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A QUANTITATIVE GENERAL EQUILIBRIUM MODEL OF THE SWEDISH ECONOMY

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PREFACE

A quantitative general equilibrium model is a useful tool primarily for two types of studies. One type is studies with a time horizon two or more decades into the future. In such studies, a model of this type can highlight various features of future economic conditions by delineating a number of resource allocations, each consistent with full equilibrium on product and factor markets. A second field of application is comparisons between actually observed resource allocations and hypothetical equilibrium allocations.

In this report a quantitative general equilibrium model of an open economy is developed and applied on Swedish data. In addition to the foreign trade flows, the model emphasizes the energy flows in the economy. The model is solved in a two-step procedure which is reiterated until a full equilibrium is reached. In the first step, technological coefficients and output prices are determined on the basis of initial values on factor prices. Then the excess demands on factor markets at product market equilibrium are determined. After that the initial factor prices are adjusted and the process repeated.

The report also contains some preliminary results, as well as a brief discussion of future directions of research in this field.

CONTENTS

INTRODUCTION *

Nobody would deny that the basic conditions for economic activity in an industrialized national economy change a great deal over a period of two or three decades. The accumulation of net investments adds substantially to the economy's stock of real capital. The set of available production technologies is likely to change and this also applies to the supply and demand conditions on international markets for goods and services. Moreover, it is possible that labor supply conditions will be quite different a couple of decades from now.

Much more difficult to foresee is how and why these basic economic conditions change, and what these changes will mean in terms of, for instance, the material standard of living, employment and energy consumption. Even if we regard the total capital stock, the technology and the labor force at some future point in time as given magnitudes, a wide range of alternative resource allocations are possible. In addition, we may want to look at these resource allocations from many different points of view.

Among the possible resource allocations it seems to be reasonable to focus on one where all commodity and factor markets are cleared, and where there are no excess profits or losses in the production system; i.e. a situation consistent with general economic equilibrium. This is not because such a situation is a very likely state of affairs; in fact it is probably a very unlikely situation in most industrialized economies. The reason is instead that a diseqilibrium will always produce forces, working through the market mechanism or through the formation of economic policy, which tend to change the allocation of resources in the economy. Thus, a resource allocation which does not satisfy the basic

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* Lars Bergman is responsible for Sections I, II, IV, V, and Andras Por for Section III.

equilibrium conditions mentioned above is a rather arbitrary description of the future state of an economy. An equilibrium allocation, on the other hand, is both feasible and can be maintained under unchanged exogenous conditions. For this reason, a general equilibrium type of model seems to be a useful tool for characterizing economic conditions a couple of decades into the future.

Another way of utilizing a quantitative general equilibrium model is to compare a solution of the model with the actual situation in the economy at a given point in time. Such a comparison indicates how close the actual allocation is to an equilibrium allocation. The model can also highlight the nature of the disequilibrium situation, that is, give some rough indications as to which sectors are "too big" and which are "too small".

The purpose of this study is to develop and apply a model which can be used to calculate equilibrium resource allocations in a small, open economy under various assumptions about world market conditions and the domestic supply of primary factors of production. In this paper, the model is used for a comparison between the actual allocation of resources in the Swedish economy 1975 and an equilibrium allocation. Thus, we try to answer the question: to what extent was there a disequilibrium in the sectoral allocation of primary factors of production in Sweden 1975, and what was the macroeconomic significance of that disequilibrium ?

The paper is organized in the following way. In Section II the structure of the model is presented and discussed. Section III deals with the solution procedure while Section IV contains some preliminary results. Finally, in Section V, future directions of research are briefly discussed.

II THE MODEL

Given estimates of the total supply of labor and capital, world market conditions and production functions at some point in time, the model determines the sectoral allocation of labor and capital. This is done in such a way that all commodity and factor markets are simultaneously cleared. In this process the endogenously determined factor and commodity price system play a strategic role. Thus, except for the use of non-energy intermediate inputs, all technological coefficients are determined as functions of relative prices; that also applies to exports, imports and household expenditures. Public consumption expenditures and total net investment, on the other hand, are exogenously determined.

The model economy has one aggregated household sector and n+4 production sectors with sector index i = 0,1,...,n+3. In the applications presented in Section IV, n=20, but in general n can be any integer between 1 and N where N is determined by the capacity of the available computer. There is only one kind of output from each of the production sectors, and each commodity is only produced in one sector. The sectors 0 and 1 are the energy sectors, producing fuels and electricity respectively. Sector n+1 is the housing sector and n+2 the public sector. The capital goods sector, n+3, is a book-keeping sector where various produced goods are combined in fixed proportions. Thus, the input-output coefficients of the capital goods sector define the composition of the economy's stock of real capital.

The present model is an extended and elaborated version of a model previously developed by one of the authors (Bergman 1978) which, in turn, was inspired by earlier work by Johansen (1959 and 1974), Restad (1976) and Førsund (1977). It would lead too far to discuss all the differences between the present model and its predecessors, but the basic differences are related to the determination of foreign trade, domestic prices and the treatment of the energy sectors. In addition, the solution technique differs considerably from the one utilized by the above mentioned authors.

In the following presentation of the model, the exact form of production, household demand and foreign trade equations are specified. The parameters of these functions have been estimated in a rather crude way. Later on, when a more complete data base has been compiled, these parameters will be reestimated. Then it might also turn out that the functional forms should be revised. That will not change the basic features of the solution procedure presented in Section III provided the reestimated production functions are of a constant returns-to-scale type.

II. 1. Exogenous and endogenous variables

To begin with, the variables and parameters of the model are defined.

A. Endogenous variables

- X_j gross output in sector $j = 0,1\ldots, n+3$
- X_{ij} use of commodity $i = 2,3,\ldots,n$
- K_j capital stock in sector $j = 0,1,\ldots, n+2$
- N_j employment in sector $j = 0,1,\ldots, n+2$
- $\bar{M}_j$ input of complementary imports* in sector $j = 0,1.$
- C_i household consumption of commodity $i = 0,1,\ldots, n+1$
- Z_i export of commodity $i = 1,2,\ldots, n$
- M_i import of commodity $i = 0,1,\ldots, n$
- P_i domestic production cost of commodity $i = 0,1,\ldots, n+3$
- P_i^D domestic price of commodity $i = 0,1,\ldots, n+3$
- W index of the level of wages in the economy as a whole.
- W_j wage rate in sector $j = 0,1,\ldots, n+2$
- R index of the return on capital in the economy as a whole.
- R_j net rate of return on capital in sector $j = 0,1,\ldots, n+2$
- Q_j "user cost" of capital in sector $j = 0,1,\ldots, n+2$
- V exchange rate (units of domestic currency per unit of foreign currency).

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*Complementary imports is meant to imply the import of commodities that cannot be (or at least are not) produced within the country.

O household consumption expenditures.

Y real gross national product.

C total real household consumption.

Z total real exports

M total real imports

B. Exogenous variables

G real public consumption

N total labor force

K total capital stock

I total net investments

D target surplus (deficit) on the current account.

P_i^{WE} price level, expressed in foreign currency, on international markets to which domestic exporters supply commodity $i = 1,2,\ldots, n$

P_i^{WI} price level, expressed in foreign currency, of imports of commodity $i = 0,1,\ldots, n$

$\bar{P}_i$ world market price, expressed in foreign currency, of complementary imports used as inputs in sector $i = 0,1$

Θ_i <u>ad valorem</u> indirect tax on commodity $i = 0,1,\ldots, n+2$

Φ_i <u>ad valorem</u> custom duty on commodity $1 = 0,1,\ldots, n$

T_{ij} energy tax parameter for sector $j = 0,1,\ldots, n+2,c$ 1/ and energy type $i = 0, 1$.

C. Parameters

a_{ij} input of commodity $i = 2,3,\ldots,n$, per unit of output in sector $j = 0,1,\ldots, n+2$

$\bar{b}_{ij}$ input of complementary imports of type $i = 0,1$, per unit of output in sector $j = 0,1$.

ρ_j, γ_j substitution parameters in the production function of sector $j = 0,1,\ldots, n+2$

$\alpha_j, a_j, b_j, c_j, d_j$ distribution parameters in the production of sector $j = 0,1,\ldots, n+2$

λ_j annual rate of technological change in sectors $j = 0,1,\ldots, n+2$

1/ The index c denotes the household sector

δ_j annual rate of depreciation in sector $j = 0,1,\ldots, n+2$

σ_j annual rate of change of world market trade with commodity $i = 1,2,\ldots, n$

ω_j index of the relative wage rate in sector $j = 0,1,\ldots, n+2$

β_j index of the relative rate of return on capital in sector $j = 0,1,\ldots, n+2$

η_i, η_{ij} expenditure and price elasticity parameters in the household demand for commodity $i = 0,1,\ldots, n+1$

ε_i, μ_i price elasticity parameters in the export and import demand for commodity $i = 0,1,\ldots, n$. 1/

$A_j, B_i, Z_i^0, X_j^0, M_i^0$ constants in the production household demand, export and import functions.

