PROCEEDINGS OF JOINT TASK FORCE MEETING ON DEVELOPMENT PLANNING FOR THE NOTEC (POLAND) AND SILISTRA (BULGARIA) REGIONS

VOLUME I

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PREFACE

In 1978, two case studies, focusing on development problems of the Notec (Poland) and Silistra (Bulgaria) regions, were initiated at the International Institute for Applied Systems Analysis (IIASA) within the Regional Development Task. Work on these studies is being carried out by IIASA scholars in collaboration with members of Polish and Bulgarian institutions, respectively, and preliminary findings were published between 1978 and 1979 by IIASA and the Systems Research Institute of the Polish Academy of Sciences.

Since 1978, considerable progress has been achieved. The intermediate stage of the case studies is now complete and the results are documented in these Proceedings (volumes I and II). The papers included in this publication were presented at the joint Notec-Silistra meeting, which took place at IIASA on 28 May-1 June, 1979. This meeting provided the first opportunity for the three groups participating in the investigations to compare approaches, models, and results, thus stimulating further methodological development and an extension of case-study activities. The final stage of the Notec and Silistra case studies is now underway.

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INTRODUCTION

THE SILISTRA AND NOTEC CASE STUDIES-- A REVIEW OF WORK UNDERTAKEN AND UNRESOLVED PROBLEMS

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The International Institute for Applied Systems Analysis (IIASA), being a research organization of multinational and multidisciplinary character, focuses not only on contributing to the advancement of science by developing and formalizing new approaches and methods of systems analysis, but also on applying these methods to solve problems of international importance. When it was decided that case study analysis should form one of the main activities undertaken by the Regional Development Task at IIASA, it was anticipated that this experience should lead to the generalization of approaches used and models implemented. The regional case studies were, therefore, chosen with the aim of embracing a broad range of regional problems and, in addition, for the purpose of analyzing how similar problems can be solved under different economic and political conditions.

SILISTRA AND NOTEC PROBLEMS

The initial negotiations for a collaborative study of the Silistra region took place at the beginning of 1977 between IIASA and the Bulgarian National Member Organization and, in the spring of the same year, a formal agreement was signed. Early in 1978 practical investigations began and, in October of that year, the first Task Force meeting took place. Preliminary investigations of the Silistra region indicated that the socioeconomic problems existing there were wide-ranging.

Although the agricultural sector is dominant in the region, not all the possibilities for agricultural growth have been exploited. It is expected that in the future the region will significantly increase agricultural production (mainly livestock products) but the optimal proportions between livestock, crops, and home-produced agricultural products have yet to be carefully defined and the effect of extending the regional irrigation system should be clarified.

Current rural-urban migration must be analyzed from the viewpoint of the relation between rural labor resources and agricultural production maximization. The use of existing housing funds in rural areas should also be considered, since rural-urban migration has led to underutilization, while in the urban areas the construction of additional housing is required as a result of the increased demand.

Although the Silistra region is some distance from the country's industrial and cultural centers and the largest proportion of the Silistra population is employed in the agricultural sector, it is still necessary to increase the level of skilled labor. The solution to the regional development problem requires

-- the maximization of regional agricultural production, taking into account not only the advantageous natural conditions for grain and meat production, but also the feasibility of increasing specialized local crop production (apricots, grapes, and vegetables);
-- the development of an irrigation system to increase local agricultural efficiency;
-- the development of a complementary local industry to stem the outflow of labor (e.g., fertilizer production, food processing, agricultural machinery production);
-- the maximization of the productive use of labor resources in local agriculture and the restriction of rural-urban migration;

-- the development of a system of settlements and services, considering primarily the need to use fully the existing housing stock in rural areas and to improve road networks, the health care system, services, etc.; and
-- the development of local agriculture and industry without creating serious environmental problems, reserving a zone for recreation.

At the end of 1977, a formal agreement was signed by IIASA and the Polish National Member Organization to organize a joint case study of the Upper Notec region. This work progressed rapidly and during 1978 two Task Force meetings took place to discuss the results: the first on May 10-12 at IIASA and the second on September 12-15 in Jablonna, near Warsaw.

The Upper Notec region specializes in agriculture, but production is limited by water shortages. Almost every second year the rainfall in inadequate during the vegetation period and this situation is aggravated by the increasing water demands from the urban population, industry, and water transport. The expansion of water supply should, therefore, be considered as a goal complementary to agricultural development.

The main goals associated with regional agriculture are

-- to intensify regional agriculture, by fully utilizing its productive capacities and by maximizing agricultural production; and
-- to reduce the disparity in agricultural development, and hence productivity levels, between the northwestern and southeastern parts of the region.

Regional agriculture is characterized by a significant differentiation between the southeast and northwest of the region. This is mainly a result of historical developments going back to the period when the northwestern part belonged to Germany, and the southeastern to Russia. In the northwest of the region, the size of farms was larger and the land was cultivated more intensively; such conditions encouraged rapid agricultural development.

Today agriculture in the northwest is more advanced. This area has a higher proportion of large socialized farms, higher capital inputs, lower labor inputs, and higher productivity. In comparison, agriculture in the southeast is backward.

The future of private farms, which cover a significant portion of the region's total area, should be considered in terms of the farmers' incentive to intensify cultivation of their land, their response to the possibilities of using additional supplies of water, the rate of dissemination of progressive methods among private farms, etc. The image of the future farm (initially of the private farm) and the capability of predicting this accurately, is of key importance for the success of the Notec project.

The existing situation in the Upper Notec river basin has been characterized in past years by a negative migration balance, which may be attributed to its predominantly rural structure. The annual net out-migration, however, has been small, amounting to approximately 0.5 percent of the total population, as compared to the natural increase of 1.1 percent annually.

Large areas of the basin are contained within the migration fields of the cities of Bydgoszcz and Torun, and to a lesser extent, within the fields of more distant provincial capitals, in particular of Poznan and Gdansk.

The future balance between sectoral demand for and supply of labor in the regional economy depends primarily on the regional agricultural situation. If changes in the structure of local agriculture only lead to structural changes in employment, this problem will not be serious. However, if local agriculture releases a significant proportion of its labor force, then migrant flows might become significant. In this case, major intraregional cities, such as Inowrocław, and adjacent cities, such as Bydgoszcz, Włocławek, and Konin, will play an important role in absorbing the excess labor supply. Therefore, the number of future jobs in regional agriculture should be estimated together with rural-urban migration flows.

Additional problems exist, of which the following should be mentioned.

-- The character of agricultural areas will alter when additional water supplies are provided; this may affect pri-private farmers' incentive to itensify land use and may stimulate structural changes in agriculture.
-- In the future the system of technical and social services must be improved and facilities for tourism should be developed.
-- The system of rural and urban settlements must be improved; certain areas should be selected for development.

A comparison of the conditions in the Silistra and Notec regions indicates that they have many common features and suffer from similar problems. Both regions are mainly agricultural, having a complementary industrial sector and few specialized services. The problem of irrigation is very important in both regions and future regional agricultural growth depends on its efficacy. Migration (rural-urban, region-region) is a common problem, which needs urgent attention. Increasing the quality of life for the populations of both regions is of special interest to the central and local authorities, and, in both cases, the development of the water supply system would make it possible to create recreation zones (on a limited scale).

At the same time differences exist between the regions:

-- in Silistra, agriculture is organized on the basis of state-owned agroindustrial complexes, whereas in Notec many different types of farms exist;
-- in Bulgaria centrally made decisions can be implemented with relative ease through the system of state farms, whereas in Poland the private sector has to be considered; and
-- the directions of agricultural specialization in Silistra have now been formulated more definitely (priority is given to the maximization of meat production), but in Notec specialization can only be defined after assessing the effectiveness of the irrigation system and the private sector.

GENERAL METHODOLOGY

During the first Notec and Silistra Task Force meetings, several proposals about the methodology of regional problem analysis were put forward.

A scheme for the system of models was presented in Albegov and Kulikowski (1978). This system is based on a four-stage analysis, in which several disaggregated sectoral models are included. The following problems are analyzed in sequence:

-- regional specialization,
-- intraregional location of sectoral activities,
-- population and labor resources, and
-- settlement systems and pollution.

A method has been devised for analyzing intersectoral relations to reduce the size of problems and to coordinate the sectoral solutions, thus allowing detailed sectoral models to be included in the model system. The method involves the preparation of "reaction" functions, derived from the near-optimal sectoral solutions, for coordinating the main blocks (models) followed by implementation of the Bellman approach to problem solution. The reaction function demonstrates the effect on the regional economy of the use of limited resources by different sectors.

The first and second and the third and fourth stages of analysis are coordinated by matching the productive activities with the number of people willing to live in the region, given the wage level and the quality of life. Furthermore, capital and natural resources are coordinated with labor resources.

The choice of the optimal solution is based mainly on information about the "environment" outside the region, in the form of the external price system. Therefore, this approach can be viewed as a "bottom-up" approach.

The main problem that occurs at this point is how to make the system operational and capable of running all the calculations necessary for providing solutions to the scenarios and for assessing the consequences.

The main aim of the approach, which was formulated by Andersson and La Bella (1979) is to

> ...organize a system of models, which would be such that they could highlight essential decision problems at different levels of decision making in the form of scenarios, all consistent within the framework of the different models...These different scenarios could then be used...as an input into discussions of the policy making process.

The hierarchy of models proposed is as follows:

-- international trade model,
-- national model,
-- regional model,
-- physical planning and allocation models, and
-- migration and commuting model.

One of the most important models in the proposed scheme is the trade-transportation equilibrium model, which has linear constraints and a nonlinear objective function that uses the "minimum reorganization" principle. The trade-transportation model produces the most essential information for the dynamic allocation model of future regional production and the employment structure.

A more important question here is whether the level of disaggregation of data in this system will be adequate to achieve a practical solution.

The models used for the Notec case study fall into three groups relating to

-- regional benefit,
-- water system expansion,
-- water resource management.

The general methodology, formulated in Albegov and Kulikowski (1978), includes two different approaches that will be combined:

-- the use of nonlinear dynamic models of aggregate consumption and benefit as a basis for calculating some important regional growth incentives; and
-- the construction of a system of models in which the most important sectors are described in detail.

The first approach is used in Kulikowski (1978a and 1978b). The second approach is employed in Kulikowski (1978) and has been developed in Makowski (1978a).

In the first approach very aggregated data from the past are used for estimating production functions, economic benefits, etc. Using this as a basis for further calculations, an optimal solution indicating the optimal interregional allocation of capital investment, or the optimal number of persons employed in each region, can be found.

This approach has the same limitations as any econometric approach in which data from the past are used to estimate future growth. However, when the above-mentioned approach is used together with detailed sectoral models, practical implementation seems possible.

In 1978 work began on the development of a system of models for an agricultural region. It was planned that the system would consist of the following models:

-- a long-term normative model of integrated development (MRI);
-- a model of regional development (MRD);
-- a model of regional agriculture (MAG); and
-- a model of the water resource system (MWS).

This work has not been completed, yet it is interesting to make a comparison of the different methodologies for regional analysis using the preliminary results.

To determine the most suitable methodology for regional growth analysis, the practical results of implementation should be obtained. This stage has not yet been reached and, therefore, for the present, the separate models of the general system should be assessed for their ability to provide practical recommendations for decision makers.

AGRICULTURE

During the second Notec and the first Silistra Task Force meetings, several reports devoted to agricultural problems were presented. In some reports agriculture is analyzed in general; and in others specific problems are considered.

In the first group, the following can be included:

1. Csaki and Propoi (1978),
2. Popchev et al. (1978), and
3. Gavrilov et al (1979).

In the first report (Csaki and Propoi), a general dynamic linear programming model for long-range planning of a diversified crop-livestock agroindustrial complex is discussed. This model is characterized by its comprehensive description of the interrelations within the complex and by its dynamic problem analysis. At the same time, it should be underlined that this model is only useful for regional analysis if conditions in the region are homogeneous. If they are not, intraregional data should be introduced.

A model of large agroindustrial complexes is also described in the report by Popchev et al. This static linear optimization model is designed to evaluate the impact of limited regional resources on the overall production process in both planning and operational activities; the usual type of constraints (land, water, labor, machinery, products, etc.) are included. The impact of the input of resources (namely, water resources) is investigated and as a result, a "loss function", which aids the decision maker in choosing the appropriate policy, is constructed.

The model described in the report by Gavrilov et al. is a further development of the work by Csaki and Propoi. After an examination of more than 60 variants of the model, several were chosen and they form the basis for the development of the agroindustrial complex Drastar.

The main advantage of this model (to be precise, this system of agricultural models) is that it provides practical results and checks the reliability of the input information. The authors are also considering the possibility of linking this model with other models (of settlements, transportation, etc.). It is important to find out whether this model can be used in other regions in Bulgaria. At least one factor--the spatial factor--should be considered in the model.

The second group of reports (devoted to particular aspects of agriculture) include

1. Manteuffel et al. (1978), and
2. Podkaminer et al. (1978).

The first report describes a static integer programming model that defines the production structure and its spatial distribution over areas suitable for irrigation. At the first stage of analysis, the state and private farms are analyzed, and at the second, regional problems in general are considered.

The model solution, which recommended irrigation for approximately 25 percent of land with an irrigation potential, was practically implemented in the Notec region.

The report by Podkaminer et al. presents a model that allows consistent agricultural production programs rationalizing water use to be determined. The scenario approach to the description of external conditions and multiobjective and vector optimization are discussed. The same methodology was used to build, in effect, a small linear programming model determining the minimum monthly water demand. The results of this model have not yet been received.

It is clear that significant advances have been made in modeling regional agriculture in general and with respect to water use. Important methodological and modeling work was carried out at IIASA, as well as at various institutes in Bulgaria and Poland. Some of the models were used to make practical recommendations to decision makers (see, e.g., the reports by Gavrilov et al. 1979 and Gouevsky et al. 1979).

In the future, intraregional differentiation should be included in the models.

WATER SUPPLY AND MANAGEMENT

The reports in which water supply and management problems are discussed fall into two categories: those dealing with models of water supply and water use and those presenting models of water distribution and control.

The first group includes the following:

1. Makowski (1978b),
2. Albegov et al. (1978),
3. Krus (1978),
4. Gouevsky et al. (1979), and
5. Somorowski (1978).

The report by Makowski describes a model of water system development, which fits into the system of models that estimate the consequences of various development alternatives from which the optimal solution with respect to an assumed criterion (criteria) is derived. The goal of the model is to maximize the net benefit to society (i.e., to integrate the supply and demand models). It is important that the model of water system development (MWD) describes the region under analysis as a number of subregions and considers agricultural water demand as well as the water demands of other sectors of the regional economy.

MWD includes

-- a model of agriculture,
-- a model of disposable water resources,
-- a model of disposable water resource use,
-- a model of the investment sequence, and
-- a model for the analysis and evaluation of scenarios.

The models employ linear programming (LP) techniques. It is planned that optimization procedures will be included to help estimate the economic consequences of different scenarios.

In the paper by Albegov et al., an LP technique is used to model the regional water supply system. This system is rather simple and is designed to be used as part of the system of regional development models. Silistra regional data were used to obtain practical results, the most important of which are a system of marginal water costs used in different subsystems and during different seasons. This indicates that important data can be obtained from relatively simple models.

The model developed by Krus could be used to analyze in aggregated form regional specialization problems related to water allocation. The aim was to construct a model to be used for

- evaluating development strategies of selected sectors of the regional economy;
- calculating the water demand function (i.e., the function describing the amount of water desired at a given price);
- finding the optimal allocation of a given amount of resources; and
- evaluating exogenous variables for more detailed models of particular sectors, such as the models of agriculture and water expansion.

The author used the Cobb-Douglas production function for describing sectoral development,a nonlinear approximation for presenting the water cost function; the water supply-demand coordination problem was then analyzed.

No practical results have yet appeared,but it seems that the general approach could be employed to determine the optimal allocation of any resource. The model can be used at the top level of regional analysis; lower-level models should provide it with aggregated input information based on detailed sectoral analysis.

In the paper by Gouevsky et al., a system of models for detailed analysis of regional water demand, which coordinates the agricultural water demand and supply models, is presented. Water is considered in great detail: agricultural consumption (crop and livestock production and processing); potable water supply (population and livestock); transportation (and other in-stream uses); and recreation (and other on-site uses).

The central model is that of agricultural water demand, of which three versions exist. In the last version, the Silistra region is subdivided into five subregions, allowing calculation of the difference in soil fertility, climatic and weather conditions, costs of water supply, etc. The interactions between the model builders and the Silistra decision makers have provided the stimuli for developing the model.

Preliminary results have already been obtained; for example, it has been shown that the level of irrigated land in the region is quite sensitive to the price of water.

The next step is to integrate the demand and supply models in order to obtain economic equilibrium solutions for the Silistra decision makers. These two models will be coordinated with other regional models in the future.

In the report by Somorowski, the future trends in agricultural growth in the Notec region are discussed. Two development alternatives--growth without extensive irrigation and growth with irrigation--are considered and the consequences of both alternatives are assessed.

In the second group of reports on water distribution and control, the following can be included:

1. Gutenbaum et al. (1978),
2. Pietkiewicz-Sałdan and Inkielman (1978), and
3. Babarowski (1978).

In the first, the problem of optimal water distribution is analyzed taking into account the multicriteria nature of this problem and the need to minimize the losses due to water shortage or excess. The model used is stochastic in character and large-scale (describing a large number of irrigated fields and 10-day periods). Efforts are made to overcome these difficulties by applying methods of multilevel optimization.

No results have yet been reported but the usefulness of this approach for solution of the operational control problem is obvious.

The report by Pietkiewicz-Sałdan and Inkielman is a further development of the previous investigation with respect to decision rules to be applied for the Upper Notec basin. Two types of rules are considered: those concerning the user policy of water demand, and those concerning the supply policy. The conditions required to achieve the optimal solution are discussed but no practical applications are demonstrated.

The report by Babarowski is also a further development of the first work (Gutenbaum et al.), in which the problem of optimizing limited water resource distribution among crops is mathematically formulated and the operational rules for problem solution are defined. Examples of practical implementation or test cases are not given.

Although the number of "water-oriented" investigations is relatively large, they are very different from the viewpoint of completeness; while some of them (Gouevsky et al,; Albegov et al.) have already provided practical results, many are still at the stage of theoretical analysis.

In comparison with other sectors of the regional economy, a significant part of the reports concentrate on the solution of current problems. This reflects the nature of the water supply problem and the traditional development of research in this field. At the same time, efforts to coordinate the water supply-demand model with other regional sectoral models are underway,but this work has not yet been completed.

POPULATION AND MIGRATION

Investigations in the field of population and migration are documented in the following papers:

1. Kulikowski (1978b),
2. Arcangeli and La Bella (1978),
3. Naumov (1979),
4. Philipov (1979), and
5. Mihailov (1979).

The report by Kulikowski describes a nonlinear, dynamic consumption model, which shows the impact of regional welfare policy on interregional migration. The optimization problem of migration is investigated by comparing aggregated consumption of the regions (i.e., government expenditures on education, health care, housing, environmental protection, etc.).

Using official statistical data, the author carried out an estimation of the parameters of regional accessibilities for 17 Polish voivodships in 1973. The results are compared with the real situation.

An econometric model, in which the migration rate depends on the interregional utilities, is presented. The main idea of this model rests on the assumption that the potential migrant bases his decision to move on the ratio of utilities and a comparison is made with another region or with the national average. This model has not yet been practically implemented.

In the report by Arcangeli and La Bella, the broad spectrum of mobility factors, migration and regional development policies is considered on the basis of Italian data. For the IIASA case study, the approach to interregional migration analysis is most important. The assumed propensity to migrate from one region to another is explained by natural factors and proved by testing the model.

Empirical analysis confirms the main points of the theory elaborated: a positive feedback exists between population and capital movement, in the sense that the drain of labor from the less developed to the more developed areas within a country causes a drain of financial resources, which, in turn, increases regional differentials and, consequently, the migration rates. In this case, only external causes and public intervention can stem the flow of human and financial resources.

Naumov's report deals with two topics: the initial hypotheses and criteria for regional population growth, and some characteristics of the demographic growth and economic activities of the Silistra regional population. Some basic initial data for population and labor resource modeling of the Silistra region are included in this paper.

Philipov uses a multiregional demographic model developed at IIASA in the Human Settlements and Services Area to estimate future population growth in the Silistra region. This work is a practically oriented study, which reveals many interesting relationships. In particular, it shows that, on the basis of existing tendencies, one can expect a further rapid growth of the regional population and a significant out-migration flow. The main problem is to combine this model with a socioeconomic model that can provide the demographic model with valuable external information.

In the report by Mihailov, an overall scheme of the optimization cycle of the system of models for integrated territorial development of the region is outlined. Migrations are modeled in detail according to their place at different stages of the optimization cycle. In this respect, the following features of the migrations and their links with the remaining subsystem models of the regions are modeled.

-- Projections are made of the population growth without migration and including migration (preliminary results for the Silistra region are obtained).
-- The required labor force is derived in terms of the regional input-output model.
-- The required labor force stemming from the subsystem optimizations is derived.
-- The place of the labor force in the strategic models for development of the region is determined and the effect of migrations is defined.
-- Expected migrations are modeled, taking into account the propensity of the population to migrate.
-- Migration regulation is modeled on the basis of reducing the expected migrations to the required effective migrations.

The main objective of the investigation is to improve both migration modeling and the system of models for integrated regional development.

It is clear that the problem of population growth and migration are the main points of concern in several reports. The high level of theoretical analysis and the practical nature of the results is also remarkable.

The central point here is the task of combining the demographic model with other models (of a socioeconomic character). Only in this way is it possible to describe the interdependence between economic growth and demographic changes.

SETTLEMENT SYSTEMS

Only one report, presented at the first Silistra Task Force meeting, analyzes the problem of settlement systems, although during the first Notec Task Force meeting this topic was given much attention.

The one report by Devedjiev et al., entitled: "Human Settlement Systems as a Device for Development and Improvement of the Settlement Network in the Silistra Region", contains important data characterizing the existing system of settlements in the Silistra region. These data provide a sound basis for analyzing regional settlements growth in the future. Such analysis will take place in the framework of the Unified Territorial Plan of Bulgaria.

The plan was conceived to

- further the development of the polycentric structure of the settlement network in the region;
- ensure efficient use of existing housing funds in each part of the region;
- form a system of settlements, labor, housing, and recreation zones, and public services; and
- organize regular public transportation between the settlements.

Analysis of the system of settlements in Silistra has revealed some tendencies for future socioeconomic development, which are discussed in the following section.

PROBLEMS FOR FUTURE DEVELOPMENT

As can be seen from the previous section, not all existing problems are dealt with by current activities and not all existing investigations have been developed satisfactorily.

Significant work on the development of a system of regional models and the analysis of problems of agriculture, water supply, migration, and, to some extent, human settlements has been accomplished. However, the problems of industrial growth and enterprise location, transportation and services development, and environmental protection have received scant attention.

The positive and negative features of the model system can only be assessed by attempting to solve the real problem; hence, the importance of the role of the case studies. The more attractive approach to problem solution appears to be to elaborate workable subsystems of the regional economy, which can be used as separate models or as modules of a more comprehensive system.

The sectoral models developed require further improvement. For example, in the agriculture models, factors such as space and water were not tested satisfactorily. From the viewpoint of generalization, the need to divide the regional territory into 30-50 subregions should be considered (to reflect better the intraregional differences in soil, farm type, technology, etc.). The same can be said for the water factor: water resources and costs depend significantly on use (potable water for irrigation, etc.), the location of consumers (distance from the water flow), and the season in which water is consumed, etc.

Development of the agriculture models is not yet complete with respect to generalization of the different types of farms (private, state, or collective) and the dependence of agriculture on the labor market, capital, and other resources.

Models of water supply must be satisfactorily linked with other models (industry, agriculture, services) in which the water demand should be specified, and the relationship between natural limits and monetary data (costs, shadow prices) should be established. This data exchange should enable water demand to be described in detail (by all sectors, in all seasons, in different subregions, etc.); it should supply all consumers with a reliable basis for the local optimization of water demand.

At the same time, workable but complex models of regional water supply must be developed. These models should be focused on a general case (few water streams, few natural and possible artificial reservoirs, etc.). The dynamics of water demand must also be taken into account.

The water planning models and water management and control models in current use should be exchanged for more up-to-date models; water management and water control models should be simultaneously tested. Reliable data are required to ascertain the effectiveness of the proposed models under different economic conditions (for state and collective farms). These models should accurately describe the existing water demand under different weather conditions and should indicate the optimal distribution of limited water resources.

The regional migration model should also be generalized. Thus, the factors determining migration should be sufficiently large in number to embrace the most common situation found in all IIASA's National Member Organization Countries.

First attempts are underway to transfer the Italian experience to Bulgarian conditions. Independent of the success of this transfer, it is necessary to combine the demographic model developed in the Human Settlements and Services Area of IIASA with the regional migration model. Without such a combination of models it is impossible to predict future changes in the regional labor force.

With regard to the system of settlements model, the implementation of mathematical models and the development of improved models of rural and urban settlements should be intensified. Unfortunately, the formal description of the problem is still not advanced in either case studies.

Some very important problems that are not represented here have yet to be solved--first, industrial growth, industrial location, and environmental protection in a regional context.

Although some countries (e.g., the USSR) have significant experience in dealing with industrial growth and location, the transfer to IIASA of national experience in the form of models and programs is a complicated and time-consuming process.

Such activities must, therefore, begin immediately.

Since the function of the regional system of models is to provide a detailed analysis of the region, detailed environmental protection models may be included in the system. Work on the Silistra case study has begun in Bulgaria, but it is still at an initial stage.

In addition to the development of regional models, three important aspects of the investigations require further attention:

-- creation of a workable system of models that includes the most important subsystems (agriculture, industry, etc.);
-- development of a model to assess the consequences of different approaches; and
-- coordination of the interests of the different authorities.

The number of subsystems that should be considered in decision making within a particular region depends on local conditions. This means that in a realistic situation only part of the full set of subsystems is important and that the system of models will have a changing structure. Therefore, only the more important subsystems need to be included in the system of models at the first stage of analysis. For the Silistra and Notec regional study, the four subsystems to be linked are: regional agriculture and industry, regional water supply, and the regional labor force. Of course, it is hoped that a regional settlement model will be included in the above system but, as earlier stated, work on this is not yet complete.

When a relatively sophisticated system of models is used, the speed of calculation of the results is important, especially if many scenarios are examined.

In order to solve this problem, some general characteristics can be used to replace a very sophisticated description of the separate subsystems. Using these characteristics, one can evaluate all the solutions for a number of scenarios and, using a multicriteria approach, choose one (or several).

The use of an aggregate description of the functioning of separate subsystems together with a cost-benefit function for every player seems to be the most promising approach to dealing with the problem of coordinating the interests of different bodies (authorities, etc.). Game theory will be employed to solve this problem.

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PART I

General Problems

STRATEGIC PLANNING FOR THE DEVELOPMENT OF A REGIONAL SYSTEM

E. Christov
O. Panov

INTRODUCTION

The Silistra Regional Development Project, which is being carried out in collaboration with the International Institute for Applied Systems Analysis (IIASA), is a pilot project designed to improve planning and management of the Silistra region in Bulgaria. A case study aimed at finding new methods and techniques for developing and managing regional systems forms a part of this project. Reports on the regional models under development were presented at IIASA's 1978 Silistra Task Force meeting. This paper describes a further stage in the work of the project--strategic planning* for the development of a regional system--and outlines the conceptual framework on which the project is based.

The problem-solving process consists of three interrelated elements (Figure 1):

-- problem analysis,
-- choice of methods for problem solution, and

*We define strategic planning as a series of planning activities resulting in the formulation of long-term (15-25 years) socioeconomic objectives and of programs for accomplishing these objectives.

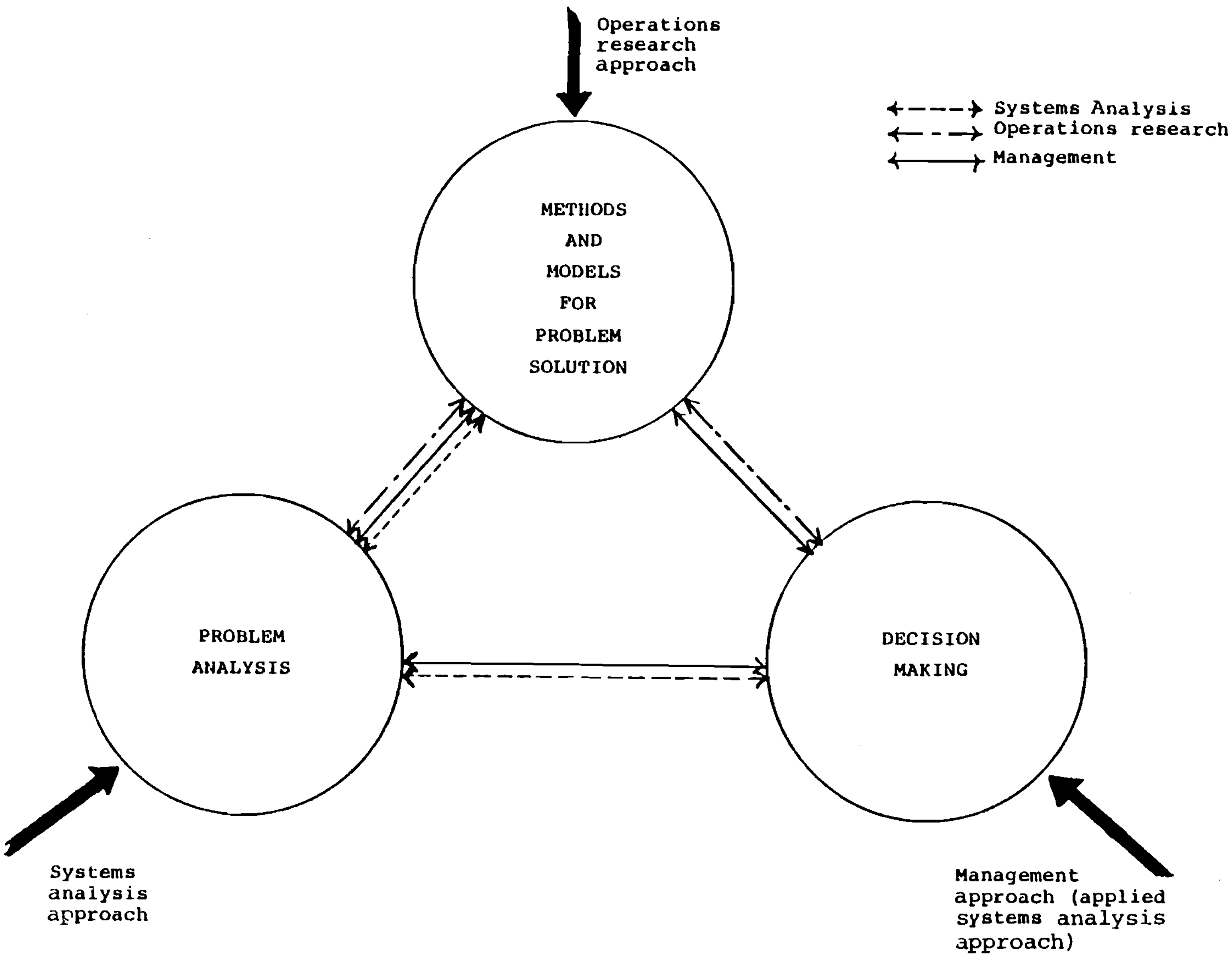

Figure 1. The problem-solving process.

-- decision making.

For such a complex interdisciplinary study it is important to select an approach that will cater for the needs of management. There are three possible approaches: systems analysis, operations research, or applied systems analysis and each places emphasis on different elements of the problem-solving process. In the systems analysis approach most emphasis is placed on problem analysis. Initially the problem is defined, the goals are set, the method of solution is determined, and the most efficient of the various solutions obtained is selected. The operations research approach, on the other hand, emphasizes the importance of the tools used to solve the problem. First, the methods to be used are evaluated, the problem is then analyzed and simplified for the purpose of model development. This approach is suitable for providing management with a variety of scientific methods for problem solving. However, the approach we have taken for our study is the applied systems analysis or management approach, which combines elements of both systems analysis and operations research. Initially, the underlying causes of the problem are examined--why is it necessary to solve the problem and by whom should it be solved? The decision-making environment is then analyzed--how can the management process be perfected? Finally, the problem is defined and the most suitable methods of solution are chosen. This approach is essential when an established decision-making procedure exists; it allows scientific tools to used directly by decision makers in the planning process.

THE PLANNING PROCESS

The planning system in Bulgaria is composed of various levels (Figure 2). The highest level consists of the top State and Party institutions--the Communist Party Congress and the National Assembly. At this level all major national decisions are approved. Every economic organization or enterprise must accept the principal figures of the national plan, which to be made law must be approved by the National Assembly. The second level consists of the Council of Ministers and the State Planning

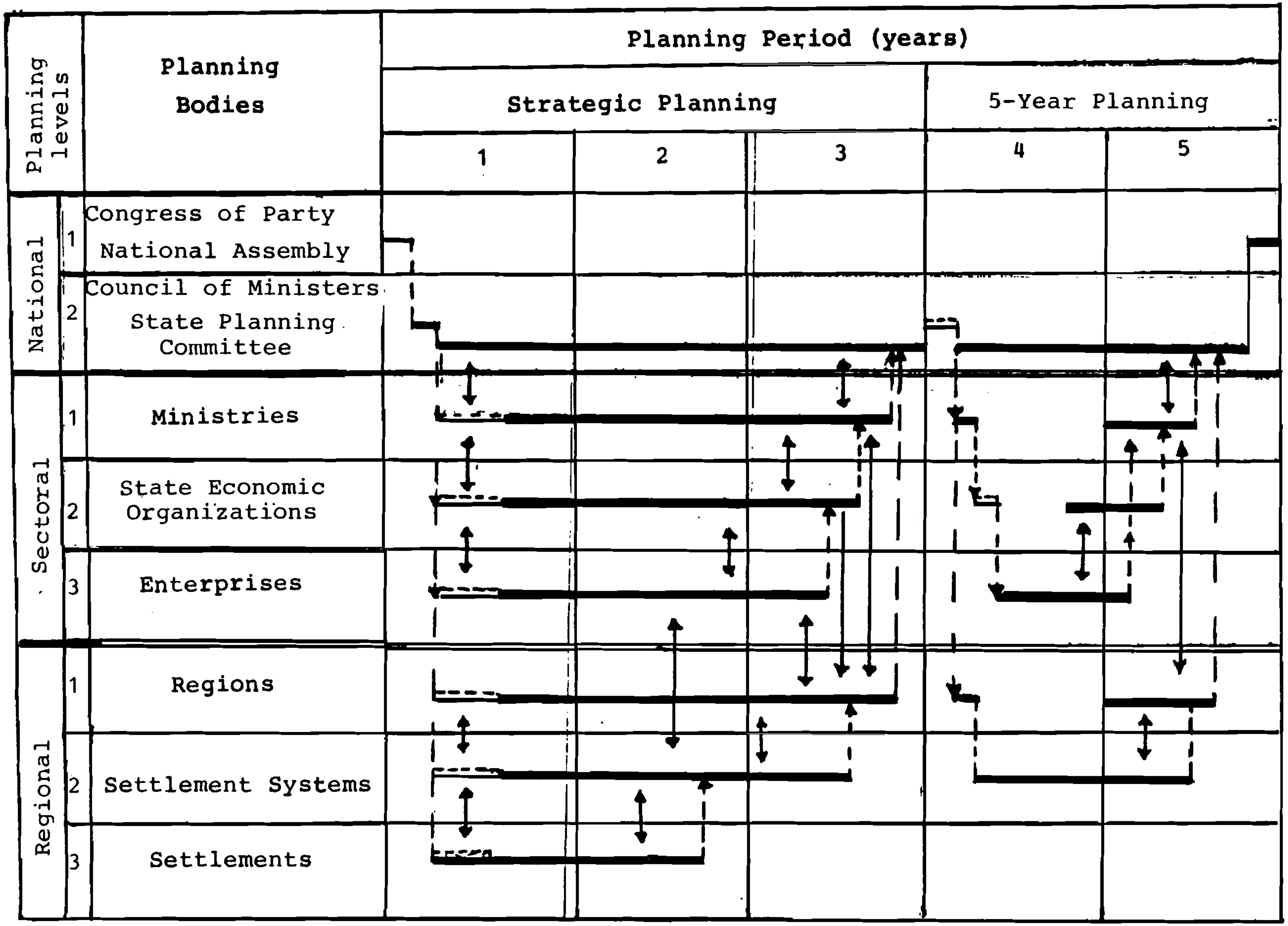

Figure 2. The Bulgarian planning system (════ setting goals and constraints; ▬▬▬ planning and programming; ---- aggregation or disaggregation; —— coordination).

Committee--it is here that national economic plans are formulated. The third level consists of the various ministries, each ministry being responsible for an economic sector. Their function is to distribute national resources and to establish production targets for the economic organizations within their sector. However, it should be noted that the economic organizations have some degree of independence in formulating their own economic policy.

Recently, another level has been introduced into the planning system--the regional level--but it is still not well developed. Planning at this level is carried out simultaneously and parallel with sectoral planning. Regional planning is organized on three sublevels:

-- the regional authorities, who manage the region as a whole (the region being the principal administrative unit);
-- the settlement systems; and
-- the individual settlements with the system.

The planning process begins after the 5-year plan is approved. For example, in 1981 approval for the eighth plan will be obtained and from 1981 onwards the same procedure will be reenacted for the ninth plan, as well as for the long-range plan covering the period up to the year 2000. This period of strategic planning is dominated by basic goals or targets set at the State level for all other levels within the national economy. Only these goals and the constraints on their achievement are considered. After these goals and constraints have been specified, the other levels begin to develop their own strategic plans relating them as far as possible to State objectives. A feedback procedure is then organized to solve the development problems that arise at each level. At present, strategic plans are formulated mainly at the national level. However, to improve the planning system, development should be considered at all levels--national, sectoral, and regional. Our project is aimed at improving strategic planning procedures in a regional context.

Global modeling studies have shown that development processes in regional systems are characterized by inertia. Exploitation of local resources, development of a local infrastructure, and production specialization cannot be achieved over the short term. Consequently, it is essential to consider regional planning in long-term time perspective.

The need for strategic planning at a regional level is especially important in Bulgaria for the following reasons. First, the country is organized into settlement systems, which elect their own local management bodies. The policies formulated by these bodies can only be implemented through the regional plan. It is therefore necessary to modify local development plans in accordance with the policy carried out by the regional and settlement system authorities. Second, in the regions a policy of self-sufficiency in the basic agricultural products is pursued. There is also a guaranteed national minimum of resources allocated to the regions according to the normatives (i.e., standards laid down by the State) for the various types of enterprise. This necessitates a complex tying together of economic and social development plans* to ensure that the needs of the population are satisfied at every point in the region. Third, at present Bulgaria's socioeconomic development planning is implemented in such a way that continuous feedback between the various sectoral and regional planning levels is assumed. This method also assumes that the overall problem of socioeconomic development will be solved at all planning levels simultaneously and will take national goals into account. At the sectoral level, a set of strategic planning documents is formulated to ensure feedback between national and sectoral management during the drafting of the plan; however, at the regional level this is not the case. Finally, regional economic growth can be accelerated if development is coordinated. A considerable saving in costs may be achieved if the scale of infrastructural development is consistent with regional conditions.

*These plans include the basic parameters not only for economic growth, but also for living standards.

ORGANIZATION OF THE PROJECT

As already mentioned, the regional system is considered to be relatively separate from the national system, despite its links with other regions, the national system, and other countries (Figure 3). The regional system is not organized spatially (i.e., according to geographic and economic conditions only), rather its organization is based on the existing system of human settlements. The basic goal of the project is to improve regional planning and management procedures. Although the region is an independent socioeconomic system with its own characteristics, its development objectives do not necessarily conflict with those of the national system; however, it _is_ important that regional and national goals are coordinated. The region is not considered as simply a place for specializing in certain activities and for exploiting national resources, but as an organizational system directly responsible for the effectiveness of its own development and a partner in the strategic development of the country as a whole.

Regional socioeconomic growth should include

- development of the natural and man-made environment as a part of the general development of the region;
- human reproduction, as related to demographic processes and development of all services provided for the population (education, health care, etc.);
- material production, which is the economic basis of development in every social system and determines the development of every region;
- spiritual production, including science, the arts, culture, and all other related activities; and
- development of the social and political structure and management system.

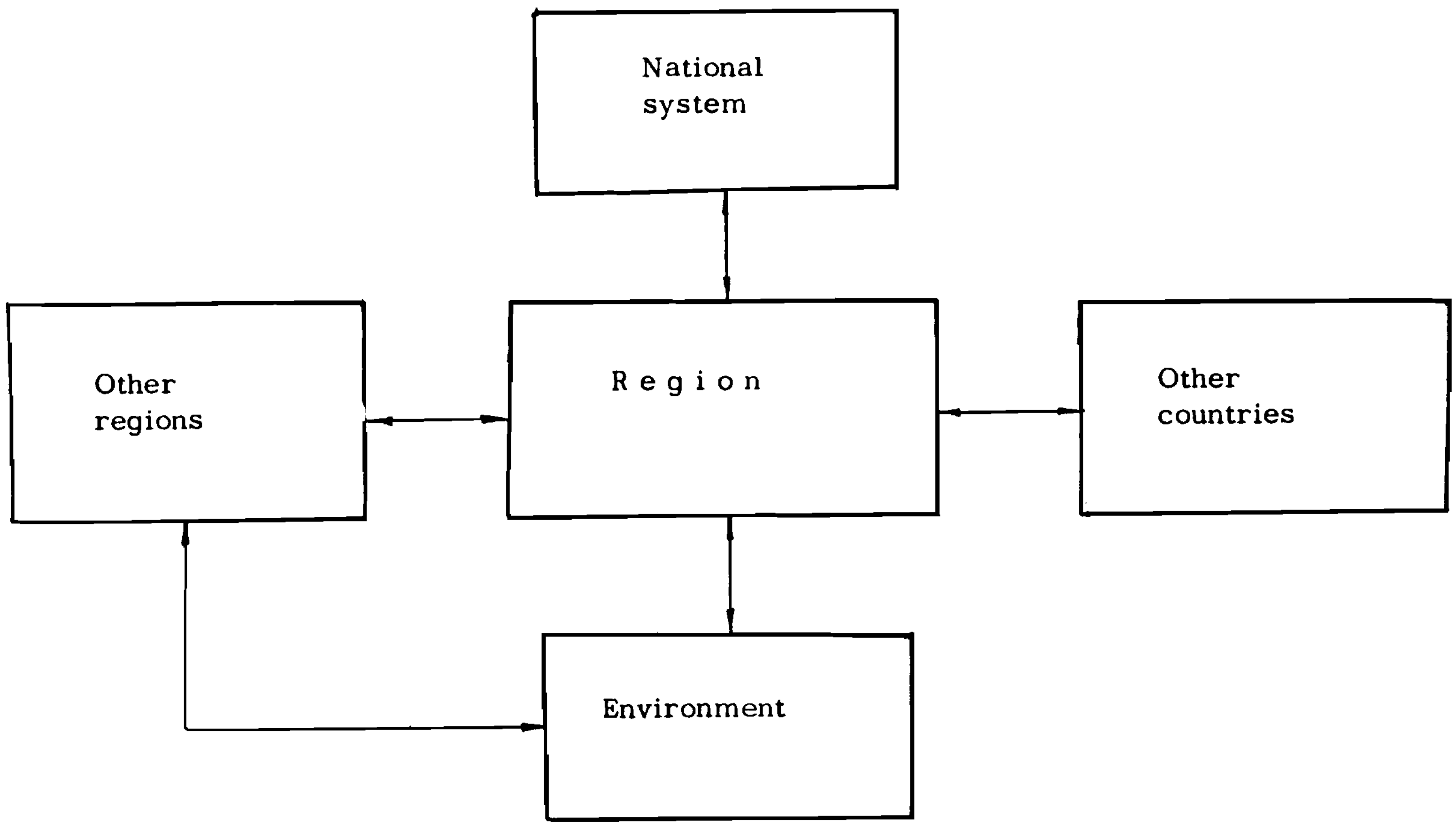

Figure 3. The interrelations of the region and other systems.

The main criterion for evaluating the development of the regional system is improvement of the living standards and "quality of life" of the regional population. The methodological difficulties in effecting this evaluation can be overcome by formulating a set of social indicators.

The Silistra case study includes not only modeling activities directly related to decision making, but also the preliminary work for the whole project. The results of the project will be subject to discussion among the central and local (regional and settlement system) authorities and specialists from various disciplines involved in the planning process. An official document for strategic planning of the region will be produced as a result of these discussions.

The project consists of four main programs:

-- development of production,
-- social development,
-- development of the management system, including the management information system, and
-- infrastructural development.

These regional programs are linked not only to national programs and long-range territorial plans, but also to similar programs for the settlement systems at the subregional level (Figure 4). In addition, there are special programs for some of the most important activities related to regional development. To ensure coordination of these activities, there is continuous interaction between the different groups of specialists involved in the project.

The program for the development of regional production is based on the proposals of all sectoral authorities (Ministries and State Economic Organizations). It includes a study of regional labor distribution and of the possibilities for introducing new types of production.

The program for social development focuses on nonproductive aspects of regional development, including cultural activities, education, health care, etc., and is aimed at improving the quality of life in the region.

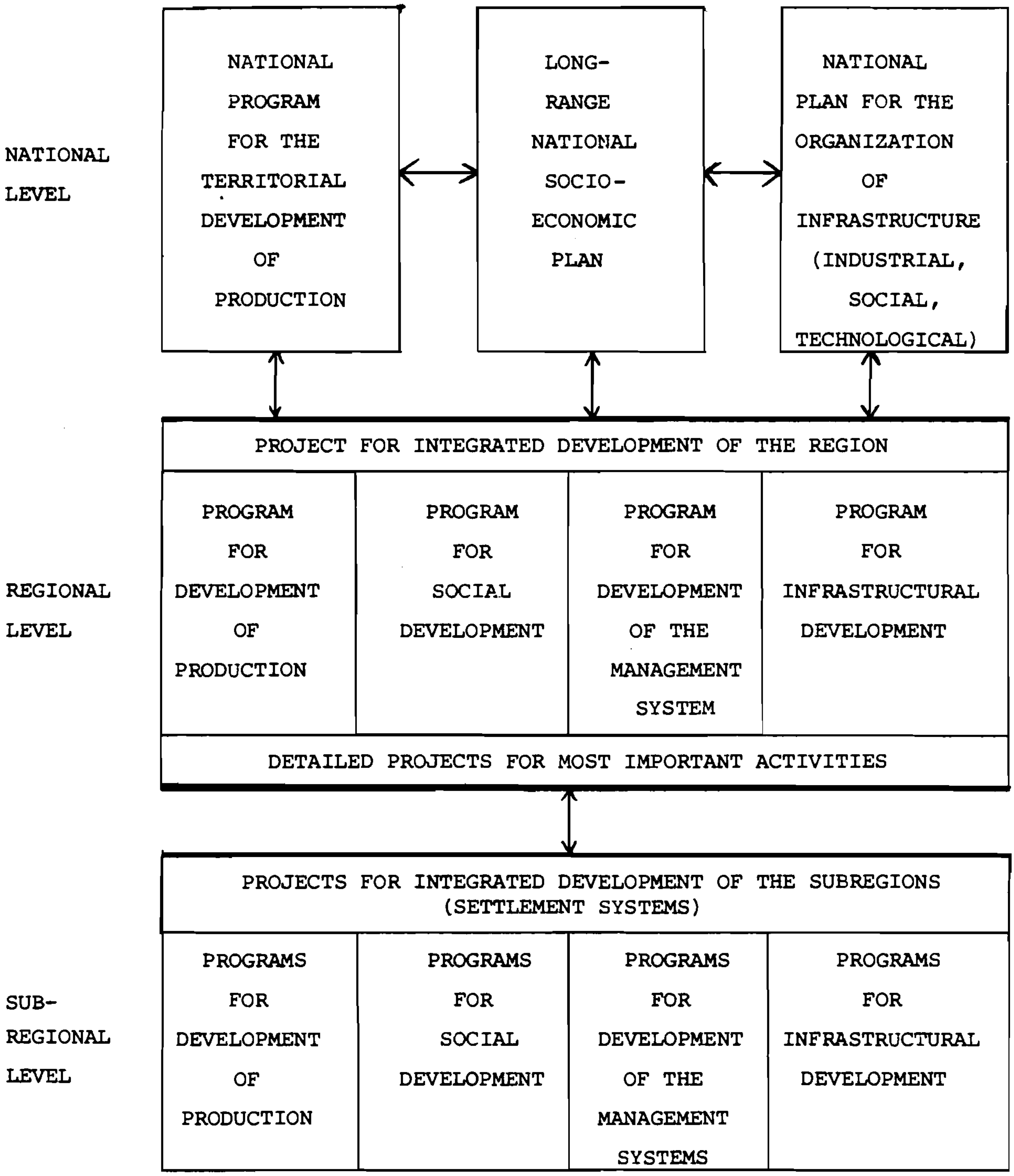

Figure 4. The structure of the Silistra project.

The management program deals with regional coordination of the various types of industrial production, including technological development and improvement in management mechanisms. A management information system, intended for use by all participants in the planning of the region, will be established in this program.

The infrastructure program deals with zonal location problems related to regional production activities, housing, recreation, transport, communications, etc. It makes an important contribution to the national plan of infrastructural organization.

The case study will provide preliminary results for the whole project and an evaluation of the effects of sectoral activities within the region, so that development of all aspects of the regional system may be integrated.

THE SYSTEM OF REGIONAL DEVELOPMENT MODELS

Work on the Silistra case study is concentrated on the development of a system of models, which is being developed in collaboration with IIASA. This system is designed to provide a set of techniques for studying alternative paths of development for the Silistra region. The system of models is constructed on three levels (Figure 5).

- At the highest level, the region is considered as one point and its links with the national system are specified. The results from this model are used indirectly for the models at a lower level.
- At the second level, each sector of the regional economy is considered, although in some models the area under analysis may exceed the boundaries of the region; for example, the regional transport system forms a part of the model of the national transportation network, and the water supply and irrigation network models each form a part of the model of the northeastern region.

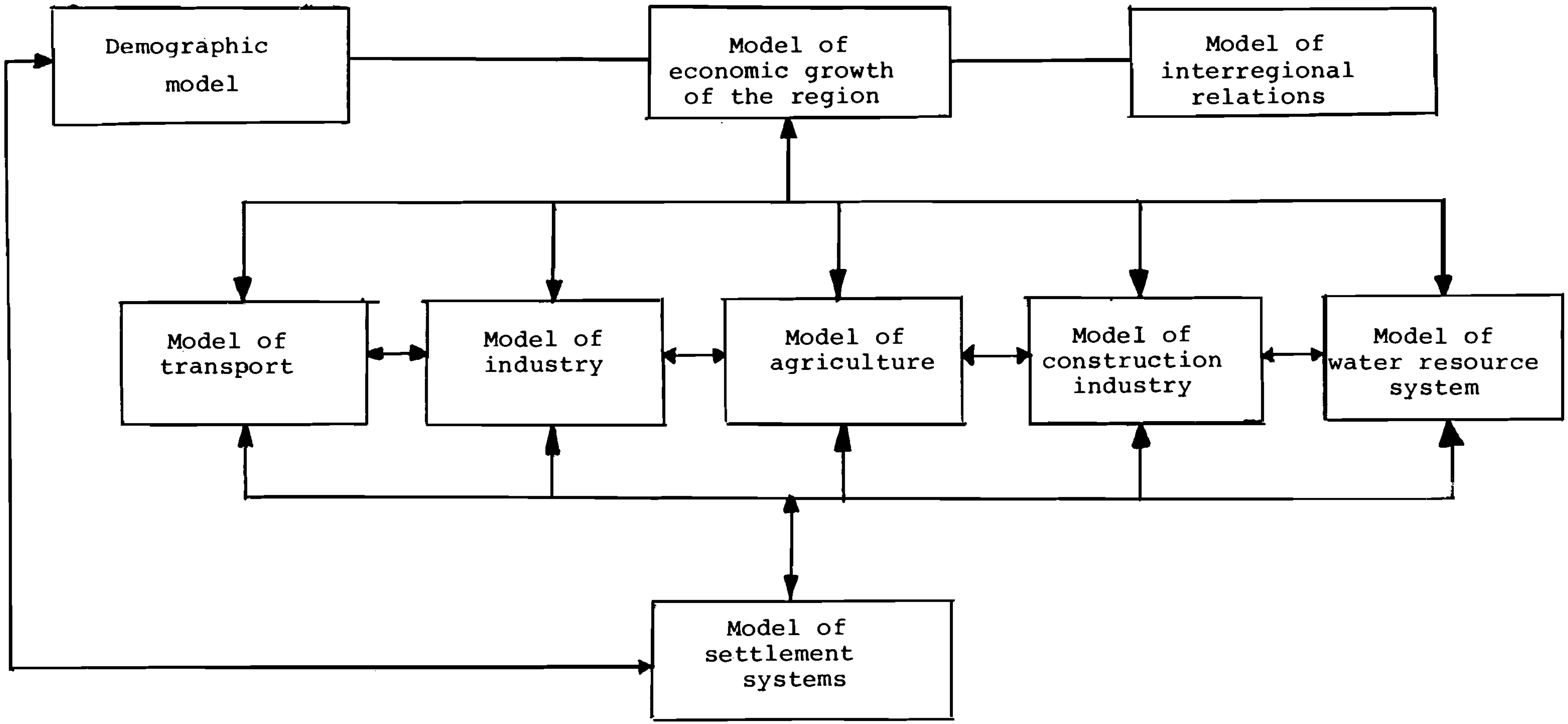

Figure 5. System of models for the Silistra region.

-- At the third level, the settlements systems are modeled. Here links with the regional and national levels are ensured by the sectoral models, which provide data on the activities undertaken in each settlement system.

It is assumed that regional modelers will have access to national models when they begin to develop their own model base, thus facilitating coordination of regional and national goals. However, for the Silistra case study, the national model is not considered in the modeling activity. This is because the region plays such a small role in the national economy that it cannot significantly influence overall national economic development. The Silistra region contains only 2.02 percent of Bulgaria's population and produces merely 1.9 percent of the gross national product and 1.4 percent of total national industrial production. Regional capital investment, which does not fluctuate annually, constitutes approximately 1.5 percent of national capital investment.

The sectoral and regional approaches to the solution of development problems are coordinated to form the optimum regional development plan. The region's full potential can only be realized if development is viewed from both standpoints.

The models of the system are not integrated by joint input or computer linkage, but by the evaluations of experts (Figure 5). The system functions on three levels. The first level consists of a regional economic growth model, an interregional relations model, and a demographic model. Each of these models is briefly described below.

The model of regional economic growth forms the core of the whole system. It will use data from the national input-output table and also from the models of demographic processes and interregional relations, providing the main constraints and data on sectoral growth for the sectoral models. Such a model, which simulates the behaviour of the region as one point, is currently being developed in Bulgaria by Panov and Petrova. Although the structure has already been determined, no real data are yet available to operate the model. The following models could be used interactively to form an alternative core model:

-- the Turnpike model, developed by Andersson;
-- the MSG model, developed by Bergman;
-- the Almon-Nyhus input-output model; and
-- the Mirror and Maltos models for interregional and intraregional location of activities, also developed by Andersson.

Both options are currently being evaluated.

The interregional relations model is based on input-output analysis and provides information on the influence of changes in other regions on the socioeconomic growth of the Silistra region, and vice versa. One problem restricting its development is the lack of essential input data.

At present only part of the demographic model is operational--that part dealing with the extrapolation of the main demographic parameters. The migration section, being developed by Philipov (Philipov 1979) is important for the functioning of this model.

The second level includes models of different sectors of the regional economy. These models are linked to corresponding models on the national level. The most important sectoral model is the model of industrial development, which deals with national industrial growth and the optimal location of industrial activities within the region. Further information on this model is given in the paper by Christov, Assa, and Panov included in these Proceedings. In addition to the main model of industrial development, a separate model of one product has been developed by Kolarov; the product studied is pulp--the pulp industry being an important sector of Silistra's economy. This study, which is a useful complement to the modeling activities relating to regional industrial development, is described in more detail elsewhere in these Proceedings.

The model of agricultural development is an essential component of the model system, since the Silistra region is geared mainly to agricultural production. It is a long-term model in which the region is considered as one point. It determines the optimal structure of agriculture in terms of specialization for a period of 15 years in the future. This model was discussed at the 1979 Silistra Task Force meeting (Gavrilov et al. 1979). In addition, a model of the optimal land allocation for the agricultural production and processing industries is now being developed by Gavrilov, Stoykov, and Milenkov and it is described in these Proceedings. The Generalized Regional Agriculture Model (GRAM), developed by Albegov, could also be used to describe agricultural development (Albegov 1979).

The model of the construction industry, based on the model developed by Panov (Panov 1972), solves optimization problems related to

-- the allocation of raw materials;
-- the location of plants producing prefabricated building materials; and
-- the selection of construction technologies.

Sectoral investment data obtained from this model provides the input for the other sectoral models; thus, the model is directly linked to the other sectoral models.

The model of the transportation network has been developed by Mihailov and Nicolov and is described in detail in these Proceedings. This model solves problems related to passenger transport and is thus linked to the demographic model. Through evaluation of the volume of goods transported, it is also coordinated with the other sectoral models.

The model of water resource development, developed by Gouevsky, consists of submodels of water demand, water supply, and water resource management. It is linked to the industry and agriculture models in the system. Further information on this model is given in the paper by Gouevsky, Genkov, Tzvetanov, and Topolsky included in these Proceedings.

The sectoral models describe the most important regional economic activities. All other sectors, including services and nonproductive activities, are described at the third level in the settlement systems model.

The simulation model of the settlement systems consists of

-- a data base;
-- a package of programs for data processing and analysis; and
-- a bank of models describing the interdependencies of the factors influencing the development of the settlement system.

It is described in the paper by Konakchiev, Grigorov, and Evtimov, which is included in these Proceedings. A set of regional development indicators has yet to be formulated, development of this model will then be complete. There are close information links between this model and all the other models of the system containing spatial data.

From the description of the model system given above, it is evident that the process of linking the models to form an interactive system is an important part of the modeling activity. It is important to obtain from each of the models realistic results that can be unified to form a consistent plan for regional development. Any large model system dealing with complex problems must inevitably simplify the relationships between factors influencing those problems, which implies that the results obtained are likely to be less useful for a practical situation. Since the objectives of regional development as a whole and regional sectoral development differ significantly, it follows that the methods used to fulfill them must also vary; however, to link models constructed on different methodological bases directly is a difficult task. Linkage is therefore achieved by evaluation by specialists of the input and output of the models. The Silistra project includes schemes to train regional planners to use the model system in a fully interactive man-machine mode.

In the near future a large-scale simulation model of the regional system fashioned specifically to fit the regional development analytical framework and incorporating a data bank will be constructed. However, only if the models of the system are sufficiently practical can we proceed to this stage of the project, for which the following activities will be undertaken:

- -- analysis of local resources;
- -- analysis of basic needs and interregional links;
- -- analysis of production capacity;
- -- determination of the optimal structure of resources;
- -- simulation of the production cycle and analysis of the results;
- -- analysis of national policy for the region; and
- -- estimation of resources required in the next period of analysis.

PROBLEMS OF PROJECT MANAGEMENT

Many problems have occurred during management of the Silistra Regional Development Project and if practical results are to be obtained, they must be solved. The major problems are outlined below.

Terminology

Each branch of science and technology has its own set of specialized terms. In interdisciplinary research this naturally presents certain communication problems. Scientists from different fields may be unfamiliar with each other's terminology; even greater confusion can occur if the same term is used to signify one thing in one discipline and yet something rather different in another. This aspect of communication should be given particular attention in the study.

Unifying the Data Base

Many scientific teams are engaged in the work of the project and it is usual for each team to build its own information system based on its particular research requirements. Exchange of information between each team is not always easily organized given that there are differing requirements and methods of data collection. In the future a data base suitable for use in other regions without the need for major modifications will be established.

Computer Hardware and Software

In Bulgaria a wide variety of computers with program packages oriented to specific problems are available. This presents difficulties for implementing the results of the modeling effort, since the transferal of model solutions from one type of computer to another can cause serious technical problems. Modeling studies designed for practical implementation should be performed on the same type of computer as that used throughout the planning process.

Coordination and Unification of the Results from each Program

In the Silistra project the development of different components of the regional system is considered. Each task or program deals with a particular set of problems and the solutions from each of them must be unified. Coordination is not an easy task for several reasons: the solutions may be based on different initial criteria: economic, social, technical, economy, national, sectoral, regional, etc.; or there may be no standard methodology for solving such a diversity of problems.

It could be said that the single overall goal of regional development is to improve the quality of life of the population, but the quality of life criterion cannot easily be defined and involves questions of emphasis and priority. Emphasis is predominantly given to

- the economic aspect of overall development of the region;
- the social aspect, being most directly connected with improving the quality of life of the population;
- the infrastructural aspect, which influences implementation of development; and
- the management aspect, determining the practicality of the project's solutions.

In the future, planning decisions relating to the aspects of regional development mentioned above must be coordinated.

Structure of the Research and Planning Teams

During the Silistra project there will inevitably be changes in the research team. Thus, a long-term plan of personnel to be engaged on the project is required. From the point of view of practical implementation, it is necessary to run a continuous training program for planners throughout the course of the project so that the planning personnel are always able to use and implement the solutions provided by the modeling activity. In Bulgaria training schemes for planners are run simultaneously with such studies.

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OPTIMIZATION OF REGIONAL DEVELOPMENT--AN INTEGRATED MODEL FOR STUDYING SOCIOECONOMIC AND ENVIRONMENTAL POLICIES

R. Kulikowski

1. INTRODUCTION

During recent years there has been an increasing demand for computerized regional planning models that deal with the allocation of natural resources, labor, and capital. Such models have had a considerable impact on regional and national production and welfare. In countries with planned economies, planners and decision makers believe that these models can help in the efficient allocation of production factors and welfare at the regional level. A decentralized management and planning system exists for this purpose. In Poland, for example, the Government and Planning Commission represents the highest decision-making unit, which cooperates with lower-level planning units of the 49 voivodships (regions). Each voivodship, in turn, cooperates with the corresponding units at the lowest community level, gmina, which usually consist of a small town and a rural neighborhood.

During the planning process (with a 5- or 10-year horizon), planners, decision makers, and the public consider each development alternative and problems related to regional economic efficiency,and its effect on welfare and equity at the regional and national levels are discussed. Obviously, the problem of efficiency versus equity cannot be properly understood without specifying the differences between regional (local) and national (global) goals and policies.

Generally, the regional planning and decision-making units in Poland are concerned with maximization of regional utilities, taking into account constraints on primary resources. For example, an increasing population will require more employment and housing. However, population size depends on local demographic factors and migrations, which in turn depend on the regional standard of living and environmental quality. The availability of new jobs depends, on the other hand, on new capital investments. In Poland, major investments are decided upon at the national level, taking into account the regional comparative advantages in production and welfare levels. In order to regulate regional welfare, the national level subsidizes local budgets for public services, such as education, health, housing, and environmental protection. A decision regarding the location of a new plant is usually accompanied by an increased subsidy to the regional budget. Sometimes, the subsidy is favored by the region much more than by the industry that would pollute the environment.

The national (central) planning and decision-making bodies, on the other hand, are also concerned with national utility and international problems such as the international balance of payments, both of which require an efficient economy. Thus, a substantial part of the national budget (created by taxes imposed on productive sectors) is spent on productive investments in those regions offering the maximum rate of return (i.e., low cost of land, water, labor, transportation, environmental charges, etc.).

A region wishing to attract industry may pursue a policy that offers low production costs at the expense of deteriorating environmental quality. Such a policy may, in the long run, produce a decrease in welfare--a completely unanticipated result. For example, if producers pay little for discharging waste, pollution increases while the population suffers and migrates from the region. Since the decreasing labor supply increases the cost of labor (in Poland it takes the form of increasing commuting or housing costs), the regional comparative advantages (to locate the industry) may decline. The industrial growth may also create a number of externalities in production and services. Increased commuting may increase traffic congestion, deteriorate access to services, etc.

Because of the changing production structure, there is also a demand for skilled labor* and an educational system that will provide training in the required skills. Increasing inequalities among different regions in per capita personal income result from urbanization, despite the fact that the existing wage system is strongly motivated by egalitarian concepts.

Besides wages and employment, which determine the per capita regional personal income, welfare depends to a large extent on aggregate consumption (services). Since there is scepticism about characterizing services in terms of expenditures, decision makers are tempted to maximize indices, which refer to the number of education, recreation, housing, etc. facilities per capita. However, such indices raise the question of "accessibility to services". Due to age structure, education level, distance, etc., the access of the population to certain services may be limited. For example, because of the distance from urban centers, the rural population has a limited access to higher education, theaters, etc. As a result, the utility, as perceived by the population, is sometimes different from that conceived by the planners, and migration follows in an unpredictable fashion.

All these factors indicate that the problem of choosing the optimum policy of regional development, taking into account the different interests of all the parties concerned, is not easy. Nor is it easy to find an optimum solution to the location of industry or public investment projects. Since any decision regarding productive or public investment changes the existing production as well as welfare structure, planners wish to know the impact of that decision in terms of costs and benefits on all the parties concerned.

For example, if an extension of a regional water system is planned, the benefits resulting from, say, economies of scale may affect industry or agriculture and may change (generally in differing degrees) the utilities of the rural and urban populations.

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*"Skilled" is used in the broadest sense to signify manual, technical, and managerial skills.

Several questions are then raised.

-- What should be the optimum size of the project?

-- How should total investment be distributed between industrial and agricultural users?

-- How much of the total investment expenditures should be paid out of regional and how much out of the central budget?

-- How much of the subsidies (for infrastructure, services, environmental protection, etc.) should accompany the regional project?

-- How much will a party concerned with the project gain from choosing to abstain or support an alternative (competing) regional project?

Using the terminology of cooperative game theory, one can also ask the following question. Will stable policies (in terms of pricing, taxes, subsidies, etc.) exist for the regions and water users? When the core of the game is empty, it is doubtful whether the project will be approved. It is, of course, difficult to give any rational answer to these questions without studying the regional project in a formal way. For example, if there is an economy of scale in water supply and the marginal water cost decreases with an increase in the supply, the water users may choose to cooperate in order to exploit the scale benefits. However, at a certain water supply level, the marginal cost may begin to increase, and each additional user must pay more unless the water price is increased. For the last case, existing users will act against admitting additional partners.

Some decision makers feel that the lack of full understanding of the possible benefits by all parties concerned is one of the main reasons why many ambitious regional projects have been rejected before they were ever fully evaluated.

Decision makers wish to have a regional model that could be used both

-- to evaluate each proposal or project from the point of view of the benefits it creates for all the parties concerned; and

-- to find the extent of maneuvers (i.e., the size of the core of the game) in terms of regional locations, subsidies, taxes, prices, charges, etc., and the national and regional benefits of such maneuvers.

When the space of maneuvers is not empty, it is still possible to take action against the factors that are not specified explicitly in the model and to provide the partners with the opportunity to bargain.

Despite the advances achieved in recent years in understanding the forces influencing agglomeration processes and regional growth, progress in regional modeling is rather slow. As a result, a certain disenchantment and critical assessment of the state-of-the-art in regional (and especially urban) modeling exists (see Sayer 1976). Many of the recognized failures in regional modeling can be attributed to

1. structural deficiencies, such as the absence of mechanisms capable of dealing with economies of scale and externalities in production and services, and their impact on location decisions and agglomeration growth;
2. the absence of interrelations among the demographic factors, economy, and environment of the region, as well as between regional and national management;
3. the lack of mechanisms enabling decision makers to see how, given the demographic, environmental, etc. constraints, the instruments of regional policy, such as taxes, subsidies, prices, charges, standards, and regulations, can be used to attain welfare and stability (i.e., public and institutional agreements and support under conflicting interests); and
4. excessive model complexity, which makes the comprehension as well as estimations of the parameters and validation of the model's accuracy over time almost impossible.

This paper may be regarded as an attempt to develop a methodology for integrated regional policy-oriented modeling that should help the model builder to avoid the shortcomings 1-4 mentioned above. The main idea underlying this methodology is to regard the region as a set of producers, consumers, and authorities of different levels that stimulate regional growth. When, in a given region, they arrive at a stable system of cooperation, and more partners can be admitted, the region can be considered as growing. If, on the other hand, centrifugal forces appear (e.g., as a result of externalities and the effect of diseconomies of scale), regional growth may decline.

The regional production functions are assumed to be homogeneous to a degree (generally) of greater unity and nonlinear input costs, which reflect the impact of externalities, and which come to an equilibrium with "of scale" production. A similar assumption is imposed on regional utilities.

The land, population, and natural resources (such as air and water) are regarded as primary factors for the regional economy. The total land area is regarded as given, but it can be allocated among different users. The total amount of water can be altered by an extension of a water system at some additional cost. The total population is assumed to be predetermined by demographic factors and interregional migrations, which depend on differences among regional utilities.

The consumers' utilities are assumed to depend on the access to employment (wages) and services (education, health, social care, recreation, etc.).

The regional utility (as perceived by the regional authority) depends, in addition, on national subsidies, the cost of environmental damage resulting from production and consumption, and the cost of migrations, housing, etc. The national utility is assumed to depend on production output, consumption, and the international trade balance.

Using an optimization technique developed in Kulikowski (1977), and Kulikowski (1978a) and (1978b), the optimum strategies of allocation of production factors and services were derived. In particular, the optimum allocation of labor by way of interregional migrations had been derived following the method developed in Kulikowski (1978c).

These strategies also enable computation of demands for water, capital, etc. In the case where a new partner (industry) is trying to enter the regional economy or where an extension of the input system (e.g., water supply or wastewater treatment system) is planned, the marginal production costs are derived and the stability of the regional system is tested.

In the case of simple (Cobb-Douglas) production and utility functions, all the model parameters can be estimated from statistical or engineering data.

As shown in the paper by Kulikowski and Krus included in these Proceedings, the methodology proposed can be used for the construction of a computerized interactive gaming model for studying alternative rural-urban development policies.

The methodology described in this paper is presently being tested in the regional pilot project in Poland concerned with the expansion of the water system for mainly agricultural irrigation. This regional development project, the Notec case study, is briefly described in section 5 (see also Albegov and Kulikowski 1978a and 1978b). The project is being carried out by a number of research institutions in Poland in cooperation with the International Institute for Applied Systems Analysis (IIASA).

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2. OPTIMUM ALLOCATION OF RESOURCES IN AN OF SCALE ECONOMY WITH EXTERNALITIES

The basic relation used in modeling production systems is the production function

$$x = f(x_1, \ldots, x_m) \quad , \tag{1}$$

where

$x_1, \ldots, x_m$ are production factors (e.g., labor, capital, water, land); and
x is the production output in natural units.

For the sake of computational convenience, it is usually assumed that f is deterministic, continuous, and differentiable and satisfied a number of further properties, such as

$$f(0) = 0 \quad , \qquad f'_{xi}(\cdot) > 0 \quad ; \tag{2}$$

$$f''_{xi}(\cdot) \leq 0 \quad ; \qquad \text{or} \tag{3}$$

$$f''_{xi}(\cdot) > 0 \quad . \tag{4}$$

If f is homogeneous, so for a positive λ,

$$f(\lambda x_1, \ldots, \lambda x_m) = \lambda^{\beta} f(x_1, \ldots, x_m) \quad , \tag{5}$$

conditions (3) and (4) become

$$\beta \leq 1 \qquad \text{or} \qquad \beta > 1 \quad . \tag{6}$$

In the case where $\beta > 1$, output increases with inputs at an increased (of-scale) rate.

A typical production function, in which the inputs and output are time dependent, is the generalized Cobb-Douglas function:

$$x(t) = Ae^{\mu t} \prod_{\nu=0}^{m} [x_\nu(t)]^{\beta_\nu} \quad , \tag{7}$$

where

A, μ, β_ν are given positive constants;

$$\sum_{\nu=0}^{m} \beta_\nu = \beta \quad , \qquad \beta \lesseqgtr 1 \quad ; \qquad \text{and} \tag{8}$$

μ characterizes neutral technological progress.

To evaluate the output value, planners use integrated, discounted within the planning period [O,T], production in monetary terms:

$$Y(x) = \int_0^T p(t)e^{-\lambda t}x(t)dt \quad , \tag{9}$$

where

$p(t)$ is the given price of $x(t)$; and

$e^{-\lambda}$ is the given annual discount rate.

In formula (9) one assumes that t is a continuous time variable. If discrete (e.g., changing once annually) variables are preferred, the sum can be used instead of the integral. The macromodel (7) and (9) can be used for studying a single-sector regional economy.

The performance of the production system cannot be fully evaluated without taking into account the discounted input (factor reward) costs (Y_ν), i.e.,

$$Y_\nu = \int_0^T e^{-\lambda_\nu t} c_\nu(x_\nu)dt \quad , \qquad \nu = 0,\ldots,m \quad , \qquad (10)$$

where

λ_ν are given positive numbers; and

$c_\nu(x_\nu)$ is a given input cost function.

As in the case of the production function, a number of properties regarding $c_\nu(x_\nu)$ can be postulated. Consider, in particular, the labor cost (c_o) as a function of employment (x_o). Statistical data indicate that in urban agglomerations $c_o(0) = 0$, $c_o'(x_o) > 0$ and $c_o''(x_o) > 0$. The increasing marginal labor cost, when the demand for labor exceeds the local supply, can be explained by the increasing cost of commuting from outside the agglomeration (see Kulikowski 1977).

On the other hand, the expanding circular agglomeration exhibits a decreasing marginal land cost ($c_\nu''(x_\nu) < 0$), because of the decreasing land rent as one moves outside the agglomeration center.

Water (and other services) costs generally exhibit more complex behavior. Typical water cost functions are shown in Figure 1. In the segment OA, increasing the use of local water resources increases the marginal production cost. At point a, an extension of the existing water system is necessary (e.g., the construction of reservoirs or of water transfers from outside the region under analysis). The decreasing marginal water cost follows along ed up to the saturation point d, where the marginal cost again begins to increase along the trajectory df.

The main goal of the producer can be formulated as follows. Maximize the revenue (9), subject to the factors reward costs Y_ν, satisfying the constraints

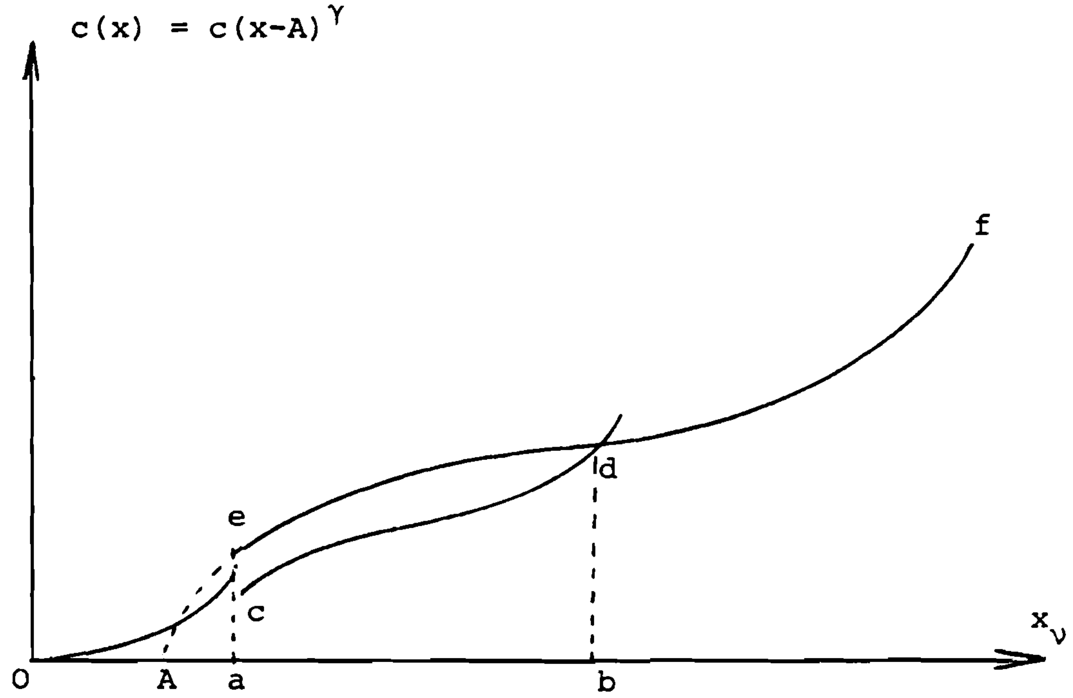

Figure 1. Input cost functions.

$$\sum_{\nu=0}^{m} Y_\nu \leq \bar{Y} \quad , \qquad Y_\nu \geq 0 \quad , \qquad \nu = 0,\ldots,m \quad , \tag{11}$$

where

$\bar{Y}$ is the expected input cost in [0,T].

If such a strategy for allocating inputs can be found, e.g.,

$$x_\nu(t) = \hat{x}_\nu(t) \quad , \quad \nu = 0,1,\ldots,m \quad , \quad t\varepsilon[O,T] \quad , \tag{12}$$

and

$$Y_\nu = \hat{Y}_\nu \quad , \qquad \nu = 0,1,\ldots,m \quad , \tag{13}$$

that maximum $\hat{Y} = Y(\hat{\underline{x}})$, subject to equations (10) and (11), is attained, it can be said that the optimum production strategy exists.

If, in addition $Y(\hat{x}) > \bar{Y}$, it can be said that the strategy is profitable.

In the case when f and c_ν, $\nu = 0,1,\ldots,m$, are concave, it is relatively easy to find conditions under which optimum and profitable (in the whole range of inputs) strategies exist. However, the concavity assumption narrows the class of production processes that can be studied. If f, c_ν are not concave (i.e., when they are of the form shown in Figure 1), the existence of optimum strategies depends greatly on the properties of the f, c_ν functions. An important problem is to find conditions under which such strategies exist. The main idea of this paper is to approximate piecewise the f, c_ν functions by relatively simple exponential functions such as (7) or

$$c_\nu(x_\nu) = \omega_\nu(t)[x_\nu(t)]^{\gamma_\nu} \quad , \qquad \nu = 0,1,\ldots,m \quad , \tag{14}$$

where

$\omega_\nu(t)$, γ_ν are given and positive.

If $\gamma_\nu > 1$ $(\gamma_\nu < 1)$, the marginal production cost

$$\bar{\omega}_\nu(t) = \frac{dc_\nu}{dx_\nu} = \gamma_\nu \omega_\nu(t)[x_\nu(t)]^{\gamma_\nu - 1} \tag{15}$$

increases (decreases) with x_ν.

Generally, different ω_ν, γ_ν may exist in each segment of the inputs range (compare Figure 1).

In a similar way, we assume that because of changes in indivisibilities, technology, organization, etc., the production function may be described by equation (7) with different (generally) β_ν and β coefficients at a different input range (compare Andersson and Marksjo 1972).

The main problem is to find out (given a set of technologies Ω, characterized by $\beta_\nu{}^i$, $\gamma_\nu{}^i$, $\nu = 0,1,\ldots,m$, $i \varepsilon \Omega$), whether a non-empty domain of inputs exist within which one can organize optimum and profitable production processes. In addition, the marginal production costs for a particular plant size should be determined. The notion of the plant size for which the optimum solution exists is important also when a new plant is designed. It provides information about the stability of planners' strategies. In the case of a large domain of stable solutions, the impact of random factors and errors, resulting from a poor estimation of model parameters, is much reduced.

It is convenient to solve the general optimization problem in two steps:

A. $$\max_{\underline{x} \varepsilon \Omega_1} Y(\underline{x}) \quad , \tag{16}$$

where

$\underline{x}$ is a vector ;

$$\Omega_1 = \{x_\nu(t) : \int_0^T e^{-\lambda_\nu t} c_\nu(x_\nu)dt \leq Y_\nu,\ x_\nu(t) \geq 0,\ \nu = 1,\ldots,m\} ; \tag{17}$$

$x_o(t)$ is a given function; and

Y_ν, $\nu = 0,1,\ldots,m$, are given values.

B. Compute $Y(\hat{\underline{x}})$ as a function of $\underline{Y} = \{Y_o,\ldots,Y_m\}$ and

$$\max_{\underline{Y}\in\Omega_2} \underline{Y}(\underline{Y}) \quad , \tag{18}$$

where

$\underline{Y}$ is a vector;

$$\Omega_2 = \{Y_\nu : \sum_{\nu=0}^{m} Y_\nu \leq \bar{Y}, \quad Y_\nu \geq 0 \quad , \quad \nu = 0,1,\ldots,m\} \quad ; \text{ and} \tag{19}$$

$\underline{Y}$ is a given value.

In the general case, the solution of problems (16) and (18) is not easy. As shown in equation (17), when f, c_ν are of the form (7) and (14), respectively, one can use the generalized Hölder inequality* to solve problem A. That result can be formulated in the form of the following Factors Coordination Theorem.

Let $\omega_o(t)[x_o(t)]^{\gamma_o}$ be integrable in [0,T], and

$$\lambda = \sum_{\nu=0}^{m} \lambda_\nu\alpha_\nu \quad , \quad \alpha_\nu = {}^{\beta_\nu}/\gamma_\nu \quad , \quad \sum_{\alpha=0}^{m} \alpha_\nu = 1 \quad , \tag{20}$$

$$Ap(t)e^{\mu t} \prod_{\nu=0}^{m} [\frac{\alpha_\nu}{\omega_\nu(t)}]^{\alpha_\nu} = W = \text{constant} \quad . \tag{21}$$

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*The Hölder inequality states that

$$\int_0^T |x_o(t)x_1(t)|dt \leq \{\int_0^T [x_o(t)]^p dt\}^{\frac{1}{p}}\{\int_0^T [x_1(t)]^q dt\}^{\frac{1}{q}} \quad ,$$

$p^{-1} + q^{-1} = 1$, and the equality sign appears iff

$|\hat{x}_1(t)|^p = C\ |x_o(t)|^q$, C is a positive constant.

