirements and workers' skills. The high-employment unemployment rate is, in turn, a principal determinant of movements in the natural and observed unemployment rates.

Other features of the model's key empirical relations, describing the direct determinants of actual labor force, hours, and unemployment, are of considerable interest. We find that labor force and hours respond inversely to changes in real wages but with rather small elasticities. Nearly all the sensitivity of labor force to real wages results from changes in participation of younger females. Since average hours are negatively related to the relative number of women in the labor force, a rise in real wages has an indirect, positive effect on hours.

With regard to the unemployment equation, unanticipated wage increases tend to reduce the actual unemployment rate, in accord with the predictions of search theory; but cyclical fluctuations in the demand for labor, as measured by the ratio of actual to high-employment employment, also have significant negative impacts on the actual unemployment rate. Thus, in our system, deviations of the actual from the natural unemployment rate are not solely due to unanticipated changes in wages.

Although our labor force equations do not themselves incorporate effects of wage surprises on participation, such effects are nonetheless present in the complete system. Since an unanticipated wage increase reduces the number of unemployed for a given labor force, and since aggregate labor force participation rises as employment increases, a positive wage surprise must have a positive indirect effect on labor force.

In using the system to generate joint estimates of labor supply, the high-employment unemployment rate, and the natural unemployment rate, we suppressed the disequilibrium elements in the unemployment equation, imposing equality between expected and actual wages and between actual and high-employment employment. We were still faced, however, with choosing a time path for real wages. Our decision to use the actual path of real wages over the sample period and assume two percent growth thereafter is admittedly arbitrary. The arbitrariness can be removed and the real wage endogenized by linking our labor supply system with a model of labor demand, then solving the expanded system subject to labor market clearance at the natural rate of unemployment and other restrictions; but this is beyond the scope of the present paper.

NOTES

1. The authors are Professors of Economics at Northwestern University and Stanford University respectively. This research was supported by the National Science Foundation under Grant DAR77-27746. Edward C. Kokkelenberg and Klaus Neusser provided valuable research assistance with the data and computations.

2. W1 is the after-tax nominal wage and PCE is the implicit GNP deflator for personal consumption expenditure. W1 is calculated as labor income from private and government employment(excluding income originating in the military and employer contributions to social security) multiplied by one minus the tax rate on labor income and divided by total hours worked by private and government employees. The tax rate is taken to be the effective rate (personal income tax plus social security tax) that would be paid by a single individual working full-time at the current nominal wage and taking the minimum standard deduction. Data on private hours per worker are from the Bureau of Labor Statistics; average hours of government employees are assumed constant at 2,000 per year.

3. See, for example, Bowen and Finegan (1969), Dernburg and Strand (1966), and Tella (1965). In a more recent study, Wachter (1972) presents evidence that the short-run elasticity of participation rates of secondary workers (females and younger and older males) can better be explained by "permanent" wages and actual relative to permanent wages, rather than by employment fluctuations. While our approach is in the more traditional vein, it bears some similarities to Wachter's. First, we test

for direct effects of real wages (current rather than permanent) on participation. Second, as we shall see in the section on the unemployment equation, the employment/labor force ratio is a function of unanticipated wage changes; thus, the latter variable has indirect effects on participation. Finally, Wachter argues that a decline in the actual relative to the permanent wage may induce higher participation rates for women, because it leads to disappointingly low earnings for their husbands. We attempt to capture this same phenomenon, but through a variable measuring the shifting age structure of the male population, as explained below.

4. These remarks may lead the reader to wonder what happens to the participation rates of other groups in the projections. Those of males age 16-19 rise gradually, while those of younger females decline. The most radical changes occur for females 45 and over, whose participation rates go up sharply. By 2000, participation rates of males and females age 45-64 are about the same. A major factor accounting for the relatively large movements in female participation is the behavior of NNIMRAT, which after peaking at 1.10 in 1981, declines to 0.69 in 2000.

5. See the papers in Phelps et al. (1970) for expositions of search theory and Darby (1976) for a synthesis. A good summary of the search and mismatch theories is found in Gordon (1978).

REFERENCES

Bowen, W.G. and T.A. Finegan (1969) The Economics of Labor Force Participation. Princeton, N.J.: Princeton University Press.

Coen, R.M. (1973) Labor force and unemployment in the 1920's and 1930's : a reexamination based on postwar experience. Review of Economics and Statistics 55(1):46-55.

Darby, M.R. (1976) Macroeconomics : The Theory of Income, Employment,and the Price Level. New York:McGraw-Hill.

Dernburg, R. and K. Strand (1966) Hidden unemployment 1953-62: a quantitative analysis by age and sex. American Economic Review 56 (March):71-95.

Easterlin, R.A. (1978) What will 1984 be like? Socioeconomic implications of recent twists in age structre. Demography 15(4):397-432.

Feldstein, M. (1975) The importance of temporary layoffs: an empirical analysis. Brookings Papers on Economic Activity 6(3).

Friedman, M. (1968) The role of monetary policy. American Economic Review 58 (March):1-17.

Gordon, R.J. (1978) Macroeconomics. Boston:Little-Brown.

Hickman, B.G. and R.M. Coen (1976) An Annual Growth Model of the U.S. Economy. Amersterdam:North-Holland.

Lucas, R.E.jr. (1973) Some international evidence on output-inflation tradeoffs. American Economic Review 63 (June):326-34.

Perry, G.L. (1977) Potential output and productivity. Brookings Papers on Economic Activity 1:11-47.

Phelps, E.S. et.al. (1970). The Microeconomic Foundations of Employment and Inflation Theory. New York:Norton.

Sargent, T.J. (1976) A classical macroeconomic model for for United States. Journal of Political Economy, 89(2):207-237.

Tella, A. (1965) Labor force sensitivity to employment by age and sex. Industrial Relations 4 (Feb.):69-83.

U.S. Department of Commerce, Bureau of the Census (1977) Projections of the Population of the United States: 1977 to 2050. Current Population Reports, Series P-25, No.704 Washington:U.S.Government Printing Office.

Wachter, M.L. (1972) A labor supply model for secondary workers. The Review of Economics and Statistics LIV (May):141-151.