Working Paper

ROBUSTNESS CRITERION FOR PLANNING
WATER SUPPLY/DEMAND SYSTEMS

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PREFACE

Water resource systems have been an important part of resources and environment related research at IIASA since its inception. As demands for water increase relative to supply, the intensity and efficiency of water resources management must be developed further. This in turn requires an increase in the degree of detail and sophistication of the analysis, including economic, social and environmental evaluation of water resources development alternatives aided by application of mathematical modelling techniques, to generate inputs for planning, design, and operational decisions.

During the year of 1978 it was decided that parallel to the continuation of demand studies, an attempt would be made to integrate the results of our studies on water demands with water supply considerations. This new task was named "Regional Water Management (Task 1, Resources and Environment Area)".

This paper is concerned with the robustness of the integrated water supply/water demand systems which is defined as the system ability to perform under different future events than originally expected at a relatively small incremental costs. It is shown how the robustness criterion may be used (in addition to the cost-effectiveness criterion) for screening long lead-time investment alternatives.

Janusz Kindler
Task Leader

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Tsuyoshi Hashimoto

1. PROBLEM DEFINITION

Suppose we have successfully developed a water demand relationship,

$$Y = f(\underline{X}, \xi) \quad , \tag{1}$$

where $\underline{X}$ is a vector of explanatory variables and ξ is a random variable. A major problem that remains is what action is to be taken now to cope with future demand which can be forecasted by (1). It is usually the case that in addition to the random factor ξ variabilities are also involved in $\underline{X}$ itself due to the future policy changes, economic and technologic uncertainties, social changes, etc.

Let Z represent a variant of future events. That is, Z may be a value (or a range of values) of an important parameter, or it may represent an alternative development scenario, either qualitatively or quantitatively specified. In the latter case, Z, may consist of values of a few (or more) factors which characterize future outcomes.

It is assumed that effects of randomness, represented by ξ, on Y are minor as compared with uncertainties involved in Z. The explanatory variables $\underline{X}$ are functions of the future event Z and thus uncertainty is involved in prediction of future water demand Y.

The following notations are introduced:

Let $a_1, a_2, \ldots, a_r$ be alternative initial actions available. Each a_i may be either structural or non-structural measures or combination thereof; e.g. it may represent developing a new major water source-like inter-basin transfer, or imposing stringent wastewater effluent standards to encourage recycling so that water demand will be suppressed.

Let $Z_1, Z_2, \ldots, Z_k$ denote variants of future events as described above.

Define the following costs:

K_i = initial costs associated with alternative a_i,

C_{ij} = additional costs which will be incurred by taking action a_i, while actual outcome is Z_j,

$SC_{ij} = K_i + C_{ij}$ = total costs for (a_i, Z_j) (2)

$LC_j = \min_i [K_i + C_{ij}]$ = costs of the most efficient alternative under the event Z_j. (3)

Assume both K_i and C_{ij} are positive (and thus so is LC_j). Note that the initial costs K_i can be defined without referring to a particular future conditions Z_j. Depending on the actual outcome Z_j, some additional costs C_{ij} are incurred. The additional costs C_{ij} may include costs of modifying the original action a_i and short-run losses due to changes in economic activities as well as costs involved in operating the system under the event Z_j. Once the total costs S_{ij} of the alternative a_i and the minimum costs LC_j under event Z_j are computed, the opportunity costs OC_{ij} are defined as

$$OC_{ij} = SC_{ij} - LC_j \qquad \forall\ i,j \quad . \tag{4}$$

The opportunity costs are the incremental costs associated with the alternative a_i of not planning cost effectively for this particular event Z_j.

2. ALTERNATIVE APPROACHES TO THE PROBLEM

A conventional approach to the problem of selecting an appropriate initial action is to specify a single variant of future events $Z = Z_j$ (as the planning conditions) and to take action a_i so that

$$K_i + C_{ij} \rightarrow \min_{a_i} \quad .$$

If the probability P_j of Z_j is specified, this single Z may be such a variant that has the maximum value of P_j or that is characterized by average values of factors describing future outcomes (symbolically $Z = \tilde{Z}$).

An alternative way is to use expected values and to take action a_i so that the expected total costs are minimized over all the possible actions:

$$K_i + \sum_j C_{ij} P_j \rightarrow \min_{a_i} \quad .$$

This approach is equivalent to minimizing the expected opportunity costs over all the possible actions as manifested below. That is,

$$K_i + \sum_j C_{ij} P_j = \sum_j (K_i + C_{ij}) \cdot P_j = \sum_j SC_{ij} \cdot P_j$$
$$= \sum_j OC_{ij} \cdot P_j + \sum_j LC_j \cdot P_j \quad ,$$

where the second summation does not depend on the alternative a_i.

