WORKING PAPER

AN OUTLINE OF I.I.A.S.A.'s FOOD AND AGRICULTURE MODEL

M. A. Keyzer

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IIASA
International Institute
for Applied Systems Analysis

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PREFACE

This paper describes the general structure of IIASA's food and agriculture model. Propositions on the existence of equilibrium are formulated and discussed, but no proofs are given. These will be published in a forthcoming report where the algorithms developed to numerically solve the model are also described.

There are three chapters:

Chapter I	serves as a general introduction to the modeling system.
In Chapter II	the model is described in a formal way.
In Chapter III	the assumptions introduced in the model specification are discussed and their realism is assessed.

Chapter I
INTRODUCTION

1. *IIASA's FOOD AND AGRICULTURE PROGRAMME*

A central task of IIASA's Food and Agriculture programme is to study the impact of national policies of both developed and developing nations on hunger and malnutrition in the world and to evaluate the consequences of new international agreements in the field of food and agriculture.

The research strategy is to develop a simulation model containing about 20 national models which interact through trade and capital flows. The model operates with a one-year time increment and has a time horizon of 15-20 years.

Country experts independently develop national models which should be linkable into one global model.

The models should therefore satisfy basic linkage requirements.

- International trade variables should follow a common commodity classification (i.c. 18 agricultural and 1 residual, non-agricultural commodity).
- Imports and exports of commodities should be generated on a yearly basis.
- Imports and exports should be functions of world market prices, which are insensitive to the absolute level of prices.

The development of a theoretical and computational modelling framework along these lines, is the subject of the present report.

2. *A SYSTEM OF INTERLINKED, OPEN EXCHANGE MODELS*

There are n commodities, indexed $i = 1, \ldots, n$ and l countries, indexed $h = 1, \ldots, l$. We consider a national model as a net import function depending on world market prices. Although a formal treatment must be postponed until Chapter II, we list the main characteristics of this net import function. Let z_i^h be the net import of commodity i by nation h and p_i^w be the world market price of commodity i; we write the net import function as:

$$z_i^h = z_i^h \, (p_1^w, \, \ldots, \, p_n^w), \qquad i=1, \, \ldots, \, n; \qquad h=1, \, \ldots, \, 1 \tag{2.1}$$

Three basic requirements are imposed on it.

(i) Net imports should be insensitive to the absolute level of prices (the functions should be homogeneous of degree zero in prices).

(ii) The net import function should be continuous at all positive prices.

(iii) The function should, at positive prices, satisfy a balance of trade condition: Let $k_h(p_1^w, \ldots, p_n^w)$ describe the nation's deficit on its balance of trade (this function should be homogeneous of degree one).

The balance of trade requirement can then be written as:

$$\sum_{i=1}^{n} p_i^w z_i^h = k_h(p_1, \ldots, p_n^w) \ , \quad \text{for all } (p_1^w, \ldots, p_n^w) > 0 \tag{2.2}$$

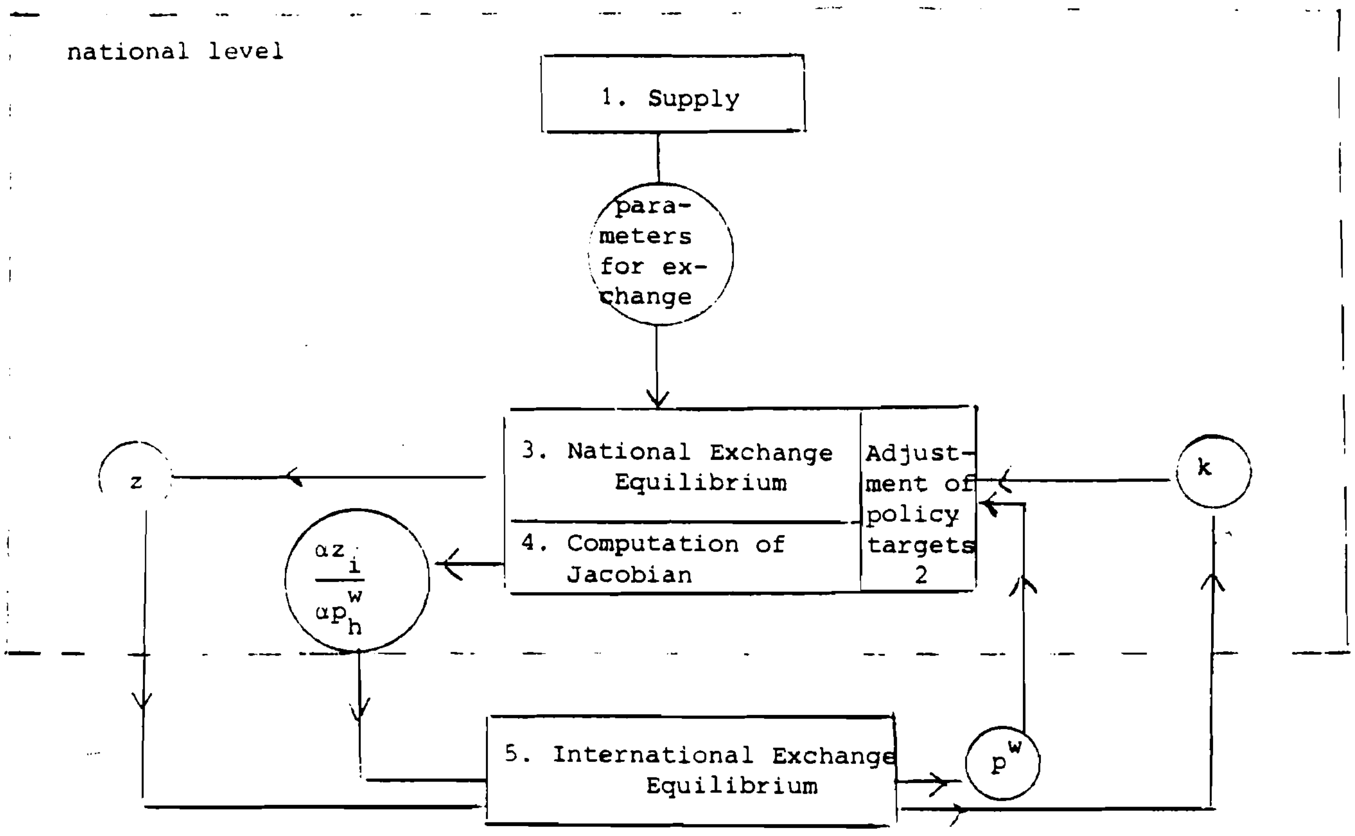
At the international level demand should not exceed supply:

$$\sum_{h=1}^{1} z_i^h \leqq 0 \qquad h = 1, \ldots, 1 \tag{2.3}$$

Depending on the assumed market conditions, the function $k_h(p_1^w, p_n^w)$ can be specified. We define as a competitive international equilibrium the solution of (2.1)-(2.3) when $\sum_{h=1}^{1} k_h(p_1^w, \ldots, p_n^w) = 0$ for all $(p_1^w, \ldots, p_n^w) \geqq 0$ i.e. when (2.3) is the only balance condition imposed at international level.

In Chapter II, para 2 a national model with domestic price policies, quota's on international trade and national buffer stocks is presented. We call this an *open exchange model* ("open" because it has international trade and "exchange" because a one period lag in supply is assumed). As long as this model possesses a *unique* solution its net imports are functions of world market prices, which satisfy requirements (i)-(iii). *Competitive international equilibrium* is one of the modes for interlinking a system of open exchange models (Chapter II, para 3). An *international buffer stock agreement* (Chapter II para 4) and a *market segmentation agreement* (Chapter II para 5) offer alternative modes. The economic realism and the institutional background for the assumptions made in Chapter II are evaluated in Chapter III.

The following diagram shows the general structure of the system's operation.



The dotted lines indicate that the international model calls for the excecution of the national models, one after the other.

3. *MODELLING ECONOMIC EQUILIBRIUM AS A COMPLEMENTARITY PROBLEM*

We shall model the economic process by first describing the behaviour of individual agents and then integrating this behaviour through the imposition of overall physical and financial balance conditions, in the Walrasian equilibrium tradition. From the mathematical point of view, we shall restrict ourselves to a class of models called complementarity problems (see Cottle (6)).In order to illustrate the applicability of this mathematical tool, we first interpret the competitive equilibrium model as a complementarity problem. Let $z_i^w = \sum_{h=1}^{l} z_i^h$, i=1, ...,n, define world net imports from commodity i; the competitive international model can be written as:

$$z_i^w = z_i^w(p_1^w, \ldots, p_n^w) \tag{3.1}$$

$$\sum_{i=1}^{n} p_i^w z_i^w = 0 \tag{3.2}$$

$$z_i^w \leqq 0 \tag{3.3}$$

$$p_i^w \geqq 0 \tag{3.4}$$

With the additional property that prices can be normalized according to $\sum_{i=1}^{n} p_i^w = 1$, that the solution p^w, should be strictly positive, and that (3.2) is satisfied at all positive prices. Equations (3.1)-(3.4) describe a complementarity problem which we can state more generally as: Find $(\psi_1, \ldots, \psi_r)$ so as to satisfy, for $k = 1, \ldots, r$:

$$\left.\begin{aligned} q_k &= q_k(\psi_1, \ldots, \psi_r) \\ \psi_k &\geqq 0 \\ q_k &\geqq 0 \\ \sum_{k=1}^{r} \psi_k q_k &= 0 \end{aligned}\right] \tag{3.5}$$

Obviously in the competitive model we have: $r = n$, $q = -z^w$, $\psi = p^w$. Examples of complementarity problems can be found in Cottle (6). Linear complementarity problems have received wide attention (see Cottle and Dantzig (7)). Bimatrix games, the optimality conditions of linear and quadratic programmes are linear complementarity problems. Kuhn Tucker optimality conditions and the equilibrium problems we shall study correspond to complementarity problems which can be nonlinear.

Typically in an economic context, ψ_k indicates some price, while q_k represents a quantity corresponding to that price and the complementarity equation ($\Sigma\ \psi_k\ q_k = 0$) is a representation of the requirement that revenue should equal expenditure (i.e. the strong version of Walras' Law).
The economic equilibrium problems which we shall study have three symplifying characteristics:

(a) $q_k = q_k(\psi_1, \ldots\psi_r)$ is homogeneous of degree zero in $(\psi_1, \ldots\psi_r)$.

(b) $(\psi_1, \ldots\psi_r) \in \Psi$, where Ψ is a nonempty, closed, bounded, convex set in the nonnegative orthant.

(c) $\Sigma_i\psi_i q_i = 0$ for all $(\psi_1, \ldots\psi_r) \in \Psi$.

The formulation as a complementarity problem is of special interest because such a problem possesses the property that, in its solution for k = 1, ..., r:

if $\psi_k > 0$ then $q_k = 0$

if $q_k > 0$ then $\psi_k = 0$

This property may seem trival from the mathematical point of view, it has proven, however, to be very useful for representing policies with fixed targets within an economic model. We consider a plan with a fixed price target and a constrained quantity for each commodity.

Let $\bar{p}_i$ be a price target for commodity i. The planner wishes to see this target realized as long as an associated quantity constraint is unbinding: $q_i > 0$. Otherwise the planner is willing to let the price rise above target. Let p_i be the price realization, then:

$$\left.\begin{aligned} q_i &= q_i(p_1, \ldots, p_n) \\ p_i &= \bar{p}_i + \psi_i \\ q_i &\geqq 0 \\ \psi_i &\geqq 0 \\ \psi_i\, q_i &= 0 \end{aligned}\right] \qquad (3.6)$$

If a solution to this problem exists, it will satisfy the planner's wishes but the fact that the target cannot be realized, does not by itself imply that the model has no solution. On the other hand, establishing existence of a solution clearly is a critical test for the consistency of both the model and the plan (cf. Chapter III, para 13).

The planner may wish to associate more than one constraint to one target, implying that as long as one constraint is not binding, the target should be realized (cf. Chapter II, para 2); one then writes

$$\left.\begin{aligned} &\psi_i\, q_{1i}\,(p) = 0 \\ \text{and} \\ &\psi_i\, q_{2i}\,(p) = 0 \end{aligned}\right] \qquad (3.7)$$

For an intuitive illustration of a case with one constraint associated to each target, we consider a price target on the world market which is strived at through the operation of an international buffer stock. The formal model is presented in Chapter II, para 4, but the role of complementarity conditions can already be seen here. As long as, for a commodity, the buffer stock is not depleted, its price should not rise above target level, because the international agency running the operation is assumed to announce that it will sell at target price as long as its stock is not depleted.

Let u_i be the final availability of stock and ψ_i the upward deviation of world market price from price target for commodity i.
We then have, for i = 1, ..., n, the requirement that:

$$\text{if} \quad u_i > 0 \Rightarrow \psi_i = 0$$

and

$$\text{if} \quad \psi_i > 0 \Rightarrow u_i = 0$$

This can also be written as:

$$\psi_i \geqq 0 \,, \quad u_i \geqq 0 \,, \quad \psi_i u_i = 0 \qquad (3.8)$$

where

$$u_i = u_i(p_1^w, \ldots, p_n^w) \qquad (3.9)$$

and

$$p_i^w = \bar{p}_i^w + \psi_i \qquad (3.10)$$

As long as the structure of (3.9) is not explicitly described, we do not know how the buffer stock operation is financed and cannot establish existence of a solution, but the example illustrates how a market regulating arrangement can be represented within the framework of a complementarity problem.

Chapter II

GENERAL FORMULATION OF THE MODEL

1. *PLAN OF THE CHAPTER*

The model is described in a general, formal way in para 1-6. An informal discussion of the empirical relevance and economic background of the main assumptions is postponed until Chapter III. Proofs of the propositions are not given here and will appear in a forthcoming reprint. As a means of introducing the main hypotheses we first present a model of a closed economy with lagged supply (para 2). We then "open" the economy by allowing international trade and by introducing a government which raises income tax and regulates the domestic market through price policies, quotas on international trade and through the operation of a buffer stock (para 3). We call this the open exchange model. If the solution to this model is unique, it will, at positive international prices, describe net imports as a continuous function with the property that the value of net imports at world market prices equals a given trade deficit and that this function is homogeneous of degree zero in international prices and trade deficit. This makes it possible to regard the open exchange model as one actor, operating on the international market. Three versions of an international model are subsequently developed.