II. 2. Produced, consumed and traded commodities

Using the symbols defined in the previous sub-section, we now come to the description of the structural equations of the model. It is then convenient to start with the commodity concepts used in the following exposition. In principle, there are n+4 commodities in the model. All of these are domestically produced and n are so called tradeable goods. However, when the model is applied on actual data, these "commodities" are not well-defined products, but rather fairly aggregated commodity groups. For convenience we assume that the composition of those commodity groups are stable over time. It should be noted that the domestically produced commodity aggregate with a given classification usually is not identical with imported commodities classified in the same way. Thus imports and domestic output with the same commodity classification are not necessarily perfect substitutes.

Here we assume that there is a finite elasticity of substitution between imported and domestically produced units with the same commodity classification. 2/. Moreover, we assume that the elasticity of substitution is the same in all domestic uses of the commodity group in question. Thus the domestic supply of a given commodity group is the sum of imports and domestic output less exports. The proportions are determined in accordance with import functions of the following type 3/:

1/ Since there is no export of commodity 0, ε_0 is not defined.

2/ Norman and Wergeland (1978) make the alternative assumption that there is always some foreign competitor that produces exactly the same commodity group as the domestic producers in a given sector. Moreover, the foreign producers are assumed to be price leaders on the world market. These assumptions call for a different approach to the treatment of foreign trade.

3/ It is easy to show that if the users of commodity i have CES-type preference functions with M_i and $X_i - Z_i$ as arguments, then eq. (1) is a necessary first order condition for an optimum choice between imports and domestic output. Moreover, the parameter μ_i is the elasticity of substitution between imports and domestic output.

$$m_i \equiv \frac{M_i}{X_i - Z_i} = \frac{M_i^0}{X_i^0 - Z_i^0} \left(\frac{P_i}{(1+\phi_i) VP_i^{WI}} \right)^{\mu_i} \quad ; \qquad i = 0,1,\ldots, n. \qquad (1)$$

Thus, there are three different commodity aggregates with index i : imported, domestically produced (and exported) and domestically consumed commodities of type i. Since the modelled economy is not assumed to be big enough to affect world market prices, these are taken as exogenously determined. The price of domestic output is determined by domestic production costs, while the price of domestically consumed units of commodity group i is determined in accordance with the following expression 1/.

$$P_i^D = \frac{m_i}{1+m_i} (1+\phi_i) VP_i^{WI} + \frac{1}{(1+m_i)} P_i \quad ; \qquad i = 0,1,\ldots,n \qquad (2)$$

II. 3. Technology and production costs

The production technology is characterized by constant returns to scale in all sectors. Labor, capital, fuel and electricity are substitutable factors of production, while the use of non-energy intermediate inputs is proportional to the level of gross output. The input of imported primary energy resources in the energy sectors is also proportional to the output levels in these sectors.

The description of the technology starts by the definition of a composite capital-labor input, F_j, and a composite fuel-electricity input, H_j, in accordance with the following equations:

$$F_j = K_j^{\alpha_j} N_j^{1-\alpha_j} e^{\lambda_j t} \quad ; \qquad j = 0,1,\ldots,n+2: \qquad (3)$$

$$H_j = \{c_j X_{0j}^{\gamma_j} + d_j X_{1j}^{\gamma_j}\}^{\frac{1}{\gamma_j}} \quad ; \qquad j = 0,1,\ldots,n+2; \qquad (4)$$

Thus the elasticity of substituion between capital and labor is unity, while the elasticity of substitution between fuel and electricity is defined by $(1-\gamma_i)^{-1}$ where γ_i is a finite non-zero number different from unity. The composite factors of production F_j and H_j can be combined to yield gross output in accordance with

$$X_j = A_j \{a_j F_j^{\rho_j} + b_j H_j^{\rho_j}\}^{\frac{1}{\rho_j}} \quad ; \qquad j = 0,1,\ldots,n+2; \qquad (5)$$

1/ For $i = n+1, n+2$ it holds that $P_i^D \equiv P_i$

Consequently the elasticity of substitution between F_j and H_j is defined by $(1-\rho_j)^{-1}$, where ρ_j is a finite non-zero number different from unity. The following expressions make the description of technology complete.

$$X_{ij} = a_{ij} X_j \quad ; \; i = 2,3,\ldots,n \; ; \quad j = 0,1,\ldots,n+3; \qquad (6)$$

$$\bar{M}_j = \bar{b}_{ij} X_j \quad ; \; j = 0,1 \qquad (7)$$

Although this might very well be a reasonable specification of the range of available techniques, several other specifications can also be defended on <u>à priori</u> grounds. The introduction of technological change and the implied equality between the elasticity of substituion between capital and energy and the corresponding parameter for labor and energy 1/ are probably the most dubious parts of the chosen specifications. However, a discussion of these issues should be postponed until a later stage when a more comprehensive set of data is available.

The choice of factor combination is determined by the assumption that producers in the private sectors maximize their profits, subject to the production functions. For the public sector it is assumed that total costs are minimized subject to the production function. However, since all production functions exhibit constant returns to scale, the first order necessary conditions for an optimum choice of factor combination is the same for all sectors. The equations are derived in the following way.

First a new variable, P_j^*, is defined by

$$P_j^* = (1-\theta_j)P_j - \sum_{i=2}^{n} P_i^D a_{ij} - V\bar{P}_j \bar{b}_{jj} \quad ; \quad j = 0,1,\ldots,n+2; \qquad (8)$$

where $\bar{b}_{jj} \equiv 0$ for $j = 2,3,\ldots,n+2$. Using this variable, the total profit in a production sector, Π_j, is defined by

$$\Pi_j = P_j^* X_j - T_{0j} P_0^D X_{0j} - T_{1j} P_1^D X_{1j} - W_j N_j - Q_j K_j \; ; \qquad (9)$$

$$j = 0,1,\ldots,n+2$$

Maximization of Π_j yields the following necessary first order conditions for an optimum choice of factor combination.

$$a_j(1-\alpha_j)\left(\frac{A_j F_j}{X_j}\right)^{\rho_j} = \frac{W_j N_j}{P_j^* X_j} \quad ; \quad j = 0,1,\ldots,n+2 \qquad (10)$$

1/ See Bergman (1978) for a brief discussion about some econometric evidence on the elasticity of substitution between energy and other factors of production.

$$a_j \alpha_j \left(\frac{A_j F_j}{X_j}\right)^{\rho_j} = \frac{Q_j K_j}{P_j^* X_j} \quad ; \; j=0,1,\ldots,n+2; \qquad (11)$$

$$b_j c_j \left(\frac{A_j H_j}{X_j}\right)^{\rho_j} \left(\frac{X_{0j}}{H_j}\right)^{\gamma_j} = \frac{T_{0j} P_0^D X_{0j}}{P_j^* X_j} \quad ; \; j=0,1,\ldots,n+2; \qquad (12)$$

$$b_j d_j \left(\frac{A_j H_j}{X_j}\right)^{\rho_j} \left(\frac{X_{1j}}{H_j}\right)^{\gamma_j} = \frac{T_{1j} P_1^D X_{1j}}{P_j^* X_j} \quad ; \; j=0,1,\ldots,n+2; \qquad (13)$$

The variable Q_j, the "user cost" of capital, is defined by

$$Q_j = P_{n+3}(\delta_j + R_j) \quad ; \quad j = 0,1,\ldots,n+2; \qquad (14)$$

where

$$P_{n+3} = \sum_{i=2}^{n} P_i^D a_{i,n+3} \; ; \qquad (15)$$

and

$$R_j = \beta_j R \quad ; \quad j = 0,1,\ldots,n+2; \qquad (16)$$

The parameter β_j reflects the fact that profit requirements sometimes differ between sectors, even if intersectoral differences in risk and uncertainty are taken into account. For instance. in Sweden there is no rate of return requirements in the public sectors and the use of capital in the housing sector is subsidized. Moreover, by suitable choice of values for β_j, an actual resource allocation can be reproduced as a solution to the model.

The wage rate in sector j is defined by

$$W_j = \omega_j W \quad ; \quad j = 0,1,\ldots,n+2; \qquad (17)$$

If labor was a homogenous factor of production all ω_j should be equal to unity in an equilibrium allocation of resources. However, labor is not a homogenous factor of production. In view of this fact, the proper approach would be to explicitly introduce several types of labor in the model. Here, a simplified approach is chosen, implying that the parameters ω_j are regarded as indicators of the composition of the labor force in the different sectors. This approach is reasonable as long as there are no significant reallocations of labor between the sectors, and the wage structure is approximately constant over time.

At this stage it should be noted that, with given values of W, R and V, it is possible to determine technological coefficients and unit production costs without bothering about the activity levels in the production sectors.1/ This is, of course, a consequence of the assumption about constant returns to scale in the production system. As will be seen in Section III, however, this feature of the model can be utilized in order to simplify the solution procedure.