Still another approach is provided by what is called the alternative predictions method (APM). An economic approach of APM described by Pawlowski (1978) is as follows. First various subsets A of $\{Z_i, \ldots, Z_k\}$ is specified. The minimum costs LC_A are defined on each subset A rather than on each event Z_j. Similarly, the opportunity costs OC_{Aj} are defined for all $Z_j \notin A$. Then, among all the "admissible" subsets of $\{Z_i, \ldots, Z_k\}$, construct an alternative prediction A so that

$$LC_A + \sum_{Z_j \notin A} OC_{Aj} \cdot P_j + \sum_{Z_k \varepsilon A} \; \sum_{Z_i \varepsilon A} OC_{ik} \cdot P_k \rightarrow \min_A \quad .$$

Here an admissible prediction is defined as one having probability of the prediction becoming true being greater than or equal to a predetermined number γ $(0 < \gamma < 1)$. Finally an action will be determined based on the constructed prediction A.

The most general way to deal with the problem of decision-making under uncertainty is to introduce a utility function which will order alternative outcomes according to decision-maker's preference of "risky" choice. In the present case, for instance, the utility of alternative a_i may be defined as a function of

K_i, C_{ij} and P_j. This approach, however, will not be treated here except for a few special cases.

3. CRITIQUES ON THE CONVENTIONAL METHODS AND THE APM

When choice of initial action is to be made under uncertainty involved in future outcomes, it is important to evaluate each alternative with respect to its performance under varying conditions rather than selecting the one which is optimal (cost-effective) under "design" conditions. This kind of consideration is particularly relevant in the case of water supply/demand systems, since water supply systems typically involve large-scale facilities with a long lead-time for design and construction, and any action of demand/supply integration has significant, long-lasting effects. It is also a well-documented fact that exclusive use of expected values to evaluate and screen alternatives which are subject to variabilities of various kinds can be quite misleading (see for example, Adams and Gemmell, 1975; Szidarovsky et al. 1976). Thus the conventional approaches are not very satisfactory. This point is illustrated by the example given later.

Use of alternative predictions method is a better approach to alternative evaluation and selection in that decisions on initial actions are made based on a broader range of information contained in alternative predictions A rather than on a single event Z. A few problems involved in APM, however, are noted. To define the costs LC_A, one-to-one correspondence is necessary between an alternative predictions A (a subset of future events $\{Z_i, \ldots, Z_k\}$) and the initial action based on it. That is, some screening procedure is already implicit in the definition of

LC_A. Also the APM does not tell how to define an initial action based on A. This two-step procedure may obscure information on alternative options available in the screening processes.

Some more general points related to the problem of screening alternative actions will be noted to motivate search for other possible methodologies.

In many cases of alternative evaluation and screening, some (if not all) of initial actions are composites of structural and non-structural measures; e.g. developing new water sources with appropriate pricing policies or with cost allocation arrangements. The probability P_j associated with each future outcome Z_j may very well be a function of initial actions. For instance, availability of water and pricing schemes will affect municipal growth, which, in turn, is one of determining factors of future water demand. Also the probability P_j most likely is evaluated more or less subjectively, as it will represent not only what is likely to occur but also which development path decision-makers prefer more to others. There is no such thing as expert's objective assessment.

Since, in general, both the probability P_j and costs vary among alternatives, some composite measures are required to allow comparisons among alternatives consisting of structural and non-structural measures. For a measure to be operational, it should also be defined in such a way that reflects behavioural characteristics of decision-makers and institutional arrangements. In particular, such a measure should stand for subjective evaluation of probabilities associated with different future outcomes and different risk behaviours.[1)]

4. ROBUSTNESS CRITERION

4.1. Concepts of Robustness

A concept of robustness has been adopted in water resources planning by Fiering from statistical literature (Fiering, 1976; Matalas and Fiering, 1976). In statistics, the robustness of a decision to accept or reject a particular hypothesis is high, if that decision would remain unchanged over a wide range of sample values or evidence on which the decision is to be based. As noted by Fiering (1976), however, a robustness measure should not simply be a physical quantity; economic considerations, i.e. costs involved in such a decision should be incorporated in it. In this respect, the concept of robustness is closely related to Stigler's concept of flexibility (Stigler, 1939), as discussed in the Appendix.

According to the above reasoning, the robustness criterion is defined for a water supply/demand system as its ability to cope with varying conditions Z_j at a relatively small increase in costs. Depending on initial action a_i and subsequent modifications and operation of the system, the costs $SC_{ij} = K_i + C_{ij}$ will be different among alternatives a_i and thus provide basis for definitions of robustness measures. The robustness criterion will supplement a conventional optimality criterion of cost-effectiveness in the processes of screening alternatives a_i.

[1)] In case of alternative predictions method, risk behaviours may be reflected in choice of γ, the admissible probability level of a prediction coming true.

4.2. Proposed Measures of Robustness

Measures are sought for representing variabilities or deviation of economic performance associated with each alternative a_i. Comparing costs C_{ij} under different events $\{Z_j\}$ within each alternative a_i, however, is not satisfactory, since reference points are different among alternatives. First define an opportunity cost ratio for alternative a_i under event Z_j:

$$ROC_{ij} = \frac{OC_{ij}}{LC_j} = \frac{SC_{ij} - LC_j}{LC_j} \quad . \tag{5}$$

Use of "the minimum" costs LC_j eliminates the arbitrariness discussed above.