- First a closed international economy, with an exogenously specified distribution function for balance of trade deficits without market regulating agreements.
- Second, an international buffer stock agreement in which nations finance an international agency, which tries to keep world market prices within a given price band, by operating a buffer stock (para 4).
- Third, an agreement on market segmentation is represented, in which one group of nations decides that it will try to keep world market prices at a fixed target level, by adjusting its net import.

Each model is presented in four components: central market regulation, demand supply, finance and price formation.

2. *A CLOSED ECONOMY WITH LAGGED SUPPLY*

2.1 *Central market regulation*

None

2.2 *Demand and supply*

There are n commodities (goods), indexed i=1,...,n. The set of normalized prices $\{p_t \in R^n_+ | \; ||p_t||_1 = 1\}$ will be denoted P. There are m actors, indexed j=1,...,m.

For each actor j, demand is specified as a function x^j of prices and revenue (or income i.e. the amount of units of account available to the actor); a specified demand of actor j at the beginning of period t will be denoted x^j_t:

$$x^j : R^n_+ \times R^1_+ \rightarrow R^n_+$$

$$x^j_t : = x^j(p, m_{jt}) \qquad (2.1)$$

For each actor j, supply is specified as a function of prices; a specified supply at the end of period t will be denoted y^j_t:

$$y^j \; : R^n_+ \rightarrow R^n_+$$

$$y^j_t := y^j(p_t) \qquad (2.2)$$

Define a vector of weights γ, $\gamma \in R^n_{++}$: $\gamma := \iota$ and a satiation level ω_t, $\omega_t \in R^1_{++}$: $\omega := \gamma . \Sigma_j y^j_{t-1}$

Five basic hypotheses are imposed on demand:

(i) *Homogeneity:* demand is homogeneous of degree zero in prices and income:

$$\forall \; p_t \in P, \; m_{jt} \in R^1_{++} : \forall \; \lambda \in R^1_{++} : x^j(p_t, m_{jt}) = x^j(\lambda p_t, \; \lambda m_{jt})$$

(ii) *Adding up:* value of demand should not exceed the revenue and be equal to the revenue whenever weighted total demand does not exceed a specified satiation level.

$$\forall \; p_t \in P; \; m_{jt} \in R^1_{++} : p_t \; . \; x^j_t \leqq m_{jt}$$

and

$$\forall \; p_t \in P, \; \{m_{jt}\}^m_{j=1}, \; m_{jt} \in R^1_{++} | \; \gamma \; . \; \Sigma_j x^j_t \leqq \omega_t : \forall_j \; : p_t \; . \; x^j_t = m_{jt}.$$

(iii) *Monotonicity:* for each good demand does not decrease as revenue increases:

$$\forall\ p_t \in P,\ m_{jt} \in R^1_{++} : \forall\ \bar{m}_{jt} > m_{jt} : x^j(p_t, \bar{m}_{jt}) \geqq x^j(p_t, m_{jt})$$

(iv) *Nonsatiation:* when for any good price drops to zero, weighted total demand exceeds the satiation level:

$$\forall\ p_t \in P, \{m_{jt}\}_{j=1}^{m}, \cdot m_{jt} \in R^1_{++} : \forall_i : \exists(\hat{p}\,|\hat{p}_k = p_{kt}, k \neq i; \hat{p}_i \leqq p_{it}) : \gamma . \Sigma_j x^j(\hat{p}, m_{jt}) > \omega$$

(v) *Continuity:*

$$\forall\ p_i \in P,\ m_{jt} \in R^1_{++} : x^j(p, m_{jt}) \text{ is continuous.}$$

Three basic hypotheses are imposed on supply:

(vi) *Lag:* supply is brought to market with a one-period lag.

(vII) *Homogeneity:*

$$\forall\ p_t \in P : \forall\ \lambda \in R^1_{++} : y^j(p_t) = y^j(\lambda p_t)$$

(viii) *Boundedness:*

$$\forall\ p_t \in P : \exists\ \alpha \in R^n_{++} : \alpha > y^j(p_t)$$

Free disposal is explicitly considered as a commodity flow i.c. as a demand category.

$$s_t := \bar{\sigma}\ (\Sigma_j(y^j_{t-1} - x^j_t),\ 0), \tag{2.3}$$

where $\bar{\sigma}$ is defined as

$$\bar{\sigma}(a,b) := (c \in R^n |\ \forall i : c_i = \max(a_i, b_i)).$$

A commodity balance is imposed:

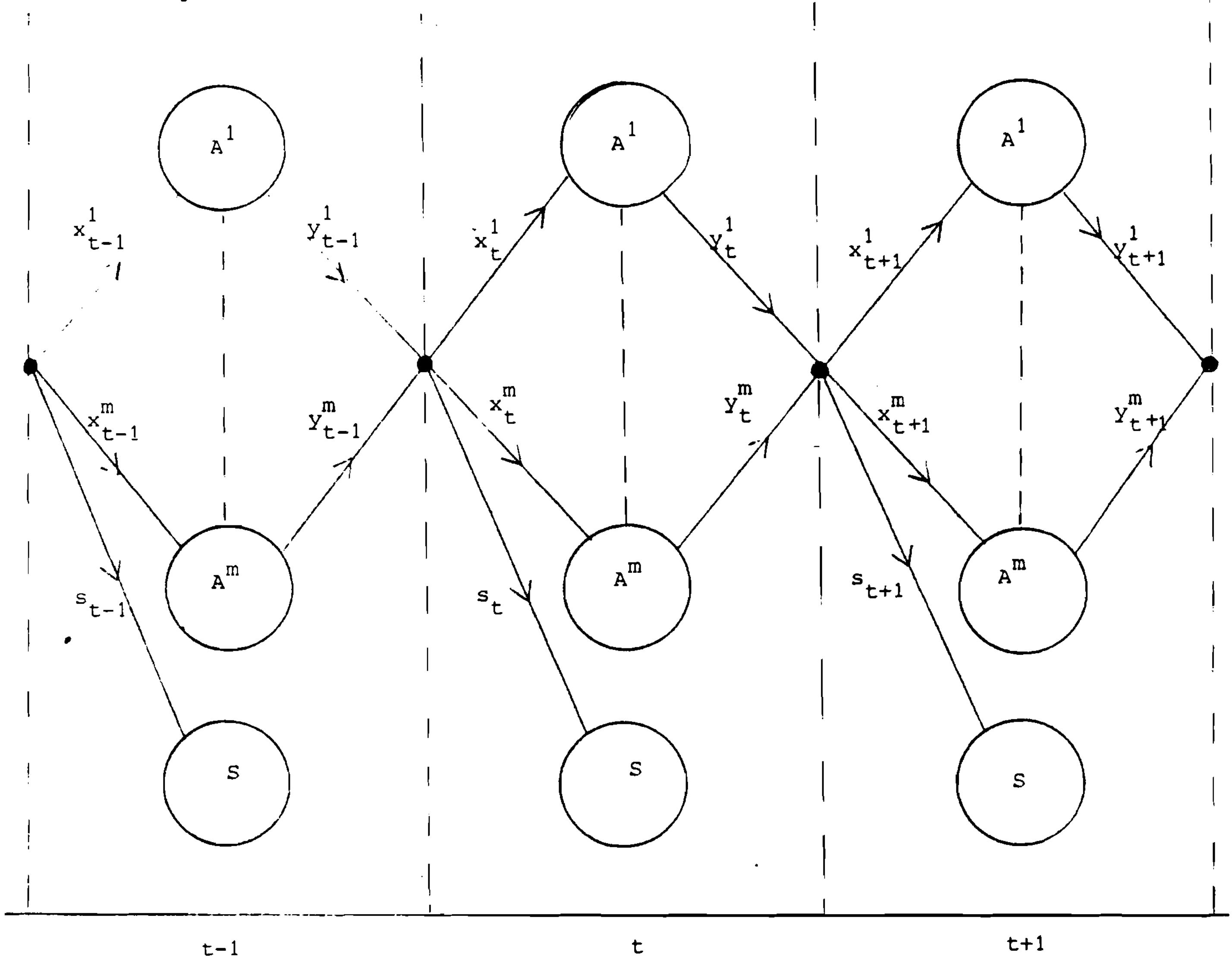
$$\Sigma_j x^j_t + s_t = \Sigma_j y^j_{t-1} \tag{2.4}$$

2.3 *Finance*

Each actor's revenue consists of the receipts from marketed supplies:

$$m_{jt} := p_t \cdot y^j_{t-1} \tag{2.5}$$

Diagram 1: *A closed economy with lagged production*



Legend:

○ : actor

→ : positive direction of a flow of goods

● : market

t : time period

$A^1, .., A^m$: income group 1, ..., m

S : free disposal activity

2.4 *Price formation*

A complementarity condition is imposed which restricts price adjustment by requiring that price of a good should be zero if its free disposal is positive:

$$p_t \cdot s_t = 0 \tag{2.6}$$

This equation can also be regarded as a financial balance equation which requires that free disposal should finance itself.
Diagram 1 describes the commodity flows in this model.

2.5 *Equilibrium in the closed economy with lagged supply*

We remark that the model can be solved sequentially for every period, at given endowments $\{y^j_{t-1}\}^m_{j=1}$. We therefore can establish equilibrium independently of the time period and formulate a proposition in which time subscripts have been dropped and x^j_t is replaced by x^j, y^j_{t-1} by y^j_{-1} etc.

Proposition 1

For all given values of $\{y^j_{-1}\}^m_{j=1}$, $y^h_{-1} \in R^n_{++}$, with demand (2.1) satisfying hypotheses 2.2i-v, free disposal (2.3) and revenue determination (2.5), the model of the closed economy possesses a solution

$$(p^*, \{x^{j*}\}^m_{j=1}, s^*),$$

satisfying
- the commodity balance (2.4)
- the price restriction (2.5)

and where $p^* > 0$.

3. *AN OPEN EXCHANGE ECONOMY WITH DOMESTIC PRICE POLICY, QUOTAS AND BUFFER STOCK*

3.1 *Central Market Regulation*

We shall now introduce an open economy model, describing a trading nation, in which a government sees it as its central goal to achieve a price target. For this it has two instruments at its disposal:

Trade instrument: net import z_t, $z_t \in R^n$, is adjusted within fixed upper and lower bound

$$\underline{z}_t \leqq z_t \leqq \bar{z}_t \tag{3.1}$$

Stock instrument: Stocks , w_{t-1}, $w_t \in R^n_+$, are bought on the market at the beginning of period t and sold at the end of the period. Stocks are adjusted within fixed upper and lower bounds

$$\underline{w}_t \leqq w_t \leqq \bar{w}_t \tag{3.2}$$

3.2 *Demand and supply*

Demand is described according to equation (2.1) under hypotheses (2.2.i-v) Supply is described according to equation (2.2) under hypotheses (2.2.vi-viii). The satiation parameters (γ, ω_t) referred to in hypotheses (2.2.ii, iv)will be specified below.

Free disposal: the surplus, once the lower bound on net import and the upper bound on stock is reached, is disposed of freely:

$$s_t := \bar{\sigma}(\underline{z}_t + w_{t-1} + \Sigma_j y^j_{t-1} - \bar{w}_t - \Sigma_j x^j_t,\ 0) \tag{3.3}$$

Buffer stock: Stock adjusts in order to keep net import within bounds. This can be formulated sequentially for each period as the minimization of the deviation from a fixed target level $\hat{w}_t$, $\hat{w}_t \in R^n_+ \,|\, \underline{w}_t \leqq \hat{w}_t \leqq \bar{w}_t$:

$$\min||w_t - \hat{w}_t|| \quad \text{over } w_t$$

$$\text{subject to} \quad \underline{z}_t \leqq w_t + s_t + \Sigma_j x^j_t - \Sigma_j y^j_{t-1} - w_{t-1} \leqq \bar{z}_t \tag{3.4}$$

Commodity balance is imposed:

$$z_t = \Sigma_j (x^j_t - y^j_{t-1}) + w_t - w_{t-1} + s_t \tag{3.5}$$

We observe that (3.3) - (3.5) combined imply that for all $p_t \in P$ net imports remain within the bounds (3.1), that stock remains below the upper bound but can fall below the lower bound (3.2), as illustrated in diagram 2.