II. 4. Final demand

Of the final demand categories public consumption and total net investments are exogenously determined. The demand for consumer goods and services by the houshold sector is determined by relative market prices and total household expenditures in the following way:

$$C_i = B_i O^{\eta_i} (T_{0c} P_0^D)^{\eta_{0i}} (T_{1c} P_1^D)^{\eta_{1i}} P_2^{D\,\eta_{2i}}, \ldots, P_{n+1}^{D\,\eta_{n+1,i}} \quad ; \qquad (18)$$

$$i = 0, 1, \ldots, n+1 \quad .$$

1/Observe that if we define

$$n_j = N_j / X_j$$

$$k_j = K_j / X_j$$

$$x_{0j} = X_{0j} / X_j$$

$$x_{1j} = X_{1j} / X_j$$

for $j = 0, 1, \ldots, n+2$, and

i) eq. (3) and eq. (4) are substituted in eq. (5)

ii) eqs. (15) - (17) are substituted in eq. (12)

iii) eq. (18) is substituted in eq. (11)

iv) eqs. (1), (2) and (9) are substituted in eqs. (11) - (14)

v) it is assumed that W, R and V are exogenously given

then eqs. (5) and (11) - (14) become a non-linear equation system with 5(n+3) equations in the 5(n+3) unknowns P_j, n_j, k_j, x_{0j} and x_{1j} $(j = 0, 1, .., n+2)$. Thus, production costs and technological coefficients can be determined without simultaneous determination of production volumes.

The problem with this formulation is that the variable 0 is in general not identical with total household expenditures defined by the market value of the goods and services actually bought by the households.1/ The advantage with (18) is that it is very easy to estimate2/, which was the deciding factor at this stage of the work.

The demand for exports is determined by

$$Z_i = Z_i^0 \left(\frac{P_i}{VP_i^{WE}} \right)^{\varepsilon_i} e^{\sigma_i t} \quad ; \quad i = 1,2,\ldots,n \; ; \tag{19}$$

which means that we assume a finite elasticity of substitution between domestically produced units of commodity group i and the commodities with the same classification supplied on international markets by foreign producers. Note that the world market price of commodity i appearing in the export demand functions, P_i^{WE} differ from the corresponding variable in the import functions, P_i^{WI}. The difference is partly due to transportation costs abroad, partly to differences in the composition of the imported and the exported commodity group i.

II. 5. Equilibrium conditions and the general level of prices

In order to close the model equilibrium conditions for all commodity and factor markets are needed. When those conditions are added all real variables in the model can be determined. The general level of prices, however, is still indeterminate3/. Thus we have to choose some numeraire for the price system.

Eqs. (20) - (25) give equilibrium conditions for the commodity markets while eqs. (26) and (27) give corresponding conditions for the factor markets. Eq . (28) is the current account balance where the exogenously determined difference between currency receipts and currency expenditures reflect the capital account surplus or deficit. By means of eq. (29) the price system is normalized so that the general price level is constant and equal to unity.

1/ That identity only holds if

i) $\eta_i = 1$

ii) $\eta_{ij} = \begin{cases} -1 & \text{when } i=j \\ 0 & \text{when } i \neq j \end{cases}$

for all i, that is, the expenditure shares are constant for all commodity groups.

2/ See Frisch (1959)

3/ Thus, multiplication of $P_0,\ldots,P_{n+2}$, W, V and 0 with some arbitrary non-zero constant does not affect the allocation of resources in the model economy.

$$X_0 = \sum_{j=0}^{n+2} X_{0j} + C_0 - M_0 \quad ; \qquad (20)$$

$$X_1 = \sum_{j=0}^{n+2} X_{1j} + C_1 + Z_1 - M_1 \quad ; \qquad (21)$$

$$X_i = \sum_{j=0}^{n+3} a_{ij} X_j + C_i + Z_i - M_i \quad ; \qquad i = 2,3,\ldots,n \qquad (22)$$

$$X_{n+1} = C_{n+1} \quad ; \qquad (23)$$

$$X_{n+2} = G \quad ; \qquad (24)$$

$$X_{n+3} = I + \sum_{j=0}^{n+2} \delta_j K_j \quad ; \qquad (25)$$

$$\sum_{j=0}^{n+2} K_j = K \quad ; \qquad (26)$$

$$\sum_{j=0}^{n+2} N_j = N \quad ; \qquad (27)$$

$$\sum_{i=1}^{n} \frac{P_i}{V} Z_i - \sum_{i=0}^{n} P_i^{WI} M_i - \bar{P}_0\bar{M}_0 - \bar{P}_1\bar{M}_1 = D^{1/} \quad ; \qquad (28)$$

$$\sum_{j=0}^{n+2} P_j X_j + \sum_{j=0}^{n} VP_j^{WI} M_j + V\bar{P}_0\bar{M}_0 + V\bar{P}_1\bar{M}_1 =$$

$$\sum_{j=0}^{n+2} X_j + \sum_{j=0}^{n} M_j + \bar{M}_0 + \bar{M}_1 \quad ; \qquad (29)$$

Recall that if W,R and V are exogenously determined, the whole price system and the technological coefficients $n_j = N_j/X_j$; , k_j/X_j, $x_{0j} = X_{oj}/X_j$ and $x_{1j} = X_{1j}/X_j$ can be determined. When prices and the technological coefficients are determined and the value of 0 in the household demand equations is given, the eqs. (20) - (25) become a linear equation system with n+4 equations in the n+4

1/If the variable $\frac{Pi}{V}$ in eq. (28) is replaced by P_i^{WE}, that implies that the export functions (19) should be interpreted as export supply rather than export demand functions.

variables $X_j, j = 0,1,\ldots,n+3$. A solution to this system together with eqs. (26) - (29) yields measures of the, positive or negative, excess demands on the markets for capital, labor and foreign currency. This set of excess demands corresponds to the given values for R,W and V. By systematic revision of these values, a general equilibrium can be reached.

II. 6. Definitions

In order to characterize the solutions of the model it is convenient to use the conventional measures of aggregate economic activity together with results for individual sectors. In the usual way real GDP is defined by

$$Y = C + G + X_{n+3} + Z - M \qquad ; \qquad (30)$$

where real private consumption is defined by

$$C = \sum_{i=0}^{n+1} C_i \qquad ; \qquad (31)$$

real exports by

$$Z = \sum_{i=1}^{n} Z_i \qquad ; \qquad (32)$$

and real imports by

$$M = \sum_{i=0}^{n} M_i + \sum_{i=0}^{1} \bar{M}_i \qquad ; \qquad (33)$$

Domestic consumption of fuels and electricity respectively are defined by

$$E_0 = X_0 + M_0 \qquad ; \qquad (34)$$

and

$$E_1 = X_1 + M_1 - Z_1, \qquad ; \qquad (35)$$

The energy tax parameters, T_{ij} and T_{ic}, finally, are defined by

$$T_{ij} = 1 + T_i + \xi_{ij} \quad ; \quad i = 0,1;\ j = 0,1,\ldots,n+2; \tag{36}$$

$$T_{ic} = 1 + T_i + \xi_{ic} \quad ; \quad i = 0,1; \tag{37}$$

II. 7. Some remarks about specialization and structural change in the model-economy

The above presented model is intended to be used is various analyses of the future state of the Swedish economy. In particular, solutions for different points in time will be compared and the differences between them in terms of the sectoral structure of the production system will be interpreted as the result of structural change processes with unspecified dynamic properties.

The Swedish economy is "small" in the sense that its producers have a limited impact on the prices on most markets for internationally traded goods and services. Moreover, it is "open" in the sense that a relatively large share of output is exported and a relatively large share of the domestic supply of goods and services is imported. This means that the sectoral structure of the economy's production system to a large extent depends on the relation between world market prices and domestic production costs.

For this reason it is important that the model can represent the relations between world market prices, domestic production costs and the economy's specialization pattern in a realistic way. If it does not work satisfactorily in this respect, it will be of limited use for the intended purposes. This subsection is therefore devoted to a brief discussion on how changes in world market conditions induce changes in the cost conditions and the sectoral structure of the model-economy's production system.1/

It is then convenient to repeat the export and import demand functions and the equation defining the domestic price level for tradeable commodity groups.

$$Z_i = Z_i^0 \left(\frac{P_i}{VP_i^{WE}}\right)^{\varepsilon_i} e^{\sigma_i t} \quad ; \quad i = 1,2,\ldots,n; \tag{19}$$

$$m_i \equiv \frac{M_i}{X_i - Z_i} = \frac{M_i^0}{X_i^0 - Z_i^0} \left(\frac{P_i}{(1+\phi_i)VP_i^{WI}}\right)^{\mu_i} \quad ; \quad i = 0,1,\ldots,n; \tag{1}$$

$$P_i^D = \frac{m_i}{1+m_i}(1+\phi_i)VP_i^{WE} + \frac{1}{(1+m_i)} P_i \quad ; \quad i = 0,1,\ldots,n. \tag{2}$$

1/ See Jungenfelt (1979) for a more general discussion on these problems.