Next, based on the correspondence between the event Z_j and the opportunity cost ratio ROC_{ij}, and on the prespecified distribution $\underline{P} = \{P_j\}$ associated with different events Z derived probability distribution $G_i(\ell)$ of the opportunity cost ratio ROC_i for the alternative a_i:

$$G_i(\ell) = Pr\{ROC_i < \ell\} \tag{6}$$

It is noted that in the range where the opportunity cost ratio is small, its exact value will not be much of a concern; the variabilities within that range may be in the same order as those resulting from the random factor ξ in (1). Also note that it is impossible to plan any system so that the opportunity cost is bounded for any extreme event. These observations suggest a method of comparing alternatives by some percentile of the opportunity cost ratio.

Based on the distribution $G_i(\ell)$ of the opportunity cost ratio, the following two measures of robustness are defined for alternative a_i:

$$R_\beta{}^i = G_i(\beta) \quad , \qquad 0 \leq R_\beta{}^i \leq 1 \tag{7}$$

$$R^i(\delta) = \frac{1}{1 + G_i{}^{-1}(\delta)} \quad , \qquad 0 \leq R^i(\delta) \leq 1 \tag{8}$$

where β and δ are prespecified values ($0 < \beta$, $0 < \delta < 1$), and $G_i{}^{-1}(\cdot)$ is the inverse function of the distribution function G_i. The value of β represents a maximum tolerable level of the opportunity cost ratio, ROC_i and δ is such a level of probability that ROC_i is less than or equal to $G_i{}^{-1}(\delta)$. Correspondence of these definitions is illustrated in Figure 1 for a continuous case. These definitions are referred to by saying that the alternative a_i has the <u>value</u> of robustness at the <u>level</u> β or δ equal to $R_\beta{}^i$ or $R^i(\delta)$. The measure R_β may be easier to evaluate based on the underlying probability P_j of variant Z_j, but when the future state space is very discrete (i.e. only a few variants are defined), it may not help very much to distinguish alternatives.

5. USES OF ROBUSTNESS

Having robustness as another criterion, alternative approaches of performance evaluation and screening of alternatives are conceivable. First it is possible to maximize $R^i(\delta)$ or $R^i{}_\beta$ over all the possible initial actions a_i. This is closely related to what is known as minimax criterion in game theory. In fact, from the specified distribution $\underline{P} = \{P_j\}$ associated with variants $\{Z_j\}$ of future outcomes, and from computed values of the

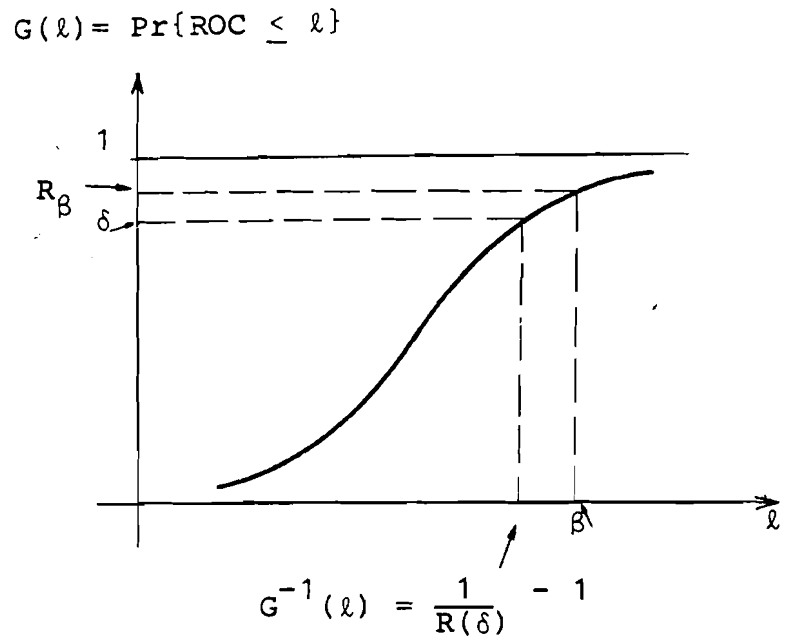

Figure 1. Correspondence between Two Measures of Robustness R_β and $R(\delta)$

opportunity costs ratio ROC_{ij} for the alternative a_i under the event Z_j, we can find, for instance, 100 δ percentile of ROC_i. Instead of minimizing the maximum regret, the robustness maximization allows minimizing a certain percentile of opportunity cost (or regret) ratio.

A more general approach, which is advocated here, is to consider trading-off between the conventional cost-effectiveness criterion (in the sense of minimizing expected total costs) and the robustness criterion. This is closely related to one of standard risk-theoretic approaches; viz. mean-variance trade-offs. Use of variance of costs (or benefits) involved in any decision vis-a-vis expected value of costs (or benefits) is one way to reflect "riskiness" into decision-making. Variance is one of composite measures of riskiness, as it is defined based on both uncertain outcomes measured in economic terms and perceived probabilities associated with them. General implications of choice of risk attributes and rules to aggregate them to give a risk measure, however, are rather deep, and beyond the scope of this paper (see Arrow , 1971; Schaefer, 1978). Our concerns here are limited to operational characteristics of the robustness measures. A few of the possible cases where the robustness measures may be relevant and useful are described below.