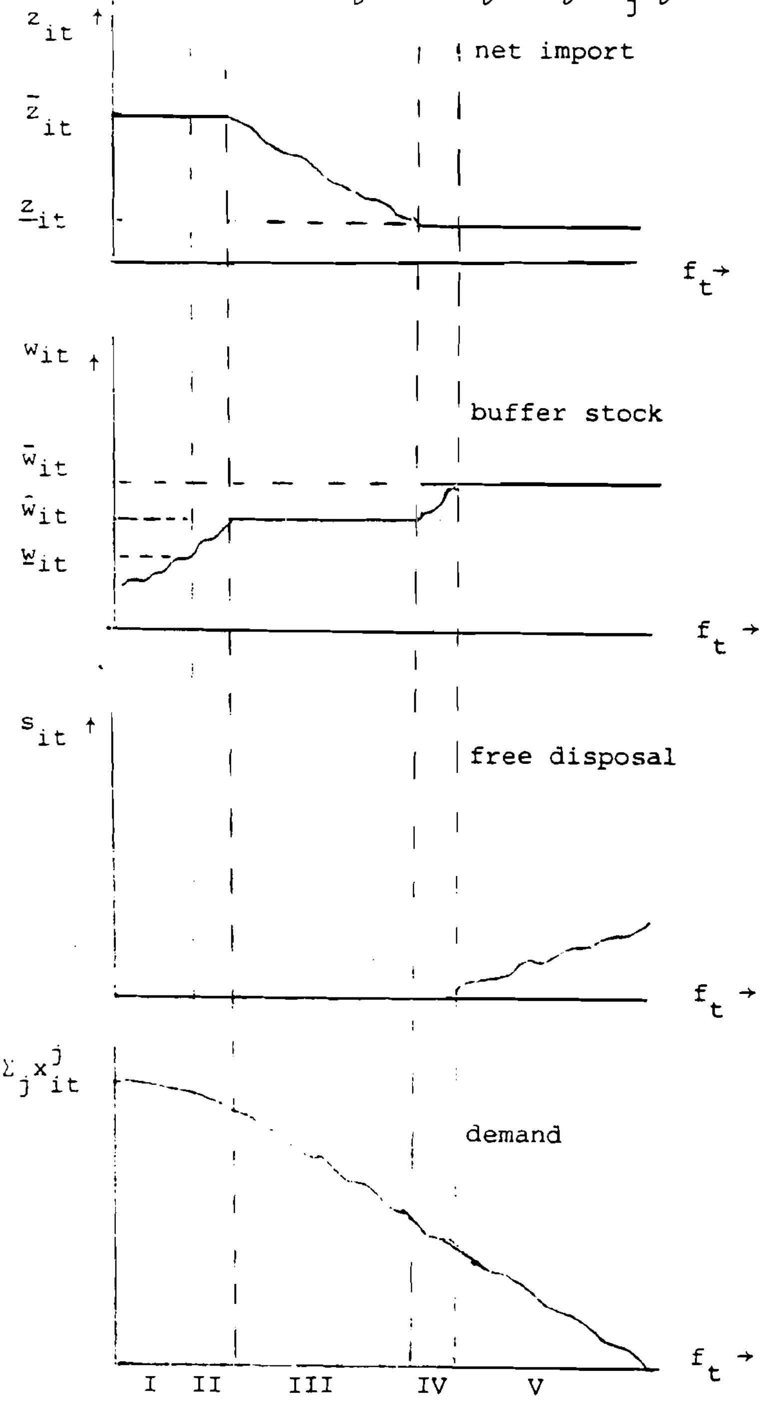
3.3 *Finance*

Government activities are financed by taxation on revenue. Distribution of tax among income groups is specified by a function of prices, each group's supplies and total tax requirements:

$$b : R^n_+ \times R^{nn}_{++} \times R^1 \rightarrow R^m$$

$$b_t := b(p_t,\ (y^j_{t-1})^m_{j=1},\ f_t) \tag{3.6}$$

Diagram 2: *Net import, buffer stock, free disposal and demand for* $p_t > 0$, $f_t < p_t - \sum_j y_t^j$ *and commodity balance satisfied.*



I . $z_{it} = \bar{z}_{it}$, $w_{it} \leq \underline{w}_{it}$, $s_{it} = 0$

II . $z_{it} = \bar{z}_{it}$, $\underline{w}_{it} \leq \bar{w}_{it} \leq \hat{w}_{it}$, $s_{it} = 0$

III . $\underline{z}_{it} \leq z_{it} \leq \bar{z}_{it}$, $w_{it} = \hat{w}_{it}$, $s_{it} = 0$

IV . $z_{it} = \underline{z}_{it}$, $\hat{w}_{it} \leq \bar{w}_{it} < w_{it}$, $s_{it} = 0$

V . $z_{it} = \underline{z}_{it}$, $w_{it} = \bar{w}_{it}$, $s_{it} \geq 0$

The function is subject to the hypotheses.

(i) *Homogeneity:* the absolute level of prices and taxation does not affect distribution of income tax over income groups (homogeneity of degree one):

$$\forall\ p_t \in P,\ \{y^j_{t-1}\}^m_{j=1},\ f_t\ :\ \forall\ \lambda \in R^1_{++}\ :$$

$$b(\lambda p_t,\ (y^j_{t-1})^m_{j=1},\ \lambda f_t) = \lambda b(p_t,\ (y^j_{t-1})^m_{j=1},\ f_t)$$

(ii) *Adding up:* the function fully distributes tax requirements:

$$\forall\ p_t \in P,\ \{y^j_{t-1}\}^m_{j=1},\ f_t\ :\ \Sigma_j b_j(p_t,\ (y^j_{t-1})^m_{j=1},\ f_t) = f_t$$

(iii) *Monotonicity:* when tax increases for one income group, it should not decrease for any, and vice versa:

$$\forall\ p_t \in P,\ \{y^j_{t-1}\}^m_{j=1},\ f_t,\ \bar{f}\,|\,f_t<\bar{f}\ :\ b(p_t,\ (y^j_{t-1})^m_{j=1},\ f_t) \leqq b(p_t,(y^j_{t-1})^m_{j=1},\bar{f})$$

(iv) *Positiveness:* each income group should have a positive after-tax revenue as long as total after-tax revenue is positive.

$$\forall\ p_t \in P,\ \{y^j_{t-1}\}^m_{j=1},\ f_t\ |\ f_t < p_t.\Sigma_j y^j_{t-1}\ :\ \forall j\ :\ b_j < p_t.y^j_{t-1}$$

(v) *Continuity:* the function is continuous with respect to prices and taxation.

Revenue itself equals receipts from marketed supplies minus tax:

$$m_{jt} := p_t \cdot y^j_{t-1} - b_{jt} \qquad (3.7)$$

We observe that tax can be positive as well as negative.

A <u>balance of trade condition</u> is imposed as an overall budget equation, which states that net imports z_t, evaluated at given international prices p^w_t, should be equal to a given deficit on the balance of trade, k_t:

For given $p^w_t \in R^n_+$, $k_t \in R^1$ | $p^w_t \cdot (\Sigma_j y^j_{t-1} + w_{t-1}) + k_t > 0$:

$$p^w_t \cdot z_t = k_t \qquad (3.8)$$

We specify the satiation parameters $(\gamma_t,\ \omega_t)$ introduced in (2.2) as:

$$\gamma_t := p^w_t,\ \omega_t := p^w_t \cdot (\bar{z}_t + \Sigma_j y^j_{t-1} + w_{t-1}).$$

The government budget equation implied by the commodity balance (3.5) and the balance of trade equation (3.6) (assuming that p_t and p^w_t are expressed in the same unit of account), is:

$$p_t \cdot (w_t + s_t) + (p^w_t - p_t) \cdot z_t = f_t + k_t + p_t . w_{t-1} \tag{3.9}$$

or

expenditures on goods + net subsidies on trade = tax receipts + trade deficit + revenue from stocks.

3.4 *Price formation*

Price realization only deviates from target under explicitly specified conditions. Let $\mu_t, \nu_t \in R^n_+$ and $\rho_t \in R^1_+$ relate price realization $p_t \in P$ to fixed price target $\bar{p}_t \in R^1_{++}$ according to:

$$p_t = \rho_t \bar{p}_t + \mu_t - \nu_t \tag{3.10}$$

Complementarity relations describe the restrictions on price adjustment. As long as either a buffer stock is unconstrained from below or a net import is unconstrained from above, price should not rise above target:

$$\begin{aligned} \mu_t \cdot (\bar{z}_t - z_t) &= 0 \\ \mu_t \cdot (w_t - \underline{w}_t) &= 0 \end{aligned} \tag{3.11}$$

As long as either a buffer stock is unconstrained from above or a net import is unconstrained from below, price should not fall below target:

$$\begin{aligned} \nu_t \cdot (z_t - \underline{z}_t) &= 0 \\ \nu_t \cdot (\bar{w}_t - w_t) &= 0 \end{aligned} \tag{3.12}$$

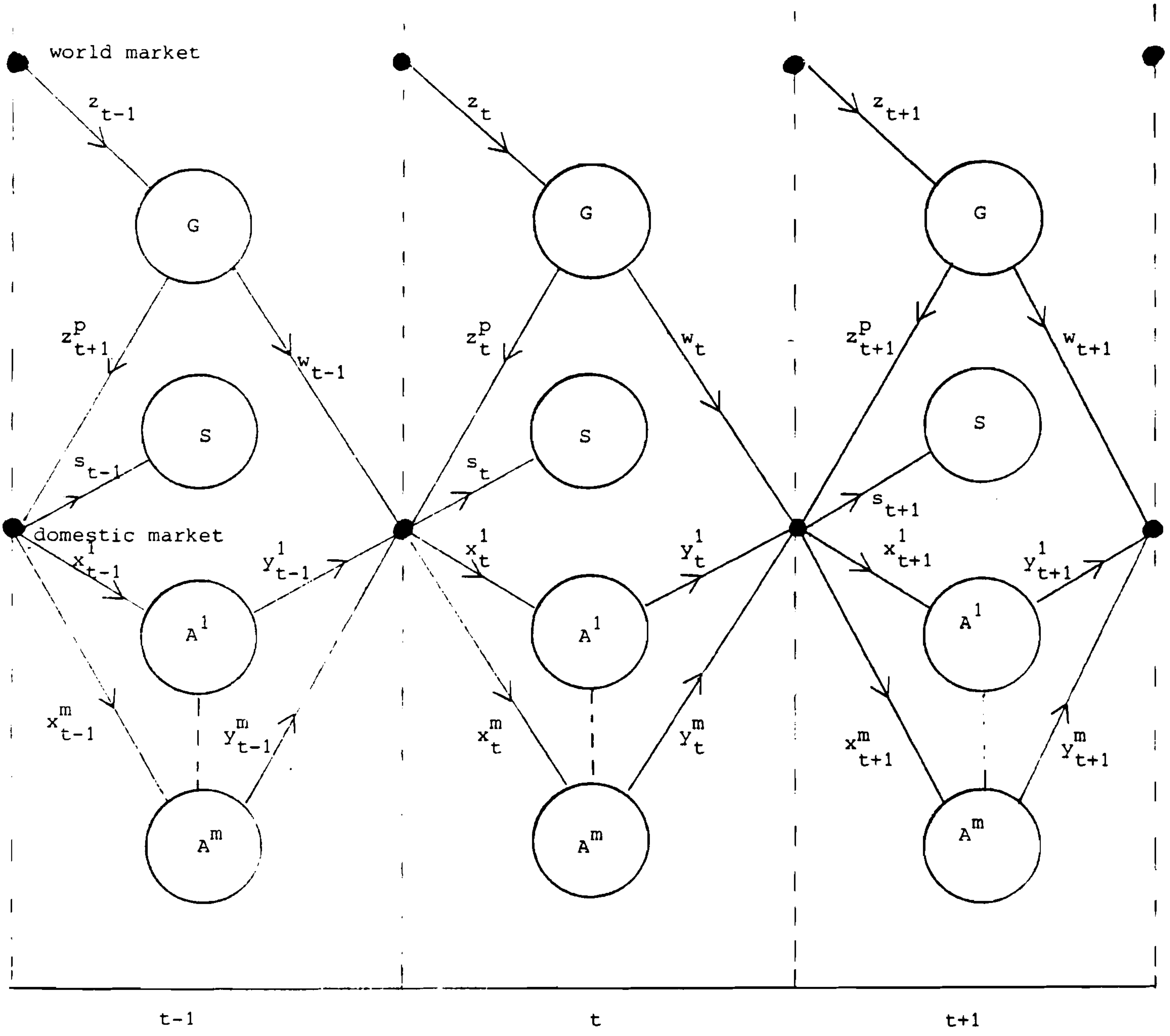
Price should be zero when free disposal is positive.

$$p_t . s_t = 0 \tag{3.13}$$

We again observe that this equation can also be looked at as a financial balance equation requiring that free disposal should finance itself. The free disposal equation (3.3) together with condition (3.13) thus describe a demand system with zero budget, which performs as a buffer stock, to prevent prices from becoming negative.

Diagram 3 illustrates the commodity flows in this model.

Diagram 3: *Open exchange economy with buffer stocks and lagged supply*



Legend: see diagram 1[1)]

G : government

S : free disposal

$A^1, .., A^m$: income group 1, .., m

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1) All variables have been defined in text, except z^p_t; $z^p_t = z_t - w_t$

3.5 *Equilibrium in the open exchange model*

As was the case with the model of the closed economy, the solution of the open exchange model can be established sequentially for each period. We therefore formulate a proposition in which the time subscript is dropped and y^j_{t-1} is replaced by y^j_{-1}, w_{t-1} by w_{-1}, z_t by z etc.

Proposition 2

With demand (2.1), satisfying hypotheses 2.2.i-v, free disposal (3.3), buffer stock (3.4) tax distribution (3.6) satisfying hypotheses 3.3.i-v, and revenue determination (3.7) the following holds:

For every given combination of supplies $\{y^j_{-1}\}^m_{j-1}$, $y^j_{-1} \in R^n_{++}$ and initial stock $w_{-1} \in R^n_+$,
for fixed

- world market prices p^w and balance of trade deficits k,
- bounds on net import: $\underline{z}$, $\bar{z}$,
- bounds and target level on stock: $\underline{w}$, $\hat{w}$, $\bar{w}$,
- price target $\bar{p}$,

such that

(i) $p^w > 0$, $p^w.(\Sigma_j y^j_{-1} + w_{-1}) + k > 0$

(ii) $p^w.\underline{z} < k < p^w.\bar{z}$,

(iii) $\bar{w} < \Sigma_j y^j_{-1} + w_{-1} + \underline{z}$

the open exchange model possesses a solution

$$(\rho^*, \mu^*, \nu^*, p^*, \{x^{j*}\}^m_1, s^*, w^*, z^*, f^*)$$

satisfying

- bounds on net import (3.1), bounds on stock (3.2), commodity balance (3.5)
- balance of trade (3.8)
- restrictions on prices (3.10 - 3.13)

and where

- $f^* < p^*.\Sigma_j y^j_{-1}$
- $p^* > 0$, $\rho^* > 0$.

3.6 *The nation as one actor, policy adjustment functions*

We formulate a proposition which establishes the possibility of describing net import as a multifunction of world market prices p^w and balance of trade deficits k.