Then, the optimum strategy (for problem 16) becomes

$$\hat{x}_\nu(t) = \{\frac{Y_\nu}{Y_o}\frac{\omega_o(t)}{\omega_\nu(t)}[x_o(t)]^{\gamma_o}\}^{\frac{1}{\gamma_\nu}} \quad , \nu = 1,\ldots,m \tag{22}$$

and this strategy is unique.

Under such a strategy the marginal production cost $\bar{p}(t)$ becomes

$$\bar{p}(t) = (Ae^{\mu t})^{-1} \prod_{\nu=0}^{m} [\frac{\omega_\nu(t)}{\alpha_\nu}]^{\alpha_\nu} \quad , \tag{23}$$

while

$$Y(\hat{x}) = W \prod_{\nu=0}^{m} (\frac{Y_\nu}{\alpha_\nu})^{\alpha_\nu} \quad . \tag{24}$$

The solution of problem B, i.e.,

$$\max \prod_{\nu=0}^{m} (\frac{Y_\nu}{\alpha_\nu})^{\alpha_\nu} \quad , \tag{25}$$

subject to

$$\sum_{\nu=0}^{m} Y_\nu \leq \bar{Y} \quad , \quad Y_\nu \geq 0 \quad , \quad \nu = 0,1,\ldots,m \quad , \tag{26}$$

is quite simple. The optimum strategy becomes

$$\hat{Y}_\nu = \alpha_\nu \bar{Y} \quad , \quad \nu = 0,1,\ldots,m \quad , \tag{27}$$

and

$$\hat{Y} = Y(\hat{\underline{Y}}) = W\bar{Y} = W\frac{Y_o}{\alpha_o} \quad . \tag{28}$$

When the price p(t) of good x is not less than the marginal production cost $\bar{p}(t)$, w > 1, and production is profitable.

Condition (21) can also be written as $p(t)/\bar{p}(t) = W$ and can be interpreted as a tendency for the producer's price to follow the change in the marginal production cost. In different regions the value of W may, of course, be different because of different input costs. Assumption (20) states that the product discount is a weighted average of input discounts (with the weights β_ν/γ_ν). Condition $\Sigma\, \alpha_i = 1$ means that input and output costs are expressed in the same monetary units (see equation (24)).

According to the Factors Coordination Theorem, when the primary resource (e.g., labor) $x_o(t)$ is exogenously given, the remainder of the production inputs, such as capital, water resources, should follow the optimum strategy (see equation (22)). When Y_ν are allocated according to equation (27), the maximum production value becomes $\hat{Y} = W\bar{Y}$. Since $Y_o[x_o] = \alpha_o\bar{Y}$, one can see that, under the optimum strategy, output is completely specified by the supply of primary resources $x_o(t)$, i.e.,

$$\hat{Y} = \frac{W}{\alpha_o} \int_0^T e^{-\lambda_o t} \omega_o [x_o(t)]^{\gamma_o} dt \quad . \tag{29}$$

The strategy (22) and (27) can also be written in terms of marginal production costs (15) as follows:

$$\hat{x}_\nu(t) = \frac{\alpha_\nu \gamma_\nu}{\alpha_o \omega_\nu(t)}\, y_o(t) \quad , \qquad \nu = 1,\ldots,m \quad , \tag{30}$$

where

$$y_o(t) = \omega_o [x_o(t)]^{\gamma_o} \quad . \tag{31}$$

The Factors Coordination Theorem can be extended to the production functions of the C.E.S. type (see equation (28)). Instead of the Hölder inequality one has, however, to use the Minkovski inequality (r is a number $0 \leq r \leq 1$):

$$\left[\int_0^T \left| \sum_{j=1}^m x_j(t) \right|^r dt \right]^{\frac{1}{r}} \leq \sum_{j=1}^m \left[\int_0^T |x_j(t)|^r dt \right]^{\frac{1}{r}} , \qquad (32)$$

which becomes an equality if

$$x_j(t) = c_j x_{j+1}(t) \quad , \qquad j = 1,\ldots,m-1 \quad , \qquad (33)$$

where

c_j are positive constants.

It is possible to extend the results obtained to the n-sector regional economy, each described by the functions (7) and (15) with generally, different $\beta_{\nu i}$, $\gamma_{\nu i}$, $\omega_{\nu i}$, $i = 1,\ldots,n$, parameters. We shall assume that the sectors are independent in the sense that their products are not used as inputs by other sectors. However, they may use the same (exogenous) inputs, which change (when $\gamma_\nu \neq 1$) the corresponding marginal production cost. This effect is usually referred to as an externality. For example, if a new factory is located in the region (with the employment L_{on}), in addition to already existing n-1 factories with employment L_{oi}, $i = 1,\ldots,n-1$, the demand for employment will increase. Thus, higher wages must be offered to compensate for the workers' foregone leisure time or the increased travel costs, which result from commuting.

As a result, the labor marginal cost $\bar{\omega}_o$ depends on the aggregate employment $\sum_i L_{oi}$, i.e.,

$$\bar{\omega}_o = \gamma_o \omega_o \left(\sum_{i=1}^n L_{oi} \right)^{\gamma_o - 1} \quad . \qquad (34)$$

The unknown parameters ω_ν, γ_ν can be estimated from statistical data. For that purpose one may compute the logarithm of (14) and use linear regression.

Assuming $y_{oi} = \bar{\omega}_o L_{oi}^{\gamma_o - 1}$, $i = 1,\ldots,n$, to be known, using equation (30) one can find the intensities of the other required (for maximum gain) production factors:

$$\hat{x}_{\nu i}(t) = \frac{\alpha_{\nu i}\gamma_{\nu i}}{\alpha_{oi}\bar{\omega}_{\nu i}(t)} y_{oi}(t) \quad , \qquad i = 1,\ldots,n \quad . \tag{35}$$

Relation (35) may also be regarded as the demand function for the νth input by the ith factory.

The ex ante marginal factor costs, in the case of many users, can be determined from the equilibrium condition between the demand and supply functions.

In order to derive the equilibrium price, it is necessary to construct the aggregate demand function:

$$x_\nu^d = \sum_{i=1}^{n} \frac{\alpha_{\nu i}\gamma_{\nu i}}{\alpha_{oi}\omega_{\nu i}(t)} y_{oi}(t) \quad , \qquad \nu = 1,\ldots,n \quad . \tag{36}$$

On the supply side, one should introduce the costs of resource development $c_\nu(x_\nu)$. It is assumed that these functions are given (e.g., in graphic form as shown in Figure 1) and can be approximated in the range of interest (e.g., a,b in Figure 1) by the function

$$c_\nu(x_\nu) = c(x_\nu - A)^\gamma \quad , \qquad a \le x_\nu \le b \quad , \tag{37}$$

where

c, A, γ are given constants.

The corresponding marginal production cost on the supply side becomes

$$\bar{\omega}_\nu = \gamma c(x_\nu - A)^{\gamma-1} \quad , \tag{38}$$

and the supply function can be written as follows

$$x_\nu^s = \left(\frac{\bar{\omega}_\nu}{\gamma c}\right)^{\frac{1}{\gamma-1}} + A \quad . \tag{39}$$

The equilibrium requires that a common price $\bar{\omega}_\nu$ exist ($\omega_{\nu i} = \bar{\omega}_\nu$, $i = 1,\ldots,n$) such that $x_\nu^d = x_\nu^s$. This price can be easily computed from formulae (36) and (39). For example, if $A = 0$,

$$\bar{\omega}_\nu = (\gamma c)^{\frac{1}{\gamma}} \left\{ \sum_{i=1}^{n} \frac{\alpha_\nu}{\alpha_{oi}} \nu i \; y_{oi}(t) \right\}^{\frac{\gamma-1}{\gamma}} \quad . \tag{40}$$

Then, plugging $\bar{\omega}_\nu$ in the place of $\bar{\omega}_{\nu i}$ in formula (36), the resources used by each regional sector $i = 1,\ldots,n$ can be derived.

It should be observed that when $\gamma > 1$ ($\gamma < 1$), $\bar{\omega}_\nu$ increases (decreases) if a new sector, with given $\gamma_{\nu i} y_{oi}(t)$ joins the regional economy. That sector will contribute to the existing economy when $\gamma < 1$, decreasing $\bar{\omega}_\nu$ and increasing all partners' profits. When the extension of the supply system is characterized by the cost function (ef) shown in Figure 1, a new sector may by attracted to the region (a,b) with decreasing marginal costs. Such a situation occurs in the case of the extension of a water system, when the construction of a channel transporting water from other regions yields decreasing marginal costs. In Guariso et al. (1978), an example of water transfer along the Mexican coast from south to north is studied. The marginal cost in dollars per m^3 was assumed equal ($bv^{-0.4}$, where v represents m^3, b is the coefficient computed for each channel). It is important to notice that the decrease in water price at the receiving end of the channel is accompanied by a change (in the Mexican case, an increase) of water price in the supplying region. This indicates that the extension of a water system by interregional water transfers must be studied in a complex model including the cost and benefits for all regions concerned.

It should also be mentioned that in the case of a closed system of regional models (which constitutes a model of the national economy), the price p(t) can be endogenized. For example, if the production functions for N regional economies are written in the form of equation (28), i.e.,

$$\hat{Y}_i = \frac{p}{\bar{p}_i} \bar{Y}_i \quad , \qquad i = 1,\ldots,N \quad , \tag{41}$$

then

$$\sum_{i=1}^{N} \hat{Y}_i = p \sum_{i=1}^{n} \bar{Y}_i/\bar{p}_i = \sum_{i=1}^{n} \bar{Y}_i = \bar{Y} \quad , \tag{42}$$

where $\bar{Y}$ can be regarded as gross national product. Then

$$p = \frac{\bar{Y}}{\sum_{i=1}^{N} \frac{\bar{Y}_i}{\bar{p}_i}} = \{ \sum_{i=1}^{n} \frac{\bar{Y}_i}{\bar{Y}} \frac{1}{\bar{p}_i} \}^{-1} \quad . \tag{43}$$

The national price (p^{-1}) is a weighted average (with the weights $w_i = \frac{\bar{Y}_i}{\bar{Y}}$, $\sum_{i=1}^{N} w_i = 1$) of $(\bar{p}_i)^{-1}$, where

$$\bar{Y}_i = \frac{\omega_o}{\alpha_{oi}} \int_0^T e^{-\lambda_o t} [x_{oi}(t)]^{\gamma_o} dt \quad , \qquad i = 1,\ldots,N \quad , \tag{44}$$

where

x_{oi} is regional employment; and

ω_o are regional wages.

Obviously, when p is set according to equation (42), a part of the regional economy (having $\bar{p}_i > p$) becomes unprofitable. In order to protect low-profit regional economies, an increase of p (over the value (43)) is needed. In the case of an open system, a more complicated system of protection can be introduced. However, this problem is outside the general scope of the paper. In order to study the impact of sectoral price distortions on the national economy, an input-output sectoral and regional model is needed. Such a model (MRI), which has been constructed for the Polish economy, is described in Bruckmann (1978). MRI can be used, in particular, for studying the impact of changes in the international terms of trade on sectoral prices, which in turn affect regional production costs. By studying the change of technological coefficients resulting from the different locations of production units and transport costs, it is possible to investigate the impact of locations on regional prices (i.e., in the present notation, the change of A coefficients in equation (21) and corresponding changes of W in equation (28) (for details see Kulikowski and Korcelli 1976). By linking MRI with the regional model, it will be possible to investigate the impact of national or international economies on regional growth.

3. REGIONAL UTILITIES, ENVIRONMENT, AND MIGRATION

When speaking about regional utilities, planners are fond of using broad concepts such as per capita consumption out of personal income and per capita aggregate consumption (i.e., education, housing, health care, environmental factors, etc.). They believe that if the regional budgets for services and wages are planned in the proper way, maximum public satisfaction will follow. However, because of the differences in service costs, scale effects, and externalities in consumption, the level of regional satisfaction may not be proportional to the expenditures; in a similar way as with a given input cost, one can produce less or more depending on the marginal production cost (see Kulikowski 1974).

In other words, the regional utility depends on the marginal prices of given inputs, such as employment and services, which determine the "marginal cost of utility", generated in the region.

The present remarks indicate that the optimum consumption strategies can be derived using a mathematical technique similar to that used in section 2. However, it is convenient to use instead of equation (14), the per capita input cost, i.e.,

$$c_\nu(x_\nu)\frac{1}{P} = P_\nu^{\gamma_\nu}\,\omega_\nu\left[\frac{x_\nu}{P_\nu}\right]^{\gamma_\nu}\frac{1}{P} = \Omega_\nu[z_\nu]^{\gamma_\nu} \quad , \tag{45}$$

where

P is the population;

$$\Omega_\nu = P_\nu^{\gamma_\nu}\,\omega_\nu P^{-1} \quad , \qquad \nu = 0,1,\dots,M \ . \tag{46}$$

$z_\nu = \frac{x_\nu}{P_\nu}$ is the per capita amount of service of νth category provided;

$P_\nu = \kappa_\nu P$ is the number of people entitled to, or demanding, the service; and

κ_ν $(0 \le \kappa_\nu \le 1)$ are given numbers.

Denoting the cost of providing the νth service per capita, within the planning period [0,T] by Z_ν, one can write (instead of equation (10))

$$Z_\nu = \int_0^T e^{-\lambda_\nu t}\,\Omega_\nu(t)\,[z_\nu(t)]^{\gamma_\nu}dt \quad , \qquad \nu = 0,1,\dots,M \ . \tag{47}$$

When the primary factor is labor, $z_o = {}^{x_o}/P_o$, although the services $(\nu = 1,\dots,M)$ do not generally coincide with the production factors introduced in section 2. However, the same symbols $(\gamma_\nu, \beta_\nu, \alpha_\nu, \lambda_\nu, A)$ are preserved for convenience in notation.

The discounted utility (in monetary terms), which is generated per consumer in the region under analysis $\nu(2)$, can be written in a form similar to equation (7) and (9), i.e.,

$$U(Z) = \int_0^T \Pi(t) A e^{-\lambda t} \prod_{\nu=0}^{M} [z_\nu(t)]^{\beta_\nu} dt \quad , \tag{48}$$

where

$$\lambda = \sum_{\nu=0}^{M} \alpha_\nu \lambda_\nu \quad , \qquad \alpha_\nu = \frac{\beta_\nu}{\gamma_\nu} \quad , \qquad \sum_{\nu=0}^{M} \alpha_\nu = 1 \quad , \tag{49}$$

and

$$\Pi(t) A \prod_{\nu=0}^{M} \left[\frac{\Omega_\nu(t)}{\alpha_\nu}\right]^{-\alpha_\nu} = V = \text{constant} \quad . \tag{50}$$

The $\Pi(t)$ is a given function, which will be called the average (national) utility price.

Then, according to the Factor Coordination Theorem for the given $\Omega_o(t)[z_o]^{\gamma_o} \varepsilon L[0,T]$, there exists the unique strategy

$$\hat{z}_\nu(t) = \left\{ \frac{\alpha_\nu}{\alpha_o} \frac{\Omega_o(t)}{\Omega_\nu(t)} [z_o(t)]^{\gamma_o} \right\}^{\frac{1}{\gamma_\nu}} \quad , \tag{51}$$

$$\nu = 1, \ldots, M \quad ,$$

under which $U(\underline{Z})$ attains the maximum value $\bar{U}(\underline{Z})$. Maximizing $\bar{U}(\underline{Z})$, subject to the constraints

$$\sum_{\nu=0}^{M} Z_\nu \leq Z \quad , \qquad Z_\nu \geq 0 \quad , \quad \nu = 0,1,\ldots,M \quad , \tag{52}$$

where

Z is the aggregate per capita consumption value,

one obtains

$$\hat{Z}_\nu = \alpha_\nu Z \quad , \qquad \nu = 0,1,\ldots,M \quad . \tag{53}$$

Then, using equations (24) and (28),

$$\hat{U} = \bar{U}(\hat{\underline{Z}}) = VZ = \frac{\Pi(t)}{\bar{\Pi}(t)} \cdot \frac{Z_o}{\alpha_o} \quad , \tag{54}$$

where the marginal cost of utility

$$\bar{\Pi}(t) = A^{-1} \prod_{\nu=0}^{M} \left[\frac{\alpha_\nu}{\Omega_\nu(t)}\right]^{\alpha_\nu} \quad . \tag{55}$$

In the case of a national system consisting of M regions, each described by equations (45,47,48,51,53-55), the national utility price Π is a weighted average of regional $\bar{\Pi}_i$ costs, $i = 1,\ldots,M$ (compare equation (43)). In formula (50), the term $\Omega_o(t)[z_o(t)]^{\gamma o}$ is exogenous. It may represent the per capita personal income ($Y_o(t)/P$ according to the notation of section 2).

If, instead of $\hat{z}_\nu$, $\hat{Z}_\nu$, a different strategy, say $\tilde{z}_\nu$, $\tilde{Z}_\nu$, $\nu = 0,1,\ldots,M$, is used, the number V in equation (54) should be replaced by

$$\tilde{V} = V \frac{U(\tilde{\underline{z}})}{\bar{U}(\hat{\underline{z}})} \frac{\bar{U}(\tilde{\underline{Z}})}{\bar{U}(\hat{\underline{Z}})} = V \prod_{\nu=0}^{M} \left(\frac{\tilde{\alpha}_\nu}{\alpha_\nu}\right)^{\alpha_\nu} \frac{U(\tilde{\underline{z}})}{U(\hat{\underline{z}})} \quad , \tag{56}$$

where

$$\tilde{\alpha}_\nu = \tilde{Z}_\nu/Z. \tag{57}$$

Obviously, $\tilde{V} \leq V$ and the equality sign holds iff $\tilde{z}_\nu = \hat{z}_\nu$, $\tilde{\alpha}_\nu = \alpha_\nu$, $\nu = 0,1,\ldots,M$. Then, if the region consists of consumers with different tastes or if employment is not coordinated with access to services, then $\tilde{\alpha}_\nu \neq \alpha_\nu$, and a decrease in regional utility follows.

The relations (51) can be used to evaluate regional demands for services, expressed in terms of average per capita income ($\Omega_o[z_o(t)]^{\gamma o}$) and prices ($\Omega_\nu$). They can be also used (in a way similar to that described in section 2) to find the benefits resulting from economies of scale (e.g., when services are used by municipal and industrial users). In order to derive the resulting marginal price, the demands of the users should be determined.

The municipal demand $x^d_{\nu m}$ becomes $x^d_{\nu m} = \hat{z}_\nu \cdot P_\nu$, where $\hat{z}_\nu$ can be expressed in terms of marginal cost $\bar{\Omega}_\nu = \gamma_\nu \Omega_\nu z_\nu{}^{\gamma_\nu - 1}$ $= \bar{\omega}_\nu \kappa_\nu$, i.e. (see equation (30)),

$$\hat{z}_\nu = \frac{\beta_\nu \Omega_o}{\alpha_o \bar{\Omega}_\nu} z_o{}^{\gamma_o} = \frac{\beta_\nu \Omega_o z_o{}^{\gamma_o}}{\alpha_o \bar{\omega}_\nu \kappa_\nu} \quad . \tag{58}$$

Then,

$$x^d_{\nu m} = \hat{z}_\nu \quad P_\nu = \frac{\beta_\nu}{\alpha_o} \frac{\Omega_o}{\bar{\omega}_\nu} z_o{}^{\gamma_o} P \quad . \tag{59}$$

Assuming industrial demand to be $x^d_\nu(\bar{\omega}_\nu)$ (given by equation (36)), the resulting price $\bar{\omega}_\nu$ can be determined such that $x^d_\nu + x^d_{\nu m} = x^s_\nu(\bar{\omega}_\nu)$, $x^s_\nu(\bar{\omega}_\nu)$ is a given supply function of the νth service (compare equation (39)).

It should be noted that municipal demand and access to services (59) depends, in addition to per capita income, on the population P, access κ_ν, and the marginal production cost

$$\bar{\omega}_\nu(x_\nu) = \gamma_\nu \omega_\nu [x_\nu]^{\gamma_\nu - 1} \quad , \tag{60}$$

which, in turn, depends on the amount of services x_ν.

When economies of scale exist in services, i.e., $\gamma_\nu < 1$, $\bar{\omega}_\nu(x_\nu)$ decreases when $x_\nu = P_\nu z_\nu$ increases and vice versa.

Generally, the existing access strategies vary among regions, due to differences in socioeconomic development. In Poland, for example, the access to water services depends greatly on the rate of urbanization. Old houses in small towns and in rural areas still use wells as the main source of potable water. In modern suburban areas, on the other hand, it is used for watering gardens, washing cars, etc. According to statistical data (Rocznik Stalystyczny 1977), the percentage of the urban population that has access to the public water system in Poland has increased from 69.5 percent in 1960, to 84 percent in 1976. In 1976 the amount of water used per urban consumer varied from 14.7 m^3 (Bialskopodlaskie voivodship) to 73.3 m^3 (Warsaw); the national average was 54.3 m^3.

The statistical evidence supports the relation (59) in which municipal water demand depends on personal income and access parameters (i.e., the rate of urbanization).

In the literature regarding public prices for services (see Mushkin 1972), examples are cited indicating that large benefits to the regional economy follow from scale effects or "internalization of externalities". However, there are no sufficiently simple techniques available to evaluate the benefits to all parties concerned. In order to use the present approach efficiently, estimates of utility parameters are needed. As shown in Kulikowski (1978c), the estimates $\bar{\gamma}_\nu$, $\bar{\Omega}_\nu$ of γ_ν, Ω_ν parameters for a given year can be derived by cross-sectional linear regression applied to the relations (obtained from equation (45)).

$$\log Z_{\nu i} = \log \Omega_\nu + \gamma_\nu \log z_{\nu i} + \xi_i \quad , \qquad (61)$$

$$\nu = 0,\ldots,M \quad , \quad i = 1,\ldots,N \quad ,$$

where

$Z_{\nu i}$ is the per capita cost of the νth service in the ith region;
$z_{\nu i}$ is per capita amount of νth service in the ith region;
N is number of regions; and
ξ_i are random variables.

In order to find estimates $\bar{\beta}_\nu$ of β_ν, $\nu = 0,1,\ldots,M$, coefficients, it is helpful to assume that decision makers maximize the utility. Indeed, using past data regarding the allocation of regional budgets over time (i.e., $Z_{\nu\tau}$, $\nu = 0,1,\ldots,M$, $\tau = -1,-2,\ldots,$), one can compute the numbers

$$\tilde{\alpha}_{\nu\tau} = z_{\nu\tau} / \sum_{j=0}^{M} z_{j\tau} \quad , \tag{62}$$

Then, using linear regression, one can find estimates $\bar{\alpha}_\nu$ of α_ν for each $\nu = 1,\ldots,M$.

The unknown numbers β_ν can be derived by the relations $\bar{\beta}_\nu = \bar{\alpha}_\nu\bar{\gamma}_\nu$, $\nu = 1,\ldots,M$.

It should be observed that the problem of estimating regional utilities is not easy. A direct approach, which uses the relations

$$\beta_\nu = \frac{dU}{U} : \frac{dz_\nu}{z_\nu} \quad , \qquad \nu = 0,1,\ldots,n \quad , \tag{63}$$

and evaluates the consumers change of utility $\Delta U/U$ (resulting from the change of $\frac{\Delta z_\nu}{z_\nu}$) cannot be applied here because of the absence of corresponding data. Therefore, an indirect approach that reconstructs the utility from past (utility maximization) strategies is used. Obviously, that approach gives the regional utilities as perceived by the decision maker rather than by the population.

The planner's standard approach is to use ex post estimates of model parameters (i.e., $\bar{\gamma}_\nu$, $\bar{\beta}_\nu$) in an ex ante sense. That approach is justified for a short planning horizon. In the long run, a special model dealing with the change in the socioeconomic system of values and preferences should be used.

The main advantage of the approach described here is that consumer welfare and demands, described by equation (59), depend on the single exogenous factor

$$z_o = \frac{x_o}{\kappa_o P} \quad , \tag{64}$$

i.e., the ratio of employment to the population of productive age $\kappa_o P$ (which is mainly predetermined by demographic factors).

In Poland, z_o depends to a large extent on the employment of women, and in rural areas, where the employment of women is higher than in urban agglomerations, z_o is usually greater (Zawadzki 1973). An increase of z_o necessitates a rise in wages and an increase in services (e.g., kindergartens). It may influence the upbringing of children and decrease labor productivity. Finally, it decreases the per capita leisure time, which was not included explicitly in the utility (48). The present social policy in Poland is strongly motivated by a desire to shorten working time (especially for the rural population). For these reasons z_o can also be regarded as a decision variable.

The present methodology can be applied for studying the impact of urbanization (i.e., the increasing waste generated by regional industry and consumers) on the environment. Assume for that purpose that the waste discharge is a known function of the output production or access to services. For example, the volume of water polluted by an industry is proportional to the production output, whereas recreational pollution depends on the amount of people having access to water resources.

In order to estimate the total discharge Q to the regional environment, one should take a linear form of different production (x_i, i = 1,...,n) and consumption ($z_\nu P_\nu$) activities, i.e.,

$$Q = P \sum_{\nu=0}^{M} \delta_\nu z_\nu \kappa_\nu + \sum_{i=0}^{n} x_i \delta_i \quad , \tag{65}$$

where

> δ_ν, δ_i are given discharge intensity coefficients.

The δ_ν, δ_i coefficients can be estimated from statistical or technical data. For example, the regional statistical data existing in Poland (Rocznik Statystyczny Wojewodztw 1978) include the volume of industrial (x_i) and municipal ($Pz_\nu\kappa_\nu$) water intakes as well as the polluted water discharges (Q_i, Q_ν); the data allow the δ_ν, δ_i coefficients to be estimated ex post. When Q must be derived in the ex ante sense, all the information regarding water-use intensities by new industries located in the region should be utilized.

Now one can compute the value of waste discharge Q as a function of population P.

The output $x_i = {}^Y i/p_i$ (compare equation (29) for T = 1) can be written as

$$x_i = \frac{W_i}{\alpha_o} \frac{\omega_o}{p_i} x_{oi}^{\gamma_o} \quad , \tag{66}$$

$$x_{oi} = z_{oi} p_o = z_{oi} \kappa_o P \quad , \qquad i = 1, \ldots, n \quad , \tag{67}$$

while $\hat{z}_\nu$, according to equation (51), becomes

$$\hat{z}_\nu = \left\{\frac{\alpha_\nu}{\alpha_o} \frac{\Omega_o}{\Omega_\nu} z_o^{\gamma_o}\right\}^{\frac{1}{\gamma_\nu}} = \left\{\frac{\alpha_\nu}{\alpha_o} \frac{\omega_o}{\omega_\nu} \frac{[\kappa_o P]^{\gamma_o}}{[\kappa_\nu P]^{\gamma_\nu}} z_o^{\gamma_o}\right\}^{\frac{1}{\gamma_\nu}} \quad . \tag{68}$$

When z_o, κ_ν are given and $\gamma_\nu = 1$, $\nu = 0, 1, \ldots, M$, Q is a linear function of population P. When

$$\gamma_o > \gamma_\nu \quad , \qquad \nu = 1, \ldots, M \quad , \tag{69}$$

Q(P) is convex.

It is usually assumed that the environmental damage function $\bar{E}(Q)$ (which represents in monetary units all possible damage to society, i.e.,health, vegetation, goods, etc.) is convex in Q (Kneese 1971). Then, under assumption (69), this function, say, $\bar{E}(Q) = \tilde{E}(P)$, is convex in P.

In order to reduce the damage $\bar{E}(Q)$, a waste treatment system can be used. According to technical data, the treatment cost function T(Q,q), where Q is the input to the treatment plant and q is the residual discharge of waste to the environment, exhibits the following properties (Ferrar 1973 and Rinaldi et al. 1977):

$T(Q,q)$ is convex with respect to Q and q (taken separately); $T(Q,\alpha q)$ is concave with respect to Q.

It is assumed that in the region studied a given scheme of regulations for environment production is being used, such as a fixed percentage (α) of waste removal.

In this case the marginal treatment cost decreases with Q, and the scale benefits may follow from externalizing the environmental damage (i.e., constructing collective waste treatment systems). Since the location of new industrial plants depends to a large extent on expected production costs, including waste treatment costs, the regions may benefit considerably from collective action in constructing and controlling the pollution protection system.

It should also be noted that, in general, regional pollution processes are interrelated. For example, consider a system of regions located along the same river, a discharge of waste at the upstream region R_i affects the downstream region R_j. In other words, the environmental damage function for R_j, $j > i$, becomes

dependent on $\sum_{k=i}^{j} a_k Q_k$, where a_k, $a_j = 1$, $a_k \geq a_{k-1}$,

$k = i - 1,\ldots,j$, are positive coefficients.

This means that generally the regional environment function $\tilde{E}_j$ depends on the upstream urbanization patterns and population (i.e., $\tilde{E}_j(b_i,P_i,\ldots,b_jP_j)$, where b_i are given coefficients). It should be noted that special models dealing with water pollution control problems have been proposed recently (compare Kindler and Ijjas 1978). In these models, it is usually assumed that alternative scenarios of future waste loads are given by the regional authorities concerned with economic planning in the area. Obviously, the methodology proposed in this paper enables endogenization of future waste loads; in other words, linkage of the type of models described in Kindler and Ijjas (1978) with the present model is possible.

Existing environmental protection systems usually restrict the discharge limits (e.g., by decreasing the admissable percentage of waste α) for the upstream dischargers. At the same time, the discharge and marginal production costs increase when the industry, or growing agglomeration, is located in an upstream region.

One of the main problems connected with regional planning, is to achieve an allocation of population P_i, $i = 1,\ldots,N$, among the regions R_i, $i = 1,\ldots,N$, such that an efficient national economic system is obtained, subject to constraints and costs imposed by the environment and demographic factors. This goal can be realized by encouraging interregional migration s_iP_i (where s_i is the migration rate) at the expense of environmental costs and migrants accommodation costs $E_i(\cdot) + C_i(s_i)$.

By extending the approach taken in Kulikowski (1978c), a corresponding model for finding the optimum migration strategy can be formulated as follows.

Assume that the projected (zero migration) population $P_i(0) = P_i$, $i = 1,\ldots,N$, is given. As a result of migrations (s_i), the population in the first year of the planning period attains the values $P_i(1+s_i)$, $i = 1,\ldots,N$. The migration optimization problem can then be formulated in the following way.

Find the strategy $\underline{s} = \hat{\underline{s}} = \{\hat{s}_1,\ldots,\hat{s}_N\}$, such that the function

$$F(\underline{s}) = \sum_{i=1}^{N} [Y_i(s_i) - E_i(\underline{s}) - C_i(s_i)] \quad ,$$

$$E_i(s) = \hat{E}_i[P_1(1+s_1),\ldots,P_N(1+s_N)] \quad , \qquad (70)$$

$$Y_i(s_i) = \frac{W_i}{\alpha_{oi}} \omega_o \, [z_{oi}P_i(1+s_i)\kappa_{oi}]^{\gamma_o}$$

attains the maximum subject to the constraint $\underline{s}\varepsilon\Omega$,

where

$$\Omega = \{s_i : \sum_{i=1}^{N} s_i P_i \leq S \quad , \quad -1 \leq s_i \leq 1 \quad , \tag{71}$$

$$i = 1,\ldots,N \quad ; \text{ and}$$

S is the maximum acceptable international migration.

Since it is hard to identify the analytical form of $E_i(\cdot)$ in simpler models, it can be assumed that under the existing environmental regulations, which fix the waste removal percentages (α_i) in each region, environmental damage can be neglected. The environmental costs are then reduced to $\bar{E}_i(Q_i) = T_i[Q_i, \alpha_i Q_i]$ and $E_i(s)$ are known convex functions.

The problem then becomes one of evaluating $C_i(s_i)$ functions. In Kulikowski (1978c), an econometric model was used for this purpose. Since it is believed that the correlation between the ratio $U_i/_U$ (U_i = regional, U = national utilities) and the relative increase of population $P_i(1+s_i)/P_i = 1 + s_i$ exists, the model was assumed in the form

$$s_i = d\left(\frac{U_i}{U}\right)^a - 1 \quad , \quad a, d = \text{are constants} \quad . \tag{72}$$

In the notation (see equation (54)), $U_i = V_i Z_i$, $U = Z$, where

Z_i is per capita aggregate consumption (from personal income and services) in monetary units in region R_i;

Z is average (national) per capita aggregate consumption; and

$V_i = \frac{\Pi}{\Pi_i}$ is the utility level at R_i.

The unknown coefficients a, d may be estimated using statistical data. For that purpose, it is convenient to write formula (72) in the form

$$\log (s_i+1) = \log d + a^{-1} \log \left(\frac{V_i Z_i}{Z}\right) , \quad i = 1,\ldots,N , \tag{73}$$

and use linear regression.

The estimates of d and a as well as V_i for different voivodships were derived in Kulikowski (1978c). The estimates varied around $a \cong 50$, $d \cong 1.0003$.

The annual cost of accommodation of one migrant in region R_i (coming from the national system characterized by Z)

$$\Delta c_i = Z_i - Z = Z\left[\frac{Z_i}{Z} - 1\right] . \tag{74}$$

Then,

$$C_i(s_i) = \Delta c_i s_i P_i , \quad \text{for } s_i > 0 . \tag{75}$$

Since equation (73) can be written in the form

$$Z_i/Z = \left(\frac{s_i+1}{d}\right)^a V_i^{-1} , \tag{76}$$

one gets

$$C_i(s_i) = Z\left[\left(\frac{s_i+1}{d}\right)^a V_i^{-1} - 1\right] s_i P_i , \quad s_i > 0 . \tag{77}$$

In the case of negative migration ($s_i < 0$), savings to the regional budget, and consequently a decrease of $C_i(s_i)$, are possible.* Since the cost function for $s_i > 0$ grows much faster than

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*It is, however, argued that $C_i(s_i)$ may decrease more slowly than the linear function (77) (e.g., because of the cost of capital, which is necessary to substitute for the outflow of labor and the fixed expenses for existing services). For this reason, in Kulikowski (1978c), $C_i(s_i)$ was assumed to be zero for $s_i < 0$.

$|C_i(-s_i)|$, the savings do not generally compensate for the cost of accommodating migrants.

Since $F(\underline{s})$ is a concave function for $\gamma_o \leq 1$, it attains at the point $\hat{\underline{s}}$ a unique maximum in the convex set Ω. It is possible to show that the maximum is also attained for $\gamma_o > 1$, although $F(\underline{s})$ is not concave for small $\underline{s}$. It can be observed that the cost $C_i(s_i)$ is minimum in the region R_j, which has the largest utility level V_j (i.e., $V_j = \max_{i \in N} V_i$). If R_j has at the same time the lowest environmental costs $E_j(\underline{s})$, it receives the largest share of migrants (i.e., $\hat{s}_j = \max_{i \in N} \hat{s}_i$). It can also be shown that $\hat{s}_j$ decreases when the urbanization of R_j increases (e.g., if γ_o is growing). In other words, the optimum solution $\hat{\underline{s}}$ allocates the population in such a way that the maximum economic efficiency characterized by $Y_i(s_i)$ is attained minus the urbanization costs $E_i(s) + C_i(s_i)$.

When the urbanization costs increase, while $Y_i(s_i)$ is constant, a decrease in migration should follow. It should also be observed that the policy aimed at increasing the job access index z_{oi}, which is a substitute for migration, may decrease $\hat{s}_i$. However, as already mentioned, this policy creates additional (kindergarten, etc.) costs, say $\tilde{c}_i$, which should be added to the right side of (70). A possible decrease in $\hat{s}_i$ is then reduced by the increased cost of $\tilde{c}_i$.

In order to derive the optimum values of $\hat{s}_i$, $i = 1,\ldots,N$, in the general case a numerical (gradient) technique can be used.

It should be observed that when the optimum values $\hat{s}_i$, $i = 1,\ldots,N$, are known, using equation (74), one can derive the necessary expenses in per capita regional consumption.

The regional differences in consumption are necessary to keep the migration level at an optimum level and thus secure maximum economic efficiency. Therefore, in the model, the equity versus efficiency problem is resolved by a compromise between the economic gain and the urbanization costs (i.e., environmental costs and population transfer costs.).

When the $\hat{s}_i$, $i = 1,\ldots,N$, values are known, the regional population $P_i = P_i(1+\hat{s}_i)$ can easily be computed for the next optimization step. The labor supplies at each region, as well as the demands for the corresponding production factors and services, can be also computed from equations (22) and (51). By repeating the calculations step by step, all the data important for planning can be explicitly derived (for details,see Kulikowski 1978c).

Obviously, operation of the model depends to a large extent on the demographic regional projection model used, which is not discussed here. A model dealing with spatial population and migration analysis, such as the model described in Willekens and Rogers (1978), can be used for this purpose. The migration rates, which are exogenous in this model, can be endogenized by using the regional development model described in this paper.

The regional development model can also be used for a situation in which several sectors are located in the same region. In this case, the intersectoral migration cost is mostly associated with vocational reeducation or retraining.

4. REGIONAL POLICIES AND STABILITY

In sections 2 and 3 a simple model of the spatial allocation of economic activities was studied. It was based on the assumption that the central decision maker is able to locate all the activities according to a single criterion.

In practice, however, the planning and decision-making process is multiobjective, reflecting the interests of many regional agents or partners (i.e., producers, consumers, and representatives of local and higher-level authorities). In Poland, for example, two versions of the national development plan are usually constructed (Zawadzki 1973). The first--the vertical plan--reflects macroeconomic aspects of national development. The second--the spatial or horizontal plan--relects regional development aspects. Several government commissions or committees evaluate each plan before the final decision is taken. The greater the number of committee members that support the project, the greater are the chances that it will be approved.

The approach can be illustrated by a simple example. Take a committee concerned with the evaluation of the annual regional budget, including subsidies S_i, $i = 1,\ldots,N$, paid out of the central budget to the N regional budgets. The committee consists of N + 1 members, representing N regions (R_i, $i = 1,\ldots,N$) and the central authority R_o. Assume that the regional benefits B_i consist, besides S_i, of $\alpha_{oi}Y_i$ representing regional wages, which are paid by the industries located in the region, minus the cost of urbanization, i.e., the population transfer cost C_i and environmental damage E_i (compare section 3):

$$B_i(s_i) = \alpha_{oi}Y_i(s_i) + S_i(s_i) - C_i(s_i) - E_i(s_i) \quad , \quad i = 1,\ldots,N \quad . \tag{78}$$

It is also assumed that the policy of R_o is principally motivated by the balance of payments (F) and productive investments ($\sum_{i=1}^{N} \alpha_{1i}Y_i$).

Providing that R_o chooses a policy that includes regional subsidies $S_i(s_i)$ and appropriate taxation levels T_i, $i = 1,\ldots,N$, then the goal function of R_o becomes

$$B_o(\underline{s}) = \sum_{i=1}^{N} \{[T_i-\alpha_{oi}-\alpha_{1i}]Y_i(s_i) - S_i(s_i)\} - F \quad . \tag{79}$$

The national utility is assumed to be

$$B(\underline{s}) = \sum_{i=0}^{N} B_i(\underline{s}) = \sum_{i=1}^{N} \{[T_i+\alpha_{1i}]Y_i(s_i) - C_i(s_i) - E_i(\underline{s})\} - F \quad . \tag{80}$$

If taxes T_i are chosen in such a way that $B_o(\underline{s}) = 0$, one gets

$$B(\underline{s}) = \sum_{i=1}^{N} [\alpha_{oi}Y_i(s_i) + S_i(s_i) - C_i(s_i) - E_i(s_i)] \tag{81}$$

and the national goal function is reduced to maximization of total consumption minus the costs of environmental damage and population transfer.

When $S_i(s_i)$ is regarded as the aggregate regional budget, consisting of aggregate consumption and investment funds, one should set $\alpha_{1i} = 0$, $i = 1,\ldots,N$, in (79) and (80). In that case, the problem of maximizing $B(\underline{s})$, subject to the constraints imposed on $s_i (s_i \varepsilon \Omega)$ and

$$\sum_{i=1}^{N} Y_i(s_i) = \sum_{i=1}^{N} [\alpha_{oi} Y_i(s_i) + S_i(s_i)] \quad , \tag{82}$$

is equivalent to equations (70) and (71). However, the corresponding solution ($\hat{\underline{s}}$) may be regarded as too detached from the partner's policies. The reason is that the subsidy S_i constitutes only a part of the regional budget, while the rest is covered by the income from services, taxes, charges, etc. (e.g., in Poland in 1975 it constituted on average 48 percent, with 18 percent representing the given share of regions in the central budget). On the other hand, the regional aggregate consumption

$$P_i \sum_{\nu=1}^{m} Z_{i\nu}(\hat{s}_i) \quad , \tag{83}$$

which is predetermined by optimum migration $\hat{s}_i$, represents a part of the regional budget. The regional fiscal policy is therefore motivated by more objectives than in the simplified model (78). The same is true with respect to the central authority (i.e., B_o in equation (80)). Moreover, a number of random factors not specified explicitly in the model, such as changes of prices, may influence the outcome of the game. For these reasons the partners of the game may wish to bargain in order to realize their goals and policies.

For example, they may agree to depart from $S_i(\hat{s}_i)$, giving more or less support to the given ith region, on the understanding that nobody's interests are hurt too much. In other words, the committee may choose to play a N+1 person game in order to find (with given taxes T_i, $i = 1,\dots,N$) the best and most acceptable regional subsidies scheme $\hat{S}_i$, $i = 1,\dots,N$.

In general, the game may be played with all the subregions constituting a region or the whole country. Another possible game consists in fixing S_i, $i = 1,\dots,N$, and finding the taxation scheme $T_i = \hat{T}_i$, $i = 1,\dots,N$. The number of partners may change when the subsidy is related to a specific project (e.g., the expansion of the regional water supply system) and it has not been decided how many regions or industries should participate. In the last case, it is important to determine how the regional and national goals can be coordinated, in order to have a stable socioeconomic system. The stability of the system depends, in particular, on the scale benefits, which the project may create by the joint actions, and on the impact of externalities, which produce centrifugal forces.

In order to investigate the existence of stable policies, it is helpful to introduce some simple concepts from game theory. In particular, one should define the characteristic function $\phi(n)$ of the game.

The numeral n belongs to the set $\tilde{R}$, which is a union of the set of regions $R = \{1,\dots,N\}$ and R_o, i.e., $n \varepsilon \tilde{R} = R \cup R_o$. The values of Φ give the total potential profit available $(B(\hat{\underline{s}}))$ to any subset of n of the parties:

$$\phi(n) = \begin{cases} 0 \ , & \text{if } n = N \text{ or } n = 0, \\ \max\limits_{s \varepsilon \Omega} B(\underline{s}) \ , & \text{if } n \varepsilon R \neq \{0\} \ . \end{cases} \tag{84}$$

The assumption of zero net profit for all subsets of regions R_i $(n \varepsilon R)$ without R_o, represents the fact that regions are not obliged to participate in the system, but at the same time they cannot play the game without R_o, or enter the system without making an agreement with R_o.

An important property of $\Phi(n)$ is that of convexity; $\Phi(n)$ is convex if

$$\phi(n \cup \{i\}) - \phi(n) < \phi(m \cup \{i\}) - \phi(m) \quad , \tag{85}$$

for all $n \subsetneq m \subsetneq \tilde{R}$ and all $i \varepsilon \tilde{R}-m$.

A simple regional policy is specified by the following rules: find $\hat{s}_i$, $i = 1,\ldots,N$, and compute $\Phi(N)$; determine $S_i(\cdot)$, $i \varepsilon R$, such that

$$\max B_i(s_i) = B_i(\hat{s_i}) = B_i^* \tag{86}$$

and

$$B_i^* = \frac{1-a}{N}\,\phi(N) \quad , \qquad 0 \leq a \leq 1 \quad , \qquad i \varepsilon R \quad . \tag{87}$$

Condition (86) indicates that the regions maximizing their own goals (B_i) also maximize the national goal ($\Phi(N) = B$). This property is called efficiency.

Condition (87) means that regions divide equally part of the total benefit. When $a = 0$ there is no profit for R_o ; when $a = 1$, there is no profit for the regions.

The regional policy (86) and (87) is rather unacceptable for small regions, which may feel that the larger regions are given too much support. They may argue that the benefits should be shared according to the improvement the region generates when it enters the system. In other words, they would prefer to replace (87) by a "lexicographic scheme" (see Shapley 1971).

For that purpose, one can introduce an ordering that, to each index number $i \varepsilon \tilde{R}$, assigns the number $\omega(i) \varepsilon \tilde{R}$. Then, one can compute the values $\Phi(n_k)$ for $n_k = \{i\colon \omega(i) \leq k\}$, $k = 0,1,\ldots,N$. The regional benefits in the lexicographic scheme become

$$B_i^* = \phi[n_{\omega(i)}] - \phi[n_{\omega(i)-1}] \quad , \qquad i \varepsilon \tilde{R} \quad , \tag{88}$$

$$\phi(n_{-1}) = 0 \quad .$$

As in the case of condition (87), $\sum_{i=0}^{N} B_i^* = \phi(N)$. If $\omega(0) = 0$, R_o has no benefit, whereas for $\omega(0) = N$, the benefits of R_i are zero.

A typical regional policy of the last type can be obtained following the Shapley scheme, where a convex combination of all possible orderings is taken into account, i.e.,

$$B_i^* = \sum_{j=i}^{(N+1)!} \lambda_j \{\phi[n_{\omega_j(i)}] - \phi[n_{\omega_j(i)-1}]\} \quad , \quad i \varepsilon \tilde{R} \quad , \tag{89}$$

where

$$\lambda_j = [(N+1)!]^{-1} \quad . \tag{90}$$

Besides efficiency, important features of the regional policy are <u>acceptability</u>: the vector of benefits $B = \{B_o, \ldots, B_N\}$ are nonnegative in the sense that $B_i \geq 0$, $i\varepsilon\tilde{R}$, and <u>stability</u>: the policy is stable with respect to $\tilde{R}$ if

$$\sum_n B_i^{\tilde{R}} \geq \sum_n B_i^n \quad , \qquad \forall n \subseteq \tilde{R} \tag{91}$$

(i.e., all subsets n of $\tilde{R}$ take advantage of the coalition with the remaining parties).

The following definition is also used. The set of $B^{\tilde{R}}$ generated by all stable and efficient policies is called the core of $\tilde{R}$.

A theorem of Shapley (1971) states that if $\phi(n)$ is convex, the core exists and it is a convex polyhedron. In addition, the policy (89) is stable and generates all the points of the core.

In Figure 2, a typical core (a,b,c,d,e) for a 3-partner (n=2) convex game is shown. Since $\phi(n)$ is convex, the parties $B_o^* + B_1^*$, $B_o^* + B_2^*$ produce less gain than $B_o^* + B_1^* + B_3^*$. In other words, $\phi(\{0,1,2\})$ is greater than $\phi(\{0,1\})$ or $\phi(\{0,2\})$. As a result, the efficient policy belongs to the core (a,b, c,d,e) lying on the plane $B_o^* + B_1^* + B_2^* = \phi(\{0,1,2\})$. On the other hand, for the points B_i^* on the (a,b,c,d,e) polyhedron $\sum_{\tilde{R}} B_i^* = \phi(N)$, $\sum_n B_i^* \geq \phi(n)$, $n \subseteq \tilde{R}$. Since these are the conditions for efficiency and stability, the core is contained in the polyhedron (a,b,c,d,e).

In order to find out whether a stable and efficient regional policy exists, one has to check whether a core of the game exists. As shown in Rinaldi et al. (1977), under the assumption that the profit functions are concave (i.e., when in our model $\alpha_{oi}Y_i(s_i) - C_i(s_i)$ is concave and $E_i(\underline{s})$ is negligible), the core exists. When $E_i(\underline{s})$ is nonnegligible, and a treatment system with a $T_i(Q_i,q_i)$ cost function is used, the existence of the core depends on the form of E_i and T_i.

Since E_i is generally convex, while T_i may be convex or concave (e.g., due to scale benefits in waste treatment), we can have cases in which stable policies exist or cases in which all policies are unstable. It is possible to prove that when environmental damage is dominant with respect to the economies of scale, no stable policies exist unless the benefits of R_i are sufficiently large or R_o is a profit-making institution.

The impact of increasing environmental damage is illustrated in Figure 2. When environmental congestion increases, the core "moves" up along the plane:

$$B_o^* + B_1^* + B_2^* = \phi(\ 0,1,2\ ) \quad , \tag{92}$$

but becomes smaller and smaller, as shown by (a,b',c',e') and (a,b",c",e"). When $B_o = 0$, there is a segment (cd on Figure 2) in which the central authority R_c realizes the main objective and it still has an infinity of options in the allocation of resources among the partners R_i . In other words, R_o (and in a similar way R_i) can realize additional goals that are not specified explicitly in B_i, $i = 0,1,\ldots,N$.

In the case of a small core (e.g., a,b",c",e"), it is possible to achieve a stable policy if the partners agree that R_o is a profit-making institution. For that purpose, an increase in taxes T_i is needed (see equation (79)).

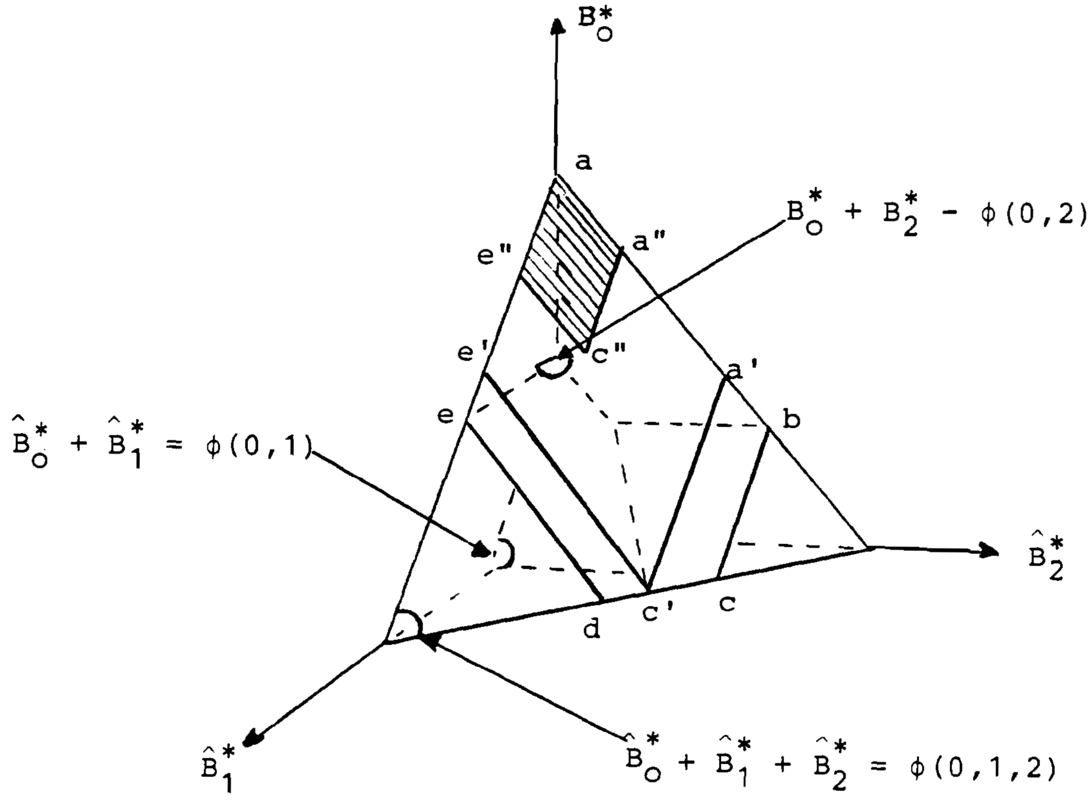

Figure 2. The core in the space of benefits.

It can also be observed that an increase in the benefit of R_i (which may be a result of a decrease in local factor prices, such as the water price) will increase the core. In other words, the core will exist only if the benefits of R_i are sufficiently high with respect to the corresponding environmental damage.

The game approach can also be applied at the regional level to investigate the existence of stable charges or taxation schemes. In Rinaldi et al. (1977), a regional environmental management system consisting of n firms (polluters) and a central authority, was studied. The benefits of polluters were assumed in the form

$$B_i(x_i) = A_i(x_i) - C_i(x_i) \quad , \qquad i \varepsilon \tilde{R} \quad , \qquad (93)$$

while the benefit accruing to the central authority was

$$B_o(\underline{x}) = \sum_{i \varepsilon \tilde{R}} C_i(x_i) - C(\underline{x}) \quad , \qquad (94)$$

where

$A_i(x_i$ is the profit of firms discharging x_i;

$C_i(x_i)$ is the tax for discharging x_i; and

$C(\underline{x})$ is the cost of discharging $\underline{x}$.

In the case when the central authority takes care of waste treatment:

$$C(\underline{x}) = \min_q \; [T(\underline{x},q) + E(q)] \quad . \qquad (95)$$

Depending on the form of $T(\underline{x},q)$, the structure of the treatment network etc., the existence of stable taxation schemes was analyzed. It is possible to observe a similarity in the models (78), (79) and (93), (94). In the more complex models, one has to take into account all the regional taxes, charges, and subsidies paid to regional budgets. Generally, many different games played simultaneously by different regional and higher-level partners should be studied.

5. NOTEC REGIONAL DEVELOPMENT--A CASE STUDY

The methodology described in sections 2, 3, and 4 has already been applied for the construction of a system of models for the regional development project in Poland--the Notec case study. Detailed information on this project can be found in Albegov and Kulikowski (1978a) and (1978b). The project is concerned primarily with large-scale irrigation of the Upper Notec region in central-northern Poland, which is adjacent to the Vistula river (although it belongs to the Odra watershed). This mainly agricultural region has an area of 6,194 km^2 and a population of 476,900. The region belongs to three voivodships (Bydgoskie, Koninskie and Włocławskie). The increasing annual regional water shortages are caused, not only by the extension of agricultural production, but also by the increasing use of water by industry and the urban population. It is therefore necessary to investigate the future water demands of the main users: industry, municipal users, and agriculture. Since the industrial and municipal water demands can be derived from equations (35)-(64), one should concentrate on the derivation of agricultural water demand. For this purpose, a water irrigation model (described in detail in Albegov and Kulikowski 1978a and 1978b) can be used.

A simplified version of the water irrigation model deals with the water system shown in Figure 3. Irrigation water W is distributed among the subregions $R_i,\ldots,R_n$ by channel L, connected to reservoir R_o. The problem is to find the optimum irrigated areas (f_i) in R_i, i = 1,...,n, which maximize the profit function

$$F(\underline{f}) = \sum_{i=1}^{n} A_i f_i - \sum_{\tau=1}^{T} \omega(\tau) W(\tau) - K \quad , \tag{96}$$

where

$$A_i = \Delta P_i - K_i - O_i ; \tag{97}$$

ΔP_i is the per hectare increase in crop-yield value resulting from irrigation;

K_i is the annual capital cost (or rent) per ha for irrigation equipment;

O_i is the annual operating cost per ha ;

$\omega(\tau)$ is the average water price $W(\tau)$ for the communities adjacent to the reservoir in time period $\tau = 1,\ldots,T$; and

K is the annual capital cost of channel L.

Function (96) should be maximized subject to the constraints imposed on the available areas in R_i

$$0 \leq f_i \leq F_i \quad , \qquad i = 1,\ldots,n \quad , \tag{98}$$

and the water flow constraints of the form

$$\sum_{i=j}^{n} q_i(\tau) f_i \geq 0 \quad , \qquad j = 2,\ldots,n \quad , \tag{99}$$

$$\sum_{i=1}^{n} q_i(\tau) f_i = W(\tau) \quad , \qquad \tau = 1,\ldots,T \quad , \tag{100}$$

where

$q_i(\tau)$ is the irrigation water requirement per at R_i; and

F_i is the maximum arable area at R_i.

For the given ω, the solution of the problems (96), (98)-(100) determines the optimum irrigated areas $\hat{f}_i$ in each subregion R_i. When ω increases, the total irrigated area

$$F = \sum_{i=1}^{n} \hat{f}_i(\omega) \tag{101}$$

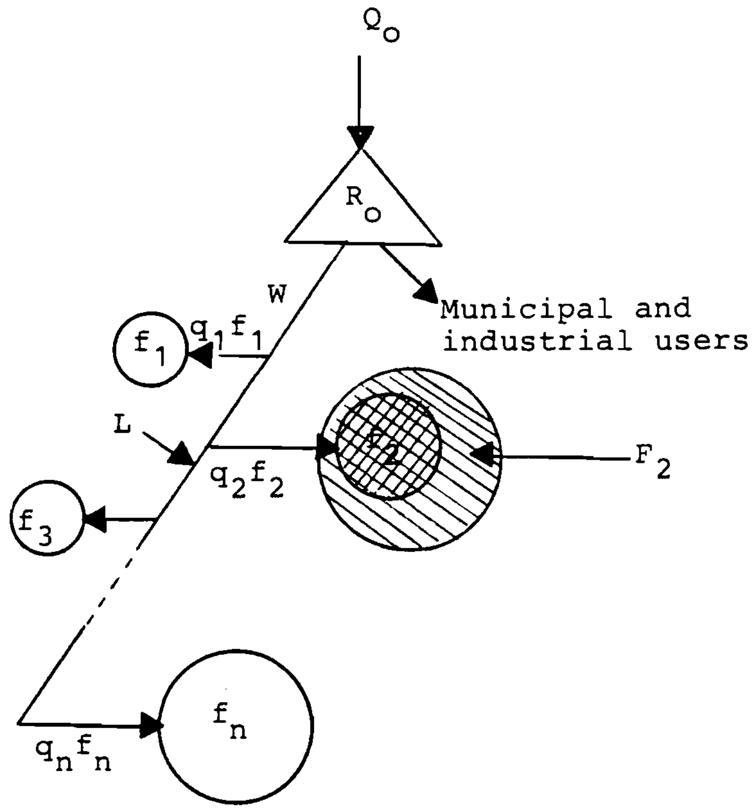

Figure 3. Simple water system.

and the amount of water consumed

$$W[\omega(\tau)] = \sum_{i=1}^{n} q_i(\tau)\hat{f}_i[\omega(\tau)] \quad , \qquad (102)$$

decrease.

The values of $q_i(\tau)$ depend on the soil and meteorological conditions and can be determined by tables or by Klatt's formulae (see Klatt 1958).

Assuming that function $W(\omega)$ for each $\tau\varepsilon[O,T]$ is known (i.e., derived numerically for different ω from the model), the corresponding irrigation demand function $W^d(\bar{\omega})$, where $\bar{\omega}$ is the marginal water cost, can be evaluated.

Assuming that the cost function of expanding R_o is of the form (37), one gets $\bar{\omega} = \gamma\omega$ and

$$W^d(\bar{\omega}) = W(\frac{\bar{\omega}}{\gamma}) \quad . \qquad (103)$$

When the agricultural, industrial, and municipal users (each characterized by the corresponding water demand functions $W_a^d(\bar{\omega})$, $W_i^d(\bar{\omega})$, $W_m^d(\bar{\omega})$) are connected to the same reservoir, it is possible to derive the resulting water price $\bar{\omega}$ by solving the equation

$$W_a^d(\bar{\omega}) + W_i^d(\bar{\omega}) + W_m^d(\bar{\omega}) = W^s(\bar{\omega}) \quad , \qquad (104)$$

where

$W^s(\bar{\omega})$ is the given supply function of the water reservoir (compare Willekens and Rogers 1978).

In the Notec case study, the water system consists of several interconnected reservoirs and channels. In order to use the methodology briefly outlined in this section, a more complicated computerized model is needed (as proposed in Albegov and Kulikowski 1978a and 1978b).

It should be observed that the solution of the optimization problem (96)-(100) also enables the benefits per ha resulting from irrigation to be computed. In the general case, the irrigation benefits will depend on the local parameters (A_i, K_i). The farmers' profit will, in addition, depend on the size of the farm and the technology used. Irrigation is generally more effective when labor-intensive technology and fertilizers are used and when there is crop specialization. A special model dealing with these problems for the Notec case study, is presently being constructed (see Albegov and Kulikowski 1978a and 1978b). The farmers' utilities (for different farm sizes) are determined and the rural-urban migration, which predetermines the labor supply for the agricultural sector, is checked. When the labor supply is below the level necessary to support the planned technology, policies discouraging the outflow of labor from the region are suggested. In particular, the required increase in aggregate consumption (i.e., services) can be derived using the methodology described in section 3.

The main problem dealt with in this paper is how to derive the regional benefits for different water supply alternatives. For example, consider the water supply cost functions of the two (cd, ef) forms shown in Figure 1. The first "cheaper" alternative (cd) consists in increasing the size of the reservoir in order to store more of the natural inflow. With the increasing water demand ($x \to b$), the saturation of supply and an increase in the marginal (and average) cost of water follows. Another alternative (ef) uses the central channel to transfer cheap water from a distant river (Vistula) basin. The project with the cd cost function may be regarded as cheaper for short-run development. For the long run, when the water demand $x > b$, the ef alternative might be the best.

In selecting the optimum alternative, it is necessary to know future water demands. Since the agricultural water demand is limited (due to the bounded arable area), future regional water demands depend, primarily, on the rate of regional urbanization and new industrial locations. These, in turn, depend on demographic factors and regional comparative advantages in production and consumption. The main idea behind the present system of models is to check all the alternative development strategies using the production and consumption submodels described in sections 2 and 3 and the regional stability concepts of section 4.

In the Notec case study, the three voivodships should agree to participate in financing the water system expansion and irrigation project. The "cheap" alternative may satisfy the demand from existing users, including that for agricultural irrigation, but may put a constraint on the future growth of regional industry and regional urbanization.

The more expensive alternative, on the other hand, will attract industry to the Notec region if production costs are lower than elsewhere. Thus, more partners (industrial sectors) should be included in the investment localization game. The model should show which industries could participate and support the project to reach a stable regional system. It is therefore necessary to compute for each industry (in particular, food processing and chemicals) the marginal production costs, water prices, and regional and national benefits. It should be noted that the computation of regional water charges (although in many regions in Poland they have not yet been levied) is very important. It provides the planners with information regarding the regional comparative advantages of different industrial locations and urban areas. Moreover, as argued in the literature (e.g., Hanke 1972), the levying of charges considerably limits the water requirements, while the cost of water meters is negligible (as compared with the cost of water system development).

The system of models constructed for the Notec case study should also enable the planners to evaluate different industrial locations in terms of the efficient utilization of local resources, such as labor, agricultural land, etc., and of regional environmental protection.

If the methodology developed proves to be useful for the Notec case study, it may also be used for the more ambitious, integrated, nationwide Government Program "Wisla" (Vistula), which has recently been announced.

In order to use the system of models briefly described in the present paper for the "Wisla" program, it is necessary to construct regional (voivodship) submodels, taking into account the interregional linkages of water supply, migration, and the environment, etc.

The integrated regional model based on the methodology proposed in this paper is shown in Figure 4. It is linked with other regional models and with the national model of the MRI type (Bruckmann 1978).

The National Decision Center cooperates with the Regional Decision Center regarding regional investments, subsidies, environmental standards, prices, taxes, etc. The main linkages among the regions (besides the flow of goods, which are not specified explicitly in the model) are migrations and transfer of natural resources (i.e., mainly water). The national model may also be linked to a global model by international trade markets.

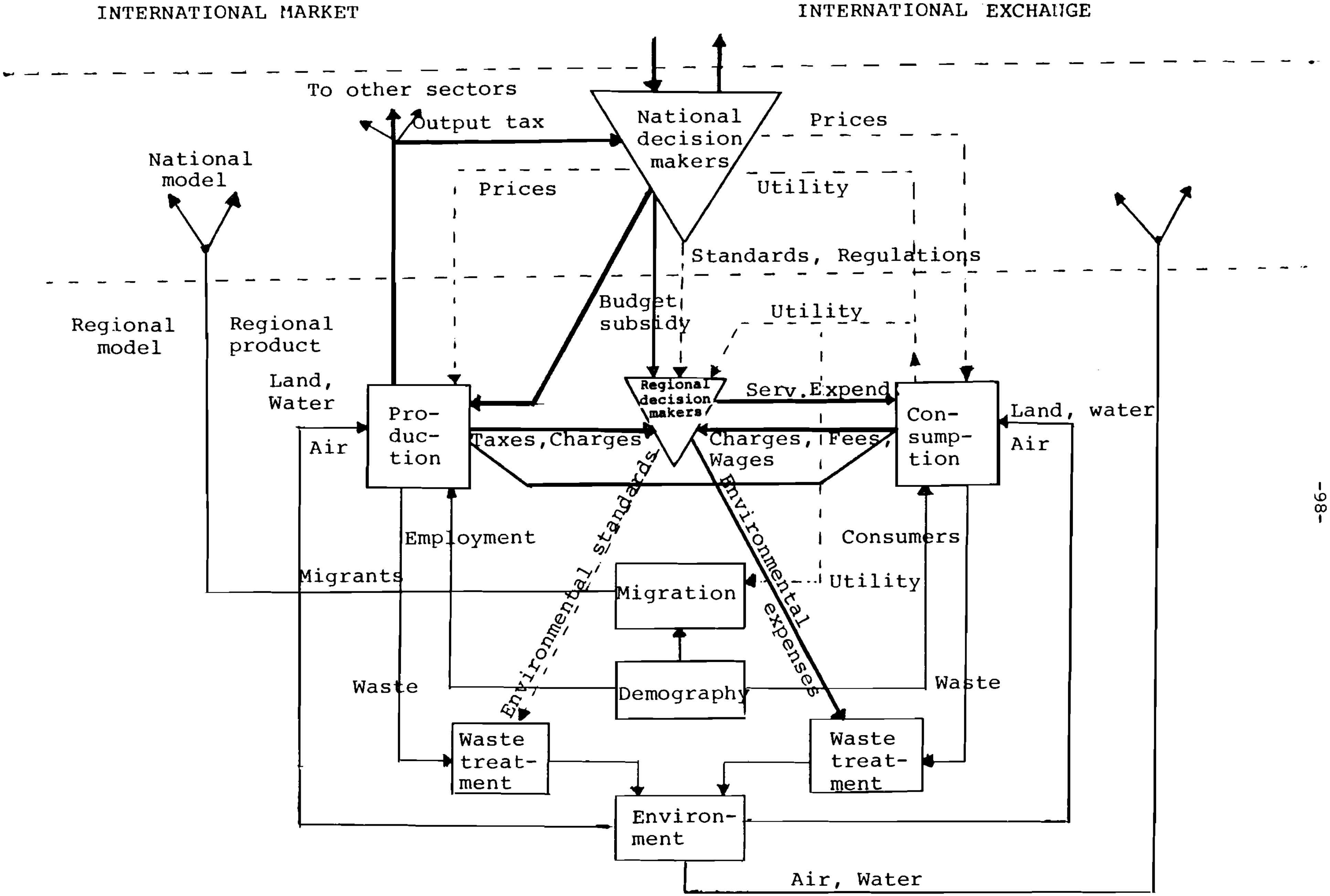

Figure 4. Integrated regional development model (━━ monetary flows; —— flow of resources labor. water, etc.); ... information flow; ∇ decision centers).

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A REGIONAL COMPUTERIZED (INTERACTIVE) PLANNING SYSTEM

R. Kulikowski
L. Krus

1. INTRODUCTION

This paper shows how a computerized, interactive model for studying rural-urban regional development policies can be constructed using the general methodology outlined in the paper by Kulikowski included in these Proceedings. The budget planning process, in particular, is modeled and optimized for different benefit and cost functions. This process, in which three decision makers participate, deals with capital expenditures and personal and aggregate consumption in the rural-urban (R-U) paradigm. Linkage of the R-U model and the model of the national economy is also studied.

A simple interactive system that is based on this R-U model was set up using the IIASA PDP 11/45 computer working under a UNIX time-sharing system. This work has produced a set of algorithms implemented as programs in FORTRAN IV. The system is operational and preliminary results have been obtained. Full documentation of the system, which is referred to as the IRUD system (interactive rural-urban development), can be found in Krus (1979).

Three groups of input data are used in the interactive system:

-- model parameters, which are evaluated on the basis of statistical data;

-- exogenous variables, all of which are input variables considered to be independent of the decision maker; and

-- decision variables, which denote the decision maker's strategies (see Figure 1).

The optimal solution is derived using an optimization procedure and the values of the endogenous variables are obtained at the output of the model. Thus, the model allows the results of different strategies and their related solution to be compared.

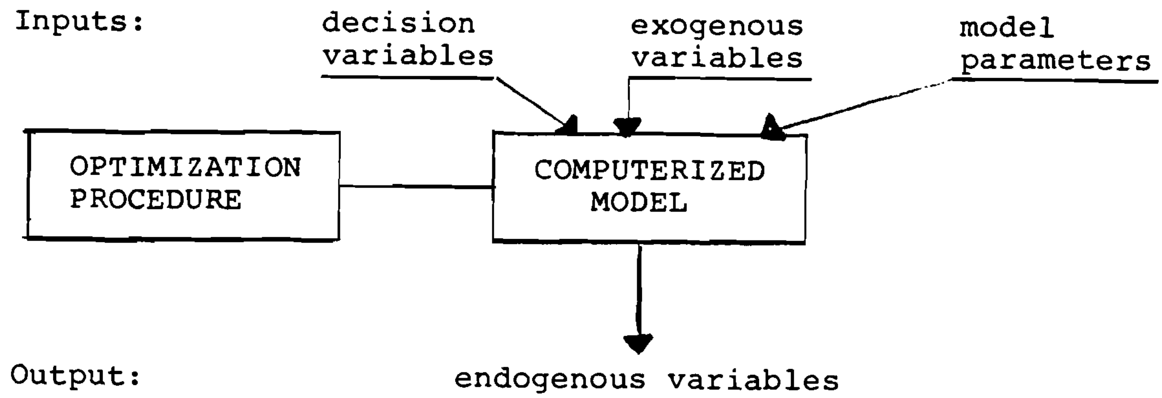

Figure 1. Computerized model and input-output information.

2. BASIC R-U OPTIMIZATION MODEL

Consider a system in which the regional authority (R_0) is concerned with the optimum allocation of resources between rural (R_1) and urban (R_2) subregions, shown in Figure 2. In particular, for each planning period, the R_0 splits the subsidy (S) received from the central budget into two parts (S_i), which subsidize the local budgets of R_i, i = 1,2. The subsidies S_i are in turn spent on capital expenditures (S_{ki}) and aggregate consumptions (S_{ci}); i.e., $S_i = S_{ki} + S_{ci}$, $i = 1,2$, while $S_1 + S_2 = S$.

Given the regional demographies, i.e., the projected (within the planning interval), total ($\bar{P}_i$), and productive ($\bar{P}_{0i}) = \kappa_{0i}\bar{P}_i$) populations, the projected labor forces at R_i (with zero migrations) can be estimated. Given the expected number of in-migrants (M_i) at R_i and employment (access) ratios ($z_{0i} = L_i/\bar{P}_{0i}$,

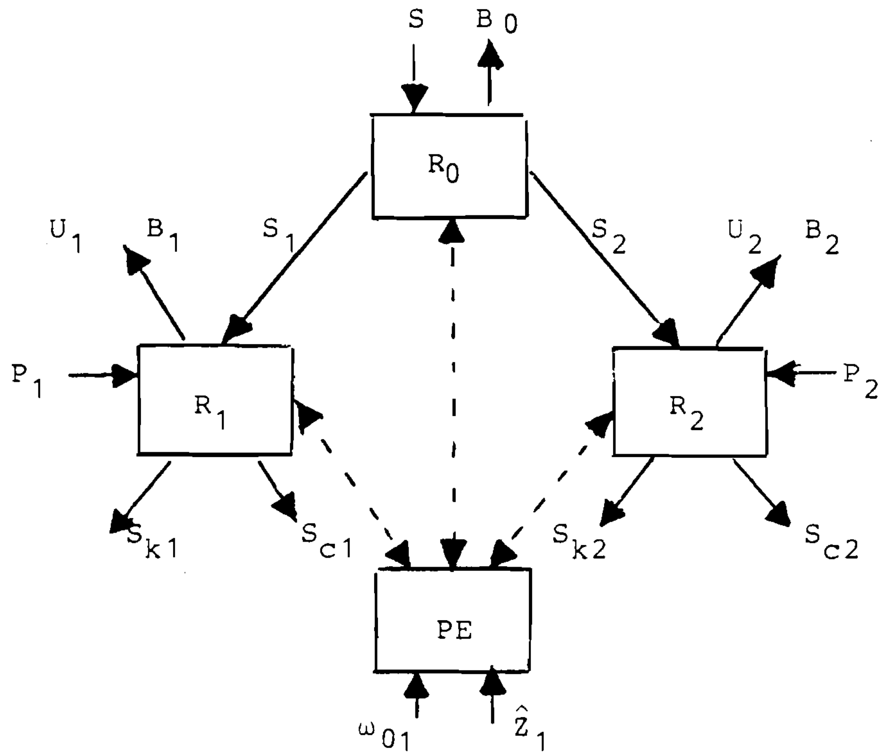

Figure 2. The model of the rural-urban system (PE is policy evaluation; ⟵ represents an exchange of resources; ←--- represents and exchange of information).

where L_i is the number of employees), it is possible to derive the projected productions ($\hat{Y}_1$), under the optimum allocation of input costs($Y_{\nu i}$, where $\nu = 0$ is labor, $\nu = 1$ is capital):

$$\hat{Y}_i = W_i \bar{Y}_i = W_i \frac{Y_{0i}}{\alpha_{0i}} \quad , \tag{1}$$

where

$$W_i = P_i a_i \left(\frac{\alpha_{0i}}{\omega_{0i}}\right)^{d_{0i}} \left(\frac{\alpha_{1i}}{\omega_{1i}}\right)^{\alpha_{1i}} \quad , \quad \alpha_{0i} + \alpha_{1i} = 1 \quad ; \tag{2}$$

$$\bar{Y}_i = Y_{0i} + \hat{Y}_{1i} \quad , \quad \hat{Y}_{1i} = \frac{\alpha_{1i}}{\alpha_{0i}} Y_{0i} \; ; \tag{3}$$

$$\hat{Y}_{0i} = \omega_{0i} L_i \left(1 + s_i \frac{\kappa_m}{\kappa_{0i}}\right), \; L_i = \kappa_{0i} z_{0i} \bar{P}_i, \; s_i = \frac{M_i}{\bar{P}_i} \; ; \tag{4}$$

α_{0i}, α_{1i} are Cobb-Douglas production function elasticities with respect to labor and capital, respectively;

ω_{0i}, ω_{1i} are labor and capital costs, respectively;

p_i is the product price;

κ_{0i} is the share of the population of productive age $(\bar{P}_{0\,i}/\bar{P}_i)$;

κ_m is the share of migrants of productive age;

s_i is the projected migration rate; and

a_i are given coefficients, i = 1,2.

It is assumed that the projected utilities (U_i), generated at R_i (under the optimum strategy) can be expressed as

$$U_i = V_i Z_i = V_i \frac{Z_{0i}}{\hat{\eta}_{0i}} \quad , \tag{5}$$

where

$$V_i = \Pi_i b_i \left(\frac{\eta_{0i}}{\Omega_{0i}}\right)^{\eta_{0i}} \left(\frac{\eta_{1i}}{\Omega_{1i}}\right)^{\eta_{1i}} \quad , \quad \eta_{0i} + \eta_{1i} = 1 \quad ; \tag{6}$$

$$Z_i = Z_{0i} + \hat{Z}_{1i} \quad , \quad \hat{Z}_{1i} = \frac{\eta_{1i}}{\eta_{0i}} Z_{0i} \quad ; \tag{7}$$

η_{0i}, η_{1i} are utility function elasticities, with respect to employment (personal income) and aggregate consumption (services), respectively;

Ω_{0i}, Ω_{1i} are the input costs ($\Omega_{0i} = \omega_{0i}\kappa_{0i}$);

Z_{0i} is consumption, which is equal to personal per capita income;

$\hat{Z}_{1i}$ is the per capita consumption of services (education, health care, etc.) supplied by government, under the optimum strategy;

$Z_i = Z_{0i} + \hat{Z}_{1i}$ is the total per capita consumption;

Π_i is the utility cost; and

b_i are positive constants, $i = 1,2$.

It is assumed that the regional system is closed with respect to migration outside the region. There is, however, considerable migration from the rural to urban areas within the given region; thus, $s_1 = -s$, $s_2 = sl$, $l = \bar{P}_1/\bar{P}_2$, and $s_i\bar{P}_1 + s_2\bar{P}_2 = 0$.

The production and consumption submodels (equations (1) and (5)) are linked by relations

$$Y_{0i}(s) = z_{0i}(s)P_1(s) = \eta_{0i}Z_i(s)P_i(s) \quad , \tag{8}$$

where

$$P_1(s) = \bar{P}_1(1-s) \quad , \quad P_2(s) = \bar{P}_2(1 + sl) \quad . \tag{9}$$

In this paper, we use a simpler version of the general model described in the paper by Kulikowski included in these Proceedings. The general model has two production and consumption factors.

The projected migration rate (s) is assumed to be dependent on the ratio of utilities U_2/U_1, according to the behavioral (econometric) relation

$$\frac{P_{2(s)}}{\bar{P}_2} = 1 + sl = d\left(\frac{U_2}{U_1}\right)^{\frac{1}{a}} = d\left[\frac{V_2}{V_1}\frac{Z_2(s)}{Z_1(s)}\right]^{\frac{1}{a}} \quad , \tag{10}$$

where d and a coefficients are determined (ex post) by the method of least squares. The relation (10) is then used in the ex ante sense within the planning interval.

Given the basic relations (1), (5), (8), and (10), one can allocate the subsidies S_i, S_{ki}, and S_{ci} in such a way that the given goals of R_i, $i = 0,1,2$, will be realized. For that purpose, the following benefit functions will be introduced:

$$B_i = Y_{0i} + S_i \quad , \quad S_i = S_{ki} + S_{ci} \quad , \tag{11}$$

where

Y_{0i} is the personal income of populations at R_i; and
S_i are the subsidies paid to R_i, $i = 1,2$, by R_0.

$$B_0 = \sum_{i=1}^{2} \left(\frac{W_i}{\alpha_{0i}} - 1\right) Y_{0i} - S_i - C(s) \quad , \tag{12}$$

where

$C(s) = [Z_2(s) - Z_1(s) + Z_h]\bar{P}_1 s$ is the urbanization cost; (13)
Z_h is the per capita cost of urban housing and infrastructure.

The benefit of R_0 consists of total production value minus total consumption and capital cost, as well as the urbanization cost $(C(s))$.

According to equation (10), one can control the number of migrants by changing the ratio of consumption in urban and rural areas. These quantities depend on rural-urban policies in income distribution (wages) and employment (z_{0i}), as well as on demographic processes and migration. All these policies and processes should be correlated to maximize the regional benefit.

In rural areas, for example, the joh access $(Y_{0i}(s))$ depends on the agricultural policy and, primarily, on structural change, which alters the number of jobs in the modern (state) and traditional (private) agricultural sectors (see section 3.3). One of the objectives of this policy is to maintain per capita consumption (Z_1) at not less than the given level $(\hat{Z}_1)$. For the implementation of such a policy, it is necessary to satisfy

$$Y_{01}(s) = \eta_{01}\hat{z}_1\bar{P}_1(1 - s) = \omega_{01}\kappa_{01}z_{01}(s)\bar{P}_1 \quad 1 - s\,\frac{\kappa_m}{\kappa_{01}} \quad . \tag{14}$$

In turn, it requires that

$$z_{0i}(s) = \bar{z}_{01}\,\frac{1-s}{1-s\frac{\kappa_m}{\kappa_{01}}} \quad , \tag{15}$$

where

$$\bar{z}_{01} = \frac{\hat{z}_1\eta_{01}}{\kappa_{01}\omega_{01}} \quad . \tag{16}$$

It should be observed that, in the model studied, $\hat{z}_1$, ω_{01} (or z_{01}) are regarded as exogenous, because the value of leisure time is not explicitly introduced in the utility function.

Urban per capita consumption should be an increasing function of s:

$$z_2(s) = \hat{z}_1\left(\frac{sl+1}{d}\right)^a \frac{V_1}{V_2} \quad . \tag{17}$$

Since, on the other hand, see equation (8), the urban per capita consumption is equal:

$$z_2(s) = \frac{Y_{02}(s)}{\eta_{02}P_2(s)} = \omega_{02}(s)\kappa_{02}z_{02}(s)\,\frac{1+sl\frac{\kappa_m}{\kappa_{02}}}{\eta_{02}(1+sl)} \quad , \tag{18}$$

for $z_{02}(s) = \bar{z}_{02}$ = constant, the following value of the urban wage level is obtained:

$$\omega_{02}(s) = \frac{\eta_{02}\bar{z}_1 V_1}{\kappa_{02}\bar{z}_{02}V_2 d^a} \frac{(sl+1)^{a+1}}{1+sl\frac{\kappa_m}{\kappa_{02}}} \quad . \tag{19}$$

When s is known, the corresponding capital cost (S_{ki}) and aggregate consumptions (S_{ci}) can be derived using the formula

$$S_{ki} = \hat{Y}_{1i}(s) = \frac{\alpha_{1i}}{\alpha_{0i}} Y_{0i}(s) = \frac{\alpha_{1i}}{\alpha_{0i}} \eta_{0i}P_i(s)Z_i(s) \quad , \tag{20}$$

$$S_{ci} = \eta_{1i}P_i(s)Z_i(s) \quad , \ i = 1,2 \quad , \quad Z_1(s) = \hat{Z}_1 \quad . \tag{21}$$

The main problem is therefore to find the optimum value of s.