First consider the case where we are concerned with a relatively rare but potentially costly event. A potentially costly event means a variant Z_j of future outcomes for which the additional costs C_{ij} of some alternatives a_i are extremely high. This may be the case, for instance, if a serious water shortage is foreseen under some development scenario without implementing a large-scale development of new water sources.

Suppose there are two alternatives a_1 and a_2, which have opportunity costs for three possible variants Z_1, Z_2 and Z_3 of events as given by the matrix in Table 1. Also given are probabilities P_1, P_2 and P_3 of different variants occurring. The variant Z_3 represent a rare but potentially costly event. The expected values and variances of the opportunity cost (or regret) are computed and given for each alternative, together with the values of robustness $R(\delta)$ computed from equation (8).

Table 1. Regret Matrix for a Simple Example

Alter-native	Varaint of future event	z_1	z_2	z_3	Expected regret	Variance of regret	G^{-1} (.995)	Robust-ness R (.995)
	Probability	.50	.49	.01				
	Opportunity cost OC_{ij}							
a_1		10	10	50	10.4	15.84	.50	.67
a_2		15	5	5	10.1	24.99	.19	.84
	LC_j	80	80	100				

If the rare event Z_3 is ignored, both a_1 and a_2 have virtually the same expected regret (which is approximately 10), and the alternative a_1 appears better because its costs have smaller variance.[2] The alternative a_1 will be picked by the risk-theoretic (mean-variance trading-off) and a_2 by the game-theoretic (i.e. minimax) approaches. If the rare event Z_3 is included in the analysis, selection is not easy based on both the mean and the variance of the regret. Use of robustness in such a case as an alternative criterion reflects the belief that we are not much concerned about the exact values of opportunity costs as long as they are small and bounded by some acceptable fraction of the costs of alternative which is most effective under each event.

The second possible use of the robustness criterion is suggested by what is called safe-fail system, as compared with fail-safe system (Rogers et al. 1976). Some initial actions may have potentially high opportunity costs under some event in future, but the costs may be reduced by modifying and supplementing the initial actions as such an event is observed. Some alternatives

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[2] A small variance of total costs by itself does not imply a good design, since it just measures deviations around the mean. Robustness, on the other hand, is based on deviations from the least-cost which serves as a reference.

permit such modifications at less cost, while others do not. By using robustness as an alternative criterion, we may choose such an initial action that avoids extremely high opportunity costs under any event, even though it may result in higher (but tolerable) opportunity costs under most events. In this respect, it is worthwhile to emphasize that an alternative which is never cost-effective under any event Z_j (and thus would never be chosen by a conventional method, even if we parameterize on future conditions as variants Z_j's of events) may still be the most robust.

Another case where the robustness criterion may be useful is when the distribution $\underline{P}$ of future events is highly uncertain. In such a case, calculation of expected costs based on estimated distribution alone will not be very meaningful. Rather it is better to use some simple measure of the deviation of economic performance.

Also in some cases, decision-makers may want costs due to forecast errors to be bounded. This may be an important consideration for water projects, since they usually have long lead times, and planning conditions often change during that period. When multiple parties are involved in developing water sources, cost variability is one of the major factors that affect participation of each and thus viability of the project. In this case, a robust plan may provide a firmer basis for cost allocation or any other arrangement necessary to implement the project.

6. IMPLICATIONS TO DECISION-MAKING AND INSTITUTIONAL ASPECTS

As stated before, the probability P_j associated with each event Z_j cannot in most cases be evaluated objectively like e.g. expert's estimates. It represents not only what is likely to occur but also which variant of future outcomes decision-makers perceive more likely than the others. It may even reflect which development path decision-makers prefer to others. Occurrence of different events largely depends on national policies or other factors which are not controllable at a regional level of planning, but neither of them is completely uncontrollable. Therefore, when multiple parties are involved in the planning process, some kind of agreement on those alternative possibilities is a necessary prerequisite.

This procedure of articulating probabilities for variants of future outcomes is not only necessary but also a desirable step. It is likely that not all of the concerned agree on a single development path; rather it is more realistic to specify alternative paths as represented by variants $\{Z_j\}$ with associated probabilities $\{P_j\}$.