To that end we assume that the bounds and target levels on buffer stock and the bounds on net import are specified as functions of world price and trade deficit, denoted

$\underline{w} : R_+^n \times R^1 \to R_+^1$ (lower bound on buffer stock), (3.14)

$\hat{w} : R_+^n \times R^1 \to R_+^1$ (target level on buffer stock), (3.15)

$\bar{w} : R_+^n \times R^1 \to R_+^1$ (upper bound on buffer stock), (3.16)

$\underline{z} : R_+^n \times R^1 \to R_+^1$ (lower bound on net import), (3.17)

$\bar{z} : R_+^n \times R^1 \to R_+^1$ (upper bound on net import). (3.18)

Moreover, the target price is assumed to be a given function of world price and trade deficit:

$$\bar{p} : R_+^n \times R^1 \to R_{++}^n \tag{3.19}$$

We assume that these functions satisfy the following hypotheses:

(i) homogeneity: $\forall (p^w,k) \in R_+^n \times R^1 : \forall \lambda \in R_{++}^1 : \underline{w}(p^w,k) = \underline{w}(\lambda p^w, \lambda k)$,

$\hat{w}(p^w,k) = \hat{w}(\lambda p^w,\lambda k)$, $\bar{w}(p^w,k) = \bar{w}(\lambda p^w,\lambda k)$, $\underline{z}(p^w,k) = \underline{z}(\lambda p^w,\lambda k)$,

$\bar{z}(p^w,k)\ \bar{z}\ (\lambda p^w,\lambda k)$

(ii) homotheticity: $\exists\ N : R_{++}^1 \to R_{++}^1,\ N(\lambda)\,|\forall(p^w.k) \in R_+^n \times R^1 : \forall \lambda \in R_{++}^1 :$

$N(\lambda)\ \bar{p}(p^w,k) = \bar{p}(\lambda p^w, \lambda k)$.

(iii) continuity: $\underline{w}$, $\hat{w}$, $\bar{w}$, $\underline{z}$, $\bar{z}$, $\bar{p}$ are continuous functions.

(iv) compatibility: $\forall (p^w,k) \in R_+^n \times R^1$: $p^w.\underline{z}(p^w,k) < k < p^w.\bar{z}(p^w,k)$

$\underline{z}(p^w,k) \leq \bar{z}(p^w,k)$

$\underline{w}(p^w,k) \leq \hat{w}(p^w,k) \leq \bar{w}(p^w,k)$

$\bar{w}(p^w,k) < \Sigma_j y^j_{-1} + w_{-1} + \underline{z}(p^w,k)$.

Let us denote the set of equilibrium net imports corresponding to a given international price and trade deficit by $Z^*(p^w,k)$ and define the set of normalized international prices and trade deficits which allow a nonnegative demand as:

$$T(\bar{y}_{-1}) .= \{(p^w,k) \in R^n_+ \times R^1 \mid \|p^w\|_1 = 1,\ p^w.\bar{y}_{-1} + k \geqq 0\} \qquad (3.20)$$

We now define the national net import multifunction by assigning to it equilibrium net imports whenever this equilibrium is defined and by artificially defining it otherwise:

$$Z' : R^n_+ \times R^1 \rightarrow \mathbb{P}(R^n),\ Z'(p^w,k) := \{z' \in R^n \mid \exists\ z^* \in Z^*(p^w,k) : z' = z^*,$$
$$\text{if } p^w > 0 \text{ and } p^w.\bar{y}_{-1} + k > 0;$$
$$\text{otherwise: } \underline{z}(p^w,k) \leqq z' \leqq \bar{z}(p^w,k),\ p^w.z' = k\} \qquad (3.21)$$

Proposition 3

If: - bounds and target on buffer stock (3.14) - (3.16)
- bounds on net import (3.17) - (3.18),
- price target (3.19)

satisfy hypotheses 3.6.i-iv

and, if

for all feasible combinations $(p^w,k) \in T(\bar{y}_{-1})$ such that $p^w > 0$, $p^w.\bar{y}_{-1}+k > 0$, the open exchange model does not possess more than <u>one</u> equilibrium solution ρ^*, μ^*, ν^*,

then

the net import multifunction Z', as defined by (3.21) possesses the following properties:

(i) <u>homogeneity</u>: $\forall(p^w,k)\in T(\bar{y}_{-1}) : \forall\lambda\in R^1_{++} : \forall z'\in Z'(p^w,k) : z'\in Z'(\lambda p^w,\lambda k)$

(ii) <u>additivity</u>: $\forall(p^w,k) \in T(\bar{y}_{-1}) : \forall\ z' \in Z'(p^w,k) ; p^w.z' = k$

(iii) <u>continuity</u>: $\forall(p^w,k) \in T(\bar{y}_{-1}) : Z'(p^w,k)$ is nonempty, bounded, closed.
$\forall(p^w,k) \in T(\bar{y}_{-1}) :$ grf $(T; Z')$ is closed.
$\forall(p^w,k) \in T(\bar{y}_{-1}) \mid p^w > 0,\ p^w.\bar{y}_{-1} + k > 0 : Z'(p^w,k)$ contains one single element.

(iv) <u>convexity</u>: $\forall(p^w,k) \in T(\bar{y}_{-1}) : Z'(p^w,k)$ is convex.

Proof is given in para 10.

It follows from proposition 3 that whenever trade deficit and international price lie in the interior of $T(\bar{y}_{-1})$, net import is a continuous function so that the nation as a whole possesses all the properties of its constituting consumers except monotonicity and nonsatiation. When considering an economy with nations as basic actors, nonsatiation can be reintroduced artificially but monotonicity cannot be restored. Thus, although price policies, quantity restrictions and taxation can also be introduced at the international level, the policies must be specified in such a way that they do not require monotonicity of the net import function.

4. *A CLOSED INTERNATIONAL ECONOMY*

4.1 *Central Market Regulation*

None

4.2 *Demand Supply*

We move to the international level and describe a closed international economy, indexed w, in which demand and supply are generated as net import by nations, indexed h, $h \in H^w$, which satisfy the conditions of proposition 3. For each nation we distinguish

supply equals total supply by income groups and buffer stock:

$$\bar{y}_t^h := \sum_j y_t^h + w_t^h \tag{4.1}$$

net import satisfies proposition 3 and is defined as:

$$z_t^h : T(\bar{y}_{t-1}^h) \rightarrow \mathbb{P}(R^n),\ z_t^h(p_t^w,\ k_{ht}) \tag{4.2}$$

Nonsatiation is introduced by stipulating that one nation indexed d^w, should possess this property. We call it the slack nation .
Let $\bar{H}^w := \{h \in H^w \mid h \neq d^w\}$
The slack nation has a net import function

$$z^d : R_+^n \rightarrow R^n,\ z_t^{d^w} := z^d(p_t^w) \tag{4.3}$$

It satisfies, for all $p_t^w \in R_+^n$ the following hypotheses:

(i) homogeneity: $\forall\ \lambda \in R_{++}^1 : z^d(\lambda p_t^w) = z^d(p_t^w)$

(ii) adding up : $p_t^w . z^d(p_t^w) \leq 0$, with equality whenever $\gamma . z^d(p_t^w) \leqq \omega_t$

(iii) continuity

(iv) nonsatiation: $\forall_i : \exists(\hat{p} | \hat{p}_k = p_{kt}^w,\ k \neq i,\ \hat{p}_i \leqq p_{it}^w) : \gamma . z^d(p_t^w) > \omega_t$

We then impose:

Free disposal:

$$s_t^w := \bar{\sigma}(-\sum_{h\in H^w} z_t^h, 0) \qquad (4.4)$$

Commodity balance:

$$\sum_{h\in H^w} z_t^h + s_t^w = 0 \qquad (4.5)$$

4.3 *Finance*

Trade deficits are distributed among nations in a way similar to the tax distribution among income groups within the nation.

Transfer distribution functions distributes a total transfer to an international agency, $\eta_t^w \in R^1$ over all nations $h \in \bar{H}^w$.

$$k_h : \underline{R}_+^n \times \prod_{h\in\bar{H}^w} (R_{++}^n) \times R^1 \rightarrow R^1,$$

$$k_{ht} := k_h(p_t^w, (\bar{y}_{t-1}^h)_{h\in\bar{H}^w}, \eta_t^w) \qquad (4.6)$$

The function satisfies the following hypotheses:

(i) homogeneity: the absolute level of prices and transfers does not influence the distribution of trade deficits over nations (homogeneity of degree one):

$$\forall \lambda \in R_{++}^1 : k_h(\lambda p_t^w, (\bar{y}_{t-1}^h)_{h\in\bar{H}^w}, \lambda\eta_t^w) = \lambda k_h(p_t^w, (\bar{y}_{t-1}^h)_{h\in\bar{H}^w}, \eta_t^w)$$

(ii) adding up: the functions fully distribute the transfer

$$\sum_{h\in\bar{H}^w} k_h(p_t^w, (\bar{y}_{t-1}^h)_{h\in\bar{H}^w}, \eta_t^w) = -\eta_t^w$$

(iii) continuity

(iv) positiveness: each nation should be allowed positive demand as long as total value of demand is positive:

$$\forall(\eta_t^w | \eta_t^w < p_t^w \cdot \sum_{h\in\bar{H}^w} \bar{y}_{t-1}^h) : \forall h \in \bar{H}^w : p_t^w \cdot \bar{y}_{t-1}^h + k_{ht} > 0$$

No international agency is introduced, so that financial balance requires:

$$\eta_t^w = 0 \qquad (4.7)$$

4.4 *Price Formation*

As in the closed economy, only free disposal introduces a restriction on price formation

$$p^w_t \cdot s^w_t = 0 \qquad (4.8)$$

Diagram 4 shows the commodity flows in this model.

4.5 *Equilibrium in the closed international economy*

As in para 1.2, equilibrium can be established sequentially for each period. We therefore drop the time subscript in the formulation of the following proposition.

Proposition 4:

At all given level of supplies $(\bar{y}^h_{-1})_{h \in H^w}$, $\bar{y}^h_{-1} \in R^n_{++}$,

with - net import (4.2), (4.3) satisfying proposition 3 and hypotheses
(4.2.i-iv) respectively,
free disposal (4.4)
- transfer distribution (4.6) satisfying hypotheses (4.3.8-iv).

the model of the closed international economy possesses a solution

$$p^{w*},\ \eta^{w*},\ \{k_h\}_{h \in \bar{H}^w} ,\ \{z^{h*} \in Z^h(p^{w*},\ k^*_h)\}_{h \in \bar{H}^w} ,\ z^{d^w}_* ,\ s^{w*}$$

satisfying

- commodity balance (4.5)
- financial balance (4.7)
- price restriction (4.8)

and where

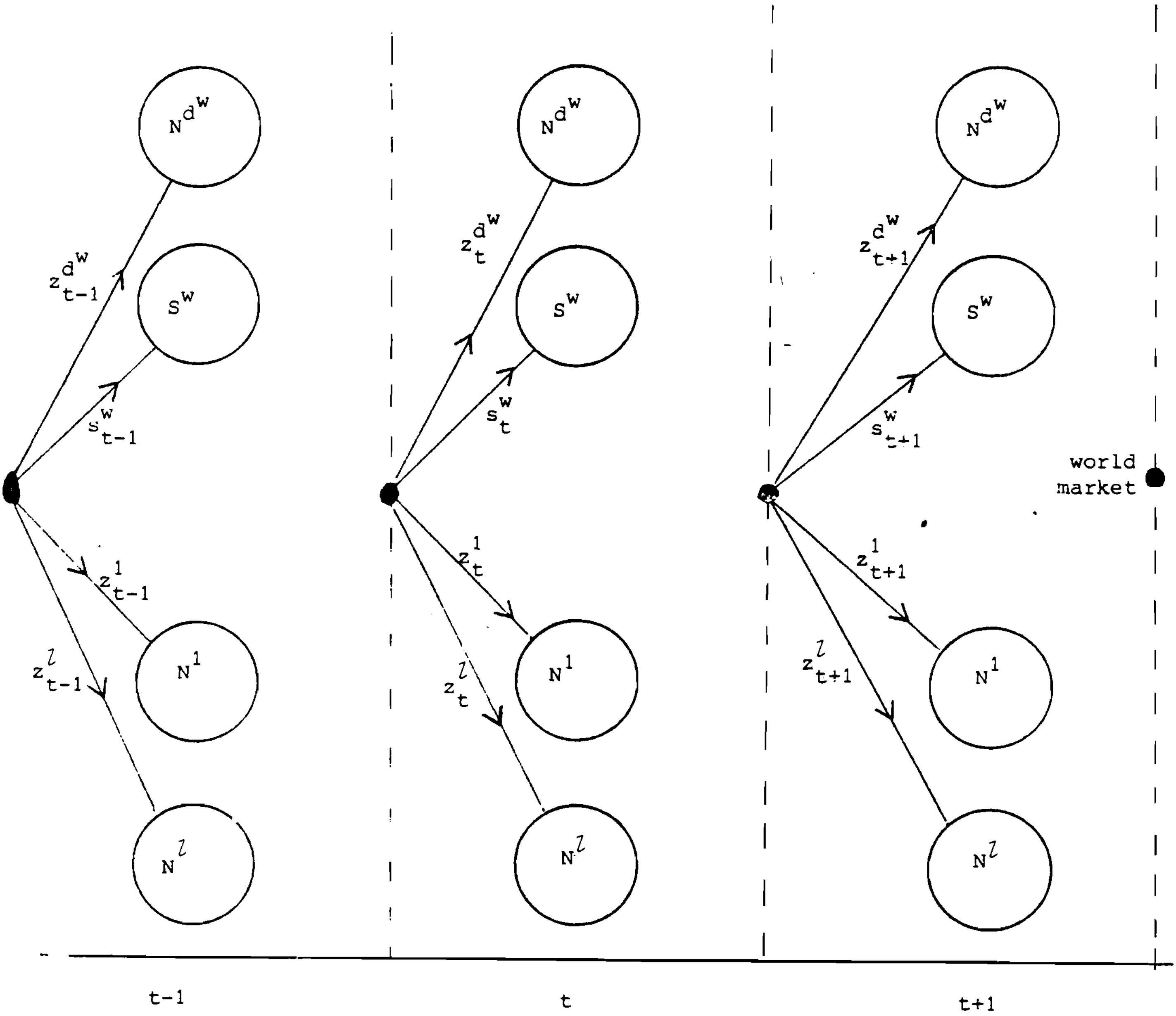
- $p^{w*} > 0,\ \eta^{w*} < p^w \ \cdot \Sigma_{h \in \bar{H}^w} \bar{y}^h_{-1}$

5. *AN INTERNATIONAL BUFFER STOCK AGREEMENT WITH A FIXED PRICE BAND*

5.1 *Central Market Regulation*

We introduce an international agency which sees it as its central goal to keep prices within between an upper and a lower bound. For this it has one instrument at its disposal: stock adjustment.