First, consider a case without aggregation over commodities. In such a case a distinction between domestically produced commodities and the corresponding imported commodities cannot be maintained, provided transportation costs are properly taken into account. Thus, the absolute values of the parameters ε_i and μ_i should approach infinity. This means that whenever the relative price variables in the foreign trade functions differ from unity, the set of tradeable goods can be partitioned into two subsets, one with the domestically produced and exported commodities and one with the imported commodities. The domestic commodity price system will coincide with world market prices expressed in the domestic currency unit. Moreover, in equilibrium the number of produced and exported tradeable goods will not be greater than the number of exogenously given factors of production, which is two in the present model.1/

It follows that it is the assumptions that the parameters ε_i and μ_i have finite values that prevents complete specialization in the model-economy. However, it is not an unreasonable assumption. In fact, in an empirical analysis it seems to be the only reasonable assumption; even in very detailed commodity classification systems, commodities within the same cell on the most disaggregated level often exhibit differences in quality, etc.

Given that imported and domestically produced commodities with the same classification are not perfect substitutes, the equilibrium specialization pattern in the model-economy reflects international differences in unit production costs. As long as the domestic economy and "the rest of the world" grows at the same rate and cost conditions develop in a uniform way, the market shares of domestic producers on domestic and international markets remain unaffected. Thus, a price-cost equilibrium is maintained as long as there is a fixed relation between domestic production costs and world market prices. A ceteris paribus decrease in the world market price of an internationally traded commodity, or an increase in the domestic production cost for that commodity, will always result in decreased market shares at home and abroad for the domestic producers of the commodity in question.

However, this is not the whole story. If we continue to consider the case where the world market price of a given commodity decreases, it is clear from eq. (2) that the domestic price of that commodity will also decrease. This price decrease will work through the input-output system and result in lower production costs, and thus increase international competitiveness, in all domestic production sectors. This mechanism is reasonable and realistic, but unlike in the real world it is the only direct link between world market prices and domestic production costs in the model-economy at given factor prices.2/

In the real world there is an additional link between these two sets of variables, related to the zero or limited intersectoral mobility of real capital. At the planning, or ex ante, stage investments can be allocated between the sectors in accordance with profitability criteria. Once an investment is made,

1/This is a standard result in international trade theory.

2/In order to attain an equilibrium, factor prices have to be adjusted so that the exogenously given current account constraint can be satisfied.

the resulting capital equipment is more or less tied to the production unit in which the investment was made. Moreover, the ex post substitutability of the factors of production is usually lower than the corresponding ex ante figure.

Thus, if factor prices change over time and/or there is technological change, a production sector will at any point in time contain a set of production units representing a range of cost conditions at the prevailing factor prices. By a suitable ordering of the production units, the supply conditions of the sector can be described by an increasing supply curve. If the world market price of the commodities produced by a sector with these features is reduced, the demand for its output will, ceteris paribus, decrease. In the same way as in the model-economy the reduction of the sector's international competitiveness will be somewhat counteracted by reduced prices on intermediate inputs. In addition, it will be counteracted by a reduction of the sector's average production costs, as a direct result of the reduction of output. This is because it is the least efficient production units that will be closed down first when the world market price of the commodity group in question is reduced. This mechanism is not incorporated in the constant returns to scale description of the production system in the present model.

To sum up, the model treats the impact of changes in world market conditions on the domestic allocation of resources in a realistic but somewhat incomplete way. The mechanisms that operate in the model-economy also operate in the real world, but not all mechanisms that operate in the real world are incorporated in the model. The consequence of this incompleteness of the model is that it tends to overestimate the impact of changes in world market conditions on the sectoral allocation of resources in the economy.This is because the lacking mechanism is one that dampens the impact on the sectoral structure rather than the opposite.

III. THE SOLUTION PROCEDURE

In solving equilibrium models, one must find solutions for excess-demand equations in two sets of markets: commodity markets and factor markets. Various techniques have been developed to solve such a system, but all are based essentially on one of two different approaches. The first approach is to reformulate the economic specification so that it can be treated as an optimization problem, which is then solved by means of mathematical programming methods. The second approach is to solve the equation system directly.

Within the general strategy of directly solving the system of nonlinear equations, we can further distinguish between two general approaches. The first is to reduce the problem to one of finding a fixed point of equations and then using a fixed point algorithm for numerical solutions. For example, the Scarf fixed point algorithm.1/ The other approach, which we have also partly taken, tries to attack the equation system in its

1/ See Scarf (1973).

original form by using relatively simple numerical techniques. For a survey of various methods within this approach, see Adelman and Robinson (1978).

Within the general strategy of directly attacking the equation system describing an equilibrium model, one has essentially a choice of two solution strategies. The first strategy is to solve for excess-demand equations in the factor markets and so substitute out the commodity markets. This approach is called the factor-market strategy. This approach is efficient if the inner cycle of our numerical procedure for clearing the commodity market (including a price adjustment mechanism) is relatively simple and the number of factors is relatively smaller than the number of commodities.

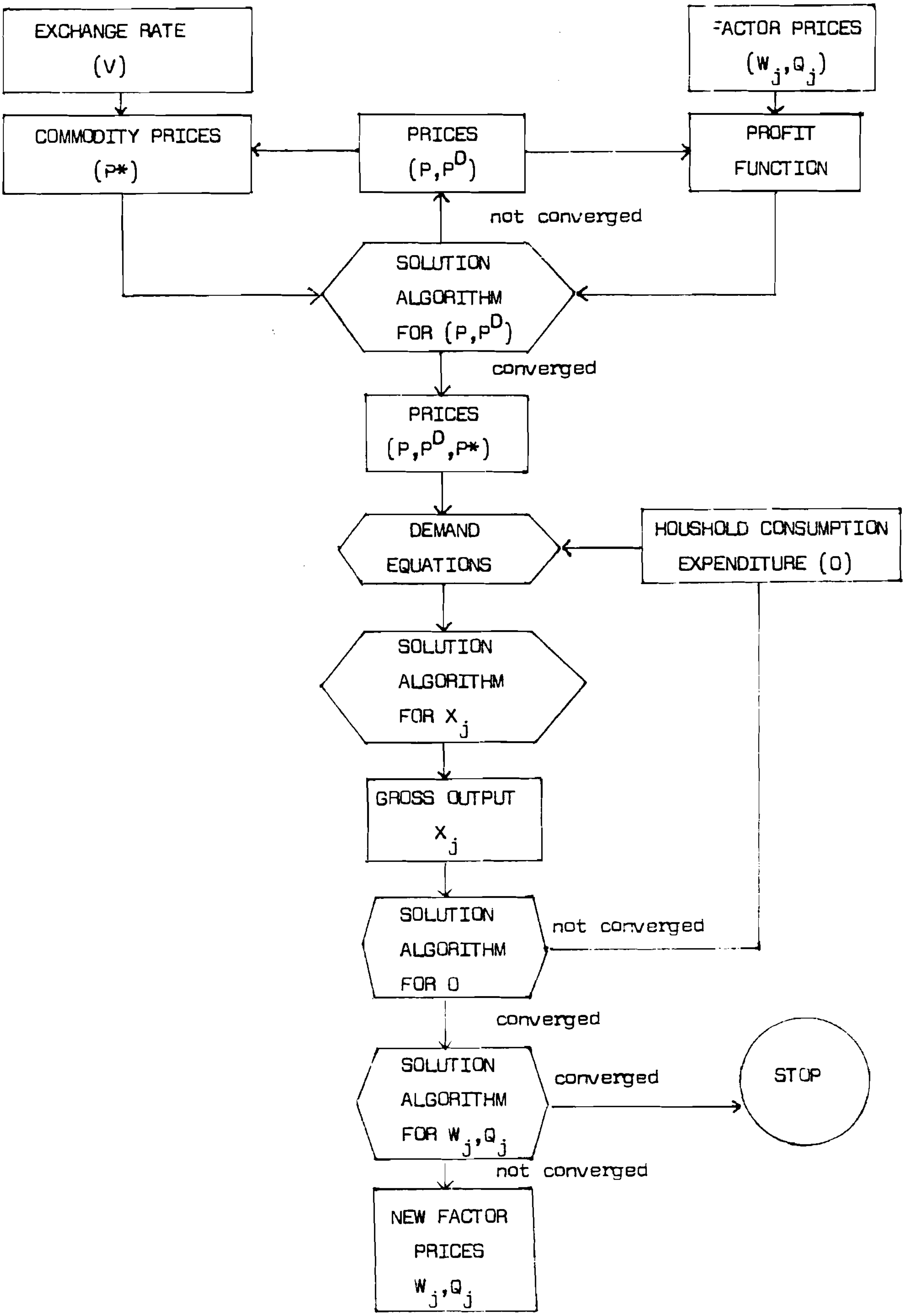
The second approach is to find the solution to the excess-demand equations in the commodity markets and so substitute out the factor markets. This approach is called the commodity-market strategy. In this approach one must first clear the factor markets in the inner cycle of the procedure and then clear the commodity markets in the outer cycle.