Another step necessary for the use of robustness criterion in alternative evaluation and selection is specification of level β or δ. Two possible ways are conceivable. One is to specify a value prior to generating information on economic performance of a set of alternatives, possibly at the same time as prior articulation of probabilities P_j . The other is to determine a value in a more or less *ad hoc* way as we evaluate values of robustness of the alternatives for which information of performance under different events Z_j has already been generated. Naturally the value of robustness $R(\delta)$ or R_β is different depending on the specified level δ or β, and so is the ordering by the robustness criterion. In the latter case, therefore, iterative procedures may be necessary to agree on the level β or δ. The specified level of β or δ reflects, to a certain extent, risk behaviours of decision-makers. If more emphasis is placed on rare but potentially very costly events, for instance, a higher value will be selected for δ, which represents the probability that the relative opportunity costs under different events are bounded by some value. How meaningful such a level is with respect to risk behaviours and which one of two possible ways mentioned above will capture this aspect better, are interesting questions, but beyond the scope of this paper. It is just noted from an operational point of view that, depending on particular cases or contexts of decision-making, either β or δ may be easier to specify, having better appeal to decision-makers.

Mention is made of levels of information utilization to clarify some more implications of the robustness criterion in decision-making processes. Given alternative initial actions available $\{a_i\}$ and variants $\{Z_j\}$ of future events with associated probabilities $\{P_j\}$, procedures of screening alternatives may be classified according to levels of information utilization. A classification is given in Table 2. One extreme is to pick a single variant without regard to probabilities and to evaluate economic performance of alternatives under this event. Another extreme is tabulation, i.e. all the alternatives with their performance are on display vis-a-vis all the possible future events. Decision-making is the easiest in the former case, and will probably be the most difficult in the latter case. Compromise must be sought between indecision due to too much unorganized information and loss of information due to screening and aggregation. A major question is how to extract useful information that can be used to make meaningful distinction among alternatives. Use of the robustness criterion may provide an answer to this.

Table 2. Alternative Screening Procedures

	Alternative approaches	Specified events	Use of probabilities P_j in specifying events?	in defining objective?	Remarks
1.	min $K_i + C_{ij}$	Z_j	no	no	single out an alternative
2a.	"	Z_j	yes: $P_j \to \max$	no	"
b.	"	$Z_j \doteq \bar{\bar{Z}}$	yes but implicit	no	"
3.	Minimax	$\{Z_1,\ldots,Z_k\}$	no	no	"
4.	min $K_i + \sum_j C_{ij} P_j$	"	no	yes but implicit	"
5a.	Alternative predictions method	"	yes	yes	partial screening
b.	Robustness-cost-effectiveness trading-off	"	yes	yes	"
6.	Tabulation	"	no	no	no screening

7. SIMPLE EXAMPLE

A simple example will illustrate possible use of the robustness criterion as compared with other methods. Table 3 provides all the information for the problem. In addition to the initial costs K_i for the alternative a_i and the additional costs C_{ij} of the alternative a_i under event Z_j, the minimum costs L_j under each event and the opportunity cost ratio ROC_{ij}, calculated according to the formulae (3) and (5), respectively, are also given in the matrix. Under the matrix are shown the probability distributions $\underline{P}$ of future events. The alternatives a_1 through a_4 do not significantly affect the probabilities, while the

Table 3. Costs and Probabilities Data for the Example Problem

Alternative initial actions

ROC_{ij} / C_{ij} (regret)	K_i	a_1 30	a_2 25	a_3 20	a_4 15	a_5 15	a_6 10	LC_j
z_1		1.72 19(31)	1.11 13(20)	0.72 11(13)	0 3(0)	0.60 15(12)	1.11 28(20)	18
z_2		1.05 15(23)	0.82 15(18)	0.55 14(12)	0 7(0)	0.36 15(8)	0.86 31(19)	22
z_3		0.29 10(9)	0.03 7(1)	0.06 13(2)	0.10 19(3)	0.16 21(5)	0 21(0)	31
z_4		0.06 4(2)	0.16 12(5)	0.25 20(8)	0.34 28(11)	0.66 38(21)	0 22 (0)	32
z_5		0 2(0)	0 7(0)	0.09 15(3)	0.56 35(18)	0.66 38(21)	0.41 35(13)	32

a_1–a_4: $\underline{p} = \begin{bmatrix} 0.05 \\ 0.20 \\ 0.35 \\ 0.25 \\ 0.15 \end{bmatrix}$ a_5: $\underline{p} = \begin{bmatrix} 0.20 \\ 0.40 \\ 0.25 \\ 0.15 \\ 0 \end{bmatrix}$ a_6: $\underline{p} = \begin{bmatrix} 0.05 \\ 0.30 \\ 0.30 \\ 0.30 \\ 0.05 \end{bmatrix}$

alternatives a_5 and a_6 do. The alternative a_1 may be a large-scale development of new water sources (e.g. inter-basin transfer) and the alternatives a_2, a_3 and a_4 may represent progressively smaller-scale developments (e.g. of local water sources). The alternatives a_5 and a_6 involve taking non-structural measures in addition to probably minor or stagewise development of new water sources.