The problem (11) - (13) can be regarded as a game, in which the three players (R_i) try to maximize the corresponding benefits (B_i) by allocation of subsidies (S_i, S_{ki}, S_{ci}). The effective solution in terms of s can be derived by maximizing

$$B(s) = \sum_{i=0}^{2} B_i(s) = \sum_{i=0}^{2} \frac{W_i}{\alpha_{0i}} Y_{0i}(s) - C(s) \quad , \tag{22}$$

where, according to equations (13) and (17),

$$C(s) = \bar{P}_1\hat{Z}_1\left[\frac{V_1}{V_2}\left(\frac{sl+1}{d}\right)^a + h\right] s \quad , \quad h = \frac{Z_h}{\hat{Z}_1} - 1 \quad . \tag{23}$$

The necessary and sufficient (due to the concavity of B (s)) condition of optimality: B(s) = 0, yields

$$\left(\frac{sl+1}{d}\right)^a\left[1 + \frac{sla}{1+sl}\right] = \frac{V_2}{V_1}\left[\frac{\bar{\sigma}(s)}{\bar{P}_1\hat{Z}_1} - h\right] \triangleq \sigma(s) \quad , \tag{24}$$

where

$$\bar{\sigma}(s) = \frac{d}{ds}\left[W_1 \frac{Y_{01}(s)}{\alpha_{01}} + W_2(s) \frac{Y_{02}(s)}{\alpha_{02}}\right] \quad . \tag{25}$$

For small s, in simpler models, one can assume that $\omega_{02}(s) \simeq \bar{\omega}_{02}$ = constant, thus $W_2(s) = W_2$= constant and $Y_{01}(s)$, $Y_{02}(s)$ are linear in s. In that case,

$$\bar{\sigma}(s) \cong \bar{\sigma} = \frac{W_2}{\alpha_{02}} Y_{02}(0) \; 1 \frac{\kappa_m}{\kappa_{02}} - \frac{W_1}{\alpha_{01}} Y_{01}(0) \frac{\kappa_m}{\kappa_{01}} \quad . \tag{26}$$

Since

$$Y_{0i}(0) = \eta_{0i} Z_i(0) \bar{P}_i \quad , \quad Z_1(0) = \hat{Z}_1 \quad , \tag{27}$$

then

$$\sigma(s) \approx \sigma \quad \frac{V_2}{V_1}\left[\frac{W_2}{\alpha_{02}} \; \frac{Z_2(0)}{\hat{Z}_1} \; \frac{\eta_{02}\kappa_m}{\kappa_{02}} - \frac{W_1}{\alpha_{01}} \; \frac{\eta_{01}\kappa_m}{\kappa_{01}} - h\right] \quad , \tag{28}$$

where

$$Z_2(0)/\hat{Z}_1 = \frac{V_1}{V_2} d^{-a} \quad . \tag{29}$$

It is noticeable that $\frac{W_i}{\alpha_{0i}} \omega_{0i} = Y_i/L_i$ are the production values per worker at R_i. These values can be derived (ex post) by statistical data. They can also be used in the ex ante sense to determine expected (within the planning interval) values of W_i, assuming that Y_i/L_i do not change much.

By solving equation (24), one can find the optimum value of s, say $\hat{s}$, and compute $C(\hat{s})$. It may be observed that the greater are the expected labor efficiencies $W_2/\alpha_{02} = Y_2/\alpha_{02}L_2$ and κ_m/κ_{02}; as compared to W_1/α_{01} and κ_m/κ_{01}, the greater is $\bar{\sigma}$ and s. An increase of V_2, κ_{02}, as compared to V_1, κ_{01}, also contributes to an increase of $\hat{s}$. As illustrated by Figure 3, where

$$y(s,\sigma) = \frac{W_2}{\alpha_{02}} Y_{02}(s) + \frac{W_1}{\alpha_{01}} Y_{01}(s) = y(0) + \bar{\sigma}s \quad , \tag{30}$$

an increase of $\bar{\sigma}$ ($\bar{\sigma}_2 > \bar{\sigma}_1$) produces an additional gain $\Delta B = B(\hat{s}_2,\sigma_2) - B(\hat{s}_1,\bar{\sigma}_1)$ for the regional economy. When modern industries are located in an urban subregion, W_2 and ΔB should be increased. The investment in a water system, which reduces the water price, affects W_1, W_2, as well as the outputs Y_1, Y_2, simultaneously.

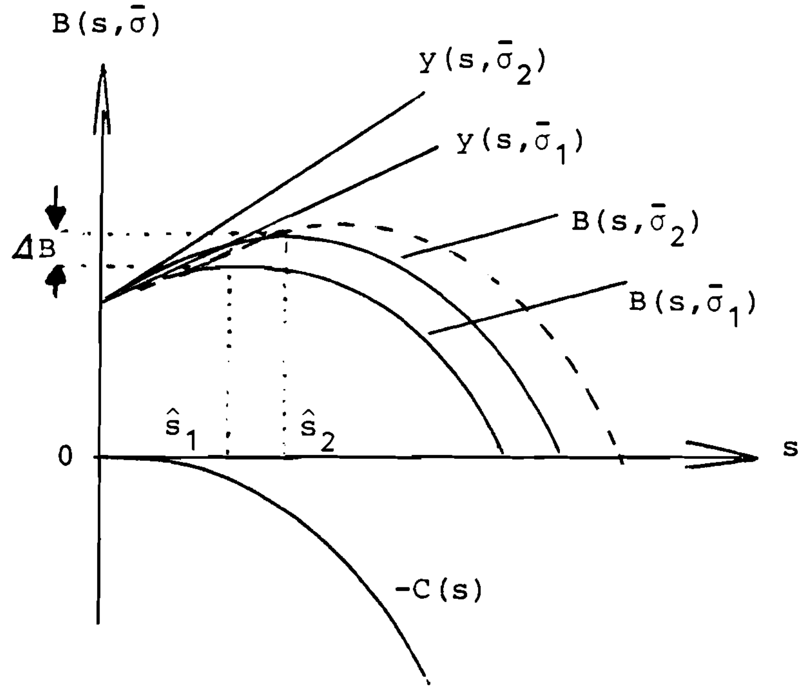

Figure 3. The impact of migration on regional benefits (--- represents the form of the B(s) function for the present case).

Under optimum $s = \hat{s}$, the necessary subsidies become

$$\hat{S}_{ki} = \frac{\alpha_{1i}}{\alpha_{0i}} Y_{0i}(\hat{s}) \quad , \quad \hat{S}_{ci} = \frac{\eta_{1i}}{\eta_{0i}} Y_{0i}(\hat{s}) \quad , \tag{31}$$

while

$$B_i(\hat{s}) = \hat{B}_i = \hat{Y}_{0i}\left(1 + \frac{\eta_{1i}}{\eta_{0i}} + \frac{\alpha_{1i}}{\alpha_{0i}}\right) \quad , \; i = 1,2 \quad , \tag{32}$$

$$B_0(\hat{s}) = \hat{B}_0 = \sum_{i=1}^{2} \hat{Y}_{0i}\left(\frac{W_i}{\alpha_{0i}} - \frac{\eta_{1i}}{\eta_{0i}} - \frac{\alpha_{1i}}{\alpha_{0i}} - 1\right) - \hat{C} \quad , \tag{33}$$

$$B(\hat{s}) = \hat{B} = \sum_{i=1}^{2} \frac{W_i}{\alpha_{0i}}\hat{Y}_{0i} - \hat{C} \quad . \tag{34}$$

According to equation (8),

$$Y_{0i}(\hat{s}) = \hat{Y}_{0i} = P_i(\hat{s})Z_i(\hat{s})\eta_{0i}\left(Z_1(\hat{s}) = \hat{Z}_1\right) \quad , \tag{35}$$

$$P_1(\hat{s}) = \bar{P}_1(1-\hat{s}) \quad , \; P_2(\hat{s}) = \bar{P}_2(1+\hat{s}l) \quad . \tag{36}$$

In the more accurate models, it is necessary to take into account that for $z_{02}(s) = \bar{z}_{02}$ = constant, $\omega_{02}(s)$ is an increasing function of s. Assuming, for the sake of simplicity, $\kappa_m = \kappa_{01} = \kappa$,

$$\omega_{02}(s) = \bar{\omega}_{02}\left(\frac{sl+1}{d}\right)^{a} \quad , \; \bar{\omega}_{02} = \frac{\eta_{02}\hat{Z}_1 V_1}{\kappa \bar{z}_{02} V_2} \quad . \tag{37}$$

Taking into account that

$$W_2(\omega_{02}) = A\left[\omega_{02}(s)\right]^{-\alpha_{02}} \quad , \tag{38}$$

where A is a constant, which is independent of ω_{02}, one obtains

$$\frac{d}{ds}\left[\frac{W_2(s)}{\alpha_{02}} Y_{02}(s)\right] = B(sl+1)^{\beta} \quad , \tag{39}$$

where

$$B = (\alpha_{02})^{-1}(\beta+1)Al\kappa\bar{z}_{02}\bar{P}_2(\bar{\omega}_{02})^{1-\alpha_{02}} \quad , \; \beta = a(1-\alpha_{02}) \quad . \tag{40}$$

Then

$$\bar{\sigma}(s) = l(\beta+1)\frac{\bar{W}_2}{\alpha_{02}}Y_{02}(0)(sl+1)^{\beta} - \frac{W_1}{\alpha_{01}}Y_{01}(0) \quad , \tag{41}$$

where

$$Y_{02}(0) = \bar{\omega}_{02}\kappa\bar{z}_{02}\bar{P}_2 \quad , \; \bar{W}_2 = A(\bar{\omega}_{02})^{-\alpha_{02}} \quad . \tag{42}$$

It can be seen that equation (8) holds for $s = \hat{s}$. The form of the B(s) function for the present case is shown in Figure 3 by the broken line. In this case, however, W_2 in equations (33) - (34) should be replaced by

$$W_2(\hat{s}) = \bar{W}_2[1+\hat{s}l]^{-a\alpha_{02}} \quad . \tag{43}$$

For the decision maker, the main problem is to evaluate losses (in terms of B_i) when the strategies, say, $\tilde{S}_{ki}$, $\tilde{S}_{ci}$, $\tilde{S}_i$, differ from $\hat{S}_{ki}$, $\hat{S}_{ci}$, $\hat{S}_i$, i = 1,2. The possible change in strategies may be attributed to the poor estimation of migration rate s. If, for example, the planner expects (with given ω_{0i}, κ_{0i}, κ_m) the s to take the value $\tilde{s}$, he will propose capital expenditures

$$\tilde{S}_{ki} = \frac{\alpha_{1i}}{\alpha_{0i}}Y_{0i}(\tilde{s}) = \frac{\alpha_{1i}}{\alpha_{0i}} \cdot \frac{Y_{0i}(\tilde{s})}{Y_{0i}(\hat{S})}Y_{0i}(\hat{s}) \quad , \tag{44}$$

instead of

$$\hat{S}_{ki} = \frac{\alpha_{1i}}{\alpha_{0i}} Y_{0i}(\hat{s}) \quad . \tag{45}$$

By denoting $\alpha_{1i}Y_{0i}(\tilde{s})/\alpha_{0i}Y_{0i}(\hat{s})$ by $\tilde{\alpha}_{1i}/\tilde{\alpha}_{0i}$, one obtains

$$\frac{\tilde{S}_{ki}}{\hat{S}_{ki}} = \frac{\tilde{\alpha}_{1i}}{\tilde{\alpha}_{0i}} \cdot \frac{\alpha_{\ i}}{\alpha_{1i}} \quad , \text{ where } \tilde{a}_{0i} + \tilde{a}_{1i} = 1 \quad , \; i=1,2 \quad . \tag{46}$$

Then

$$\tilde{\alpha}_{1i} = \frac{\alpha_{1i}\tilde{S}_{ki}}{\alpha_{0i}\hat{S}_{ki} + \alpha_{1i}\tilde{S}_{ki}} \quad , \quad \tilde{\alpha}_{02} = 1 - \tilde{\alpha}_{1i} \quad . \tag{47}$$

As a result of the $\tilde{S}_{ki}$ strategy, output production decreases; i.e., the W_i coefficients become

$$\tilde{W}_i = W_i\left(\frac{\tilde{\alpha}_{0i}}{\alpha_{0i}}\right)^{\alpha_{0i}}\left(\frac{\tilde{\alpha}_{1i}}{\alpha_{1i}}\right)^{\alpha_{1i}} \leq W_i \quad , \; i=1,2 \quad , \tag{48}$$

where the equality sign follows iff $\tilde{\alpha}_{0i} = \alpha_{0i}$.

A similar situation appears when the planner spends on aggregate consumption

$$\tilde{S}_{ci} = \frac{\eta_{1i}}{\eta_{0i}}Y_{0i}(\tilde{s}) = \frac{\eta_{1i}Y_{0i}(\tilde{s})}{\eta_{0i}Y_{0i}(\hat{s})}Y_{0i}(\hat{s}) = \frac{\tilde{\eta}_{1i}}{\tilde{\eta}_{0i}}Y_{0i}(\hat{s}) \quad , \tag{49}$$

instead of

$$\hat{S}_{ci} = \frac{\eta_{1i}}{\eta_{0i}}Y_{0i}(\hat{s}) \quad . \tag{50}$$

The corresponding decrease of utilities follows, i.e.,

$$\tilde{V}_i = V_i\left(\frac{\tilde{\eta}_{0i}}{\eta_{0i}}\right)^{\eta_{0i}}\left(\frac{\tilde{\eta}_{1i}}{\eta_{1i}}\right)^{\eta_{1i}} \leq V_i \quad , \tag{51}$$

where

$$\tilde{\eta}_{1i} = \frac{\eta_{1i}\tilde{S}_{ci}}{\eta_{0i}\hat{S}_{ci} + \eta_{1i}\tilde{S}_{ci}} \quad , \; \tilde{\eta}_{0i} = 1 - \eta_{0i} \quad , \; i=1,2 \quad . \tag{52}$$

The impact of nonoptimum strategies on production output and utilities can also be seen when one uses the relations (1)-(5); i.e.,

$$\tilde{Y}_i = \tilde{W}_i\left[Y_{0i} + \tilde{S}_{ki}\right], \quad \tilde{U}_i = \tilde{V}_i\left[Z_{0i} + \tilde{Z}_{1i}\right] \quad . \tag{53}$$

When

$$\tilde{S}_{ki} = \hat{S}_{ki} = \frac{\alpha_{1i}}{\alpha_{0i}}\hat{Y}_{0i} \quad , \; \tilde{Z}_{1i} = \hat{Z}_{1i} = \frac{\eta_{1i}}{\eta_{0i}}\hat{Z}_{0i} \quad , \tag{54}$$

then

$$\tilde{Y}_i = \frac{W_i}{\alpha_{0i}}\hat{Y}_{0i} = \hat{Y}_i \quad , \; \tilde{U}_i = \frac{V_i}{\eta_{0i}}\hat{Z} = \hat{U}_i \quad . \tag{55}$$

In order to derive $\tilde{Z}_i = \tilde{Z}_{0i} + \tilde{Z}_{1i}$, one should observe that strategies $\tilde{S}_{ki}$, $\tilde{S}_{ci}$ generate (according to equation (44)) the wage funds

$$\tilde{Y}_{0i} = \frac{\alpha_{0i}}{\alpha_{1i}}\tilde{S}_{ki} \quad , \; i=1,2 \quad . \tag{56}$$

Then, per capita income (from wages) becomes

$$\tilde{Z}_{0i} = \frac{\tilde{Y}_{0i}}{P_i(\tilde{s})} = \frac{\alpha_{0i}}{\alpha_{1i}}\,\frac{\tilde{S}_{ki}}{P_i(\tilde{s})} \quad , \tag{57}$$

while

$$\tilde{Z}_{1i} = \frac{\tilde{S}_{ci}}{P_i(\tilde{s})} = \frac{\alpha_{0i}}{\alpha_{1i}} \frac{\tilde{S}_{ki}}{P_i(\tilde{s})} \quad , \tag{57}$$

while

$$\tilde{Z}_{1i} = \frac{\tilde{S}_{ci}}{P_i(\tilde{s})} \quad , \quad i = 1,2 \quad . \tag{58}$$

Then,

$$\tilde{Z}_i(\tilde{s}) = (\tilde{S}_{ki}\alpha_{0i} + \tilde{S}_{ci}\alpha_{1i})[\alpha_{1i}P_i(\tilde{s})]^{-1} \quad , i = 1,2 \quad . \tag{59}$$

The value of $\tilde{s}$, which results from this strategy (according to equation (10)), can be derived as the solution of

$$\left(\frac{1+\tilde{s}l}{d}\right)^a = \frac{\tilde{V}_2\tilde{Z}_2(\tilde{s})}{\tilde{V}_1\tilde{Z}_1(\tilde{s})} \quad . \tag{60}$$

After solving equation (60), it is possible to derive

$$\tilde{B}_i = \tilde{Y}_{0i}\left(1 + \frac{\eta_{1i}}{\eta_{0i}} + \frac{\alpha_{1i}}{\alpha_{0i}}\right), \ i=1,2 \quad , \tag{61}$$

$$\tilde{B}_0 = \sum_{i=1}^{2} \tilde{Y}_{0i}\left(\frac{\tilde{W}_i(\tilde{s})}{\alpha_{0i}} - \frac{\eta_{1i}}{\eta_{0i}} - \frac{\alpha_{1i}}{\alpha_{0i}} - 1\right) - \tilde{C} \quad , \tag{62}$$

$$\tilde{B} = \sum_{1=i}^{2} \frac{\tilde{W}_i(s)}{\alpha_{0i}}\tilde{Y}_{0i} - \tilde{C} \quad , \tag{63}$$

where

$$\tilde{Y}_{0i} = \frac{\alpha_{0i}}{\alpha_{1i}} \tilde{S}_{ki} \quad , \quad \tilde{C} = (\tilde{Z}_2 - \tilde{Z}_1 + Z_h)\bar{P}_1\tilde{s} \quad ; \tag{64}$$

$$\tilde{W}_1(s) = \tilde{W}_1 = \text{constant, and } \tilde{W}_2(s) = \tilde{W}_2(1+\tilde{s}l)^{-a\alpha_{02}} \quad . \tag{65}$$

When $\tilde{Z}_i = \hat{Z}_1$, $\tilde{B} \le \hat{B}$, whereas generally $\tilde{B}_i \neq \hat{B}_i$, $i = 0,1,2$. The equality signs follow when $\tilde{S}_{ki} = \hat{S}_{ki}$, $\tilde{S}_{ci} = \hat{S}_{ci}$, $\tilde{S}_i = \hat{S}_i$, $i = 1,2$.

In order to use the proposed model effectively (for decision making), it is advisable to implement it in the computerized interactive form, shown in Figure 2. The policy evaluation box (PE) receives the information regarding the proposed strategies ($\tilde{S}_{ki}$, $\tilde{S}_{ci}$, $\tilde{S}_i$), expected W_i (prices), wages, demographic factors, etc. and computes the resulting benefits β_i, utilities, employment, migration, etc. The computed values are monitored for the decision makers, who are represented in Figure 2 by R_i boxes. By comparing the resulting $\tilde{B}_i$ with optimum values, the decision makers may change their strategies. Using that system, it is also possible to investigate the impact of industrial investments, which (due to indivisibilities) cannot be made equal to the optimum values.

3. EXTENSIONS OF THE R-U MODEL

3.1. The Impact of Commuters

In order to include in the basic R-U model the labor force commuting from rural to urban centers, one can use an econometric relation in the form of equation (10):

$$1 + s_c l_c = d_c \left(\frac{\omega_{02-c}}{\omega_{01}}\right)^{1/a_c} \quad , \tag{66}$$

where

c is the transport cost (as paid by commuters); and
s_c is the ratio of commuters to rural employees.

The coefficients d_c, a_c are determined by standard econometric methods, using ex post data. The relation (66) indicates that commuters react mostly to the net urban $(\omega_{02}-c)$ to rural (ω_{01}) wage ratio.

The transport cost is the product of average commuting distance $\bar{r}$, N working trips, and fare plus traveling time cost b, i.e., $c = N\bar{r}b$.

The average commuting distance can be derived from the commuters density function $n(r)$, which can be computed using existing statistical data. The average distance can be defined as the ratio of the moments of $n(r)$, i.e.,

$$\bar{r} = \int_0^{\infty} rn(r)dr / \int_0^{\infty} n(r)dr \quad . \tag{67}$$

One should observe that commutings change the income (from wages? Y_{0i} at both R_i subregions. Then, equation (11) should be replaced by

$$\bar{B}_i = \bar{Y}_{0i} + S_i \quad , \; i=1,2 \quad , \tag{68}$$

where

$$\bar{Y}_{01} = \omega_{01}\kappa_{01}\bar{z}_{01}\bar{P}_1\left[1 - s_c - s\frac{\kappa_m}{\kappa_{01}}\right] + Y_0(s,s_c) \quad ; \tag{69}$$

$$\bar{Y}_{02} = \omega_{02}(s)\kappa_{02}\bar{z}_{02}\bar{P}_2\left[1 + s_c l_c + sl\frac{\kappa_m}{\kappa_{02}}\right] - Y_0(s,s_c) \quad ; \tag{70}$$

$$Y_0(s,s_c) = \omega_{02}(s)\kappa_{02}\bar{z}_{02}\bar{P}_2 s_c l_c \frac{\kappa_m}{c\kappa_{02}} \quad ; \tag{71}$$

$$l_c = \frac{\bar{P}_1}{\bar{P}_2} \frac{\bar{z}_{01}\kappa_{01}}{\bar{z}_{02}\kappa_{02}} \quad . \tag{72}$$

As a result, per capita rural income

$$Z_1(s,s_c) = \frac{\bar{Y}_{01}}{\eta_{01}\bar{P}_1(1-s)} \geq \hat{Z}_1 \quad , \tag{73}$$

where

$\hat{Z}_1$ is the given minimum per capita income of farmers (excluding commuters).

It should also be noted that the migration rate s is determined, according to equation (17), by $\hat{Z}_1$ rather than by $Z_1(s,s_c)$. Consequently, (compare equation (19)), the urban price level ($\omega_{02}(s)$) depends on s, but does not depend on s_c; thus,

$$\omega_{02}(s) = \bar{\omega}_{02} \left(\frac{1+sl}{d}\right)^a \quad , \qquad \bar{\omega}_{02} = \frac{\eta_{02}\hat{Z}_1 V_1}{\kappa_{02}\bar{z}_{02}V_2} \quad . \tag{74}$$

Then, equation (12) should be replaced by

$$\bar{B}_0 = \sum_{i=1}^{2} \left[\left(\frac{W_i(s)}{\alpha_{0i}} - 1\right)\right] Y_{0i}(s,s_c) - S_i] - \bar{C}(s,s_c) \quad , \tag{75}$$

where

$$Y_{01}(s,s_c) = \omega_{01}\kappa_{01}\bar{z}_{01}\bar{P}_1\left[1 - s_c - s\frac{\kappa_m}{\kappa_{01}}\right] \quad ; \tag{76}$$

$$Y_{02}(s,s_c) = \omega_{02}(s)\kappa_{02}\bar{z}_{02}\bar{P}_2\left[1 + s_c l_c + sl\frac{\kappa_m}{\kappa_{02}}\right] \quad , \tag{77}$$

$$W_1(s) = W_1 = \text{constant} \quad ; \tag{78a}$$

$$\bar{W}_2(s) = \bar{W}_2 \left[\omega_{02}(s)\right]^{-\alpha_{02}} \quad , \qquad \bar{W}_2 = \text{constant} \quad ; \tag{78b}$$

$$\bar{C}(s,s_c) = C(s) + C_c(s,s_c) \quad ; \tag{79}$$

$$C_c(s,s_c) = [\bar{\omega}_{02}(s) - \omega_{01} + Z_c]\ \bar{P}_1 s_c \quad ; \text{ and} \tag{80}$$

Z_c is the (per capita) cost of urban infrastructure required because of commuting.

The commuting cost function $C_c(s,s_c)$ consists of the term $\bar{\omega}_{02}(s) - \omega_{01}$, which represents the wage difference between the urban and rural employee, and the urban cost of infrastructure (in transport and services mostly).

In order to find the optimum migration and relation commuting rates it is convenient to derive first the $s_c(s)$ relationship. Using equations (66) and (74), one obtains

$$\left(\frac{1+s_c l_c}{d_c}\right)^{a_c} = \frac{\omega_{02}(s)-c}{\omega_{01}} = \left(\frac{1+sl}{d}\right)^a \frac{\bar{\omega}_{02}}{\omega_{01}} - \frac{c}{\omega_{01}} \quad , \tag{81}$$

or

$$s_c(s) = \left\{\left[\frac{\bar{\omega}_{02}}{\omega_{01}} \left(\frac{1+sl}{d}\right)^a - \frac{c}{\omega_{01}}\right]^{\frac{1}{a_c}} d_c - 1\right\} l_c^{-1} \quad . \tag{82}$$

The optimum value of $s = \hat{s}$ can be derived by maximizing the function

$$\bar{B}(s,s_c) = \sum_{i=0}^{2} \bar{B}_i[s,s_c(s)] = \tilde{B}(s) \quad . \tag{83}$$

Then, the value $\hat{s}_c = s_c(\hat{s})$ can be derived. It is possible to observe that when $a_c = a$, $c \ll \omega_{01}$, $s_c(s)$ can be approximated by the linear function

$$s_c = \left[\frac{d_c}{d}\left(\frac{\bar{\omega}_{02}}{\omega_{01}}\right)^{\frac{1}{a}} (1+sl) - 1\right] l_c^{-1} \quad . \tag{84}$$

In that case $\tilde{B}(s)$ is a concave function and a unique strategy $s = \hat{s}$ exists.

3.2. Capital Cost and Labor Substitution

In the basic R-U model, the different costs of capital per worker (at R_1 and R_2) were not taken into account. As a result, the total goal function (equation (22)) was equal to the gross regional product minus the urbanization cost. If, instead, one decides to maximize total consumption, the $B_0(s)$ and $B(s)$ should be replaced by $\bar{B}_0(s)$ and $\bar{B}(s)$, in which W_i should be replaced by $\bar{W}_i = W_i - \alpha_{1i}$, $i = 1,2$. In that case, the corresponding value of $\bar{\sigma}$ as well as $\hat{s}$ (for $\alpha_{11} \simeq \alpha_{12}$) decreases.

It should also be noted that the outflow of labor from agriculture might require capital substitution (to keep the agricultural production value at a given level $Y_1 = Y_1^*$). Such a situation occurs when regional agricultural production is less than demand while the price of imported food is much higher than the local production cost. In this case, the policy aimed at reducing the cost of capital for agriculture (ω_{11}) may be advisable. The cost of capital increase (as a result of reducing ω_{11}) becomes

$$C^*(Y_1^*) = \hat{Y}_{11}(\bar{s}) \left[\frac{Y_1^*}{\hat{Y}_{11}(\bar{s})} - 1\right] = \hat{Y}_{11}(\bar{s}) \left[\left(\frac{Y_1^*}{\hat{Y}_1(\bar{s})}\right)^{\frac{1}{\alpha_{11}}} - 1\right] \tag{85}$$

$$= \frac{\alpha_{11}}{\alpha_{01}} Y_{01}(\bar{s}) \left[\left(\frac{\alpha_{01} Y_1^*}{W_1 Y_{01}(\bar{s})}\right)^{\frac{1}{\alpha_{11}}} - 1\right] \quad ,$$

where

$$\bar{s} = s\frac{\kappa_m}{\kappa_{01}} + s_c \quad . \tag{86}$$

The total goal function (22) should, in the present case, be supplied with the term

$$\Delta B(Y_1^*) = [Y_1^* - \frac{W_1}{\alpha_{01}} Y_{01}(s)] \pi - C^*(Y_1^*) \quad , \tag{87}$$

where

π is the export to import price ratio.

The optimum value of $Y_1^* = \hat{Y}_1^*$ can be derived by solving the equation $\Delta B'(Y_1^*) = 0$, which yields

$$\hat{Y}_1^* = a^* Y_{01}(\bar{s}) \quad , \tag{88}$$

where

$$a^* = \pi^{\alpha^*} W_1^{\frac{1}{\alpha_{01}}} \frac{1}{\alpha_{01}} \quad , \qquad \alpha^* = \frac{\alpha_{11}}{\alpha_{01}} \quad . \tag{89}$$

Then, for $Y_1^* = \hat{Y}_1^*$,

$$\Delta B(\hat{Y}_1^*) = b^* Y_{01}(\bar{s}) \quad , \tag{90}$$

where

$$b^* = (\pi W_1 - \alpha_{11})\alpha_{01}^{-1} + (\pi W_1)^{\alpha^*} - \pi a^* \quad . \tag{91}$$

3.3. Structural Changes in Agriculture

This far, agriculture has been regarded as a single, uniform sector with production function (1). In some countries, such as Poland, agriculture consists of modern (state owned) and traditional (privately owned) farms with different acreages (for statistical classification, private farms can be grouped in five size categories with different production functions). In addition, a relatively small collective farm sector exists, which for the sake of simplicity may be grouped with the state sector.

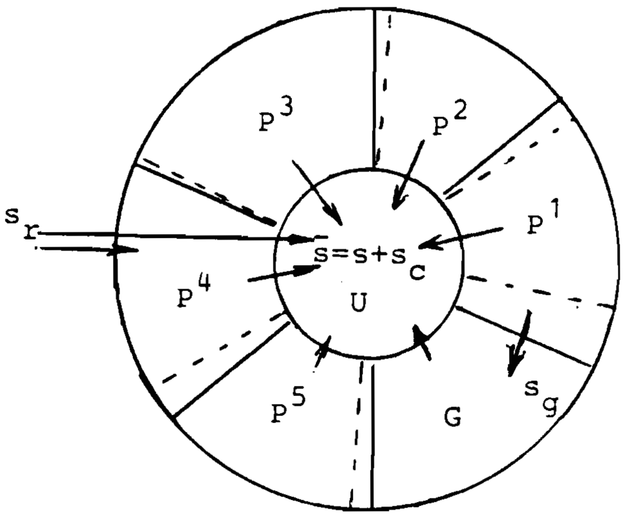

Figure 4. Urbanization diagram (U is the urban part of the region; G is the state sector of agriculture; P^i, i = 1,...,5, is the private sector of agriculture; ---represents transfers of land; and ← represents the direction of migration).

The private (P^i, $i = 1,\ldots,5$) and state (G) sectors are shown on the urbanization diagram (Figure 4), where U represents the urban part of the region. The urbanization process is characterized by land between the sectors and migration of labor. In particular, there is considerable migration of labor from the small size farms P^i to the urban areas in the form of permanent or commuting migrants ($\bar{s}^i = s^i \; L \; s_c^i$) accompanied by intersector migrations. There is also an outflow of labor (L_p) from the private sector ($P = \{P^i\}$) to the state sector (G) at rate s_g; i.e., $L_p(s_g) = \bar{L}_p(1-s_g)$. The migrants' moves are accompanied by the corresponding transfers of land. Generally, the ratio of state to private farm acreage (A_g/A_p) characterizing the structural changes in agriculture increases with migration.

In the present model, we are concerned with the optimum policy of structural change. It is assumed that, for the state sector, the ratio $v_g \triangleq A_g/L_g(S_g)$, where $L_g(s_g) = \bar{L}_g \; (1 + s_g l_g)$

and $l_g = \bar{L}_p/\bar{L}_g$, does not change much (in Poland it is presently around 8 ha per worker). This means that state sector employment is determined by the available acreage, which in turn (in Poland) depends mainly on the area of land given up by ageing private farmers (in accordance with the law, farmers of nonproductive age may exchange land for a retirement pension).

Denoting the $\bar{A}_p/\bar{L}_p$ ratio by $\bar{v}_p$ (presently it is around 3.5 ha per workers), the expected (within the planning interval) acreage given up by the retired private farmers becomes $A_\omega = L_\omega \bar{v}_p$, where L_ω is the expected increase of farmers in the post productive age (derived from demographic projections).

It will be assumed that only a part $A_{\omega g} = \xi A_\omega$, where ξ is a decision variable ($\xi \in [0,1]$), is used to create the new state farms and the remaining land is rented or sold to the private farmers.

Then, the total state sector acreage $A_g(\xi)$ becomes

$$\bar{A}_g + \xi A_\omega = v_g \bar{L}_g (1+s_g l_g) \quad , \tag{92}$$

and

$$s_g(\xi) = \left[(\bar{A}_g + \bar{v}_p \bar{L}_\omega \xi)(\bar{L}_g v_g)^{-1} - 1 \right] l_g^{-1} \quad . \tag{93}$$

Within the private sector, one obtains

$$L_p(s_g) = \bar{L}_p (1-s_g) \quad , \tag{94}$$

$$A_p(\xi) = \bar{A}_p - L_\omega \bar{v}_p \xi \quad . \tag{95}$$

Obviously, the wage (i.e., the income) of private farmers (ω_{01}) depends on farm size v_p (for details, see Kulikowski 1978), so both the values for the planning period should be given (although they are not necessarily equal to the existing values of $\bar{\omega}_{01}$, $\bar{v}_p$). Then, the expected number of jobs (J_1) in the {P,G} system is equal:

$$J_1(\xi) = \frac{\bar{A}_g}{v_g} + \frac{\bar{A}_p}{v_p} - \xi L_\omega \left(\frac{\bar{v}_p}{v_p} - \frac{\bar{v}_p}{v_g}\right) , \tag{96}$$

while the access index $z_{01}(\xi)$ becomes

$$z_{01}(\xi) = \frac{J_1(\xi)}{\kappa_{01}\bar{P}_1(1-\bar{s})} . \tag{97}$$

An important problem regarding the policy of structural change is to find an optimum value for ξ. One of the main objectives is to keep the farmer's income (Z_{01}), or $\hat{z}_1 = Z_{01}/\eta_{01}$) at a given predetermined level.

Such a policy requires that ξ is the solution of equation (8), for i = 1, i.e.,

$$\omega_{01}\kappa_{01}z_{01}(\xi) = \eta_{01}\hat{z}_1 , \tag{98}$$

which, after taking into account equation (97), becomes

$$\hat{\xi} = \left[\frac{\bar{A}_g}{v_g} + \frac{\bar{A}_p}{v_p} - \frac{\eta_{01}\hat{z}_1}{\omega_{01}}\bar{P}_1(1-\bar{s})\right]\left[L_\omega\bar{v}_p(v_p^{-1} - v_g^{-1})\right]^{-1} . \tag{99}$$

When ξ is known, using the equations (92), (94), and (95), one can find the values of $A_g(\hat{\xi})$, $A_p(\hat{\xi})$, and $L_g(\hat{\xi})$, $L_p(\hat{\xi})$.

It is then possible to replace the aggregated agricultural sector in the basic R-U model. In particular, assuming the same rural income (ω_{01}), one can write

$$\hat{Y}_1 = \hat{Y}_p + \hat{Y}_g , \tag{100}$$

where

$$\hat{Y}_p = W_p \omega_{01} L_p(\hat{\xi}) \quad , \tag{101}$$

$$\hat{Y}_g = W_g \omega_{01} L_g(\hat{\xi}) \quad , \tag{102}$$

and

W_p, W_g are given numbers.

When the capital cost assigned to the rural regional budget is spent on the state farm sector only, one should replace $\tilde{S}_{k1}$ in equation (44) by

$$S_{k1} = \frac{\alpha_{1g}}{\alpha_{0g}} \omega_{01} L_g(\hat{\xi}) \quad . \tag{103}$$

In the last case, the capital investments in the private sector are financed out of farmer's revenue, which depends on the output $(\hat{Y}_p)$ and input prices, regulated by the government agencies. In order to substitute the outflow of private labor by capital the government may, however, give the farmers low-percentage credits or decrease the cost of agricultural services. In that case, the corresponding cost of capital can be derived by the method described in section 3.2.

3.4. Linkage of the R-U Model with National Economic Model

So far we have discussed a regional model in which it is assumed that the region is closed, with respect to in-migration to other regions. A possible extension of this model consists in introducing rural (M_r) and urban (M_u) out-migrants, which change the projected (P_1, P_2) populations as follows:

$$\bar{P}_1 = \tilde{P}_1 - M_r \quad , \qquad \bar{P}_2 = \tilde{P}_2 + M_u \quad . \tag{104}$$

The R-U projected (without migrants) population is equal:

$$\tilde{P}_r = \tilde{P}_1 + \tilde{P}_2 \quad , \tag{105}$$

while the expected R-U population with migrants becomes

$$P_r(s_r) = \tilde{P}_r\ (1+s_r) \quad , \qquad s_r = \frac{M_u-M_r}{\tilde{P}_r} \quad . \tag{106}$$

In a similar way, the population in the rest of the country becomes

$$P_c(s_r) = \tilde{P}_c(1-s_c) \quad . \tag{107}$$

It will be assumed that the new (r-c) system (consisting of R-U and the rest of the country) is closed with respect to migration, so $s_r = s_c l_c$, $l_c = \tilde{P}_c/\tilde{P}_r$. Assume also that the ratio $m = M_u/M_r$ is known, so one can derive

$$\bar{P}_1(s_r) = \tilde{P}_1 - \frac{\tilde{P}_r s_r}{m-1} \quad , \tag{108}$$

$$P_2(s_r) = \tilde{P}_2 + \frac{m\tilde{P}_r s_r}{m-1} \quad . \tag{109}$$

Then using the method described in section 2, where P_1, P_2 should be replaced by $\bar{P}_1(s_r)$, $\bar{P}_2(s_r)$, one can find the optimum strategies.

Generally, s_r is unknown in the ex ante sense. One can, however, regard the r-c system in the same way as the basic R-U system. Such an approach allows the form of the basic equations (1) - (60) to be preserved, but with a change in the indices from i = 1,2 to j = c,r, respectively (in other words, the sector with index i = 1 is regarded as the rest of the country, while i = 2 corresponds to the whole R-U regional system studied so far). Using that approach, $\hat{Z}_c$ is regarded as given, therefore

$$Z_r(\hat{s}_r) = \frac{\hat{Z}_c V_c}{V_r} \left(\frac{\hat{s}_r l_r + 1}{d_r} \right)^{a_r} \quad , \tag{110}$$

is predetermined.

On the other hand, Z_r is determined by R-U migration s, i.e.,

$$Z_r = \hat{Z}_1 \frac{P_1(\hat{s})}{P_r} + \hat{Z}_2 \frac{P_2(\hat{s})}{P_r} \quad . \tag{111}$$

When $\hat{Z}_{\varsigma}$ and $\hat{Z}_1$ are fixed, the sole parameter that can be altered is $\hat{Z}_2$. The value of $\hat{Z}_2$ can be changed by altering $\bar{\sigma}$, i.e., by locating such industries in the urban economy, which produces the labor efficiency $Y_2/L_2 = (W_2/\alpha_{02})\omega_{02}$.

Indeed, the value of Y_2/L_2 can be regarded (in microeconomic terms) as the averaged sum of individual labor efficiencies $Y_j/L_j = F$, $j = 1,\ldots,N$, where N is the number of factories.

Obviously,

$$\frac{Y_2}{L_2} = \sum_{j=1}^{N} \frac{L_j}{L_2} F_j \quad . \tag{112}$$

Usually, a part $N' < N$ represents existing or given factories, while $\{N-N'\}$ represents new factories, with given F_j indices, planned within the optimization interval. By selecting the location of new factories (i.e., F_j and employment, L_j), one can determine the proper value of Y_2/L_2 and, consequently, $\hat{Z}_2$ as well as $\hat{Z}_1$.

4. DESCRIPTION OF THE IRUD SYSTEM

4.1. Input Data for the IRUD Model

Input data for the IRUD model are presented in Table 1. They include parameters, as well as exogenous and decision variables, for the production, consumption and migration submodels. The decision variables represent the players' strategies. The player R_0 on the regional level divides the total amount of subsidies between the rural and urban subregions. The players R_1 on the rural level and R_2 on the urban level divide the subsidies obtained between capital rent and aggregate consumption. Policy variables form the fourth group of input data. They are selected from the model parameters and exogenous variables and include quantities that can be assumed by the central planner. The notation presented in Table 1 refers to the model description in the lefthand column and to the computer printout in the right-hand column.

Two versions of the model have been considered in the IRUD system. In the first version, the following simplifications have been made: the labor costs Y_{0i} $(i = 1,2)$ are linear with respect to migration rate s; the ω_{01}. ω_{02} are constant; and housing costs are neglected (parameter $h = 0$, $Z_h = 0$). In the second version ω_{02} is considered as a function of s and is calculated on the basis of equation (19). Housing costs are taken into account in the total migration cost function (C(s)). Then, the relation (13) is replaced by

$$C(s) = [Z_2(s) - Z_i(s) + Z_h] \cdot \bar{P}_1 \cdot s =$$

$$= Z_i(s) \cdot [(\frac{sl+1}{d})^a + h] \cdot \bar{P}_1(s) \quad , \qquad (113)$$

where

$$h = 1 + \frac{Z_h}{Z_i(s)} \quad . \qquad (114)$$

Table 1. Input information for the IRUD model.

Magnitude	Notation in model description	Notation in computer printout
1. MODEL PARAMETERS		
<u>Production function:</u>		
Parameters	W_1, W_2	W1, W2
Elasticities with respect to:		
-- labor	α_{01}, α_{02}	alfa01,alfa02
-- capital	α_{11}, α_{12}	alfa11,alfa12
<u>Utility function:</u>		
Parameters	V_1, V_2	V1, V2
Elasticities with respect to:		
-- personal income	η_{01}, η_{02}	eta01,eta02
-- services	η_{11}, η_{12}	eta11,eta12
<u>Migration submodel:</u>		
Share of population of productive age	κ_{01}, κ_{02}	kappa01, kappa02
Share of migrants of productive age	κ_m	kappam
Parameters of the behavioral relation describing the migration rate as a function of utilities' ratio	a,d	a,1
Parameter of housing costs	h	h
2. EXOGENOUS VARIABLES		
Rural population	P_1	P1
Urban population	P_2	P2
Minimum rural consumption per capita	z_1	z1
Average wage: rural and urban	ω_{01}, ω_{02}	omega01, omega02

Table 1. (continued)

	Magnitude	Notation in model description	Notation in computer printout
3.	DECISION VARIABLES (players' strategies)		
	R_0 level:		
	Division of the value of total subsidies	S	S
	-- rural subregion	S_1	Soi
	-- urban subregion	S_2	
	R_1 level (rural):		
	Division of the S_1 value between		
	-- capital rent	S_{k1}	Ski
	-- aggregated consumption	S_{c1}	Sci
	R_2 level (urban):		
	Division of the S_2 value between		
	-- capital expenditures	S_{k2}	Ski
	-- aggregated consumption	S_{c2}	Sci
4.	POLICY VARIABLES (selected from the model parameters and exogenous variables)		
	Minimum rural consumption per capita	Z_1	Z1
	Average wages	ω_{01}, ω_{02}	omega01, omega02
	Housing costs (in migration cost function)	Z_h, h	Zh, h

4.2. Information Obtained from the Model

The quantities obtained as a result of the model runs are presented in Table 2. They include the values of the goal functions and main endogenous quantities describing production, consumption, and migration.

Table 2. Output information (quantities calculated in the model).

Magnitude	Notation in model description	Notation in computer printout
1. Goal functions:		
1. Total	B	B
2. Player R_0	B_0	Bo
3. Players' R_1, R_2	B_1, B_2	Bi
2. Migration submodel:		
1. Migration rate	s	s
2. Migration	M	M
3. Cost of migration	C(s)	C
4. Population dependent on migration	$\bar{P}_1(s), \bar{P}_2(s)$	Psi
3. Production submodel:		
1. Production value	$Y_1(s), Y_2(s)$	Yi
2. Wages fund	$Y_{01}(s), Y_{02}(s)$	Yoi
3. Employment dependent on migration	$L_1(s), L_2(s)$	Lsi
4. Index of labor access	z_{01}, z_{02}	zo
4. Consumption submodel:		
1. Total per capita consumption	Z_1, Z_2	Z
2. Consumption out of personal income	Z_{01}, Z_{02}	Zoi
3. Aggregated consumption	Z_{11}, Z_{12}	Z1i

4.3. Structure of the Interactive System

The general structure of the system is presented in Figure 5. The blocks on the left side of the scheme refer to the user of the system. On the right side, the subroutines used in particular blocks of the system are given. The system starts operation with the initialization of the input data. The standard values are assigned to the exogenous variables and model parameters by DATA statement. The following INIT block enables the user to interactively update the initial values. The values assigned to the particular quantities are visualized on the screen or obtained in printouts. If the user requests that any of the values be changed, the system lists the notation of the variables in sequence. The user can decide which variable should be changed and can assign a new value for the particular variable. After this, the updated data are visualized and at the user's request, the procedure can be repeated (for example to correct mistakes). This program block utilizes the unit subroutine to obtain the updated initial values, which are input in the OPTIM block, to calculate the optimal solution. First, the optimal migration rate $\hat{s}$ is calculated under the assumption that the model maximizes the total benefit function B.

The optimization problem consists in finding the zero point of the function F(s), which is derivative of B with respect to s. For this the Newton iterative method is used. The Newton subroutine uses two additional subroutines: fmig, which calculates the values of the F(s) function and dmig, which calculates the values of the derivative F'(s). Having the optimal migration rate $\hat{s}$, the optimal values of decision strategies are derived and all endogenous variables are calculated by the model relations. Optimal results obtained in this way are stored in the memory. The value of total subsidies are used in the subsequent DIAL block. This block enables the players to introduce the strategies interactively. DIAL starts with the presentation of the total subsidies. The R_0 player can divide the subsidies between the rural and urban subregions. The information is then presented for players R_1 and R_2 and the rural and urban subsidies are visualized. The players R_1 and R_2 can divide the

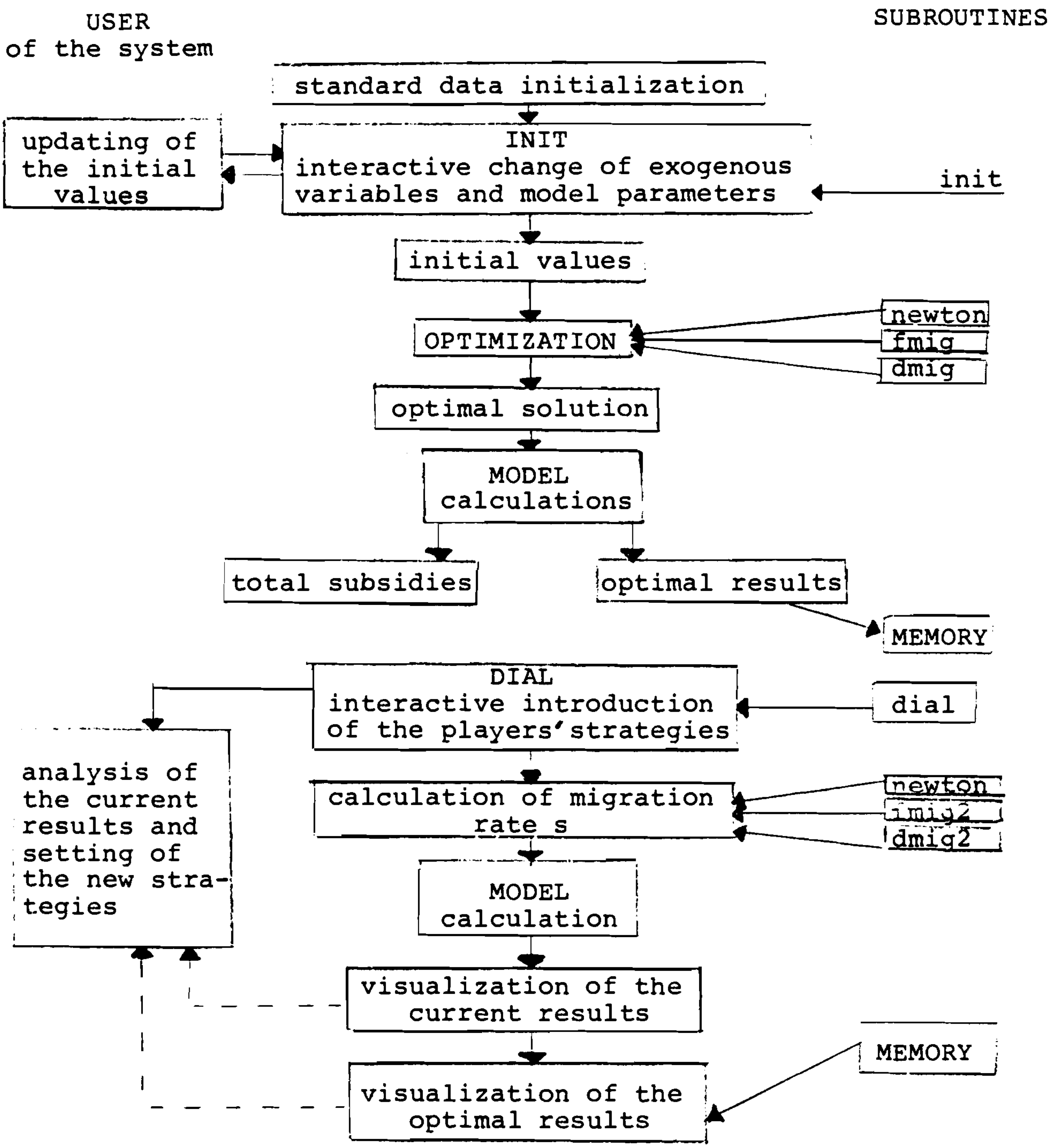

Figure 5. The structure of the IRUD system.

subsidies into capital rent and aggregated consumption in a similar way. All the introduced values of subsidies are shown on the screen and can be improved on request. The players' strategies and the initial values are used in the next block, which calculates the resulting migration rate $\tilde{s}$. The $\tilde{s}$ value is found as the solution of equation (60). For this reason, the Newton method is used to find the zero point of the function

$$F_2(\tilde{s}) = \frac{1+\tilde{s}l}{d}^{s} - \frac{\tilde{V}_2}{\tilde{V}_1} \frac{\alpha_{12} \tilde{Z}_2(\tilde{s})}{\alpha_{11} \tilde{Z}_1(\tilde{s})} \quad . \qquad (115)$$

The Newton method utilizes the subroutines fmig 2 and dmig 2. fmig 2 calculates the values of the $F_2(\tilde{s})$ function, while dmig 2 derives the values of the derivative $F_2'(\tilde{s})$. Using the migration rate $\tilde{s}$, the endogenous quantities are calculated by model relations. The values of the quantities, called current results, reflect the decision strategies and in general are not equal to the optimal results. The current results are visualized on the screen. The user can analyze these results and new strategies can be introduced. The system runs in this loop as long as the user does not decide to finish play. Then the optimal results are shown.

4.4. Operation of the System

The IRUD system has been implemented as a set of FORTRAN IV subprograms. Under the UNIX system, the subprograms have been compiled and already linked using the ftn compiler and the executable program has been obtained. The user, to operate the IRUD system, should type on the terminal the name of the file that contains the executable program (it is assumed that the user has already logged in the UNIX system and can use the terminal). An example of operation of the IRUD system is presented in Table 3. The example shows the information exchange between the system and the user. The information introduced by the user is underlined. At first, the IRUD system gives the initial values of exogenous variables and model parameters on the screen. It asks the user whether he wishes to change the values. The user answers by

Table 3. An example of information exchange between the IRUD system and the user.

```
     start

     initial values:

     population     P1=     420000.     P2=     550000.
     W1= 0.520     W2= 1.510
     alfa01= 0.52     alfa02= 0.68
     alfa11= 0.48     alfa12= 0.32
     kappam= 0.681
     kappa01= 0.525     kappa02= 0.561
     wages: omega1=     2457.0     omega2=     2873.0 zl/month
      eta01=0.850     eta02=0.850
      eta11=0.150     eta12=0.150
     total rural consumption per capita Z1=     15606.
 do you change the initial values ?,
 (write please t(true) if yes or f(false) elsewhere)
t
 you can change the values interactively,
 if you change particular value, follow please its description
  by t, elsewhere by f
 rural population P1
f
 urban population P2
f
 W1
t
 write please the value(values) with decimal point (separating by commas)
0.7,
 W2
f
 alfa01, alfa11  (rural)
f
 alfa02, alfa12  (urban)
f
 wages omega1, omega2
f
 rural consumption Z1
f
 parameter a
f
     initial values:

     population     P1=     420000.     P2=     550000.
     W1= 0.700     W2= 1.510
     alfa01= 0.52     alfa02= 0.68
     alfa11= 0.48     alfa12= 0.32
     kappam= 0.681
     kappa01= 0.525     kappa02= 0.561
     wages: omega1=     2457.0     omega2=     2873.0 zl/month
      eta01=0.850     teta02=0.850
      eta11=0.150     teta12=0.150
     total rural consumption per capita Z1=     15606.
 do you change the initial values ?,
 (write please t(true) if yes or f(false) elsewhere)
f
     PLAYER Ro
you have S=   12536.6 mln zl  subsidies to divide on rural and urban part
 write the value of rural subsidies, please
5500.0,
     PLAYER R1
you have So1=     5500.0 mln zl  subsidies to divide on capital rent and
 aggregate consumption,
write the value of capital rent,  please
4600.0,
```

Table 3. (continued)

```
    PLAYER R2
you have So2=    7036.6 mln zl  subsidies to divide on capital rent and
 aggregate consumption,
write the value of capital rent,  please
5900.0,
    decision variables (mln zl) :
    total subsidies: rural S1p=    5500.0     urban S2p=     7036.6
    subsidies  Skp1=    4600.0      Skp2=     5900.0
    subsidies  Scp1=     900.0      Scp2=     1136.6
 do you change your strategies?
(answer by t if yes or f elsewhere)
f


     results of the players strategies
     total subsidies S=   12536.6 mln zl
     subsidies Soi            mln zl
     rural          5500.000           urban          7036.572

     subsidies Ski            mln zl
     rural          4600.000           urban          5900.000

     subsidies Sci            mln zl
     rural           900.000           urban          1136.572

     migration rate           s= 0.0932
     migration M=     41247.
     cost of migration             c=  297.3 mln zl
     goal function B          30333.4 mln zl
     goal function Bo           2004.0 mln zl
     goal functions Bi
     rural         10498.196           urban         17831.189

     wages fund Yoi            mln zl
     rural          4998.196           urban         10794.617


     production value Yi            mln zl
     rural          6727.799           urban         23902.867

     population dependent on migration Psi
     rural        378752.812           urban        591247.187

     employment dependent on migration Lsi
     rural        169522.328           urban        313105.253

     index of labor access zo
     rural             0.458           urban             0.524

     total consumption per capita Z in zl
     rural         15572.680           urban         20179.697

     consumption out of personal income Zoi
     rural         13196.460           urban         18257.367

     aggregate consumption Z1i
     rural          2376.220           urban          1922.330
```

Table 3. (continued)

```
 do you continue the play
(answer please by t if yes or f elsewhere)
 f

     optimal solution:

     total subsidies S=   12536.6 mln zl
     subsidies Soi              mln zl
     rural          5620.543           urban          6916.028

     subsidies Ski              mln zl
     rural          4718.480           urban          5029.339

     subsidies Sci              mln zl
     rural           902.062           urban          1886.192

     migration rate              s= 0.0825
     migration M=     34651.
     cost of migration              c=  204.5 mln zl
     goal function B           30411.2 mln zl
     goal function Bo            2074.5 mln zl
     goal functions Bi
     rural         10732.230           urban         17604.437

     wages fund Yoi               mln zl
     rural          5111.687           urban         10683.408


     production value Yi               mln zl
     rural          6881.117           urban         23734.553

     population dependent on migration Psi
     rural        385348.531           urban        534651.500

     employment dependent on migration Lsi
     rural        173371.562           urban        310324.594

     index of labor access zo
     rural             2.459           urban             1.530

     total consumption per capita Z in zl
     rural         15606.000           urban         21507.852

     consumption out of personal income Zoi
     rural         13265.101           urban         18281.674

     aggregate consumption Zli
     rural          2340.900           urban          3226.178
```

typing "t" if yes or "f" elsewhere. If the user types "t", the system starts to list the initial values using the notation given in Table 1. Each variable listed has to be followed by "t" typed by the user if he wants to change it or "f" elsewhere. The new value should be typed using decimal points and should be separated by commas. At the end of the above procedure, the updated initial values are shown on the screen.

Next time the system asks the user whether he wishes to change the initial values. The procedure can be repeated to correct mistakes. If the user assumes as correct the change of the initial values, he types "f" and the game starts. During the game, the system calculates the optimal results. The total subsidies S are shown for the R_0 player. Then, the user types the value S_1 of rural subsidies. The value $S - S_1$ belongs to the urban subregions. The information for the player R_1 consists in the rural subsidies. The user introduces the R_1 strategy by typing the value of capital rent. The R_2 strategy is introduced in a similar way. All the values should be typed with decimal points in million zlotys. The strategies introduced then appear on the screen. If the user notices any mistake in the values introduced, he can repeat the DIAL procedure by typing "t". If the strategies are correct, the user types "f" and the results of the players' strategies are presented on the screen. At the first, the subsidies are shown. After this, all output quantities are visualized using the notation given in Table 2. Because of the limited size of the screen, the results are divided into two parts. To obtain the second part, the "cr" key should be used. The results are followed by the question to the user as to whether he wants to continue the game. If so, the user types "t" and the system goes back to the DIAL procedure. New strategies can be typed and new results can be derived. If the user decides to finish the game, he types "f". The system shows the optimal results. The results are divided into two parts and the "cr" key should be used to obtain the second part. This completes the system run.

4.5. Examples of Computation Runs and Data Used

The values of exogenous variables and parameters assumed to test the model calculations and the IRUD system are presented in Table 4. The values are based on statistical data for 1973 on the Bydgoskie voivodship, which forms the greatest part of the Upper Notec region. Examples of the results for first version of the model are presented in Figures 6a, 6b, 7a, and 7b.

Table 4. Data for experimental calculations.[a]

1. EXOGENOUS VARIABLES		
Rural population	$P_1 = 420{,}000.$	
Urban population	$P_2 = 550{,}000.$	
Rural per capita consumption	minimum Z_1 15,600 zl/month	
Average wage: rural	$\omega_{01} = 2{,}457$ zl/month	
urban	$\omega_{02} = 2{,}873$ zl/month	
2. MODEL PARAMETERS		
Production Function:		
Parameters	$W_1 = 0.52$	$W_2 = 1.51$
Elasticities with respect to: labor	$\alpha_{01} = 0.52$ (0.7)	$\alpha_{02} = 1.51$
capital	$\alpha_{11} = 0.48$ (0.3)	$\alpha_{12} = 0.32$
Utility Function:		
Elasticities with respect to: personal income	$\eta_{01} = 0.85$	$\eta_{02} = 0.85$
services	$\eta_{11} = 0.15$	$\eta_{12} = 0.15$
	$V_1 = V_2 = 1$	

3. EMPLOYMENT -- MIGRATION SUBMODEL

Share of population of productive age

rural -- $\kappa_{01} = 0.525$, urban -- $\kappa_{02} = 0.561$

Share of migrants of productive age $\kappa_m = 0.685$

Parameters of the behavioral relation describing the migration rate as a function of the utilities ratio U_2/U_2:

$a = 5.25$, $d = 1$

[a] All the values are based on data for the Bydgoskie voivodship, 1973.

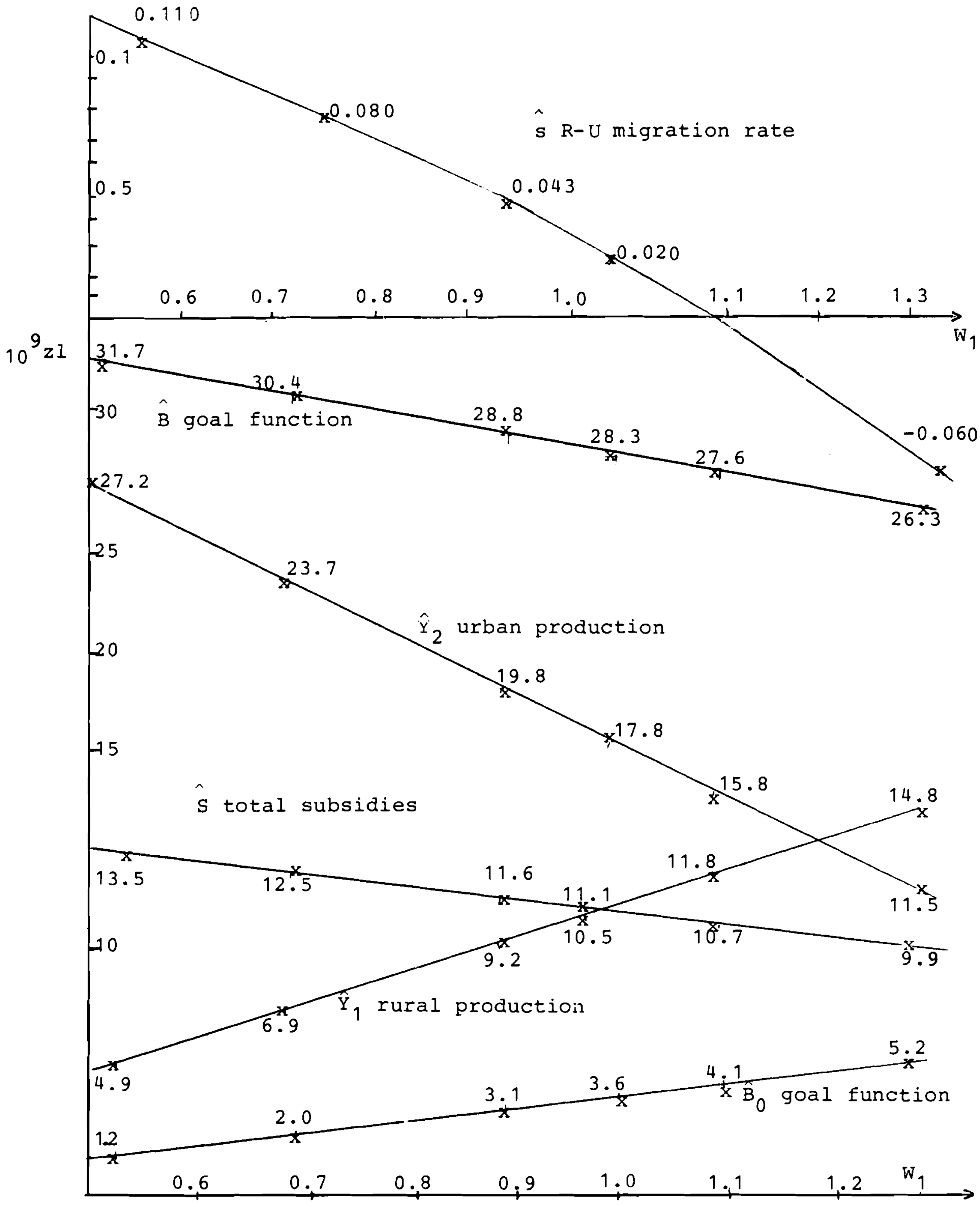

Figure 6a. Optimal solution as a function of the W_1 parameter.

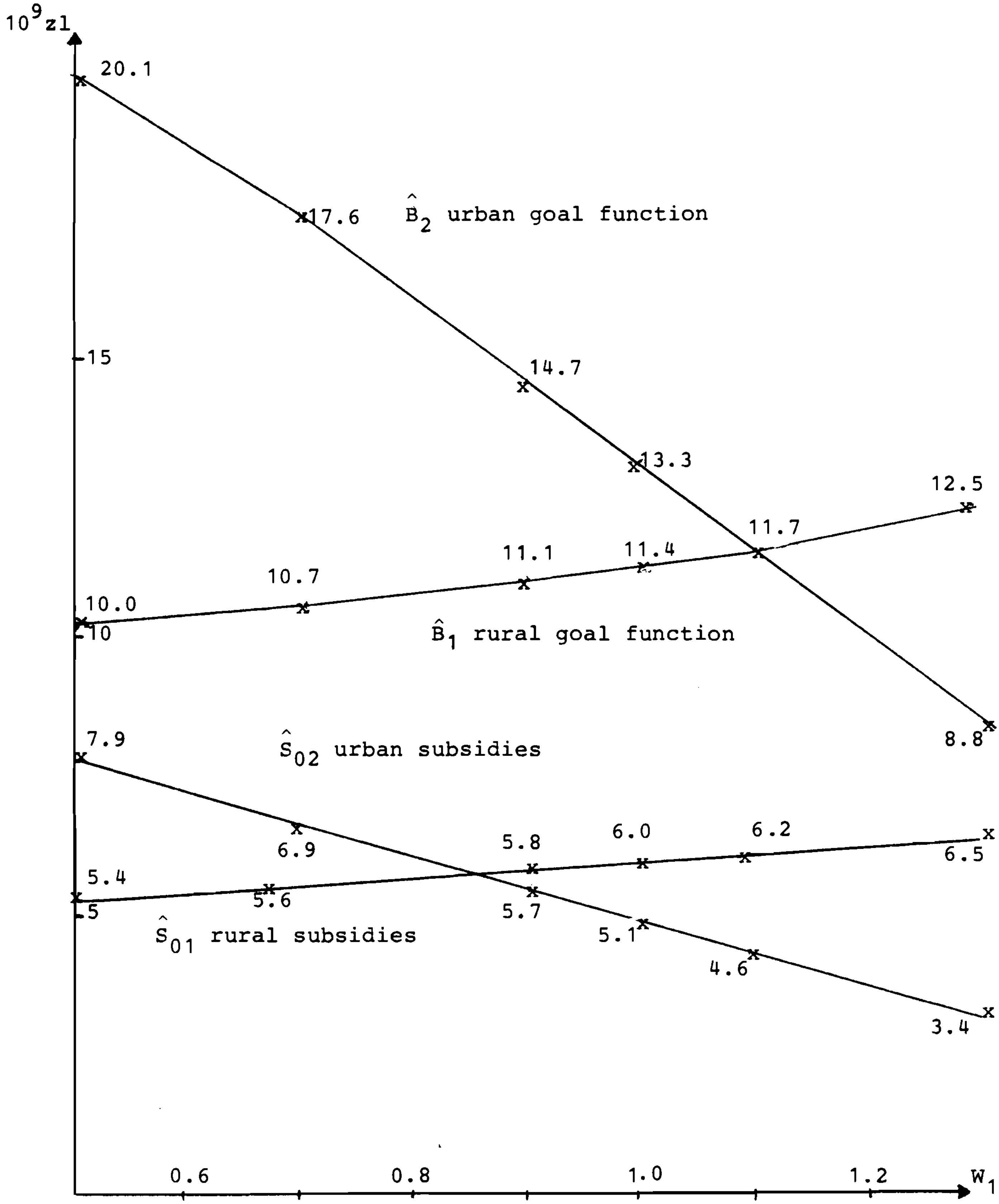

Figure 6b. Optimal solution as a function of the W_1 parameter.

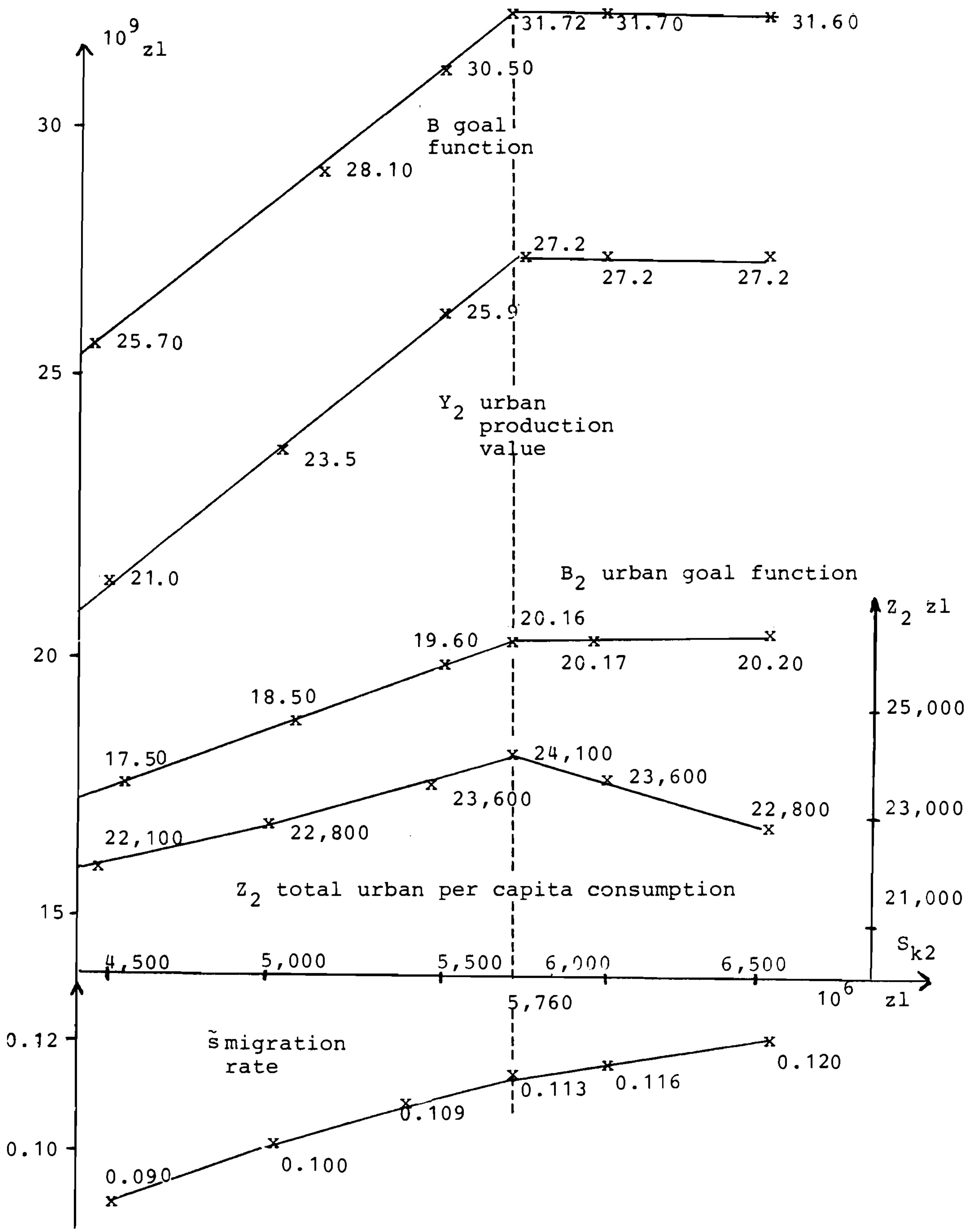

Figure 7a. Model results as a function of S_{k2} strategy.

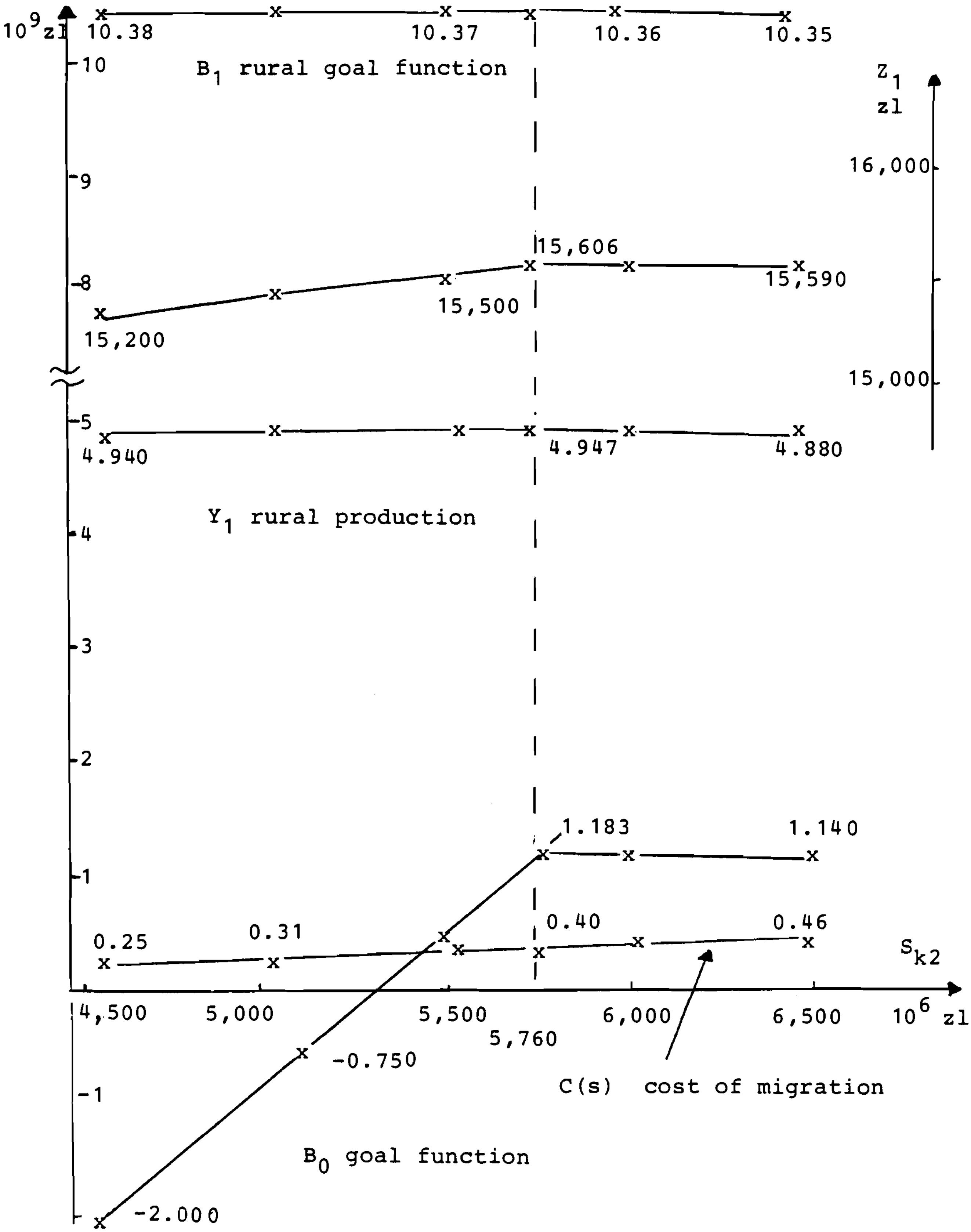

Figure 7b. Model results as a function of S_{k2} strategy.

In the proposed linkage of models within the Notec project, the parameters W_1 and W_2 are of great importance. The parameters are interpreted as the production efficiency of the rural and urban economies, respectively. Subsidies on water system development can be considered as stimulating an increase in production efficiency. The optimal solution, which is dependent on W_1, is presented in Figures 6a and 6b. The model equations are analyzed under conditions of an increase in W_1 from 0.52 (current state) to 1.3. The efficiency of W_2 for urban production is equal to 1.5. The values of other parameters and exogenous variables are presented in Table 3. Figure 3 shows changes in the optimal migration rate $\hat{s}$, the goal functions $\hat{B}$ and $\hat{B}_0$, the total subsidies $\hat{S}$, and the rural and urban production values $\hat{Y}_1$, $\hat{Y}_2$, respectively. The optimal migration rate $\hat{s}$ is equal to 0.113 in the current state and decreases as W_1 increases. It is equal to 0 for $W_1 = 1.1$. The total goal function $\hat{B}$, defined as the sum of production values minus migration costs, decreases but goal function $\hat{B}_0$, which takes into account the value of subsidies, increases. If the W_1 parameter increases, a less optimal value of subsidies is needed. In Figure 6b, the $\hat{B}_1$ and $\hat{B}_2$ goal functions are presented as well as the subsidies $\hat{S}_{01}$, $\hat{S}_{02}$. If, in the current state, the optimal division of subsidies for $W_1 = 0.52$ is 7.9 and 5.4 for the urban and rural subregions, respectively, for $W_1 = 0.9$ the subsidies should be divided in equal parts.

Figures 7a and 7b show how the model results depend on the R_2 player's strategy, namely, on the value of S_{k2}. S_{k2} denotes the urban subsidies on capital rent. The other strategies (S_{01}, S_{02}, and S_{k1}) have been assumed constant and equal to the optimal values. In Figure 7a, the B and B_2 goal functions, the urban production value Y_2, the total urban per capita consumption Z_2, and the migration rate $\tilde{s}$ are presented. An increase of capital rent S_{k2} causes an increase of migration to the urban economy so that an increasing rate of migration can be observed.

The optimal derived value of subsidies S_{k2} is equal to $\hat{S}_{k2}$ = 5,760 million zloty. For $S_{k2} \leq S_{k2}$, the constraint defined by the linear relation of the urban production with respect to capital rent is active. For $S_{k2} \geq S_{k2}$, the production value is constrained by the linear relation with respect to the migration rate s. In Figure 7b, the rural goal function B_1, rural production Y_1, and total rural per capita consumption are presented as well as goal function B_0 and migration cost C(s).

The results obtained lead us to propose a more complicated second version of the model mentioned in the section 4.1. The preliminary optimal solution obtained from the second version of the model is presented in Table 5.

5. CONCLUDING REMARKS

The first, simplified version of the IRUD system has been set up and partially tested. The system is running on the PDP 11/45 computer under the UNIX time-sharing system and preliminary results based on approximate data have been obtained. The system enables the interactive change of model parameters and exogenous variables as well as the interactive introduction of decision variables. It can be considered as a tool for game playing among the regional, urban, and rural authorities. The results obtained can be used on a basis for establishing a computerized system for more advanced policy evaluation. It will link the models being developed for the Notec project.

ACKNOWLEDGEMENT

The authors are grateful to the staff of IIASA, especially to Professor Murat Albegov for his interest and support and to Professor Andrzej Wierzbicki for valuable discussions. Prof. Wierzbicki's idea of assessing a multiobjective solution based on objective reference points will be very fruitful when applied to the problems described in the paper. The authors are also grateful to Ms. Bozena Lopuch from the Food and Agriculture Program as well as to Mr. Lutz Blencke from the Computer Service Department for introducing them to the UNIX system and for continuous assistance.

Table 5. Preliminary results of the second version of the model.

```
     initial values:

     population      P1=     420000.       P2=     550000.
     W1= 0.520      w2= 1.510
     alfa01= 0.52      alfa02= 0.68
     alfa11= 0.48      alfa12= 0.32
     kappa= 0.540
     wages omega1=    2457.0 zl/month
      eta01=0.850      eta02=0.850
      eta11=0.150      eta12=0.150
     total rural consumption per capita Z1=    15606.
     coefficient  a= 5.250
 do you change the initial values ?,
 (write please t(true) if yes or f(false) elsewhere)
f
     optimal solution:

     total subsidies S=12268.3 mln zl

     subsidies Soi            mln zl
     rural         5688.200          urban          6580.146

     subsidies Ski            mln zl
     rural         4775.279          urban          4785.561

     subsidies Sci            mln zl
     rural          912.921          urban          1794.585

     migration rate           s= 0.0715

     migration M=    30013.

     cost of migration           c= 2024.2 mln zl

     goal function B      21829.2 mln zl

     goal function Bo      -5781.6 mln zl

     goal functions Bi
     rural       10861.419          urban         16749.463

     wages fund Yoi             mln zl
     rural         5173.218          urban         10169.317

     production value Yi          mln zl
     rural         5173.218          urban         18680.230

     population dependent on migration Psi
     rural       389987.125          urban        580012.875

     employment dependent on migration Lsi
     rural       175458.500          urban        260952.703

     total consumption per capita Z in zl
     rural        15606.000          urban         20626.959

     consumption out of personal income Zoi
     rural        13265.101          urban         17532.916

     aggregate consumption Z1i
     rural         2340.900          urban          3094.044
```

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PART II

Models of Agriculture

SYSTEM OF MODELS FOR DEVELOPING AGRICULTURAL PRODUCTION IN A REGION (THE SILISTRA CASE STUDY)

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A. Kehayov

INTRODUCTION

This paper describes a system of models that can be applied to the problem of developing agricultural production in a given region, in this case Silistra. It summarizes our research on the application of quantitative methods in the analysis of development strategies for the Drastar agroindustrial complex. We develop the ideas put forward in Carter et al. (1977) and Gavrilov et al. (1979a) and lay the foundations for a broader analysis of the problems resulting from the development of agricultural production. At the same time, this study forms part of the general research carried out within the framework of the development program for the Silistra region.

Although the proposed system of models reflects the characteristics of the Silistra region, it can be used to study other regions in the country. Thus, it is a useful tool for developing an interactive procedure for researching and planning the development of agriculture. For a general idea of the scope and organization of the system of models, see Figure 1.