We have chosen the factor-market strategy (Fig.1). Our approach is to solve numerically the two equilibrium conditions in the factor markets (eqs. (26) and (27)) as a function of factor prices W_j and Q_j. As a subproblem, however, we must solve for market-clearing prices in the commodity markets with given factor prices W_j and Q_j. Observe that all wage rates (W_j) depend only on the unknown W. Further, note that assuming prices are given, all "user-costs" of capital (Q_j) depend only on unknown R. To equilibriate conditions (26) and (27) in the factor markets, we have only to adjust variables W and R.

It is easy to see that our system is homogenous in exchange rate (V) and all prices. Therefore, we can fix unknown V to any reasonable value and solve the system for unique market-clearing prices in both markets. At the end of the solution procedure, the price system can be normalized by means of eq. (29) so that the general price level is constant and equal to unity.

We come now to the details of exactly how we derive the factor excess demands given initial values for variables W, R and V. We shall first consider the price adjustment mechanism and indicate how, given an initial set of variables W, R and V, we derive the prices p^*, p and p^D (Part 1), then discuss commodity demand and current account balance (Part 2) and finally, examine the excess-demand equation in the factor markets (Part 3), along with the solution procedure.

FIGURE 1


EXCHANGE RATE
(V)
FACTOR PRICES
(W j ,Q j)
COMMODITY PRICES
(P*)
PRICES
(P,P D)
PROFIT
FUNCTION
not converged
SOLUTION
ALGORITHM
FOR (P,P D)
converged
PRICES
(P,P D,P*)
DEMAND
EQUATIONS
HOUSHOLD CONSUMPTION
EXPENDITURE (O)
SOLUTION
ALGORITHM
FOR X j
GROSS OUTPUT
X j
SOLUTION
ALGORITHM
FOR O
not converged
converged
SOLUTION
ALGORITHM
FOR W j ,Q j
converged
STOP
not converged
NEW FACTOR
PRICES
W j ,Q j

Price Adjustment: Part 1

Using the definition (8), total profit in a production sector can be written:

$$\pi_j = P_j^* X_j - T_{oj} P_o^D X_{oj} - T_{1j} P_1^D X_{1j} - W_j N_j - Q_j K_j$$

Under the assumption of exogenous prices, the profit-maximizing behavior can be depicted by the first order conditions (10-13) and the production functions (3)-(5).

We are now able to solve the conditions (3)-(5) and (10)-(13) explicitly for profit maximizing values. Using these profit maximizing values we obtain the following function:

$$P_j^* X_j = X_j A_j^{-\rho_j} \left\{ a_j^{\frac{1}{1-\rho_j}} \left[e^{-\lambda_j t} \alpha_j^{-\alpha_j} (1-\alpha_j)^{-(1-\alpha_j)} Q_j^{\alpha_j} W_j^{(1-\alpha_j)} \right]^{\frac{\rho_j}{\rho_j - 1}} + \right.$$

$$\left. + b_j^{\frac{1}{1-\rho_j}} \left[c_j^{\frac{1}{1-\gamma_j}} (T_{oj} P_o^D)^{\frac{\gamma_j}{\gamma_j - 1}} + d_j^{\frac{1}{1-\gamma_j}} (T_{1j} P_1^D)^{\frac{\gamma_j}{\gamma_j - 1}} \right]^{\frac{\rho_j(\gamma_1 - 1)}{\gamma_j(\rho_j - 1)}} \right\}^{\frac{\rho_j - 1}{\rho_j}}$$

By setting the profit function equal to zero we obtain the commodity prices as a function of factor prices. Moreover, P_j^* can be interpreted as the unit production cost net of the cost for non-energy intermediate inputs. The "net unit cost" P_j^* becomes

$$P_j^* = A_j^{-\rho_j} \left\{ a_j^{\frac{1}{1-\rho_j}} \left[e^{-\lambda_j t} \alpha_j^{-\alpha_j} (1-\alpha_j)^{-(1-\alpha_j)} Q_j^{\alpha_j} W_j^{(1-\alpha_j)} \right]^{\frac{\rho_j}{\rho_j - 1}} + \right.$$

$$\left. + b_j^{\frac{1}{1-\rho_j}} \left[c_j^{\frac{1}{1-\gamma_j}} (T_{oj} P_o^D)^{\frac{\gamma_j}{\gamma_j - 1}} + d_j^{\frac{1}{1-\gamma_j}} (T_{1j} P_1^D)^{\frac{\gamma_j}{\gamma_j - 1}} \right]^{\frac{\rho_j(\gamma_j - 1)}{\gamma_j(\rho_j - 1)}} \right\}^{\frac{\rho_j - 1}{\rho_j}}$$

Now, substituting P_j^* into equation (8) we obtain a uniquely defined equation system for unknowns P_j and P_j^D. The solution technique is based on the method of successive approximations (for example: Gauss-Seidel).

Commodity excess demand and currect account balance: Part 2

Recall that assuming variables W, R and V are given and prices are determined by the solution procedure discussed above, the technological coefficients $n_j = N_j/X_j$, $k_j = X_{oj}/X_j$ and $X_{1j} = X_{1j}/X_j$ are also determined.

When the value of the household consumption expenditure O is given, the eqs.(20)-(25) become a linear equation system in the variables X_j. The coefficient matrix of this linear system does not depend on variables O. By inverting the coefficient matrix and multiplying it by the right-hand side, which depends explicitly on O, we obtain explicit expressions for all variables X_j as a function of the variable O. By substituting these expressions for the variables X_j in equation (28), we obtain one equation in one variable (O). This equation can be solved by various methods. We have chosen the golden-section method.

Labor and capital excess demand: Part 3

From the solution of Part 1 and Part 2 equations, we have the employment (N_j) and capital stock (K_j) in all sectors. We can thus define the labor and capital excess demands

$$\Delta K = \sum_{j=0}^{n+2} K_j - K$$

$$\Delta N = \sum_{j=0}^{n+2} N_j - N$$

A simple adjustment of variables W and R for the next iteration can be achieved in accordance with the following rules:

if $\Delta K < 0$	$R_{new} < R_{old}$
if $\Delta K > 0$	$R_{new} > R_{old}$
if $\Delta K = 0$	$R_{new} = R_{old}$
if $\Delta N < 0$	$W_{new} < W_{old}$
if $\Delta N > 0$	$W_{new} > W_{old}$
if $\Delta N = 0$	$W_{new} = W_{old}$

In order to clear the factor markets, we can also use any solution procedure which numerically estimate the inverse of the Jacobian matrix.

We tested both solution methods and did not find any significant difference between their convergence-speed.

IV. SOME PRELIMINARY RESULTS

In this section some preliminary results are presented. After a brief discussion of the data base in sub-section IV.1, the following sub-section deals with a comparison of the actual allocation of resources in Sweden 1975 and an equilibrium allocation, as determined by the model. In sub-section IV.3 results from some comparative-statics experiments where the supply of capital and labor are varied, are presented. This means that, for the moment, we refrain from using the model for long term projection; instead we concentrate on computing equilibria under various conditions for a given point in time.

IV. 1. The data base

The bulk of the data base is obtained from an (unpublished) input-output table for Sweden. The data refers to the inter-sectoral flows in 1975.* The original table had 34 sectors, of which were industrial sectors. For this study some industrial sectors were disaggregated, while some nonindustrial sectors were aggregated. The resulting sectoral break-down can be seen in Table 1. The sectors 2-15 can be characterized as "industrial sectors." In addition the data base comprises estimates of the capital stock and the employment (measured in man-hours) in each of the production sectors. From this data the input coefficients for (non-energy) intermediate inputs can be obtained from the input-output data. Moreover, given assumptions about the elasticity of substitution between types of energy and between energy and primary factors of production, and using the neoclassical theory of distribution, all the parameters of the production functions can be derived from the distribution data contained in the input-output table. Coupled with employment and capital stock data the same source can be used for derivation of the depreciation rates as well as the sector specific wage and profitability parameters (ω_j and β_j, respectively).

The parameters of the household demand equations were initially estimated in the way proposed by Frisch (1959). However, the fact that this type of demand system does not satisfy the budget constraint identically (see p.10) turned out to create problems; the comparative static analysis was disturbed by substantial variations in the error term.[1/] For this reason it was simply assumed that all own-price elasticities were minus unity and all cross-price elasticities were equal to zero.

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*The author is grateful to Mr. Bengt Roström at the Swedish Central Bureau of Statistics for providing the input-output data for this study.

1/By "error term" is meant the difference between the expenditure variable, 0, in the demand equations, and the market value of the consumer goods basket actually bought by the households, $\sum_i P_i C_i$.

Table 1. The production sectors.