Values of expected total costs EC_i and values of robustness $R^i(\delta)$ at level $\delta = .75$ are computed and given in Table 4, together with ordering of the alternatives by each criterion. The expected total costs do not vary much among the alternatives,

Table 4. Expected Total Costs EC and Robustness $R(\delta)$ of Alternatives

	a_1	a_2	a_3	a_4	a_5	a_6
EC_i	38.75	35.15	35.15	35.35	34.90	35.35
ordering	⑥	②	②	④	①	④
$R^i(.75)$	.599	.671	.714	.746	.625	.538
ordering	⑤	③	②	①	④	⑥
remarks	dominated	dominated	trade-offs	trade-offs	trade-offs	dominated

and it may not be easy to make a decision based on this criterion alone, except that probably the alternative a_1 will be excluded from further consideration. Introduction of the robustness criterion reveals three out of six alternatives are dominated with respect to these criteria by one or more alternatives.[3)] Those inferior alternatives being eliminated, trade-offs exist among the alternatives a_3, a_4 and a_5 as illustrated by Figure 2.

Other methods are also applied to see how results can be different. First note that there exists some ambiguity in specifying a single variant Z_j in the approach 2a shown in Table 2, since the probability distributions are different depending on alternatives. If the variant Z_3 is selected based on the distribution corresponding to the alternatives a_1 through a_4, the

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3) This dominance is a weaker concept than first-degree stochastic dominance (Whitmore and Findley, 1978).

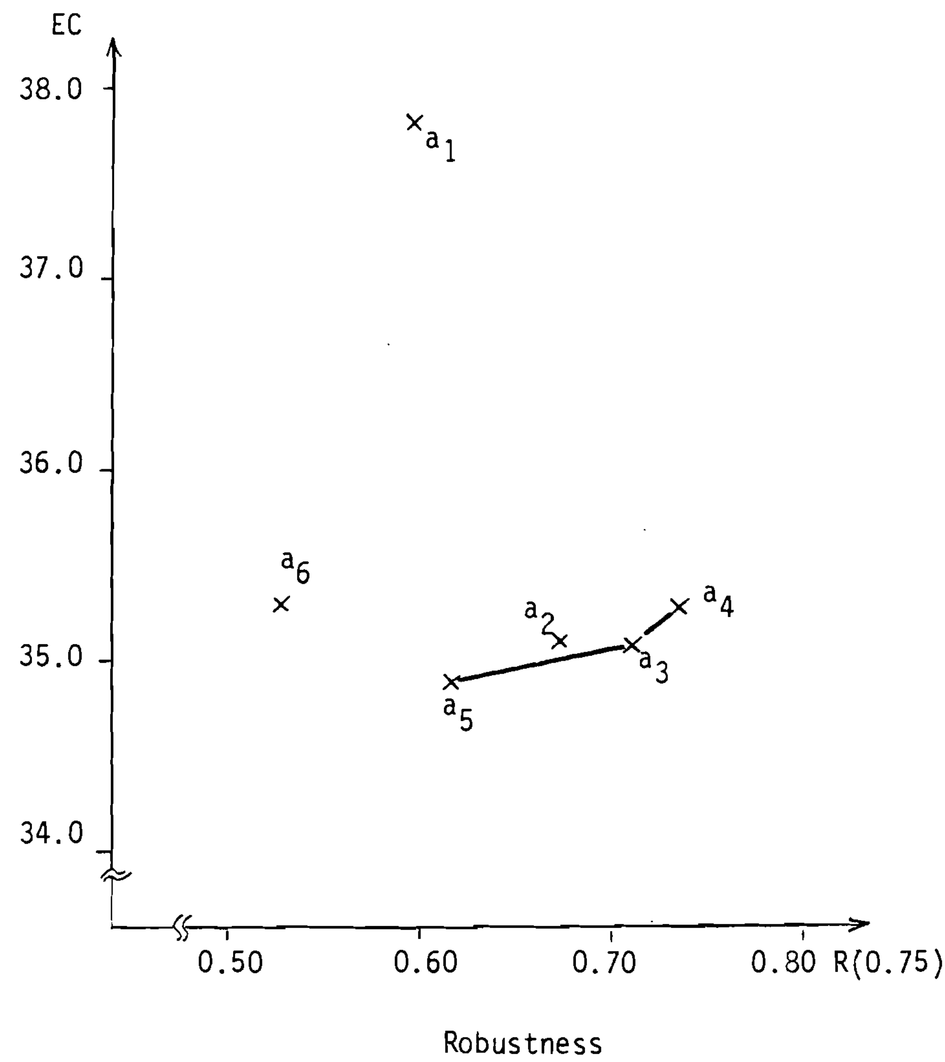

Figure 2. Total Cost-Robustness Trade-Offs

alternative a_6 is found to be the least-cost among all (a_2 among structural alternatives) under this event. Both of them are inferior solutions, if the robustness is taken into account, and all the other alternatives are near optimal. If the variant z_2 is selected instead, since it has the maximum probability when the alternative a_5 is implemented, the alternative a_4 becomes the least-cost. A conventional criterion of minimizing expected total costs leads to selection of the alternative a_5; minimization of maximum regret or of maximum relative regret dictates adoption of the alternative a_3 or a_4, respectively.

How to make a final selection based on the information presented in Table 4 and Figure 2 is a remaining question. Probably other non-economical criteria, e.g. environmental quality should also be introduced. Based on the economic criteria alone, however, the alternative a_3 which is never cost-effective under any particular event, may still be the final selection.