Individually, most of the models are optimizational in character, but the system as a whole runs as a simulation model. To arrive at a practical solution, it combines two approaches--the

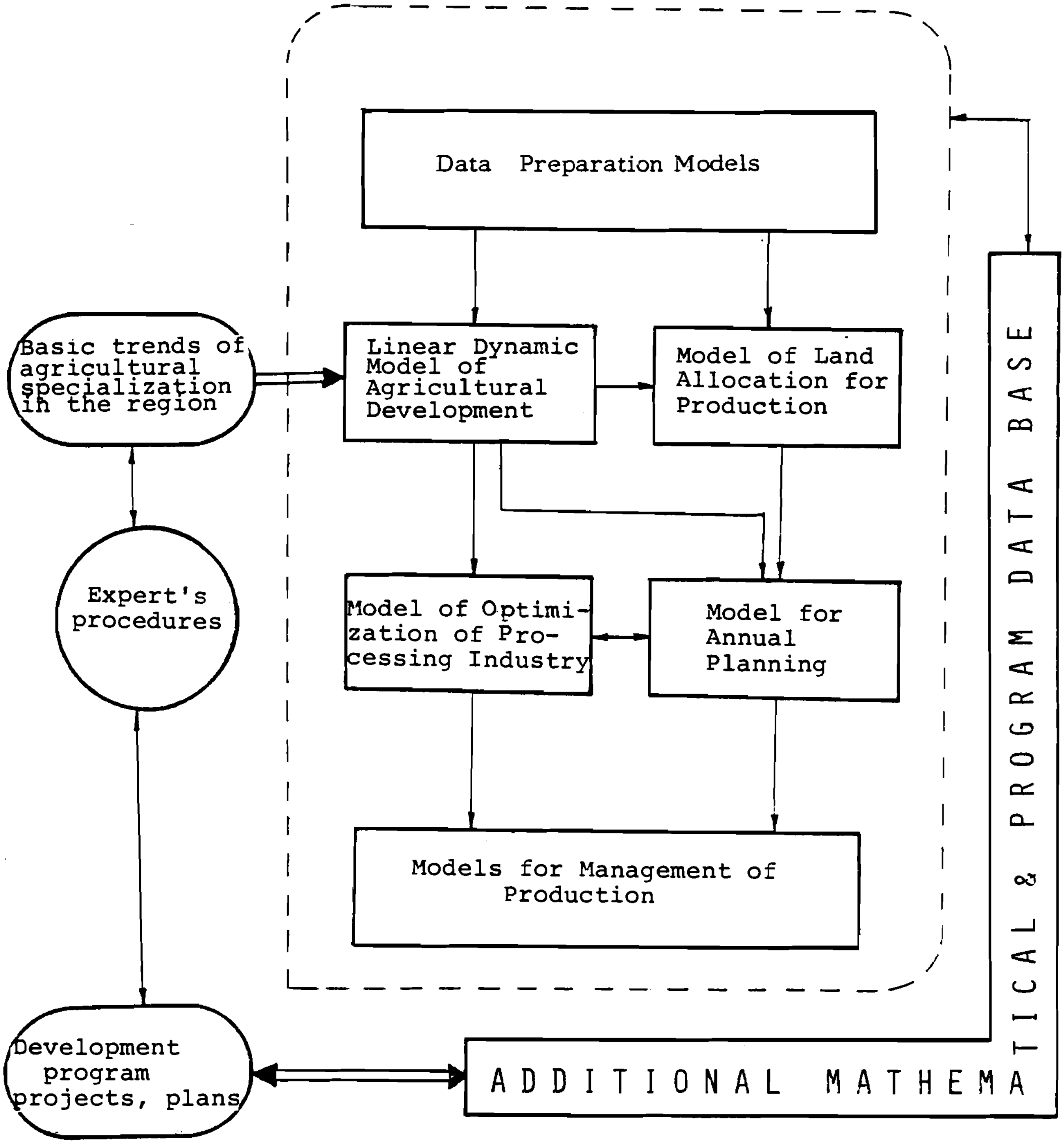

Figure 1. Scope and organization of the proposed system of models.

formal and the informal. Informal procedures allow experience and intuition to be used in the evaluation of the results. The overall solution is chosen after the results from all of the models have been analyzed and compared. Only by using such an approach is it possible to achieve a development plan that is both feasible and socially acceptable. However, this approach can create serious difficulties for the researcher, as we have experienced with the development of Drastar. These difficulties are principally related to procedures for the exchange of information between models and specialists.

A brief description of the models of the system follows. This includes attempts to clarify the interrelations between the models and the assumptions on which they are based.

THE LINEAR DYNAMIC MODEL OF AGRICULTURAL PRODUCTION

The linear dynamic optimization model of agricultural production development occupies the central place in the proposed system of models. This is not only because it was the first model to be constructed but also because of its function--to outline the possible strategies for the agricultural development of the Silistra region. It has been developed for the conditions of the agroindustrial complex Drastar, which manages the arable land and most of the processing industry of Silistra. A detailed account of the experience gained during the development period is given in Gavrilov et al. (1979b) and Gavrilov et al. (1980). The purpose of the model is to find both a production structure for the complex and the means to achieve it, making the most efficient use of the available land, and thus obtaining the maximum net income from production.

The model simulates the development of regional agriculture for a time horizon of 15 years. This is described by five 1-year and five 2-year periods. Two conditions have been considered in choosing the time horizon. It must be sufficiently long to allow the results from investment to become evident and yet it must not lead to a large increase in the model's size. Simulations using this model in the Drastar complex show that it is possible to carry out the analysis over a 20-year period maintaining the same degree of detail.

It is a lumped parameter model; i.e., it examines the development of the region as one point in space. Such an approach allows the size of the problem to be reduced; this is necessary because the model, at its final stage of modification, includes approximately 910 constraints and 1,200 variables. Therefore, if we examine 10 regions, the model will include more than 10,000 constraints and 15,000 variables. In the operation of such a large-scale model, serious computational difficulties can occur as a result of its clumsiness. The approach allows us to work with parameters aggregated for the whole region and therefore to construct an aggregated model. Thus, one of the disadvantages of using a detailed, long-term model is avoided; i.e., the accumulation of serious mistakes at the stage where the plan is finally determined. It has been shown that aggregated models usually provide more accurate predictions than nonaggregated models. The most likely explanation for this is that the underestimates of production for each subregion often cancel the overestimates (and vice versa) to produce a more accurate prediction for the whole region. The most essential assumption of the model is that the region functions under given economic conditions. In the most recent version of the model, a parameter representing irrigation is included.

Because of the need to develop a fodder production base, so that only the minimum of animal-feed would have to be purchased, the first block of the model represents crop growing. If $x(t)$ is a vector representing crops, crop growing in general is described by

$$\sum_{i=1}^{n} x_i(t) = X(t) \qquad (1)$$

and

$$\sum_{i \in I_1} x_i(t) = X^1(t), \qquad (2)$$

where

$X(t)$, $X^1(t)$ are respectively the total area of arable land and the areas of this land that can be irrigated; and

I_1 is the variety of crops cultivated in irrigated areas (there are also a number of assumptions about crop rotation, the dynamics of crop development, and so on, but for the sake of brevity they are not discussed here).

The annual development of the different breeds of livestock is described in general terms by

$$U(t+1) = B(t)y(t) + u(t), \quad t = 0,1,2,\ldots,T-1 , \qquad (3)$$

where

$y(t)$ is the vector representing various groups and breeds of animals;

$B(t)$ is the matrix of the passage of the animals from one group to another every year; and

$u(t)$ is the vector of animals purchased.

Constraint (3) is accompanied by different initial boundary conditions.

The distribution of land between crop production for human consumption, for forage, and for exports is

$$C(t)x(t) - Dy(t) + v(t) - v(t+1) + w^1(t) - w^2(t) - p(t) = 0, \qquad (4)$$

where

$C(t)x(t)$ is the vector of fodder production;

D is the matrix with elements reflecting the consumption of different types of fodder;

$v(t)$ is the vector of food stocks;

$w^1(t), w^2(t)$ are, respectively, the vectors representing the purchases and sales related to crop production; and p(t) is the vector of the food consumption of the population.

If F is a matrix of the various technologies used in livestock production and the vector z(t) represents the final products from stockbreeding, then

$$z(t) = Fy(t) \quad . \tag{5}$$

Certain constraints are used to reflect the necessary production capacities (in terms of agricultural machinery, buildings, and equipment for stockbreeding, processing plants for fodder, meat and dairy products, storage).

The economic activities of the complex are represented by constraints relating to the funds available for capital investment, their exhaustion, the means of financing the maintenance of production, and the formation of the final economic results.

If we denote the net income of the complex during the year t by J(t), the basic criterion used for evaluating each solution to find the optimum is

$$J = \sum_{t=1}^{T} \frac{1}{(1-e)^{t-1}} J(t) \rightarrow \max \quad . \tag{6}$$

To preserve the unity of the proposed models, the details of the various versions of the linear dynamic model will not be discussed.

LAND ALLOCATION MODEL FOR THE AGRICULTURAL PRODUCTION AND PROCESSING INDUSTRIES

The second model of the system--the land allocation model for regional agricultural production and processing industries--is directly linked to and an extension of the linear dynamic model of agricultural production development. In this paper, we discuss some of its basic characteristics and its function within the framework of the model system. It is described in more detail in these Proceedings in Gavrilov et al. "Modeling Land Allocation for the Agricultural Production and Processing Industries of the Silistra Region".

In contrast to the linear dynamic model, the model of land allocation accounts for the spatial characteristics of the Silistra region. The region is divided into several subregions (in the Silistra project we plan to have 10), which are based on the boundaries of existing settlement systems (Carter et al. 1977). Thus, the purpose of the model is to determine which of the development options prepared by the linear dynamic model should be implemented in a subregion.

Every unit is characterized by its area of arable land, by the number of suitable sites it has available for the construction of farms and processing plants, by its available labor resources (number, level of education, possibilities for training them to obtain new qualifications), by the level of development of its technical infrastructure, and so on. All these features are considered when the unit is being evaluated for development.

If $x^i(t)$ is a vector representing the various agricultural crops in subregion j during year t, then

$$\sum_{i=1}^{n} x_i^j(t) = X^j(t) \quad , \tag{7}$$

$$\sum_{i \in I_1} x_i^j(t) \leq X^{1j}(t) \quad , \tag{8}$$

$$\sum_{j=1}^{m} x_i^j(t) = x_i(t) \quad , \quad i=2,\ldots,n \quad , \tag{9}$$

where

$X^j(t)$ is the arable land available in subregion j;

$X^{1j}(t)$ is the land available for irrigation; and

$x_i(t)$ as in equation (1) is the area of land used for cultivating crop i in the whole region.

If $y^i(t)$ is the vector representing the type of animals bred in subregion j during year t, then

$$\sum_{j=1}^{m} y^j(t) = y(t) \quad . \tag{10}$$

The land distributed between crop production and stockbreeding for every region is given by constraints similar to equation (4).

The number and capacities of fodder processing plants and the expenses related to the processing of fodder and its transportation to and from the plants are determined in the model. To calculate the total expenses incurred by regional fodder production, the above costs should be added to the costs of the other agricultural production activities. L(t) represents total expenses in a given period, and the criterion used for evaluating the effectiveness of the solutions is

$$L(t) \rightarrow \min \quad . \tag{11}$$

As previously mentioned in the model, the region is divided into several subregions. As a result, there is a danger of a corresponding increase in the size of the model. However, this is avoided by using a recursive rather than a dynamic approach to problem solution. To avoid the regular distribution of a small volume of production among regions during the first years of the period under analysis and the unremunerative distribution of the rapidly increasing production during the later years, the solution is made taking a negative step (one step) in time beginning from the last year of the time horizon (i.e., the fifteenth year). An algorithm that formulates the constraints for the next step on the basis of the results from the preceding one has been constructed.

DATA PREPARATION MODELS

The linear dynamic model of agricultural production and the land allocation model assume that highly developed technologies are used for crop growing, stockbreeding, and forage and food processing. It is necessary to include parameters for many technologies since the choice between alternative development strategies is made within these models. The former model requires averaged indicators for the region as a whole and the latter requires indicators for each subregion.

Data preparation models are therefore included in the model system to provide the technological and economic data for the two models. These data preparation models consist of a combination of specialized models of the various technologies, which are divided into two main classes--crop growing and stockbreeding. They are essentially simulation models that have an optimizational capability, and they function interactively. The results from these models must have a certain order and format for input into the linear dynamic and land allocation models.

Models of crop-growing technologies should be developed for the basic crops of a region, such as maize, barley, wheat, soybeans, sunflowers; it would also be useful to have such models for some secondary crops, such as tobacco, hemp, beans. The models will consist of two parts:

-- submodels of separate crops; and
-- submodels of various technologies (for a given series of operations and activities over time, require certain types of machinery, equipment, and resources).

The submodels may function together or independently.

The main function of these optimization submodels is to provide results on which the choice of the most suitable hybrid or variety of a certain crop may be based. The submodels of crop growing require the data on

-- natural and climatic conditions, such as soil type, average weekly temperatures for day and night, humidity, rainfall, winds, sunshine, possibilities for installing irrigation systems; and

-- the hybrids or varieties available, the soil and climatic conditions and technology they require, and the productivity that can be expected from them.

The hybrid is chosen according to the modeling objective, for example, maximal output, minimal loading over a given period, or high resilience. The constraints on the choice must also be specified, for example, scarcity of resources, labor use, irrigation requirements, harvesting time.

The submodels of technologies should be interactive simulation models, whose results will be used for the construction of various types of technologies. These submodels will require data on

- -- a given hybrid or variety, including its yield possibilities;
- -- machinery systems and the possibilities for combining them;
- -- normatives for resource use (i.e., certain regulations set by the authorities), including the possibilities for substituting one resource for another; and
- -- resource scarcities at certain times.

With a given value for the n-1th parameter (if a technology is characterized by n parameters), this submodel should allow parameter n to be calculated under all technological conditions.

In contrast to the data preparation models of crop-growing technologies, all of which have a similar structure, the stockbreeding models describe each subbranch in detail. The differences between the subbranches occur mainly in relation to the dynamics of herd development and the level of their aggregation.

It has been necessary to construct the models of stockbreeding technology in this way because the commodity dynamics of agricultural land allocation and development greatly increases the dimensions of the models, making them difficult to solve. Nevertheless, it is necessary to include some detail in order to be able to determine the possibilities for exchanging components of animal-feed rations. This may require some nonlinear relationships to be included in linear models, thus creating additional difficulties. The principal parameters in this type of model are

- -- productivity (meat, milk, wool);
- -- possible systems of machines, equipment; and
- -- available resources (labor, natural, financial).

The model should determine

- -- the various combinations of animal-feed components;
- -- economic characteristics and indicators;
- -- necessary additional expenses; and
- -- natural indicators, such as volumes of production.

The main part of the model consists of a description of animal-feed rations for the various livestock types. The use of alternative combinations of feed components is simulated under conditions of different productivity levels. The nutritional requirements may be specified exogenously or the structure of the ration may be fixed within given limits. If optimization of the feed ration is required, the following criteria can be used:

-- minimum of fodder; or

-- minimal use of other resources for given levels of output.

It would be useful to examine the possibilities for interaction between two or more models describing different types of animals.

MODEL FOR OPTIMIZING THE STRUCTURE OF THE PROCESSING INDUSTRY

The results of the linear dynamic models of agricultural production development determine the production structure of the agriculture of the complex, but not of the processing industry. The complex's final products are sold on both the internal and the international market. Since there are considerable fluctuations in the prices of meat and dairy products on the international market, it is necessary to optimize the production structures of meat and milk processing enterprises several times annually.

Optimization of the production structure of the milk, meat, and vegetable and fruit processing industries is required to determine the economic indicators of these enterprises more exactly. In the model for optimizing the production structure of the complex, the indicators are highly aggregated and are based on data taken from a given enterprise with a fixed production structure.

This mathematical model includes the following elements.

-- The production capacities of the enterprises are known (annual optimization allows the capacities of some subdivisions to be increased at expense of others; this will be dealt with more thoroughly in the following section).

-- The number of animals of various breeds in each group, and hence the volume of meat production in the enterprises during the period under analysis, is known.

-- The volume of goods produced is related to the varieties specified in trade agreements both at home and abroad.

-- The composition of every type of product is determined according to national and international standards.

-- Labor and other resources required for meat processing are not considered as constraints but are calculated within the model.

-- For every type of meat, several cuts, to be processed in two or three ways, are specified.

The results of the model should indicate the type and volume of products that have to be produced in order to obtain the maximum net income from sales in accordance with international prices. Because of certain differences in the conditions of sale on various markets (e.g., EEC, Middle East), the products for these markets differ both in type and composition as well as in price. The objective function that accounts for this is

$$J = a(bz+cy) + a_1 dz_1 - M - T \,, \qquad (12)$$

where

a is a coefficient for changing the currency from one market into a currency accepted in another market;

a_1 is a coefficient for changing leva into the required foreign currency;

b is a vector representing the price of 1 ton of meat of a given cut and processed in a given way;

c is a vector representing the price of one animal;

d is a vector representing the prices on the internal market (in which auxiliary costs relating to the sale are included, e.g., for transportation, storage, handling);

z is the volume of meat of a given cut processed in a given way that is sold on various markets;

z_1 is the volume of meat sold on the internal market;

y is the number of animals of a given type sold on a given market;

M are the expenses of materials for the processing industry; and

T are the expenses of labor required for the processing industry.

There are four major groups of constraints in this model. They are given below. The volume of meat processed depends on the capacity of the enterprise. So if we denote the vector representing the annual operational time of the various production lines by S, the matrix representing the processing and production of certain types of meat for the international market by A, and the matrix representing the production of certain types of meat for the internal market by A_1, then

$$Az + A_1 z_1 = S \quad . \tag{13}$$

It is assumed that the number of groups of specialized workers corresponds to the number of sections in the production process. On this basis, a group of labor resource constraints is constructed to calculate the labor required for the production process. The volume of raw materials is calculated from a group of constraints that is disaggregated according to the type of raw materials and the purpose for which it is used. A third group of constraints represents the number of animals to be sold or slaughtered (the numbers are taken from the linear dynamic model). These constraints account for the relation between the number of animals that are bred and the volume of final production that can be expected (see Figure 2). The fourth group of constraints is related to the material and labor costs for processing. The models for optimizing the production structure in the other processing industries contain similar constraints, assumptions, and objective functions.

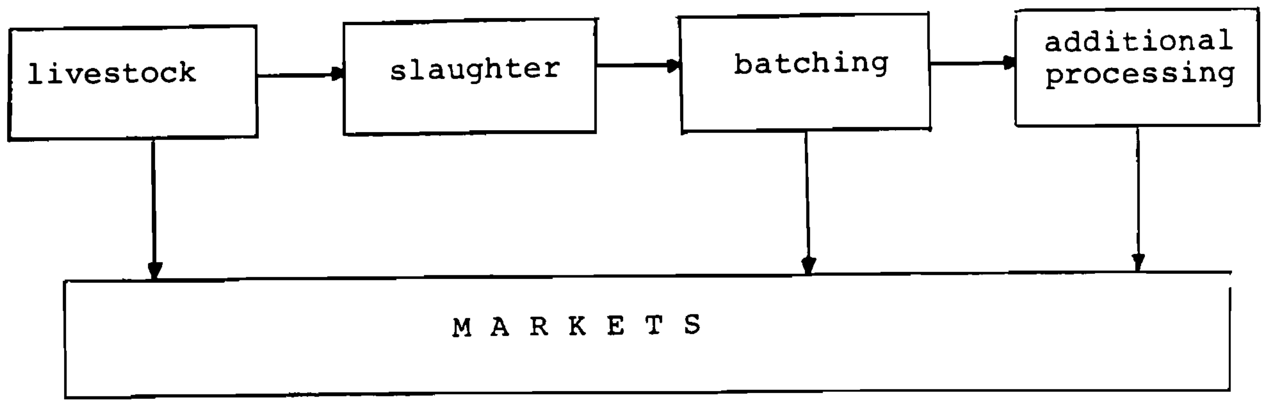

Figure 2. The links between livestock raising and output of livestock products.

OPTIMIZATION MODEL FOR ANNUAL PLANNING IN THE COMPLEX

The model for optimizing the production structure of the complex in a given year is directly linked to all the models described above, with the exception of the data preparation models (see Figure 1). Its purpose is to tailor the results of the linear dynamic model for optimizing the production structure over a number of years to meet the requirements of the annual plan for the given year.

As a result of changes in some conditions during the running of the linear dynamic model, there are changes in the production structure and volume of this model. The prices of the raw materials purchased by the complex and the agricultural goods produced are affected accordingly.

This alters the initial position from which the complex undertakes production activities during a given year. Taking into account the level of the previous year's production and the principal changes in the production structure as determined by the linear dynamic model, the model for optimizing the production structure for the next 1-year period is formulated. This model is similar to the linear dynamic model in form (every block corresponds to one year) and in its internal structure (it contains submodels for crop growing, stockbreeding, and so on).

MANAGEMENT MODEL SYSTEM

The most detailed results from all the models are input into the group of models for short-term management (planning). These are postoptimizational models, which are more helpful for agricultural planning than more aggregated models. This system includes models for optimizing tractor stocks, animal-feed, and the transportation network.

LINKAGE OF THE MODELS

Additional software has been developed to link the models and to transform this model system into a tool for the decision maker. The software will

-- reorganize the output of one model for input into the other models of the system; and

-- process the results of these models to give the results a more convenient format.

In addition, the software will be used to integrate the models with the data base. Its development has been necessary for three reasons, which are explained below.

The need to reduce the size of each model should lead to elimination of all links of secondary importance. However, the specialists, in analyzing the solutions, require detailed information about the characteristics of certain cases. A compromise on the degree of detail to be included in the model may, therefore, have to be worked out.

The standard form in which results are obtained from the computer could be inconvenient or of no value to the decision maker. Therefore, these results have to be transformed into a suitable form--tabular, textual, graphic. The information may be processed so that it can be presented in different ways to coincide with the professional interests of the user.

If a variety of time periods are used, each indicator must be aggregated and disaggregated. Here we have the problem of choosing the method of approximate aggregation and disaggregation.

CONCLUSIONS

Although in its final form, the system of models will be a useful tool for the planning and management of agricultural production in the Silistra region, it cannot solve all the problems. In defining the development strategy for agricultural production, it is assumed that the general direction of the development of regional production is predetermined, since the management hierarchy predetermines specialization within each region. However, beyond the scope of the proposed system of models to solve the problems of specialization within the region, although some advice about this problem can be offered.

In modeling the development of agricultural production at a national level, the potential of the system may be revealed. This would allow for an optimal structure to be found for the agricultural production and processing industries over a long period. Without such a modeling activity, it may be possible to determine agricultural specialization in different regions using informal procedures. A future task will be to search for a way to link modeling of agricultural production development on a national and a regional level.

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MODELING LAND ALLOCATION FOR THE AGRICULTURAL PRODUCTION AND PROCESSING INDUSTRIES OF THE SILISTRA REGION

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The model presented in this paper forms part of a system of models designed to help solve problems of regional agricultural development, especially in relation to the Silistra region in Bulgaria. Its purpose is to determine the most effective land allocation for the agricultural production and processing industries of a region. These activities comprise crop production, stockbreeding, the forage and meat and milk processing industries, and the fruit and vegetable canning industries.

The volume of production from these industries is assumed to be prescribed by the long-term regional agricultural production plan (Gavrilov et al. 1979). The current costs of production, transportation, and related capital investment should be kept to a minimum by the most efficient use of the resources of each subregion (the region is divided into 10 subregions for the purpose of the analysis). Land allocation is not planned for one year only but for the period of the development program.

The model, as a tool for analysis and planning, can provide insight into evaluating procedures for model development. It is solved recursively with a negative step in time starting from the final year of the analysis discussed in Gavrilov et al. "Systems of Models for Developing Agricultural Production in a Region", included in these Proceedings. There are several reasons for taking this approach; the most important being that, for a dynamic model, there could be serious computational difficulties if the number of subregions considered is too large and the time horizon (say 15-20 years) is too long. This recursive procedure avoids the occurrence of inconsistencies in the solution. For example, it can be used to find the best location for production at the most developed stage of operation of the agrarian complex and to determine the ways of achieving this location.

As has been mentioned previously, the land allocation model has been developed within the framework of the development program for the Silistra region. Therefore, the first quantitative results of this model will be implemented in the agroindustrial complex "Drastar", which manages the arable land of the region. Most of the general conditions relating to the linear dynamic model maintain their validity for this model (Gavrilov et al. 1979). The linear dynamic model and its relations to the other regional development models in the system are briefly discussed in these Proceedings in Gavrilov et al., "System of Models for Developing the Agricultural Production of a Region". Some of the most important general conditions specified in the model are mentioned below.

1. The region is divided into K subregions and each is indicated by k. The subregions are differentiated according to two criteria: the spatial characteristics of the land (i.e., soil, climate, geography) and the administrative and economic unit. Differentiation based on the first criterion leads to narrow specialization within each subregion. The second allows the model to be used to aid the planning and management process.

   Since the model will be implemented for the Silistra region, where the spatial conditions are relatively uniform, the first approach does not play a significant role. However, the second is most important. The human settlement is the basic subregional unit and, for the purpose of the case study, we take 10 subregions, each corresponding to existing settlements. The land allocation model can be used to clarify problems related to the formation of human settlements.

2. Within each subregion, it is possible to grow a variety of crops; for Silistra these are described in Gavrilov et al. (1979). Although variations in the soil quality are not specified in the model, the general soil, climatic, and geographical conditions must be given. The crops most appropriate to the conditions of each subregion can then be chosen; for example, hemp must be placed in areas near the Danube -- its cultivation would be impossible in the interior of the region. Other crops are subject to similar types of constraints.

3. It is assumed that technologies will be developed for growing various types of crops under specific conditions. The yields and their related costs will differ according to the technology used and also according to the soil, climatic, and geographical conditions.

4. Fruit and vegetable growing is specified exogenously by subregions and years. The resources required for this production (capital investment, machinery, labor, etc.) are also specified exogenously without being optimized.

5. The constraints on the development of stockbreeding in the regions relate to geographical features (e.g., rugged terrain is more suitable for sheep than cattle), to the availability of certain resources not specified in the model, to the qualifications and customs of the population, and to ecological considerations. Thus, the most suitable type of stockbreeding is determined for every subregion.

6. The production capacities for the stockbreeding industry are specified exogenously by years and subregions. The lifespan of the machinery is also specified exogenously.

7. A location has to fulfill many criteria in order to be considered suitable for stockbreeding facilities. For our case, it is assumed that there are several suitable locations available for a variety of agricultural

activities. Every location is characterized in the model by factors such as the capital required for its development and size.

8. The volume of water required for irrigation is not derived from the model. It is prescribed exogenously by subregions and years. The model determines the size and location of the area to be irrigated and the net cost of the water required.

9. The location of factories for processing concentrated fodder is determined exogenously. It should be noted that sunflower and soybean processing is not carried out within the Silistra region.

10. The volume of processed fodder production is determined by the model. Each subregion processes the fodder it requires. Deliveries between subregions are only carried out for unprocessed fodder.

11. It is assumed that there are sufficient labor resources for agriculture at every point in all subregions, but the structure of the labor force must be solved in the model.

12. Given certain constraints, it is assumed that labor can be temporarily transferred from one subregion to another.

13. At peak periods (mainly during harvesting) part-time workers from other sectors can be hired to supplement the agricultural labor force. Unskilled workers are chosen for this seasonal work so that the increased labor costs can be kept to a minimum.

14. The costs represented in the model include not only the production costs of fodder but also transporation costs and related capital investment. Capital investment depends on the date of redemption, the volume of output, and additional capital investment. This last factor is dependent on the features of the location with respect to auxiliary facilities such as water supply and electricity installation.

The constraints included in the land allocation model are given below.

For the land by subregions:

$$\sum_{i=1}^{n} (x_{ik}^{1} + x_{ik}^{2}) = X_k, \quad k = 1,2,..,K \quad , \tag{1}$$

$$\sum_{i=1}^{n} x_{ik}^{1} \leq X_k^{1}, \qquad k = 1,2,...,K \quad , \tag{2}$$

where

x_{ik}^1 is the area of irrigated land occupied by crop i in subregion k;

x_{ik}^2 is the area of nonirrigated land occupied by crop i in subregion k;

X_k is the total area of arable land in subregion k; and

X_k^1 is the total area of irrigated land in subregion k.

For crop rotation by subregions:

$$\underline{b} \sum_{i \in I_1}^{n} x_{ik}^{1} - \sum_{i \in I_2} x_{ik}^{1} \leq 0, \quad k = 1,2,...,K, \tag{3}$$

$$\sum_{i \in I_2} x_{ik}^{2} - \bar{b} \sum_{i \in I_1} x_{ik}^{1} \leq 0, \quad k = 1,2,...,K, \tag{4}$$

where

I_1, I_2 are crops cultivated by hoeing and crops not cultivated by hoeing, respectively; and

$\underline{b}$, $\bar{b}$ are the lower and upper limits of ratios of mixed crops cultivated by hoeing to achieve effective crop rotation (many of the previous studies assume that $\underline{b} = 0.2$ and $\bar{b} = 0.8$, although the values of the parameters are still subject to discussion).

The following constraints are introduced for rotating sunflowers with other crops:

$$5(x^1_{i^*k} + x^2_{i^*k}) \leq X_k, \quad k = 1,2,\ldots,K, \tag{5}$$

$$5x^1_{i^*k} \leq X^1_k, \quad k = 1,2,\ldots,k, \tag{6}$$

where

i^* is the area used for sunflower growing.

For regional crop specialization:

$$0.8\ A^1_i \leq \sum_{k=1}^{K} x^1_{ik} \leq 1.2\ A^1_i, \quad i = 1,2,\ldots,n, \tag{7}$$

$$0.8\ A^2_i \leq \sum_{k=1}^{K} x^2_{ik} \leq 1.2\ A^2_i, \quad i = 1,2,\ldots,n, \tag{8}$$

where

A^1_i, A^2_i are the volumes of irrigated and nonirrigated crops, respectively, planned for crop i for a given region (these quantities are obtained from the solution of the linear dynamic model).

For subregional stockbreeding specialization:

$$\sum_{k=1}^{K} y^{jp(j)}_{s(j)k} = Y^{jp(j)}_{s(j)}, \quad j = 1,2,\ldots,J,\ s(j) = 1,2,\ldots,S_j,\ p(j) = 1,2,\ldots,P_j, \tag{9}$$

$$y^{jp(j)}_{s(j)k} \leq B^{jp(j)}_{s(j)k}, \quad \text{for } j,\ p(j),\ s(j),\ k, \tag{10}$$

$$y^{jp(j)}_{s(j)k} \geq \underline{B}^{jp(j)}_{s(j)k}, \quad \text{for } j,\ p(j),\ s(j),\ k, \tag{11}$$

where

$y^{jp(j)}_{s(j)k}$ is the number of livestock of type j, group s(j), breed p(j), raised in subregion k (livestock type refers to sheep, pigs, poultry, etc.; group refers to specialization within type j, for example, for cattle: cows, heifers;

$Y^{jp(j)}_{s(j)k}$ is the number of livestock of type j, group s(j), breed p(j) to be raised on a given area in subregion k (the numbers are taken from the linear dynamic model solution for all years of the period under analysis;

$B^{jp(j)}_{s(j)k}$ are the constraints related to the different types, groups, and breeds of livestock by regions; and

$\underline{B}^{jp(j)}_{s(j)k}$ is the number of stockbreeding facilities available for a given group of animals.

For the nutritional requirements of the livestock and the production of fodder:

$$a^{1r}_{ik} x^{1}_{ik} + a^{2r}_{ik} x^{2}_{ik} - \sum_{j=1}^{J} \sum_{s(j)=1}^{S_j} \sum_{p(j)=1}^{P_j} b^{jp(j)}_{s(j)ik} Y^{jp(j)}_{s(j)k} - \bar{x}^{r}_{ik}$$

$$- \sum_{\substack{k=1 \\ k_1 \neq k}}^{K} \bar{\bar{x}}^{r}_{ikk_1} + \sum_{\substack{k=1 \\ k_1 \neq k}}^{K} x^{r}_{ik_1k} - \bar{z}^{r}_{ik} = 0, \quad i=1,2,\ldots,n \quad k=1,2,\ldots,K, \quad r=1,2,\ldots,R_i, \qquad (12)$$

where

a^{1r}_{ik} is the volume of fodder of type r obtained from crop i in subregion k on 1 decare of irrigated land;

a^{2r}_{ik} is the volume of fodder of type r obtained from crop i in subregion k on 1 decare of nonirrigated land;

$b^{jp(j)r}_{s(j)ik}$ is the volume of fodder of type r obtained from crop i, consumed by animal type j, group s(j), breed p(j) in subregion k;

$\bar{x}^{r}_{ik}$ is the volume of fodder of type r sold outside the region under analysis;

$\bar{x}^{r}_{ik_1k}$ is the volume of fodder of type r obtained from crop i, sold by subregion k_1 to subregion k; and

$\bar{z}^r_{ik}$ is the stock of fodder of type r.

In addition to constraint (11), other constraints are also valid for some of the variables in the model:

$$\sum_{k=1}^{K} \bar{x}^r_{ik} \geq \bar{X}^r_i, \; i = 1,2,\ldots,n \quad r = 1,2,\ldots,R \,, \tag{13}$$

$$\sum_{k=1}^{K} \bar{z}^r_{ik} = Z^r_i, \; i = 1,2,\ldots,n \quad r = 1,2,\ldots,R_i, \tag{14}$$

where

$\bar{X}^r_i$, Z^r_i are determined by the linear dynamic model and represent the volumes of fodder of type r for sale and for stock, respectively.

The volume of concentrated fodder transported (d) to factory 1 for processing including transportation costs can be expressed as

$$d^r_{ik} = a^{1r}_{ik}\, x^1_{ik} + a^{2r}_{ik}\, x^2_{ik} - \bar{x}^r_{ik} - \bar{z}^r_{ik}, \; r \in [Q], \tag{15}$$

$$d^r_{ik} = \sum_{l=1}^{L} d^{rl}_{ik}, \tag{16}$$

where

[Q] is the set of concentrated fodder.

If

$$c^r_{ik} = \sum_{j=1}^{J} \; \sum_{s(j)=1}^{S_j} \; \sum_{p(j)=1}^{P_j} b^{jp(j)r}_{s(j)ik} \; y^{jp(j)}_{s(j)k} \;, \; r \in [Q], \tag{17}$$

is the volume of concentrated fodder of type r, consumed by the livestock in subregion k, then c^{rl}_{ik} represents the corresponding volume that should be produced in factory 1.

If we introduce the variable $e^r_{ikk_1}$,

$$e^r_{ikk_1} = \sum_{\substack{k_1=1\\k_1\neq k}}^{K} x^r_{ik_1k} - \sum_{\substack{k_1=1\\k_1\neq k}}^{K} \bar{x}^r_{ik_1k}, \tag{18}$$

that is nonnegative when the corresponding quantity of fodder is transported to the subregion and negative when the same quantity of fodder is taken out, then taking into account (15), (16), (17), and (18), equation (12) can be expressed as

$$d^r_{ik} - c^r_{ik} + e^r_{ikk_1} = \emptyset. \tag{19}$$

For the processing capacities for concentrated fodder:

$$\sum_{k=1}^{K} \sum_{i=1}^{n} \sum_{r=1}^{R_i} \sum_{l=1}^{L} d^{rl}_{ik} - \sum_{l=1}^{L} w_l = 0, \tag{20}$$

where

w_l is the capacity of factory l; and

L is the number of factories.

For the labor resources required:

$$\sum_{i=1}^{n} (h^1_{ik} x^1_{ik} + h^2_{ik} x^2_{ik}) + \sum_{j=1}^{J} \sum_{s(j)=1}^{S_j} \sum_{p(j)=1}^{P_j} h^{jp(j)}_{s(j)k} y^{jp(j)}_{s(j)k} +$$

$$\sum_{l=1}^{L} h_l w_l + \sum_{\substack{k_1=1\\k_1\neq k}}^{K} h_{kk_1} - h_k \leq H_k, \quad k = 1,2,\ldots,K, \tag{21}$$

$$h_{kk_1} \leq H_{kk_1}, \tag{22}$$

where

h^1_{ik}, h^2_{ik} are the labor resources required for the cultivation of 1 decare of crops;

$h^{jp(j)}_{s(j)k}$ are the labor resources required for stockbreeding;

h_1 are the labor resources required for processed fodder production;

h_{kk_1} are the labor resources that can be transferred temporarily to subregion k from subregion k_1 and vice versa;

h_k is the labor that can be transferred from industry to agriculture within the subregion; and

H_k are the total labor resources of the subregion required to carry out farming activities.

Constraint (21) refers to peak periods of activity during the year.

The objective function is expressed as

$$\bar{0} = \sum_{j=1}^{J} \sum_{k=1}^{K} (f^1_{ik} x^1_{ik} + f^2_{ik} x^2_{ik}) + \sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{s(j)=1}^{S_j} \sum_{p(j)=1}^{P_j} \tag{23}$$

$$((f^{jp(j)}_{s(j)k} + g^{jp(j)}_{s(j)k}) y^{jp(j)}_{s(j)k} - g^{jp(j)}_{s(j)k} \bar{B}^{jp(j)}_{s(j)k}) +$$

$$\sum_{k=1}^{K} \sum_{i=1}^{n} \sum_{\substack{r=1 \\ r\in[K]}}^{R_i} \sum_{l=1}^{L} (s^{rl}_{ik} d^{rl}_{ik} + \bar{s}^{rl}_{ik} \bar{c}^{rl}_{ik} + \sum_{k=1}^{K} \sum_{k_1=1}^{K} \sum_{i=1}^{n} \sum_{r=1}^{R_i} s^r_{kk_1} \cdot$$

$$(\bar{x}^r_{ik_1k} + x^r_{ik_1k}) + \sum_{k=1}^{K} \sum_{\substack{k_1=1 \\ k_1\neq k}}^{K} s_{kk_1} h_{kk_1} + \sum_{l=1}^{L} (f_l + g_l) w_l$$

$$+ \sum_{k=1}^{K} s_k h_k ,$$

where

f^1_{ik}, f^2_{ik} are the reduced production costs per decare for crop i;

$f^{jp(j)}_{s(j)k}$ are the production costs for livestock;

f_1 are the production costs for the fodder industry;

$g^{jp(j)}_{s(j)k}$ is the reduced additional capital investment for constructing cattle-breeding and processing facilities for the fodder industry;

$s^{rl}_{ik}, \bar{s}^{rl}_{ik}$ are the costs for transporting concentrated fodder from subregion k to factory l for processing and from factory l to subregion k to subregion k_1 for consumption, respectively;

$s^r_{ikk_1}$ are the costs for transporting a given amount of fodder from subregion k to subregion k_1;

s_{kk_1} are the costs for transporting labor from subregion k to subregion k_1; and

s_k are the costs related to the hiring of temporary labor from other subregions.

The problem is to find out:

$$\bar{0} \rightarrow \min,$$

taking into account constraints (1) - (22). At present, the model contains approximately 550 constraints and 600 variables. Some information problems relating to the technological indicators still remain.

As has already been mentioned, the model will be implemented in the Silistra region. Figure 1 presents a general block diagram of the implementation process. The development of this model represents an important stage in the development of the model system. By linking the linear dynamic model (the first model to be developed) to the land allocation model, it will be possible to fulfill some of the requirements of regional management and planning.

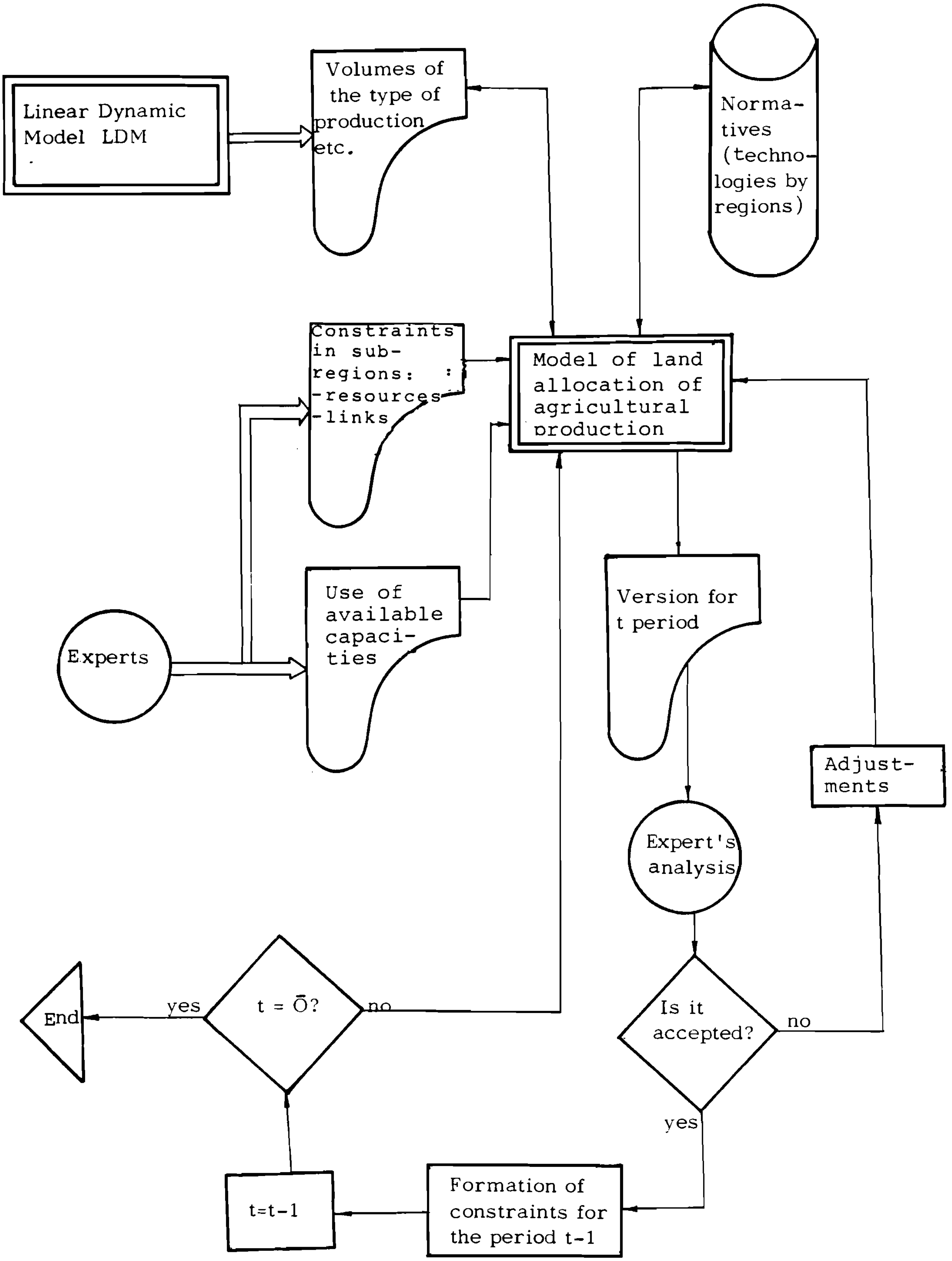

Figure 1. Implementation of the land allocation model.

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Gavrilov, G., S. Stoykov, C. Milenkov, and A. Kehayov (1979) Modelirane Razvitieto na Proizvodstvoto v Agropromishlenia Komplex Drastar (Modeling the Development of Production in the Agroindustrial Complex Drastar). Sofia: Institut za Socialno Upravlenie (in Bulgarian).

IMPLEMENTATION OF THE GENERALIZED REGIONAL AGRICULTURE MODEL (GRAM) FOR THE UPPER NOTEC REGION

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A. Straszak

INTRODUCTION

The implementation of the Generalized Regional Agriculture Model (GRAM) is discussed in this paper. The first version of GRAM was developed in 1978 and, at that stage, the possibility of implementing the solutions for the Upper Notec region case study was considered. A thorough analysis of GRAM indicated that the model is an excellent tool with which to obtain much valuable information about issues of importance to regional and subregional authorities. Modifications were proposed in order to adapt the model to the economic and agricultural conditions specific to the Upper Notec region, while maintaining its generality; these were incorporated into the basic version of GRAM (Albegov 1979), which has finally been implemented.

The paper consists of four main sections. First some general considerations about the role of GRAM and its links with the other models of the Upper Notec region are discussed. The implemented version of GRAM is then briefly described and basic data and results are presented. The final section contains some concluding remarks about the future development of the model.

THE ROLE OF THE MODEL

The model is primarily intended to determine regional agricultural specialization, i.e., the structure of crop and livestock production in disaggregated form (distributed among subregions, sectors, land qualities, technologies, etc.). In addition, basic financial and resource flows are assessed. The model operates under specified natural conditions and a given agricultural policy. This policy, which may be considered at the subregional and regional level, determines some prices (mainly on the internal state market) and the distribution of resources such as capital investment, fertilizers, and water. It is important to note that the policy-making structure is explicitly represented in the model, mainly in the subregional structure.

An in-depth analysis of the impact of a given agricultural policy may principally be made for a short (say,one-year) planning horizon. For analysis over the long-term (say, 5 years) the fluctuations of important coefficients, such as prices on domestic and world markets, make use of this model less preferable.

The model can also reveal the points in which there are critical resource needs; e.g., it may indicate a water supply system to which additional water flows should be directed.

Application of the model can extend beyond the regional level to the national level, with a division of the country into regions. Thus, both regional and national agricultural specialization may be determined.

LINKAGE OF GRAM TO THE OTHER MODELS OF THE UPPER NOTEC REGION

The place of GRAM in a system of general models for regional development is described in detail by Albegov (1979); hence, it is not mentioned here. However, since the models developed for particular regions may differ to some extent from the general scheme, we briefly show how GRAM is linked to the other models developed for the Upper Notec case study.

The main links between GRAM and the other models of the system are shown in Figure 1 (the other models are described in these Proceedings). However, the functions of GRAM and the models developed by Podkaminer et al. and Makowski to some extent overlap.

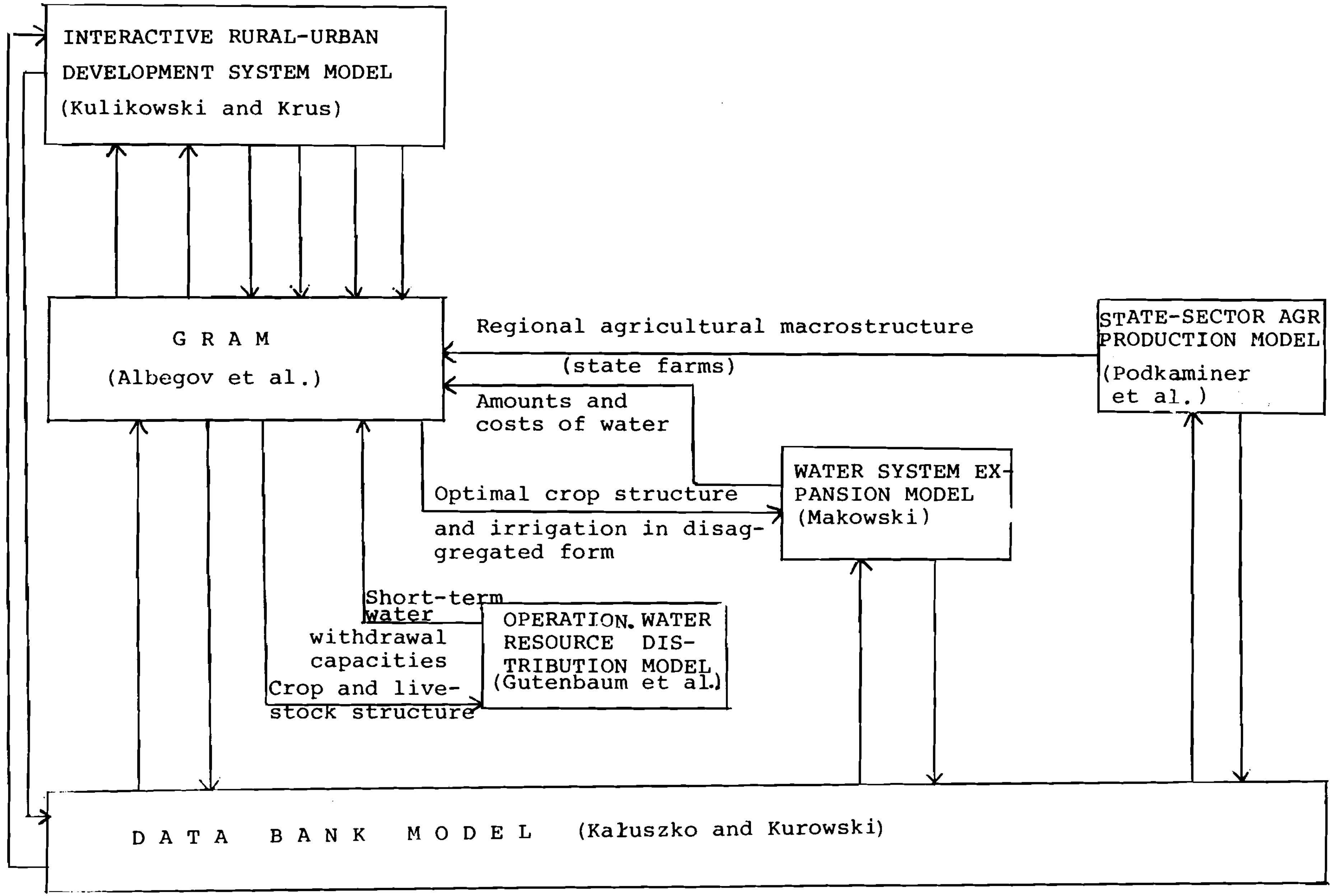

Figure 1. The main links between GRAM and the other models of the Upper Notec region (papers in which these other models are discussed are included in these Proceedings).

DESCRIPTION OF GRAM

The model description presented in this section includes definitions of: the indices, coefficients, decision variables, constraints, and objective functions.

Indices

The values and meanings of the indices used in the model for the Upper Notec case study are given below.

α is type of land differentiated according to soil quality (fertility):

$\alpha = 1$ -- weak (W),
$\alpha = 2$ -- medium minus (M),
$\alpha = 3$ -- medium plus (P),
$\alpha = 4$ -- good (G);

f is type of fertilizer used:

$f = 1$ -- nitrate (N),
$f = 2$ -- phosphate (P),
$f = 3$ -- potassium (K),
$f = 4$ -- calcium (Ca);

i is crop type:

$i = 1$ -- wheat,
$i = 2$ -- rye,
$i = 3$ -- barley,
$i = 4$ -- oats,
$i = 5$ -- other grains,
$i = 6$ -- sugar beet,
$i = 7$ -- potatoes,
$i = 8$ -- maize,
$i = 9$ -- carrots & forage beets,
$i = 10$ -- beans, etc.,
$i = 11$ -- clover, etc.,
$i = 12$ -- flax, etc.,
$i = 13$ -- meadows and pastures;

ω is the index of crop-rotation group I^{ω} for the above set of crops, $I = \{1,2,\ldots,19\}$:

$I^1 = \{1,2,5\}$,
$I^2 = \{3,4,12\}$,
$I^3 = \{7,8\}$,
$I^4 = \{6,11\}$,
$I^5 = \{9,10\}$,
$I^6 = \{13\}$;

β_i is the second crop, the best (or only) successor of the first crop i.

j is livestock type:

j = 1 -- cattle (milk),	j = 4 -- pigs,
j = 2 -- cattle (beef),	j = 5 -- horses,
j = 3 -- sows,	j = 6 -- sheep,

j = 7 -- poultry;

k is livestock-breeding specialization for meat, wool, leather, eggs, etc.): it is assumed initially that k = 1;

l is type of market on which a particular commodity is sold (purchased):

l = 1 -- internal state market,
l = 2 -- internal private market,
l = 3 -- external (export) market;

m is livestock product type:

m = 1 -- meat,	m = 3 -- milk,
m = 2 -- leather,	m = 4 -- eggs,

m = 5 -- wool;

n are feed components:

n = 1 -- nutrition units,	n = 6 -- preserved forage,
n = 2 -- proteins,	n = 7 -- grains stalks,
n = 3 -- dry mass,	n = 8 -- starchy roots,
n = 4 -- green mass,	n = 9 -- potatoes,
n = 5 -- hay,	n = 10-- other crops,

n = 11 -- milk;

p is type of property ownership

p = 1 -- state farm,
p = 2 -- collective farm,
p = 3 -- private farm;

s is type of crop-production technology:

s = 1 -- relatively efficient current technology (P),
s = 2 -- modern, highly efficient technology with high fertilizer use without irrigation (F),
s = 3 -- modern, highly efficient technology with high fertilizer use with irrigation (R);

s' is type of livestock- and poultry-breeding technology: in the study, $s'_1 = 1$;

r is the subregion corresponding to an administrative (voivodship) or soil quality division:

r = 1 -- Bydgoszcz (B)
r = 2 -- Włocławek (W)
r = 3 -- Konin (K);

Coefficients

The following coefficients are included:

$a_{fiprs\alpha}$ is fertilizer required to produce one unit of crop i on land α on property p in subregion r, using technology s;

$\hat{a}_{fjk}$ is manure obtained from one unit of livestock j of specialization k, expressed in units of fertilizer f;

$b_{iprs\alpha}$ is labor required to produce one unit of crop i on land α on property p in subregion r, using technology s;

$b_{jkprs'}$ is labor required to produce one unit of livestock j of specialization k on property p in subregion r, using technology s';

$c_{iprs\alpha}$ is capital investment required to produce one unit of crop i on land α on property p in subregion r, using technology s;

$\bar{c}_{iprs\alpha}$ is additional capital investment required to produce one unit of crop i on land α on property p in subregion r, when technology s is used for land improvement;

$c_{jkprs'}$ is capital investment required to produce one unit of livestock j of specialization k on property p in subregion r, using technology s';

$d_{iprs\alpha}$ is annual water supply required to produce one unit of crop i on land α on property p in subregion r using technology s;

$\hat{d}_{iprs\alpha}$ is water demand at peak periods to produce one unit of crop i on land α on property p in subregion r, using technology s;

$d_{jkprs'}$ is annual water supply required to produce one unit of livestock j on property p in subregion r, using technology s';

$\hat{d}_{jkprs'}$ is water demand at peak periods to produce one unit of livestock j of specialization k on property p in subregion r, using technology s';

$e_{iprs\alpha}$ is machinery required to produce one unit of crop i on land α on property p in subregion r, using technology s;

$\left.\begin{matrix} f^{max}_{njk} \\ f^{min}_{njk} \end{matrix}\right\}$ are maximum and minimum demands for feed component n to produce one unit of livestock j of specialization k;

g_{in} is content of feed component n required to produce one unit of crop i;

g_{mn} is content of feed component n required to produce one unit of livestock product m;

$h_{mjkps'}$ is output of livestock product m per unit of livestock j of specialization k bred on property p, using technology s';

n_i is number of nutrition units in one unit of crop i;

n_{mjk} is number of nutrition units in one unit of livestock product m obtained from livestock j of specialization k;

$u_{iprs\alpha}$ is average yield of crop i on property p in subregion r from one unit of land α, using technology s;

$u^1_{iprs\alpha}$ is average yield of second crop i on property p in subregion r from one unit of land α, using technology s;

B is maximum amount of labor available in the whole region;

B_{pr} is maximum amount of labor available on property p in subregion r;

C is total amount of external and internal capital investment available in the whole region;

C_{pr} is total amount of external and internal capital investment available on property p in subregion r;

D	is maximum annual water supply available for the whole region;
$\hat{D}$	is maximum water supply available at peak periods for the whole region;
D_{pr}	is maximum annual water supply available on property p in subregion r;
$\hat{D}_{pr}$	is maximum water supply available at peak periods on property p in subregion r;
E	is maximum machinery available in the whole region;
F_i^{min}, F_i^{max}	are minimum and maximum consumption levels of crop i in the whole region;
F_m^{min}, F_m^{max}	are minimum and maximum consumption levels of livestock product m in the whole region;
F_{ipr}^{min}, F_{ipr}^{max}	are minimum and maximum production of crop i on property p in subregion r;
G_f	is maximum volume of fertilizer f available in the whole region;
G_{fpr}	is maximum volume of fertilizer f available on property p in subregion r;
H_{il}	is maximum volume of purchase of crop i on market l for livestock consumption;
I_{il}	is maximum volume of purchase of crop i on market l for human consumption;
I_{ml}	is maximum volume of external purchases of livestock product m on market l for human consumption;
$\bar{I}_{il}$	is sale limitation of crop i on market l;
$\bar{I}_{ml}$	is sale limitation of livestock product m on market l;
$L_{\omega pr}^{max}$, $L_{\omega pr}^{min}$	are maximum and minimum areas of land that, in accordance with crop rotation, could be used for growing crops of group ω on property p in subregion r;

$L^{min}_{pr\alpha}$, $L^{max}_{pr\alpha}$	are minimum and maximum areas of useable land α on property p in subregion r;
$L^{min}_{prs\alpha}$, $L^{max}_{prs\alpha}$	are minimum and maximum areas of land α on property p in subregion r that can be improved with the use of technology s (irrigated, drained, etc.);
L^{m}_{pr}	is area of meadows and pastures on property p in subregion r;
L_{pr}	is maximum possible arable land use on property p in subregion r;
M^{min}_{jpr}, M^{max}_{jpr}	is minimum and maximum possible production of livestock j on property p in subregion r;
P^{l}_{i}	is unit price of home-produced crop i on market l;
P^{l}_{m}	is unit price of home-produced livestock product m on market l;
P^{imp}_{il}	is unit price on crop i purchased for forage on market l;
$\bar{P}^{imp}_{il}$	is unit price of crop i purchased for human consumption on market l;
$\bar{P}^{imp}_{ml}$	is unit price of livestock product m purchased for human consumption on market l;
$S_{iprs\alpha}$	is production cost of one unit of crop i on land α on property p in subregion r, using technology s;
$S_{jkprs'}$	is maintenance cost for one unit of livestock j of specialization k on property p in subregion r, using technology s' (excluding feeding costs);
W_{p}	is minimum wage on property p.

Decision Variables

The model contains the following decision variables.

P_{iprl} is purchase of crop i for forage on market l by property p in subregion r;

Q_{iprl} is purchase of crop i for human consumption on market l by property p in subregion r;

Q_{mprl} is purchase of livestock product m for human consumption on market l by property p in subregion r;

R_{iprl} is sale of crop i on market l by property p in subregion r;

R_{mprl} is sale of livestock product m on market l by property p in subregion r;

W_{ipr} is consumption of crop i on property p in subregion r;

W_{mpr} is human consumption of livestock product m on property p in subregion r;

$X_{iprs\alpha}$ is volume of first harvest of crop i on land α on property p in subregion r, using technology s;

$X_{jkprs'}$ is number of livestock j of specialization k on property p in subregion r, using technology s';

$Y_{iprs\alpha}$ is volume of second harvest of crop i on land α on property p in subregion r, using technology s;

Z_{ipr} is consumption of crop i on property p in subregion r;

Z_{mpr} is livestock consumption of livestock product m on property p in subregion r.

Constraints

The constraints are grouped according to land use, production and feed balances, resources, etc.

Land-Use Constraints

The following constraints related to the different aspects of land use are introduced.

a. The availability of arable land is expressed as

$$\sum_{\substack{s,\alpha \\ i\in I-I^6}} \frac{X_{iprs\alpha}}{u_{iprs\alpha}} \leq L_{pr} \quad , \tag{1}$$

for all p, r. Full utilization of land is assumed; in general, '=' may also be replaced by "≤".

b. The availability of land of a particular quality is expressed as

$$L_{pr\alpha}^{min} \leq \sum_{i,s} \frac{X_{iprs\alpha}}{u_{iprs\alpha}} \leq L_{pr\alpha}^{max} \quad , \tag{2}$$

for all p,r,α.

c. The minimum and maximum areas of land occupied by crops from groups $I^1,\ldots,I^5$ in accordance with the limitations imposed by crop rotation are expressed as

$$L_{\omega pr}^{min} \leq \sum_{s,\alpha} \frac{X_{iprs\alpha}}{u_{iprs\alpha}} \leq L_{\omega pr}^{max} \quad , \tag{3}$$

for all p, r, ω = 1,...,5.

d. The minimum and maximum areas of land that can be improved (e.g., by irrigation) are expressed as

$$L_{prs\alpha}^{min} \leq \sum_{i} \frac{X_{iprs\alpha}}{u_{iprs\alpha}} \leq L_{prs\alpha}^{max} \quad , \tag{4}$$

for all α, p, r, s = 2, 3.

e. The availability of pastures and meadows is expressed as

$$\sum_{\substack{s,\alpha \\ i\in I^6}} \frac{X_{iprs\alpha}}{u_{iprs\alpha}} \leq L_{pr}^{m} \quad , \tag{5}$$

for all p, r.

Livestock Product and Crop Balances

a. The livestock product balance is expressed as

$$\sum_{j,k,s'} h_{mjkps'} X_{jkprs'} - W_{mpr} - Z_{mpr} - \sum_{l} R_{mprl} = 0 \quad , \quad (6)$$

for all m, p, r.

b. The crop balance is expressed as

$$\sum_{s,a} (X_{iprs\alpha} + Y_{iprs\alpha}) - W_{ipr} - Z_{ipr} - \sum_{l} R_{iprl} = 0 \quad , \quad (7)$$

c. The balance of first and second harvests is expressed as

$$\sum_{s,\alpha} \frac{X_{iprs\alpha}}{u_{iprs\alpha}} - \sum_{\substack{s,\alpha \\ i\in\beta_i}} \frac{Y_{iprs\alpha}}{u^{1}_{iprs\alpha}} \geq 0 \quad , \quad (8)$$

for all i, p, r.

Feed Balances

Feed components n are balanced at a regional level for state farms and at a subregional level for collective and private farms.

a. The feed balance for state farms is expressed as

$$\sum_{j,k,r,s'} f^{min}_{njk} X_{jkprs'} \leq \sum_{i,r} g_{in} Z_{ipr} + \sum_{i,r,l} g_{in} P_{iprl}$$

$$+ \sum_{m,r} g_{mn} Z_{mpr} \leq \sum_{j,k,r,s'} f^{max}_{njk} X_{jkprs'} \quad , \quad (9)$$

for all n, p = 1.

b. The feed balance for collective and private farms is expressed as

$$\sum_{j,k,s'} f^{min}_{njk} X_{jkprs'} \leq \sum_{i} g_{in} Z_{ipr} + \sum_{i,l} g_{in} P_{iprl}$$

$$+ \sum_{m} g_{mn} Z_{mpr} \leq \sum_{j,k,r,s'} f^{max}_{njk} X_{jkprs'} \quad (10)$$

for all n, r, p = 2,3.

Minimum Requirements for the Production of Crops and Livestock Products

a. The minimum requirement for crop production is expressed as

$$F^{min}_{i} \leq \sum_{p,r} W_{ipr} + \sum_{p,r,l} Q_{iprl} \leq F^{max}_{i} \quad , \quad (11)$$

for all i.

b. The minimum requirement for livestock products is expressed as

$$F^{min}_{m} \leq \sum_{p,r} W_{mpr} + \sum_{p,r,l} Q_{mprl} \leq F^{max}_{m} \quad , \quad (12)$$

for all m.

Production Limits for Crop and Livestock Products

a. The production limit for crops is expressed as

$$F^{min}_{ipr} \leq \sum_{s,\alpha} (X_{iprs\alpha} + Y_{iprs\alpha}) \leq F^{max}_{ipr} \quad , \quad (13)$$

for all i, p, r.

b. The production limit for livestock products is expressed as

$$M_{jpr}^{min} \leq \sum_{k,s'} X_{jkprs'} \leq M_{jpr}^{max} \quad , \tag{14}$$

for all j, p, r.

Resource Constraints

a. Labor at a subregional level is expressed as

$$\sum_{i,s,\alpha} b_{iprs\alpha} (X_{iprs\alpha} + Y_{iprs\alpha}) + \sum_{j,k,s'} b_{jkprs'} X_{jkprs'}$$

$$\leq B_{pr} \quad , \tag{15}$$

for all p, r.

b. Labor at a regional level is expressed as

$$\sum_{i,p,r,s,\alpha} b_{iprs\alpha} (X_{iprs\alpha} + Y_{iprs\alpha}) + \sum_{j,k,p,r,s'} b_{jkprs'}$$

$$X_{jkprs'} \leq B \quad . \tag{16}$$

c. Total water consumption at a subregional level is expressed as

$$\sum_{i,s,\alpha} d_{iprs\alpha} (X_{iprs\alpha} + Y_{iprs\alpha}) + \sum_{j,k,s'} d_{jkprs'} X_{jkprs'}$$

$$\leq D_{pr} \quad , \tag{17}$$

for all p, r.

d. Peak-period water consumption at a subregional level is expressed as

$$\sum_{i,s,\alpha} \hat{d}_{iprs\alpha} (X_{iprs\alpha} + Y_{iprs\alpha}) + \sum_{j,k,s'} \hat{d}_{jkprs'} X_{jkprs'}$$

$$\leq \hat{D}_{pr} \quad , \tag{18}$$

for all p, r.

e. Total water consumption at a regional level is expressed as

$$\sum_{i,p,r,s,\alpha} d_{iprs\alpha} (X_{iprs\alpha} + Y_{iprs\alpha}) + \sum_{j,k,p,r,s'} d_{jkprs'}$$

$$X_{jkprs'} \leq D . \tag{19}$$

f. Peak-period water consumption at a regional level is expressed as

$$\sum_{i,p,r,s,\alpha} \hat{d}_{iprs\alpha} (X_{iprs\alpha} + Y_{iprs\alpha}) + \sum_{j,k,p,r,s'} \hat{d}_{jkprs'}$$

$$X_{jkprs'} \leq \hat{D} . \tag{20}$$

g. Machinery supplies (and horses) at a regional level are expressed as

$$\sum_{i,p,r,s,\alpha} e_{iprs\alpha} (X_{iprs\alpha} + Y_{iprs\alpha}) \leq E . \tag{21}$$

h. Fertilizer supplies at a subregional level for collective and private farms are expressed as

$$\sum_{i,s,\alpha} a_{fiprs\alpha} (X_{iprs\alpha} + Y_{iprs\alpha}) - \sum_{j,k,s'} \hat{a}_{fjks'} X_{jkprs'}$$

$$\leq G_{fpr} , \tag{22}$$

for all r, f, p = 2, 3.

i. Fertilizer supplies at a regional level are expressed as

$$\sum_{i,p,r,s,\alpha} a_{fiprs\alpha} (X_{iprs\alpha} + Y_{iprs\alpha}) - \sum_{j,k,p,r,s'} \hat{a}_{fjks'}$$

$$X_{jkprs'} \leq G_f , \tag{23}$$

for all f.

Purchase and Sale Limits

a. The purchase of crops for livestock consumption is expressed as

$$\sum_{p,r} P_{iprl} \leq H_{il} \quad , \tag{24}$$

for all l, $i \in \bigcup_{\omega=1}^{5} I^{\omega}$.

b. The purchase of crops for human consumption is expressed as

$$\sum_{p,r} Q_{iprl} \leq I_{il} \quad , \tag{25}$$

for all l, $i \in \bigcup_{\omega=1}^{5} I^{\omega}$.

c. The purchase of livestock products is expressed as

$$\sum_{p,r} Q_{mprl} \leq I_{ml} \quad , \tag{26}$$

for all m,l .

d. The sale of crops is expressed as

$$\sum_{p,r} R_{iprl} \leq \bar{I}_{il} \quad , \tag{27}$$

for all l, $i \in \bigcup_{\omega=1}^{5} I^{\omega}$.

e. The sale of livestock products is expressed as

$$\sum_{p,r} R_{mprl} \leq \bar{I}_{ml} \quad , \tag{28}$$

for all m,l.

Financial Constraints

a. Capital investment availability at a subregional level is expressed as

$$\sum_{i,s,\alpha} c_{iprs\alpha} X_{iprs\alpha} + \sum_{i,s,\alpha} \bar{c}_{iprs\alpha} X_{iprs\alpha} + \sum_{j,k,s'} c_{jkprs'} X_{jkprs'} \leq C_{pr} , \qquad (29)$$

for all r, p.

b. Capital investment availability at a regional level is expressed as

$$\sum_{i,p,r,s,\alpha} c_{iprs\alpha} X_{iprs\alpha} + \sum_{i,p,r,s,\alpha} \bar{c}_{iprs\alpha} X_{iprs\alpha} + \sum_{j,k,p,r,s'} c_{jkprs'} X_{jkprs'} \leq C . \qquad (30)$$

c. The minimum income of collective and private farms at a subregional level is expressed as

$$\sum_{i,l} P_i^l R_{iprl} + \sum_{m,l} P_m^l R_{mprl} - \sum_{i,s,\alpha} S_{iprs\alpha} X_{iprs\alpha} - \sum_{j,k,s'} S_{jkprs'} X_{jkprs'} - \sum_{i,l} P_{il}^{imp} P_{iprl} - \sum_{i,l} \bar{P}_{il}^{imp} Q_{iprl} - \sum_{m,l} P_{ml}^{imp} Q_{mprl} \geq W_p B_{pr} ,$$

for all r, p = 2, 3. (31)

Objective Functions

The objective functions applied are related to regional output, which may be expressed in monetary and nonmonetary terms.

In implementing the model, six objective functions, all to be maximized, were proposed.

Regional profit is expressed as

$$\begin{aligned} & \sum_{i,p,r,l} R_{iprl}\, P_i^l \\ & - \sum_{i,p,r,s,\alpha} S_{iprs\alpha}\, X_{iprs\alpha} \\ & + \sum_{m,p,r} R_{mprl}\, P_m^l \\ & - \sum_{j,k,p,r,s'} S_{jkprs'}\, X_{jkprs'} \\ & - \sum_{i,p,r,l} P_{iprl}\, P_{il}^{imp} - \sum_{i,p,r,l} Q_{iprl}\, \bar{P}_{il}^{imp} \\ & - \sum_{m,p,r,l} Q_{mprl}\, P_{ml}^{imp} \,. \end{aligned} \tag{32}$$

Regional agricultural production in monetary terms is expressed as

$$\begin{aligned} & \sum_{i,p,r,s,\alpha} X_{iprs\alpha}\, P_i^1 + \sum_{m,j,k,p,r,s'} h_{mjkps'}\, X_{jkprs'}\, P_m^1 \\ & - \sum_{i,p,r,l} P_{iprl}\, P_{il}^{imp} \\ & - \sum_{i,p,r,l} Q_{iprl}\, \bar{P}_{il}^{imp} \\ & - \sum_{m,p,r,l} Q_{mprl}\, P_{ml}^{imp} \,. \end{aligned} \tag{33}$$

Regional agricultural production in nutrition units is expressed as

$$\sum_{i,p,r,s,\alpha} n_i \, [X_{iprs\alpha} - \sum_{l} (P_{iprl} + Q_{iprl})]$$
$$- \sum_{m,j,k,p,r,s'} n_{mjk} \, (h_{mjkps'} \, X_{jkprs'} - \sum_{l} Q_{mprl}) \quad . \tag{34}$$

Production of livestock products in monetary terms is expressed as

$$\sum_{m,p,r,l} R_{mprl} \, P_m^l - \sum_{m,p,r,l} Q_{mprl} \, P_{ml}^{imp}$$
$$- \sum_{i,p,r,l} P_{iprl} \, P_{il}^{imp} \quad . \tag{35}$$

Production of livestock products in nutrition units is expressed as

$$\sum_{m,j,k,p,r,s'} n_{mjk} \, (h_{mjkps'} \, X_{jkprs'} - \sum_{l} Q_{mprl}) \quad . \tag{36}$$

Export production in monetary terms is expressed as

$$\sum_{i,p,r} (R_{ipr3} \, P_i^3 - Q_{ipr3} \, \bar{P}_{i3}^{imp} - P_{ipr3} \, P_{i3}^{imp} \tag{37}$$
$$+ \sum_{m,p,r} (R_{mpr3} \, P_m^3 - Q_{mpr3} \, P_{m3}^{imp}) \quad .$$

The choice of the objective function is very important because of its effect on the results obtained. It, therefore, requires a detailed analysis of the intentions of policy-making authorities and of the relevance of a specific goal for the region under analysis.

BRIEF PRESENTATION OF NUMERICAL RESULTS

For implementation, the sets of indices, i.e., the types of crop, livestock, etc., were as given above. This resulted in a linear programming problem of 3,438 columns (variables) and 968 rows (constraints and objective function). The first objective function, i.e., regional profit (32), was applied.

Since a large amount of data was used, it is not all listed. Only land and labor resources are presented to give some indication of the region's size (see Table 1).

Table 1. Land and labor resources.

Land/Labor	Ha/Persons (approx.)
Total area of region --	235,000
in subregions:	
Bydgoskie voivodship[a]	200,000
Włocławskie voivodship[a]	45,000
Koninskie voivodship[a]	50,000
in property types:	
state-owned	75,000
collective	15,000
private	205,000
Labor resources	420,000

[a]Note that Bydgoskie, Włocławskie, and Koninskie are the adjectival forms of Bydgoszcz, Włocławek, and Konin.

The main results for production of crops, livestock, and livestock products are given below.

a. Crops, in tons (approx):

wheat	95,000
rye	115,000
barley	72,000
oats	35,000
other grains	37,500
sugar beets	710,000
potatoes	715,000
maize	35,000
forage beets	140,000

a. (continued)

beans	42,000
clover	150,000
flax	63,000
products from meadows and pastures	230,000

b. Livestock, in units (approx):

cattle (milk)	140,000
cattle (beef)	360,000
pigs	630,000
horses	38,000
sheep	22,000
poultry	4,300,000

c. Livestock products (approx):

meat (in tons)	95,000
leather (in tons)	6,630
milk (in 10^3 liters)	392,000
eggs (10^3)	170,000
wool (in tons)	90

The pattern of regional agricultural specialization obtained from the model is given briefly in Tables 2 and 3; the dominant subregions, technology types, etc. for a particular crop/livestock type are underlined.

One of the important features of the solution to the livestock specialization problem is the region's self-sufficiency with respect to the agricultural products considered.

The most limiting constraints are those concerning the minimum income requirements for collective and private farms.

Table 2. Crop specialization.

Crop	Subregion	Property type	Technology	Soil quality
Wheat	B, W, K	S	P	W, M, P
Rye	B, W, K	P, C	P	W, M
Barley	B, W, K	S, P	P	W, M, P, G
Oats	B, K	P, S	P	W, P
Other grains	B, K	P, S	P	W, P
Sugar beet	W, B, K	P, C, S	R, P	G
Potatoes	B, W, K	P, C	P, F	M, G
Maize	W, K	S, P, C	P	M, P, G
Forage beets, etc.	W, B, K	P, C	P	G, M
Beans, etc.	B, W, K	C, P	P	P, W, M
Clover, etc.	B, K	S	P	W
Flax, etc.	B, W, K	P, C	P	P, M, G
Meadows, etc.	B, W, K	P, C, S	R, F	G

Table 3. Livestock specialization.

Livestock type	Property type	Subregion
Cattle (milk)	P, C, S	B, K, W
Cattle (beef)	P, C, S	B, K, W
Pigs	P, C	B, K, W
Horses	P	B, W
Sheep	P	K, B
Poultry	P, S, C	B, W, K

CONCLUDING REMARKS

The first results obtained from GRAM confirm the authors' belief that the model is a useful tool for determining not only regional agricultural specialization but also the most important regional resource flows (capital, fertilizers, labor, machinery, water, etc.). The results seem compatible with agricultural theory and practice; in the future, it is only necessary to reconsider some minor issues.

The large size of GRAM implies that difficulties could occur in analyzing the results and the interrelations of agriculture and the other sectors of the regional economy. However, this task was greatly facilitated by the model's interactive mode and clearly defined (reflecting the existing administrative, economic, and other divisions) structure. Use of the SESAME/DATAMAT LP system made it possible to compute the model efficiently and to present the results in an elegant and compact form.

In the future, some mechanisms accounting for the stochastic nature of regional agriculture will be included in GRAM. It is also expected that a multicriteria approach to the solution of the model will be formulated and that a dynamic model based on GRAM will be developed.

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NORMATIVE MODELING OF STATE-SECTOR REGIONAL AGRICULTURAL PRODUCTION UNDER VARIOUS WATER SUPPLY ALTERNATIVES--APPLICATION TO THE UPPER NOTEC REGION

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INTRODUCTION

The problem of determining optimal agricultural production patterns allowing for a limited water supply has been formulated mathematically in several ways (see De Ridder 1974, Dudley et al. 1971, Gisser and Pohoryles 1977, Pomerada 1978, etc.). The approach differs according to whether the emphasis is placed on the agricultural, economic, or hydrological aspects of the problem. The mathematical formulation adopted in this paper is based mainly on the methodology of deterministic linear programming developed by Heady and his associates (see Heady and Dvodskin 1977, Heady and Eyvidson 1973, Heady et al. 1973, Nicol and Heady 1975, etc.).

However, the model presented takes into account the random nature of water demand in specific months of the vegetation period. Thus, the production pattern determined in the model will be feasible in weather conditions deviating from the average.

BASIC ASSUMPTIONS

Linear programming is generally used to determine the set of all feasible agricultural patterns of a region. Moreover, linear programming models tend to treat the whole region as virtually one economic unit and do not consider the possibility of a lack of spatial homogeneity in climate, soil, and production equipment.

Such models are usually static in character. However, the pattern sought is the annual agricultural production structure and the model solution is expected to have long-term significance. The year to which the model applies is assumed to be typical for a longer time period, in which technological and economic conditions are constant. Since the yearly program to be determined is expected to be feasible for both the preceding and following years of the time period, making short-term solutions impossible, this model must differ from a purely static model.

VARIABLES AND CONSTRAINTS

Four groups of variables are included in the model:

-- total quantities of particular animal feed produced;
-- livestock-breeding activities;
-- land-use activities (with specified rotations); and
-- total directly productive water use in certain months of the vegetation period (under the most likely weather conditions).

Land-use activities are divided into three subgroups:

-- rotations including at least one crop requiring irrigation;
-- rotations excluding crops requiring irrigation, but grown on soils suitable for irrigation; and
-- rotations recommended for land unsuitable for irrigation.

The basic soil categories for land-use activities are defined.

Five groups of constraints are introduced:

-- the availability of the basic means of production;
-- the availability of water for direct use in agricultural production in specific months of the vegetation period, under various weather conditions;
-- the production and economic targets set for the region's state-owned farms;
-- the interrelations between activities that guarantee the long-term feasibility of the region's annual program (i.e., reproduction equations for the herd structure and environmental preservation constraints); and
-- the balances of feeds of particular types.

DESCRIPTION OF WATER AVAILABILITY CONSTRAINTS

For the sake of brevity, only the water availability constraints included in the model are given. These constraints reveal some important assumptions that have been adopted. The following notation is used:

x_{ijk} is the area of land of soil category i under rotation j involving at least one crop requiring irrigation and using technology k;

W_r is the level of livestock production activity r;

$q_{ijkt\tau d}$ is the water required directly for productive purposes in activity x_{ijk} in month t under average monthly temperature τ and effective precipitation d;

q_{rt} is the water required directly for productive purposes in activity W_r in month t; and

$Q_{t\tau d}$ is the upper limit of the water supply for directly productive agricultural use in month t under average monthly temperature τ and effective precipitation d.

The model constraints include a subset of the following inequalities:

$$\sum_{ijk} x_{ijk} q_{ijkt\tau d} + \sum_{r} W_r q_{rt} \leq Q_{t\tau d} \quad , \qquad t = 1,2,3,4,5,6, \tag{1}$$

$$\tau = 1,2,\ldots,I_t, \quad d = 1,2,\ldots,I_t \quad .$$

The coefficients q_{rt} are estimated according to livestock-breeding knowledge. To assess the coefficients $q_{ijkt\tau d}$, the Klatt-Press theory on the relationships between optimum rainfall for various crops, average monthly temperatures, and soil texture should be applied.

OBJECTIVE FUNCTION

The problem of determining an objective function for the water-use programs has not been completely solved. There are many aspects of the problem that make simple proposals questionable. Therefore, the objective function adopted in this study--to maximize the volume of net agricultural production expressed in corn equivalent units--is considered to be provisional.

MODEL FOR THE UPPER NOTEC REGION

The simplified model for the Upper Notec region presented above consists of 171 variables and 87 constraints. Among the latter, there are 27 constraints of type (1) reflecting the climatic properties of the Upper Notec region. The specification of the model required that over 5,000 coefficients be assessed. A number of versions of the model were solved, from which information about the influence of the changes in the available means of production upon the optimum production pattern has been obtained. Selected results are presented in the Appendix.

LINKAGE OF THE MODEL TO THE WATER SYSTEM DEVELOPMENT MODEL

The model and its specific solutions may be utilized while constructing models dealing with many aspects of the region's water system development (see, e.g., Makowski 1979).

The pattern of agricultural production that is taken to be endogenous in Makowski (1979) may be identified with an optimum solution to a variant of the model presented. Both models indicate that the quantities of water needed for agricultural production must be related to each other. Obviously, some problems occur because of the differences in the levels of spatial aggregation adopted. However, a relatively simple way of disaggregating the solutions of the presented model so as to fit the requirements of the water system development model has been worked out using an auxiliary linear programming model (Makowski 1979), in which agricultural production patterns are determined for particular subregions. The spatial distribution of production factors is therefore fully acknowledged. The objective function of the auxiliary model is defined so as to secure the minimum possible differences between the optimum (aggregated) regional production pattern and the pattern resulting from the disaggregation of the acceptable subregional patterns.

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APPENDIX: OPTIMUM PRODUCTION RESPONSE TO WATER SUPPLY IN THE UPPER NOTEC REGION--SELECTED RESULTS

Several versions of the basic model were solved using an IBM 370 computer. One of these was concerned with studying the response of the region's state-sector farming to water supply under some additional assumptions, from which the most important are

- -- labor force availability: 25,000 men;
- -- land under state ownership: 187,400 ha; and
- -- supplies of water $Q_{t\tau d}$ are identical for all t, τ, and d.