Number	
0	Energy[a/]
1	Agriculture, fishing, basic food
2	Forestry, wood, pulp and paper
3	Mining and quarrying
4	Other food, beverages, liquor and tobacco
5	Textile, clothing and leather
6	Paper products
7	Chemical products[b/]
8	Non-metallic mineral products except petroleum and coal
9	Metals
10	Fabricated metal products
11	Non-electrical machinery, instruments, photographical and optical equipment and watches
12	Transport equipment except ships and boats
13	Electrotechnical products
14	Shipyards
15	Printing and miscellaneous products
16	Hotel and restaurant services, repairs, letting of premises other than dwellings, and private services other than bank, insurance and business services
17	Construction
18	Wholesale and retail trade, communications
19	Transport and storage
20	Financial and insurance services
21	Housing services
22	Public services
23	Capital goods[c/]

a/In this study fuels and electricity are aggregated to one single energy commodity. See p.

b/Excluding petroleum refineries and asphalt and coal products.

c/The capital goods sector is not a production sector but a "book-keeping" sector which aggregates different kinds of capital goods, primarily machinery and buildings, in fixed proportions to an aggregate capital good used in all "real" production sectors.

As was mentioned above (p.10) the budget constraint is satisfied identically in this case. In a later version of the model, the system of household demand equations will be replaced by a system with "better" properties.

The parameters in the import functions, i.e. the elasticities of substitution between imported and domestically produced commodities with the same classification, was calculated on the basis of a study by Hamilton (1979). In that study import equations of the same type as those in this study were estimated, but the commodity classifications in the two studies were not identical. Accordingly Hamilton's results could be used only after some aggregation and, in some cases, disaggregation. One can say that the estimates finally arrived at have some empirical ground, but very little can be said about the estimates in terms of conventional statistical criteria.

The export price elasticities were calculated on the basis of the import price elasticities in accordance with the following reasoning: "If domestically produced and imported commodities with the same classification are substitutes on the Swedish market, that should also be the case on other national markets. Moreover, in view of the fact that Sweden primarily trades with other highly industrialized countries, the price elasticities in the export functions should not differ much from those in the import functions. However, since Swedish producers can be expected to have a relatively stronger position on the domestic than on the international market, the absolute values of the price elasticities should be somewhat lower in the import than in the export functions." The problem is, of course, to determine the magnitude of "somewhat". This is obviously deep water, but some "guesstimation" was carried out, the result of which can be seen in Table 2.

As could be seen in the presentation of the model, it treats the demand for energy in a fairly elaborated way. For the moment we are not primarily concerned with energy demand, and in order to reduce the number of parameters to be estimated, fuels and electricity are aggregated into one single type of energy, produced by an aggregated energy sector.

Formally this means that the variable H_j in eq. (4) is replaced by the aggregated energy input, made up by fuels and electricity in sector-specific fixed proportions. The elasticity of substitution between the aggregated energy input and primary factors of production was assumed to be .25 in all sectors.

This assumption is somewhat crude, but defendable on the basis of the econometric literature in this field.[1/] Moreover, the assumption about the elasticity of substitution between energy and primary factors of production turned out to have an insignificant impact on the results presented in the following sections.

Since we, at the moment, only are concerned with the situation in 1975, we do not have to worry about the time-dependent parameters.[2/] The data base contains estimates of all exogenous

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1/See Bergman (1978) for a brief review.

2/That is, the rates of productivity increase and the growth of world market trade.

variables except the prices on international markets where Swedish exporters compete (the variables P_i^{WE}). The input-output table contains implicit price-indices for Swedish exports which is not exactly the magnitudes we are looking for. However, lack of time prevented more ambitious data collection work, and it was assumed that the price indices of Swedish exports in fact did coincide with the relevant world market price indices.

Table 2. Import and export price elasticities.

Sector number	Sector	Import price elasticity	Export price elasticity
0	Energy	1.0	-
1	Agriculture, fishing, basic food	1.5	-2.5
2	Forestry, wood, pulp and paper	0.8	-1.5
3	Mining and quarrying	1.0	-2.0
4	Other food, beverages, etc.	1.0	-2.0
5	Textile, clothing, leather	1.5	-3.0
6	Paper products	0.3	-0.6
7	Chemical products	1.0	-1.5
8	Non-metallic mineral products	0.5	-1.0
9	Metals	0.8	-1.5
10	Fabricated metal products	1.5	-2.5
11	Nonelectrical machinery, etc.	1.0	-2.0
12	Transport equipment	1.0	-2.0
13	Electrotechnical products	0.8	-1.2
14	Shipyards	1.0	-1.5
15	Printing and miscellaneous	0.8	-1.2
16	Hotels, restaurants, etc.	0.2	-0.3
17	Construction	-	-
18	Wholesale and retail trade, etc.	0.2	-0.3
19	Transport and storage	0.2	-0.3
20	Financial and insurance services	0.2	-0.3
21	Housing services	-	-
22	Public services	-	-
23	Capital goods	-	-

IV. 2. An equilibrium allocation of resources in 1975

In 1975 the Swedish economy entered a period which later has been called the years of the "cost-crisis". A combination of substantial wage increases and low rate of productivity growth made Swedish production costs increase considerably faster than world market prices. Consequently the international competitiveness of the trade-exposed part of the economy was reduced and net exports dropped substantially. In particular that was the case for the shipyards and some parts of the iron and steel industry.

The bad performance of the Swedish economy induced a number of policy measures including a couple of devaluations, and influenced the wage negotiations in 1977 and 1978 so that quite moderate nominal wage increases were agreed upon; for many groups the nominal wage increase was smaller than the increase in the cost of living. During 1979 many of the symptoms of a price-cost disequilibrium have disappeared and during the first part of 1979 the rate of growth of industrial production was high in relation to postwar standards.

The data for 1975 reveal signs of an emerging disequilibrium situation; the average rate of profit in the private sector was very low and intersectoral profit differentials were substantial. In a few sectors, for instance the iron and steel sector, losses were reported and profit rates were very low in the shipyards and the textile and clothing industries. The intersectoral profit differentials indicate that a different allocation of the economy's stock of capital could yield a larger total output of goods and services. In the following we regard the intersectoral profit differentials as signs of a disequilibrium in the sectoral allocation of resources in Sweden, and we set out to estimate the nature and significance of that disequilibrium. In order to estimate the magnitude of potential reallocation gains, or to get a rough measure of the degree of disequilibrium in the economy, the above presented model can be solved under the assumption that profit rates, and thus the marginal productivity of capital, is equalized within the private sector1/ of the economy. It was also assumed that the supply of labor was 2% higher than actual employment in 1975. In terms of the symbols used in the presentation of the model, the parameters β_0, $\beta_1, \ldots, \beta_{20}$ are set equal to unity, and the value of the exogenous variable N is set equal to 1.02 times the actual employment in 1975.

Below, results from such calculations are presented.2/ It should be noted that the intersectoral wage differentials, observed in the data for 1975, are assumed to reflect intersectoral differences in the composition of the labor force and are therefore imposed on the wage structure in the model economy. That

1/It is assumed that, like in the real world, the rate of return requirement on capital used in the public sector is equal to the rate of depreciation. Moreover, like in the real Swedish economy, the use of capital in the housing sector is subsidized in the model economy.

2/The idea of comparing an actual allocation with an efficient allocation, using a general equilibrium type of model, is not new. In a study based on input-output data for 1957 [Werin, 1965] used a linear programming model for such purposes. The results indicated that the actual allocation did not differ much from an efficient allocation. However, the model was quite rigid; all technological coefficient as well as the composition of household consumption was fixed, and the resource allocation could be changed only through changes in the pattern of foreign trade.

is, the observed values of the parameters ω_j are taken as given. In the same way the actual 1975 system of indirect taxes and subsidies is kept unchanged in the model calculations. However, the 1975 data reveal a deficit on the current account but the model calculations are carried out under the assumption of a zero current account deficit.

Table 3 contains the results for some macro-economic variables as well as for factor prices and the exchange rate. According to these results an allocation where the marginal productivity of capital is equalized within the private sector of the economy and total employment is 2% higher than it was in 1975 would yield a 4% larger GNP than the actual 1975 allocation. Since public consumption and net investments are exogenously determined in the model and assumed to be equal to actual 1975 values, the incremental GNP is divided between household consumption and net exports. The division between these final demand components is such that the share of household consumption in GNP is equal in both allocations.

Table 3. Actual and computed values for selected macroeconomic variables, factor prices and the exchange rate 1975.

	Actual value (1)	Equilibrium value (2)	(2):(1) (3)
GNP*	278.9	290.0	1.04
Household consumption*	144.7	151.0	1.04
Industrial production*	234.3	239.2	1.02
Wage rate**	1.00	.97	.97
Profit rate (%)***	.038	.068	1.79
Exchange rate****	1.00	1.03	1.03

*Expressed in 10^9 skr in actual purchasers' prices in 1975.
**Index of the wage level at a given sectoral wage structure.
***Net profits in the private sector in relation to the replacement value of the total capital stock in the private sector.
****Index. Domestic currency per unit of foreign currency.