Diagram 4: *A "competitive" international economy*



Legend: see diagram 1

N^1 , N^l : nation 1, d^w, l

S^w : free disposal activity

Stock instrument: Stocks u_{t-1}, u_t, $u_t \in R^n_+$, are brought on the market at the beginning of period t and sold at the end of the period. Stocks are adjusted within fixed upper and lower bounds:

$$\underline{u}_t \leqq u_t \leqq \bar{u}_t \tag{5.1}$$

5.2 *Demand Supply*

Net import, z_t^h by nations h, $h \in H^w$ is as described by (4.2), (4.3).

Free disposal:

$$s_t^w := \bar{\sigma}(- \sum_{h \in H^w} (z_t^h) - \bar{u}_t + u_{t-1}, 0) \tag{5.2}$$

Buffer stock: stock adjusts in order to keep commodity balance. This can be formulated sequentially for each period as the minimization of a deviation from a fixed target level, $\hat{u}_t$, $\hat{u}_t \in R^n_+ \mid \underline{u}_t \leqq \hat{u}_t \leqq \bar{u}_t$:

$$u_t := \hat{u}_t + u_t^+ - u_t^-$$

where u_t^+, $u_t^- \in R^n_+$ are optimal in

$$\begin{aligned} &\min \quad ||u_t^+||_1 + ||u_t^-||_1 \text{ over } u_t^+, u_t^- \\ &\text{subject to: } \hat{u}_t + u_t^+ - u_t^- - u_{t-1} + \sum_{h \in H^w} z_t^h + s_t^w = 0 \end{aligned} \tag{5.3}$$

Commodity balance: although implied by (5.3) is imposed for the sake of completeness:

$$u_t - u_{t-1} + \sum_{h \in H^w} z_t^h + s_t^w = 0 \tag{5.4}$$

5.3 *Finance*

Buffer stock is financed by nations. Total transfer η_t^w is distributed among them according to transfer distribution functions (4.6). A financial balance is imposed which requires that that total transfer should equal the value of the net increase in stocks, valued at current prices.

$$p_t^w \cdot (u_t - u_{t-1}) = \eta_t^w \tag{5.5}$$

5.4 *Price Formation*

Price realization only deviates from target under explicitly specified conditions.

Let $\underline{p}_t^w, \bar{p}_t^w \in R^n_{++}$ be fixed bounds within which the agency tries to keep the international prices.

Let $\tilde{p}_t^w, \mu_t^w, \nu_t^w \in R^n_+$, $\rho_t^w \in R^1_+$ relate price realization to price target according to:

$$\left.\begin{array}{l} p_t^w = \tilde{p}_t^w + \mu_t^w - \nu_t^w \\ \text{where} \\ \rho_t \underline{p}_t^w \leqq \tilde{p}_t^w \leqq \rho_t \bar{p}_t^w \end{array}\right\} \qquad (5.6)$$

As long as limits on stocks are ineffective, prices should remain within the bounds:

$$\left.\begin{array}{l} u_t^w \cdot (u_t - \underline{u}_t) = 0 \\ \nu_t^w \cdot (\bar{u}_t - u_t) = 0 \end{array}\right\} \qquad (5.7)$$

As soon as stock level drops below target, the upper price bound becomes target and as soon as stock level rises above target, the lower price bound becomes target:

$$\left.\begin{array}{l} (\rho_t^w \bar{p}_t^w - \tilde{p}_t^w) \cdot u_t^- = 0 \\ (p_t^w - \rho_t^w \underline{p}_t^w) \cdot u_t^+ = 0 \end{array}\right\} \qquad (5.8)$$

Price target should be such that, at target prices the value of the stock should equal the value of target stock

$$\tilde{p}_t^w \cdot (u_t - \hat{u}_t) = 0 \qquad (5.9)$$

Price should be zero when free disposal is positive

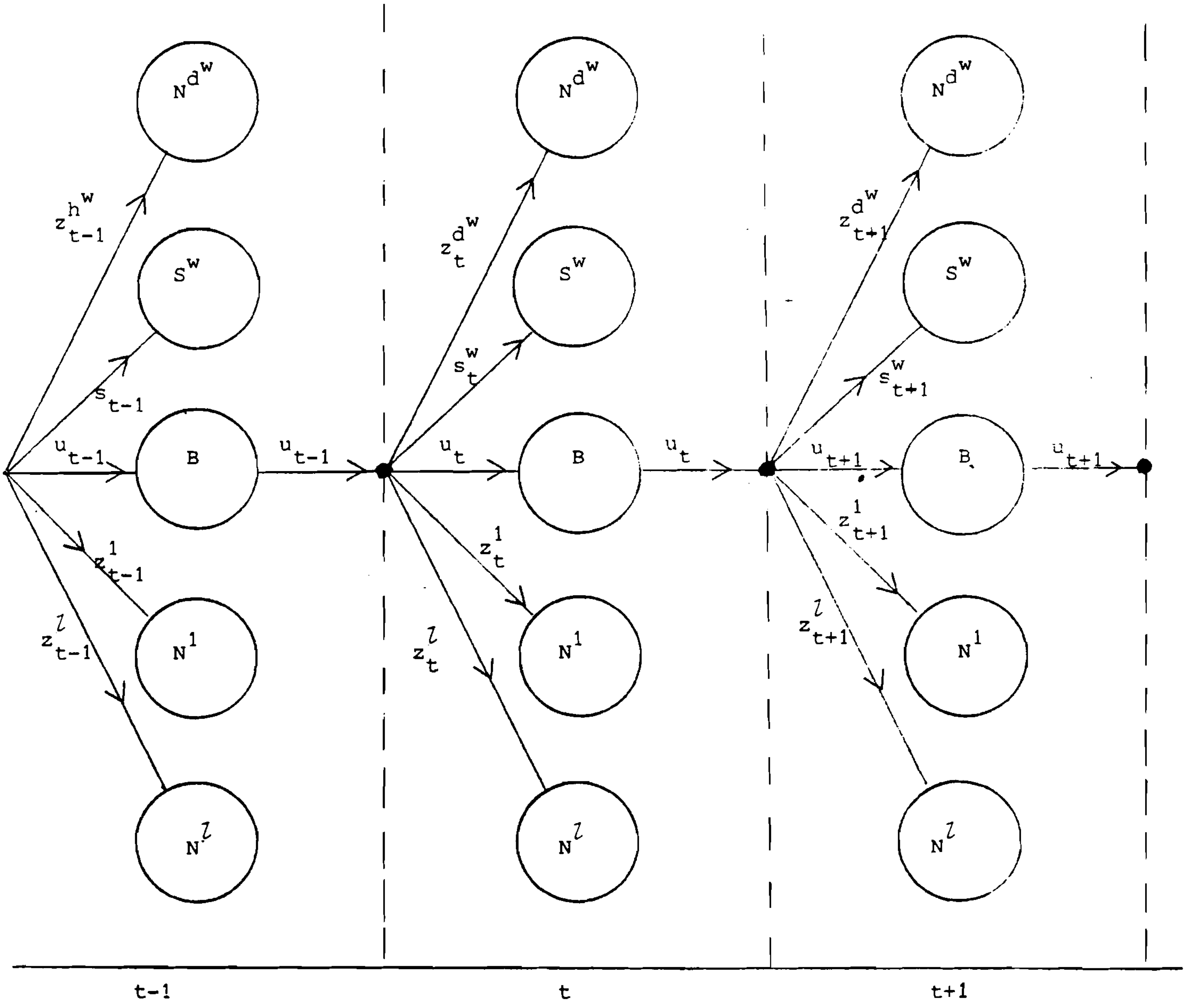
$$p_t^w \cdot s_t^w = 0 \qquad (5.10)$$

Diagram 5 illustrates the commodity flows in the model.

5.5 *EQUILIBRIUM UNDER THE INTERNATIONAL BUFFER STOCK AGREEMENT*

We observe that restriction (5.8) - (5.10) can alternatively be looked at as financial balance conditions. Restricion (5.9) is of special interest since it implies a kind of value preservation.

Diagram 5: *An international economy with buffer stocks*



Legend: see diagram 1

N^1, N^l : nation 1, ..., d^w, ..., l

S^w : free disposal activity

B : buffer stock agency

We call $(\tilde{p}^w_t.\hat{u}_t)$ the ex ante commitment. We also note that (5.8) implies that whenever a price is within the band the corresponding stock is at target level. Substitution of (5.7).(5.8) into (5.5) yields an explicit transfer function:

$$\eta^w_t = \tilde{p}^w_t.\hat{u}_t + \mu^w_t.\underline{u}_t - \nu^w_t.\bar{u}_t - p^w_t.u_{t-1} \tag{5.11}$$

We see that the transfer, which we shall refer to as ex post commitment is equal to:

ex ante commitment + financial consequences of price deviation from target - value of initial stock.

Since equilibrium can be established sequentially for each period, we again drop the time subscript in the proposition on existence of equilibrium.

Proposition 5:

With net import (4.2), (4.3), free disposal (5.2), buffer stock (5.3), transfer distribution (4.6), the following holds:

For every given combination of supplies

$$\{\bar{y}^h_{-1}\}_{h \in H^w},\ \bar{y}^h_{-1} \in R^n_{++}$$

and initial stock

$$w_{-1} \in R^n_+,$$

for fixed - bounds and target level on stock: $\underline{u}$, $\hat{u}$, $\bar{u}$

- bounds on price target $\underline{p}^w$ $\bar{p}^w$

such that

$$\bar{u} < \sum_{h \in H^w} \bar{y}^h_{-1} + u_{-1}$$

the model of the international buffer stock agreement possesses a solution:

$$\rho^{w}*,\ \mu^{w}*,\ \nu^{w}*,\ \tilde{p}^{w}*\ ,\ \eta^{w}*,\{k^*_h\}_{h \in \bar{H}^w},\{z^h* \in Z^h(p^w*,\ k^*_h)\}_{h \in \bar{H}^w}, z^{d^w}*, s^v*, u*, u^+*, u^-*$$

satisfying

- commodity balance (5.4), bounds on stocks (5.1),
- financial balance (5.5)
- price restrictions (5.6) - (5.10).

and where

- $\eta^{w*} < p^{w*} \cdot \sum_{h \in H^w} \bar{y}^h_{-1}$

- $p^{w*} > 0, \rho^{w*} > 0.$

6. *AN INTERNATIONAL AGREEMENT ON MARKET SEGMENTATION*

6.1 *Introduction*

A buffer stock agreement cannot, in the long run, keep prices away from a "natural" equilibrium level: stocks will get overfilled or depleted within a few periods. A more adequate way to meet price targets in the long run is to have net import itself adjust. We now describe an agreement in which one group of nations strives at a price target on the international market by adjusting its net trade with that market.
This segments the world into two international economies: first the economy of the rest of the world (economy indexed w = I), and second the economy of this group (economy indexed w = II).

6.2 *Model of Economy I*

6.2.1 *Central Market Regulation*

Central aim of the agreement among members of economy II is to achieve a fixed price target. To reach this target, there is one instrument: adjustment of net trade.
Trade instrument: net import by economy I is adjusted within fixed upper and lower bound:

$$\underline{z}^I_t \leq z^I_t \leq \bar{z}^I_t \qquad (6.1)$$

6.2.2 *Demand – Supply*

Net import by nations h, h $\in H^I$ is as described by (4.2), (4.3)

Free disposal

$$s^I_t := \bar{\sigma}\,(-\sum_{h \in H^I} z^h_t + \underline{z}^I_t, 0) \qquad (6.2)$$

Commodity balance:

$$z^I_t = \sum_{h \in H^I} z^h_t + s^I_t \qquad (6.3)$$

6.2.3 *Finance*

The nations in economy I do not take part in the agreement and are therefore not involved in its financing. They merely share trade deficits among each other according to transfer distribution function (4.6).

Financial balance:

$$\eta_t^I = 0 \tag{6.4}$$

6.2.4 *Price formation*

Let $\mu_t^I, \nu_t^I \in R_+^n$, $\rho_t^I \in R_+^1$ relate price realization $p_t^I \in P$ to fixed price target $\bar{p}_t^I \in R_{++}^1$ according to

$$p_t^I = \rho_t^I \bar{p}_t^I + \mu_t^I - \nu_t^I \tag{6.5}$$

Complementarity relations describe the restrictions on price adjustment. As long as upper bound on aggregate net import is ineffective, price should not rise above target and vice versa:

$$\left.\begin{array}{l} \mu_t^I \;.\; (\bar{z}_t^I - z_t^I) = 0 \\ \nu_t^I \;.\; (z_t^I - \underline{z}_t^{II}) = 0 \end{array}\right\} \tag{6.6}$$

Price should be zero when free disposal is positive:

$$p_t^I \;.\; s_t^I = 0 \tag{6.7}$$

6.3 *Model of Economy II*

6.3.1 *Central Market Regulation*

None

6.3.2 *Demand - Supply*

Net import by nations h, $h \in H^{II}$ is as described by (4.2), (4.3).