Thus, any version, say the nth, corresponds to the basic model, with inequalities (1) taking the following form:

$$\sum_{ijk} x_{ijk} q_{ijkt\tau d} + \sum_{r} W_r q_{rt} \leq Q^{(n)} \quad , \qquad t = 1,2,\ldots,6, \qquad (2)$$
$$\tau = 1,2,\ldots,I_t, \quad d = 1,2,\ldots,I_t \quad .$$

The characteristics of the optimum solutions of nine model variants to state-sector farming in the Upper Notec Region are given in Table 1. They are in the form of ratios: $C^{(n)}/C^{(2)}$, where $C^{(n)}$ refers to variant n and $C^{(2)}$ to the basic model.

With respect to the water supply per unit of cultivated land, the basic model reflects the present situation of state-sector farming in the Upper Notec region.

Table 1. Characteristics of the optimum solutions of the nine models.

n	$\frac{Q^{(n)}}{Q^{(2)}}$ [a]	$\frac{P_c^{(n)}}{P_c^{(2)}}$ [b]	$\frac{P^{(n)}}{P^{(2)}}$ [c]	$\frac{M^{(n)}}{M^{(2)}}$ [d]	$\frac{L^{(n)}}{L^{(s)}}$ [e]	$\frac{V^{(n)}}{V^{(2)}}$ [f]
1	0.50	0.98	0.72	0.96	0.22	1.04
2	1.00	1.00	1.00	1.00	1.00	1.00
3	1.50	1.01	1.03	1.00	1.69	1.68
4	2.00	1.03	1.13	1.04	2.23	3.06
5	2.50	1.04	1.38	1.07	2.83	3.90
6	3.00	1.05	1.45	1.10	3.43	4.74
7	4.00	1.08	1.69	1.15	4.60	6.55
8	8.00	1.18	1.34	1.28	9.28	17.37
9	12.00	1.23	1.27	1.39	14.55	23.21

[a] Q is total monthly water supply;
[b] P_c is net agricultural production in corn equivalent units;
[c] P is net agricultural production (in current prices);
[d] M is net meat production;
[e] L is irrigated land; and
[f] V is net vegetable production.

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PROCEEDINGS OF JOINT TASK FORCE MEETING ON DEVELOPMENT PLANNING FOR THE NOTEC (POLAND) AND SILISTRA (BULGARIA) REGIONS

VOLUME II

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PART III

Models of Water Systems and the Environment

COORDINATION OF TASKS WITHIN THE GENERAL STUDY OF THE UPPER NOTEC PILOT PROGRAM

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INTRODUCTION

The Governmental Program PR-7.06 was established to develop and implement a water resource management system in the Upper Notec agricultural region in Poland. This program considers the development of water resources in conjunction with the socio-economic development of the whole region.

The program was planned in two stages. In the first stage (1976-1979) the design of the system was accomplished. The second stage (after 1979) is concerned with the practical implementation of the system. This paper describes the coordination of activities during the design stage.

CHARACTERISTICS OF THE PROGRAM

The design work, which began in 1977, makes possible the updating of data on the territorial features, the structure, functions, and organization of the water resource management program.

The Upper Notec region is situated in the Notec river basin above the point where the Bydgoskie canal joins the river. The borders of the communities located in the river basin define the boundary of the region, within which 31 communities are included in an area of 4,900 sq km. The structure of land use is given in Table 1.

Table 1. The structure of land use in the Upper Notec region.

Type of land use	Area (in ha)	%
Agricultural use:	353,845	72.2
-- arable land	303,006	61.8
-- orchards	5,798	1.2
-- grassland	44,861	9.2
Forests	80,477	16.4
Other	55,036	11.4
Total	490,258	100.0

The water and land reclamation system and the agricultural-economic system have certain interrelated elements that define the structure of the water resource management system. For the water and land reclamation system, these elements include water bodies, reservoirs, water intakes, sewage treatment plants, internal water system for industrial plants, navigation and municipal sectors, and also irrigation and drainage systems serving one or several agricultural units. The corresponding elements of the agricultural-economic system include all agricultural production units (state, cooperative, and private farms), services and the food-processing industry. The region is divided into subregions in which there are water and land reclamation subsystems covering partial basins and agricultural-economic subsystems.

Detailed examination of the natural and economic conditions of the region have revealed that

- -- agriculture is being developed intensively over a considerable area of the region;
- -- considerable water deficits occur during the vegetation period;

-- significant urbanization taking place in the region and chemical and food industries are being developed; and
-- the level of water pollution is a considerable hazard to the region.

The aim of the water and land reclamation program is to develop a system of facilities for satisfying the water requirements of agriculture and other economic sectors. This requires the rational use of water resources, taking environmental factors into account. Therefore, the elaboration and implementation of a plan for agricultural development of the region is of primary importance. The establishment of a management center in the system should be considered as a stage of implementation. The center would control water distribution using modern data collection and processing methods and techniques, and optimization procedures; in addition, hydrological and meteorological surveys (including also a study on water quality) would be undertaken.

In designing the Upper Notec water resource management pilot system, a specific arrangement of tasks was required because of the close relationship between water supply and socioeconomic development.

GENERAL STUDY

The General Study of water resources and land reclamation for the pilot system constitutes the final document of the first stage of research and design and forms the basis for accomplishing further more detailed activities at a later stage. The Institute for Land Reclamation and Grassland Farming (IMUZ) at Falenty is the body coordinating the tasks of developing and implementing the system. Two design organizations are responsible for carrying out the tasks of the General Study: the Central Research and Design Bureau for Irrigation, Drainage Land Improvement, and Water Supply for Agriculture (BIPROMEL) and the Design and Technological Bureau for the Agricultural Industry and Agriculture Enterprise Organization (BIPROZET).

Initially, two topics were defined for study. The first (PR-7.06.01) was an examination of the natural and economic features of the region and a forecast of agricultural production potential with and without irrigation; and the second (PR-7.06.02), to be performed in eight detailed tasks, consisted in an analysis of the state of land reclamation and water resources and the general design of the water and land reclamation system. Data obtained independently of the development forecasts of the region from the Voivodship Boards (provincial authorities) in Bydgoszcz, Włocławek, and Konin were taken into account. Information from these two studies was used in the General Study (PR-7.06.03). This consists in five detailed tasks concerned with

-- planned economic development of the region, taking into consideration alternative plans for the development and distribution of agricultural production, and the development plans for forestry and forest density, for the agricultural industry, and for services and the economic infrastructure (performed by BIPROZET);

-- water resource management, taking into account forecasts of surface and groundwater resources, water demand forecasts, water balances, and additional water resources from natural river flows, from water storage reservoirs, and from water transfers outside the basin (performed by BIPROMEL);

-- technical solutions to the water and land reclamation system, considering water bodies, reservoirs, transfer canals, water intakes, water supply and sewage treatment systems in rural areas, river regulation, flood control, land reclamation systems, subsoiling, recultivation, erosion prevention, fish ponds, schemes for protecting water against pollution, schemes for data collection and processing, and system control (performed by BIPROMEL);

-- analysis of alternative water and land reclamation systems, taking into account forest and fishing resources, recreational use of rivers, lakes, and reservoirs, and the organization of water resource management (performed by BIPROMEL and BIPROZET); and

-- optimization of the water resource management system, taking into account changes in the structure of agriculture, in agricultural organization and management systems, and in the relations between agriculture and the food processing industry of the region (performed by BIPROZET, BIPROMEL, and IMUZ).

COORDINATION OF TASKS

In Figure 1, the organizational structure of the tasks in the program is shown. The main coordinator and the leading design organizations cooperate closely with the researchers to fulfill the tasks in the General Study, as indicated in Figure 2. The greatest difficulty encountered during the performance of the tasks was the lack of information available in certain research areas before the study was undertaken.

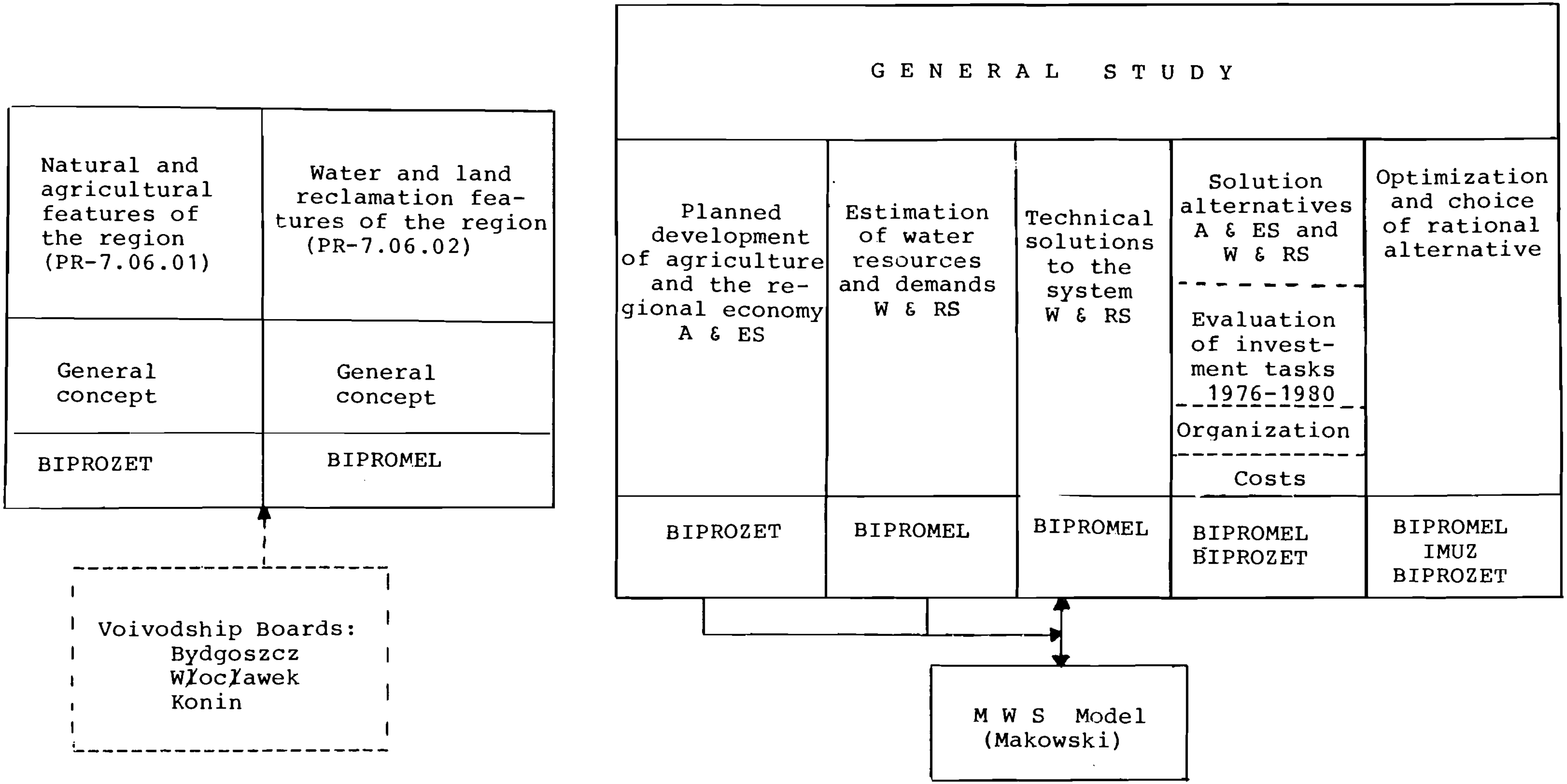

Figure 1. General study of water resource management and land reclamation.

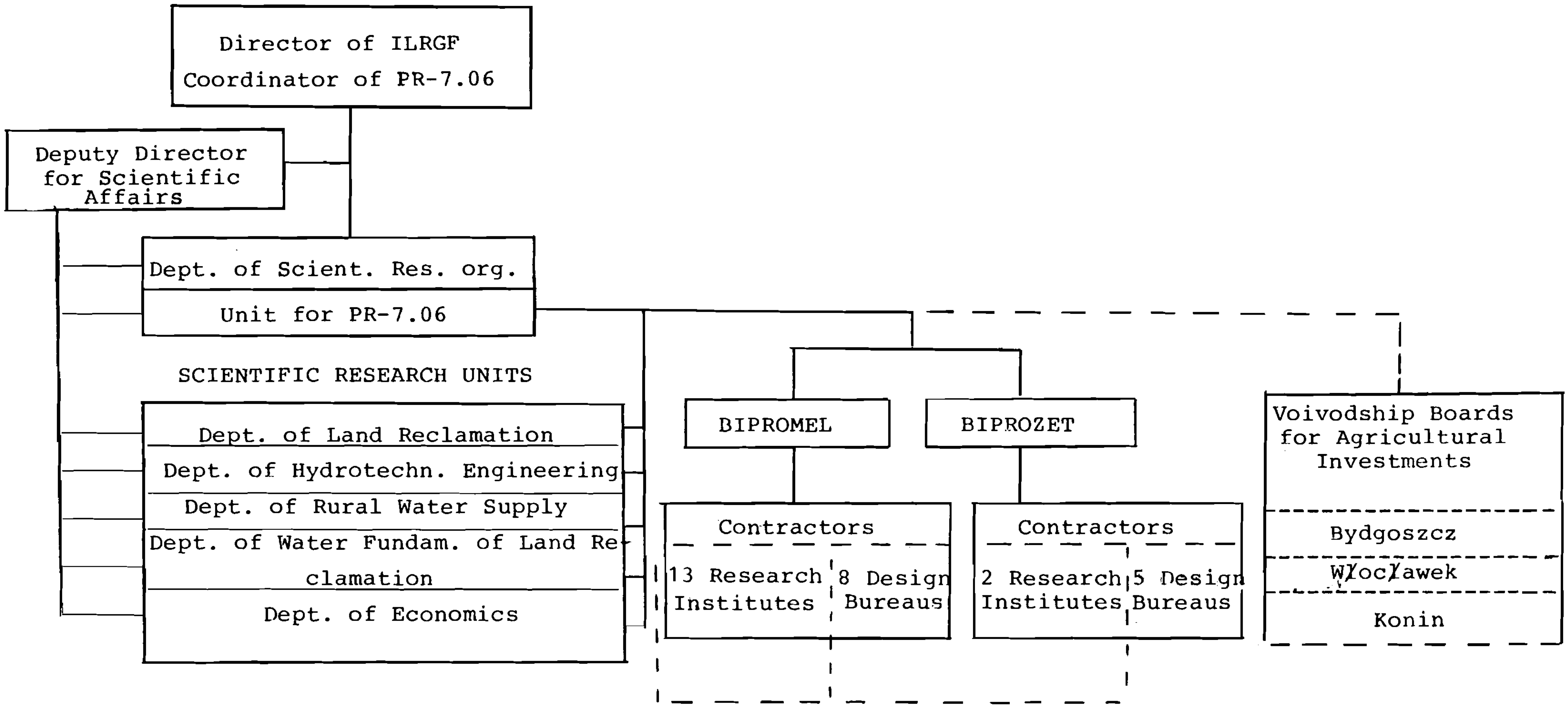

Figure 2. Scheme of coordination within PR-7.06.

PLANNING WATER RESOURCE DEVELOPMENT IN THE SILISTRA REGION

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V. Genkov
I. Tzvetanov
B. Topolsky

INTRODUCTION

In recent years remarkable progress has been made in methodologies for planning water resource development. This has been possible mainly by using systems analysis techniques to solve the complex problems involved in the design and operation of modern water resource projects and water use facilities. The Silistra case study -- launched in 1977 by the International Institute for Applied Systems Analysis (IIASA) in collaboration with various Bulgarian institutions -- provides an excellent framework for the application of various methodologies and systems analysis techniques. Moreover, because of the practical importance of the problems to be solved, it is necessary to refine existing methodologies and to develop new ones in order to satisfy the ever-increasing demands of decision making in a highly socialized environment.

The water resources of the Silistra region have been the subject of development for many years. Detailed geographical, geological, and economic studies have been carried out and a number of alternative strategies for developing the region's water resources have been proposed. However, the results of these studies have not been unified to form a comprehensive evaluation of the effects of these strategies on the entire socio-economic process of the region.

The study of agricultural water demands carried out in 1977 by IIASA's Water Demand Group (Gouevsky and Maidment 1977; Gouevsky et al. 1980) was a turning point in the application of systems analysis to water resource development of the Silistra region. The study group has provided a methodology for comprehensive analysis of irrigation and livestock potable water demands as well as for forecasting these demands in the future. The model is being further extended and refined in Bulgaria.

In 1978 IIASA's Regional Development Task initiated a case study to evaluate water supply alternatives for one of the irrigation systems in the region. The results of this study are available in Albegov and Chernyatin (1978). The model has been put on the computer at the Institute for Social Management, Bulgaria. Both the demand and the supply models have recently been linked together.

The aim of this paper is: (a) to outline and discuss a system of models for water resource development and water resource control in the Silistra region; (b) to propose principles for coordinating and integrating these models into a unified system; (c) to suggest and implement methodologies for planning water resource development in conditions of uncertainty; (d) to discuss problems of water resource management in conditions of a water deficit.

A SYSTEM OF MODELS FOR WATER RESOURCE DEVELOPMENT

Water Use in the Silistra Region

Water is being used at present and will be widely used in the future for the following activities in the Silistra region:

-- agriculture (crop and livestock production and processing),

-- potable water supply (population and livestock),

industry,

-- transportation (and other in-stream uses), and

-- recreation (and other on-site uses).

Agriculture and the potable water supply are assumed to account for the greatest water demand in the region. This is because a vast irrigation system will be developed in the near future to meet the feed requirements of livestock and the ever-increasing potable water requirements of the population. For irrigation and the supply of potable water to the population and livestock, water transfers over a distance of up to 60-80 km from the water source are required. Substantial capital investments are needed to provide this supply; thus, a trade-off between decreasing water demands and increasing water supply has to be made.

Industry is the next most important user. Two types of enterprise exist: heavy water users (food and kindred products, wood processing) and other users (machine building, various municipal services). It is expected that in the future residuals generated from industry may also cause some upstream-downstream conflicts as well as negative environmental effects.

Water use for transportation and recreation is limited for the time being to areas near the River Danube. However, in developing the region, access to these services will be provided for the population of the central and southern part of the region. In this respect there may be conflicts between agriculture and these two particular users. A comprehensive demand analysis of the users should be carried out to avoid such conflicts.

Water resource development in the Silistra region is taking place with significant alterations in the regional economic, social, and environmental policies, needs, and technologies. Important factors influencing the overall development of the region are: the scarcity of water within the region; the high cost of providing water for various localities; large water demands for agricultural activities, which are spread throughout the region; the existence of other important users (potable water supply, industry), whose water demands have to be met by a guaranteed supply; the land of the region will be used to build water transfer facilities to supply adjacent regions. All these factors necessitate a thorough examination of both user's water demand and water supply alternatives in order to reveal those most preferable from the economic, social, and environmental point of view.

Purpose of Modeling

The main purpose of modeling the water resources in the Silistra region is to carry out a comprehensive systems analysis of water resource development and management and to evaluate the influence of water resources on the entire social and economic development of the region. This goal will be achieved by fulfilling the following tasks:

-- determination of optimal water demands over time and in space for the various users and coordination of these demands with those for other resources that are used for a given production process or service;

-- evaluation of the development potential of water supply systems that satisfies optimal water demands over time and in space and generation of parameters for the design of water supply systems;

-- identification of key variables for the planning and management of water resource development in the region;

-- determination of optimal control policies for water resources in extreme situations (deficit of water resources); and

-- development or implementation of a methodology flexible enough for application in other regions.

Basic Models for the Development and Control of Water Resources

The above stated tasks related to modeling water resource development can be achieved by constructing a single model incorporating all features of development and control policies. However, such an approach would lead to a cumbersome model that would be difficult to use. Moreover, it is difficult to adapt such a model for application in other regions with different characteristics. Therefore, in this paper we propose a system of models selected according to three criteria.

-- The models should describe comparatively independent processes of water utilization.

-- The number of models should be the minimum necessary to solve the regional development problems adequately.

-- A water resource process should only be modeled if it is essential for regional development.

Based on these criteria and the previous discussion of the demand and supply aspects of water resources, it is suggested that the following should constitute the system of models for the development and control of water resources in the Silistra region (Figure 1):

M1. Agricultural water demand model;
M2. Irrigation and livestock unit water requirements model (an auxiliary model to M1);
M3. Potable water demand model;
M4. Industrial water demand model;
M5. Transportation and recreation water demand model;
M6. Water demand models of the adjacent regions;
M7. Potable water supply model;
M8. Water supply model for irrigation, industry, transportation, and recreation; and
M9. Water resource management model.

As can be seen from Figure 1, the various models involved in water resource development constitute a three-tiered hierarchical system:

--national level models (general guidelines for water resource development and management);

--regional level models (agriculture, environment, construction industry, economic development, industry, a healthcare system, migration processes, settlement systems); and

--regional water resource models, which in turn form a three-level system of models: demand models, M1,...,M6; supply models, M7, M8; and a water resource management model M9.

There are three types of links between any two models: amount of water required (denoted by a single arrow); economic indicators such as cost, capital investment, marginal values (denoted by a double arrow); various parameters such as technologies, resource

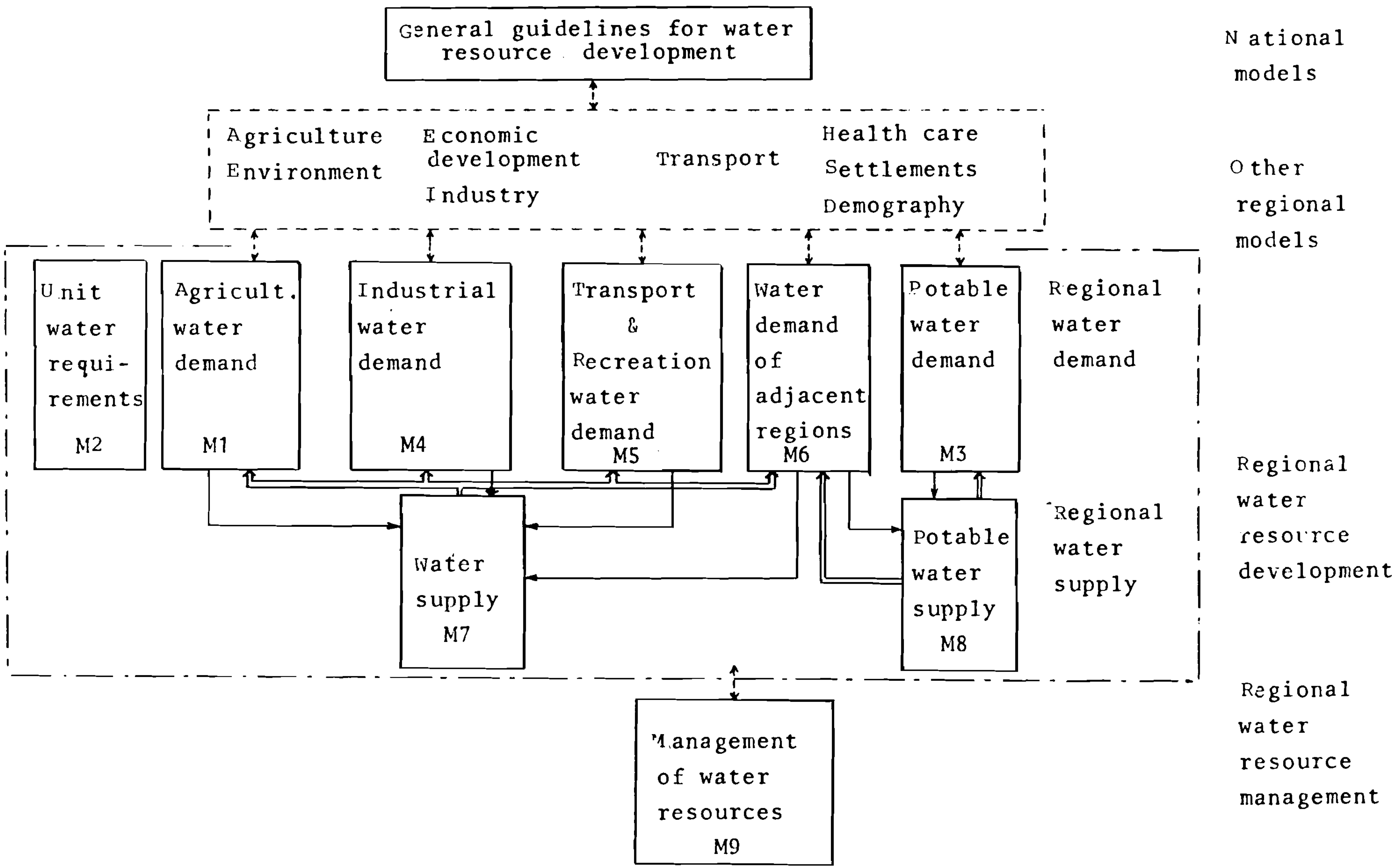

Figure 1. A system of water resource models of the Silistra region.

constraints, production targets, and so on, to be exchanged between water resource models and the other regional models (denoted by a dotted two-way arrow).

The agricultural water demand model M1, the supply models M7 and M8, and the water resource management model M9 are programming models; i.e., a solution for several key variables, such as water demands, production alternatives, water supply alternatives, management policies, and so on, is sought by maximizing (minimizing) a certain performance index subject to a number of constraints reflecting technological processes, production targets, physical and financial constraints. All these models are explained in detail elsewhere (Gouevsky et al. 1980; Albegov and Chernyatin 1978; Tzvetanov and Petrov 1976).

The others are statistical or simulation models. M2 is a statistical model that determines the water requirements of the irrigation unit using a modified Penman equation (Doorenbos and Pruitt 1977) to determine evapotranspiration from data on temperature, humidity, wind and sunshine duration or radiation (if available). The models M3, M4, and M5 for determining potable, industrial, transportation, and recreation water demands are intended to be simulation models. If, however, some of the regional industrial models are developed as programming models, then industrial water demand may be derived from these models by introducing a "water submodel" into them. M3, M4, and M5 are in the process of development.

Coordination of the Regional Demand and Supply Models

To facilitate the analysis of water resource development, the proposed models M1,...,M8 have been divided into two categories: demand and supply models. This, however, calls for a procedure for coordinating (integrating) the models. For the present, two coordination principles are envisaged: marginal values equilibrium and cost (capital investments) sharing.

The Marginal Value Equilibrium Principle

The marginal value equilibrium principle stems from economic interpretation of the well-known Lagrange multipliers. Suppose a water user such as agriculture, industry, or transportation is considered. The benefit B(t) of this user is a function of the quality $Q_d(t)$ of water used at time t, t=1,...,T and of production alternatives and other resources x(t) involved in the production process of the user, i.e.,

$$B(t) = B[Q_d(t), x(t)] \quad . \tag{1}$$

Under the assumption that the user is maximizing the benefit, the water demands $Q_d(t)$ can be derived by solving the following optimization problem:

$$\max_{[Q_d(t), x(t)] \in R_d} \{B[Q_d(t), x(t)]\} \quad , \tag{2}$$

where

R_d is a set of admissible values for $Q_d(t)$ and x(t).

Let $[Q_d^o(t), x^o(t)]$ be an optimal solution of (2). Then the first derivative

$$\frac{\partial B(t)}{\partial Q_d^o(t)} = m_B^o[Q_d^o(t)] \quad , \tag{3}$$

is the marginal value of water demand Q_d^o at time t.

To meet the demand $Q_d(t)$ the user has to make use of some water supply facilities that exist or that should be specially developed. The cost C(t) of developing water supply facilities, such as reservoirs, pumping stations, canals, is a function of the amount of water $Q_s(t)$ supplied as well as of the existing technologies Y(t) to produce this amount

$$C(t) = C[Q_s(t), Y(t)] \quad . \tag{4}$$

It is reasonable to assume that the supply facilities should be developed at minimum cost. Hence, the amount of water to be supplied to the user can be derived by solving the following optimization problem:

$$\min_{[Q_s(t),\ Y(t)] \in R_s} \{C | Q_s(t),\ Y(t)\} \quad . \tag{5}$$

where

R_s is a set of all admissible values of $Q_s(t)$ and $Y(t)$.

If $[Q_s^o(t),\ Y^o(t)]$ is an optimal solution of (4), then the first derivative

$$\frac{\partial C(t)}{\partial Q_s^o(t)} = m_c\ [Q_s^o(t)] \quad . \tag{6}$$

Assuming that (2) and (5) incorporate all important factors of socioeconomic behavior of both the water demand user and the water supply unit, the marginal values (3) and (6) are actually marginal social benefits and marginal social costs, respectively. Suppose the overall utility function $U(t)$ for the use of water resources is

$$U(t) = B(t) - C(t) \quad . \tag{7}$$

Since $U(t)$ is actually the net benefit in the system, it is reasonable to maximize it:

$$\max_{\substack{[Q_d(t),\ x(t)] \in R_d \\ [Q_s(t),\ Y(t)] \in R_s \\ Q_d(t) = Q_s(t)}} [B(t) - C(t)] \quad . \tag{8}$$

The Lagrangian of (8) is

$$\max_{\substack{[Q_d(t),\ x(t)] \in R_d \\ [Q_s(t),\ Y(t)] \in R_s \\ m(t) \geq 0.}} [B(t) - C(t) + m(t)\ (Q_d(t) - Q_s(t))] \quad . \tag{9}$$

The necessary conditions for optimality are

$$\frac{\partial B(t)}{\partial Q_d(t)} = m(t), \quad \frac{\partial C(t)}{\partial Q_s(t)} = m(t), \quad Q_s(t) = Q_s(t), \qquad t = 1,\ldots,T. \tag{10}$$

It follows from (10), (3), and (6) that at the optimal solution point $[Q_d^o(t),\ Q_s^o(t),\ m^o(t)]$ the following conditions must hold:

$$Q_s^o(t) = Q_s^o(t) \quad , \qquad t = 1,\ldots,T, \tag{11}$$

$$m_B^o(Q_d^o(t)) = m_C^o(Q_s^o(t)) \quad . \tag{12}$$

Note that (11) and (12) determine the socially optimal level of water resource development. In many cases, however, a socially optimal level of development may be considered to be any solution of (11) and (12) if (11) holds true, but (12) is an inequality, i.e.,

$$Q_s^o(t) \ ' \ Q_s^o(t) \quad , \qquad t = 1,\ldots,T, \tag{13}$$

$$m_B^o(Q_d^o(t)) - m_C(Q_s^o(t)) \geq \beta \ (\leq\beta) \quad , \tag{14}$$

where

β is the nonnegative number.

(14) implies that the supply-demand system is in quasi-equilibrium, i.e., that there is a pressure from the demand/supply side to increase supply/demand but the decision makers hold-up this process for reasons external to the water resource system. For example, in coordinating the potable water supply model M7 with the potable water demand model M3, the marginal value of demand may be less than the marginal value of supply but these two models are still considered to be "in equilibrium" because this service and the cost of providing it are socially justified.

The marginal value equilibrium principle has certain advantages as well as some limitations when one tries to use it as a vehicle for coordination. The major advantage is that, in applying this principle, the decision makers maximize net benefit in the water resource system. However, a decomposition of supply and demand models and subsequent coordination through the marginal values is possible if the overall utility function is separable. In addition, a maximum net benefit in the system will occur only if the marginal demand and supply values are assigned as prices to users and supply units, which may be rather difficult in some situations.

Cost-Sharing Principle

The cost-sharing principle is another approach to be applied in case the marginal values do not function adequately in a certain water resource system. The key idea of this approach is implemented in the following iterative procedure for sharing costs of development of water supply among users.

1. Assuming certain water demands $Q^1_{di}(t)$ of user i, $i = 1,\dots,n$, the supply unit minimizes the cost of development of water facilities as follows:

$$\min \{C[Q_s(t), Y(t)]\},$$

subject to

$$Q_s(t) = \sum_{i=1}^{n} Q^1_{di}(t), \tag{15}$$

$$Y(t) \in R_s.$$

As a result, the cost of supply $C^1(t)$ is obtained.

2. The cost $C^1(t)$ is shared among users i as follows:

$$C_i^1(t) = k_i(t)\ 1_i(t)\ l_i(t)\ \frac{Q_{di}^1(t)}{\sum_{i=1}^{n} Q_{di}^1(t)}\ C^1(t), \tag{16}$$

$$k_i(t)\ l_i(t)\ \frac{Q_{di}^1(t)}{\sum_{i=1}^{n} Q_{di}^1(t)} = 1, \quad t = 1,\ldots,T, \tag{17}$$

where

$C_i^1(t)$ is the cost share due from user i;

$k_i(t)$ is a coefficient indicating the additional cost burden of user i; and

$l_i(t)$ is a cost grant coefficient (if the ith user's supply is considered to be very important but at the same time the user cannot bear a high supply cost).

3. Each user i solves its own optimization problem to determine a new value of water demand Q_{di}^2:

$$Q_{di}^2(t) = \arg \max_{[Q_{di}(t),\ x_i)t)] \in R_d} \{B[Q_{di}(t),\ x_i(t)]\},$$
$$C_i(t) = C_i^1(t) \tag{18}$$

if

$$\sum_{i=1}^{n} Q_{di}^2(t) = Q_s(t), \tag{19}$$

the process is terminated. Otherwise the procedure goes to 2.

It can easily be seen that the cost-sharing principle is very close to that of the marginal value. The differences are that

-- the supply cost for a certain user can be corrected by the decision makers in the system to achieve an equity in cost sharing; and

-- the cost-sharing principle operates according to the cost of supply, which includes capital investment and operation and maintenance costs, instead of according to the price of water, or marginal value of water supply, as does the marginal value principle.

PLANNING OF WATER RESOURCE DEVELOPMENT UNDER CONDITIONS OF UNCERTAINTY

The planning of water resource development may consist of two stages: strategic planning, which outlines the general behavior of the system over the distant future, and calendar planning, which determines the sequence of events that should take place over time to achieve the goals of the strategic plan. To facilitate the discussion, only the first type of planning activity is considered here; i.e., the problem is to outline what should happen in water resource development, without specifying the exact time intervals at which the development has to take place.

Let $\bar{V}^p$ be a vector* of planning indicators of a certain water-related activity in an agricultural system. For example, such indicators may be net benefit, irrigation investments, or amount of irrigated land. These indicators depend on production alternatives $\bar{x}^p$, on the amount of resources $\bar{R}^p$ used in the process, as well as on some "soft" parameters $\bar{w}^p$, such as management skills and political and social attitudes:

$$\bar{V}^p = f(\bar{x}^p, \bar{R}^p, \bar{w}^p) \quad . \tag{20}$$

In real world applications neither $\bar{x}^p$, $\bar{R}^p$, or $\bar{w}^p$ can be determined or predicted exactly and hence the real value $\bar{V}^r = f(\bar{x}^r, \bar{R}^r, \bar{w}^r)$ differs from $\bar{V}^p$; i.e., $\bar{V}^p \neq \bar{V}^p$. It should be

*Vector quantities are indicated an upper bar.

noted that all planning techniques such as mathematical (deterministic or stochastic) programming assume that the determined $\bar{V}^p$ equals $\bar{V}^r$ (the assertion $\bar{V}^p \neq \bar{V}^r$ makes these techniques meaningless).

Furthermore, the best value $\bar{V}^p$ does not generally coincide with the mathematical expectation of the respective planning indicator or with its most probable or upper/lower value. To cope with this difficulty in the planning process two approaches have been suggested by Lichtenstein (1976).

The first approach determines $\bar{V}^p$ by introducing a penalty function $F(\bar{V}^p, \bar{V}^r)$ for each $\bar{V}^r \neq \bar{V}^p$. Here is one simple piecewise linear penalty function:

$$F(\bar{V}^p, \bar{V}^r) = \begin{cases} a_1 + b_1(\bar{V}^r - \bar{V}^p), & \text{if } \bar{V}^r > \bar{V}^p \\ 0 & , \text{if } \bar{V}^r = \bar{V}^p , \\ a_2 + b_2(\bar{V}^p - \bar{V}^r), & \text{if } \bar{V}^r < \bar{V}^p \end{cases} \tag{21}$$

where

a_1, a_2, b_1, and b_2 are coefficients determined by a comprehensive economic analysis of the consequences of achieving the values of the planning indicators $\bar{V}^p$.

This approach determines the best value of a certain planning indicator $\bar{V}_1^p$ by minimization of the expected losses $E[F(V_1^p, V_1^r)]$ due to increasing or decreasing the value of V_1^p, i.e.,

$$\min_{V_1^p} \{E[F(V_1^p, V_1^r)]\} \quad . \tag{22}$$

The second approach determines $\bar{V}^p$ by using the so-called a priori level of reality, which is based on the following idea. Let P_1^o $(0 \leq P_1^o \leq 1)$ be an a priori level of reality for $V_1^r \geq V_1^p$; i.e., P_1^o measures the probability of having $V_1^r \geq V_1^p$. Suppose the planning indicator 1 should be maximized in the system. The set $B_{p^o_1}$ defined as

$$B_{p^o_1} = \{\bar{Y}^r \in D \;/\; P(V_1^r \geq V_1^p) \leq P_1^o\} \quad , \tag{23}$$

is called a set of sufficiently real plans. To find the best value of V_1^p, the following maximization problem has to be solved:

$$\max_{V_1^p \in B_{p^o_1}} V_1^p \quad . \tag{24}$$

The value of P_1^o can be determined either by experts' evaluations or by the penalty function method. Lichtenstein (1976) has shown that both methods--penalty function and the a priori level of reality--are equivalent and either can be used for solving practical planning problems.

The methodology of finding the best planning indicators for the agricultural water demand model M1 is based on the integrated optimization-simulation model (OSM) discussed in Gouevsky et al. (1979).

Optimization Model

The optimization part of OSM for agricultural water demand in the Silistra region is described in detail in Gouevsky et al. (1980). The objective function of the model maximizes the difference between the value of marketed crop and livestock products and their production costs:

$$Z = \max\ (\bar{b}_1\bar{x}_1 - \bar{c}_1\bar{x}_2 - \bar{c}_2\bar{x}_3) \quad , \tag{25}$$

where

$\bar{x}_1$ are amounts of crop and livestock products consumed in the system or exported;

$\bar{b}_1$ are benefits per unit of these products;

$\bar{x}_2$ are amounts of crop and livestock production and processing activities;

$\bar{c}_1$ are costs per unit of these activities;

$\bar{x}_3$ are quantities of input resources; and

$\bar{c}_2$ are costs of supplying the resources.

The objective function Z is maximized subject to the following constraints, which reflect the resource availability, material balances, and production targets:

$$A \bar{x} \leq \bar{B} \quad , \tag{26}$$

where

$\bar{x}$ is $\bar{x}_1$, $\bar{x}_2$, $\bar{x}_3$;

A is a matrix having elements a_{ij} that indicate per unit of production activity or per unit consumption of resources; and

$\bar{B}$ is a righthandside vector of exogenous upper and lower limits of resources or production targets.

The model (25) and (26) can also be considered as a planning and forecasting tool because it specifies the scale of development of all production alternatives and resources involved in the system. As such the model has the disadvantage of assuming that any planning indicator, say the objective function Z^P or the scale of development of any resource or production alternative x_1^P, obtained by the optimization model (25) and (26), equals the real obtained value Z^r or x_1^r of these indicators. One way to cope with this disadvantage is to combine the model (25) and (26) with a simulation model of the crucial exogenous components of the vector $\bar{B}$, the coefficients of the matrix A, and the objective function Z according to the methodology described by (21) and (22) or (23) and (24).

Simulation Model

The objective function (25) together with constraints (26) form a deterministic optimization model. However, there are several key parameters in the model that are stochastic by nature and whose values substantially change the model behavior. Such parameters are crop yields, which vary according to the climatic and meteorological conditions (precipitation, solar radiation, frost, hail) and to management practice and policies being implemented at present or in the future. The same considerations apply to benefit and cost coefficients of the objective function and to the production targets included in the righthandside vector.

To reveal the impact of all these parameters on the behavior of the agricultural water demand model of the Silistra region, a sensitivity analysis was carried out. The analysis indicated that crop yields have the greatest influence on planning indicators, which are assumed to be the system's net benefit from irrigation and irrigation capital investment. Other indicators such as irrigated land, volume of water for irrigation, number of animals, land areas of various crops are influenced by the basic planning indicators. Indeed, crop yields depend on various technological and management factors. There are two methods of determining the crop-yield probability distribution characteristics.

The first is based on the history of the process; i.e., crop-yield records are used to obtain probability distributions. This approach has been extended in the "self-organization" algorithms (Ivakhnenko 1970, 1975) based on the generalized Kolmogorov-Gabor polynomial. To obtain the polynomial parameters, the problem is divided into a number of subproblems each solving two-argument equations; i.e., the principle of argument grouping is applied.

To determine an appropriate model for the crop-yield coefficients, several subsequent levels of selection (stages) are employed. At each stage new intermediary arguments are brought in and a new set of two-argument equations is generated. Each generated equation, considered as a potential forecasting model is in competition with the remaining equations. A specific feature of this approach is that, according to the principle of nonfinal solution, more than one model is selected at each stage. The models' parameters are then evaluated in accordance with the least square error criterion. This criterion is chosen on the basis of the external addition principle; i.e., the criterion must be independent of the procedure for creating the intermediary equations. The algorithm describing the above idea is programmed in PL/1 language as a subroutine package and is available at the Central Management and Computer Center, Ministry of Agriculture, Sofia (Topolsky 1979).

The second approach to determine crop-yield coefficients assumes that the crop-yield probability characteristics are determined by taking into account possible future changes in production technologies or management policies. This approach usually involves adopting experts' evaluations. It has been chosen to determine the probability distribution of crop yields in the Silistra agricultural water demand model M1.

Let a_{ij} be the yield of crop i obtained using technology j. Suppose the probability distribution of yield a_{ij} (assuming a_{ij} is independent) is

$$P\ (a_{ij} \le d_{ij}^{min}) = 0 \quad , \tag{27}$$

$$P\ (a_{ij} \le d_{ij}^{max}) = 1 \quad , \tag{28}$$

$$P\ (a_{ij} \le q_{ij}) = 0.5 \quad , \tag{29}$$

where

$$q_{ij} = (d_{ij}^{min} + d_{ij}^{max}) \ / \ 2 \ . \tag{30}$$

(27) indicates that the crop yield d_{ij}^{min} will be obtained in any case, while (28) indicates that a crop yield $a_{ij} > d_{ij}^{max}$ cannot be obtained in the system during the considered planning horizon.

Therefore, the probability distribution of crop yields in the interval $d_{ij}^{min} \le a_{ij} \le d_{ij}^{max}$ can be approximated as follows:

$$P(a_{ij}) = P(a^s_{ij} < a_{ij}) \tag{31}$$

$$= a_{ij}(d^{max}_{ij} - d^{min}_{ij})^{-1} - d^{min}_{ij}(d^{max}_{ij} - d^{min}_{ij})^{-1}$$

where

a^s_{ij} is yield of crop i using technology j obtained from simulation run s.

Assuming that the value $P(a_{ij}) \in [0,1]$ is simulated by a random number generator, i.e.,

$$P(a_{ij}) = G^s \quad ,$$

the values of a^s_{ij} from the simulation run s are

$$a^s_{ij} = d^{min}_{ij} + (d^{max}_{ij} - d^{min}_{ij})\ G^s \quad , \tag{32}$$

$$G^s \in (0,1],\ s = 1,\ldots,k \quad .$$

This is the basic crop-yield simulation equation that has been incorporated in the integrated optimization-simulation model of the Silistra agricultural water demand model M1.

Integrated Optimization-Simulation Model

A combination of the optimization model (25) and (26) and the simulation model (32) for deriving the coefficients a_{ij} of the matrix A can substantially improve the reliability of achieving planning indicators of the agricultural system. The integrated optimization-simulation model can be described by the following three-stage procedure.

1. Generate k sets of simulated parameters a^s_{ij}, s = 1,...,k, using the simulation equation (32). The number can be determined by the following expression suggested by Lichtenstein (1976):

$$k = \ln\ (1-P^d_1)/\ln(1-P^o_1) + 0.5 \quad , \tag{33}$$

where

P_i^d is a desired probability of finding the best value of the planning indicator 1; and

P_1^o is the a priori level of reality to achieve the best value of the planning indicator 1, $0 \leq P_1^o \leq 1$.

A characteristic of (33) is that the number of simulations k does not depend on the number of parameters in the optimization model (25) and (26) or on the probability distribution of these parameters. The only requirement is that the number of simulated parameters should not be less than 15 to achieve an accurate solution.

2. For each $s = 1,\ldots,k$:

$$\max Z$$

subject to

$$A^s\bar{x} \leq \bar{B} \quad , \tag{34}$$

where

$$A^s = \{a_{ij}^s, a_{ij}\}, \; s = 1,\ldots,k \quad .$$

The result of this stage is that the sets of value $F = \{Z^s / s = 1,\ldots,k\}$ and $E = \{\bar{x}^s \; / \; s = 1,\ldots,k\}$ are obtained.

3. Find the best value, Z^p or x_1^p, of the planning indicators by maximization or minimization (depending on the type of indicators) over the sets F or E, i.e.,

$$Z^p = \max_s(\min) \; Z^s \quad , \quad x_1^p = \max_s(\min) x_1^s \quad . \tag{35}$$

The last item of the procedure may be slightly different if the best value of two or more competing planning indicators is to be found. Suppose Z^p (net benefit) and x_1^p (irrigation investments) are to be planned. Both Z^p and x_1^p are random variables. Also, Z^p is not a deterministic function of x_1^p and vice versa. In addition, the probability of achieving a net benefit from irrigation Z^p decreases as the probability of reducing irrigation investment x_1^p increases.

To deal with the problem of finding the best values for competing planning indicators, item 3 of the above procedure can be modified according to the decision maker's preference using one of two alternative approaches both requiring assignment of priority to Z^p and x_1^p.

The first approach begins with finding the best value for the indicator that is given priority. The model (34) is changed by adding the constraint that this indicator should not be less (or greater if the indicator is to be minimized), than the best value found. The best value of the second indicator is obtained by following steps 1,2, and 3.

The other approach assumes that the best value of the second indicator is based on simulation runs in which the first indicator has values not less (or greater in case of minimization) than its best value.

Results of Applying the Optimization-Simulation Model

The methodology for planning in conditions of uncertainty has been applied to determine the best values of two planning indicators of the agricultural water demand model M2. These indicators are annual net benefit generated in the region's agricultural system from irrigation and irrigation investment to develop facilities such as sprinklers and field canals.

An expert analysis was carried out to reveal the intervals in the variation of crop yields, i.e., to determine the values d_{ij}^{min} and d_{ij}^{max} and the respective probabilities (see (27),(28), (29), and (30)) of crop yields a_{ij}.

Altogether 48 coefficients of crop yields a_{ij} were assumed to vary and they were simulated according to (32). Finally, using the procedure of the optimization-simulation model, the best values of the two planning indicators annual net benefit Z^p and irrigation investment x_1^p were obtained. Although the model generates the values of all other variables, to keep the paper short only the values of Z^p and x_1^p are discussed.

As Z^p and x_1^p are competing indicators they were given priority, $Z^p > x_1^p$. The model was run once using the mean values of crop yields. Initial values of $Z^o = 50 \times 10^6$ leva (lv) and

$x_1^o = 320 \times 10^6$ lv were obtained. A group of experts analyzed these values and assigned the a priori probability of achieving Z^o and x_1^o for the assumed planning horizon to be 0.3 and 0.4, respectively. The desired probability P_1^d of finding the best values for both planning indicators (see (33)) was assumed to be 0.9. By substitution of these probabilities in (33) the following number of simulations k was obtained: for the annual net benefit indicator, k=7, and for the irrigation investment indicator, k=5.

In Figure 2, the results of applying the integrated optimization-simulation model for the annual net benefit indicator are given.

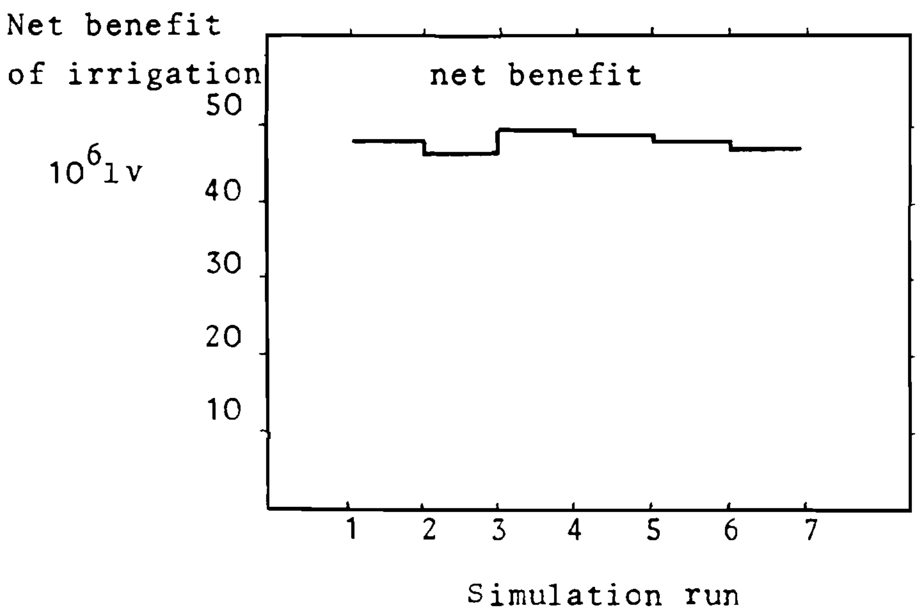

Figure 2. Results of annual net benefit simulation .

The best value of $Z^p = 48.7 \times 10^6$lv is found by applying (35) to values Z^s in Figure 2. An additional constraint $Z^s \geq 48.7 \times 10^6$ lv was added to (34) and the procedure (33), (34), and (35) was again implemented. The results are shown in Figure 3. This figure indicates that the best planning value of irrigation investment x_1^s is 249×10^6 lv, which corresponds to 79,000 ha irrigated land.

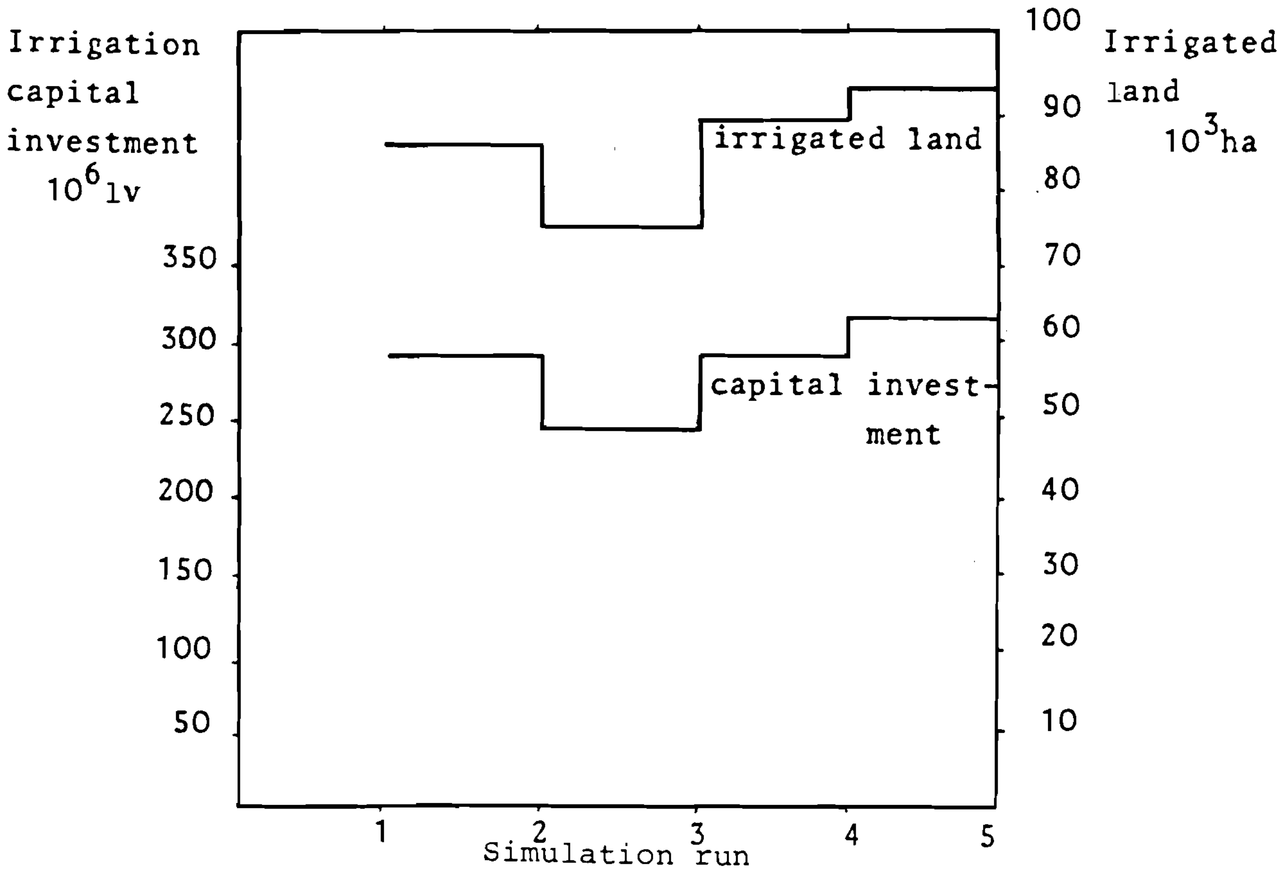

Figure 3. Results of irrigation investment simulation.

The results also indicate that varying irrigation investment by 25 percent changes the annual net benefit by only 1.5 percent. The reason being that the optimization carried out by (34) changes the production process pattern of the agricultural system when crop yields vary. For example, a reduction in crop yields results in the introduction of crops that are more drought-resistent (and thus reduces the area of land requiring irrigation), changes animal diets, and restructures the pattern of livestock specialization in order to maximize the net benefit. However, this has certain limits imposed by the market structure and the food demand of the population that must be satisfied.

MANAGEMENT OF REGIONAL WATER RESOURCES

One of the management problems that the Silistra regional water authorities may face in the future is how to allocate a limited amount of water among users. Water may become scarce even if the water resource system is developed according to forecasts for the future. This may happen as a result of unexpected demand, changes in the characteristics of the supply facilities, or failure in the system.

In allocating scarce water resources, the sytem should be capable of solving the following problem. Which users should be given water and how much should each user obtain? Most of the methodologies that have been developed so far to answer this question usually assume that priority is given to users according to the loss that will occur to each user if the amount of water allocated to it is less than demanded. Therefore, the key parameter to be determined is the user's loss function, which indicates how much the user would fail to gain if its allocation of water is reduced.

There are two approaches to determining the loss function of any water user. The first approach (Popchev et al. 1978) uses the marginal value of water demand associated with demand programming models. The integral of the dual value m_{Bi} of the user i,

$$\int_0^{Q_{di}^o} m_{Bi}\, dQ_{di} = B_i^o \quad , \tag{36}$$

is the maximum value of net benefit that the system would obtain if the amount of resource Q_{di} equals the demand Q_{di}^o.

If Q_{di} varies in the interval $[0, Q_{di}^o]$, the net benefit B_i in the system will be

$$B_i = \int_0^{Q_{di}} m_{Bi}\, dQ_{di} \quad . \tag{37}$$

A loss function $L_i(Q_{di})$ of user i with respect to the amount of water allocated Q_{di} is the difference $B_i^o - B_i$ for all values of $Q_{di} \in [0, Q_{di}^0]$, i.e.,

$$L_i(Q_{di}) = B_i^o - B_i = B^o - \int_0^{Q_{di}} m_{Bi}\, dQ_{di} \quad . \tag{38}$$

The second approach can be applied when the water demand is obtained by statistical or simulation models; i.e., no marginal values of water demand are available. The loss function $L_i(Q_{di})$ is determined by evaluating the user's benefit B for the extreme values in the interval 0, Q_{di}^o and, if possible, for several values of Q_{di} within the interval. Then, an estimate of the benefits is made and $L_i(Q_{di})$ is obtained again by (38).

Having obtained the loss function $L_i(Q_{di})$ for all users, the system management unit may require a minimum total loss -- caused by scarcity of water --in the system. To achieve this goal, the following minimization problem has to be solved:

$$\min \sum_{i=1}^{n} L_i(Q_{di}) \quad ,$$

subject to (39)

$$\sum_{i=1}^{n} Q_{di} < Q_s, \; Q_{di} \in T \quad .$$

where

Q_s is total amount of water available in the system; and

T is a set of values of Q_{di} that reflect the topology of the water resource system.

Large-scale optimization problems (39) can be solved by employing a hierarchical concept consisting in grouping the water users in subsystems k, k = 1,...,p, according to type of water use (agriculture, industry, and so on), then in performing the allocation process in two stages: bottom-up and top-down.

First, the bottom-up stage is carried out. The loss functions $L_i(Q_{di})$ of the users belonging to a given subsystem are derived and problem (39) is solved separately for all subsystems. As a result, a generalized loss function $L^k(Q_d^k)$ for the respective subsystem is obtained. Then the total loss $\sum_{i=1}^{n} L^k(Q_d^k)$ for all subsystems in minimized subject to constraints on the amount of water available Q_s in the system.

The top-down stage begins with finding the amount of water Q_d^k to be allocated to each subsystem k. This amount is fixed as a constraint for the respective subsystems and, using the loss functions $L_i(Q_{di})$, optimization is carried out to determine the optimal amount of water to be allocated to each user i.

This two-stage approach has been intensively studied by Tzvetanov (1975) and Popchev et al. (1976a, 1976b, 1976c). Some of the proposed algorithms are computerized.

CONCLUSION

The paper proposes a system of models for water resource development in the Silistra region, Bulgaria. The system contains three types of models: water demand, water supply, and water management models. Water demand is determined either by programming models (agriculture) or by statistical and simulation models (industry, potable water demand, transportation, and recreation). The water supply models for meeting potable and other water demands are programming models. It is intended that the water demand and water supply models will be coordinated by application of the marginal value or the cost-sharing principle.

Special attention has been paid to the planning of water resource development under conditions of uncertainty. As an example, the agricultural water demand model is considered. An integrated optimization-simulation model has been applied to determine the best values of planning indicators such as net benefit from irrigation and irrigation investment of the Silistra agricultural system. The results have shown that uncertainties about crop yields caused by weather variations and management policies, result in comparatively high variations of some of the planning indicators. For example, a variation in crop yields from 6 percent to 16 percent may change irrigation investment by 25 percent or more. Therefore, to find the best planning value of the irrigation investment is of considerable significance for decision makers.

The water resource management model is linked to the planning model of water resource development through the loss function concept. This function is an integral of the marginal value of water and it is used to determine priorities among users. The loss function obtained by water demand programming models has the advantage that it takes into account all resources utilized in a certain production process as well as other modeled economic characteristics of the process.

There will be further efforts to create the other water demand and water supply models that are not described in detail here. The solutions of all models in the system will be coordinated in order to make a comprehensive study of the water resources in the Silistra region and to assist decision makers in water resource management.

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MODELING THE SILISTRA WATER SUPPLY SYSTEM

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In 1978 a collaborative study was initiated to develop a model for solving water supply problems in the Silistra region. The research teams participating are from the Regional Development (RD) Task and the Resources and Environment (REN) Area at the International Institute for Applied Systems Analysis (IIASA), from the Sofia Institute for Water Projects (SIWP), and from the Sofia Institute for Social Management (SISM), Bulgaria. SIWP's approach to selecting the "optimal" water supply system for Silistra was confined to assessing a few alternative systems. However, using this approach, the probability of finding the optimal solution is not high. The role of IIASA has therefore been to apply a mathematical modeling approach to the problem.

There are two main objectives in modeling the Silistra water supply system. The first is to develop a model to determine the least-cost water supply system that meets given water demands--a somewhat traditional goal for many water resource models. The second objective is to describe formally the links between the water supply model and the other regional models under development at IIASA. Since irrigated agriculture is a major sector of the Silistran economy, much attention is paid to interfacing the water supply model with a model of agriculture.

Silistra is not a large region, being only 2,700 km^2 in area, and it is located in the northeastern part of Bulgaria. The soil quality and number of days of sun per year help to create a situation that is favorable to the intensive development of agriculture. The River Danube is the only source of water for irrigation and household and industrial consumption.

The Silistra water supply system is divided into two separate subsystems: irrigation water supply and water supply for household and industrial consumption. Such a division is required because of the different levels of water quality demanded by the various water users. With regard to industry, the Silistra region consists mainly of food enterprises. As is well known, both the food industry and the public require water quality to be of potable water standard (i.e., of higher quality than that needed for agriculture). Only 10-15 percent of total regional water demand is taken up by household and industrial users, for whom the only water sources are stream terrace waters, which are limited in quantity. Furthermore, the potable water supply requires the creation of a watermain system, whereas irrigation water is withdrawn directly from the Danube streamflow. Thus, the irrigation water supply is not only a separate part, but also the most important part, of the Silistra water supply system. For this reason the irrigation water supply system only is considered.

From the geographical point of view, the water supply system of the Silistra region is divided into three unconnected irrigation systems--Tutrakan, Malak Preslavets and Silistra (Figure 1). We deal only with the water supply system for Malak Preslavets for several reasons. First, this system is the most representative in terms of the irrigation area (approximately 60 percent) and the elements it contains, such as reservoirs, pumping stations, canals, etc. Second, since the

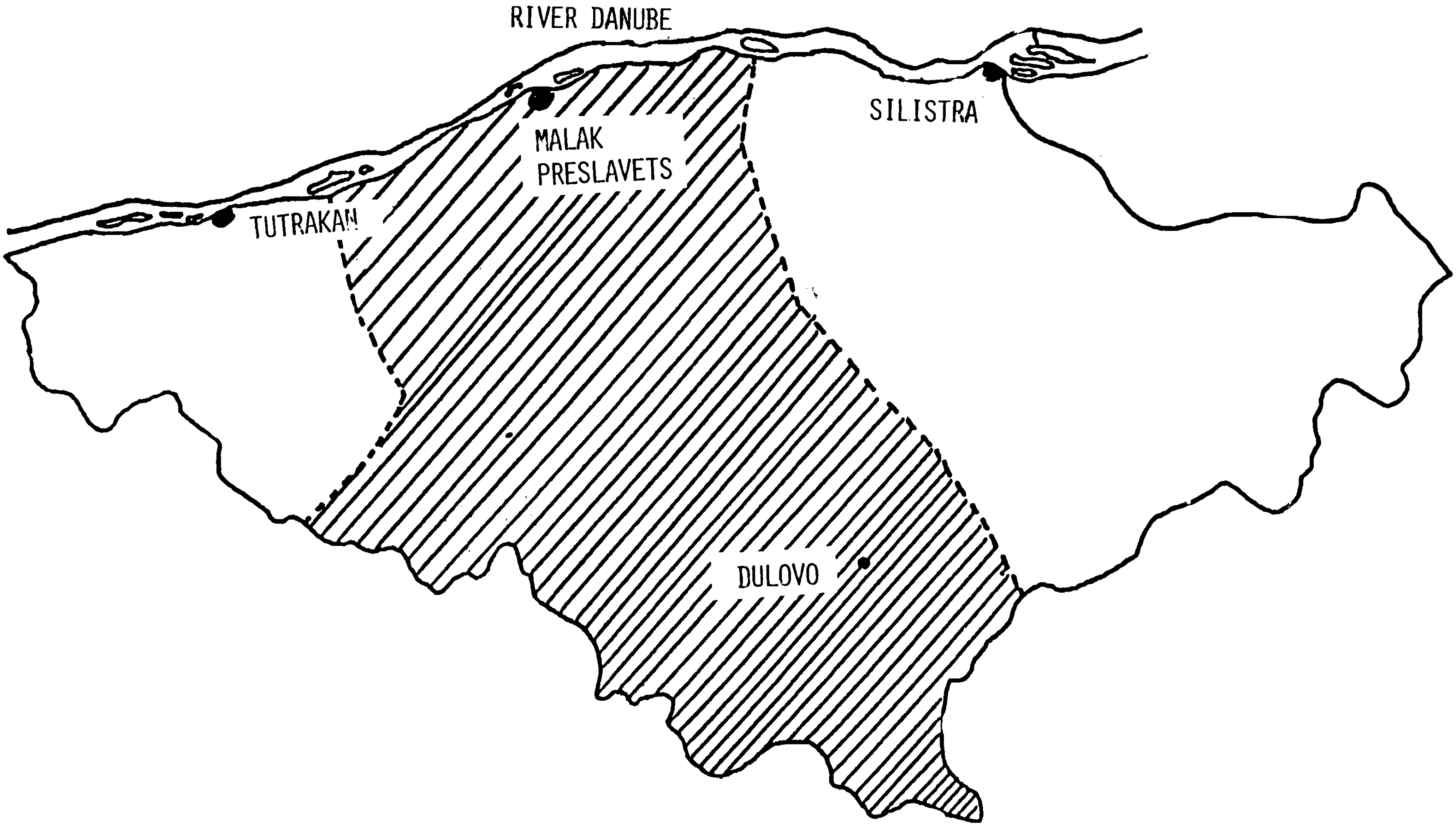

Figure 1. The three water supply systems of the Silistra region.

Tutrakan system is almost completed, it does not require modeling. Finally, the topography of Silistra subregion makes it unsuitable for reservoir construction. However, the locations of canal tracks have already been decided upon and the development of a water supply system is merely a matter of construction.

The detailed scheme of the water supply system modeled is shown in Figure 2. It consists of three reservoirs, six pumping stations, ten canals, and nine distributing canals. The following notation is introduced to formally describe the model.

w_j^i is the irrigation water flow for irrigated area j in time period i ($i=1,\ldots,8$; $j=1,\ldots,12$);

y_s^i is the water flow in canal s ($s=1,\ldots,10$) in time period i;

x_p^i is the water flow in distribution canal p ($p=1,\ldots,9$) in time period i;

S_k^i is the active water storage in reservoir k ($k=1,2,3$) in time period i; and

τ^i is the length of time period i.

Obviously, the water supply system first on water demands w_j^i, which are assumed to be predetermined and should correspond with the actual water demand by the end of the planning period (1990). One year was divided into nine time periods (Figure 3). While modeling, the 3-month time period of December, January, and February was omitted because the whole water supply system does not operate during these months. For each irrigated area the general irrigation timetables were specified (Figure 3). The sixth--the first 10 days of August--is the most intensive irrigation period for all areas.

The water resources available were considered as unlimited because of the abundance of water at the Silistra site of the Danube. This allowed consideration of within-year regulation of water resources only. Since the Silistra region is small, the transit time delays for canals were not taken into account.

The first set of constraints reflects the balance relations between the predetermined water demands for different irrigated areas and the water flows in canals:

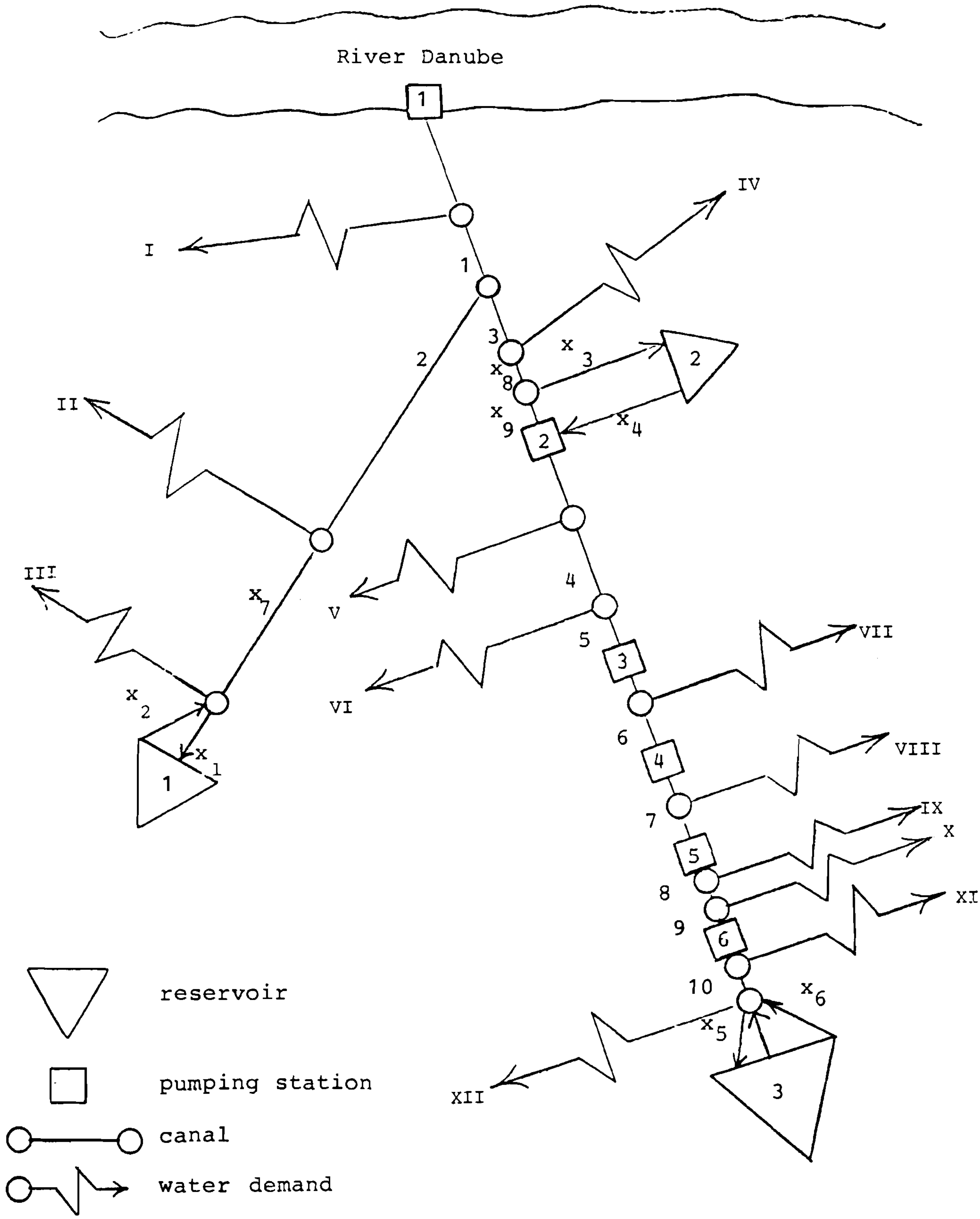

Figure 2. Scheme of the water supply system (I-XII represent the areas irrigated; x_1-x_9 represent the water flows in different distribution canals).

Number of period (i)	0	1	2	3	4	5	6	7	8
Months	Dec Jan Feb	Oct Nov March April	May	First 20 days of June	Last 10 days of June	July	First 10 days of Aug	Last 20 days of Aug	Sept
Length of period in months (τ^i)	3	4	1	$\frac{2}{3}$	$\frac{1}{3}$	1	$\frac{1}{3}$	$\frac{2}{3}$	1
	Out of operation	No irrigation					The most intensive irrigation		

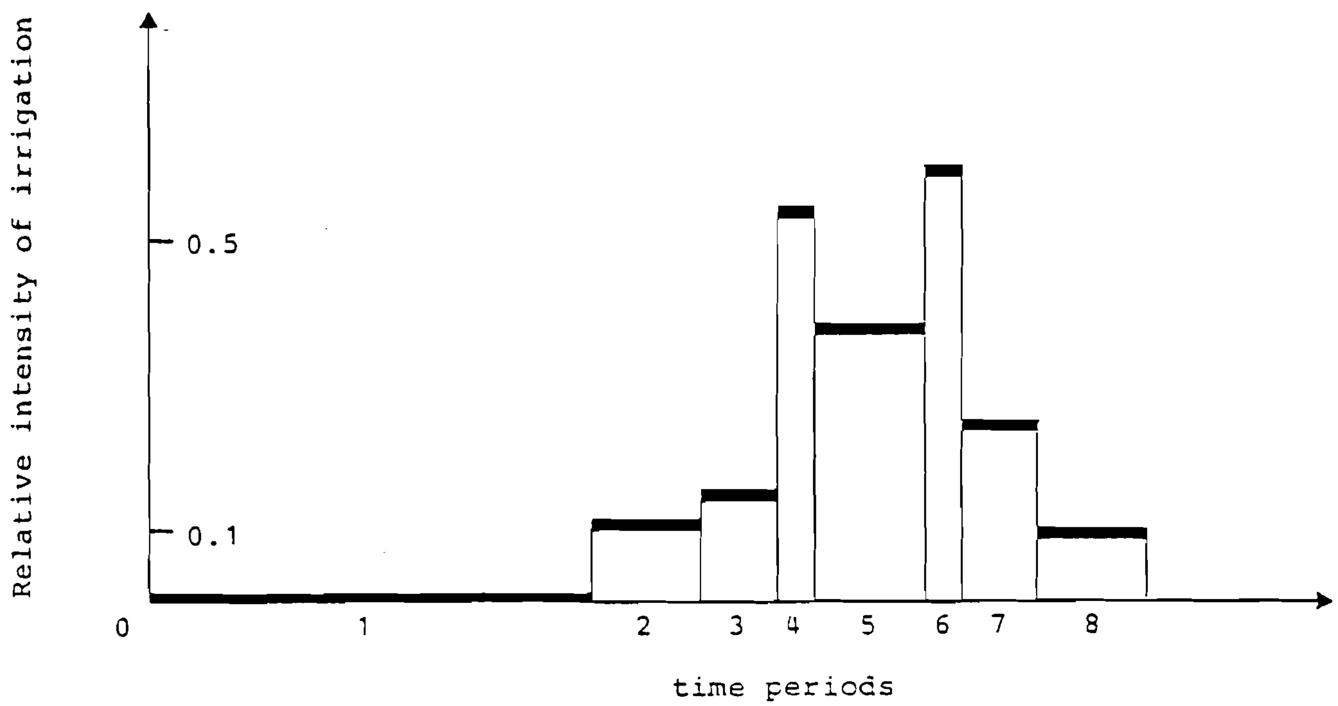

Figure 3. Time periods and a timetable for irrigation.

$$\left\{\begin{array}{ll} y_1^i - y_2^i - y_3^i = w_1^i \quad , & i = 1, \ldots, 8 \quad , \\ y_2^i - x_7^i = w_2^i \quad , & \\ x_7^i + x_2^i - x_1^i = w_3^i \quad , & \\ y_3^i - x_8^i = w_4^i \quad , & \\ x_9^i - y_4^i = w_5^i \quad , & \\ y_{r-2}^i - y_{r-1}^i = w_r^i \quad , & r = 6, \ldots, 11 \quad , \\ x_6^i - x_5^i + y_{10}^i = w_{12} \quad . & \end{array}\right. \tag{1}$$

The water storage in reservoirs is described as follows:

$$\left\{\begin{array}{l} S_k^{i+1} = S_k^i + \tau^i (x_{2k-1}^i - x_{2k}^i) \quad , \\ \qquad k = 1, 2, 3 \quad , \quad i = 1, \ldots, 7 \quad , \\ S_k^1 = S_k^8 + \tau^8 (x_{2k-1}^8 - x_{2k}^8) \quad . \end{array}\right. \tag{2}$$

The second relation in equation (2) emphasizes that the water stored in any reservoir at the beginning and end of the same year must be equal--a condition of the annual cycle.

The following set of constraints reflects the fact that the release of water from any reservoir cannot be greater than the volume of water stored therein.

$$\begin{array}{l} \tau^i x_{2k}^i - S_k^i \le 0 \quad , \\ \qquad k = 1, 2, 3 \quad , \quad i = 1, \ldots, 8 \quad . \end{array} \tag{3}$$

Finally, some obvious physical constraints are

$$x_3^i - x_8^i \leq 0 \quad , \quad i = 1, \ldots, 8 \ ,$$

$$x_9^i + x_3^i - x_4^i - x_8^i = 0 \quad ,$$

$$x_p^i \geq 0 \quad , \qquad p = 1, \ldots, 9 \tag{4}$$

$$y_s^i \geq 0 \quad , \qquad s = 1, \ldots, 10$$

$$s_k^i \geq 0 \quad , \qquad k = 1, 2, 3.$$

As stated above, one of our modeling objectives is to find the least-cost water supply system. In the model under analysis the measure of total costs associated with the establishment and operation of the water supply system is the reduced annual cost associated with

-- the establishment of reservoirs, pumping stations, and canals;
-- losses resulting from the submerged arable land;
-- the operation of reservoirs, pumping stations, and canals; and
-- electrical energy required for pumping stations.

Thus, the objective function E can be written as follows --

capital and operating costs of reservoirs:

$$E = \sum_k c_k \max_i s_k^i \ ;$$

capital and operating costs of canals and pumping stations:

$$+ \sum_s a_s \max_i y_s^i + a_{11} \max_i x_6^i + a_{12} \max_i x_9^i \ ; \tag{5}$$

costs of electrical energy for pumping stations:

$$+ \sum_{\alpha,i} e_\alpha \tau^i y_\alpha^i + e_2 \sum_i \tau^i x_9^i \quad , \qquad \alpha = 1, 5, 6, 7, 9 \ .$$

Thus, the mathematical model for the water supply system consists of the set of relations (1) - (5). The quantities y_s^i, x_p^i, S_k^i are decision variables. The water demands w_j^i and cost coefficients c_k, a_s, e_α are initial data provided by SIWP. By introducing auxiliary variables, the objective function E can easily be reduced to a linear function of decision variables. From the mathematical point of view, this means that the model presents a linear programming problem.

The version of the Silistra water supply model described above was developed and implemented on the Pisa IBM 370 at IIASA at the beginning of 1979.* It was then transferred to the ICL 1904 installation at SISM. The results of running the model on both computers are more or less consistent and some of them are given in Table 1. Some of the parameters and the reduced annual costs can be used for hydrotechnical calculations of the whole water supply system.

A sensitivity analysis was performed for basic parameters with respect to the variations in water demands w_j^i and cost coefficients c_k, a_s, e_α. The model runs indicated that the optimal basic parameters are sensitive to water demands. When varying coefficients c_k and a_s, it was mainly the variations in price of land that were taken into account. This is an interesting problem, because the price of land is determined by rather subjective values. It is sufficient to say that for some reservoirs and canals, approximately 60 percent of the total capital costs stem from submerging the land. Running the model indicated that the basic parameters of the water supply system are virtually insensitive to a decrease in land price of up to 50 percent. The same results were obtained when increasing the price of electrical energy (or coefficients e_α) by up to 50 percent. Some results of sensitivity analysis are shown in Figure 4.

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*The program for running the model was prepared by Andras Por.

Table 1. Basic parameters of the water supply system (lv represents leva).

Danube pumping station	--	38.0	m^3/sec
Canal 2	--	15.8	m^3/sec
Canal 3	--	21.3	m^3/sec
Reservoir 1	--	$5.2x10^6$	m^3
Reservoir 2	--	$2.5x10^6$	m^3
Pumping station 2.	--	21.8	m^3/sec
Pumping station 3	--	16.4	m^3/sec
Pumping station 4	--	15.0	m^3/sec
Pumping station 5	--	12.4	m^3/sec
Pumping station 6	--	10.9	m^3/sec
Reservoir 3	--	$131.7x10^6$	m^3
Canal from reservoir 3	--	38.6	m^3/sec
Reduced annual cost	--	$29.4x10^6$	lv/year

The results of the sensitivity analysis can be used to quantify the accuracy requirements for initial data (i.e., we require very accurate data on water demands w_j^i). On the other hand, it is fortunate that the model is fairly robust with respect to the variations in cost coefficients c_k, a_s, e_α since they are the main source of uncertainty for the problem of the Silistra water supply. In other words, we may reduce the accuracy of some economic data.

Another group of results obtained are related to interfacing the water supply model with other regional models, especially with the agriculture model. These results are presented in Table 2. Coordination of the regional models is mainly realized through the mechanism of marginal water costs distributed spatially.