It was mentioned above that the symptoms of a marked price-cost disequilibrium which became apparent during 1976 and 1977 induced devaluations of the Swedish currency and insignificant or negative real wage increases. The model results give some support for such a strategy; in comparison with an equilibrium allocation, wages were too high and the currency over valued. in Sweden already in 1975.

According to these results it seems that the material standard of living in Sweden in 1975 could have been significantly higher if the available real capital had been distributed over the production sectors in such a way that the marginal productivity of capital had been equalized within the private sector of the economy. This conclusion also holds if the assumption that employment could be increased by 2% is relaxed. In that case, the increase in GNP is, however, reduced to 2.5%. It should also be noted that equalization of the marginal productivity of labor in the economy, that is, relaxation of the assumption of a fixed wage structure, only led to a GNP-increase by approximately 1%. Moreover, variations in the assumption about the elasticity of substitution between energy and primary factors of production did not affect the results very much. Thus, different sectoral allocations of capital seem to be the crucial difference between the actual and the equilibrium allocations.

However, there is an income distribution problem connected with the evaluation of the equilibrium allocation. As can be seen in Table 3, the more efficient allocation of the capital stock leads to a higher rate of profit and at the same time a lower wage rate. Since factor supplies are given and constant, this means that the share of capital in the national income is considerably higher in the equilibrium allocation than in the actual allocation. Since capital incomes tend to be more unevenly distributed than labor incomes, the equilibrium allocation is not necessarily preferable to the actual allocation from a social welfare point of view.

Next we turn to the results obtained for the individual sectors, summarized in Table 4. In order to explain the results, a step-by-step reasoning is applied in spite of the fact that the results are obtained from an equilibrium model where the values of the endogenous variables are simultaneously determined. The starting point is that the equilibrium rate of profit is considerably higher than the actual 1975 rate of profit. The primary effect of higher rate of return requirements is that the production costs in the very capital-intensive sectors Energy and Housing services increases. The cost increase leads to higher output prices for these sectors, and the demand is reduced.

From the other sectors' point of view the reduced production in the Energy and Housing service sectors means that the supply of capital increases. Consequently they tend to change their technology in a capital-intensive direction. This tendency can be seen in Table 4; the ratio between equilibrium and actual values is generally higher for capital than for employment and production.

However, in the Metals sector and the Shipyards the increase in the capital intensity is relatively small. This is because the very low, or negative, profit rates in these sectors call for a reduction rather than an increase of the capital intensity. Thus, two counteracting forces are operating; increasing supply of capital tend to increase capital intensity while the adjustment of the marginal productivity of capital to the normal level within the private sector works in the other direction. The result is a slight increase in the capital-intensity.

No matter if the "size" of a sector is defined in terms of employment, capital stock or production, the Metals sector and the Shipyards would be considerably smaller in an equilibrium allocation than they actually were in 1975. These two sectors

Table 4. Actual and calculated employment (N_i), capital stock (K_i) and production (X_i) levels 1975[1/].

i	Sector	$\frac{N_i^0}{\overline{N}}$ (%)	$\frac{N_i^*}{N_i^0}$ (%)	$\frac{K_i^0}{\overline{K}}$ (%)	$\frac{K_i^*}{K_i^0}$ (%)	$\frac{X_i^0}{\overline{X}}$ (%)	$\frac{X_i^*}{X_i^0}$ (%)
0	Energy	1.0	.81	7.9	.88	4.3	.89
1	Agriculture, fishing, etc.	7.0	1.07	4.5	1.15	6.2	1.11
2	Forestry, wood, pulp, etc.	4.9	1.14	3.4	1.22	6.7	1.15
3	Mining and quarrying	.5	1.05	.9	1.10	.9	1.04
4	Other food, beverages, etc.	.9	1.06	.7	1.14	4.2	1.08
5	Textile, clothing, leather	1.9	.97	.7	1.04	2.5	.98
6	Paper products	1.1	1.00	1.8	1.08	1.5	1.04
7	Chemical products	2.1	1.04	1.3	1.11	3.3	1.05
8	Non-metallic mineral prod.	1.1	.97	.9	1.05	1.1	1.00
9	Metals	2.2	.83	2.4	.89	3.1	.87
10	Fabricated metal products	3.0	1.03	.9	1.10	2.7	1.04
11	Non-electrical machinery,etc	4.5	1.05	1.6	1.12	4.7	1.06
12	Transport equipment	2.5	1.04	1.1	1.12	4.1	1.06
13	Electrotechnical products	2.4	.79	4.3	1.09	2.5	.93
14	Shipyards	1.2	.65	.5	.70	1.3	.66
15	Printing and miscellaneous	2.1	1.08	.9	1.15	2.0	1.09
16	Hotels, restaurants, etc.	9.8	1.17	2.5	1.27	5.4	1.17
17	Construction	9.9	1.00	1.4	1.07	8.3	1.00
18	Wholesale and retail trade	7.3	1.00	6.3	1.07	8.7	1.02
19	Transport and storage	5.8	.92	6.0	.96	3.6	.95
20	Finance and insurance	3.6	1.05	.8	1.12	4.3	1.06
21	Housing services	.7	.84	32.3	.90	5.2	.90
22	Public services	25.0	1.00	14.1	1.02	13.2	1.00

1/0 = actual 1975 values, * = calculated values, — = aggregated actual values.

have in fact had great difficulties during the last few years and are now in a period of contraction. This result indicates that in spite of many far-reaching simplifications and weaknesses in its empirical basis, the model is able to elucidate some important aspects of reality.

However, Table 4 also contains some less satisfactory results. Thus, the calculated production levels in Agriculture, Forestry and, to some extent, Mining and quarrying, are considerably higher than the corresponding actual values. Since no significant expansion of these sectors has taken place since 1975, this result seems to be questionable.

In a narrow sense the reason is that the actual rates of profit in Agriculture and Forestry were quite high in 1975. Thus, in spite of the increase in the average rate of profit in the private sector, the profit rate equalization within that sector tends to reduce the rate of return requirement in Agriculture and Forestry. Consequently, the production costs in these sectors tend to decrease, which, in turn, induce a demand expansion.

There is, however, reason to believe that the profit rates revealed by the 1975 data are deficient for sectors highly dependent on natural resources. In these sectors there is a substantial amount of land rents in total capital income, while the capital stock estimates refer to physical capital and neglect natural resources. Thus, the capital income and the capital stock measures are not compatible, and if land rents are not excluded from the capital income measure, the rate of profit will be overestimated. Since no such adjustment of the capital income data was made when the data base used in this study was prepared, we have probably seriously overestimated the actual 1975 profitability of the natural resource based sectors.

In Table 5 the results for domestic production costs, import prices expressed in the domestic currency unit and the domestic price level for each of the commodity groups can be seen. It should be noted that the corresponding actual 1975 values were all unity. The most marked differences between actual and calculated values are found for Energy, Metals, Housing services, Transport, Forestry, Hotels and restaurants and Finance. In the cases of Energy, and Housing services the basic reason is the high capital intensity in conjunction with substantially increased rate of return of capital, while the cost increase in the transport sector partly depend on that factor, partly on the energy price increase. For the remaining of the somewhat extreme sectors, the major factor behind the production cost difference is the profit rate equalization within the private sector.1/

1/In Finance and insurance "unallocated bank services" were treated as capital income in the banking sector. This led to a very high profit rate for this sector in the data base.

Table 5. Calculated production costs, import prices and domestic prices for different commodity groups.

	Commodity produced by sector	Production cost	Import price	Domestic price
0	Energy	1.26	1.03	1.20
1	Agriculture, fishing, etc.	.97	1.03	.98
2	Forestry, wood, pulp, etc.	.87	1.03	.88
3	Mining and quarrying	.97	1.03	.99
4	Other food, beverages, etc.	.97	1.03	.97
5	Textiles, clothing, leather	1.04	1.03	1.04
6	Paper products	.97	1.03	.98
7	Chemical products	.99	1.03	1.00
8	Non-metallic products	1.06	1.03	1.06
9	Metals	1.20	1.03	1.13
10	Fabricated metal products	.99	1.03	1.00
11	Non-electrical machinery, etc.	.99	1.03	1.00
12	Transport equipment	.98	1.03	1.00
13	Electrotechnical products	1.12	1.03	1.08
14	Shipyards	1.05	1.03	1.04
15	Printing and miscellaneous	.91	1.03	.92
16	Hotels, restaurants, etc.	.77	1.03	.78
17	Construction	1.00	1.03	1.00
18	Wholesale and retail trade	1.07	1.03	1.07
19	Transport and storage	1.30	1.03	1.26
20	Finance and insurance	.75	1.03	.78
21	Housing services	1.15	-	1.15
22	Public services	.97	-	.97
23	Capital goods	.95	-	.95

The results presented in Table 5 should also be interpreted with an eye on the estimates, or the assumptions made, of the price elasticity parameters in the import and export functions. If the ratio between the domestic cost and the import price is close to unity, the numerical value of the price-elasticity parameter is fairly unimportant. The same applies for the corresponding parameter in the export functions when the relation between domestic production cost and the price, expressed in the domestic currency unit, on international markets where domestic exporters compete is close to unity. It should be noted that as a consequence of the assumptions made, import prices coincide with the prices charged by the domestic exporters' foreign competitors.