Free disposal

$$s_t^{II} := \bar{\sigma}\;(-\sum_{h \in H^{II}} z_t^h - z_t^I,\; o) \tag{6.8}$$

Commodity balance

$$\sum_{h \in H^{II}} z_t^h + z_t^I = 0 \tag{6.9}$$

6.3.3 *Finance*

Transfer distribution is effectuated according to (4.6)

Financial balance requires that the value of net imports by economy I should be covered by transfers:

$$\eta_t^{II} = p_t^{II} \cdot z_t^{I} \qquad (6.10)$$

6.3.4 *Price restriction*

Price should be zero when free disposal is positive:

$$p_t^{II} \cdot s_t^{II} = 0 \qquad (6.11)$$

6.4 *Equilibrium under the Market Segmentation Agreement*

We observe that equilibrium can be established sequentially, first for economy I and then for economy II. Since it can also be established sequentially in time, we drop time subscripts in the formulation of the proposition on existence.

Proposition 6:

With net import (4.2), (4.3), free disposal (6.2), (6.8) and transfer distribution (4.6), the following holds:

For every given combination of supplies

$$\{\bar{y}^h_{-1}\}_{h\in H^I}, \{\bar{y}^h_{-1}\}_{h\in H^{II}},$$

for fixed - target price, $\bar{p}^I$

- bound on net imports by economy I, $\underline{z}^I$, $\bar{z}^I$

such that:

$$\underline{z}^I \leqq 0 < \bar{z}^I < \sum_{h\in\bar{H}^{II}} \bar{y}^h_{-1}$$

the model of economy I possesses a solution

$$\rho^{I*}, \mu^{I*}, \nu^{I*}, p^{I*}, \eta^{I*}, \{k^*_h\}_{h\in\bar{H}^I}, \{z^{h*} \in Z^h(p^{I*}, k^*_h)\}_{h\in\bar{H}^I}, s^{I*}, z^{I*},$$

which satisfies

- bounds on net imports (6.1), commodity balance (6.3),
- financial balance (6.4)
- price restrictions (6.5) - (6.7),

where

$$p^{I*} > 0, \ \rho^{I*} > 0$$

and the model of economy II possesses a solution

$$p^{II*}, \eta^{II*}, \{k_h^*\}_{h \in \bar{H}^{II}}, \{z^{h*} \in Z^h(p^{II*}, k_h^*)\}_{h \in H^{II}}, \dot{s}^{II*},$$

which satisfies

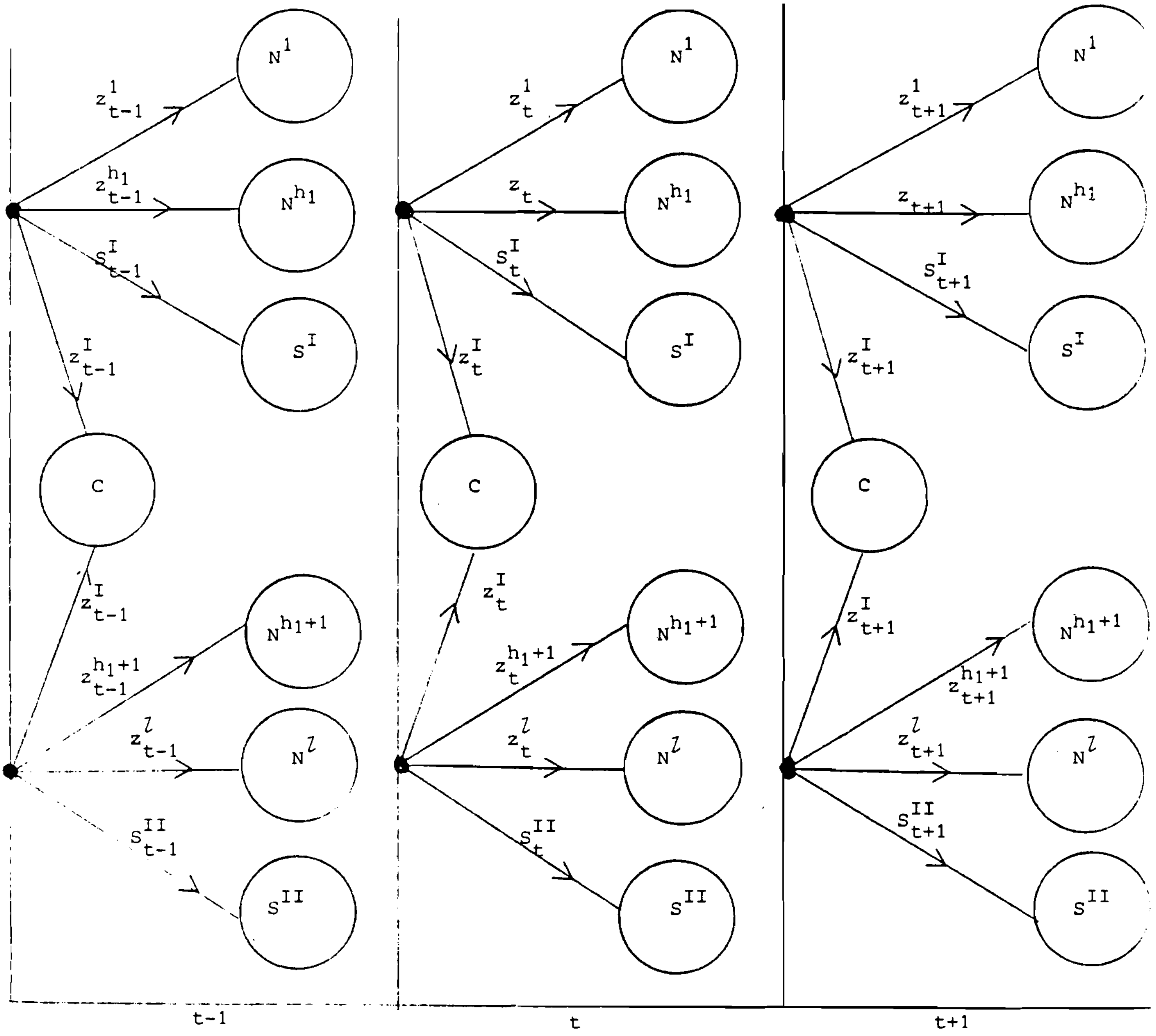
- commodity balance (6.9)
- financial balance (6.10)
- price restriction (6.11)

and where

$$p^{II*} > 0.$$

Diagram 6 describes the commodity flows in the model.

Diagram 6: *An international agreement on market segmentation*



Legend: see diagram 1

$N^1, \ldots, N^{h_1}$: nations member of economy I

$N^{h_1+1}, \ldots, N^l$: nations member of economy II

S^I : free disposal economy I

S^{II} : free disposal economy II

C : agency managing the agreement.

Chapter III

ECONOMIC REALISM OF THE ASSUMPTIONS

1. *GENERAL EQUILIBRIUM MODELS*

The national and the international models in Chapter II are general equilibrium models in the Walrasian tradition. They describe individual behaviour of certain actors (consumers, nations, etc.) and then integrate this behaviour by imposing aggregate balance equations (quota, limits on stocks, financial constraints, etc.). They are general and not partial because they keep a comprehensive account of expenditures on goods and services. They are equilibrium models because overall physical balances and financial constraints determine the level of adjustment variables (e.g. prices and taxation). In the literature the term "disequilibrium model" also is used to indicate that prices are not the only adjusting variable. We do not follow that convention. We shall not enter the debate whether or not money should be considered as one of the goods in the models. Several conditions under which money can be left out of the model, the conditions for a dichotomy between money and other goods, are summarized in Negishi (2.5), but clearly, in general, money plays a role of its own. In the applications to food and agriculture which were primarily envisaged for the models of Chapter II, money only is a unit of account and not a store of value[1]. It is for that reason that the national deficit was called deficit on the balance of trade. We thus only consider goods and services and disregard all monetary "commodities"; we shall see below how the model can be given a more general interpretation (cf. para 9), but up to that point, goods are considered to be objects, the quantity of which can be measured physically and which are desired by income groups. Goods differ by physical characteristics or by

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[1] From the price normalization rule applied in II, para 2 follows that 1 unit of account = $\sum_{i=1}^{n} p_i$, but one could formulate more generally: one unit account = $\sum_{i=1}^{n} p_i\alpha_i$; $\alpha_i > 0$. It is also possible to select a nonlinear index as unit of account, see para 8.4 below.

location in space; services are treated as goods. We suppose that the number of different goods is finite; goods are not distinguished by their location in time: we only consider present goods and no demand and supply for future goods. The main assumptions underlying the models will now be discussed and minor generalizations will be shown. Paragraphs 2-8 are rather technical and directly relate to Chapter II, para 1, 2, 6. Para 9-11 are general and can be read independently.

2. *LAGGED SUPPLY*

We have assumed a one period lag in supply (hypothesis II, 2.2.vi). From a theoretical point of view this approach is not uncommon because it is quite possible to graft a competitive or an oligopolistic supply module onto the exchange model through a multifunction $y = y(p_t, p_{t-1}, \dots)$ satisfying homogeneity and boundedness conditions (see e.g. Jasckold-Gabszewicz (15). If one looks however at supplies as production capacities and considers the actual production as this capacity minus increase in the buffer stock, then some adjustment of actual production is already present in the open exchange model described in Chapter II, para 2. Note that demand covers both demand for inputs and for final goods. It is doubtful whether input demand can be formulated as a continuous function when production plans are generated in linear programmes but this is a matter we do not further dwell upon. The hypotheses II.2.2.vii, viii on boundedness and homogeneity do not require further comment.

3. *DEMAND*

Demand functions have been introduced directly into the model of Chapter II, para 2, without any derivation from utility maximization. This is done for the sake of simplicity. As pointed out by Barten (3) and others the *homogeneity* requirement (II.2.2.i) and the *adding up* requirement (II.2.2.ii) are the only requirements from utility theory which survive aggregation over consumers with differing preferences. Because we wish to set up a national model such aggregation cannot be avoided. The homogeneity requirement suffers from the shortcomings of the dichotomy discussed in para 1 above, as long as money is not explicitly taken into consideration. The adding up requirement implies that *savings* are disregarded. It is possible to overcome this limitation by adding a savings function to the demand system.

$$g_{jt} := g_j(p_t, m_{jt}) \tag{3.1}$$

The function should be homogeneous of degree one in (m_{jt}, p_t) and should satisfy

(i) $m_{jt} > g_{jt} \geqq 0$. for all $m_{jt} > 0$, $p_t \in P$

(ii) $m_{jt} = p_t . x_t^j + g_{jt}$, all $m_{jt} \geqq 0$, $p_t \in P$

An example of an extended expenditure system (which does not satisfy (i) however), is the extended linear expenditure system by Lluch and Powell ( ).

A most essential and restrictive assumption on the demand system is the monotonicity requirement that all goods have a *nonnegative propensity* with respect to income (II.2.2.iii). The condition obviously only is imposed on the short run propensity. Empirical evidence as reported in Powell (27), Nasse (24), Brown and Deaton (5), fortunately suggests that from an empirical point of view the assumption is not restrictive.

For *nonsatiation* condition II.2.2.iv, to hold it is sufficient that one income group satisfies it. This is not a very restrictive assumption expecially for consumer goods.

Satiation may occur but import quota should be binding first. The reason to impose the nonsatiation assumption is that we wish the balance of trade equation to be satisfied; when a price is zero, taxation looses grip in demand and if a good with zero price does not violate a quota constraint, a zero price is compatible with equilibrium and the balance of trade equation can be violated.

4. *INVESTMENT*

One saving has been introduced, it is natural to consider investment. Investment can be regarded as a component of either demand by income groups or buffer stock demand. We observe that an aggregate investment function can also be introduced, which is dependent on prices and can be limited by savings.

5. *TAXATION*

Taxation covers in the model direct as well as indirect taxes, but does not cover tariffs and subsidies on international trade. The tax distribution function implies a variable rate of taxation and monotonicity condition (II.3.3.iii) implies that when the taxation rate increases for some income group, it does not decrease for any groups. One might object to this that income transfers to one group imply higher taxes for the others but this can be taken care of by specifying tax functions such as:

$$b_{jt} := -b_{1j}(p_t) + b_{2j}(p_t, (y^j_{t-1})^m_1, f_t) \tag{5.1}$$

Here $b_{1j}(p_t)$ is an (indexed) income transfer. Obviously only the second component is required to exhibit weak monotonicity with respect to taxation. As mentioned in para 3 above, private savings can be regarded as a voluntary tax and thus as a component of (5.1). In this way different types of taxes can be handled separately. Even tariff receipts can be decomposed into margins due to quota and margins due to the difference between target prices and world market prices, and both can be redistributed according to separate rules.

6. *BALANCE OF TRADE*

The balance of trade equation is the budget equation of the nation. Irrespective of the policies pursued by the nation's government, this equation has to hold. It is formulated in terms of an international unit of account (one international unit of account = $\sum_{i=1}^{n} p^w_i$). This is not restrictive because the balance of payments holds by definition.The limitation comes in with the requirement that the trade deficits are given for each nation, add up to zero for all world market prices and possess homogeneity property (II.3.3.i,ii). This makes international capital flows, foreign exchange reserves, international transfers of profits, interest and wages exogenous to the national models and the homogeneity requirement points at the dichotomy assumption referred to in para 1. International capital and income transfers are thus considered to be indexed, exchange rates have no implications for the model and foreign exchange reserves do not adjust internally. This brings us to the adjustment mechanism of the national model. Taxation is the variable which adjusts until balance of trade is satisfied. In a more general application of the model the foreign exchange demand would be an obvious candidate as an adjusment variable.

7. *POLICY TARGETS*

The open exchange model considers policy targets on domestic prices, on net imports and on buffer stocks. There is a hierarchy between these targets. The quotas on net imports and the limits on stocks have to be satisfied in any equilibrium. The price target has to be satisfied as long as it does not violate limits on stocks and the stock target finally has to be satisfied only as long as the price target is not endangered by the effectiveness of a quota.

The realism of such a construction is hard to assess. National governments do impose quotas on international trade and domestic price policies or tariffs are also quite common, both in developing and in developed countries. If a quota should only be allowed to overrule the target level of buffer stocks, but not the target level of prices, then we would have a model in which quotas only appear as parameters in the demand function for stocks but not as restrictions on the model as a whole. This would produce a very simple structure of a national model with domestic price and buffer stock policy only. In such a model the first task of a buffer stock, demand stabilization would still be performed but not the second one, price stabilization.