Two types of marginal water costs--seasonal and mean annual--were evaluated when modeling. By definition, the seasonal unit

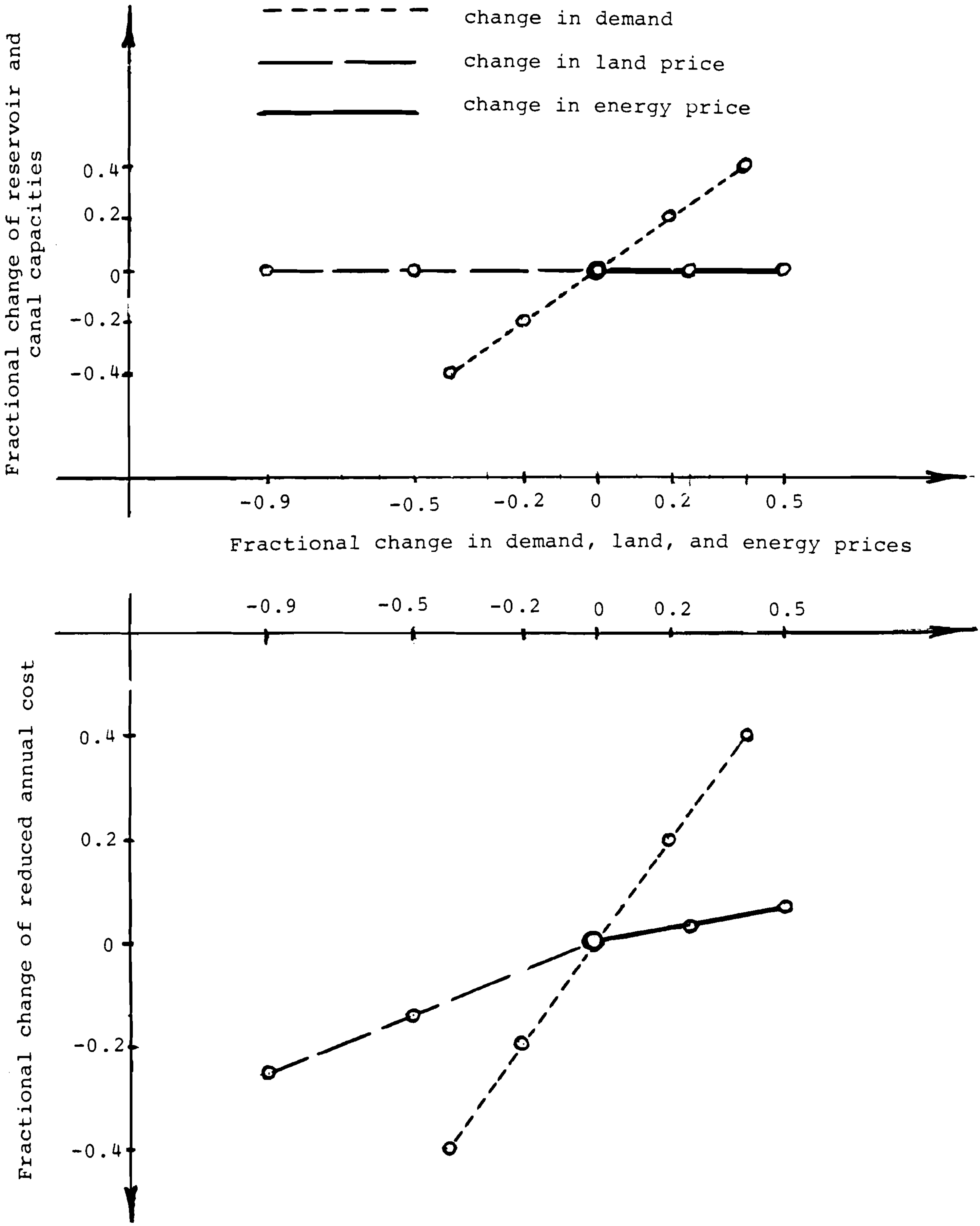

Figure 4. The sensitivity of the system to data variations.

Table 2. **Seasonal and annual marginal** water costs (10^{-2} lv).

PERIOD / AREA IRRIGATED	2	3	4	5	6	7	8	MEAN ANNUAL
I	0,59	0,59	2,89	1,01	10,7	0,59	0,59	2,67
II	0,59	0,59	2,89	2,89	12,8	0,59	0,59	3,57
III	0,59	0,59	2,89	2,89	12,8	0,59	0,59	3,53
IV	0,59	0,59	3,77	3,77	10,7	0,59	0,59	3,6
V	0,63	0,63	3,81	3,81	15,9	0,63	0,63	4,54
VI	0,63	0,63	3,81	3,81	23,9	0,63	0,63	5,69
VII	0,66	0,66	3,84	3,84	38,8	0,66	0,66	8,03
VIII	0,75	0,75	5,58	5,58	49,5	0,75	0,75	10,5
IX	1,82	1,95	5,63	5,63	49,6	5,63	1,62	11,5
X	1,82	1,95	5,63	5,63	49,6	5,63	1,95	11,7
XI	2,16	2,16	5,84	5,84	49,8	5,84	2,16	11,8
XII	2,16	2,16	5,84	5,84	14,7	5,84	2,16	6,16

cost p_j^i of water in irrigated area j is the increment of the optimal value of objective function E caused by the unit increment of water consumption in irrigated area j in time period i. The seasonal unit water costs obtained by running the model, shown in Table 2, depend essentially on the geographical location of the irrigated area and the season of water consumption.

The mean annual unit cost m_j of water for irrigated area j was determined as follows. The additional unit of water consumed by area j in a year was distributed over all the time periods as follows:

$w_j^i + \Delta w_j^i$ is the varied water demand for irrigated area j in time period i,

$$\Delta w_j^i = \frac{w_j^i}{\sum_i \tau^i w_j^i} \quad . \tag{6}$$

By definition m_j is the increment of the optimal value of objective function E when varying the water demand only in area j according to equation (6). As shown in Table 2, the mean annual unit cost of water varies about 4.5 times, depending on the location of the irrigated area.

How can these marginal water costs be used to coordinate the water supply model and the agriculture model? The principal scheme of interaction of the two models is shown in Figure 5, in which it is evident that the agriculture model uses marginal water costs as one of its inputs and gives water demands as an output. For the water supply model, water demands are the main input and the marginal water costs are one of its outputs. To start such an iterative process, the water demands or marginal costs must be set a priori and the process should be repeated until it converges. At present, the collaborative efforts of IIASA and SISM are directed towards the numerical realization of such an interaction.

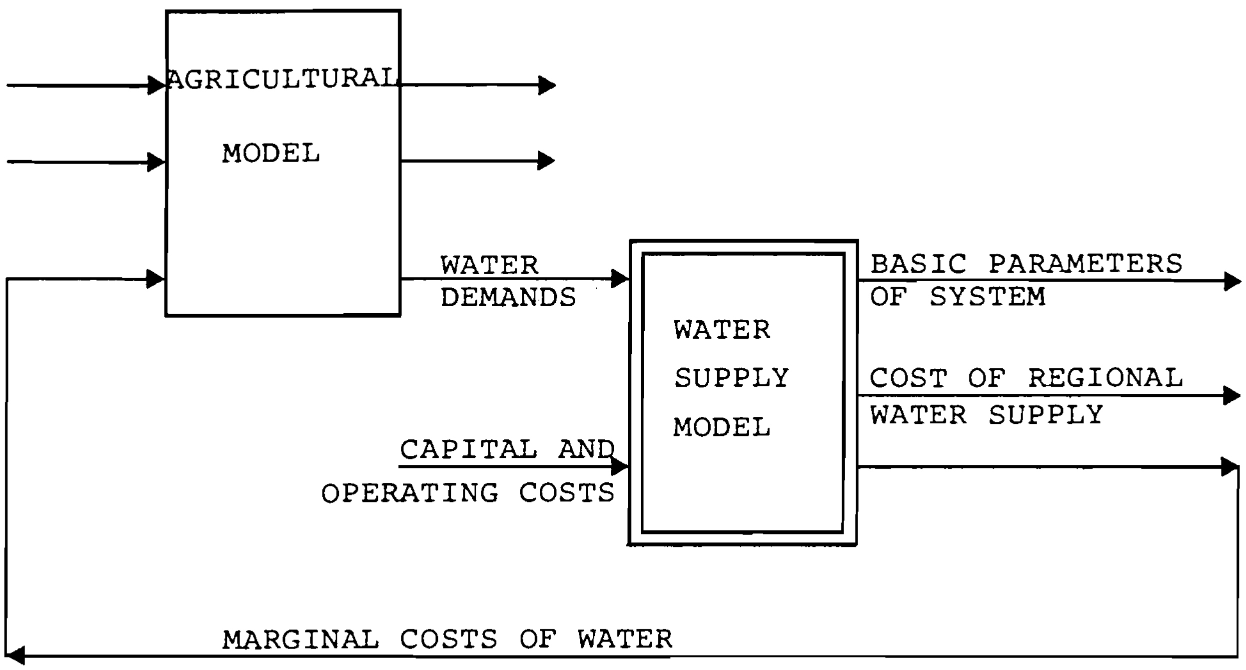

Figure 5. Interfacing the water supply model with the agriculture model.

MODELING THE EXPANSION OF THE WATER SYSTEM IN THE UPPER NOTEC REGION (PRELIMINARY RESULTS)

M. Makowski

1. INTRODUCTION

The work documented in this paper forms a further stage in the modeling effort being carried out within the framework of the Polish governmental program PR-7, which deals with water resource development. This stage of the modeling activity is directed towards developing a computer-based model (referred to as MWD) for use in planning the expansion of a large multi-basin surface water resource system in an agricultural region --the Upper Notec region, Poland.

The model will provide information useful for planning the size and location of irrigated fields as well as for determining the optimal configuration of the water system, which is composed of several rivers, reservoirs, and transfers of water both within and outside the region. It is assumed that the water system will meet the demands of all users (i.e., not only agricultural, but also municipal and industrial users). The demands of the municipal and industrial groups are considered here as exogenous decision variables, whereas agricultural water demand is optimized within the model. Thus, MWD is aimed at solving problems of water demand and supply. Although the model is being developed specifically for the Upper Notec region, it will be suitable for application in other regions. Its assumptions are given in Makowski (1978).

The Upper Notec region is described in detail in Albegov and Kulikowski (1978a and 1978b). It is a diversified agricultural region of about 6,000 km^2, 1,100 of which require irrigation. Existing water sources cannot meet the demands of the irrigation plan proposed for the region. However, the region offers possibilities for the construction of over 20 reservoirs and for water transfers within the region and from the Vistula river.

Despite the agricultural character of the region, the possibilities for industrial development should be examined. It is assumed, however, that a heavy water-consuming industry will not be located here. Since the industrial and municipal surface water requirements are expected to be small in comparison with those of agriculture, the inclusion of a water demand function for agriculture only seems to be reasonable.

For planning purposes, the region has been divided into 12 subregions (Figure 1), out of which 11 are connected by a water system. The structure of the whole watershed can be represented by a network whose nodes are subregions; see Figure 2 in which the single arrows represent water flows in rivers and the double arrows represent water transfers. Numbers near the nodes correspond to the number of reservoirs that could possibly be constructed in a given subregion.

2. MWD AS A PART OF THE MODEL SYSTEM

The model presented in this paper is intended to form part of the system proposed by Kulikowski (1978), which will be used for comprehensive planning of the whole region. Several models of regional subsystems are currently being developed and it is hoped that at least one of them will be linked with MWD; see the papers by Albegov et al., Kulikowski and Krus, Kulikowski, and Podkaminer et al. included in these Proceedings, and also Sosnowski and Tolwinski (1979). For MWD to be implemented, it should be linked to a model of regional agriculture. It is

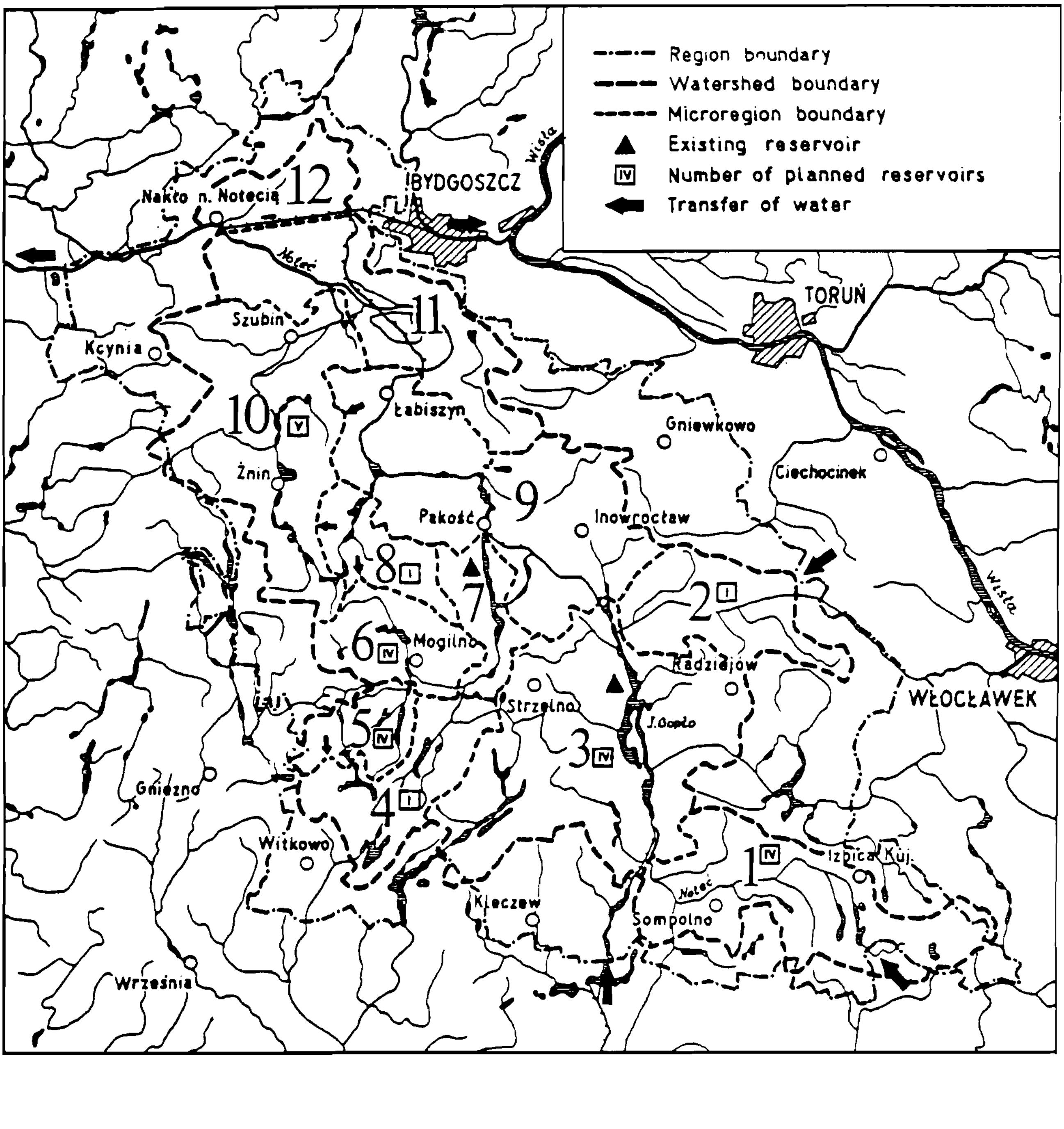

Figure 1. The Upper Notec region.

therefore expected that the model described by Albegov et al. and/or the model proposed by Podkaminer et al. will be directly linked with it. The problem of linkage is briefly discussed in the concluding section.

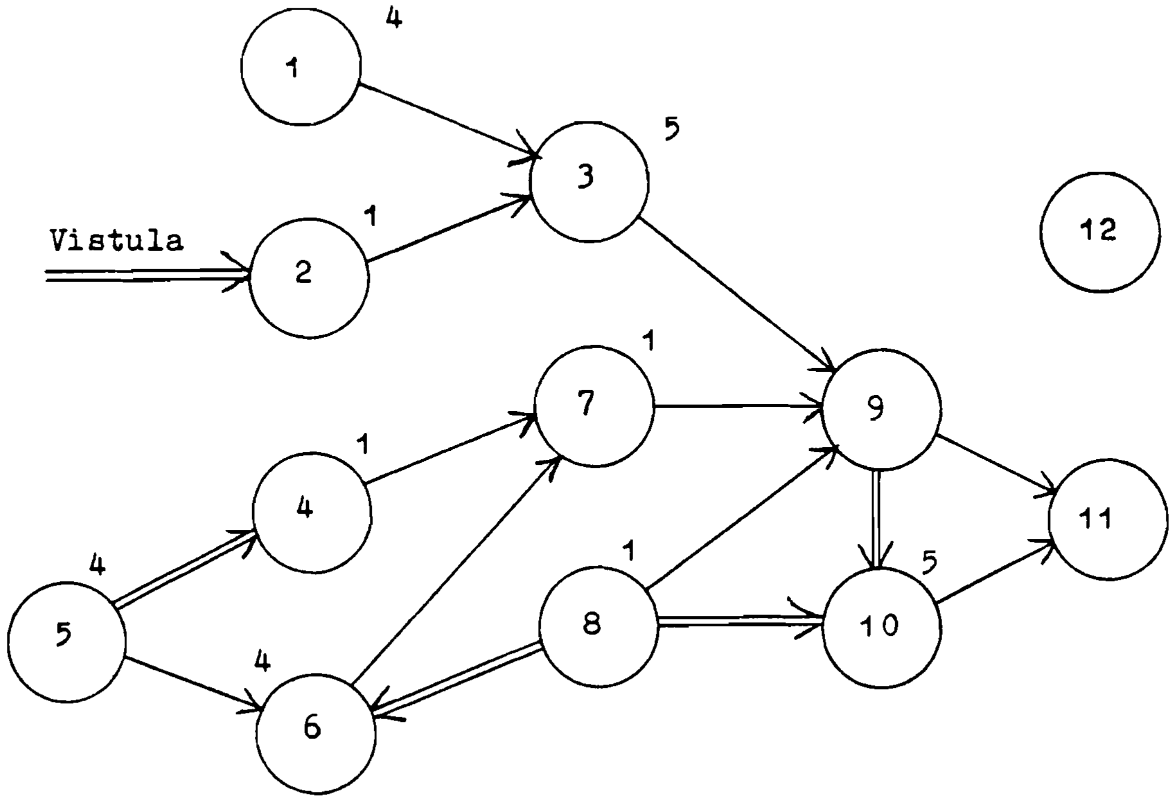

Figure 2. The structure of the watershed (← water flows in rivers; ⇐ water transfers; the number inside node is subregion, the number outside is the potential number of reservoirs).

2.1. Structure of the Model

As proposed in Makowski (1978), MWD will be composed of four interlinked submodels: MWD0, MWD1, MWD2, and MWD3. Two of these, MWD0 and MWD3, will be developed at a later stage.

In this paper the results of running MWD1 and MWD2A are discussed. MWD1 is used to find the optimal allocation of water resources among subregions. MWD2A is a simplified (linearized) version of MWD2, which is used to determine the optimal structure of the water supply system.

We decided to use a simplified and faster linear version of MWD2 in order to make a preliminary evaluation of several scenarios, some of which will be selected for more detailed study. It is expected that the MWD submodels will be used in many scenario analyses and also in iterative procedures together with at least an agriculture model. Thus, we have tried to create a model of water demand and supply that would produce an evaluation of a given scenario in less than 1 minute.

After linkage of MWD1 and MWD2A we can determine

-- the optimal irrigation network for given disposable resources (using only current flows or rivers,or given water transfers and/or reservoirs);
-- the optimal supply for a given irrigation network; and
-- the optimal irrigation network and water supply system.

Thus, the model can be used as a demand-oriented, supply-oriented, or demand/supply-oriented model.

2.2. Model Use

The model is designed to provide information useful for decision making. Hence, it will be used to analyze several scenarios rather than to derive a single, optimal solution. Scenarios are classified as follows:

-- those for determining an optimal solution for various values of exogenous variables and model parameters (e.g., crop structure, required irrigation flow, maximum area requiring irrigation in each subregion, value and increase of crop production, interest rate);
-- those for evaluating the consequences that result from using suboptimal values of decision variables such as given irrigation areas in particular subregions or a

given reservoir capacity. Additional constraints may also be introduced (e.g., those related to investment volume or minimum area requiring irrigation). Scenario analysis with a comprehensive examination of results plays an essential role in the use of the model.

2.3. Structure of the Computer Program

Since MWD is to serve many purposes, it will be composed of a set of subroutines that may be used for different configurations depending on the requirements. Special care has been taken to have a flexibly organized collection of packages that will allow efficient memory use. All segments are written in FORTRAN, with the exception of MEMORF (in COMPASS), and the following are used in most applications:

MRW is the main program, which drives all routines;
MEMORG performs computer memory arrangements during executions of a program;
DATAIN reads data and performs its initial processing and aggregation;
RESIN reads data concerned with reservoirs and tranfers;
DPRINT prints data and indices that can be computed from initial data;
TRANS arranges the data for a small test run;
BOUND computes the value of constraints;
GOALF computes the value of a goal function; and
REPORT interprets the results.

For the MWD1 and MWD2 submodels, the PDQLP Fast Linear Programming System designed on Control Data Computer System has been adopted. To enable the LP routine package within MRW to be efficiently used, the PDQLP program has been modified and now acts as a subroutine linked to MRW. Three segments have been created for linking MRW and PDQLP:

MATGEN generates a matrix for the LP problem using data provided by DATAIN;

NAMES generates names of rows and columns; and
DECODER decodes the LP results.

By using the applied approach, the time-consuming process of initial processing and preparation of data for medium-size LP problems is avoided, and it is possible for nonspecialists to use the package of programs. Note that by using a subroutine generating a matrix for the LP program, to change one parameter (which may result in a considerable change in the LP matrix, thus requiring the rearrangement of whole matrix) now needs replacement of only one record in the input stream. Moreover, that record is easy to find and interpret. Thus, the process of performing various scenario analyses has become really simple, especially if data are stored in the form of an UPDATE library.

Other segments are being prepared for both linear--with regularization--and nonlinear problems (see concluding section). All segments are implemented on a CDC CYBER 73-16 computer.

3. GENERAL MATHEMATICAL DESCRIPTION

3.1. Decision Variables

The decision variables are classified into two groups.

3.1.1. *Decision Variables that are Subject to Optimization*

In MWD1, the decision variables subject to optimization, are the areas to be irrigated in ith subregion f_i. In MWD2A, they are represented by capacities of kth reservoirs (denoted by v_k) and technical possibilities of lth water transfers (denoted by x_l).

3.1.2. *Exogenous Decision Variables*

The following variables belong to the group of exogenous decision variables: crop structure, maximum areas subject to irrigation in each subregion, irrigation flows (compare Makowski 1978), parameters for statistical evaluation of data concerning natural flows, increase in crop production as a result of irrigation, crop prices, interest on capital, municipal and industrial water use.

3.2. Objective Function

The goal functions used as a measure of performance for evaluating different scenarios, are given below.

The goal function for MWD 1 can be formulated as

$$H(F) = H(f_1,f_2,\ldots,f_N) = \sum_{i=1}^{N} cn_i f_i \quad , \tag{1}$$

where

$H(F)$ is the goal function for MWD1;

f_i is the area to be irrigated in the ith subregion;

N is the number of subregions; and

cn_i is the expected income of the ith region after irrigation.

The method of calculating cn_i is described in Makowski (1978). Therefore we only need to mention that the following factors are considered: increase in crop production as a result of irrigation, crop prices, additional agricultural costs related to irrigation, investments required for irrigation technology and water intakes, depreciation rate, interest rate, operational costs. In other words, the cn_i are equal to a difference between annual income resulting from irrigation and annual irrigation costs. Note that no water costs are involved. Increased production from irrigation allows the introduction of other types of production in fields that are no longer needed for less efficient crops, we have therefore decided to include additional income. The latter calculation will be referred to as "full income", whereas the former is considered as "basic income". Since calculation of cn_i is expected to be discussed further, we have performed a scenario analysis using both sets of cn_i.

In determining cn_i, it is assumed that an agricultural water user will not pay for the water consumed. But obviously the cost of providing water should be considered when the water system expansion has been decided upon. Thus, the following goal function is proposed for MWD2:

$$L(V,X,F) = H(F) - \sum_{k=1}^{K} R_k(v_k) - \sum_{l=1}^{L} S_l(x_l) \quad , \tag{2}$$

where

V,X denotes vectors of reservoir capacities and water transfer volume;

K,L are the numbers of reservoirs and water transfers under consideration; and

R_k,S_l are the annual costs of reservoirs and water transfers (Makowski 1978).

For MWD2A, the goal function may be written as

$$L(F,V,X) = \sum_{i=1}^{N} cn_i f_i - \sum_{l=1}^{L} cnt_l x_l - \sum_{k=1}^{K} cnr_k v_k \quad , \tag{3}$$

where

cnt_l is the annual cost of the ith transfer of 1 m^3/sec; and
cnr_k is the annual storage cost for 1 m^3 in the kth reservoir.

3.3. Constraints

Most of constraints reflect a restriction on the water balance for every 10-day period and for every subregion. Formal presentation of all these constraints does not help to clarify the problem (see Makowski 1978). We start by assuming that the initial disposable flow for irrigation QDP in the ith subregion and the tth period is known. QDP are computed as differences between natural flow and the amount of water used by industrial and municipal users, taking into account minimal water flow governed by biological constraints and water that is withdrawn and returned to the system.

The irrigated areas f_i are constrained by the following two sets of inequalities:

$$0 \leq f_i \leq fm_i \quad , \tag{4a}$$

$$\sum_{j \in K_i} ql_{jt} f_j \leq qdp_{it} + \sum_{j \in K} qz_{it} + \sum_{l \in K_i} x_l \quad , \tag{4b}$$

where

K_i is a set of indices of regions that are located in regions upstream relative to the ith region;

ql_{jt} is the average irrigation flow for jth region and tth 10-day period computed for a given crop structure and their irrigation timetables (Makowski 1978);

qz_{it} is the water released from reservoirs and used in ith region;

qdp_{it} is the initial disposable flow;

x_l is water transferred to the lth region (if water is transferred from lth region, then x_l is negative); and

fm_i is the area liable to irrigation in ith region.

The volumes of water transfers x_l are exogenous variables for MWD1. Additional flow from reservoirs qz_{it} should be such that more water is not used during the whole vegetation period than has been stored in the previous season in all reservoirs located upstream; i.e., the following inequality should hold,

$$\sum_{t=IV}^{KV} 10 \cdot 24 \cdot 3600 \cdot qz_{it} \leq \sum_{j \in L_i} v_j \quad , \tag{5}$$

where

v_j is the filling of jth reservoir;

L_i is a set of indices of reservoirs that are located in regions upstream relative to ith region; and

IV, KV are the numbers of the first and last 10 days of the vegetation period, respectively.

Finally, constraints on maximal filling of reservoirs and on water transfer capacities are also included.

Since we try to determine the final structure of irrigated areas and the water supply system in preliminary runs, no constraints on the total amount of investment have been imposed. Should the amount of investment be unacceptable, the relevant constraints may be included.

4. ANALYSIS OF PRELIMINARY RESULTS

4.1. Data

All data for preliminary runs were provided by the Design Agency BIPROMEL. They will be refined in the near future, particularly those data used for determining goal function parameters and those of irrigation flow. The results presented in this paper should not be considered as final, since they will be discussed at many institutions in Poland together with the method of goal function calculation and the data used.

Data used by MWD occupy about 300 records (80 column punch cards); therefore, only a short draft of data and parameters is given (see Appendixes) to illustrate the problem of initial disposable flows and unit costs of water transfer or storage.

Analysis of QDP (initial disposable flows) that ranges from a negative value (a deficit of water already occurs) up to 31.3 m^3/sec indicates that it is really necessary to consider both a spatial and temporal allocation of water. Costs of unit water transfer or storage also differ considerably according to the various projects.

Cultivation of grain, rape, beet, potatoes, fiber crops, pulses, maize, fodder crops, vegetables, fruits, grass, and two types of forest were considered, with respect to irrigation (see Appendix D).

4.2. Scenarios

Two groups of scenarios have been examined. Each scenario is denoted by an alphanumeric character, where the first letter indicates the cn (parameter of the MWD1 goal function) calculation. A refers to full income,whereas B indicates that basic income has been used (compare section 3.2).

In each group the following scenarios were examined:

1. Irrigation of all areas requiring irrigation was assumed using water from current flows only.
2. Each subregion tries to perform a locally optimal strategy without consideration of linkages with other regions.

3. Only current flows may be utilized for optimal water allocation.
4. Same as 3rd scenario, but use of 2 existing reservoirs is allowed.
5. Same as 4th plus possibility of construction of 24 additional reservoirs.
6. Same as 5th, with a transfer of water from the Vistula river, and 4 intraregional water transfers are examined.

4.3. Discussion of Results

Selected results are listed in Table 1. Scenarios A1, A2, B1, and B2 are known to be infeasible, but they provide useful information.

For irrigation of all liable areas, it is necessary to supply over 418 mln m^3 of water annually in addition to that which is available from rivers. This is simply infeasible since the total capacity of all planned reservoirs only amounts to 211.87 mln m^3 and the water transfer from the Vistula totals approximately 95 mln m^3. It is expected that the annual income resulting from irrigation (assuming zero price for water) will equal 2,775 x 10^6 złoty (1 U.S. \$ = 32 zł, according to the official exchange rate), if full income is calculated. Information about an increase in production of particular crops is also available.

Scenarios A2 and B2 show what would happen if every subregion attempted to perform a locally optimal strategy without examining the consequences that occur downstream. These results may also serve as an argument for establishing institutional arrangements for the operation of a whole water system.

Let us now briefly discuss results of the eight feasible runs. It should be noted that the scenarios of groups A and B determine total areas that differ very little in comparison with the huge differences between respective cn_i, which are listed in Table 2. Although the total areas for scenarios A are greater than those for scenarios B, much greater differences were expected.

Table 1. Aggregated results.

Scenario	F(ha)[a]	F/FM[b]	H[c]	L[c]	V[d]	Remarks
A1	115,310	1.000	2,775	-	-418	infeasible
A2	39,028	0.338	994	-	-57	infeasible
A3	21,280	0.185	569	-	0	
A4	41,804	0.363	1,087	1,070	67	
A5	58,448	0.507	1,523	1,309	205	
A6	83,777	0.727	2,110	1,887	206	+95 mln m^3 from Vistula
B1	115,310	1.000	946	-	-418	infeasible
B2	39,028	0.338	317	-	-57	infeasible
B3	20,332	0.176	189	-	0	
B4	41,797	0.362	353	336	67	
B5	50,321	0.436	435	396	129	
B6	77,959	0.676	689	592	129	+95 mln m^3 from Vistula

[a] F is the sum of irrigated areas (in ha).

[b] FM is the sum of all areas requiring irrigation.

[c] H and L (in zł x 10^6) are the goal function values for MWD1 and MWD2, respectively.

[d] V (in m^3 x 10^6) is the water volume used from reservoirs (if negative denotes a deficit).

A comparison of scenarios A3 and A4 (B3 and B4) indicates that both income and areas irrigated may be doubled if only two existing reservoirs are used optimally. To increase those indices by about the same values (see A5 and B5), almost all of the other 24 reservoirs should be constructed with approximately double the capacities of the two existing reservoirs. This may be easily explained if it is pointed out that the former reservoirs are to be used only in "critical" 10-day periods. However, with increasing water consumption, the critical period is extended.

Table 2. Values of cn coefficients (in zł/ha).

Subregion	A	B
1	25,890	8,648
2	18,432	8,545
3	27,883	9,845
4	17,054	2,768
5	20,437	5,194
6	24,895	10,690
7	22,775	8,379
8	15,663	3,431
9	24,579	9,714
10	24,063	9,039
11	27,734	5,921

Scenarios A6 and B6 proved that the transfer of water from the Vistula is profitable despite the fact that it is the most expensive transfer (compare data in Appendix C and in Table 3). Such a solution should be expected, since the water that might be transferred within the region could also be used without being transferred. However, if water is transferred, both transfer and storage costs usually have to be considered and, hence, very often conveyance is not profitable.

It is interesting to note that not all the possibilities for water transfer and storage are realized. The amounts of water transferred,and the planned capacities of reservoirs that are not fully utilized in all scenarios, are listed in Table 3. Reservoirs that are to be fully utilized are not listed (their parameters are given in Appendix B).

From an analysis of the presented results, the following conclusions can be formulated.

1. It is impossible to irrigate all areas that require irrigation assuming the given crop structure and irrigation inflow, even if all reservoirs and the water transfer from the Vistula are built according to data presented in Appendix B. The reason being that the proposed locations of reservoirs and water transfers

Table 3. Utilization of reservoirs and transfers.

Water Storage or Transfer[a,b]	Max[c]	Cost[d]	A5	A6	B5	B6
T99I02	10.00	0.54	-	max	-	max
T08I10	5.00	0.23	-	0.69	-	0
T08I06	5.00	0.20	-	0	-	0.33
T05I04	2.00	0.11	-	0	-	0
T09I10	3.00	0.17	-	max	-	max
8Folusz	79.00	2.32	72	73.1	0	0.596
3Slawsko	0.32	1.91	max	max	0	0
5Mlynek	3.23	2.12	max	max	0	0
6Padniewo	0.20	2.02	max	max	0	0
6Izdby	0.20	1.00	max	max	max	0

[a] Txxlyy signifies a transfer of water from region xx (e.g., 99 stands for Vistula) to region yy.

[b] The names refer to reservoirs and the preceding numbers correspond to the region in which the reservoir is located.

[c] Max signifies maximal value of an annual transfer or storage of water in mln m^3.

[d] Cost signifies the annual cost for 1 m^3 of water storage or transfer.

do not provide the conditions necessary to meet the water demand. One may try to examine the possibility (and efficiency) of irrigation of all fields by increasing transfers of water from outside the region and/or by changing the location and size of reservoirs. The latter approach does not seem feasible.

2. An increased water transfer from the Vistula may be profitable. It would provide water more cheaply than from the Folusz reservoir. Thus, it is recommended that the possibilities and costs of increasing a transfer up to 20 or 30 m^3/sec should be examined. It is expected that a greater water transfer may cause a decrease in the number of reservoirs. Since the Folusz reservoir is the most expensive (and also the largest), its construction may be cancelled. Costs of reservoirs differ considerably (from 0.15 up to 2.31 zł/m^3); hence, others may also become unprofitable.

3. An additional evaluation of a policy for replacing expensive reservoirs with cheaper water transfers should be performed. Such a replacement will make the value of the goal function increase. On the other hand, the construction of reservoirs might be justified for other reasons, i.e., flood control purposes or the need for water storage.

4. Intraregional water transfers are not fully utilized. However, it is recommended that this option should be reexamined if it makes possible an increase of water.

5. The difference in goal function values for MWD1 and MWD2 (H and L, respectively) is relatively small for almost all relevant scenarios. It shows that the water cost is insignificant, at least in relation to the scenarios considered in this paper. The situation may change if greater water resources are available.

Let us now comment briefly on the results of water allocation, which is equivalent to considering the locations of irrigated fields (Table 4). It is not surprising that water allocation closely follows changes in cn_i coefficients. One may compare the results given in Table 4 with cn_i structure (Table 2). Note that the irrigation of fields in subregions 4 and 8 is not sufficiently profitable for the available water resources to be used. A small share of the fields in subregion 5 is irrigated only in one scenario (A3). The results discussed illustrate a broader problem that is mentioned below.

Let us ignore for a moment the difference between A and B group scenarios. The cn coefficients are determined by a crop structure that reflects a rational agricultural policy. A decrease in the area of irrigated fields (according to scenario A6, which has the largest irrigation project, only 72.7 percent of the area requiring irrigation is irrigated) may result in substantial changes in crop structure. Thus, another run of MWD would be required. Hence, because MWD appears sensitive to crop structure, one should link MWD with an agriculture model. The latter would determine an optimal crop structure for a given amount of water disposable for irrigation. Note also that

Table 4. Irrigated areas (in ha).

Sub-region / Scenario	1	2	3	4	5	6	7	8	9	10	11
A1 and B1	7,665	7,390	19,911	6,640	4,220	8,740	5,870	5,020	16,350	20,330	13,174
A2 and B2	230	141	2,283	356	688	955	4,366	39	16,350	436	13,174
A3	230	0	2,114	0	688	0	0	0	5,074	0	13,174
A4	230	0	7,279	0	0	955	3,380	0	16,350	436	13,174
A5	230	0	16,742	0	0	1,536	4,443	0	16,350	5,973	13,174
A6	5,062	7,390	19,911	0	0	1,343	5,390	0	16,350	15,157	13,174
B3	0	0	2,283	0	0	0	0	0	15,351	436	2,262
B4	0	0	7,395	0	0	955	3,487	0	16,350	436	13,174
B5	0	0	10,436	0	0	2,071	2,317	0	16,350	5,973	13,174
B6	871	7,390	19,911	0	0	3,041	3,749	0	16,350	13,443	13,124

location of irrigated areas may lead to intraregional agricultural specialization. The problem of numerical stability and interpretation of a unique, optimal solution can be seen in Table 5.

Table 5. Numerical stability.

Scenario	Iter[a]	5%[b]	10%[b]
A3	7	4	4
A4	66	13	29
A5	131	24	51
A6	224	74	109
B3	6	1	3
B4	68	8	31
B5	143	34	36
B6	169	70	90

[a] Iter is the number of LP iterations.

[b] 5% and 10% denote the iteration numbers during which a goal function's value is less than the value of the optimal solution.

4.4. Programs Statistics

Information contained in Table 6 may be useful for evaluating the MWD program size and efficiency.

Note that no forcing in PDQLP was used; i.e., the typical pricing rule of choosing the most positive modified cost was applied. This has been done mainly to determine whether it is necessary to deal with the problem of regularization. Application of mild forcing (variable enters a basis only if the objective value will be improved by so doing) will reduce both the number of iterations and the CPU time.

Since several scenarios have been run in one batch, it is not possible to calculate the exact cost of running a particular scenario. Cost also depends greatly on the priority assigned to the batch; the price for 1 systems second (combination of memories use and CPU time) ranges from 1.1. to 4.5 zł; i.e., higher priority entails a shorter waiting time. Thus, the cost of one run for

Table 6. Program statistics.

Scenario	DIM1[a]	DIM2[b]	CORE[c]	CPU[d]	Iter[e]
A1, A2	-	-	13,520	1.4	-
A3	154x198	11x121	16,278	6.9	7
A4	277x209	134x132	34,181	9.3	66
A5	301x209	158x132	37,636	24.9	131
A6	306x209	163x132	38,363	52.0	224

[a] DIM1 is the dimension of a scenario problem.

[b] DIM2 is the dimension of an equivalent aggregated problem.

[c] CORE is the maximum number of words (60 bits) required during the running of the program (memory use is controlled by the MEMORF routine).

[d] CPU is the central processor unit time in seconds.

[e] Iter is the number of the LP iterations.

the greatest scenario A6, with the lowest priority, may be evaluated as about 300 zł. This cost may be compared with investment costs (listed in Appendixes B and C) in order to determine the saving made from relatively little expenditure in examining the various alternatives. On the other hand, however, one run of MWD2 will require much more of both time and money. Hence, an evaluation should be made, using a simplified MWD2 model, of as many scenarios as possible, but only a limited number of them should be subject to full examination by MWD2.

5. CONCLUSIONS

Despite the fact that the data used were preliminary, it is evident that development of the water system for mainly irrigation purposes is profitable. However, there are still some problems to be solved and they are outlined below.

5.1. Model Verification

Manteuffel et al. (1978), reached quite opposite conclusions from those presented in this paper. Thus, not only all the data, but also the method used to calculate the parameters, should be verified as planned (see section 4.1).

5.2. Linkage of MWD and the Agriculture Model

Because of the goal function, the problem can be solved by maximizing the benefit resulting from irrigation; thus,all water will be used whenever it is profitable. It need not necessarily be so if the goal function is the efficiency of agriculture. The final results of Podkaminer et al., which are included in these Proceedings, were not known at the time of preparation of this paper. However, preliminary results showed that a large increase in water used for irrigation caused a relatively small increase in production (in grain units) and substantial changes in agricultural production structure. Changing a goal function for the agricultural model may produce a completely different solution. Obviously decreasing water supply, while at the same time maintaining production targets, will require additional costs and/or a change in production structure. Hence, a comprehensive analysis of the effects of irrigation is really needed. It is clear that MWD should be used together with an agriculture model (see section 4.3.), but the methodology for linkage has not yet been elaborated. This will not be an easy task to accomplish, partly because both models deal with different spatial and temporal aggregations.

5.3. Numerical Stability

The problem of numerical stability also seems to be an important issue. The figures given at the end of section 4.3. indicate that many different solutions may be obtained, even though their goal function values are almost the same. We have not paid much attention to this problem mainly because we have been unable to develop a suitable program so soon after receiving the results discussed in the paper. We will try to deal with this problem,and a set of routines for finding a solution to the relevant regularized problem will be developed. The criterion for regularization has yet to be defined.

5.4. Additional Points

It should be emphasized that very little water is assumed to leave the region. For example, according to scenario A6, only an amount of water equal to minimal flow (2.94 m^3/sec) is left for the Notec river downstream regions.

A number of other scenarios should also be examined answering, among other questions, the following: is it profitable to irrigate all fields requiring water, if additional water is supplied from outside the region?

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Appendix A: INITIAL DISPOSABLE FLOWS (in m^3/sec) [a]

10-day period[a] \ Sub-region		1	2	3	4	5	6
April	1	1.990	1.720	7.130	1.390	1.570	3.270
	2	1.270	1.920	6.560	1.020	1.730	3.350
	3	1.410	1.520	5.800	0.730	1.410	2.720
May	1	1.340	1.190	5.110	0.620	1.160	2.230
	2	1.540	0.920	4.800	0.640	0.950	1.920
	3	1.540	0.790	4.610	0.620	0.850	1.740
June	1	1.380	0.930	4.620	0.540	0.950	1.840
	2	0.700	0.870	3.560	0.410	0.900	1.640
	3	0.370	0.680	2.740	0.350	0.760	1.340
July	1	0.220	0.440	1.990	0.330	0.570	1.020
	2	0.230	0.260	1.540	0.300	0.430	0.820
	3	0.230	0.200	1.400	0.190	0.380	0.740
August	1	0.250	0.130	1.270	0.170	0.330	0.650
	2	0.170	0.110	1.150	0.740	0.320	0.580
	3	0.120	0.060	0.980	0.300	0.280	0.500
Sept	1	0.060	0.040	0.870	0.490	0.260	0.480
	2	0	0.060	0.860	0.600	0.270	0.500
	3	0.020	0.140	1.020	0.190	0.330	0.600

[a] The months April-September are divided approximately into three 10-day periods.

Appendix A (continued)[a]

10-day period[a] \ Sub-region		7	8	9	10	11
April	1	7.280	0.790	23.880	2.820	31.330
	2	7.140	0.810	23.570	2.830	28.400
	3	5.930	0.600	20.030	2.730	24.220
May	1	5.100	0.630	18.110	2.690	22.040
	2	4.600	0.700	17.180	2.680	22.090
	3	4.320	0.470	15.510	2.190	19.180
June	1	4.430	0.120	14.310	1.670	16.230
	2	4.060	0.690	12.100	1.060	13.210
	3	3.570	0.020	10.710	0.670	11.640
July	1	3.060	0.160	9.670	0.650	10.420
	2	2.700	0.330	9.380	0.560	10.540
	3	2.460	0.360	8.980	0.510	10.330
August	1	2.300	0.350	8.560	0.460	9.810
	2	2.190	0.360	8.330	0.270	9.410
	3	2.240	0.450	8.530	0.210	9.370
September	1	2.380	0.490	8.700	0.150	9.650
	2	2.530	0.500	8.900	0.090	10.330
	3	2.280	0.460	8.790	0.070	9.640

[a] The months April-September are divided approximately into three 10-day periods.

APPENDIX B: RESERVOIR INFORMATION

Reservoir	Cap [a]	Inv [b]	Oper [c]	Amor [d]	Cost [e]	Loc [f]
GOPLO	21.10	316.10	1.00	0	0.15	3
PAKOSC	45.85	1,375.50	1.00	0	0.30	7
POLONISZ	0.30	3.50	3.00	2.00	0.58	1
MODEROWSKI	4.50	60.63	2.00	1.10	0.42	1
BRDOWSKI	5.80	35.30	2.00	1.20	0.19	1
LUBOTYNSKI	1.60	31.49	2.00	1.10	0.61	1
SKULSKI	4.30	56.09	3.00	1.60	0.60	3
OSTROWSKI	1.30	35.11	2.00	1.20	0.86	3
BUDZISLAW	0.90	7.28	2.00	1.20	0.26	3
SLAWSKO	0.32	12.76	3.00	1.80	1.91	3
BACHORZE	0.45	8.04	3.00	2.00	0.89	2
MLYNEK	3.23	75.34	7.00	2.10	2.12	5
KRUCHOWSKI	0.97	14.91	2.00	1.20	0.49	5
OSTROWICKI	1.26	27.35	2.00	1.20	0.69	5
SZYDLOWSKI	6.10	98.00	2.00	1.00	0.48	5
WIECNOWSKI	4.60	40.00	2.00	1.10	0.27	6
WIENIECKI	0.89	6.21	2.00	1.10	0.22	6
IZDBY	0.20	4.17	3.00	1.80	1.00	6
PADNIEWO	0.20	8.45	3.00	1.80	2.03	6
KAMIENICKI	13.34	71.86	2.00	1.00	0.16	4
FOLUSZ	79.30	1,874.00	8.00	1.80	2.32	8
OCWIECKI	0.90	5.07	3.00	1.20	0.24	10
ZNINSKI	7.75	46.50	4.00	1.30	0.32	10
ZEDOWO	5.06	56.94	2.00	1.10	0.35	10
BOZEJWICKI	0.57	9.05	3.00	1.80	0.76	10
BRZYSKORZ	1.08	14.29	3.00	1.80	0.64	10

[a] Cap is the capacity in $m^3 \times 10^6$.
[b] Inv are the investments required in zł $\times 10^6$.
[c] Oper is the annual operation cost (as a percentage of investment).
[d] Amor is 100/T, where T is the exploitation period.
[e] Cost is annual storage cost for 1 m^3 of water (in zł).
[f] Loc is the subregion in which the reservoir is located.

APPENDIX C: WATER TRANSFER INFORMATION

From[a]	To[b]	Flow[c]	Inv[d]	Oper[e]	Amor[f]	Cost1[g]	Cost2[h]
Vistula	2	10	320.0	13	3	5.12	0.539
8	10	5	59.5	15	3	2.14	0.225
8	6	5	53.5	15	3	1.93	0.202
5	4	2	16.5	10	3	1.07	0.113
9	10	3	38.0	10	2	1.52	0.160

[a] From is the subregion or river from which the water is taken.
[b] To is subregion to which water is delivered.
[c] Flow is transfer capability (in m^3/sec).
[d] Inv is investment required in zł x 10^6.
[e] Oper is annual operational cost (as a percentage of investment).
[f] Amor is 100/T, where T is the exploitation period.
[g] Cost1 is annual transfer cost for 1 m^3/sec (in zł x 10^6).
[h] Cost2 is annual cost of transfer for 1 m^3 (in zł).

APPENDIX D: CROP STRUCTURE[a]

Sub-region	Grain	Rape	Beets	Potatoes	Fibers	Pulses	Maize
1	0.2374	0.0326	0.0718	0.0718	0	0.0196	0.1200
2	0.2422	0.0568	0.0568	0.0338	0	0.0068	0.1529
3	0.3320	0.0286	0.0824	0.0959	0.0025	0.0095	0.2049
4	0.4157	0.0873	0.0467	0.0813	0	0.0271	0.1521
5	0.5877	0.0735	0	0.0284	0	0	0.0569
6	0.2883	0.0675	0.1247	0.1613	0.0137	0.0446	0.2265
7	0.1789	0.0085	0.1908	0.0204	0	0.0290	0.3186
8	0.2908	0.0857	0.0378	0.1335	0	0.0916	0.3207
9	0.3076	0.0446	0.0813	0.0807	0.0031	0.0226	0.2043
10	0.2897	0.0315	0.1220	0.0821	0	0.0172	0.2017
11	0.0751	0.0258	0.0516	0.0053	0	0.0038	0.0759

[a] The numbers of the table correspond to the share of irrigated fields for a given crop in a given subregion.

APPENDIX D (continued)[a]

Subregion	Fodder	Vegetables	Orchards	Grass	Meadows	Forest1	Forest2
1	0.0261	0.0196	0.0300	0.0183	0.2348	0.1174	0.0007
2	0.0609	0.0568	0	0.0325	0.2828	0.0176	0
3	0.0377	0.0457	0.0050	0.0442	0.0703	0.0407	0.0006
4	0.0136	0.0090	0.0075	0	0.1175	0.0422	0
5	0.0521	0.0853	0	0.0687	0	0.0474	0
6	0.0195	0.0114	0	0	0.0423	0	0
7	0.1584	0.0375	0	0	0.0579	0	0
8	0.0299	0	0	0.0100	0	0	0
9	0.0520	0.0373	0.0159	0.0037	0.1431	0.0037	0
10	0.0462	0.0320	0.0025	0.0089	0.1495	0.0167	0
11	0.0281	0.0091	0	0.0053	0.6285	0.0213	0.0701

[a] The numbers of the table correspond to the share of irrigated fields for a given crop in a given subregion.

OPERATIONAL DISTRIBUTION OF WATER RESOURCES FOR AGRICULTURE--AN ATTEMPT TO OBTAIN A NUMERICAL SOLUTION

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INTRODUCTION

A mathematical formulation of the problem of operational distribution of water resources is given in Gutenbaum et al. (1978), Pietkiewicz-Sałdan and Inkielman (1978), and Babarowski (1978).

Maximization of the expected value of agriculture production is adopted as the main objective for water distribution. Other objectives related to the operation of a regional water distribution system are taken into account by introducing guarantee constraints. These constraints include demands of all users (with the exception of agriculture), ecological and flood control requirements, etc.

It is assumed that the system will

-- store water in separate reservoirs; and
-- distribute water among individual users and at particular intakes.

The horizon for water distribution optimization is one year, i.e., the shortest period necessary for assessing the effects of control upon the value of the performance index adopted.

The mathematical formulation of the problem is:

determine

$$\max_{u = \{x,s,q,q_a\}} G(q_a) \quad , \tag{1}$$

subject to the constraints

$N(u,r) = 0$ are the equations of network balances; (2)

$P\{u \geq u_{gu}\} \geq \alpha_u$ are the guarantee constraints; and (3)

$u \in A_w$ where A_w is the set of feasible solutions; (4)

where

$G(q_a)$ is the objective function;

x is the vector of flows in the water network (m^3/10-day period);

s is the vector of water volume stored in the reservoir (m^3);

q is the vector of amounts of water supplied to users through the water network (m^3/10-day period);

q_a is the vector of amounts of water supplied to "irrigation complexes" (m^3/10-day period);

r is the vector of rainfall (mm/10-day period);

u_{gu} is the vector of required levels; and

α_u is the vector of guarantee probabilities (i.e., specified probabilities of meeting the guarantee constraints).

By introducing (i) decision rules for operational control, (ii) decomposition into J partial watersheds, and (iii) time discretization, it is possible to present the problem in the form shown in Figure 1.

A substantial part of the problem is represented by block a:

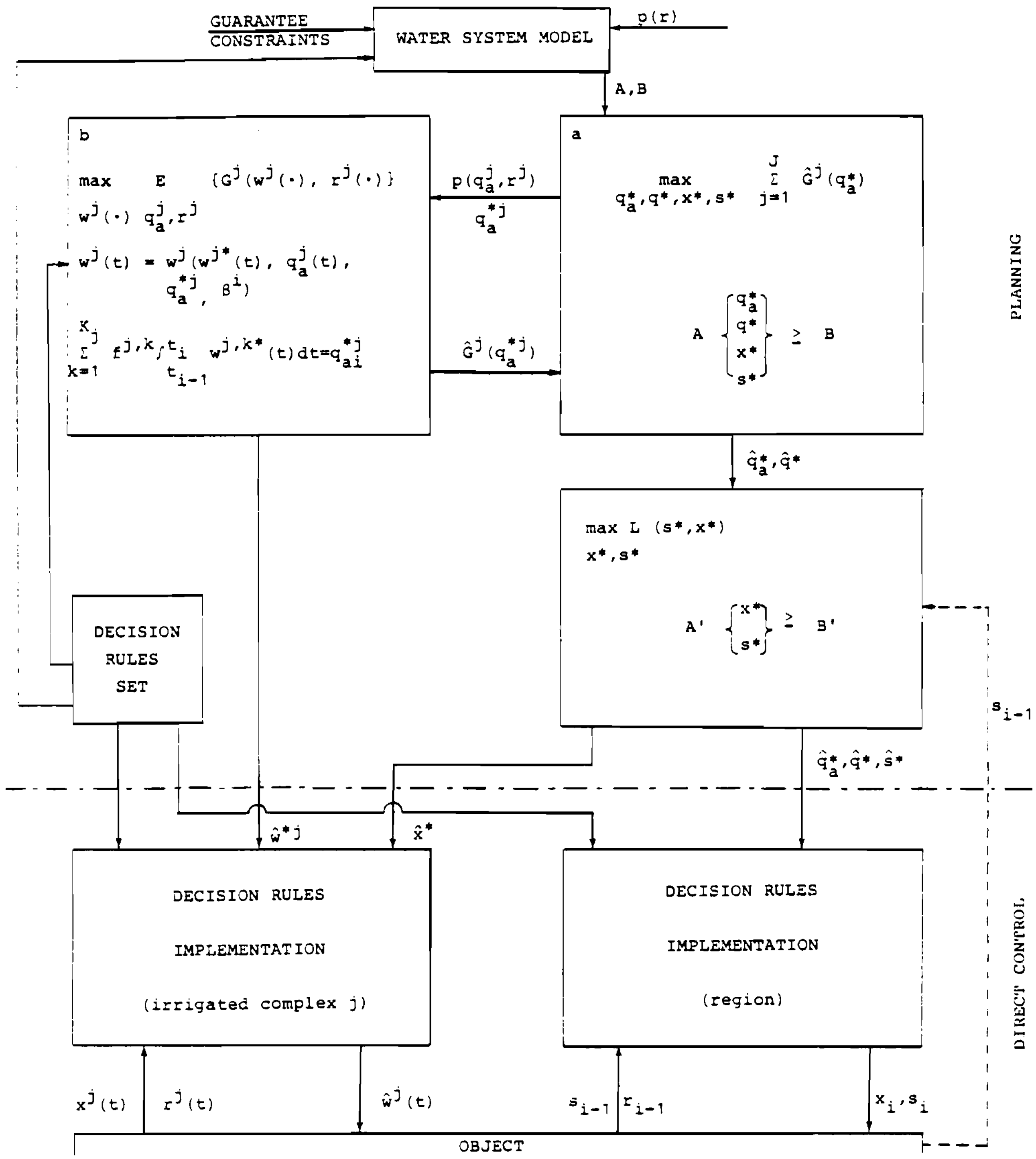

Figure 1. A block diagram of the water distribution problem.

"maximization of the expected value of total agricultural production with respect to the parameters of decision rules applied and subject to the guarantee constraints, given in deterministic form yet derived on the basis of the stochastic characteristics of rainfall"

The following notation is used in Figure 1:

$\hat{G}^j(\cdot)$ is a functional representing the locally optimized expected value of agricultural production in partial watershed j;

$$\hat{G}^j(q_a^*) = \max_{w^j(\cdot)} E\{G^j(w^j(\cdot), r^j(\cdot)\}; \quad (5)$$

E is the expected value from q_a^j and r^j (components of the vectors q_a and r for partial watershed j);

$w^j(\cdot)$ is the intensity of irrigation for irrigation system j (time-dependent local variables given as vector K_j);

K_j is the variety of crops in partial watershed j (or irrigation system j);

f^{kj} is the area of crop k in irrigation system j;

$r^j(\cdot)$ is local (time-dependent) rainfall in partial watershed j;

$\cdot i$ is the index of intervals resulting from time-discretization;

s_i^j, r_i^j, x_i^j are components of the vectors s,r,x for partial watershed j and interval i;

q_a^*, q^*, s^*, x^* are the vectors of parameters of the decision rules:

- q_a^* is the guaranteed water supply for agricultural users,
- q^* are the values of guaranteed water supply for other users,
- s^* is the planned amount of water stored in the reservoirs (the expected values),
- x^* are the threshold values of guaranteed flows in the network considered;

$p(r)$ is the probability density function of the rainfall vector; vector;

$p(q_a^j, r^j)$ is the joint probability density function of the variables q_a^j and r^j;

A,B are the matrix and vector, respectively, of coefficients occurring in the constraints determining the region Ω of acceptable solutions; and

$L(s^*,x^*)$ is the linear form of the parameters s^* and x^*.

At a local level (J block b representing partial watersheds), the guaranteed water supplies for agricultural users q_a^*, aggregated for each partial watershed, are distributed among individual users.

If the values of supplies provided for all the users q_a^* are given, then the problem represented by block a consists in selecting an arbitrary point within the acceptable region Ω.

The reasoning behind such a formulation is as follows. Up to the present, models of soil type and yield factors, which provide the basis for determining agricultural production, have not been constructed. Hence, the relations that make possible an evaluation of the performance index, and consequently determination of the optimal values of q_a^*, are not available.

On the other hand, water needs are known for individual plants. Therefore, agricultural experts recommend that the plants' water demands should be satisfied entirely or almost entirely under high guarantees (i.e., with high probability). The lack of sufficient water resources is a problem that can only be solved by selecting the areas for irrigation.

Under fixed demands, determined by the needs of agricultural users, any solution belonging to the acceptable region could be taken.

It should be noted that for balanced supply and demand, the points of region Ω only differ from one another according to the planned reservoir states s^* and guaranteed flows x^*. For different s^* and x^*, higher or lower guarantees in some groups of constraints are obtained.

Direct maximization of the guarantees for satisfying particular objectives results in a nonlinear problem. However, by introducing some linear performance indices depending upon s^* and x^*, it is possible to improve some guarantees.

In this paper some examples of linear performance indices are given. They are such that the problem of determining a solution belonging to the region Ω does not become more difficult and that the maximization of any of the indices simultaneously results in a solution advantageous for some group of guarantees.

AN EXAMPLE

A water system consisting of two reservoirs and two agricultural users is taken as an example (Figure 2).

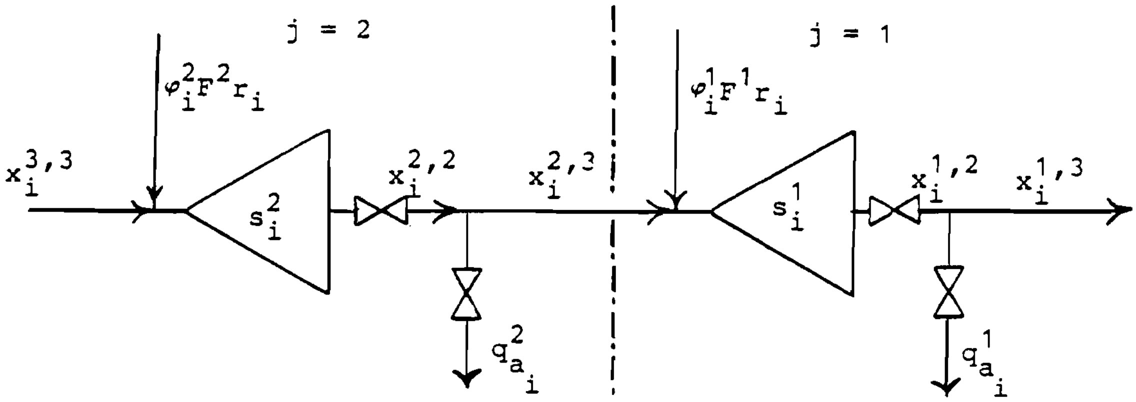

Figure 2. Example of the water network.

This simple network has been chosen to allow the results to be interpreted intuitively (difficult enough even for this network), rather than to simplify computations.

Ten-day periods of rainfall r_i (i.e., for each period i) are considered random variables. The rainfall is assumed to be the same for both partial watersheds. Water resources of a given partial watershed are given by the inflow x_i and rainfall. The relationship between these variables is decribed by a linear rainfall-runoff model. In the example presented it is assumed that this relation has the form $r^j_{fi} = \varphi^i_j F^j_{r_i}$, $j = 1,2$, where

φ^j is the runoff coefficient; and
F^j is the area of partial watershed j.

Operational control of they system under analysis is reduced to control of flow at the points indicated in Figure 2, in which the following notation is used:

- $q^j_{a_i}$ is the water consumption in partial watershed j during the 10-day period i (m^3/10-day period);
- $\hat{q}^j_{a_i}$ are the requirements of user j in 10-day period i (mm/10-day period);
- γ_j is the coefficient proportional to the irrigated area of user j;
- x_i^{*j} are the decision-rule parameters of user j, given in units of flow;
- $x_i^{j,2}$ is the reservoir outflow; and
- $x_i^{j,3}$ is the outflow of partial watershed j.

The user's consumption is given by the following rule (Figure 3):

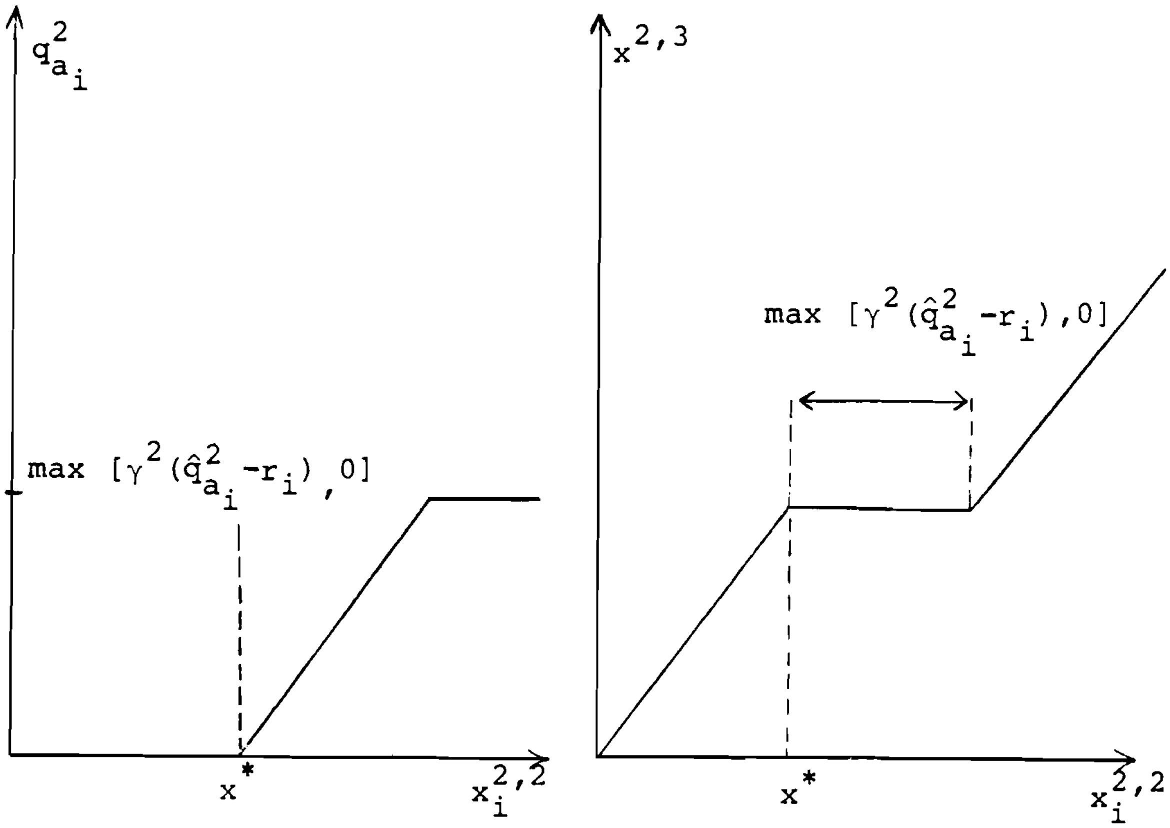

Figure 3. Agricultural user's consumption and partial watershed outflow versus reservoir release for decision rule (6).

$$q^j_{a_i} = \begin{cases} 0 & \text{for } r_i \ge \hat{q}^j_{a_i} \text{ or } r_i < \hat{q}^j_{a_i} \text{ and } x_i^{j,2} \le x_i^{*j} \\ x_i^{j,2} - x_i^{*j} & \text{for } r_i < \hat{q}^j_{a_i} \text{ and } x_i^{*j} \le x_i^{j,2} \le x_i^{*j} + \gamma^j(\hat{q}^j_{a_i} - r_i) \\ \gamma^j(\hat{q}^j_{a_i} - r_i) & \text{for } r_i < \hat{q}^j_{a_i} \text{ and } x_i^{j,2} \ge x_i^{*j} + \gamma^j(\hat{q}^j_{a_i} - r_i) \end{cases} \tag{6}$$

Moreover

$$x_i^{j,3} = x_i^{j,2} - q^j_{a_i} \ . \tag{7}$$

The reservoir outflow is governed by the rule

$$x_i^{j,2} = s^j_{i-1} - s_i^{*j} + Ex_i^{j,s} \ , \tag{8}$$

where

$$x_i^{1,s} = \gamma_i^1 F^1 r_i + x_i^{2,3} \ ; \tag{9}$$

$$x_i^{2,s} = \gamma_i^2 F^2 r_i + x_i^0 \ ; \quad \text{and} \tag{10}$$

$Ex_i^{j,s}$ is the conditional expected value computed for known r_{i-1} .

When transforming stochastic constraints into deterministic constraints for computation of the outflow of the upper partial watershed, a simplifying assumption is made that the needs of the user of this partial watershed are completely satisfied.

The higher the guarantee for a user, the smaller the error resulting from this assumption. The magnitude of this error can be estimated for all the cases discussed.

In the relations mentioned above, the variables determining the state of the network are linear functions of rainfall or the derivatives of stochastic variables. These relations are substituted in the stochastic guarantee constraints:

$$P\{s_i^j \geq s_{min,i}^j\} \geq \alpha_{s,i}^j \quad , \tag{11}$$

$$P\{s_i^j \leq s_{max,i}^j\} \geq \alpha_i^{s,j} \quad , \tag{12}$$

$$P\{x_i^j \geq x_{min,i}^j\} \geq \alpha_{x,i}^j \quad , \qquad \text{for } i=1,\ldots,I,\ j=1,\ldots,J, \tag{13}$$

$$P\{x_i^j \leq x_{max,i}^j\} \geq \alpha_i^{x,j} \quad , \tag{14}$$

$$P\{q_{a_i}^j \geq \gamma^j \ (\hat{q}_{a_i}^j - r_i)\} \geq \alpha_{q,i}^j \quad , \tag{15}$$

where

$s_{min,i}^j$, $s_{max,i}^j$, $x_{min,i}^j$, $x_{max,i}^j$, $\gamma^j(\hat{q}_{a_i}^j - r_i)$ are the guaranteed levels; and

$\alpha_{s,i}^j$, $\alpha_i^{s,j}$, $\alpha_{x,i}^j$, $\alpha_i^{x,j}$, $\alpha_{q,i}^j$ are the guarantee probabilities.

As a result, linear and deterministic guarantee constraints are obtained.

In general, the contraints have the following form:

$$[\text{linear expression with the variables } s^*, x^*] \geq [\text{guaranteed levels}] + \text{buffer} \quad (16)$$

Some of the constraints (those imposed on the reservoir states) are shown in Figure 4.

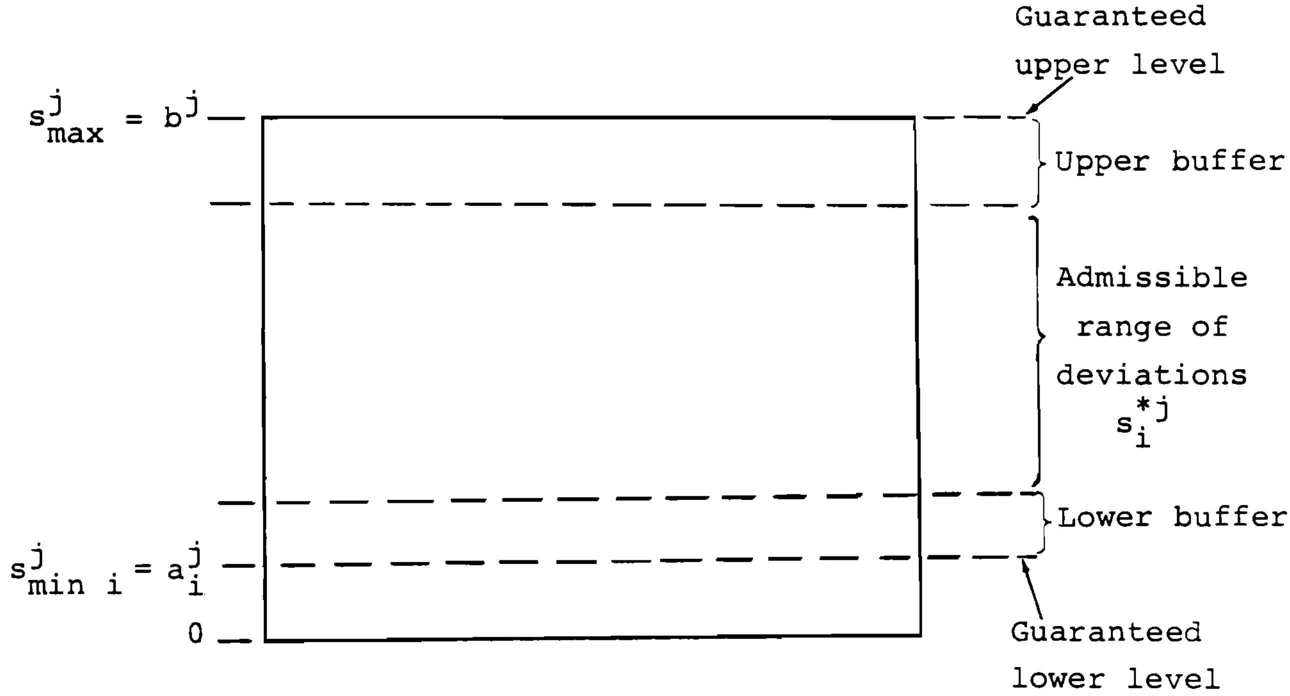

Figure 4. Deterministic guarantee constraints on reservoirs.

The buffer is a nonlinear monotone function of the guarantees α:

$$\text{buffer} = F^{-1}_{r_i}(\alpha_i) - Er_i \quad , \qquad (17)$$

where

α_i is the guarantee probability;

$F_{r_i}^{-1}(\cdot)$ is the inverse distribution function; and

Er_i is the expected value of rainfall.

Of course, $F_{r_i}^{-1}(\alpha)$ is a nonlinear function of α. Hence, each linear expression with the variables s* occurring in the performance index can be considered as a requirement for maximizing a weighted sum of nonlinear functions of the appropriate guarantees.

For instance, the use of the performance index

$$\min \left[\sum_i s_i^{*1} + 2 \sum_i s_i^{*2} \right] , \tag{18}$$

where

i is a subscript representing the time interval,

can be understood as a requirement for maximizing a nonlinear monotone function of guarantees appearing in the constraints imposed on the maximum reservoir states. The weighting coefficient is 1 for the first reservoir, and 2 for the second. All these constraints are used to determine the minimum amount of water stored in the reservoirs.

If in the vegetative period the user's guarantees are to be maximized, then the following performance index is used for the crops considered in this example:

$$\max \left[s_1^{*1} = s_{19}^{*1} + s_1^{*2} - s_{19}^{*2} + s_4^{*2} - s_{13}^{*2} - \sum_{i=5}^{13} x_i \right] \tag{19}$$

Description of Data Used in the Example

The network considered does not represent any real part of the Upper Notec water system. However, all the numerical data are consistent with those representative of this region.

-- Twenty periods are taken into account: 1 nonvegetative period with a duration of seventeen 10-day periods; 18 vegetative periods, each of 10 days; and one more non-vegetative period, distinguished for computational purposes.

-- The runoff coefficient is taken from Ostromecki (1973) for low-lying regions of Poland.

-- The areas of partial watersheds are 1,000 km^2 each, i.e., about one-sixth of the total area of the Upper Notec region.

-- The area to be irrigated represents between 5 and 50 percent of the given partial watershed area.

-- The reservoirs have maximum capacities of 10^8 m^3 (twice that of the Lake Pakosc).

-- The minumum amounts of water in the reservoirs vary from $2 \cdot 10^6$ m^3 to $8 \cdot 10^6$ m^3.

-- The ratio of maximum to minimum flows is 20:1, 20:2, or 20:4.

-- The agricultural users' requirements for potatoes and sugar beet are taken from Klatt (1958) and are evaluated for every 10-day period.

-- The stochastic characteristics (the density functions) of rainfall in the Bydgoskie voivodship are taken from Ostromecki (1973).

-- The guarantees (probabilities) for agricultural users are 0.8-0.9.

-- The ecological guarantees are 0.85-0.98.

A linear programming problem with 49 variables and 178 inequality constraints was formulated on the basis of the data given above.

RESULTS OF COMPUTATIONS

Computations were performed for several specially chosen guarantee levels (hence, principally irrigated areas) and performance indices. The most characteristic results are shown in Figure 5. Those depicted were obtained under the assumption that

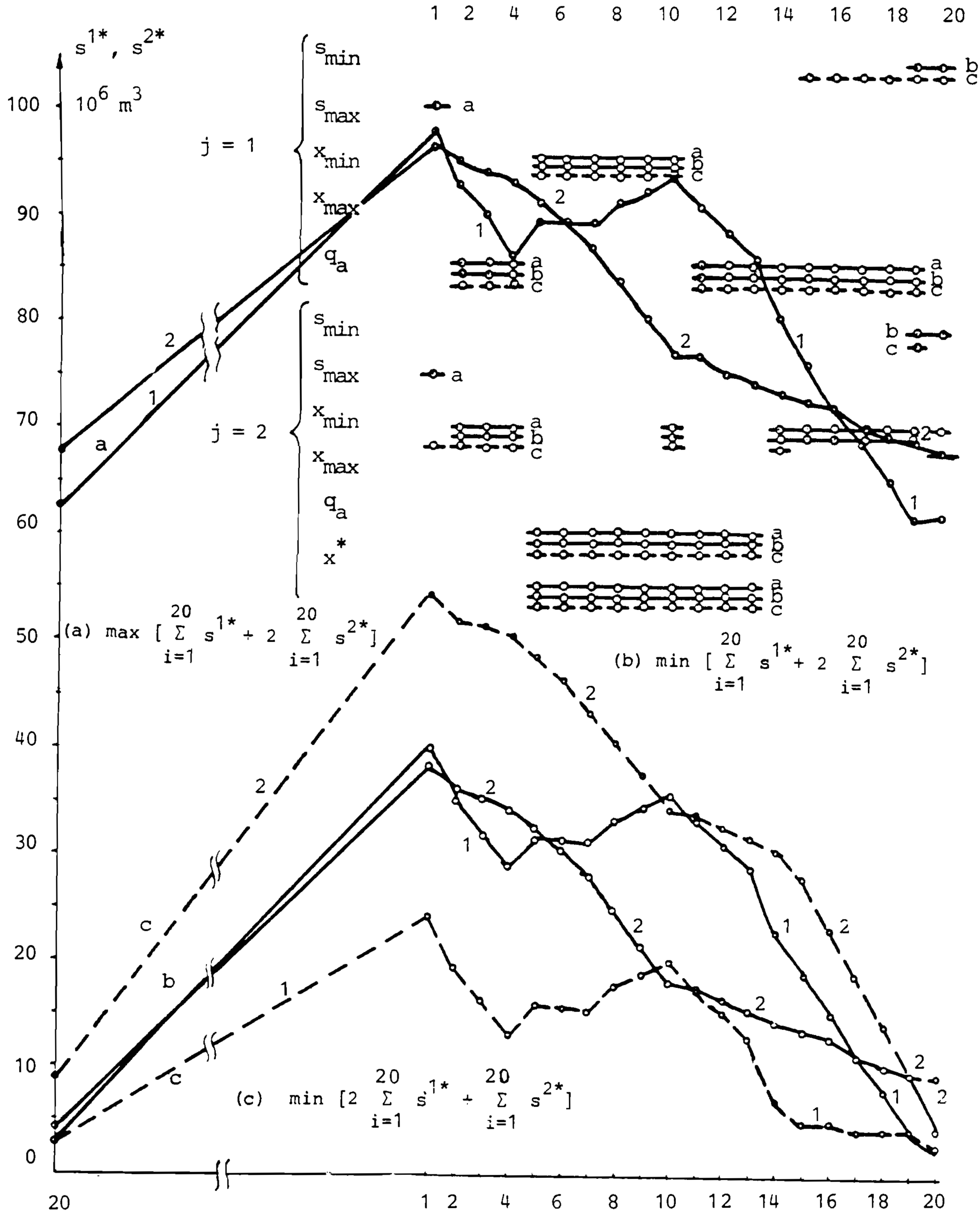

Figure 5. The solutions for $\gamma^1 = \gamma^2 = 20 \cdot 10^3$ ha and for the performance indices (20), (21), and (23) (in the upper part of the figure the active constraints for each of the solutions obtained and the constraints representing required levels are indicated).

the irrigated areas in both partial watersheds were equal (200 km^2 each). The trajectories a, b, c shown correspond to performance indices (20), (21), and (23). On the basis of these results, it is possible to analyze the influence of the form of the performance index upon an optimal solution.

By maximizing the sum of the expected states s* with the weighting coefficients,

$$\max \left[\sum_i s_i^{*1} + 2 \sum_i s_i^{*2}\right] \quad , \tag{20}$$

the most flat trajectory of the expected reservoir state s_i^{*2}, i = 1,2, is obtained. The trajectory of states s_i^{*1}, i=1,...,20, determined on the basis of the computed s_i^{*2} is as flat as possible. In this case the constraints on the upper bounds of the reservoir states in the first 10-day period are active.

The trajectories s^{*2}, although resulting from the maximization problem, are monotonically decreasing functions over the second to nineteenth 10-day period because of the agricultural users' requirements and because the minimum flows x^1_{min} and x^2_{min} have high guarantees.

For each partial watershed and each 10-day period, the constraints on the minimum flows or user consumption are always active. These constraints determine the minimum release.

Because of the performance index used, the guarantee that the reservoirs states s_i^j are not less than the given guarantee levels a_i^j is as high as possible. The weighting coefficient for the guarantees associated with the second reservoir is greater than that for the first.

For the first to nineteenth 10-day periods, minimization of the same performance index,

$$\min \left[\sum_i s_i^{*1} + 2 \sum_i s_i^{*2}\right] \quad , \tag{21}$$

results in identical, but lower, solutions for both reservoirs.

Such a situation occurs because during the ninteenth 10-day period (the end of the vegetative period), the constraints on the lower water levels become active. Moreover, as in the case of the maximization problem, the constraints determining the minimal release (those imposed on the minimal flows and user consumptions) are also active. The only difference occurs at the twentieth 10-day period (nonvegetative period). In this case, when minimizing the reservoir states, the constraints on the lower water levels become active, as opposed to the case of maximization. Hence, the constraints on minimal flows with the guaranteed levels $x^1_{min\ 20}$, $x^2_{min\ 20}$ are not active. Because of the performance index used, the guarantees that the states s^j_i do not exceed the upper levels b^j_i are maximized. The weighting coefficient for the second reservoir is twice that for the first.

The effect of a change in the weighting coefficients' values corresponding to s^{*1}_i and s^{*2}_i (i.e., those associated with the reservoir states in the first 10-day period) on an optimal solution has also been examined.

Depending on the assumed weighting coefficients for s^{*1}_i and s^{*2}_i, the solution obtained is the same as that corresponding to the performance index

$$\max\ [s^{*1}_1 + 2\ s^{*2}_1 - \sum_{i=2}^{J} s^{*1}_i - 2 \sum_{i=2}^{J} s^{*2}_i]\ , \tag{22}$$

or that resulting from

$$\min\ [s^{*1}_1 + 2\ s^{*2}_1 - \sum_{i=2}^{J} s^{*1}_i - 2 \sum_{i=2}^{J} s^{*2}_i]\ . \tag{23}$$

Hence, the requirement of a high flood control guarantee in the first 10-day period results in low reservoir states for the remaining periods.

If the weighting coefficients are interchanged, e.g.,

$$\min\ [2 \sum_{i=1}^{J} s^{*1}_i + \sum_{i=1}^{J} s^{*2}_i]\ , \tag{24}$$

then the flattest trajectory of s_i^{*1}, $i = 1,\ldots,19$, is obtained. The trajectory of s_i^{*2}, $i = 1,\ldots,19$, computed for those values of s_i^{*1} is also as flat as possible.

For the last 10-day period, the values of s_{20}^{*1} and s_{20}^{*2} are determined by the constraints on minimum reservoir states as a_{20}^1 and a_{20}^2.

In Figure 6, a solution obtained for performance index (20) is shown again in order to make some comparison possible.

The following solutions are also plotted in Figure 6 --

(i) those corresponding to the maximization of user's guarantees (with the same weighting coefficients for both users but increasing over time):

$$\max \left[\sum_{i=1}^{18} i \underbrace{(s_i^{*1} - s_{i+1}^{*1} + s_i^{*2} - s_{i+1}^{*2})}_{\text{the lefthand sides of the appropriate constraints determined by the first user}} + \sum_{i=4}^{12} i \underbrace{(s_i^{*2} - s_{i+1}^{*2} - x_i^{*2})}_{\text{the lefthand sides of the appropriate constraints determined by the second user}} \right] ; \quad (25)$$

(ii) those corresponding to the case of maximizing

$$\max \sum_{i=5}^{13} x^{*2} \quad . \quad (26)$$

The amount of water intended for the user of the first partial watershed but stored in the second watershed (the cutoff parameters of the decision rule of the user of the second partial watershed) are maximized.