8. CONCLUSIONS

Some aspects of the problem of planning under uncertainty have been addressed with reference to water supply/demand systems planning. General relationships involved in the problem are

Z	→	$\underline{X}$	→	Y	→	Planning
variant of future event		explanatory variables		forecasted water demand		

Of course, a particular plan affects the realization of variants of future events.

A concept of robustness has been introduced to account for varying ability of alternative actions available at present to cope with different future events as represented by Z. Robustness of a water supply/demand system was defined as its ability to perform under different future events at relatively small incremental costs. A couple of measures of robustness were proposed, based on opportunity costs (or regret) under different events and probabilities associated with the events. These measures are relatively simple and easy to compute. Moreover they may have better intuitive appeal in many cases than, for instance, variance.

A new approach to the problem of screening alternatives involves using robustness as a criterion to supplement a conventional optimality criterion of cost-effectiveness. Possible cases to which this approach may be relevant and its implications to decision-making and institutional aspects were discussed.

Use and validity of this approach were illustrated by a simple numerical example and comparisons were also made between this and other more conventional approaches.

Possible applications of the robustness criterion are not limited to water resources planning. Actually the concept of robustness discussed in this paper is quite general and important to many problems which have to do with decision-making under uncertain future conditions.

In this paper, only a surface has been scratched of a potentially important and fruitful area of study. Many important issues around the subject are left for further work. No discussion was given on how the costs of each alternative under different events can be computed, how the different events are defined or how the probabilities associated with them can be evaluated. How the decision-making problem using robustness may be treated within formal analytical frameworks (e.g. two-stage process or Bayesian analysis) is a remaining question.

Also the ability of water resources systems to serve changing needs over time may be measured by other criteria. Search for these criteria and efforts to quantify them so that they can be used in planning processes should be continued.

APPENDIX

Flexibility and Robustness

One intuitive notion of flexibility dictates that a plan which will permit more options in the future is a more flexible one. This concept of flexibility based simply on availability of options has not been an operational tool for decision-making processes. Some options, though available, may be extremely costly to implement.

Stigler (1939) presented a cost-related concept of flexibility. He discussed a case of industrial plants which are subject to variations in demand for their products. He called a plant flexible, if it could produce a wide range of output quantities by incurring relatively small increase in cost, even though it may not have a minimum average cost for a certain output, e.g. target quantity, q_T (See Figure A1). This concept of flexibility is also relevant to describing performance of water resources systems which are subject to variable conditions, but yet not very operational by itself.

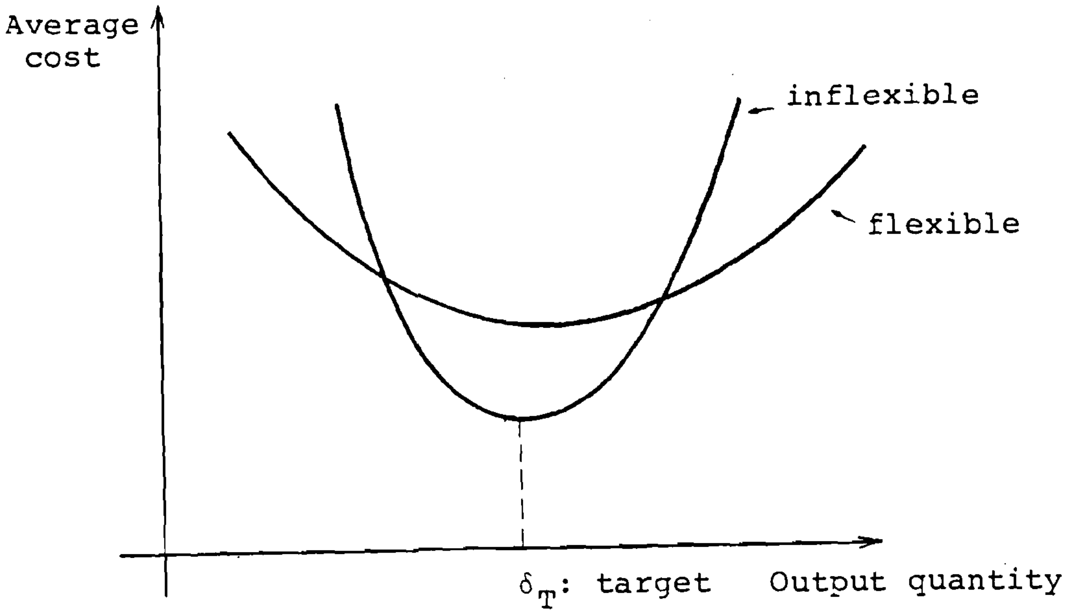

Figure A.1 Stigler's Concept of Flexibility

Marschak and Nelson (1962) presented a couple of measures of flexibility based on pay-offs (costs and/or revenue) of alternative decisions, and showed that each of them accords with the Stigler's concept. One measure is restated in our terms as follows.