If, on the other hand a component of stock demand should overrule price policy, this component should be taken as part of minimum stock demand and if it should also overrule quotas it should be treated as part of the balance of trade deficit. The hierarchical formulation is therefore more general than might appear at first sight. The model would however, gain in generality if a decoupling was made possible between the central price target which is realized through a system of tariffs/levies and the price targets supported by the buffer stock. One would let a price drop below or rise above the central target until certain bounds are reached. Within these bounds stocks would remain at their target level and only when the bounds are reached would the buffering start. This would represent a buffer stock policy with a price band.

General equilibrium models with tariffs are a standard tool of international trade theory (see e.g. Kemp (19) or Negishi (25) and the computation of equilibrium with tariffs has been studied by Shoven and Whalley (30)[2]. Quotas have received much less attention. The reason for this is probably the fact that in equilibrium the tariff equivalent of a quota and the quota equivalent of a tariff can easily be computed (see e.g. S. Bhagwati "on the equivalence of tariffs and quotas" in (4). In a model with one single utility maximizing consumer per nation, Dixon (8), Ginsburgh and Waelbroeck (13) and Takayama and Judge (34), introduce quota explicitly. Only Ginsburgh and Waelbroeck treat the implication of a combination of tariffs and quotas. The case with several income groups does not seem to have attracted much attention.

From an empirical point of view, tariffs and quotas have the same effect: a change in domestic price. But in a dynamic sense a quota is rather different from a tariff, especially under retaliation, see Rodriguez (29) and Fishelson, Flatters (12), Sweeney, Tower, Willet (32) and Ohta (25). Quotas and tariffs are often imposed on very specific commodities so that it may be very hard to measure them at an aggregate level. One often has to ascribe a margin between domestic and world market prices to tariffs and to quotas according to some prespecified rule. Quotas nevertheless permit to introduce goods with a limited tradability into the model. Due to infrastructural restrictions, import and export capacities are restricted in the short run. To reflect this, quotas can be introduced as "flexibility constraints" and serve as a useful calibration device for a simulation model. Because of the complexity of the price-quantity interaction tariffs cannot play this role so effectively

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[2] Within the context of the Scarf algorithm. In this approach domestic and international equilibrium are treated simultaneously so that the domestic equilibrium is not required to be unique.

Buffer stocks have, according to Turnovsky (36) mainly been studied in a partial equilibrium context and not in a multicommodity general equilibrium framework as in Chapter II, para 2 and 4. We observe that the term "stock" can be replaced by "demand" because we do not need, in the existence proof, the property that demand is carried over to the next period. The combination of quotas and buffer stocks is of special interest because it becomes possible to describe the behaviour of marketing boards, buying up surplusses on the domestic market and selling on the world market, according to some perceived relation between exports and world prices, in an attempt to maximize net foreign exchange receipts.

8. *POLICY ADJUSTMENT FUNCTIONS*

8.1 *Domestic targets as functions of world market prices and trade deficits*

In Chapter II, para 3 target adjustment functions were introduced in the open exchange model, the targets being functions of world market prices and trade deficits. Price targets were required to be generated by homothetic functions and quantity targets by functions which were homogeneous of degree zero. These restrictions were imposed in order to obtain net import functions which are homogeneous of degree zero in world market prices. We give two examples of such functions.

Let $o^w \in R+$ be an index of world market prices

$$o^w_t := o^w(p^w_t) \tag{8.1}$$

which is strictly positive for all $p^w_t \in P^w$ and homogeneous of degree one in p^w_t. The simplest example of such a function would be $o^w_t := \sum_i p^w_{it}$, but more generally we have:

$$o^w_t := \sum_{i=1}^{n} \bar{g}_i \, p^w_{it} \tag{8.2}$$

This index may now be used to specify target adjustment functions. Homogeneity of degree zero is then obtained by expressing all prices in terms of the index or by multiplying all fixed price targets by this index, for example:

$$\bar{p}_{it} := (1 + \tau_i)p^w_{it} \qquad i = 1, \ldots, h \qquad \text{(tariff)} \qquad (8.3)$$

$$\bar{p}_{it} := o^w_t \cdot \bar{\bar{p}}_{it} \qquad i = h+1, \ldots, n \qquad \text{(levy)} \qquad (8.4)$$

this yields the same net import as:

$$\bar{p}_{it} := (1 + \tau_i)\ (p^w_{it} / o^w_t) \qquad i = 1, \ldots, h \qquad (8.5)$$

$$\bar{p}_{it} := \bar{\bar{p}}_{it} \qquad i = h+1, \ldots, n \qquad (8.6)$$

Equation system (8.3), (8.4) is homogeneous of degree one, while (8.5), (8.6) is homogeneous of degree zero.

8.2 *Domestic targets as functions of domestic prices*

In the open exchange model (Chapter II, para 2) both price and quantity targets are functions of world market prices only. One can imagine however, policy targets to be also a function of domestic prices,(more generally of the vector (ρ_t, μ_t, ν_t). For the quantity targets (quota and buffer stocks) no problem arises as long as the target adjustment functions are continuous, bounded for all world market prices and domestic prices , homogeneous of degree zero both in p^w_t and in (ρ_t, μ_t, ν_t) and satisfy the constraints imposed on them. It is more difficult but also more interesting to investigate the consequences of taking price targets as functions of price realizations.

If, however, the price adjustment function is not adequately specified, it is possible to obtain solutions with several degrees of freedom; for example, if one chooses $\bar{p}_t = p_t$ as a specification, all possible price targets $\bar{p}_t(\bar{p}_t \in P)$ are in the equilibrium set. It would be interesting if one could use this degree of underdeterminateness of the model in order to specify new constraints, e.g. policies across markets, specifying for example that the price target on market i should be adjusted in order to keep the price on market j at target level when quota on market j are effective and buffer stocks are depleted. This would imply complementarity conditions of the form:

$$\mu_{kt}(w_{it} - \hat{w}_{it}) = 0$$

Such restrictions fall outside the scope of our present methodology and further research is needed in this direction, but the specific case will be handled now in which all price targets are tied to one index of current prices $o(p)$, $o(p) \in R^{1}_{++}$.

8.3 *Alternative normalization rules*

The discussion will proceed in terms of the open exchange model but applies to the international models as well. Throughout Chapter II prices have been constrained to the "simplex" $P: = \{p \in R^{n}_{+} | \; ||p||_{1} = 1\}$. From an economic point of view this seems to yield a highly unrealistic unit of account. We therefore wish to consider a more general normalization rule and unit of account.

Let $o(p_t) = \bar{d}_t$ be this normalization rule (and $o(p_t)/\bar{d}_t$ the unit of account), and let

$$\tilde{P}\,(\bar{d}_t): = \{p_t \in R^{n}_{+} | \; o(p_t) = \bar{d}_t\}$$

be the price set ,
where

(i) $\bar{d}_t$ is strictly positive,

(ii) $o(p_t)$ is homogeneous of degree one in p_t,

(iii) for all $p_t > 0$; $o(p_t) > 0$

Let the price targets be formulated as functions homogeneous of degree one in $o(p_t)$, then for $o(p_t) = \bar{d}_t$, $\bar{p}_t$ is fixed.

We can then proceed with existence proof and computation keeping price realization constrained to the "old" normalization ($p_t \in P$). The existence proof is unaltered since we only need the property that the price target should be bounded and positive. We owe this flexibility to the introduction of the scaling factor on price target (ρ_t). Let p^{*}_{t} be the equilibrium price. We know that $p^{*}_{t} > 0$ and thus $o(p^{*}_{t}) > 0$ (by iii) and can renormalize from $p^{*}_{t} \in P$ to $p^{**}_{t} \in \tilde{P}(\bar{d}_t)$ by dividing

$$p^{*}_{t} \; , \; \rho^{*}_{t}, \; \mu^{*}_{t}, \; \nu^{*}_{t} \text{ by } o(p^{*}_{t})^{t}/\bar{d}_t$$

We observe that, when quantity targets are functions (homogeneous degree zero) of the new unit of account, $(o(p_t)/\bar{d}_t)$, one can again set $o(p_t) = \bar{d}_t$, calculate the level of the targets and proceed with the existence proof (and with the algorithm) but the approach works only as long as all targets are functions of the same unit of account.

9. *FIXED PRICE EQUILIBRIUM IN THE OPEN EXCHANGE MODEL*

In recent years several authors have described allocations of resources in an economy where prices are fixed at a value which may not achieve equilibrium between supply and demand in the classical sense. Barro and Grossman (2), Drèze (9), Malinvaud (21) have designed such models, especially for macroeconomic applications, e.g. description of Keynesian unemployment, functioning of centrally planned economies, inflation. The models are based on the principle that quantity constraints (rationing schemes) are imposed on the individual actors. The issue under study is whether a given rationing scheme can generate an equilibrium at an arbitrarily fixed price level or within bounds on that level. We have seen that the open exchange model also describes price rigidities. But instead of rationing, net import and buffer stock adjustment generate a price rigidity. The degree of generality of the model is enhanced by the fact that limits can be imposed on the quantity adjustment so that it only prevails over price adjustment within a certain range.

We summarize the main differences between the open exchange model and the equilibrium models with quantity rationing, as described by Grandmont, Laroque, Younes (14), as follows:

Open exchange model	*Quantity rationing model*
- Individual actors only receive prices and taxes as signals.	- Individual actors receive rations and fixed prices as signals.
- Price rigidity is effectuated through an adjustment of net demand, either from the international market or from the buffer stock.	- The price rigidity is implemented through an adjustment of rations.

- Both price and quantity adjustment.	- Quantity adjustment only.

The open exchange model can also be interpreted along macro economic lines. Suppose that there are three goods: first, money, is taken to be an untradable good for which the central bank pursues a buffer stock policy. Its target price is one and the normalization rule is expressed as a price index: the central bank tries to maintain the purchasing power of money at a stable level. Second, labour is also an untradable good. In terms of the price index, wages are rigid; the rigidity is implemented by an unemployment scheme which buys up labour at fixed (indexed) price. Third, a price and a demand stabilization policy are implemented in the commodity market. The questions to be asked might be "what would be the consequences of cutting into the unemployment scheme, of a tight money policy, etc?". The model can in principle answer this type of question. A more difficult and yet unsolved question would be "Which price target should the central bank pursue in order to reduce unemployment without reducing real wages?". This is a question "across markets"; its answer poses the problems referred to in para 8.3 above. The international model can be given analogous macro economic interpretation.

The importance of buffer stocks as a demand stabilization tool has already been stressed in 1946 by J.M. Keynes (20), who wrote: "Superimposed on the meaningless short period swings affecting particular commodities and particular groups of producers, there is the fundamental malady of the trade cycle. Fortunately the same technique of buffer stocks, which has to be called into being to deal with the former problem, is also capable of making a large contribution to the trade cycle itself".

10. *UNIQUENESS OF DOMESTIC EQUILIBRIUM*

The desire to obtain the capability of individually solving national models and to keep the dimensions of the international model as low as possible, has led us to impose the uniqueness requirement. As explained in Arrow-Hahn, Chapter 9 (1), there is no prior reason why a general equilibrium model should possess only one single equilibrium. On the other hand multiplicity of solutions points to the fact that the model is not fully specified because it does not explain the choice between solutions.

We now turn to the assumptions which are specific to the international model.

11. *ONE INTERNATIONAL MARKET*

There is only one international market in the model; transportation costs are disregarded as well as discrimination. This simplification is introduced for the sake of computational convenience. If a trade matrix had to be generated we would in principle have to find the terms at which each nation trades with each other nation in each good. In a partial equilibrium setting these problems have been handled by Takayama and Judge (34) with quadratic programming algorithm; in a general equilibrium setting this is an immense task however, although not fundamentally impossible. FAO data (10) suggest that international transportation costs (the US$ margin between export value including cost-insurance-freight" and "free on board" import values) amounts to 8-9% of total import value for agricultural commodities in the period 1971-1976 and less than 6% for overall trade.

On the basis of these considerations it was decided to disregard the matrix of international trade. The margins between import prices and export prices as well as the differences between countries in unit values on import and export, can be handled by treating "transportation" as a complementary demand/supply for nonagricultural services.

12. *NO INTERNATIONAL CAPITAL MARKET*

International prices are expressed in an international unit of account. Countries can provide one another with amounts of units of account which are not the payment for goods and services, this is the deficit/surplus on the balance of trade. The international system is closed, so the deficits have to add up to zero. The deficit plays at the international level the role of the income transfer at the national level (cf. para 5 above). Again all objections against the dichotomy assumptions can be raised here.

13. *INTERNATIONAL AGREEMENTS*

13.1 *Internal vs. external agreements*

We distinguish between internal and external agreements. In an external agreement a group of countries agrees to influence the state of the rest of the world while in an internal agreement targets are formulated which do not directly affect other countries although there may be an indirect impact. A cartel typically is an external agreement while a customs union or a bilateral trade agreement are internal agreements. Internal agreements can be modelled without changing the basic structure of the competitive model. The countries with the agreement can be seen as a group which operates as a unit on the world market, facing world market prices and balance of trade restrictions just as a country does. In external agreements the countries making the agreement explicitly try to influence the value of the parameters they face from the outside i.c. the world market prices. We are not in a position to endogenously generate international agreements and the applicability of possible agreements is hard to assess. The main purpose in modelling them lies in testing their consistency and evaluating their consequences.

13.2 *Tests for an international agreement*

A new international agreement has its origin in an idea, an abstraction. Modelling can be a helpful tool in developing this idea. Once a mathematical model of the agreement has been formulated, one can perform tests on it.

We distinguish:

(i) tests on the logical consistency of the agreement.

(ii) tests on the explicitness of the agreement.

Ad (i): logical consistency

(1) Internal logical consistency.

Here the consistency of targets and constraints must be evaluated. Participants should not be unwilling to carry the financial consequences of the agreement and targets should not be conflicting (if they do not stand in a hierarchical relation to each other). The conditions on the parameters of the models in Chapter II, para 4, 5 mainly reflect this.