The performance indices (25), (26) result in a solution such that, in the nonvegetative period, the amount of water available in both reservoirs is maximized. Using the performance indices (21) and (24), the effect of the irrigation area in both partial watersheds on the solution is examined (Figure 7). The

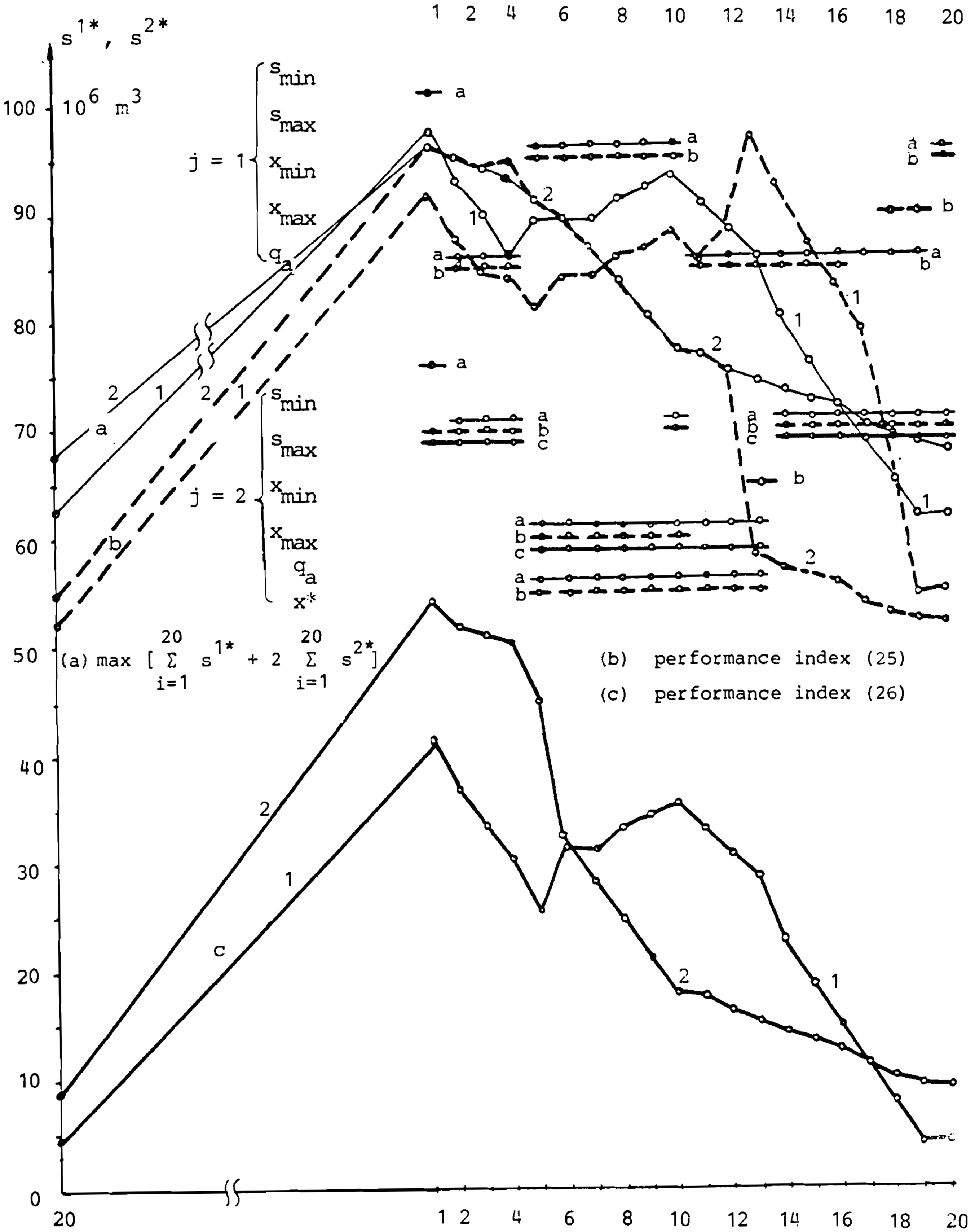

Figure 6. The optimal solutions for $\gamma^1 = \gamma^2 = 20 \cdot 10^3$ ha and for the performance indices (20), (25) and (26).

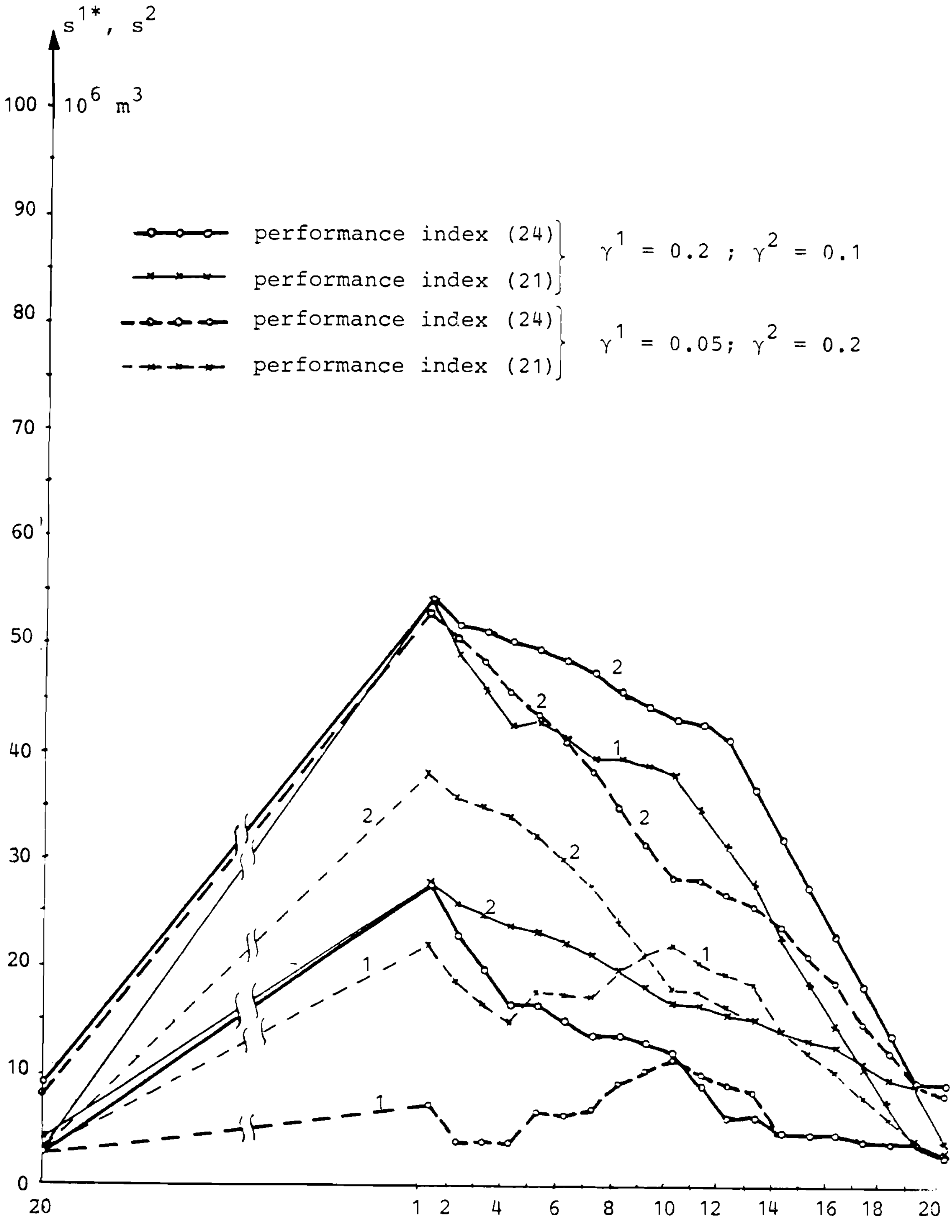

Figure 7. The optimal solutions for different irrigated areas and for the performance indices (21) and (24).

smaller the irrigated area of a given partial watershed, the smaller is the magnitude of the trajectories s^* associated with this partial watershed. In the case where the guarantee constraints determined by the users are not active, the same results are obtained.

APPLICATIONS OF THE METHOD CONSIDERED FOR DESIGN PURPOSES

The results obtained for the alternative performance indices allow the required capacities of particular reservoirs to be estimated.

In particular, performance indices of the type

$$\min \left[k_1 \sum_i s_i^{*1} + k_2 \sum_i s_i^{*2}\right] \quad , \tag{27}$$

make it possible to compute the minimum total capacity of both reservoirs such that the assumed guarantees are satisfied. Hence, the desired objectives of a water distribution system can be achieved.

To perform the computations, not only the magnitudes of trajectories s^*, but also the values of the upper and lower buffers, resulting from the guarantees selected, are taken into account.

The relationship between the minimum volumes of reservoirs b^1 and b^2 (for γ^1 and $\gamma^2 = 200\ km^2$) is shown in Figure 8.

To determine the minimum reservoir capacities, the problem can be modified. The problem that results is also linear. For this new problem the upper guarantee levels b^1 and b^2 (i.e., the minimum capacities of reservoirs) are additional decision variables (besides s^* and x^*).

The performance index

$$\min \left(k_1 b^1 + k_2 b^2\right) \quad , \tag{28}$$

yields the same solution for the trajectories s* as that obtained for the performance index

$$\min\ [k_1 \sum_{i=1}^{20} s_i^{*1} + k_2 \sum_{i=1}^{20} s_i^{*2}] \quad . \tag{29}$$

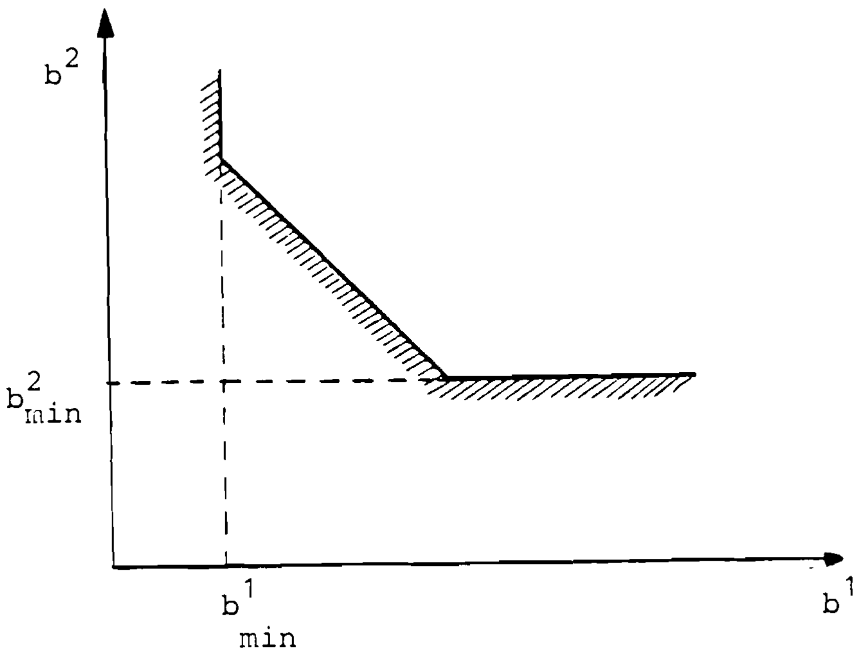

Figure 8. Reservoir capacities required to satisfy all constraints (16).

However, because of this modification it is possible to determine the values of b^1 and b^2 directly.

To determine sufficient reservoir capacities characterized by the lowest construction costs, the relations between the minimum volumes of individual reservoirs is derived analytically using Figure 8:

$$b = b^1 + b^2 \quad , \tag{30}$$

where

$$b^1 \geq b^1_{min} \quad ; \qquad \text{and} \tag{31}$$

$$b^2 \geq b^2_{min} \quad . \tag{32}$$

Using this relation, the problem

$$\min_{b^1, b^2} \; [K^1(b^1) + K^2(b^2)] \quad , \tag{33}$$

subject to (31), or

$$\min_{b^1} \; [K^1(b^1) + K^2(b-b^1)] \quad , \tag{34}$$

subject to

$$b^1 \min \leq b^1 \leq b - b^2 \min \quad , \tag{35}$$

must be solved.

Similarly, a modified optimization problem can be used to determine the irrigated areas γ^1 and γ^2 of the partial watersheds.

If the relations among buffers and between γ^1, γ^2, occurring in constraints (16), are approximated with linear functions, then the righthand sides of these constraints can be written as linear functions of both the parameters mentioned.

Hence, it is possible to formulate a linear optimization problem such that not only s* and x* but also γ^1 and γ^2 are decision variables.

Assuming that the performance index is

$$\max \; [\gamma^1 + \gamma^2] \quad , \tag{36}$$

it is possible to compute the area of irrigation in a given region as well as to determine the values of γ^1 and γ^2 independently (the areas of specific partial watersheds to be irrigated).

In the example presented, it is assumed that only one kind of crop is grown in each partial watershed. In a real environment, different kinds of crops can be grown in the same partial watershed. Therefore, the structure of crops grown in each partial watershed has to be optimal (subject to the desired overall crop structure) with respect to some economic and social criteria.

The authors believe that this part of the design task should be solved taking into account aspects such as the structure of the water network, the long-term planning horizon divided into 10-day periods, and the stochastic nature of rainfall. An attempt to formulate and solve this problem will be made in the near future.

CONCLUSIONS

The numerical example presented above is aimed at demonstrating the possibility of applying "the chance constraint programming method" in order to optimize a multireservoir water network. Using this method, it has been possible to reduce the problem to a linear programming one. The solution obtained satisfies all the desired objectives of the water distribution system represented by the guarantee constraints. In addition, several linear optimization criteria allowing priority to be given to any of the objectives have been presented. In the optimization problem resulting from the numerical example, there are 49 variables and 178 inequality constraints. The matrices are sparse (about 1 percent of nonzero elements).

It has been verified that such a formulation of the water distribution problem is also useful for designing a water network intended to perform tasks determined by the constraints. The total reservoir capacities in a given region is of prime importance.

The authors have indicated the possibility of applying the method discussed to optimization of the irrigation area in a particular partial watershed and the crop structure of a given region. The authors will focus on these problems in the future.

The main disadvantage of the method described in this paper is the arduous data processing required to solve the linear programming problem.

A computer program for processing the data has to be adjusted to the requirements of a given water network and the decision rules selected. In the example, ReVelle decision rules were applied. The use of these decision rules results in an inefficient distribution of water. However, it is possible to use other decision rules, for example, rules that govern the reservoir outflow resulting from the user's consumption. In this case, the reservoir states are not stabilized (the variance of the states is very large); hence, the necessary reservoir capacities can be larger than those corresponding to the ReVelle decision rules. It is possible that a more economical distribution of water will result in higher costs for construction of the water system.

Thus, it should be emphasized that generally the operational control strategy has an important effect on investment.

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PART IV

Models of Industry, Transportation, and Energy

MODELING THE LOCATION AND DEVELOPMENT OF INDUSTRY IN THE SILISTRA REGION

E. Christov
I. Assa
O. Panov

INTRODUCTION

Regional industry is a complex system characterized by uncertainty. It is complex because many of its variables are linked by nonlinear and stochastic relationships and it is uncertain because of the lack of knowledge about certain factors that cannot be formalized and that to a large extent influence the final solution (i.e., the intuition and experience of the decision maker).

This paper deals with modeling the development and location of industry in a region, using an approach based on systems analysis and simulation techniques. The location and development of industry in a region cuts across two broad areas of modeling--modeling the development of the industrial sector and of the whole region.

Current theoretical and applied research in modeling the regional development of industry is focused on developing models that solve small-scale problems; the reasoning behind this approach being that the national and regional industrial systems function almost independently. This approach, which is based on the monodisciplinary treatment of each subsystem in the region, contains some inherent disadvantages that have become evident with the increasing complexity of the system and changing management requirements.

In our research, however, we analyze regional industry using the input-output approach--inputs and outputs are disaggregated to deal with the interrelations between the regional and national management levels. In this way have made a considerable methodological progress. Although input-output models are a useful analytical tool,they are not without disadvantages. For example, they do not account for the location problem, which requires the assignment of industry to specific areas within the regional system. The location problem requires the solution of the regional transportation problem.

To a large extent,transportation can affect the solutions of both the input-output and location models. Usually, this type of problem is solved by allocating a certain transportation capacity to serve industrial growth. Investment in transportation will naturally affect the pattern of growth in several industrial sectors. Such conditions of interdependency and dynamism can be fully accounted for using an interactive modeling approach, which requires a large set of interconnected models to deal with regional problems. This, however, is not out intention. We wish to develop a set of models that is strictly limited to solving industrial development and location problems (Figure 1). Three principal components will be solved interactively; they are

-- an industrial location model;
-- an industrial input-output model; and
-- an industrial transportation model.

In addition, some econometric and statistical models will be constructed to indicate trends and provide forecasts of industrial growth within the region over a 20-year period. The set of models will

-- provide decision makers with a tool to aid them in regional analysis;
-- generate a number of production development options; and
-- assist in integrating regional and national policies.

We take as the subject of our study the Silistra region, Bulgaria, in which industrial output amounts to 1.4 percent of the gross national industrial output. Sectoral output is given

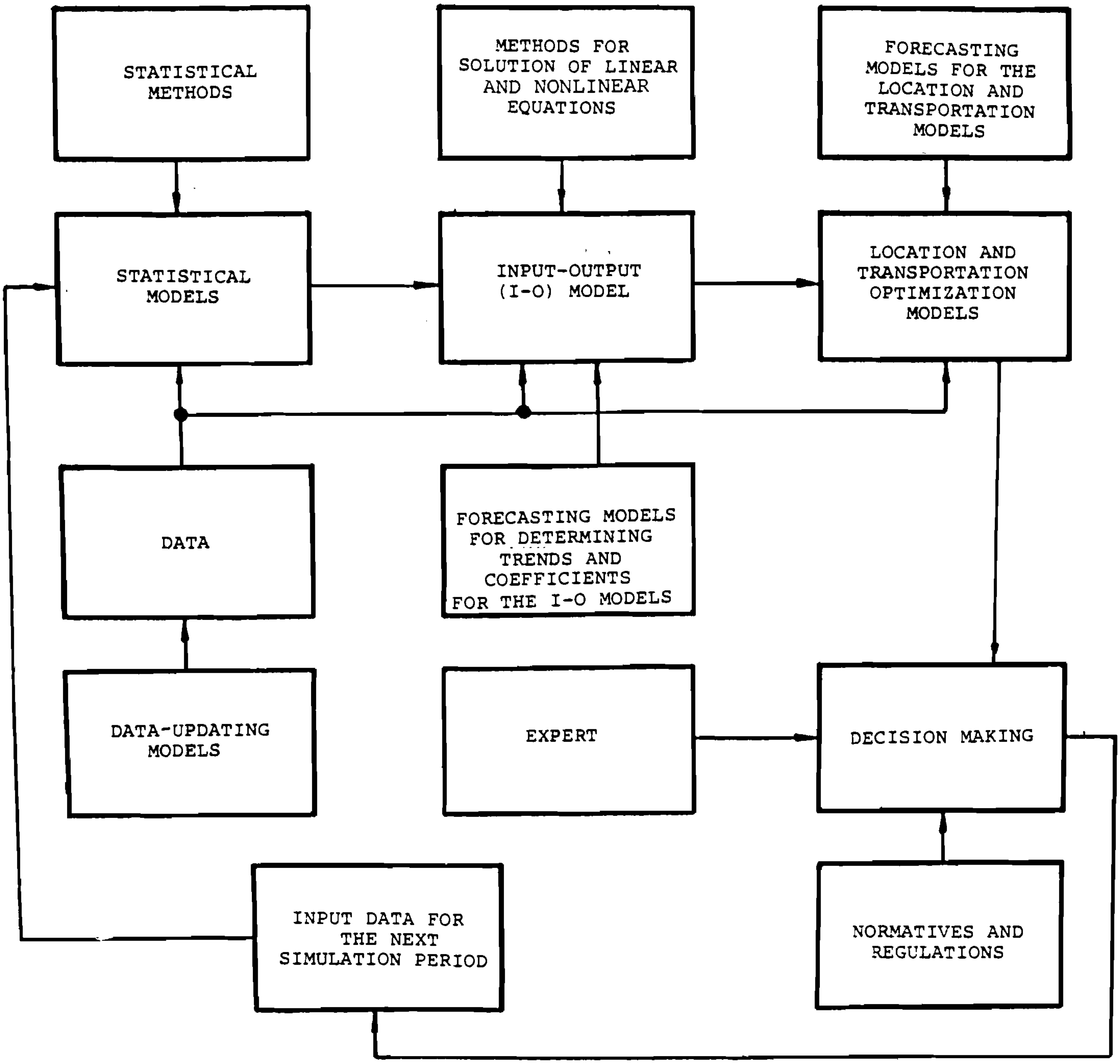

Figure 1. The general scheme of the set of industrial development and location models.

below as a percentage of national sectoral output:

Machine-building industry	--	2.00%
Chemical industry	--	0.20%
Timber industry	--	0.90%
Pulp and paper industry	--	2.00%
Textile industry	--	0.15%
Food processing industry	--	1.80%
Electronic calculators	--	100.00%

The main problem in modeling the industrial development of the Silistra region is the lack of appropriate information. Preliminary analysis indicates that

-- the geographic, historic, and demographic features of the region make it a suitable area in which to implement new industrial development policies;
-- long-term changes can be expected in the structure and location of industry over the next 20 years; and
-- the industry in the region is mainly concentrated in three subregions (the Silistra region consists of 10 subregions in total).

Since these conditions affect not only economic development but also methods of strategic planning and management, a tool that can provide new possibilities for analyzing the location and development of industry is needed.

ORGANIZATION OF THE RESEARCH

The set of models will be constructed in three stages.

First Stage

The development of regional systems depends on their ability to interact with the national economy. The first stage in our research is therefore to examine strategic plans at the national level in relation to those conditions that influence the decision-making process within the region.

The comparative advantages of the region must be assessed. This is a difficult task, which can only be solved by interaction between the national and regional management levels. At this stage we generate and compare the national and regional development scenarios. All options for the development of the productive sectors within the region are constrained by

-- the economic conditions existing in the region;
-- the geographic, historic, and demographic features of the region; and
-- the function of the region (i.e., in terms of specialization) as determined by national planners.

The way in which these constraints can influence the development options is illustrated below. Silistra has been selected as a region suitable for agricultural specialization and this will provide a stimulus for the development of the food processing industry in the region. The decision to build a canal between the Danube and the Black Sea with a large harbor near Silistra will considerably extend the possibilities of developing the region. For example, the region would become an appropriate location for machine-building industries, which require easily assessible heavy transportation facilities. All possible alternatives should be thoroughly examined at this stage. The most promising options would then form the basis for more detailed analysis at the second stage of research.

The problems at this stage are ill-defined and few mathematical methods and techniques can be used directly. Interaction between the two management levels can be facilitated by the use of econometric and statistical models. It should be emphasized that for this part of the research, the initial condition of each industry within the region will be examined. However, the modeling activity will be severely constrained by the lack of suitable data. Data collection surveys must therefore be organized to provide essential information, in addition to which Monte Carlo simulation techniques could also be used.

Second Stage

At the second stage, the less effective development options will be eliminated and policies will be formulated and implemented. Optimization (location and transportation) and forecasting models will be developed. The latter are necessary for estimating the values of the parameters that have to be forecasted for each time period of the 20-year horizon. The former models examine the alternative paths of development. If a large number of possible options needs to be explored, it may be necessary to use a computer simulation scheme.

In current management theory and practice there is a serious lack of communication between managers and experts, which can be related to the following factors. First, managers require simple and realistic models that are easy to understand and operate. Second, model building, data collection, model verification and implementation are interdependent activities; model building cannot therefore be considered solely the domain of the expert. Third, since regional socioeconomic problems are composed of many interconnected elements, they should be tackled by an interdisciplinary team of experts.

The most appropriate approach at this stage is that of interactive simulation using a set of models composed of an integer optimization (location) model, an input-output model, a transportation model, and statistical and econometric models. A first-cut model that is simple and yet reflects the main research objectives should be constructed fairly rapidly to solve the problems initially defined. This model will aid our understanding of the problems and interactions related to policy making.

Our first-cut model is an integer optimization model that can determine the structure of industrial production and the location of industry in the Silistra region. Industrial production is disaggregated into the different products by each type of industry. The major constraints are on labor use, natural resources, and production capacity. The zero-one variables in the model indicate the rejection or selection of a certain development option.

Third Stage

At the third stage the problem of the regional production infrastructure will be solved. The infrastructure contains a number of auxiliary facilities for the main industrial sectors and for the activities carried out in the human settlements.

All three stages are linked by an iterative procedure and a number or reasonable and practical solutions may be obtained for the decision maker by generating several scenarios.

There are advantages and disadvantages in both the simulation and optimization approaches. Simulation models allow a problem to be described in detail and are generally applicable but they often provide solutions that require further analysis and are therefore less efficient. Conversely, optimization models offer greater analytical power but are able to handle only a limited amount of information. However, since the strength of one approach complements the weaknesses of the other, we use both to solve the problem of industrial location and development.

DESCRIPTION OF THE FIRST-CUT MODEL

The following notation is used in the first-cut integer optimization model.

Indices and Coefficients

Q is the production capacity of a certain development option for a given industry;

W^{max}, W^{min} are the upper and lower bounds of the production capacity of a given industry;

L^{max}, L^{min} are the upper and lower bounds of labor resources;

G is the quantity of a given type of resource;

h is the international price of a given product;

e is the unit cost of a given product for a certain development option;

q is the capital investment required for a given development option;

d are the accommodation costs per person in a given subregion;

b is the labor required for a certain development option;

g is the quantity of a given resource required for a certain development option;
n is the number of subregions;
m is the number of industries in the region;
r is the number of development options;
p is the number of products;
s is the number of resources available;
T is the number of periods over the time horizon;
z are the zero-one variables; and
Z is the value of the objective function.

Constraints

The constraints on labor in each subregion are

$$L_j^{min} \leq \sum_{i=1}^{m_j} \sum_{l=1}^{p_{ij}} \sum_{k=1}^{r_{ij}} b_{ilkj} \, z_{ilkj} \leq L_j^{max} \quad , \qquad (1)$$

$$j = 1,\ldots\ldots,n \quad .$$

The constraints on production capacity in each industry are

$$W_i^{min} \leq \sum_{l=1}^{p_{ij}} \sum_{k=1}^{r_{ij}} \sum_{j=1}^{n} Q_{ilkj} z_{ilkj} \leq W_i^{max} \quad , \qquad (2)$$

$$i = 1,\ldots\ldots,m \quad .$$

The constraints on each type of resource are

$$\sum_{i=1}^{m} \sum_{l=1}^{p_{ij}} \sum_{k=1}^{r_{ij}} \sum_{j=1}^{n} g_{ilkj}^{f} \, z_{ilkj} \leq G_f \quad , \qquad (3)$$

$$f = 1,\ldots\ldots,s \quad .$$

The objective function is

$$Z = \sum_{i=1}^{m} \sum_{l=1}^{p} \sum_{k=1}^{r} \sum_{j=1}^{n} c_{ilkj} z_{ilkj} \quad , \qquad (4)$$

where

c_{ilkj} is a known function of the parameters h,e,q,d,t.

The objective function for the whole optimization horizon is a sum of the objective functions of each period:

$$Z_{\Sigma} = \sum_{t=1}^{T} Z_t \quad . \tag{5}$$

Group constraints reflect the interdependence of the development options and interrelations can be expressed by integer inequalities of the following form:

$$\sum_{i=1}^{m} \sum_{l=1}^{p} \sum_{k=1}^{r} \sum_{j=1}^{n} z_{ilkj} \leq N \quad , \tag{6}$$

where

N is an integer number.

THE ALGORITHM FOR SOLVING THE FIRST-CUT MODEL

The algorithm is designed to solve problems of the following type (Figure 2):

minimize

$$Z = c^t x \quad , \tag{7}$$

subject to the constraints

$$\begin{aligned} -b + Ax &\geq 0 \ , \\ 0 \leq x \leq 1 & \text{ are integers.} \end{aligned} \tag{8}$$

The basic terms of the algorithm are defined below. The partial solution is a subset S with k variables of specified values of zero and one. Free variables are not included in S. To complete the partial solution S, it is combined with some free variables. A partial solution is augmented when subset S is increased by a free variable with a value of 1. An element of S is underlined if a value of zero is assigned to the element.

The steps of solution of the algorithm are as follows.

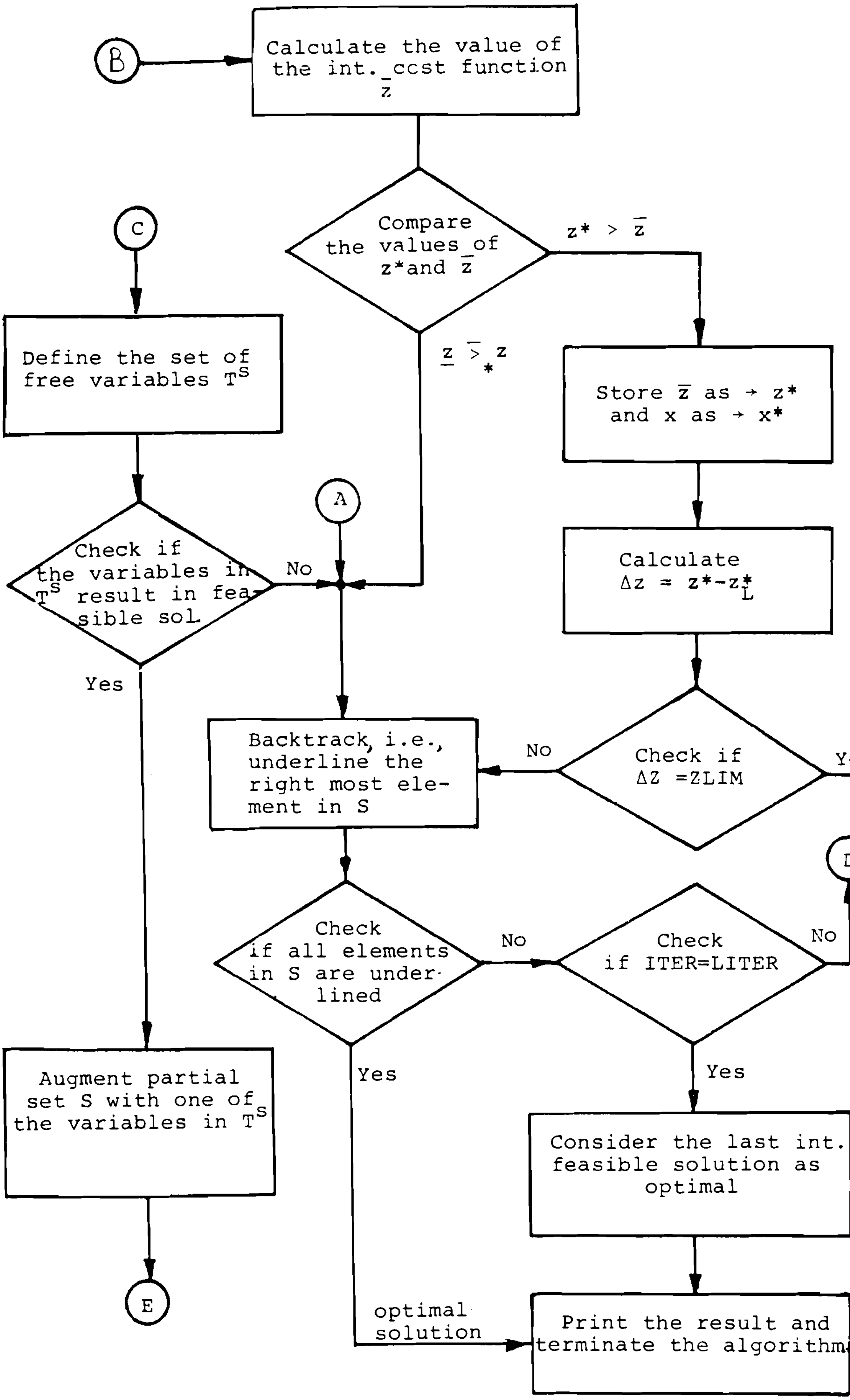

Figure 2. Flow chart of the solution of the algorithm for the first-cut model.

Step 1

The initial solution is obtained by solving the integer problem as a linear dual problem. The partial solution is constructed from the variables equal to unity in the linear solution vector x. The optimal value of the objective function z_L^* is considered as a low bound value of the integer objective function.

Step 2

A feasibility check should be made.

$$y_i^s = -b_i + \sum_{j=1}^{n} a_{ij}x_j^s \quad , \qquad j \in S \quad , \tag{9}$$

$$i = 1,\ldots,m+n_s \quad ,$$

where

n is the number of integer variables;
m is the number of constraints;
n_s is the number of surrogate constraints;
a_{ij} are the entries of matrix A; and
b_i are the entries of vector b.

If $y_i^s \geq 0$ for all i, proceed to step 12; otherwise proceed to step 3 and try to generate a feasible partial solution x^s.

Step 3

Define a set of free variables that can improve the feasibility and the current value of the objective function.

$$j \in T^s \text{ if } (cx^s + c_j < z^*) \quad , \tag{10}$$

where

$$a_{ij} > 0 \text{ for all constraints with } (y_i^s < 0;\ j \in S) \quad .$$

Step 4

Check that the variables in set T^S can be used to obtain a feasible solution.

$$y_i^T = y_i^S + \Sigma_{j\in T^S} a_{ij} \quad , \qquad \text{for all } y_i^S < 0 \quad . \qquad (11)$$

If any $y_i^T < 0$, continue to step 11. If all $y_i^T > 0$, proceed to step 5.

Step 5

Select the variable in set T^S that maximizes the expression

$$\sum_{i=1}^{m+n_S} \min(y_i^S + a_{ij}) \text{ over all } j \in T^S \text{ and } a_{ij} > 0 \quad . \qquad (12)$$

Proceed to step 6.

Step 6

Solve the integer problem as linear by fixing the variables in the new partial set S to their integer values. If the dual is unbounded, no feasible solution exists for the generated partial set and the free variables. Proceed to step 11. If the optimal solution of the restricted dual linear problem is obtained, proceed to step 7.

Step 7

Compare the values of the linear solution z' and the best integer solution found at this stage. If $z' > z^*$, no integer completion that can lead to a value more optimal than z^* exists. Proceed to step 9. If $z' < z^*$, a possible completion that can improve z^* exists. Proceed to step 8.

Step 8

Check that the variables in the linear solution have zero-one values. If this is the case, proceed to step 13, otherwise proceed to step 9.

Step 9

Construct a surrogate constraint with the following form:

$$u(Ax+b) + z^{*} - c^{t}x \geq 0 \quad , \tag{13}$$

where

u is the dual vector.

Add the constraint to the integer table only if z' < z*. Proceed to step 10.

Step 10

Rearrange the variables in the partial solution according to the value of the coefficients in the generated surrogate constraint. Proceed to step 11.

Step 11

A new partial set is constructed by underlining the rightmost element in S. Proceed to step 12. If all elements in S are underlined, the algorithm terminates, and the vector of the last solution is the optimal one.

Step 12

Calculate the value of the integer objective function for the current feasible solution.

$$\bar{z} = \sum_{j \in s} c_j x_j^s \quad . \tag{14}$$

Proceed to step 13.

Step 13

Compare the values of the current feasible solution z and the best solution stored z. If $\bar{z} < z^{*}$, store the value of z as the best solution. Proceed to step 14. If $\bar{z} > z^{*}$, the current feasible solution is not better than that which has already been found. Return to step 11.

Step 14

Replace the righthand value of the objective function that is included in the table with the value of the new found z*. Return to step 11.

THE COMPUTER PROGRAMS FOR THE FIRST-CUT MODEL

Several programs have been developed to facilitate operation of the system of models for the development and location of industry. The statistical models and forecasts are calculated by a program designed for regression, correlation, and factor analysis on the basis of the least square techniques with automatic enrichment of short time series using the Monte-Carlo simulation technique.

The input-output model will be solved by a program that uses the efficient bifactorization method for the inverse. In the program the sparsity of the input-output matrix and the special structure of the distribution of the nonzero elements in the matrix are taken into account.

The transportation problem will be solved by a program that uses methods and techniques from graph theory. This program can solve three types of transportation problem--transshipment, capacity-rated transshipment, and generalized transportation problems.

The programs, written in FORTRAN IV, for the solution of the first-cut integer optimization model can be adapted to any computer system. The advantage of using nonstandard programs is that the system of models is more easily constructed and logical, informational, and program compatibility can be achieved.

CONCLUSIONS

Three types of industrial development models are discussed in this paper: forecasting, input-output, and optimization models. They can be linked using an interactive man-machine procedure in which simulation and optimization are used as complementary rather than alternative approaches. Man-machine procedures will be developed for cases when expert intervention is required so that the set of models may be operated interactively (Figure 3).

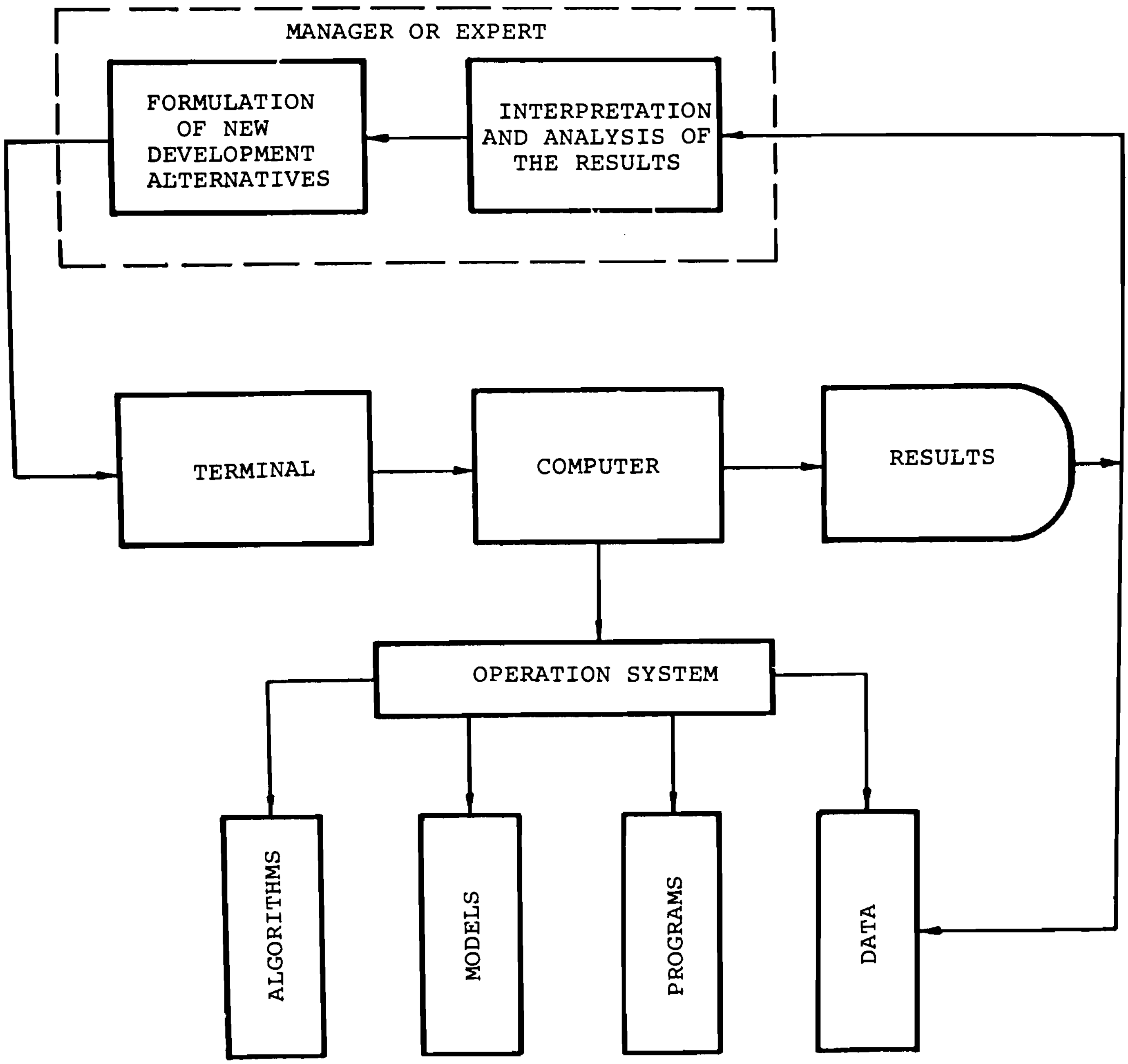

Figure 3. Man-machine interaction.

A ONE-PRODUCT ECONOMIC MODEL--
A STUDY OF PULP

N. Kolarov

INTRODUCTION

In this paper the product is considered as a combination of technological and economic properties.

The possibility of constructing a model of one product has already been discussed in Kolarov (1979). The main assumptions of this model are as follows.

-- At every point of time the product can be characterized by a set of parameters whose quantitative meaning will determine an n-dimensional vector of state of the product.

-- To comply with the dimensional value of the product vector, we can assume an n-dimensional space of the product.

-- The product has a multifunctional economic structure, which may change over time. The vector of product state is described by 16 basic economic indicators (PBEI).

The model is primarily intended to serve as an analytical tool for forecasting the internal structural changes of a given product, based on extrapolation from past trends. However, the forecasts would not be complete without consideration of certain information from multiproduct analysis using an input-output model in which the given product can be included as one of the elements. In this way a certain degree of communication is achieved between the one-product level 'one-product model) and the multiproduct level (input-output model).

Given the current accellerating demand for pulp and paper and the increasing prices of the basic raw materials required for its production, pulp plays a strategic role in the regional economy of Silistra. For this reason it was selected to demonstrate the implementation of the one-product model. It is a convenient product for study since there are only a limited number of pulp varieties and the available pulp data can be easily organized.

To provide a more accurate forecast of the future development of the given product, it was proposed that the model should be solved on two levels. Two models operate simultaneously, one in which pulp is considered as a global product and the other in which pulp is considered as a local product. We believe that such a division will allow different development alternatives for the local product to be studied within the framework of its global development. Thus, local and global development may be compared. The basic forecast curves of the product (PBEI) are compared. The other parameters related to decision-making may also be compared. The possibilities for influencing product development are explored by means of simulation. This helps transform the decisions resulting from the use of the set of simulation techniques for every forecast period into forecast values of the state vector for the 1980-1990 period. The global model of pulp is referred to as WPULP and the local (Bulgarian model as BPULP.

ONE-PRODUCT MODEL SPECIFICATION

Model Variables

Basic Economic Indicators for the Product (PBEI)
(vector of state of product)

U is product utility index;

D is product demand for the product (metric tons);

P is product price (U.S. dollars or leva);

S is product supply;

MS is product market share as a percentage of product turnover to the total market turnover;

V is the product output;

V^c is the production capacity;

E/V is the capital required for plant and equipment per unit of product;

L/V is the labor required per unit of product;

M/V are the raw materials required per unit of product;

R/V is the research and development (R & D) required per unit of product;

C is product cost per unit of the product;

G/C is marginal revenue;

Π/C is marginal profit;

I/C is capital investment;

B/C is credit.

Decision variables

Q is product quality index;

A is market promotion expenditure;

O is depreciation rate of plant and equipment;

W are wages and salaries;

M_1 is raw material expenditure--wood--in m^3;

M_2 is raw material expenditure--chemicals--in tons;

M_3 is raw material expenditure--energy--in kW;

N is structure of final product;

E is desired additional plant and capacity;

IF is internally generated capital investment;

B_1 is investment credit;

B_2 is commercial credit;

GA is government aid (subsidies).

Exogenous Variables

Y is consumer income index;

P_M is raw materials price index;

E_{t-1} is total capital in t-1;

t is time variable.

The System of Equations for the Product Model

The model is based on 16 equations that are classified into three groups--production, market, and finance sectors--and on several equations related to the selected input decision variables. Most of the equations are solved simultaneously.

The regression coefficients are symbolized with small Latin letters, one for each equation, matched with a numerical symbol according to the number of governing variables, for instance: u - $u_1, u_2, \ldots, u_4$; d - $d_0, d_1, d_2, \ldots, d_4$, etc.

The Market Sector

The utility function is

$$U_t = u_0 + u_1 D_t + u_2 Q + u_3 (R/V) + u_4 U_{t-1} \quad . \qquad (1)$$

The demand function is

$$D_t = d_0 + d_1 U_t - d_2 P_t + d_3 Y_t + d_4 D_{t-1} \quad . \qquad (2)$$

The price-formation function is

$$P_t = P_0 \, D_t^{p_1} \, S_t^{p_2} \, C_t^{p_3} \, \Delta E_t^{p_4} \quad . \qquad (3)$$

The supply function is

$$S_t = S_0 \, D_t^{s_1} \, V_t^{s_2} \, V_t^{cs_3} \, M_{1t}^{s_4} \, N_t^{s_5} \quad . \qquad (4)$$

The market-share function is

$$MS_t = Q_0 + Q_1 A_t + Q_2 B_{2t} + Q_3 MS_{t-1} \quad . \qquad (5)$$

The Production Sector

The production function is

$$V_t = V_0 \, V_t^{cv_1} \, L_t^{v_2} \, M_{1t}^{v_3} \, 1^{v_4 t} \quad . \tag{6}$$

The production-capacity function is

$$V_t^c = f_0 \, M_{1t}^{f_1} \, L_t^{f_2} \, E_{t-1}^{f_3} \, \Delta E_t^{f_4} \quad . \tag{7}$$

The capital function is

$$(E/V_t) = 1_0 + 1_1 Q_t + 1_2 (E/V)_{t-1} + 1_3 (1E+B_1)_t \quad . \tag{8}$$

The labor function is

$$(L/V)_t = 1_0 + 1_1 (R/V)_t + 1_2 W_t + 1_3 L_t + 1_4 \Delta E_t \quad . \tag{9}$$

The raw-material function is

$$(M/V)_t = m_0 + m_1 M_{1t} + m_2 M_2 + m_3 + m_4 \Delta E_{t-1} \quad . \tag{10}$$

The research and development function is

$$(R/V)_t = V_0 + V_1 V_{1t} + V_2 (\Pi/C)_t + V_3 GA_t + V_4^t \quad . \tag{11}$$

The cost function is

$$C_t = c_0 + c_1 (B/C)_t + c_2 \Theta_t + c_3 W_t + c_4 N_t \quad . \tag{12}$$

Financial Sector

The product-revenue function is

$$(G/C)_t = g_0 + g_1 P_t + g_2 S_t - g_3 C_t + g_4 (G/C)_t \quad . \tag{13}$$

The profit function is

$$(\Pi/C)_t = y_0 + y_1 P_t + y_2 S_t + y_3 (G/C)_t + y_4 Q_t \quad . \qquad (14)$$

The investment function is

$$(I/C)_t = i_0 + i_1 (\Pi/C)_t - i\Delta E_t + i_3 B_{1t} + i_4 GA_t \quad . \qquad (15)$$

The credit function is

$$(B/C)_t = B_0 + B_1 \Delta E_t + B_2 B_2 - B_3 (B_1 B_2)_{t-1} \quad . \qquad (16)$$

In addition to the equations of the model, several equations and optimization procedures determining the decision-making conditions are included for some of the variables. They consist of an equation representing desired new plant and equipment (ΔE):

$$\Delta E_t (T) = d_0 \log \left(\frac{\Pi}{(1+d) k \; P_E \; E_{t-1}} \right) \quad , \qquad (17)$$

where

$\Delta E(T)$ is the decision to invest in new plant and equipment t with investment lag;
k is the interest rate on credit;
$\alpha_0 \alpha_1$ are positive constants; and
P_E is the value of assets (buildings, plant, equipment);

a procedure for determining the optimal structure of final products (N^O); a procedure for determining the optimal depreciation rate (Θ); a procedure for determining the wage level (W); and a procedure for determining the credit required for capital investment (B_1).

The input-output specification of the model is shown below.

INPUT

Decision Variables

quality index (Q)
market promotion expenditure (A)
depreciation rate (Θ)
wages and salaries (W)
raw material 1-- wood (M_1)
raw material 2--chemicals (M_2)
raw material 3--energy (M_3)
structure of final product (N)
new plant and equipment decision (ΔE)
internally guaranteed capital investment (IF)
investment credit (B_1)
commercial credit (B_2)
government aid (GA)

Exogenous variables

consumers' income index (Y)
raw materials price index (P_M)
total capital (E_{t-1})
time variable (t)

→ THE ONE-PRODUCT MODEL →

OUTPUT

Basic Economic Indicators (PBEI)

utility (U)
demand (D)
price (P)
supply (S)
market share (MS)
output (V)
production capacity (V^c)
capital (E/V)
labor (L/V)
raw materials (M/V)
research and development (R/V)
cost (C)
revenue (G/C)
marginal profit (Π/C)
investment (I/C)
credit (K/C)

(t+1) ← Feedback ← (t)

ESTIMATION OF THE PARAMETERS OF THE ONE-PRODUCT MODEL OF PULP

The data base of the model consists of time series reflecting the quantitative changes in the product variables for pulp production and sales during the period 1965-1978 (15 observations). The primary information concerning the development of the pulp industry is easily related to pulp as a final product. All data for the pulp industry can be based on the unit of measurement of the final product. A degree of aggregation is inevitable due to difficulties in collecting and presenting some of the data.

The model parameters have been based on the time-series data for the product. After testing the equations and comparing of their feasibility, it became evident that, in general, the linear equations respond better to the processes described, and in some cases the log-linear forms are the most suitable.

The regression equations selected for both the global pulp (WPULP) and Bulgarian pulp (BPULP) are presented in Table 1.

The decision variables of the model are determined according to the most realistic hypothesis of their changes over the period under analysis, i.e., 1980-1990. For our study we have taken a scenario analysis approach. The two main scenarios used for the solution of the WPULP and BPULP models are given in Tables 2 and 3 and the results are presented in Tables 4 and 5.

Table 1. Equations of the Global and Local Models of Pulp.

No.	Equations	R
WPULP		
1	$U_t = 0.717 + 0.004\ D_t + 0.006(R/V)$	
	$+ 0.004\ Q_t + 0.0171\ U_{t-1}$	0.979
2	$D_t = 71.977 + 1006.087\ U_t - 0.218\ P_t$	
	$+ 0.687\ Y_t + 1.064\ D_{t-1}$	0.999
3	$\log P_t = 1.251 + 0.503 \log D_t - 0.194 \log + S$	
	$+ 0.290 \log C_t - 0.068 \log \Delta E_t$	0.998
4	$\log S_t = 0.421 - 0.452 \log D_t + 1.673 \log V_t$	
	$- 0.209 \log V^c + 0.147 \log M_{1t}$	
	$- 0.005 \log N_t$	1.000
5	$MS = 0.677 + 0.054\ A_t + 0.001\ B_{2t}$	
	$+ 0.246\ MS_{t-1}$	0.985
6	$\log V_t = 6.480 + 1.138 \log V_t^c + 0.015 \log L_t$	
	$+ 0.262 \log M_{1t}$	0.981
7	$V_t^c = 599.521 - 0.225\ M_{1t} + 0.699\ L_t$	
	$+ 0.144\ E_{t-1} - 2.469\ \Delta E_{t-1}$	0.997
8	$E/V_t = 121.028 + 1.069\ \Theta_t + 0.587\ (E/V)_{t-1}$	
	$+ 0.004\ (IF_t + B_1)$	0.985
9	$L/V_t = 0.158 + 0.001\ (R/V) - 0.0002\ W_t$	
	$+ 0.0001\ L_1 - 0.0002\ \Delta E$	0.976

Table 1. (continued)

No.	Equations	R
WPULP (continued)		
10	$M/V_t = 95.219 + 0.116\ M_1 + 0.004\ M_2$	
	$+ 0.011\ M_3 + 3.899\ \Delta E_{t-1}$	0.996
11	$R/V = 3.154 + 0.0003\ V_t + 0.054\ (\Pi/C)$	
	$+ 0.0002\ GA + 0.0975\ t$	0.959
12	$C_t = 216.189 - 189.055\ (B/C)_t - 0.604\ \Theta_t$	
	$-0.007\ W_t + 6.198\ N + 1.760\ P_M$	0.998
13	$G/C = 1.059 + 0.002\ P_t - 0.0001\ S_t$	
	$- 0.002\ C_t + 0.008\ (G/C)_{t-1}$	0.999
14	$\Pi/C_t = 70.231 + 0.076\ P_t - 0.054\ S_t$	
	$+ 48.742\ (G/C)_t + 0.589\ Q_t$	0.965
15	$I/C_t = 0.144 + 0.11(\Pi/C)_t - 0.024\ \Delta E_t$	
	$+ 0.0041\ B_{1t} + 0.0002\ GA$	
	$- 0.0001\ E_{t-1}$	0.849
16	$B/C_t = 0.044 - 0.009\ \Delta E + 0.0001\ B_1$	
	$+ 0.0001\ B_2 - 0.0001\ B_{1,t-1}$	0.885
BPULP		
1	$V_t = 0.055 + 0.0007\ D_t - 0.054\ (R/V)_t$	0.999
	$+ 0.004\ Q + 0.510\ V_{t-1}$	
2	$D_t = 128.769 + 365.436\ V_t - 0.321\ P_t$	0.999
	$+ 2.730\ Y_t + 0.572\ D_{t-1}$	
3	$P_t = 599.902 + 0.541\ D_t - 0.538\ S_t$	0.996
	$+ 2.160(C/V) + 0.181\ \Delta E$	

Table 1. (continued)

No.	Equations	R
BPULP (continued)		
4	$S_t = 29.526 + 0.580\ D_t + 1.250\ V_t$	
	$- 0.483\ V^c - 0.134\ M_1$	
	$- 3.868\ N$	0.999
5	$MS_t = 2.442 + 0.004\ A_t - 0.001\ B_2$	
	$- 0.221\ MS_{t-1}$	0.957
6	$\log V_t = 4.274 + 0.498 \log + 1.822 \log$	
	$+ 0.451 \log - 0.964 \log$	0.999
7	$V^c_t = 111.556 + 1.129\ W_t + 0.202\ M_1$	
	$- 22.564\ E_{t-1} - 2.010\ \Delta E_{t-1}$	0.987
8	$E/V_t = 58.291 - 0.031\ V_t + 1.249\ \Theta_t$	
	$+ 0.007(IF+B_1)$	0.992
9	$L/V_t = 0.368 - 0.017(R/V) - 0.0001\ W_t$	
	$- 0.0004\ \Delta E_{t-1}$	0.974
10	$M/V_t = 53.494 + 0.074\ M_1 + 0.042\ M_2$	
	$- 0.004\ M_3 + 0.409\ \Delta E_{t-1}$	0.984
11	$R/V_t = 1.183 - 0.004\ V_t - 0.052(\Pi/C)$	
	$+ 0.001\ GA + 0.209\ t$	0.985
12	$C_t = 68.442 - 38.592(B/C)_t + 0.887\ \Theta_t$	
	$+ 0.233\ W - 1.181\ N$	
	$+ 0.636\ P_M$	0.991
13	$G/C_t = 2.066 - 0.001\ P_t + 0.002\ S_t$	

Table 1. (continued)

No.	Equations	R
BPULP (continued)		
	$+ \ 0.0003(C/V)_t - 0.997(G/C)$	0.996
14	$\Pi/C_t = 16.125 - 0.007 \ P_t + 0.041 \ S_t$	
	$- \ 0.052(C/V)_t + 0.376 \ Q$	0.965
15	$I/C_t = 1.037 - 0.002(\Pi/C) - 0.001 \ \Delta E$	
	$+ \ 0.0001 \ B_1 - 0.00001 \ GA$	
	$+ \ 0.014 \ E_{t-1}$	0.997
16	$B/C_t = 0.77 - 0.0006 \ \Delta E + 0.0001 \ B_1$	
	$- \ 0.0001 \ B_2 + 0.00001 \ B_{1,t-1}$	0.930

Table 2. Scenario 1: Values of the decision variables for the WPULP model.

Decision variables WPULP		1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
Product quality index	Q	2.5	2.4	2.2	1.8	1.6	1.4	1.2	1.1	1.0	1.0	1.0
Market promotion expenditure (%)	A	7.5	7.8	8.1	7.8	8.0	8.5	9.1	9.3	9.5	9.7	10.0
Depreciation rate (%)	O	10.5	11.0	11.0	12.0	13.0	14.0	15.0	16.0	17.0	18.0	18.0
Wages & salaries (U.S. doll/yr)	W	6,500.0	6,750.0	7,200.0	7,600.0	8,100.0	8,400.0	8,600.0	8,900.0	9,000.0	9,200.0	9,400.0
Wood (mill m^3)	M_1	1,800.0	1,850.0	1,880.0	1,900.0	1,910.0	1,900.0	1,850.0	1,800.0	1,800.0	1,800.0	1,780.0
Chemicals (mill U.S. doll)	M_2	500.0	515.0	530.0	540.0	550.0	560.0	560.0	565.0	570.0	580.0	600.0
Energy (mill kW/h)	M_3	4,800.0	5,000.0	5,100.0	5,200.0	5,400.0	5,800.0	6,400.0	6,700.0	7,000.0	7,300.0	7,600.0
Structure of final product (mill tons)	N	5,100.0	5,200.0	5,400.0	5,500.0	5,650.0	5,900.0	6,300.0	6,800.0	6,900.0	7,100.0	7,400.0
Additional plant & equipment (mill tons)	E	10.0	10.4	10.9	11.5	13.0	13.0	13.5	14.0	14.0	14.5	15.0
Internally generated capital investment (mill U.S. doll)	IF	4.5	4.5	5.0	5.5	6.5	6.5	6.0	6.0	5.0	6.0	6.0
Investment credit (thous U.S. doll)	B_1	1,500.0	1,550.0	1,500.0	1,550.0	1,650.0	1,750.0	1,700.0	1,700.0	1,700.0	1,750.0	1,800.0
Commercial credit (thous U.S. doll)	B_2	1,500.0	1,500.0	1,650.0	1,650.0	1,700.0	1,750.0	1,700.0	1,650.0	1,550.0	1,650.0	1,700.0
Governmental aid (thous U.S. doll)	GA	500.0	550.0	580.0	600.0	620.0	630.0	630.0	680.0	700.0	720.0	750.0
Consumer price index	Y	300.0	300.0	350.0	400.0	420.0	450.0	450.0	480.0	500.0	500.0	500.0
Raw materials price index	P_M	100.0	102.0	104.0	106.0	109.0	111.0	114.0	118.0	120.0	123.0	125.0
Total capital (mill U.S. doll)	$E_{(t-1)}$	5,800.0	5,900.0	5,880.0	6,120.0	6,160.0	6,210.0	6,240.0	6,270.0	6,300.0	6,350.0	6,380.0

Table 3. Scenario 2: Values of the decision variables for the BPULP model.

Decision variables BPULP		1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
Product quality index	Q	4.0	3.0	3.0	2.0	2.0	2.0	2.0	1.0	1.0	1.0	1.0
Market promotion expenditure (thous lv)	A	150.0	175.0	200.0	220.0	250.0	290.0	330.0	370.0	410.0	400.0	400.0
Depreciation rate of plant & equip. (%)	O	7.0	10.0	11.0	11.0	12.0	13.5	15.0	17.0	18.0	18.0	18.0
Wages and salaries (lv/yr)	W	1,745.0	1,790.0	1,810.0	1,834.0	1,846.0	1,858.0	1,875.0	1,900.0	1,915.0	1,940.0	1,960.0
Wood (mill m^3)	M_1	900.0	1,030.0	1,840.0	1,200.0	1,250.0	1,250.0	1,280.0	1,310.0	1,300.0	1,300.0	1,300.0
Chemicals (thous lv)	M_2	3,850.0	3,910.0	3,940.0	4,170.0	4,310.0	4,550.0	4,650.0	4,900.0	5,200.0	5,500.0	5,700.0
Energy (mill kw/h)	M_3	12,300.0	14,110.0	15,200.0	16,300.0	18,100.0	19,100.0	20,450.0	22,320.0	2,400.0	26,000.0	27,500.0
Structure of final product (thous tons)	N	23.0	24.0	25.0	25.0	26.0	27.0	29.0	30.0	32.0	33.0	35.0
Additional plant & equipment (thous tons)	E	30.0	30.0	30.0	30.0	20.0	20.0	20.0	10.0	10.0	10.0	10.0
Internally generated capital investment (thous lv)	IF	4,000.0	4,500.0	5,200.0	6,000.0	6,400.0	6,600.0	6,700.0	7,000.0	7,100.0	7,200.0	7,500.0
Investment credit (thous lv)	B_1	3,000.0	3,500.0	4,100.0	4,550.0	4,700.0	5,000.0	5,000.0	5,100.0	4,850.0	5,000.0	5,100.0
Commercial credit (thous lv)	B_2	9,000.0	9,200.0	9,350.0	9,420.0	9,550.0	10,110.0	10,350.0	10,420.0	11,180.0	1,210.0	1,230.0
Government aid (thous lv)	GA	1,000.0	1,200.0	1,100.0	1,000.0	700.0	350.0	350.0	300.0	200.0	200.0	200.0
Consumer price index	Y	100.0	103.0	107.0	111.0	114.0	117.0	119.0	122.0	126.0	130.0	132.0
Raw mat. price index	P_M	100.0	106.0	111.0	114.0	118.0	125.0	140.0	148.0	155.0	160.0	165.0
Total capital (thous lv)	$E_{(t-1)}$	18.5	20.6	22.5	24.6	26.0	27.0	27.0	28.0	29.0	30.0	31.0
Time	t	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990

Table 4. Results of simulation--the WPULP model.

PBEI; WPULP		1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
Product utility index	U	0.88	0.89	0.90	0.90	0.90	0.91	0.92	0.93	0.93	0.94	0.94
Demand (mill tons)	D	165.00	172.80	183.00	194.00	205.60	218.00	231.00	245.00	259.00	275.30	291.80
Price (U.S. doll/tons)	P	420.00	435.00	440.00	448.00	456.00	467.00	480.00	496.00	508.00	520.00	525.00
Supply (mill tons)	S	155.00	165.00	174.00	182.00	193.00	205.00	216.00	231.00	247.00	262.00	280.00
Market share (%)	MS	2.05	2.07	2.09	2.11	2.15	2.21	2.24	2.27	2.31	2.42	2.43
Output (mill tons)	V	160.00	168.80	178.00	187.80	198.20	210.00	222.00	237.00	251.00	266.30	282.20
Production capacity (mill tons)	V^C	182.00	186.40	195.80	215.90	227 70	241.50	255.30	262.00	288.60	306.00	324.00
Capital (U.S. doll/ton)	E/V	345.00	345.00	350.00	352.00	354.00	355.00	360.00	362.00	365.00	367.00	370.00
Labor (man/d/ton)	L/V	0.10	0.09	0.09	0.08	0.07	0.07	0.07	0.06	0.06	0.06	0.05
Raw materials (U.S. doll/ton)	M/V	55.00	58.00	62.00	67.00	75.00	82.00	88.00	94.00	99.00	100.00	106.00
Research and development (U.S. doll/ton)	R/V	4.31	4.72	5.30	5.40	5.45	5.50	5.50	5.60	5.70	5.80	6.00
Cost per unit (U.S.doll/ton)	C	390.00	405.00	408.00	420.00	434.00	438.00	445.00	462.00	469.00	478.00	490.00
Revenue per unit (in U.S. doll)	G/C	1,073.00	1,074.00	1,078.00	1,067.00	1,051.00	1,066.00	1,079.00	1,074.00	1,083.00	1,088.00	1,071.00
Marginal profit (%)	Π/C	7.30	7.40	7.80	6.70	6.50	6.60	7.90	7.40	8.30	8.80	7.10
Investment per unit (in U.S. doll)	I/C	0.10	0.11	0.13	0.11	0.09	0.09	0.10	0.12	0.14	0.15	0.12
Credit per unit (in U.S. doll)	B/C	0.05	0.05	0.06	0.05	0.04	0.04	0.05	0.06	0.06	0.07	0.06
Time	t	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990

Table 5. Results of simulation--the BPULP model.

PBEI: BPULP		1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
Product utility index	U	0.56	0.60	0.64	0.68	0.70	0.74	0.78	0.81	0.84	0.86	0.90
Demand (thous tons)	D	530.00	560.00	590.00	630.00	670.00	720.00	750.00	780.00	810.00	840.00	880.00
Price (lv/ton)	P	330.00	360.00	375.00	390.00	400.00	420.00	440.00	460.00	480.00	500.00	520.00
Supply (thous tons)	S	205.00	230.00	250.00	270.00	295.00	310.00	330.00	345.00	360.00	380.00	405.00
Market share (%)	MS	2.10	2.40	2.60	2.70	2.80	2.80	3.00	3.00	3.10	3.20	3.40
Output (thous tons)	$\dot{V}$	200.00	220.00	250.00	260.00	270.00	270.00	280.00	290.00	300.00	300.00	310.00
Production capacity (thous tons)	V^c	205.00	225.00	260.00	265.00	270.00	275.00	285.00	290.00	310.00	310.00	315.00
Capital (lv/tons)	E/V	111.00	120.00	135.00	140.00	145.00	150.00	153.00	155.00	158.00	160.00	160.00
Labor (man/days/ton)	L/V	0.10	0.10	0.09	0.09	0.08	0.08	0.08	0.07	0.07	0.07	0.06
Raw materials (lv/ton)	M/V	140.00	140.00	150.00	158.00	160.00	160.00	160.00	165.00	170.00	175.00	175.00
Research and development (lv/ton)	R/V	2.20	2.30	2.40	2.60	2.90	3.00	3.10	3.20	3.40	3.60	3.80
Cost (lv/ton)	C	350.00	360.00	360.00	365.00	370.00	370.00	375.00	380.00	385.00	390.00	400.00
Revenue per unit (in lv)	G/C	1.03	1.05	1.07	1.07	1.07	1.08	1.08	1.07	1.08	1.09	1.10
Marginal profit (%)	Π/C	4.80	5.20	6.50	7.00	7.00	7.50	8.00	7.50	8.00	8.50	9.00
Investment per unit (in lv)	I/C	0.11	0.14	0.17	0.20	0.22	0.25	0.26	0.28	0.30	0.30	0.32
Credit per unit (in lv)	B/C	0.06	0.08	0.10	0.10	0.11	0.10	0.11	0.12	0.12	0.12	0.13

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MODELING THE TRANSPORTATION SYSTEM AS A PART OF THE SYSTEM OF REGIONAL DEVELOPMENT MODELS

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INTRODUCTION

Transportation is a large-scale system composed of many elements and closely connected to the other systems of the region. For this reason, the modeling of the transportation system will make an important contribution to the regional development modeling activity being carried out at the International Institute for Applied Systems Analysis. This activity is devoted to the creation of a system of regional models, which are to be used to help solve problems of medium-and long-term socioeconomic development at the regional level.

At present, there are two major objectives in developing a transportation model. First, the model must meet the requirements of the other regional subsystems in order to be linked to the model system; it has therefore been based on the main ideas presented in Mihailov (1979). Second, the different modes of transport within the transportation system are to be modeled using the same criterion. We have thus concentrated on obtaining practical results and on testing the models, but less attention has been paid to the linkage of the regional to the national model system. This will be accomplished at a later stage. The results obtained here should be considered as preliminary.

THE TRANSPORTATION MODEL AND ITS LINKS WITH THE REGIONAL SYSTEM OF MODELS

The interdependencies of the transportation model and the other regional models are defined in Figure 1, which presents in graphic form the ideas mentioned previously in Mihailov (1979). The transportation model considered in this paper would be applicable to any system of regional models. The five principal stages required to establish links between the transportation and the other regional subsystems are outlined below.

Stage 1: Determination of Total Traffic Volume from the Intraregional Input-Output Forecasting Model

The interrelations between productive and nonproductive consumption within the region may be described by producers and users and by nodes (subregions); in this paper national and local production are considered together.

$$(X_{ir}^{t} + I_{r}^{t}) = \sum_{r=1}^{s} \sum_{j=1}^{n} (a_{ij}X_{jr}^{t} + b_{ij}X_{jr}^{t} + E_{r}^{t}) + (\alpha_{i}Y_{1}^{t} + \beta_{i}Y_{2}^{t}) \quad , \tag{1}$$

$$i = 1,2,\ldots,n \quad , \qquad j = 1,2,\ldots,n \quad ,$$

$$r = 1,2,\ldots,s \quad ,$$

where

- X_{ir}^{t}, X_{jr}^{t} is the volume of products i and j produced in subregion r during year t;
- s is the number of subregions;
- I_{r}^{t} is the import to subregion r from other regions during year t;
- E_{r}^{t} is the export from subregion r to other regions during year t;
- a_{ij} is the volume of product i needed for the production of 1 unit of product j;

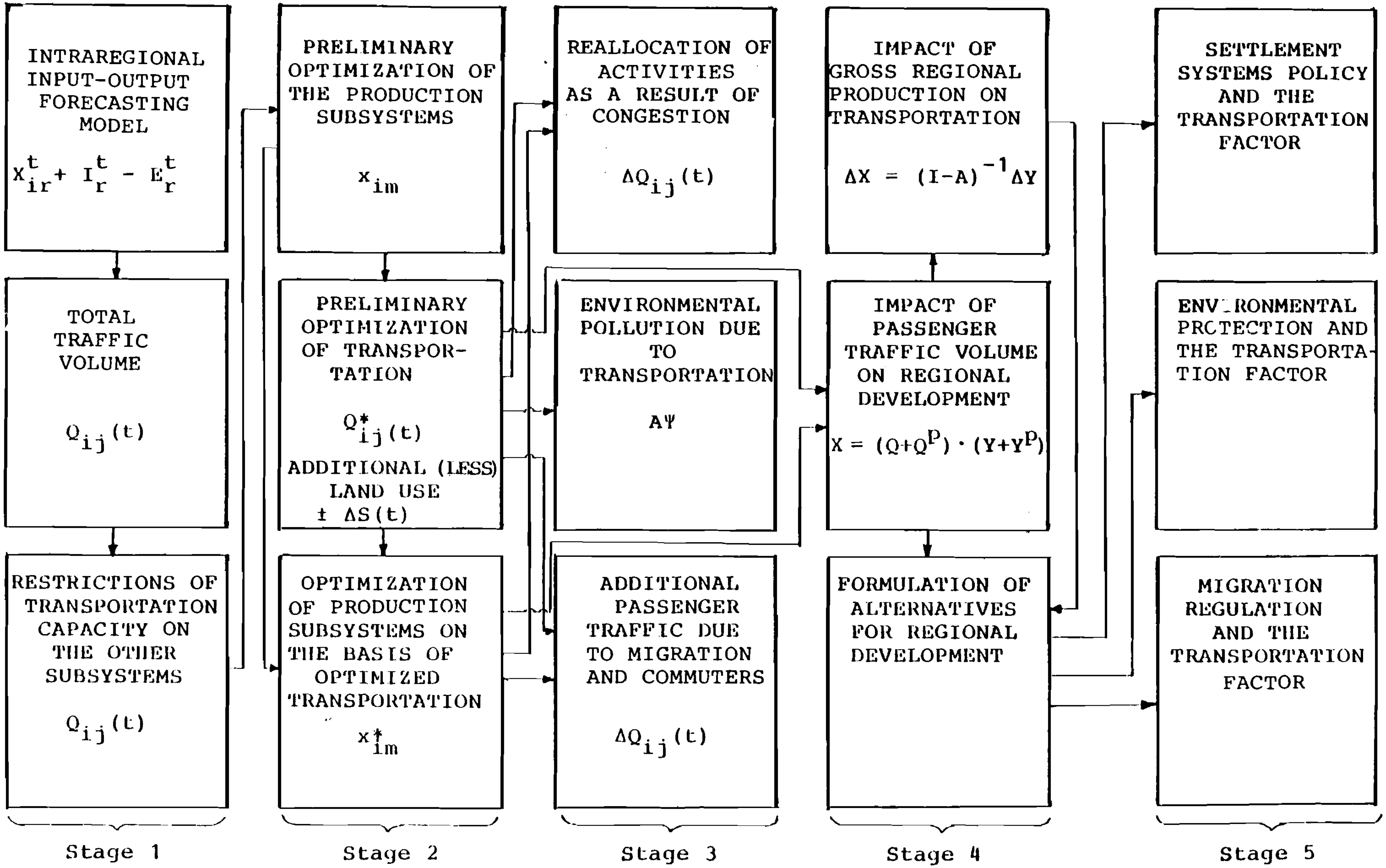

Figure 1. The interdependence of the transportation model system and the regional system of models.

b_{ij} are the capital investment coefficients;

$\alpha_i Y_1^t$ is the volume of product i for final nonproductive consumption by the population during year t (α_i are coefficients of the various products); and

$\beta_i Y_2^t$ is the volume of product i for consumption by the nonproductive subsystems during year t (β_1 are coefficients of the various products produced).

Thus, the required freight traffic volume within the region can be derived by identifying the territorial location of the producing and consuming points. On this basis a proposed transportation network for the Silistra region has been drawn (Appendix A). Within this network existing and possible modes of transport connect the producing to the consuming points. The total traffic volume consists of freight traffic volume and passenger traffic volume:

$$Q(t) = Q^f(t) + Q^p(t) \quad , \tag{2}$$

where

Q is total traffic volume;

Q^f is freight traffic volume; and

Q^p is passenger traffic volume.

The freight traffic volume consists of:

$$Q^f_{ijp} = Q^{f1}_{ijp} + Q^{f2}_{ijp} + Q^{f3}_{ijp} + Q^{f4}_{ijp} \quad , \tag{3}$$

where

Q^{f1}_{ijp} is the traffic ensuring the productive and nonproductive consumption of load p within the region;

Q^{f2}_{ijp} is the traffic ensuring the export of goods from the region under analysis to the other regions.

Q^{f3}_{ijp} is the traffic ensuring the import of goods from other regions; and

Q_{ijp}^{f4} is the transit traffic.

The passenger traffic volume may be derived in terms of a formula suggested in Mihailov and Nicolov (1974):

$$Q_{ij}^{p}(t) = N_o(P_o+P_n)S_n \quad , \qquad (4)$$

where

N_o is a coefficient of migration and commuting in the initial year of the analyzed period (km per capita);

P_o is the size of population in the initial year of the analyzed period;

P_n is the annual population growth rate; and

S_n is the elasticity coefficient expressing the dependence of migration and commuting on factors such as wages, housing, tourism, etc.

The general scheme of the transportation system components is presented in Figure 2. Using an input-output forecasting model to specify the total traffic volume, we can exogenously prescribe the volume and directions of traffic $Q_{ijp}(t)$ and also the restrictions on capital investments for the transportation subsystem:

$$\sum_j b_{ij}X_{jr}(t) \leq B(t) \quad , \qquad (5)$$

where

B(t) is capital investment during year t.

It also enables us to prescribe the constraints on labor within the transportation subsystem:

$$\sum_j v_{qj}X_{jr}(t) \leq L(t) \quad , \qquad (6)$$

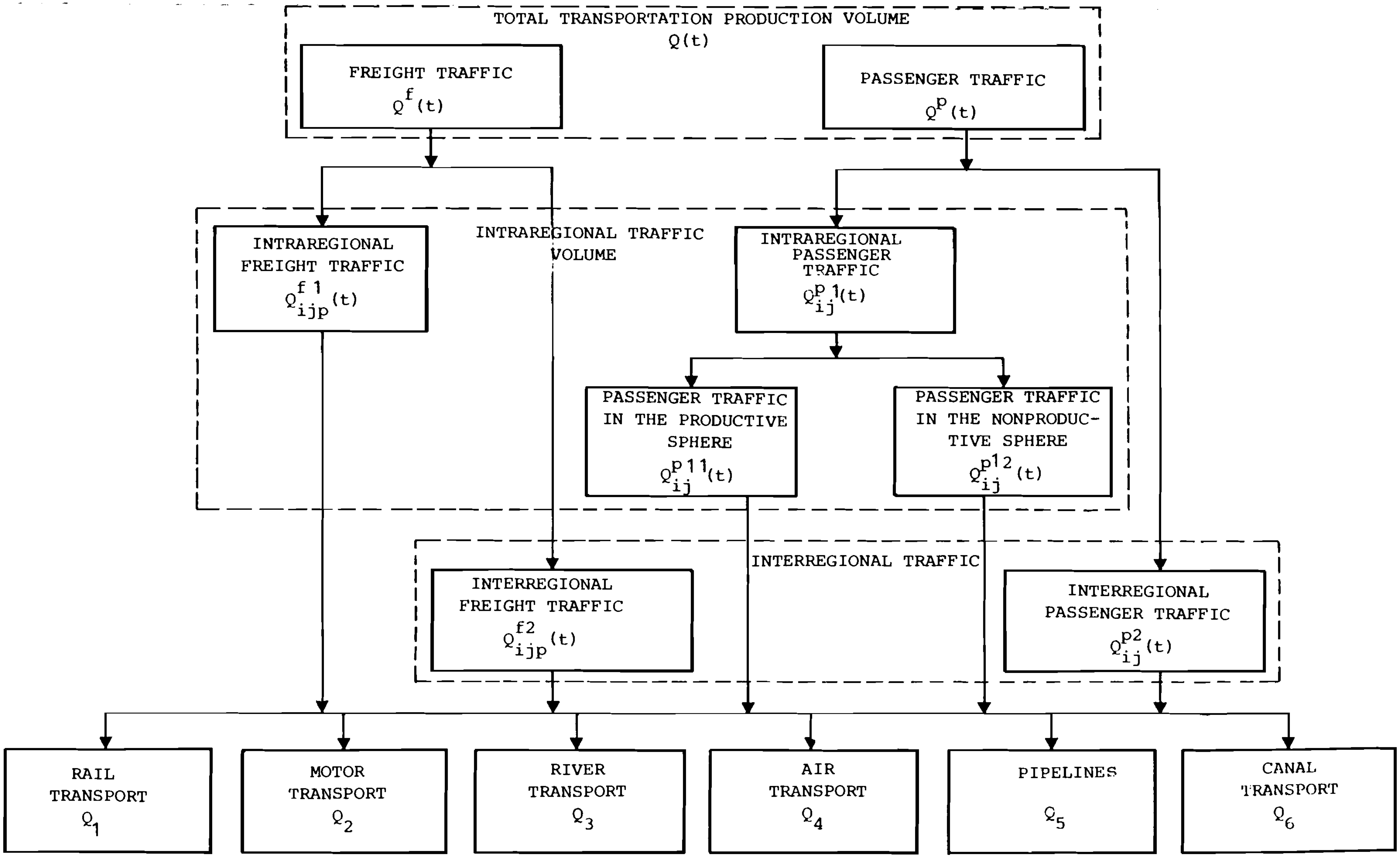

Figure 2. Transportation subsystem components.

where

v_q are labor coefficients of qualification groups of type q; and

L(t) is labor available in year t;

and to assign the restrictions on transport capacity for the other subsystems of the region:

$$\sum_i \sum_m \sum_n a_{ji} X_{jm}(t) \leq Q_{ij}(t) \quad , \tag{7}$$

where

i are products;

m are subregions; and

n are subsystems.

Stage 2: Preliminary Local Optimization of Transportation and the Other Regional Subsystems

Preliminary local optimization of the transportation and other regional subsystems can be achieved after satisfying the following conditions.

Optimization of each production subsystem in terms of the local criteria

Production subsystems are optimized in terms of local criteria (i.e., minimizing total costs--production and transportation). As a result of this, the optimal volume, structure, and territorial distribution of production and resources can be derived. Thus, the transportation costs involved in this distribution process have to be taken from the nonoptimized transportation subsystem. The reason for this being that it is not possible to optimize transportation before deriving the volume of the production and resources of the other subsystems and their territorial distribution. Generally for each production subsystem:

$$\min \{ \sum_i \sum_m [\underbrace{a_{ji} x_{jm}(t)}_{\text{local resource costs}} + \underbrace{v_{ji} \nu_i x_{jm}(t)}_{\text{wage costs}} + \underbrace{c_j x_{jm}(t)}_{\text{intermediate production costs}}$$

$$+ \underbrace{\alpha K_{jm}(t)}_{\text{annual capital investment}} + \sum_l \underbrace{(d_{ilm} + \alpha K_{ilm})\ x_{ilm}(t)]\}}_{\text{annual transportation costs}} \quad , \qquad (8)$$

subject to constraint (7),
where

- x is the production volume;
- ν_i is the average wage in sector i;
- α is a coefficient for reducing the capital investment for the period under analysis to annual costs;
- K is capital investment;
- l are modes of transport; and
- d are transport operational costs.

Preliminary optimization of the transportation subsystem

Preliminary optimization of the transportation subsystem will be treated in detail in the next section of this paper, but here it is necessary to mention that the transportation costs in objective function (8) have to be derived from optimization of the transportation subsystem (* = optimal):

$$\sum_j \sum_m \sum_l (d^*_{ilm} + \alpha K^*_{ilm}) \cdot x_{ilm}(t) \Rightarrow Q^*_{ij}(t) \quad . \qquad (9)$$

Total traffic volume must be prescribed for the transportation subsystem following optimization of the other production subsystems:

$$\sum_i \sum_m \sum_n X^*_{jm}(t) \leq Q^*_{ij}(t) \quad . \qquad (10)$$

As a result of the optimization of the other subsystems, there is additional migration and commuting, which changes the volume of passenger traffic:

$$Q^{*}_{ij}(t) = Q^{f}_{ij}(t) + Q^{p}_{ij}(t) \pm \Delta \sum_{j} \sum_{s} v_{qj} X_{jr}(t) \quad . \qquad (11)$$

Optimization of the production subsystems using the optimized transportation costs

Optimization of the production subsystemsusing optimized transportation costs can be achieved using objective function (8), in which the costs resulting from optimization of the transportation subsystem (9) are included. Costs (or profit) expressing land occupied by (or released for) the transportation subsystem have to be added (or reduced to annual costs):

$$\sum_{j} \sum_{m} \sum_{l} (d^{*}_{ilm} + \alpha K^{*}_{ilm}) \; x_{ilm}(t) \pm \Delta \; \alpha \sum_{j} \sum_{m} \sum_{l} u_{ilm} X_{ilm} \; , \qquad (12)$$

and optimization of the production subsystems (8) will be subject to

$$\sum_{i} \sum_{m} x_{im}(t) \leq S(t) \pm \Delta \; S(t) \quad , \qquad (13)$$

where

u_i is the unit price of land; and
S is the total available land.

Stage 3: Determination of the Effects on the Transportation Subsystem Resulting from the Concentration of Economic Activities at One Point in the Region

The concentration of economic activities at one point in the region has some effects on the transportation subsystem. These effects are discussed below.