An initial action a_1 is more flexible than an alternative a_2, if

(i) given any number $\theta > o$, there exists a future action a_1' such that $C(a_1'|a_2) - C(a_1'|a_2) > \theta$,

and if

(ii) there exists a number $\theta^* \geq o$, such that for all future actions a' $C(a'|a_1) - C(a'|a_2) \leq \theta^*$,

where $C(a'|a)$ is the cost associated with the future action a', given the initial action a. This definition is based on the unboundedness of cost difference as seen in (i) above. Such an action a_1', however, may never be taken under any event, or the event which calls for the action a_1' may occur only rarely. In other words, there may be the case where the initial action a_1 turns out to have lower costs by a large amount, but the probability of such a case occurring is extremely small. This implies that some probabilistic concept should be incorporated in operational measures.

If the boundedness of the payoff is assumed, on the other hand, the definition of flexibility based on the cost differences as given above may be modified as follows. If the maximum amount by which the initial action a_1 has a higher cost than a_2, is smaller than the maximum amount by which a_2 has a higher cost than a_1, then the alternative a_1 is more flexible. This flexibility criterion is equivalent to the minimax criterion applied to a two-alternative case. This definition may contradict the intuition that a flexible design is the one with payoffs more insensitive to different future conditions (i.e. a flatter cost curve). In the payoff matrix in Table A1, the initial action a_1 has a higher cost than a_2 by the amount 10 at most, and the initial action a_2 has a higher cost than a_1 by 15 at most. Thus the alternative a_1 is more flexible according to the above criterion, while the alternative a_2 has, in a sense, a flatter cost curve.

Insensitivity of payoff, however, may not be a desirable characteristic by itself. Consider the alternatives a_1 and a_2 having total cost curves as portrayed in Figure A2. The costs are given as functions of some parameter, q, whose exact value in future is unknown. The alternative a_1 is called more flexible than a_2 according to the Marschak and Nelson's measure. It may also be conjectured that the larger the uncertainty involved in

Table A1: Payoff (Cost) Matrix

		Future actions		
		a_1'	a_2'	a_3'
Alternative initial actions	a_1	10	20	0
	a_2	5	10	15

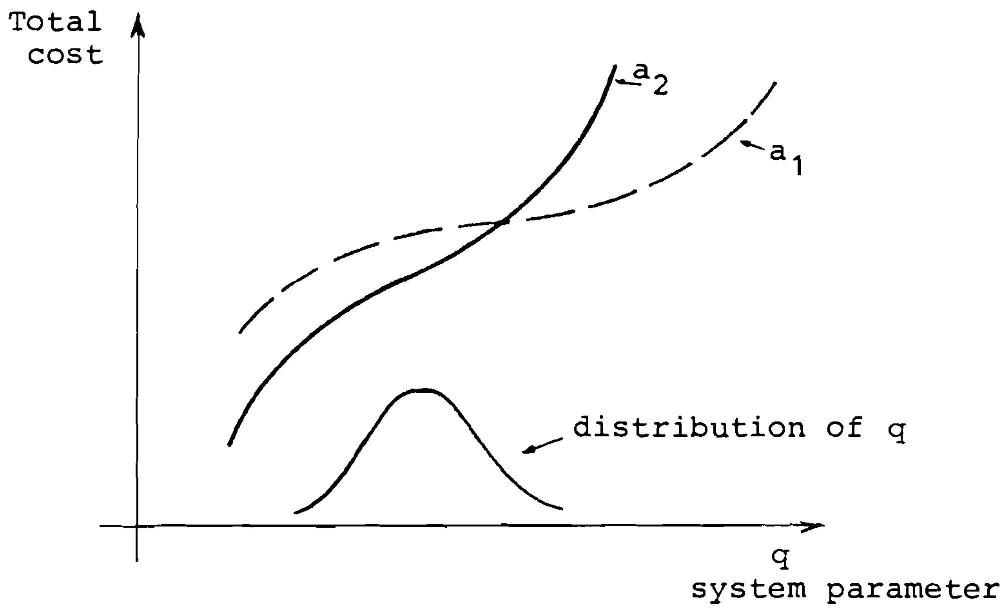

Figure A.2 Total Cost Curves for Alternative Designs

future events, the greater the advantage of the more flexible alternative. However, which alternative is found more desirable, given a level of uncertainty (or conversely, given an information level) is another question. If some analysis enables to specify the parameter with some distribution as illustrated in Figure A2, the alternative a_1 appears less desirable.

Given a specified distribution of parameters characterizing possible future conditions, another candidate for a measure of flexibility is defined. Intuitively an alternative which has a higher probability of being less costly may be called more flexible. This definition, however, is an equivalent of the "wider-range-of-options" measure of flexibility as presented above, and thus has the same deficiency. Amounts by which one alternative is less costly than another are not taken into account.

Our measures of robustness as presented in this paper are based on both cost-differences and a specified distribution associated with variable future events. The cost-differences are computed by using minimum total costs as references so that the concept of "regret" is duly represented. Rather than computing the expected regret based on the specified distribution, the measures of robustness are defined to account for variability in total costs.

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