The results indicate that the price elasticity parameters are quite important in Forestry and Electrotechnical products. Together these sectors accounted for 21% of total exports and 9% of total imports in 1975, and the ratio between calculated production costs and world market prices (expressed in the domestic currency unit) is much different from unity. However, for several important foreign trade sectors, the import and export price elasticities are not that important. That is the case for Transport equipment (11% of exports, 8% of imports), Non-electrical machinery (17% of exports, 14% of imports) and Chemical products (6% of exports, 11% of imports).

It is quite obvious that the model gives a very simplified picture of the Swedish economy. Nevertheless it seems to be able to elucidate some important aspects of the economic situation in 1975; an overvaluated currency, a somewhat too high wage level and too much resources tied to a couple of sectors with significantly reduced international competitiveness. Some of the results are dubious, but at least part of the problem can be overcome if land rents are treated in a more satisfactory way.

IV. 3. Some comparative statics

Clearly the assumptions made about the supply of capital and labor are quite strategic for the results obtained from the model. In order to indicate how sensitive the results are to these assumptions some comparative statics experiments were carried out. Thus, the assumptions about the supply of capital and labor were both varied ±10%. The resulting impact on GDP and relative factor prices can be seen in Diagrams 1 and 2. The diagrams indicate that the GDP-estimate is not very sensitive to the assumptions about capital and labor; when the supply of capital or labor was varied between 90% and 110% of the original values, the GNP-estimate only varied between 93% and 107% of the value obtained in the solution presented in the preceding sub-section.

However, variations in the supply of capital and labor had a significant impact on relative, and to some extent also absolute, factor prices. These variations were then reflected in the commodity prices, but it was only for Energy and Housing services that the variations in capital and labor supply had a significant impact on production costs.

DIAGRAM 1

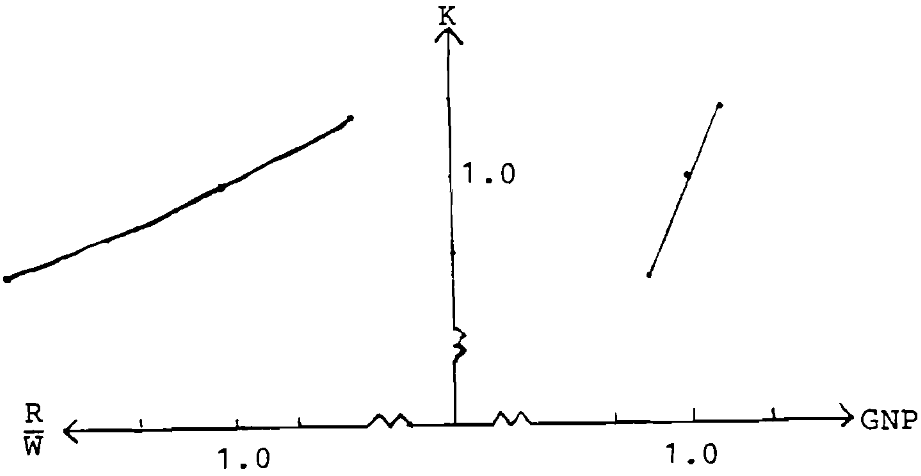
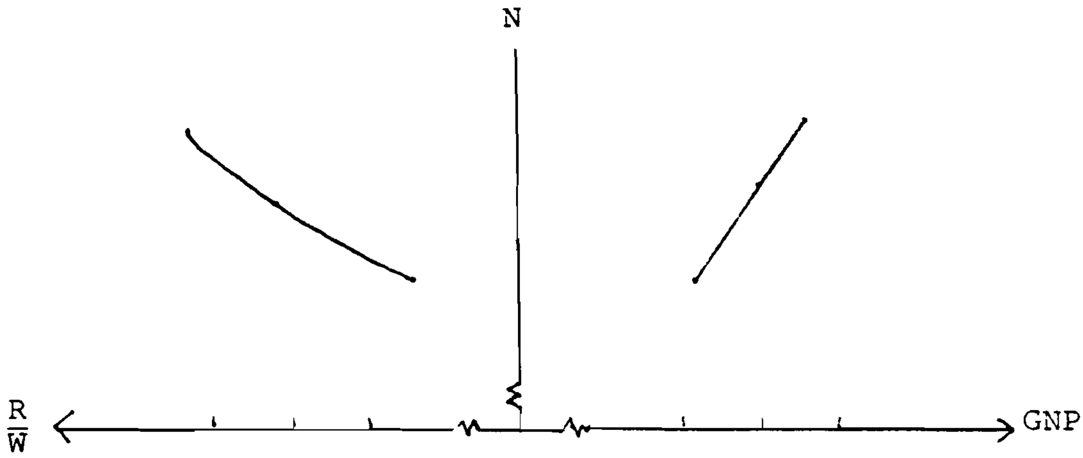


DIAGRAM 2



As was briefly mentioned in the preceding sub-section, the sensitivity of the solution with respect to the elasticity of substitution between energy and primary factors of production as well as to the wage-structure was investigated. In both cases the impact on GDP, private consumption and the sectoral structure was rather limited. Of course the elasticity of substitution between energy and primary factors of production turned out to be an important determinant of energy demand and thereby the size of the energy sector, but within reasonable limits the values of these parameters did not have a significant influence on the main features of the solution. In the same way, a complete wage equalization between the production sectors lead to increased production costs and reduced demand in some low-wage sectors, but again the main conclusions from the model simulation remained unaffected.

V. SOME FUTURE DIRECTIONS OF RESEACH

It seems reasonable to conclude that the model in its present version works satisfactorily, although a lot more work has to be put into the estimation of parameters. In that work it is quite possible that less restrictive functional forms for the description of technology and household demand will be chosen. The nested CES-Cobb-Douglas production functions used in the present version imply a number of restrictions on the substitutability of various inputs. In light of empirical evidence these restrictions might turn out to be unrealistic.

A rather attractive alternative approach to the specification of technology is to use cost rather than production functions. By using, for instance, a generalized Leontief cost function, the restrictions on the elasticities of substitution, implied by the conventional production functions, can be relaxed.1/ However, it is reasonable to let the final choice of technology representation be determined by econometric considerations.

As was mentioned in connection with the presentation of the model, the fact that the constant-elasticity household demand equations do not satisfy the budget constraint led to problems which were overcome with some very heroic assumptions about the parameters in the equations. This is reason enough to replace the demand system with a system with "better" properties, but again the final choice of specification should be governed by econometric considerations.

More fundamental changes of the model would be related to the treatment of capital and capital formation. In the present version of the model, capital is malleable, and accordingly the model can only highlight the properties of alternative long-run

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1/ In a modern variant of the Norwegian MSG-model, focused on the energy flows in the economy, the production structure is represented by generalized Leontief cost functions. See equilibrium allocations of a given amount of capital. However, in many cases one is interested in how the economy can move from a given short-run equilibrium, or even disequilibrium, to some kind of long-run equilibrium. A malleable capital model cannot say very much about such problems.

In order to improve the model in this respect, the capital stock should be regarded as tied to the sectors in which the investments were made. Moreover, a distinction between the _ex ante_ and the _ex post_ substitutability of inputs would also represent an improvement of the model. Once such a development of the model is made, it is a natural step to distinguish between different vintages of capital, and to include investment functions and explicit intertemporal links. That would, however, require that sectoral investments are determined within the model. This can be done either by incorporating explicit investment functions for the production sectors, or by turning the model into an optimization model for determination of preferred multi-sectoral growth paths.

A methodology for estimating the properites of the production technology in such a way that _ex ante_ and _ex post_ elasticities of substitution can be distinguished has been developed by Fuss [1977]. In an extended version of the present model Fuss' approach could very well be incorporated. The basic econometric problem in the estimation of production structures with _ex ante_ substitutability differing from _ex post_ substitutability seems to be that explicit expressions for the formation of investors' expectations are required. In addition, the total capacity in the production sectors has to be decomposed into "vintages", and the use of inputs has to be assigned to these vintages. Such data is not easily available, and it thus seems that the practical problems connected with a more realistic treatment of capital are significant.

However, if a vintage capital approach could be implemented in a model with explicit intertemporal links, the treatment of the relation between world market prices, domestic production costs and structural change in the model would be considerably improved. (See the discussion on p 13f.). Moreover, with a model elaborated along these lines the comparison of actual and hypothetical equilibrium resource allocations would be focused on investments rather than the allocation of the entire capital stock. This means that results from calculations in the same spirit as those presented in this paper might become more useful from an economic policy point of view.

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