(2) Consistency with a model of the real world.

An agreement can be internally consistent but inconsistent with a given, consistent model of the real world. This can obviously be due to an inadequacy of the real world model but if that model is thought to be realistic, such an inconsistency points at a theoretical weakness of the agreement itself. The existence proof checks whether there are such weaknesses. The main test is that t actions to be taken by the actors in the model should be defined for all relevant[4] states of the world (the functions should be defined).

Ad (ii): explicitness of an agreement

The fact that an agreement has passed consistency tests does not imply that its consequences are clear. It is the modeller's task to clarify these consequences by making them explicit.

(1) Explicitness of direct consequences.

When an agreement has direct logical consequences, which are not expressed in the agreement itself, again analysis can help. We take the buffer stock agreement as an example: if only target prices and limits on stocks are introduced and no financial commitments, the agreement is consistent but insufficiently specified. By substituting the complementarity conditions within the budget equations, one can however derive the explicit specification of both the a priori commitment and the effective commitment. Any other specification would lead to inconsistencies. The analysis can thus also serve as a tool to investigate direct logical consequences of an agreement in order to make them an explicit part of the agreement itself.

(2) Uniqueness of the consequences.

In the previous example the financial consequences could be made explicit because the demand for buffer stock was related to the state of the world in a one to one fashion (net increase in buffer stocks = surplus on world

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[4] Obviously for states of the world which are irrelevant (= cannot occur) the modeller is free to formulate artificial actions (such as free disposal) if this simplifies his proofs.

market). If the agreement is not so elaborated (i.e. if to one surplus on the market corresponds a whole range of net increases in stocks) a consistent solution can exist but the agreement itself is inadequately specified. Here analysis cannot help unveiling the implicit consequences, but the modeller can report that the agreement does not yet describe a concrete course of action and explain which degrees of freedom the model still possesses. Here the analysis of uniqueness of the solution is the relevant issue.

(3) Explicitness of indirect consequences.

Although theory can provide interesting insights, it is felt that the general problem of investigating consequences of international agreements on the world economy can only be handled through the development of fully specified numerical models. The outcome of such models strongly depends on the numerical values and functional specifications which are assumed, so that only very few lessons can be drawn from general theoretical models. One needs to run the model on the computer and evaluate the results. The theoretical models must therefore be solvable numerically.

13.3 *Internal agreements*

In Chapter II no attention was given to internal international agreements. The reason for this is that for internal agreements the open exchange model (Chapter II, para 3) more or less applies with the participating nations as the basic actors. The problem for such an interpretation of the model lies in the first place in the monotonicity requirement imposed on the actors ($\frac{\Delta z}{\Delta k} \geqq 0$). The second problem, which is more severe, is that we require internal equilibrium to be unique. We have seen that this is the case as long as no buffer stock reaches its bound but in general we can only establish uniqueness in the absence of quota. The open exchange model has shown how a common price, quota and buffer stock policy can be modelled as an internal agreement. Two kinds of internal agreements still have to be discussed: a trade agreement and a scheme of compensatory finance.

- In an economy with lagged production we can look at a *trade agreement* as an international redistribution of endowments before exchange. By itself such an agreement therefore has the same effect as an international income transfer, indexed, with quantities traded as weights. There in practice often exists a relation between quotas and trade agreements: a country which imports a commodity under a trade agreement usually is not willing to let that commodity be reexported and vice versa for the exporter. These two effects must be distinguished

A trade agreement will have as its first consequence a higher degree of pseudo-autarky: after the redistribution of endowments and at "normal" world market prices the net demands of the nations will be smaller, so that the income effect of a change in competitive world market prices could be smaller in absolute terms. We shall see in the appendix to this chapter that the sign of such an effect is unclear (depending on price policy, quotas, endowments of the income groups in the nation, etc.), it therefore is unclear what the overall effect on the competitive market will be. It however, seems to be a plausible conjecture that a trade agreement destabilizes the competitive market (in the sense that the effect on equilibrium prices of shift in supply increases) if the exporter is less price sensitive than the importer and vice versa, the argument being that an exporter has a perverse wealth effect (he gets richer as price goes up and might therefore have a higher demand for his own product). It is rather easy to build counter examples for this conjecture and only empirically based models can give answers.

The second effect of the trade agreement, the imposition of quotas, is clearly a decrease of the volume of goods tradable on the competitive market. Especially when overall supply is subject to fluctuations, and the trade agreement is rigid, the agreement will increase fluctuations on the competitive market. Under monopoly the outcome could be quite different. Consider the buffer stocks agency as such a monopolist. Here a trade agreement could see to it that initial stocks are sufficient for the realization of price targets and thus for price stabilization. In summary we can state that trade agreements can easily be implemented within the models of Chapter II but that the effects of such agreements are not easy to predict on theoretical grounds only.

- A *scheme of compensatory financing* is an arrangement by which countries agree to let international income transfers compensate fluctuations in export earnings. The International Monetary Fund operates such a scheme (compensatory financing facility (CFF). The European Community also operates such a scheme as part of the Lomé Agreement (STABEX)[5]. One might think of it as an explicit specification of the transfer distribution function, (II.4.6 e.g.

$$k_h := \frac{\delta_h(\hat{p}^w \cdot \hat{y}^h_{-1}) + (1-\delta_h)(p^w . \bar{y}^h_{-1})/o(p^w)}{\sum_k (\delta_k(\hat{p}^w . \hat{y}^k_{-1}) + (1-\delta_k)(p^w . \bar{y}^k_{-1}/o(p^w)} \cdot p^w . (\sum_k \bar{y}^k_{-1}) - p^w . \bar{y}^h_{-1}$$

where

k_h	compensatory financing transfer to nation h		- $(k_h \in R)$
$\hat{p}^w$	target (possibly average)	world price	$(\bar{p}^w \in R^n_+)$
p^w	actual	world price	$(p^w \in R^n_+)$
$\hat{y}^h_{-1}$	target (possibly average)	supply, nation h	$(\bar{y}^h \in R^n_{++})$
$\bar{y}^h_{-1}$	actual supply, nation h		$(\bar{y}^h \in R^n_{++})$

δ_h = 1 for countries supported by the scheme, and 0 for countries supporting it[6].

$o(p^w)$ price index.

This scheme stabilizes the purchasing power of total supplies, not of exports. This seems more rational because exports themselves are depending on the transfer. Obviously the effect of such a scheme on equilibrium prices cannot be predicted theoretically. For a discussion of the strengths and weaknesses of the IMF's scheme, see Junz and Mc Avoy ( ).

13.4 *Buffer stock agreements and the New International Economic Order*

Schemes of compensatory financing do not aim at changing world market prices. National governments receive the compensation and it depends on

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[5] The IMF scheme applies to all countries and to the balance of trade in general while the EC scheme only applies to 12 commodities and 16 countries (see Stakhovitch (31).

[6] The distinction supporting-supported does not imply that the sign of k_h should b

the national policy whether or not individual producer and consumer groups obtain any compensation. It has the character of an aid to the nation and the country giving aid can easily discriminate between receivers of aid. Developing countries raise objections to such aid schemes and demand price policies on international markets. In the 1960s the "no aid but trade" position gained wide support especially under the influence of Prebisch's view that there is a secular decline in terms of trade for primary commodity exporters (= the developing countries) (28, 16). The emphasis on trade naturally led to concern for export earnings, import payments and prices faced by the developing nations. Until 1973 trade liberalization on the part of the developed countries (preferential treatment), and compensatory financing schemes were the main concerns. After the "oil crisis" and the increase in agricultural prices in 1972-1973 the emphasis has shifted to the discussion on the "New International Economic Order". See for example U.N. World Food Conference 1974 (37), P.H. Trezise (35), C. Michalopoulos, L.L. Perez (22), and D.L. McNicol (23). From this emerged the so-called Integrated Programme put forward by the developing nations at the UNCTAD IV conference in 1976. The central feature of this programme is the establishment of buffer stocks for 18 core commodities spanning the main exports of primary commodity producers. Agreements for separate commodities would have to be coordinated financially by a Common Fund[7]. The developed nations are not enthousiastic about this proposal (cf. Junz and McAvoy (17). The experience with commodity agreements has been rather disappointing (see Johnson (16)) and the introduction of an integrated commodity agreement implies that a monopolistic force would be given to the nations controlling it, the power of which could eventually have an important influence in international affairs. We shall presently return to this issue. A buffer stock scheme as is being proposed, cannot keep lon term prices far away from an equilibrium level because stocks would either get overfilled or depleted. But price stabilization itself can have favourable effects even when it is around a "secular equilibrium level". It can be argued that prices reach income groups within a nation more easily than aid flows do and price stabilization through buffer stock operations also has a stabilizing effect on aggregate demand (as was discussed in para 12 above).

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[7] IMF already operates such a buffer stock scheme.

The market segmentation agreement could supplement the buffer stock agreement by absorbing structural surpluses and deficits on commodity markets.
It has not been our aim to pass any judgement on the desirability of specific international agreements, only to sketch a background for the international models listed in Chapter II.

13.5 *Monopolistic interpretation of an international (external) agreement*

Alternative interpretations can be given to the external agreements. Instead of visualizing a dialogue between developed and developing nations which would finally result in an integrated scheme, one can also think of a cartel being set up unilaterally (as was the case with OPEC). The market segmentation model (Chapter II, para 6) seems especially suited for representing this. The model, however, has the limitation that it lacks the capability of representing competing cartels. The buffer stock model allows to consider independent cartels for different groups of commodities simply by having the transfers distributed according to membership of the cartel. It can therefore describe cooperation as well as confrontation between cartels (e.g. the use of a food agreement against an oil agreement).

13.6 *Structural interpretation of the agreement*

A third interpretation of the agreement is to assume that it does not represent the explicit will of the participants but forms a structural characteristic of the market: it is the "invisible" hand which acts as if there would exist an agreement. This interpretation turns the specification of the structure of the market into an empirical problem. This is a very hard task especially because the econometrics of models with complementarity conditions and unequality constraints are not yet well-established. Some steps in this direction have been taken e.g. by Fair and Jaffee (11)and Hartley and Mullela (18) but only in a partial equilibrium framework.

In our system national models can be formulated, estimated and calibrated independently for given time series of international prices and deficits on the balance of trade. Once the national models are fully specified, the exogenous prices become endogenous through the international market (and therefore stochastically disturbed). The competitive model does not leave any scope for calibration at this level because the commodity balance does not contain any unknown coefficients. The buffer stock model permits some calibration since adjustment functions can be specified for the parameters (price targets, commitments, limits on stocks). Likewise, the compensatory financing scheme also has a structural interpretation as an in-built stabilizer but here more serious econometric estimation is possible because the functions involved can be made continuously differentiable and thereby suitable for maximum likehood estimation.

Appendix

NOTATION AND LIST OF VARIABLES

Notation

R^n	n-dimensional real vector space
R^n_+	nonnegative n-dimensional real vector space
R^n_{++}	positive n-dimensional real vector space
t	subscript for time period
a.b	scalar product: $\Sigma_i a_i b_i$
$\bar{\sigma}(a,b)$	$: (c \in R^n \mid \forall_i : c_i = \max (a_i, b_i))$
$\underline{\sigma}(a,b)$	$:= (c \in R^n \mid \forall_i : c_i = \min (a_i, b_i))$
$\mathbb{P}(R^n)$ $\mathbb{P}(R^n_+)$ $\mathbb{P}(R^n_{++})$	power set in R^n R^n_+ R^n_{++}
$\|\|a\|\|_k$	$:= (\Sigma_i \|a_i^k\|^{1/k})$, $k = 1, 2, \ldots$
:=	"is equal by assignment to"
=	"is equal to"
ι	vector all components equal 1

SUBSCRIPTS, SUPERSCRIPTS		RANGE
i	commodity	$i = 1, \ldots, n$
j	income group	$j = 1, \ldots, m$
h	nation	$h = 1, \ldots, l$, or $h \in H^w$
w	international economy	$w = I, II, \ldots$
k	(depending on context)	-
d^w	subscript of slack country	-

VARIABLE	MEANING	DIMENSION
b	direct tax by income group	R^1
f	total direct tax	R^1
g	savings	R^m
k	trade deficit	R^1
m	income of income group	R^1_+
o	price index	R^1_+
p	(domestic) price	R^n_+
p^w	international price	R^n_+
$\bar{p}$	domestic target price	R^n_{++}
$\bar{p}^w$	(upper bound on) international target price	R^n_{++}
$\underline{p}^w$	lower bound on international target price	R^n_{++}
$\tilde{p}^w$	international target price	R^n_+
s	domestic free disposal	R^n_+
s^w	international free disposal	R^n_+
u	level international stock	R^n_+
$\underline{u}$	lower bound on international stock	R^n_+
$\hat{u}$	target level on international stock	R^n_+
$\bar{u}$	upper bound on international stock	R^n_+
u^+	upward deviation of stock from target	R^n_+
u^-	downward deviation of stock from target	R^n_+
w	domestic buffer stock	R^n_+
$\underline{w}$	lower bound on domestic buffer stock	R^n_+
$\hat{w}$	target level on domestic buffer stock	R^n_+
$\bar{w}$	upper bound on domestic buffer stock	R^n_+

y	supply by income group	R^n_{++}
$\bar{y}$	supply by nation	R^n_{++}
x	demand by income group	R^n_+
z	net import by nation	R^n
$\underline{z}$	lower bound on net import	R^n
$\bar{z}$	upper bound on net import	R^n
z^w	net import by economy w	R^n
$\underline{z}^w$	lower bound on net import by economy w	R^n
$\bar{z}^w$	upper bound on net import by economy w	R^n
γ	satiation weight for demand	R^n_+
μ	upward deviation of domestic price from target	R^n_+
μ^w	upward deviation of international price from target	R^n_+
ν	downward deviation of domestic price from target	R^n_+
ν^w	downward deviation of international price target	R^n_+
ρ	scaling factor on domestic price target	R^1_+
ρ^w	scaling factor on international price target	R^1_+
ω	satiation level for demand	R^1_+

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