The need to relocate production and service activities

If production and service activities have to be relocated, Stage 2 should be repeated, taking into account the additional or reduced transportation costs:

$$\Delta \sum_{i} \sum_{m} (d_{ilm}+\alpha K_{ilm}) x_{ilm}(t) \quad . \tag{14}$$

The Contribution of the Transportation Subsystem to Pollution of the Environment

The degree of environmental pollution may be expressed by forming a matrix $A\psi$ consisting of n vectors $(0,\ldots,0,\psi_1, 0\ldots,0)$ representing the pollutants emitted by transport mode 1 in forming 1 unit of traffic volume. Hence, the additional expenditures required by sector i to purify the unit of product produced in sector j to conform to the statutory regulations will be

$$\Delta \sum_{i} \sum_{m} e_{ilm} x_{ilm} \quad , \tag{15}$$

where

e are coefficients for processing the pollutants.

Analysis of the Effect of Increased Passenger Traffic on the Transportation Subsystem

To determine the increased passenger traffic and its effect on the transportation system, required and expected migration and commuting should be compared. The procedure is similar to equation (11). The needed migration is determined by optimization of the other regional subsystems.

Stage 4: Determination of the Effect of the Transportation Subsystem on Regional Development

The results of the previous stages allow us to study the effect of the transportation system on regional development as a whole by means of an impact analysis model. Analysis will be made of

- the effect of passenger traffic volume on regional development; and
- the effect of gross regional production on transportation.

Analysis of the effect of passenger traffic volume on regional development

Passenger traffic (with the exception of that in the productive sector) represents services for the population as final production. By specifying the percentage share of final production, we can determine its influence on gross (productive plus nonproductive) regional production. In matrix form, this can be expressed as

$$X - AX = Y + Y^p \quad , \qquad (16)$$

where

X is a vector of total input;
A is a n x n matrix of the input coefficients;
Y is a vector of final production; and
Y^p is a vector of passenger traffic volume.

If I is an identity matrix:

$$(I-A)X = Y + Y^p \quad , \qquad (17)$$

and if (I-A) is negative, the reverse matrix will express the gross regional production as a function of passenger traffic that is a part of final production:

$$X = (I-A)^{-1} \cdot (Y+Y^p) \quad . \qquad (18)$$

Thus, if

$$(I-A)^{-1} = Q + Q^p \quad , \qquad (19)$$

then, in

$$X = (Q+Q^p) \cdot (Y+Y^p) \quad , \qquad (20)$$

the coefficients q_{ij} and q^p_{ij} (i,j = 1,2,...,n) of matrix $(Q+Q^p)$ will be direct and indirect requirements of 1 unit of final production (and of transportation) for the total output of the region.

Analysis of the Effect of Gross Regional Production on the Transportation Subsystem

Gross regional production has a significant effect on the development of the transportation subsystem. It is possible to analyze this effect by treating transportation as an intermediate product as part of gross regional production in the intraregional input-output forecasting model. Hence, if

$$x_i = \sum_j a_{ij} x_j + Y \quad , \qquad i = 1,2,\ldots,t-1 \quad , \tag{21}$$

where

Y is final production,

represents distribution of production without transportation, then,

$$x_t = \sum_j a_{t_j} x_j + Q^p \quad , \qquad t = n \quad , \tag{22}$$

where

Q^p is passenger traffic as final production,

represents distribution of production including transportation. In matrix form, the equation including transportation will be

$$X = AX + Y \quad , \tag{23}$$

with the solution:

$$X = (I-A)^{-1} \cdot Y \quad , \qquad X = X_1, X_2, \ldots, X_t \quad . \tag{24}$$

Then the increase in traffic volume as a function of gross regional production may be expressed as

$$\Delta X_t = (I-A)^{-1} \cdot \Delta Y_t \quad , \tag{25}$$

and the interdependencies will appear as shown in Figure 3.

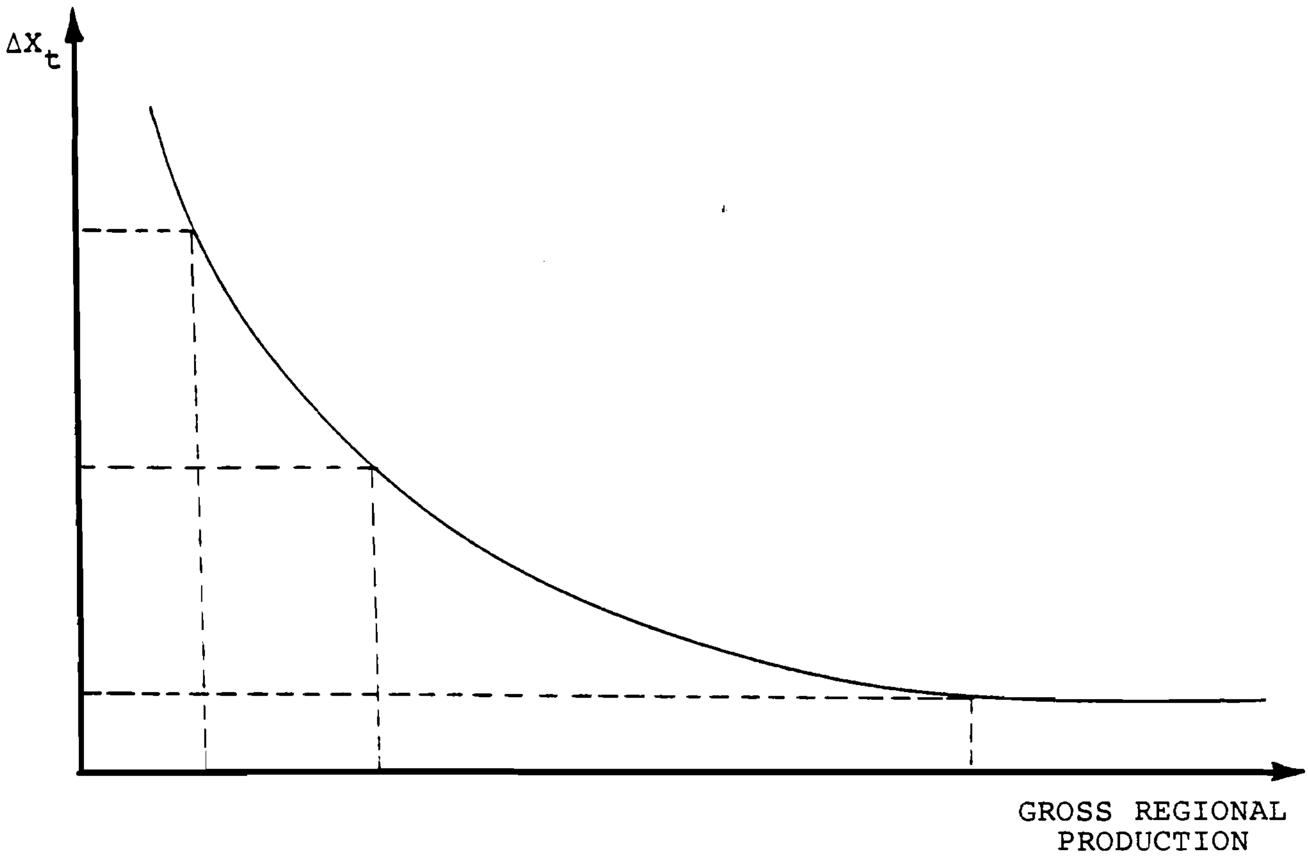

Figure 3. Dependence of total regional production on traffic volume.

With respect to the transportation subsystem, the analyses discussed above allow us to formulate alternatives for development of the region as a whole.

Stage 5: Determination of the Influence of the Transportation Subsystem on Total Optimization of the Region

Transportation is a subsystem of strategic importance for the region. This is because of its specific spatial and intersectoral links with the other subsystems and also because the units of one type of transport may be substituted for units of another type. In planning regional development, special consideration must be given to the transportation subsystem. The final stage of optimization of the regional system is concerned with planning. Here, the influence of transportation can be seen in three areas of policy making--migration and commuting, environmental protection, and human settlements.

The Influence of the Transportation Subsystem in Regulating Migration and Commuting

At this stage planners have to choose between two courses of action. Production units can be constructed close to existing settlement systems, so that a labor supply is readily available. Alternatively, these units can be built in an area without an indigenous labor supply, in which case the labor force would be encouraged to migrate or commute to the newly constructed production area, and residential, social, transportation, and other facilities would have to be developed. If the second option were chosen, the optimal size of migration and commuting flows would have to be determined. These flows will be determined when the marginal value of income due to the migration and commuting is equal to marginal total costs incurred by migration:

$$\frac{dP^m}{dC^m} = \frac{dP^m}{d(D+S+T)} = 0 \quad , \tag{26}$$

where

P^m is the additional income from migration and commuting;

C^m are additional costs caused by migration and commuting;

D are additional costs for housing;

S are other additional social costs; and

T are additional costs for the development of the transportation subsystem.

Transportation in Relation to Environmental Protection in the Region

The degree of environmental pollution is determined by the volume of pollutants released by all transport units and by the concentration of these units at one point in the region. There is the problem of how to make environmental protection regulations effective if statutory pollution levels are exceeded. In such a case two alternative measures can be taken:

-- dispersal of transportation units from one point in the region; and
-- development of nonpolluting transport technologies.

The low boundary of the effectiveness of these measures can be determined when the marginal value of income resulting from the above measures is equal to the marginal costs for environmental protection:

$$\frac{dP^{env}}{dC^{env}} = 0 \quad , \tag{27}$$

where

P^{env} is the additional income from nonpolluting transport technologies; and

C^{env} are additional costs caused by migration and commuting.

The Importance of the Transportation Subsystem on the Construction Policy of the Settlement Systems

With respect to the settlement systems construction, one of the major problems is whether or not the settlements should be concentrated at various points in the region. This involves consideration of their optimal size and location. Transportation and other social and infrastructural facilities have to be considered in these terms. The optimal size and location of the settlements are determined on the basis of production and social (including transportation) costs:

$$\sum_i \sum_j \sum_r S^r_{ij} x^r_{ij} + \sum_i \sum_j \sum_r \sum_l d^r_{ilj} x^r_{ilj} \rightarrow \min \quad , \qquad (28)$$

where

- i are settlement units ($i = 1,2,\ldots,m$);
- j are settlement system activities (social and productive);
- r are alternatives for developing settlement systems ($r = 1,2,\ldots,k_i$);
- l are consumers of production and users of services in the settlement units ($l = 1,2,\ldots,k$);
- S^r_{ij} are social costs of activities j in settlement unit i under settlement system development alternative r;
- x^r_{ilj} are the total number of activities j in settlement unit i under development alternative r; and
- d^r_{ilj} are transportation costs for activities j in settlement unit i under development alternative r.

The problem is subject to

$$\sum_i \sum_r x^r_{ilj} \geq b_{lj} \quad , \qquad (29)$$

a restriction on all users' and consumers' needs for activity j;

$$\sum_r a^r_{ij} x^r_i \geq \sum_l \sum_r x^r_{ilj} \quad , \qquad (30)$$

the total number of activities in settlement unit i is greater or equal to the commodities and services produced by activity j shipped from the settlement unit; and

$$x_i^r = \begin{matrix} 1 \\ 0 \end{matrix} \quad , \qquad (31)$$

zero-one constraints.

MODELING THE OPTIMAL DEVELOPMENT OF THE REGIONAL TRANSPORTATION SUBSYSTEM

In order to link the transportation model to the other subsystem models, the transportation subsystem has to be optimized. For the Silistra region, it has been optimized hierarchically at two levels: local optimization of each mode of transport and global optimization of the regional transportation subsystem as a whole. It was not possible to solve all problems simultaneously, the process has therefore to be carried out in stages (Figure 4).

Stage 1: Definition of the Initial State of Each Mode of Transport and the Transportation Subsystem

The initial state of the transportation subsystem ($G_{i(0)}$) in the Silistra region can be determined by defining the initial state of each type of transport:

$$G_{i(0)} = G_{1(0)} + G_{2(0)} + G_{3(0)} + G_{4(0)} + G_{5(0)} + G_{6(0)} \quad , \qquad (32)$$

where

G_1 is rail transport;
G_2 is motor transport;
G_3 is river transport;
G_4 is air transport;
G_5 are pipelines; and
G_6 are canal systems.

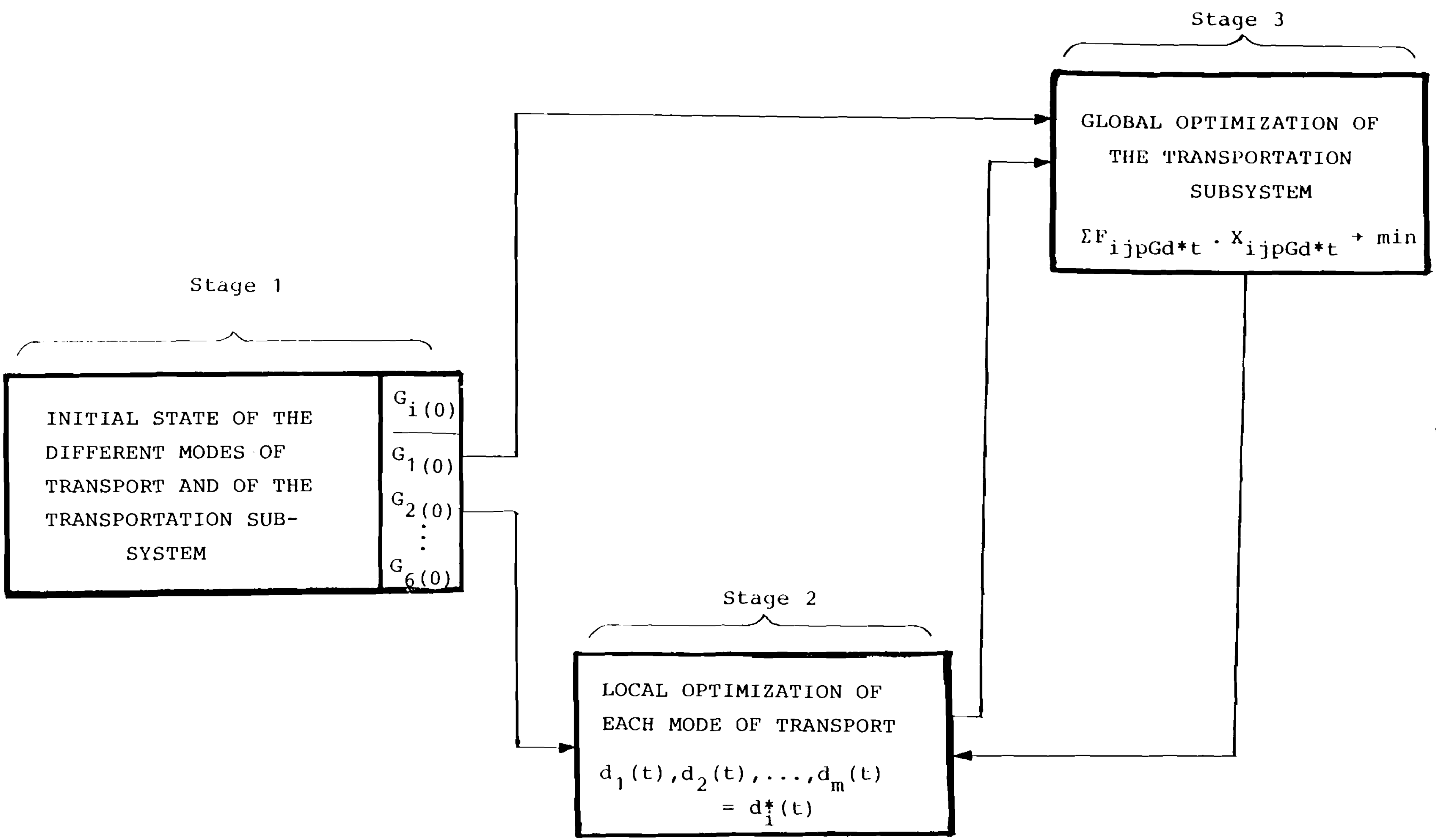

Figure 4. Block scheme of the transportation subsystem optimization.

The technological and economic indicators (d_0 and f_0, respectively) and the feasible traffic volume (x_0) are defined for each component of the transportation network.

Stage 2: Local Optimization of the Development of Each Mode of Transport

The technological development of the components of the regional transportation network (G_i) can be described by m parameters: $d_1, d_2, \ldots, d_m$. The vector functions D(t) are determined in terms of these parameters for the technological development alternatives of the transportation network:

$$D(t) = [d_1(t), d_2(t), \ldots, d_m(t)] = f[Q(t)] \quad ; \qquad (33)$$

i.e., the definition of each of these vector functions ($d_i(t)$) is a function of traffic volume (Q(t)). Possible schemes of technological development tasks for different modes of transport are given in Appendix B.

Since the potential traffic volume will considerably increase the number of development alternatives, it is necessary to analyze different alternatives for traffic volume at different intervals: $Q(t) + \Delta Q(t)$; i.e., each alternative should allow for some reserve capacity.

The choice of the optimal alternative for each type of transport is made in the same way as for the consecutive intervals of $Q(t) + \Delta Q(t)$. (See Appendix C for the indicators related to the optimal development of a rail section and Appendixes D and E for the optimal development of motor transport.)

In general it is necessary, from the set of M possible combinations of tid, for tid to be defined so that subset $M_1 \in M$ will ensure minimum transportation costs:

$$\sum_{(M_1)} (C_{tid} + \frac{1}{T_n} K_{tid}) x_{tid} \rightarrow \min \quad , \qquad (34)$$

where

x_{tid} is a vector of the traffic volume in year t concerning component i of the regional transportation network and variant d for technological development;

K_{tid} is the capital investment required given the above conditions;

C_{tid} is current expenditure given the above conditions; and

T_n is an indicator representing productive life of the transportation plant (in years), from which, using the coefficient K_{tid}, the total capital investment is reduced to annual costs.

The objective function is satisfied using constraints (35) and (36). Constraint (35) represents the required traffic between the points of production and use:

$$\sum_j x_{jpyt} = Q(y,p,t) \quad , \tag{35}$$

where

j are components of the transportation network; and
y are transport intersections;

and constraint (36) represents the compatibility of traffic volume and transportation capacities (g).

$$\sum_j x_{tid} \leq g_{tid} \quad , \tag{36}$$

As a result of the above, the technological modes for each $Q(t) + \Delta Q(t)$ were optimized--d^*.

Stage 3: Global Optimization of the Regional Transportation Subsystem

Optimization of regional transportation subsystem was based on preliminary optimization of each type of transport at Stage 2. The realization and redistribution of capital investments for the transportation subsystem is dependent on the efficiency of each mode of transport. Global optimization was performed in terms of the objective function--minimum reduced annual transportation costs--for conditions where:

- $Q(t)$ is a vector of traffic volume as a function of time, prescribed for each component of the transportation network and obtained from the solution of the other regional subsystem models;
- a_{ip} is the volume of load p (i.e., the type of passenger traffic) at production point i, prescribed from $Q_{ij}(t)$;
- b_{jp} is the volume of load p at user's point j, prescribed from $Q_{ij}(t)$;
- $D(t)$ is the vector of different development alternatives for each component of the transportation system;
- $d^{*}(t)$ are the technological development alternatives for different intervals of traffic volume $Q(t) + \Delta Q(t)$ and are prescribed from preliminary optimization at Stage 2;
- $C_{ijpGd^{*}t}$ is the vector of current transportation costs for shipping load p from user's point i to j by means of transportation mode G using variant d^{*}, which is preliminarily optimized for given intervals of traffic volume $Q(t) + \Delta Q(t)$, in year t of the period under analysis;
- $K_{ijpGd^{*}t}$ is the vector of capital investment given the above conditions;
- T_n is the indicator representing the productive life of the transport production plant; and
- X_{ijpGdt} is the potential traffic volume, which has to be distributed among the different modes of transport.

The objective function is:

$$\sum_i \sum_j \sum_p \sum_G \sum_d \sum_t \left(C_{ijpGd^{*}t} + \frac{1}{T_n} K_{ijpGd^{*}t}\right) X_{ijpGdt} \rightarrow \min \quad . \tag{37}$$

The model was solved taking into account the following constraints. For the total volume of production:

$$\sum_i \sum_p \sum_G \sum_d \sum_t X_{ijpGdt} = a_{ip} \quad , \qquad \begin{aligned} i &= 139 \; , \\ p &= 17 \; , \\ G &= 6 \; , \\ d &= 1,2,\ldots,m \; , \\ t &= 1,2,\ldots,T \; . \end{aligned} \tag{38}$$

For the total volume of consumed products:

$$\sum_j \sum_p \sum_G \sum_d \sum_t X_{ijpGdt} = b_{jp} \quad , \qquad \begin{aligned} j &= 139 \; , \\ p &= 17 \; , \\ G &= 6 \; , \\ d &= 1,2,\ldots,m \; , \\ t &= 1,2,\ldots,T \; . \end{aligned} \tag{39}$$

For the balance between the produced and consumed products:

$$\sum_{i=139} a_i = \sum_{j=139} b_j \quad , \tag{40}$$

and nonnegative constraints for the variables are:

$$X_{ijpGdt} \geq 0 \quad . \tag{41}$$

As a result of global optimization at stage 3, there is feedback to the technological development alternatives d at stage 2, and the procedure has to be repeated. An example of the input data required to satisfy the objective function is presented in Appendix F.

The distribution of commodities among different modes of transport is presented in Appendix G and the main planning indicators resulting from solution of the model are shown in Appendix H. The algorithm for the distribution of commodities among the various modes of transport is described in Appendix I.

As can be seen from the objective function, the model was solved as a linear model. However, the nonlinear dependencies of transportation costs on traffic volume were considered when each mode of transport was preliminarily optimized. For a given interval of the traffic volume $Q(t) + \Delta Q(t)$, the costs related to each mode of transport were calculated according to the share that is dependent on the growth in traffic volume and the share that is conditionally constant. For this purpose, the national shares for rail and motor transport were used: 52 and 48 per cent and 87 and 13 per cent, respectively.

Figure 5 shows the dependence of rail transport costs on the freight traffic volume and on the average train capacity.

$$C = -0.5257 - 0.0632\, P_l + 0.0022\, Q_m + 0.0015\, Q_n \quad , \qquad (42)$$

where

P_l are tons per km;
Q_m is the freight traffic volume in tons; and
Q_n is the average train capacity.

IMPROVEMENTS IN MODELING THE TRANSPORTATION SUBSYSTEM

Further work on modeling the regional transportation subsystem has to be directed towards:

-- investigating the extent to which optimization of the regional transportation subsystem depends on intra-regional and national factors;
-- investigating the possibility of constructing a model with nonlinear dependence of transportation costs on the traffic volume;
-- developing in more detail the sequence for optimizing different loads and total load volume;
-- developing more effective procedures for solving the model using constraints on capacity, capital investment, labor, and their independencies; and
-- performing sensitivity analyses on the results of the other regional subsystem models that affect the optimal development of the transportation subsystem.

$$C = -0.5257 - 0.0632\,P_1 + 0.0022\,Q_m + 0.0015\,Q_n$$

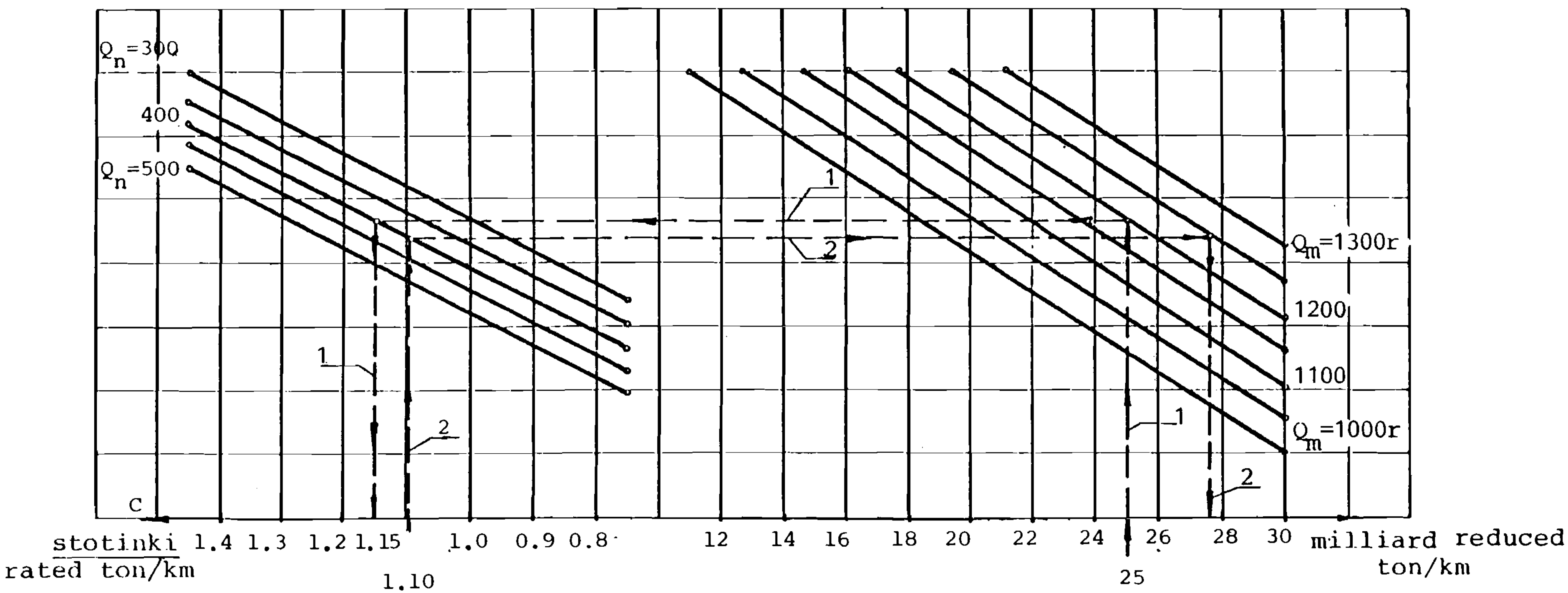

Figure 5. The dependence of rail transport costs on the freight traffic volume and on the average train capacity.

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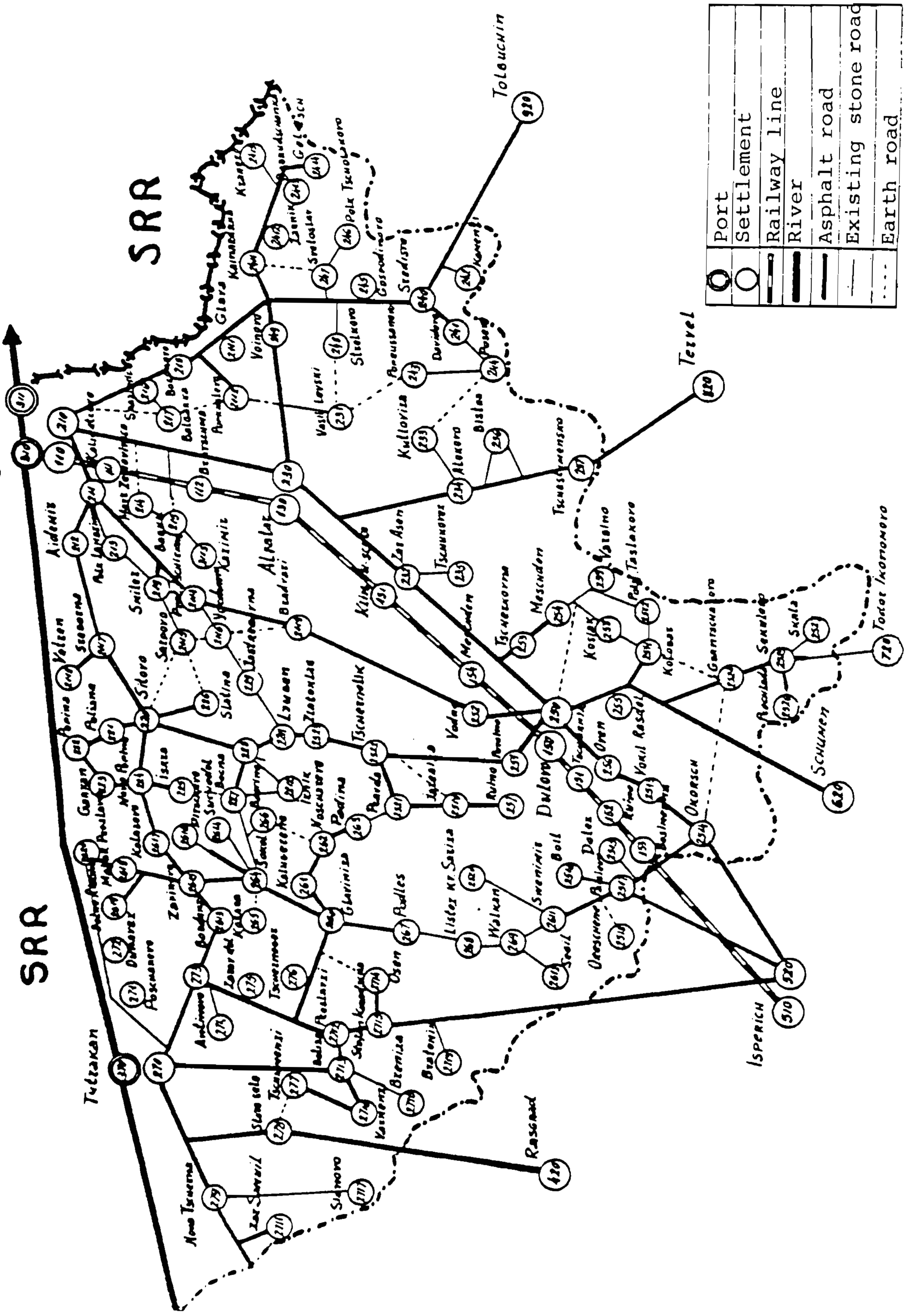
APPENDIX A: THE PROPOSED TRANSPORTATION NETWORK FOR THE SILISTRA REGION



APPENDIX B: SCHEMES OF TECHNOLOGICAL DEVELOPMENT TASKS FOR DIFFERENT MODES OF TRANSPORT

Rail Transport

$$D_1 = d_{10} + d_{11} + d_{12} + d_{13} + d_{14} + d_{15} + d_{16} + d_{17} \quad ,$$

where

d_{10} is the initial state of single railtrack;
d_{11} is the construction of additional intersections;
d_{12} is the reconstruction of stations;
d_{13} is the extension of the tracks within the stations;
d_{14} is the replacement of diesel by electric trains;
d_{15} is the construction of double sections;
d_{16} is the automation of the sections; and
d_{17} is the construction of double railtrack.

Motor Transport

$$D_2 = d_{20} + d_{21} + d_{22} + d_{23} + d_{24} + d_{25} + d_{26} \quad ,$$

where

d_{20} is the initial motor transport capacity;
d_{21} is the use of trailers;
d_{22} is the supply of new trucks with greater load capacity;
d_{23} is the supply of new specialized trucks;
d_{24} is the supply of new machinery for loading and unloading;
d_{25} are new technologies used; and
d_{26} is the construction of new repair facilities.

River Transport

$$D_3 = d_{30} + d_{31} + d_{32} + d_{33} + d_{34} + d_{35} ,$$

where

d_{30} is the initial state of the port;
d_{31} is the supply of additional port machinery;
d_{32} are new technologies used at the port;
d_{33} is the extension or reconstruction of existing ports;
d_{34} is the construction of new quays; and
d_{35} is the extension and reconstruction of storage facilities.

Motor Transport--Roads

$$D_4 = d_{40} + d_{41} + d_{42} + d_{43} + d_{44} ,$$

where

d_{40} is the initial state of the roads in the region;
d_{41} is new road surface;
d_{42} is the reconstruction of existing roads;
d_{43} is the construction of a new section of road; and
d_{44} is the construction of a main road.

Air Transport

$$D_5 = d_{50} + d_{51} + d_{52} + d_{53} \quad ,$$

where

d_{50} is the initial state of air transport;
d_{51} is the reconstruction of airports;
d_{52} is the use of new types of aircraft; and
d_{53} is the construction of a new airport.

Pipelines

$$D_6 = d_{60} + d_{61} + d_{62} + d_{63} \quad ,$$

where

d_{60} is the initial state of the regional pipeline system;

d_{61} is the construction of additional pumping stations;
d_{62} is the construction of additional double pipelines; and
d_{63} is the construction of new pipelines.

Canal Systems

$$D_7 = d_{70} + d_{71} \quad ,$$

where

d_{70} is the initial state of the canal system; and
d_{71} is the construction of a new canal system.

APPENDIX C: INDICATORS RELATED TO OPTIMAL DEVELOPMENT OF A SECTION OF THE RAILWAY TRANSPORTATION NETWORK (LOAD NUMBER 17)

Total Tons/km by Parts of Section	
Nonstandard rail transport	0
Rail transport	3,092,297,838
Motor transport	491,249,673

Tons Transported Bimodally	
From nonstandard to normal rail	0
From normal to nonstandard rail	0
From nonstandard rail to motor transp.	0
From motor transp. to nonstandard rail	0
From rail to motor transport	179,365
From motor transport to rail	255,839
Total tons bimodally transported	435,204

Type of Station	Tons Loaded	Tons Unloaded
Nonstandard rail	0	0
Rail	10,333,064	10,409,538
Motor transport	5,974,422	5,897,948
River transport	0	0

APPENDIX C (continued)

Cost of Transport Operations	Leva
Cost of transport by rail	29,487,377
Cost of transport by road	12,948,873
Cost of bimodal transport	558,135
Cost of initial transport by rail	12,629,013
Cost of final transport by rail	10,842,297
Cost of initial transport by road	3,305,792
Cost of final transport by road	2,448,804
Annual costs per ton/km by rail	1.72
Annual costs per ton/km by road	3.86
Annual costs per ton by rail	502.77
Annual costs per ton by road	308.47
Annual costs per ton bimodally transported	128.25

Distance	km
Average distance of goods transported by rail	292.03
Average distance of goods transported by road	79.83

APPENDIX D: INDICATORS FOR OPTIMAL DEVELOPMENT OF MOTOR TRANSPORT WHEN THE LOAD IS ROCK (USE OF TRUCK CAPACITY--50%)

INDICATORS	TYPE OF TRUCKS			
	ZIL-555	DRAZ-256	MAZ-503	BELAX
	4.5 tons	12.0 tons	7.5 tons	28.0 tons
Volume of loads per day	3922.000	3922.000	3922.000	3922.000
Type of load	1.000	1.000	1.000	1.000
Distance transported	2.000	2.000	2.000	2.000
Coefficient of road	2.000	2.000	2.000	2.000
Duration of working day	16.000	16.000	16.000	16.000
Time taken for loading	0.036	0.046	0.066	0.086
Time taken for unloading	0.034	0.034	0.034	0.034
Maximum speed	16.000	16.000	16.000	16.000
Time taken per journey	0.320	0.330	0.350	0.370
Journeys per day	50.000	48.500	45.700	43.200
Trucks needed	17.400	6.700	11.400	3.200
Revenue per journey	1.800	4.800	3.000	11.200
Expenditure per journey	1.330	2.660	1.810	6.360
Expenditure per km:	2.331	0.666	0.452	1.590
fuel	0.062	0.059	0.044	0.196
oil materials	0.008	0.010	0.007	0.032
tires	0.018	0.079	0.040	0.202
wages	0.129	0.217	0.165	0.299
depreciation	0.036	0.124	0.079	0.385
maintenance and repairs	0.025	0.034	0.028	0.140
average Costs	0.054	0.144	0.090	0.336
Expenditure per 100 levs revenue	73.640	55.480	60.320	56.800
Revenue per ton/km	0.200	0.200	0.200	0.200
Costs per ton /km	0.147	0.111	0.121	0.114
Revenue per ton	0.400	0.400	0.400	0.400
Costs per ton	0.295	0.222	0.241	0.227
Normative speed	15.940	10.910	13.930	10.910
Normative time for loading and unloading	0.217	0.317	0.283	0.467
Normative hours per journey	0.468	0.683	0.571	0.833
Normative hours per day	23.382	33.131	26.081	36.036

APPENDIX E: INDICATORS FOR OPTIMAL DEVELOPMENT OF COMMODITIES AMONG DIFFERENT MODES OF TRANSPORT WHEN THE TYPE OF LOAD IS SAND (USE OF TRUCK CAPACITY--50%)

INDICATOPS	TYPE OF TRUCKS			
	ZIL-555	DRAZ-256	MAZ-503	BELAX
	4.5 tons	12.0 tons	7.5 tons	28.0 tons
Volume of loads per day	59.000	59.000	59.000	59.000
Type of load	1.000	1.000	1.000	1.000
Distance transported	8.000	8.000	8.000	8.000
Coefficient of road	1.600	1.600	1.600	1.600
Duration of working day	16.000	16.000	16.000	16.000
Time taken for loading	0.036	0.046	0.066	0.086
Time taken for unloading	0.034	0.034	0.034	0.034
Maximum speed	16.000	16.000	16.000	16.000
Time taken per journey	1.070	1.080	1.100	1.120
Journeys per day	15.000	14.800	14.500	14.300
Trucks needed	0.900	0.300	0.500	0.100
Revenue per journey	5.670	15.120	9.450	35.280
Expenditure per journey	4.319	8.890	5.880	22.080
Expenditure per km:	0.269	0.556	0.368	1.380
fuel	0.057	0.055	0.040	0.183
oil	0.008	0.010	0.007	0.037
tires	0.018	0.079	0.040	0.202
wages	0.084	0.141	0.104	0.173
depreciation	0.036	0.124	0.079	0.385
maintenance and repairs	0.025	0.034	0.028	0.140
average costs	0.043	0.113	0.071	0.265
Expenditure per 100 leva revenue	76.030	58.820	62.260	62.600
Revenue per ton/km	0.158	0.158	0.158	0.158
Costs per ton/km	0.120	0.093	0.098	0.099
Revenue per ton	1.260	1.260	1.260	1.260
Costs per ton	0.958	0.741	0.785	0.789
Normative speed	15.940	10.910	13.930	10.910
Normative time loading/ unloading	0.217	0.317	0.283	0.467
Normative hours per journey	1.221	1.783	1.432	1.933
Normative hours per day	18.252	26.420	20.830	27.619

APPENDIX F: INPUT DATA FOR DISTRIBUTION OF COMMODITIES AMONG DIFFERENT MODES OF TRANSPORT

	MOTOR TRANSPORT													
No sections	GAZ-53A		ZIL-130		IFA-W50		MAZ-500		Skoda-M'		Skoda MTTN		Volvo-89	
	1 KAT	2 KAT	1 KAT	2 KAT	1 KAT	2 KAT	1 KAT	2 KAT	1 KAT	2 KAT	1 KAT	2 KAT	1 KAT	2 KAT
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
10	1.41	1.51	1.23	1.29	1.20	1.25	1.19	1.24	1.20	1.25	1.12	1.17	0.00	0.00
20	1.91	2.10	1.54	1.66	1.48	1.58	1.44	1.54	1.45	1.54	1.34	1.43	0.00	0.00
30	2.41	2.69	1.86	2.03	1.75	1.90	1.70	1.84	1.70	1.84	1.56	1.69	0.00	0.00
40	2.92	3.29	2.17	2.40	2.03	2.23	1.95	2.14	1.94	2.14	1.78	1.95	0.00	0.00
50	3.42	3.88	2.48	2.77	2.31	2.56	2.20	2.45	2.19	2.43	1.99	2.21	0.00	0.00
60	3.92	4.47	2.80	3.14	2.58	2.88	2.46	2.75	2.44	2.73	2.21	2.47	0.00	0.00
70	4.42	5.07	3.11	3.51	2.86	3.21	2.71	3.05	2.68	3.02	2.43	2.73	0.00	0.00
80	4.92	5.66	3.42	3.88	3.13	3.53	2.96	3.35	2.93	3.32	2.65	2.99	0.00	0.00
90	5.42	6.26	3.74	4.25	3.41	3.86	3.22	3.65	3.18	3.61	2.86	3.25	0.00	0.00
100	5.92	6.85	4.05	4.62	3.69	4.18	3.47	3.96	3.43	3.91	3.08	3.51	0.00	0.00
110	6.42	7.44	4.36	4.99	3.96	4.51	3.72	4.26	3.67	4.20	3.30	3.77	0.00	0.00
120	6.92	8.04	4.68	5.36	4.24	4.84	3.98	4.56	3.92	4.50	3.52	4.03	0.00	0.00
130	7.42	8.63	4.99	5.73	4.51	5.16	4.23	4.86	4.17	4.80	3.73	4.29	0.00	0.00
140	7.92	9.22	5.30	6.10	4.79	5.49	4.49	5.17	4.42	5.09	3.95	4.55	0.00	0.00
150	8.43	9.82	5.62	6.47	5.06	5.81	4.74	5.47	4.66	5.39	4.17	4.81	0.00	0.00
160	8.93	10.41	5.93	6.84	5.34	6.14	4.99	5.77	4.91	5.68	4.39	5.07	0.00	0.00
170	9.43	11.01	6.25	7.21	5.62	6.46	5.25	6.07	5.16	5.98	4.60	5.33	0.00	0.00
180	9.93	11.60	6.56	7.58	5.89	6.79	5.50	6.37	5.40	6.27	4.82	5.59	0.00	0.00
190	10.43	12.19	6.87	7.95	6.17	7.11	5.75	6.68	5.65	6.57	5.04	5.85	0.00	0.00
200	10.93	12.79	7.19	8.33	6.44	7.44	6.01	6.98	5.90	6.87	5.26	6.11	0.00	0.00
210	11.43	13.38	7.50	8.70	6.72	7.77	6.26	7.28	6.15	7.16	5.47	6.37	0.00	0.00
220	11.93	13.97	7.81	9.07	7.00	8.09	6.51	7.58	6.39	7.46	5.69	6.63	0.00	0.00
230	12.43	14.57	8.13	9.44	7.27	8.42	6.77	7.89	6.64	7.75	5.91	6.89	0.00	0.00
240	12.93	15.16	8.44	9.81	7.55	8.74	7.02	8.19	6.89	8.05	6.13	7.15	0.00	0.00
250	13.43	15.76	8.75	10.18	7.82	9.07	7.27	8.49	7.14	8.34	6.34	7.41	0.00	0.00
260	13.93	16.35	9.07	10.55	8.10	9.39	7.53	8.79	7.38	8.64	6.56	7.67	0.00	0.00
270	14.44	16.94	9.38	10.92	8.38	9.72	7.78	9.09	7.63	8.93	6.78	7.93	0.00	0.00
280	14.94	17.54	9.69	11.29	8.65	10.05	8.04	9.40	7.88	9.23	7.00	8.19	0.00	0.00
290	15.44	18.13	10.01	11.66	8.93	10.37	8.29	9.70	8.12	9.53	7.21	8.45	0.00	0.00
300	15.94	18.72	10.32	12.03	9.20	10.70	8.54	10.00	8.37	9.82	7.43	8.71	0.00	0.00

					RAILWAY TRANSPORT					
No sections	Skoda-MTS		KRAZ-256		Direct Scheme		Indirect scheme 1		Indirect scheme 2	
	1 KAT	2 KAT	1 KAT	2 KAT	Electr. train	Diesel train	Electr. train	Diesel train	Electr. train	Diesel train
1	16	17	18	19	20	21	22	23	24	25
10	0.00	0.00	0.00	0.00	1.388	1.397	2.452	2.461	3.516	3.525
20	0.00	0.00	0.00	0.00	1.436	1.454	2.500	2.518	3.564	3.582
30	0.00	0.00	0.00	0.00	1.484	1.511	2.548	2.575	3.612	3.639
40	0.00	0.00	0.00	0.00	1.532	1.568	2.596	2.632	3.660	3.696
50	0.00	0.00	0.00	0.00	1.580	1.625	2.644	2.689	3.708	3.753
60	0.00	0.00	0.00	0.00	1.627	1.682	2.691	2.746	3.755	3.810
70	0.00	0.00	0.00	0.00	1.675	1.739	2.739	2.803	3.803	3.867
80	0.00	0.00	0.00	0.00	1.804	1.876	2.868	2.940	3.932	4.004
90	0.00	0.00	0.00	0.00	1.862	1.944	2.926	3.008	3.990	4.072
100	0.00	0.00	0.00	0.00	1.920	2.011	2.984	3.075	4.048	4.139
110	0.00	0.00	0.00	0.00	1.978	2.078	3.042	3.142	4.106	4.206
120	0.00	0.00	0.00	0.00	2.036	2.145	3.100	3.209	4.164	4.273
130	0.00	0.00	0.00	0.00	2.094	2.212	3.158	3.276	4.222	4.340
140	0.00	0.00	0.00	0.00	2.152	2.279	3.216	3.343	4.280	4.407
150	0.00	0.00	0.00	0.00	2.210	2.346	3.274	3.410	4.338	4.474
160	0.00	0.00	0.00	0.00	2.268	2.413	3.332	3.477	4.396	4.541
170	0.00	0.00	0.00	0.00	2.326	2.480	3.390	3.544	4.454	4.608
180	0.00	0.00	0.00	0.00	2.384	2.547	3.448	3.611	4.512	4.675
190	0.00	0.00	0.00	0.00	2.442	2.614	3.506	3.678	4.570	4.742
200	0.00	0.00	0.00	0.00	2.500	2.681	3.564	3.745	4.628	4.809
210	0.00	0.00	0.00	0.00	2.558	2.748	3.622	3.812	4.686	4.876
220	0.00	0.00	0.00	0.00	2.616	2.815	3.680	3.879	4.744	4.943
230	0.00	0.00	0.00	0.00	2.674	2.883	3.738	3.947	4.802	5.011
240	0.00	0.00	0.00	0.00	2.732	2.950	3.796	4.014	4.860	5.078
250	0.00	0.00	0.00	0.00	2.790	3.017	3.854	4.081	4.918	5.145
260	0.00	0.00	0.00	0.00	2.848	3.084	3.912	4.148	4.976	5.212
270	0.00	0.00	0.00	0.00	2.906	3.151	3.970	4.215	5.034	5.279
280	0.00	0.00	0.00	0.00	2.964	3.218	4.028	4.282	5.092	5.346
290	0.00	0.00	0.00	0.00	3.022	3.285	4.086	4.349	5.150	5.413
300	0.00	0.00	0.00	0.00	3.080	3.352	4.144	4.416	5.208	5.480

APPENDIX G: DISTRIBUTION OF THE TOTAL TRAFFIC VOLUME AMONG DIFFERENT MODES OF TRANSPORT IN THE SILISTRA REGION

Section numbers		Section	Traffic volume in tons		Expenditures in 1,000 leva		Tasks for transportation network development		
From	To		1978	1985	1978	1985	Task Indicators	Year	Capital Investments
510	110	Isperih-r*-Silistra-r	72	82	1296	1460			
510	310	Isperih-r-Silistra-c*	30	91	518	1538			
510	150	Isperih-r-Dulovo-r	40	56	864	997			
150	510	Dulovo-r-Isperih-r	21	26	378	463			
510	130	Isperih-r-Alphatar-r	55	60	990	1068			
130	510	Alphatar-r-Isperih-r	20	22	360	392			
150	110	Dulovo-r-Silistra-r	39	44	702	783	D13-110	1982	35500
150	310	Dulovo-r-Silistra-p*	28	38	483	642	D13-150	1982	29000
310	150	Silistra-c-Dulovo-r	19	21	328	355	D14-310-150	1983	52500
110	150	Silistra-r-Dulovo-r	17	20	306	356			
110	510	Silistra-r-Isperih-r	38	95	684	1691	D14-110-510	1983	49800
310	510	Silistra-c-Isperih-r	33	37	569	625			
130	110	Alphatar-r-Silistra-r	22	25	396	445	D14-130-110	1983	48500
130	310	Alphatar-r-Silistra-c	49	57	845	963			
110	130	Silistra-r-Alphatar-r	41	49	738	872	D15-110-510	1985	115800
310	130	Silistra-c-Alphatar-r	19	27	328	456			
520	210	Isperih-m*-Silistra-m	175	160	3238	2896			
520	310	Isperih-m-Silistra-c	145	250	2538	4263	D31-310	1981	152000
210	520	Silistra-m-Isperih-m	121	110	2239	1991			
310	520	Silistra-c-Isperih-m	185	300	3238	5115			
250	210	Dulovo-m-Silistra-m	156	140	2886	2534			
250	310	Dulovo-m-Silistra-p	165	170	2888	2899			
210	250	Silistra-m-Dulovo-m	138	125	2553	2263			
310	250	Silistra-c-Dulovo-m	172	185	3010	3154			
230	210	Alphatar-m-Silistra-m	75	60	1388	1086			
230	310	Alphatar-m-Silistra-c	245	250	4288	4263			
210	230	Silistra-m-Alphatar-m	158	140	2923	2534			
310	230	Silistra-c-Alphatar-m	162	170	2835	2899			
310	370	Silistra-c-Tutrakan-p	542	550	8943	8800	D31-370	1982	85500
370	310	Tutrakan-c-Silistra-p	921	1200	15197	19200	D34-310	1985	185600
520	270	Isperih-m-Tutrakan-m	285	210	5273	3801	D41-520-270	1981	88800
520	370	Isperih-m-Tutrakan-c	95	200	1663	3410	D33-370	1984	112000
270	520	Tutrakan-m-Isperih-m	92	70	1702	1267	D22-520	1982	112000
370	520	Tutrakan-c-Isperih-m	228	240	3990	4092			
250	220	Dulovo-m-Sitovo-m	125	90	2313	1629	D41-250-220	1982	34500
220	250	Sitovo-m-Dulovo-m	235	220	4348	3982	D22-250	1982	36800
250	230	Dulovo-m-Alphatar-m	188	175	3478	3168	D22-230	1983	41500
230	250	Alphatar-m-Dulovo-m	155	135	2868	2444	D42-230-250	1983	72800
230	240	Alphatar-m-Sredishte-m	122	100	2257	1810			
240	230	Sredishte-m-Alphatar-m	225	210	4163	3801	D42-230-240	1983	68100
230	232	Alphatar-m-Zar Assen-m	75	60	1388	1086	D42-230-232	1984	72500
232	230	Zar Assen-m-Alphatar-m	265	250	4903	4525			
210	220	Silistra-r-Sitovo-m	325	310	5931	5565	D22-220	1983	72500
220	210	Sitovo-m-Silistra-r	115	100	2099	1795			
270	2615	Tutrakan-m-Zaphirovo-m	118	100	2183	1810			
2615	270	Zaphirovo-m-Tutrakan-m	125	110	2313	1991	D41-2615-270	1983	42800
210	2615	Silistra-m-Zaphirovo-m	225	205	4163	3711			
2615	210	Zaphirovo-m-Silistra-m	118	105	2183	1901	D26-210	1984	118000
270	260	Tutrakan-m-Glaviniza-m	275	255	5088	4616			
260	270	Glaviniza-m-Tutrakan-m	68	55	1258	996			
520	260	Isperih-m-Glaviniza-m	235	210	4348	3801			
260	520	Glaviniza-m-Isperih-m	128	100	2368	1810	D24-520	1984	38400
250	260	Dulovo-r-Glaviniza-m	256	240	4672	4308			
260	250	Glaviniza-m-Dulovo-m	85	70	1551	1257	D43-260-250	1983	82500
230	260	Alphatar-m-Glaviniza-m	225	205	4163	3711	D43-230-260	1984	91800
260	230	Glaviniza-m-Alphatar-m	125	115	2313	2082	D24-230	1984	48600
			8424	8700	150928	151372			

	1978	1985
Total traffic volume (tons), including different transp. schemes	8424	8700
rail transport	373	529
river transport	1463	1750
rail - river transport	178	221
motor - river transport	1397	1765
motor - rail transport	781	720
motor transport	4232	3715
Total expenditures - leva, including different transp. schemes	150928	151372
rail transport	6714	8527
river transport	24140	28000
rail - river transport	3071	4579
motor - river transport	24450	30095
motor - rail transport	14253	12925
motor transport	78300	67246

*r - rail transport
*c - canal (river) transport
*p - pipeline
*m - motor transport

APPENDIX H: MAIN INDICATORS FROM THE SOLUTION OF THE TRANSPORTATION MODEL

Type of traffic and mode of transport	Total traffic volume		Transportation costs 1,000 leva		Transportation costs per ton	
	1978	1985	1978	1985	1978	1985
FREIGHT TRAFFIC--Thousand tons						
Total rail transport:	12245	14850	9509	11783	1.27	1.27
Intraregional traffic	1180	2620	982	1863	1.20	1.14
Interregional traffic	8420	9460	7215	8400	1.17	1.15
Traffic in transit	2645	2570	1312	1477	2.02	1.74
Total motor transport:	36550	34750	140620	130350	0.26	0.27
Intraregional traffic	30150	28450	124650	111380	0.24	0.26
Interregional traffic	4520	5140	12350	13526	0.37	0.38
Traffic in transit	1880	1160	4821	2829	0.39	0.41
Total river transport:	6150	8490	50935	74100	0.12	0.11
Intraregional traffic	190	255	1240	2120	0.15	0.12
Interregional traffic	2850	3640	21650	32140	0.13	0.11
Traffic in transit	3110	4595	28045	39840	0.11	0.12
PASSENGER TRAFFIC--Million passengers						
Total rail transport:	19255	23780	17184	20205	1.12	1.18
Intraregional traffic	1185	3650	984	2829	1.20	1.29
Interregional traffic	14250	16180	13150	15558	1.08	1.04
Traffic in transit	3820	3950	3050	3211	1.25	1.23
Total motor transport:	98810	106024	263565	300050	0.37	0.35
Intraregional traffic	58645	62184	154235	159446	0.38	0.39
Interregional traffic	22115	24220	64120	68750	0.34	0.35
Traffic in transit	18050	19620	45210	46180	0.40	0.42
Total river transport:	1287	1465	31140	35130	0.04	0.04
Intraregional traffic	24	25	120	130	0.20	0.19
Interregional traffic	855	968	17640	18150	0.05	0.05
Traffic in transit	408	472	13380	16850	0.03	0.03
Total air transport:	124	136	13850	14240	0.01	0.01
Interregional traffic.	124	136	13850	14240	0.01	0.01

APPENDIX I: ALGORITHM FOR THE DISTRIBUTION OF COMMODITIES AMONG DIFFERENT MODES OF TRANSPORT

This algorithm is a modified Ford-Fulkerson method for determining the shortest distance in a set, in which data on annual transportation costs, rather than on transportation distance, are used. For this purpose a set of commodity destinations are used to indicate the sections and intersections of the different modes of transport. It is assumed that no constraints on capacity exist and the initial, final, bimodal, and transit expenditures are predetermined.

For this task

-- the point of origin and the destination of the commodities transported are known;

-- different modes of transport may be used;

-- commodities may be transported by more than one mode of transport; and

-- to obtain minimum annual costs, the distribution of commodities among the different modes of transport must be defined.

The following method for sketching the transportation set is therefore chosen.

-- Each subregion consists of a given number of intersections for different mode of transport.

-- The sections between the subregional points of intersection have a value equal to the transportation costs for 1 ton of goods transported across a given section by a given mode of transport.

-- The sections within the subregion (i.e., for each mode of transport) have a value equal to the loading and unloading costs of bimodal transport.

The total volume of commodities transported is derived by aggregating each commodity load for each mode of transport. The following procedures are used.

1. The distance between the intersections is D_i (i is the number of intersections; the initial intersection is $D_{iO} = 0$).
2. The following inequality has to be satisfied:

$$D_i + p_{ij} < D_j \quad , \tag{1}$$

where

p_{ij} is the distance between the sections i and j.

If it is satisfied,

$$D_j = D_i + p_{ij} \quad . \tag{2}$$

3. Procedure 2 must be repeated until the inequality is satisfied for all intersections. For this commodities distribution algorithm, the following inequality was chosen after each iteration:

$$A_j^{(m)} > A_i^{(m)} + P_{ij} \quad , \tag{3}$$

where

$A_j^{(m)}$ is the potential of apex j at iteration t; and

P_{ij} is the reduced (i.e., annual) transport cost for sections i to j.

The initial iteration for all apexes (with the exception of i0 has a given potential $A_j = \infty$. If the above inequality is satisfied at the following iteration for apex j, this apex is given a potential of

$$A_j = A_i + P_{ij} \quad . \tag{4}$$

After all sections are eliminated from apex i, the symbols can be taken out. This procedure continues until all the symbols are eliminated. Thus, the lowest expenditures for the transported commodities occur between sections i and j included in equation (4).

If the next apex on the line is i, it can be derived from the previously denoted apexes having the potential $A_j = \infty$ and which are changed with (m_i) or apex m with i(m). Therefore, i(1) is always equal to i0. These apex symbols retain their place until the end of the procedure.

The sequence of the apex analysis is the following: if apex i with m_i symbols is treated and some of the apex potentials jl are changed, the following apex to be treated is not i+1 but

$$\beta = \min_{(jl)} [m(jl), m(i)+1] \quad . \tag{5}$$

This procedure can be illustrated in the following way. If in the apex line under analysis some of the apex potentials denoted with (^) are changed:

$$i0, \ldots, \hat{in}, \quad in+1, \ldots, \hat{ik}, \quad \hat{il}, \; il+1, \ldots, \hat{ip} \quad , \tag{6}$$

the solution is reached when $\beta = M + 1$.

The formal description of this algorithm is given below. The following notation is used.

l is the number of the last apex in the line;

p(i) is the number of the apex in the line, following apex i;

q(j) is the number of the intersection, preceding apex j in the shortest way;

δ(j) symbolizes the apex; and

i is the number of the apex being treated.

1. All apexes receive potentials:
   $A_i = \infty$ and $\delta(i): = p(i): = 0$.
2. $A_{i0}: = 0$, i: = 1: = il .
3. For the next section (itj), equation (4) has to be satisfied. If it is breached, one can go on to point 8.
4. If $A_j = \infty$, then p(l): = j and l: = j, move to point 6. If $A_j \neq \infty$, move to point 5.
5. If δ(j) = 0, then p(j): = p(i), p(i): = j, move to point 6. If δ(j) ≠ 0, move to point 7.
6. δ(j): = 1.
7. $A_j: = A_i + P_{ij}$; q(j): = i .
8. If the section i,j is the last section, move to point 9, otherwise, to point 3.
9. If p(i) ≠ 0, then T: = i, i: = p(i), p(s): = δ(s): = 0, move to point 3. If p(i) = 0, the procedure is completed.

In this algorithm, annual transportation costs are used as an indicator of the transported commodities; in this way different modes of transports can be considered as comparable. Annual transportation costs are calculated as follows.

-- The transportation costs are divided according to the main elements of the transportation process, referring to 1 ton for loading, unloading, transfer, and transit operations and referring to 1 ton per km for transport operations.
-- Current transportation costs and capital investment are included in annual costs.
-- The costs are calculated for different commodities, taking into account factors such as the vehicle used and its load capacity.

THE POTENTIAL OF LOCAL RENEWABLE ENERGY SOURCES IN THE SILISTRA REGION

M. Albegov
T. Balabanov

INTRODUCTION

In recent years there has been increasing interest in the use of renewable energy sources, such as methane generated from the decomposition of organic matter or solar heating and drying in agricultural areas. This interest has been sparked off by a number of developments:

-- the rapid rise in fossil fuel prices;

-- forecasts of future scarcities of fossil fuels;

-- changes in agricultural production technology have resulted in a greater concentration of stock-breeding activities, thus increasing the volume of manure to be disposed of; and

-- environmental protection regulations, which force livestock producers to bear manure processing costs.

Rural areas usually have large supplies of crop and animal wastes, which in theory should be suitable for conversion into a usable source of energy. At present, the conversion process that seems to hold the greatest potential is anaerobic digestion. This process, which transforms complex organic matter to methane and other gases, has several advantages.

-- It is the simplest and most practical method for producing energy from agricultural wastes in a form suitable for efficient use.

-- It creates a stabilized residue (sludge) that retains the fertilizer value of the original material.

-- It saves the amount of energy required to produce an equivalent amount of nitrogen-containing fertilizer by synthetic processes.

Indirect benefits of methane generation include

-- the potential for partial sterilization of waste during fermentation, with a consequent reduction in the hazard to public health from fecal pathogens; and

-- a reduction, due to the fermentation process, in the transfer of fungal and other plant pathogens from the year's crop residue to the following year's crop.

One of the features of anaerobic digestion is that it can easily be combined with the use of solar energy collectors, which could be used to provide heat for the digester. In addition, solar heating and cooling can be combined with biogas produced, in order to satisfy local heating and cooling demands (Figure 1).

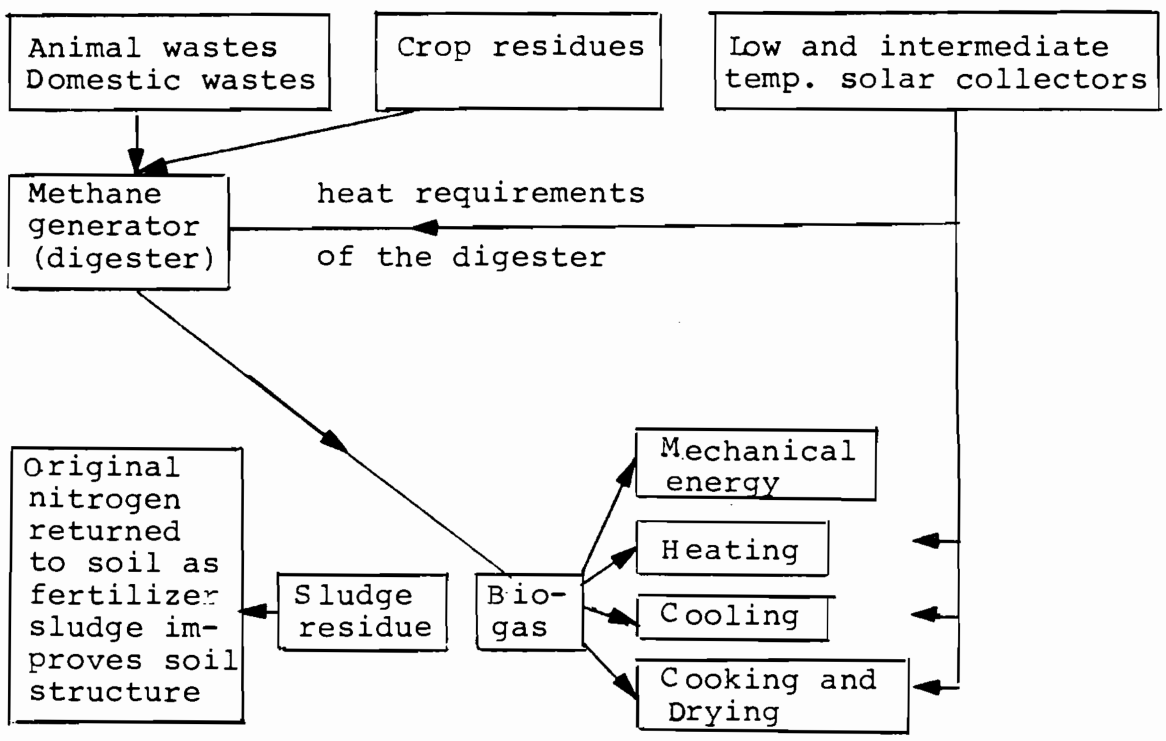


Figure 1. Linkage of biogas production and solar energy utilization.

This report is devoted to the analysis of problems of utilizing alternative energy sources for replacing conventional fuels on a regional scale.

ANAEROBIC FERMENTATION

Anaerobic fermentation takes place in water containing no dissolved oxygen. After a stream has been completely denuded of its dissolved oxygen by decaying vegetation, methane is produced. Anaerobic digestion involves a series of reactions brought about by a mixed culture of bacteria. Two stages of decomposition occur in the digestion of organic matter: liquefaction and gasification.

The bacteria are the most essential ingredients of the process. Liquefaction of organic matter occurs when enzymes from the bacteria catalyze the hydrolysis of complex carbohydrates to simple sugars and alcohols, of protein to peptides and amino acids, and of fats to fatty acids. This hydrolyzed organic matter is further broken down to organic acids by the anaerobic bacteria.

At the gasification stage, some of the simple organic compounds are converted to carbon dioxide and methane and also to an inert organic residue, known as digested sludge. Liquefaction and gasification occur simultaneously in a well-balanced system. If, however, methane-forming bacteria are absent, the digestion process may only succeed in liquefying the material and may render it more offensive than the original matter. Alternatively, if liquefaction occurs at a faster rate than gasification, the resulting accumulation of acids may inhibit the methane-forming bacteria from digesting the material, and the process will fail.

The optimal solid content of the dry inputs is 7-9 percent by weight. The ratio of carbon to nitrogen (C/N) in the inputs should be higher than 1:30. The C/N ratios for various inputs are:

cow manure	25.0
cow urine	0.8
grass clippings	12.0
cabbage waste	12.0
wheat straw	128.0

Poultry and pig manures have a lower C/N ratio than cow manure. Small quantities of vegetable matter added to animal sludge can, in certain circumstances, substantially benefit the process. The range of optimal pH is 6.0-8.0.

The temperature of the process directly affects the rate of microbial growth. As the temperature is decreased below the optimal range of 33-38°C, the net microbial growth rate decreases. The methane-forming bacteria are very sensitive to sudden temperature changes and, for optimum stability during the process, the temperature should be controlled.

The digester capacity can be determined by taking into account the time required to complete digestion, the temperature, and the volume of material. The average gas production is 0.37-0.5 m^3/kg of dry input. The time required for digestion is:

24°C	human sewage	20 days
	light vegetable waste	70 days
	cow dung	50 days
32°C	human sewage	11-15 days
	light vegetable waste	48 days
	cow dung	28 days

The gas composition is 60-70 percent methane, 30-40 percent CO_2, and 1-4 percent SO_2, with a calorific value of biogas of between 5,100 and 6,500 kcal/m^3.

POTENTIAL FOR BIOGAS PRODUCTION

Biogas could be produced from the following agricultural residues:

- -- animal wastes, including bedding, waste feed, poultry litter and manure;
- -- crop waste, sugar cane, thresh, crop stubble, straw, and spoiled fodder;
- -- slaughterhouse waste, fish waste, leather and wood wastes;
- -- by-products of agriculture-based industries, such as oil cakes, waste from fruit and vegetable processing, bagasse and press-mud from sugar factories, sawdust, tobacco waste and seeds, rice brau, and cotton dust from the textile industry;

-- forest waste;

-- waste from aquatic growth such as algae, seaweed, and water hyacinths; and

-- waste from the pulp and paper industry.

The estimated yield of residues from the major crops is given in Table 1 and the estimated quantities of manure and biogas produced from animal waste are given in Table 2.

Table 1. Residue coefficients of various crops[a]

Crop	Residue coefficient (kg dry residue/kg harvested crop)
Soybeans	0.55-2.60
Corn	0.55-1.20
Cotton	1.20-3.00
Wheat	0.47-1.75
Barley	0.82-1.50
Rice	0.38-1.25
Rye	1.20-1.95
Oats	0.95-1.75
Grain sorghum	0.50-0.85
Sugarbeet	0.07-0.20

[a]The residue coefficient is the ratio of the weight of dry residue matter to the recorded harvested weight at field moisture. For grains and straw, field moisture content is assumed to be 15 percent.

SOURCE: Based on data taken from U.S. Academy of Sciences (1977).

Table 2. Estimated manure and biogas production from animal waste.

Manure and biogas	Dairy cattle	Beef cattle	Swine	Poultry
Manure production (kg/day/1,000 kg live weight)	85.000	58.000	50.000	59.000
Volatile solids (kg dry solids/ day/1,000 kg)	8.200	5.900	5.900	12.800
Biogas production (m^3/kg for inputs)	0.300	0.415	0.450	0.530

SOURCE: Based on data taken from U.S. Academy of Sciences (1977).

The conversion efficiencies for various inputs are given in Table 3. The capacity utilization factor of the digester is 90 percent.

Table 3. Conversion efficiencies of various residues (as %).

Material	Without heating	20% CH_4 used for heating	37.5% CH_4 used for heating
CROP RESIDUES			
Dry leaf powder	40.0	32.0	25.0
Sugarcane thresh	69.0	55.0	43.0
Maize straw	77.0	62.0	48.0
Straw powder	88.0	71.0	55.0
Paper pulp	65.0	52.0	40.0
Grass clippings	71.0	57.0	44.0
MANURE			
Cattle, beef	56.0	45.0	35.0
Cattle, dairy	17.5	14.0	11.0
Swine	71.0	57.0	45.0
Sheep	17.5	14.0	11.0
Poultry	47.5	38.0	30.0

THE FEASIBILITY OF USING RENEWABLE ENERGY SOURCES ON A REGIONAL SCALE

The objective of the study is to assess the feasibility of converting existing residues into useful energy for either local or nonlocal consumption and of implementing solar heating and cooling. An evaluation will be carried out of

- -- residue characteristics, including the utility or cost of obtaining them, infrastructural requirements relating to collection and transportation, and energy content of residues as a substitute for feedstock (collection consists of the following operations: pickup, baling or compression, and loading);
- -- limitations on obtaining the residues, including location and seasonal nature of production;
- -- the type and capacities of equipment required to collect and transport residues;
- -- total production and consumption of energy in the area;

-- energy requirements for processing plants and livestock farms in the area; and
-- the feasibility of using the residues for the production of biogas and of solar heating and cooling.

Analytical Approach

The analysis will be undertaken in four stages.

Stage 1: Survey of the Region and Selection of a Site for the Digester.

Selection of the site should be based on the subdivision of the region as specified in the other models of the regional system, but also taking into account

-- logging sites and concentration of manure;
-- places of intensive agricultural activity and the total quantity of residue produced;
-- the location of sawmills, paper mills, and food processing plants;
-- the distribution of transport equipment; and
-- the possibility of using digested sludge residue.

Stage 2: Preliminary Evaluation of Residue Production in the Subregions.

Information is required on certain factors influencing the energy value of residues as a feedstock. This includes determining the cost of collecting the residues and transporting them to one or more processing plants within the region under analysis. Information is required on

-- types of residues generated;
-- quantity and cost of residue production;
-- seasonal availability of residues;
-- location of residues;
-- condition of residues;
-- usual means of residue disposal;
-- alternative uses of residues; and
-- possibility of using digested sludge residue as a substite for inorganic fertilizers.

Residue type refers to the origin of the residue. In certain cases (for example, sugar), there will be two types: mill residue or bagasse and field residue. Information of the type and quantity will be obtained from the agriculture and industry models of the regional system.

The residue condition refers mainly to its ash and water content at the time of collection.

Information on the usual means of disposal will include treatment costs (per ton of residue) for cases where, in addition to normal crop maintenance or field preparation, a further operation is necessary.

Additional information on collection, transportation, and processing should be obtained for each type of residue, so that the energy utility of each type as a feedstock can be assessed. The types of residues generated and their condition, their concentration per unit area (ton per ha), and their distance from the site of a potential conversion facility should also be determined.

Use of digested sludge residue can be defined in terms of its potential for replacing inorganic fertilizers. In Table 4 a set of conversion measures developed for sewage sludge residue in England is given:

Table 4. A set of conversion measures developed for sewage sludge residue.

Organic material (sewage sludge)	Equivalent measure of inorganic fertilizer
1 kg nitrogen	0.85 kg nitrogen
1 kg phosphorus	0.70 kg phosphorus
1 kg potassium	0.75 kg potassium

SOURCE: Poole (1977).

The cost of inorganic nitrogen fertilizer is dependent upon the cost of hydrogen, which at present is derived from natural gas. Hence, in evaluating the feasibility of biogas production, the costs should be compared with those of inorganic fertilizer production.

Stage 3: Evaluation of the Subregion.

The subregions should be classified according to their general characteristics and energy requirements:

-- the size and distribution of the population among rural and urban communities -- this information will be obtained from the population model;
-- existing and planned energy supply, distribution, and utilization facilities and grids;
-- the practicality of substituting conventional with unconventional (biogas and solar heating/cooling) fuels; and
-- siting of crop processing facilities, with quantities and types of crops processed and the quantity and type of energy to be used at each facility.

Energy types (fuels) and requirements for a subregion must be assessed in order to determine which of these needs might be satisfied by biogas or solar energy, and to what extent energy export to other areas might be necessary. Energy consumption figures have to be obtained for

-- processing facilities, such as papermills and food processing plants;
-- livestock farms (with their heating, cooling, and hot water demands; and
-- other community needs.

Information on energy requirements should be received from other models for each month of every year of the investigated period.

Stage 4: Evaluation of the Economic Feasibility of Unconventional Energy Sources.

To evaluate the economic feasibility of unconventional energy sources, a dynamic linear optimization model will be constructed for the subsystem shown in Figure 2. The period of analysis will be the same as for the other models of the regional system--up to 15-20 years for each subregion.

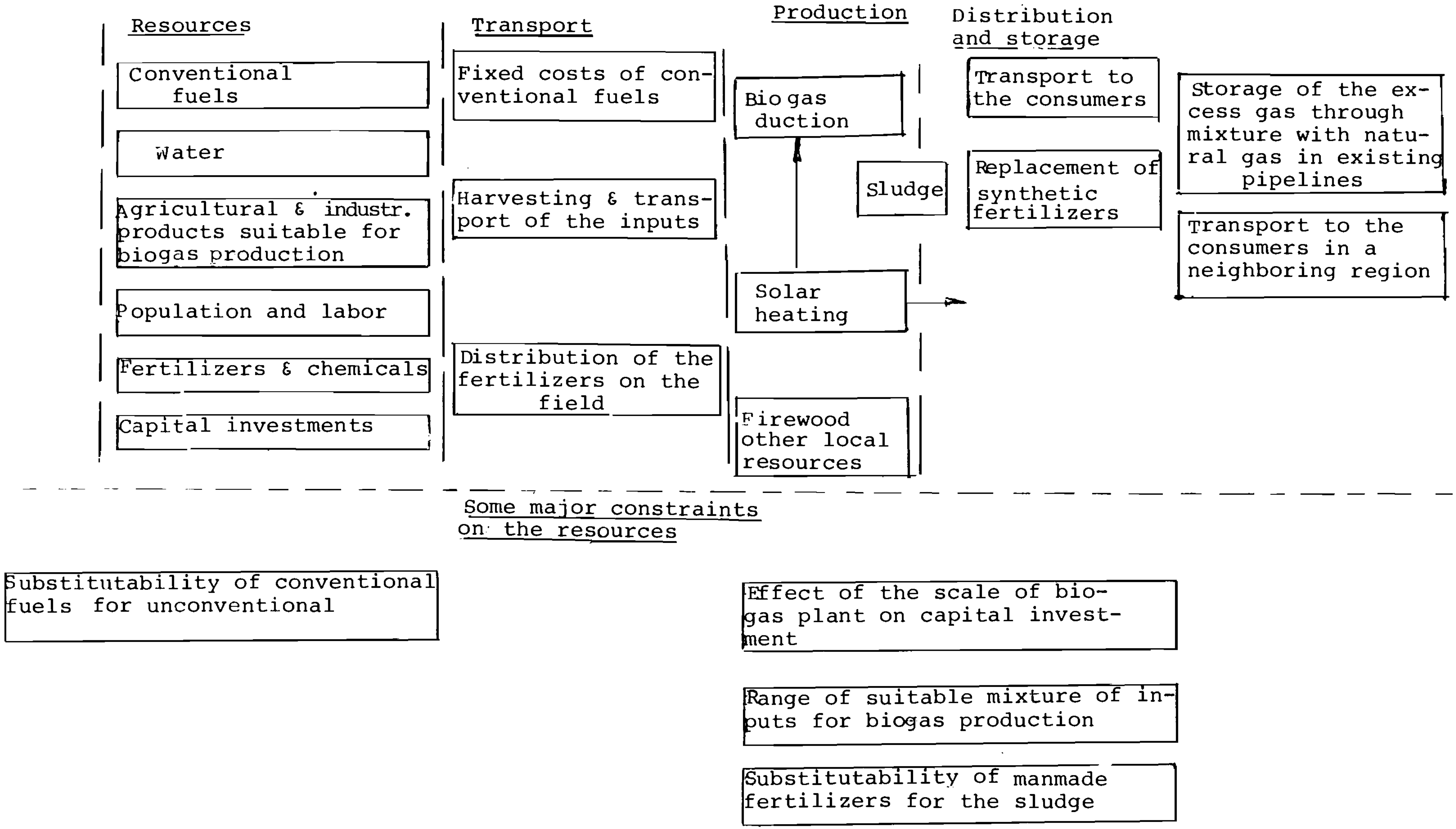


Figure 2. Content and constraints of the energy subsystem.

Assumptions

1. Several residues may be available at one point.
2. The efficiency or inefficiency of biogas for heating biomass can be resolved outside the model.
3. The biogas obtained from the digesters can be transported without compression up to 1 km and with compression up to 15-20 km.
4. The fertilizers obtained from the digester can be stored for certain seasons of the year; it may be possible to provide additional storage capacity to satisfy interseasonal demand.
5. A linear combination of different residues leads to a linear combination of outputs.
6. Consumption of final products can vary from season to season.
7. Consumption outside the region under analysis is considered indirectly--at some border points.

Indices and Coefficients

i is the type of residue available for anaerobic digestion:
$i = 1,2,\ldots,I^1$ is nontransportable animal waste,
$i = I^1+1, I^1+2,\ldots,I^2$ is transportable crop waste;

t are the seasons;

j is the type of output (biogas, fertilizers, algae, etc.);

r are points where consumption of products is concentrated;

n is the type of digester (i.e., capacity) selected for each point d;

d is a point at which the digester may be located;

l are points at which residue is located:
$l = 1,2,\ldots,L^1$ is nontransportable residue,
$l = (L^1+1), (L^1+2),\ldots,L^2$ is transportable residue;

N_{indt} is the capacity of digester n in period t at point d, when residue i is used;

b_i is the consumption of residue i per unit of digester capacity;

d_{in} is a coefficient for standardizing digester capacity n when residue i is used (it is assumed that each type of installation can process 100 percent of the residue produced at the point under analysis);

N_{dt} is the standardized capacity of digester at point d in period t;

l_{ji} is the output of product j per unit of input i;

F_{jrt} is the volume of consumption of product j in center r in period t;

G_{ilt}^{max} are the maximum residues i in season t at point l;

k is a coefficient indicating the percentage of additional storage capacity required for interseasonal demand;

A_{ld} is a vector of matrices of costs of processing and shipment of residues from point l to d ($A_{ld} = A_{1ld}$, $A_{21d}, \ldots, A_{ild}$); for $i \in I^1$, this cost could be zero or close to zero;

B_{jn} are construction and operational costs of digester n (per unit of biogas production, j = 1);

C_{rd} is a vector of matrices of transportation costs for shipment of product j from point d to point r

$(C_{dr} = C_{1dr}, C_{2dr}, \ldots, C_{jdr})$;

D_{jd} are construction and maintenance costs of storage facilities for product j at point d; and

E_{jrt} is the price of product j at point r in period t.

Variables

X_{iltnd} is the quantity of residue i at point l used in season t by digester n at point d;

Y_{jrndt} is the quantity of product j delivered to center r in period t from digester n at point d;

Z_{jdt} is the quantity of product j stored at point d in period t; and

$\hat{Z}_{jd}$ is the size of facility required for storing product j at point d.

Balance between Residues and Production Capacities

The sum of local and delivered residues should satisfy the requirements of digester n at point d in period t:

$$\sum_{\substack{i\in I^2 \\ r\in L^2}} X_{iltnd} + \underset{\substack{(i\in I_1^1 \\ l=L^1)}}{X_{iltnd}} = \sum_i b_i N_{indt} \;, \qquad \text{for all } n,d,t. \tag{1}$$

Consumption of residues is constrained. For nontransportable residues (l = d):

$$\sum_{\substack{n \\ (l=d)}} X_{iltnd} \le G_{ilt}^{max} \;, \qquad \text{for all } l,t,i\in I. \tag{2}$$

For transportable residues:

$$\sum_{n,d} X_{iltnd} \le G_{ilt}^{max} \;, \qquad \text{for all } l,t,i\in I^2. \tag{3}$$

Balance of Production Capacities

Only one type of digester can be constructed at one point. This constraint may be expressed as

$$\sum_{i,n} N_{indt} = \{^{0}_{N_{dt}} \;, \tag{4a}$$

or as

$$\sum_{i,n} d_{in} X_{indt} = N_{dt} \;, \qquad \text{for all } d,t. \tag{4b}$$

The use of (4b) instead of (4a) generally leads to a noninteger solution, but, for a linear problem, the partial integer solution could be expected in the majority of cases. What is very important is that the solution can be obtained much more easily.

Balance of Production and Shipment

Shipment of every product j from the point d in any period t is restricted by the production capacity of the digester:

$$\sum_{i,l,n} X_{iltnd} \cdot l_{ji} \geq \sum_{r,n} Y_{jrndt} \quad , \qquad \text{for all } j,t,d. \quad (5)$$

Consumption Constraint

At point r in period t, the consumption of product j could be constrained:

$$\sum_{n,d} (Y_{jrndt} - Z_{jdt}) \leq F_{jrt} \quad , \qquad \text{for all } j,r,t \quad . \qquad (6)$$

Storage Capacity

The capacity for storage of product j at point d should be equal to maximum seasonal requirements:

$$Z_{jd} \geq K \cdot Z_{jdt} \quad , \qquad \text{for all } j,t,d \quad . \qquad (7)$$

Objective Function

The difference between income and costs should be maximized. Income could be calculated on the basis of existing prices (for fertilizers and gas) when the quantity of final products is known. The sum of expenditures includes costs of residue preparation and delivery to digesters, construction and operation of digesters, storage, and transportation of final products to consumers:

$$\{ \sum_{j,r,t} E_{jrt} \sum_{n,d} Y_{jrndt} - \sum_{i,l,d} A_{ild} \sum_{t,n} X_{iltnd} - \sum_{n,j=1} B_{jn} \sum_{r,d,t} Y_{jrndt}$$

$$- \sum_{j,r,d} C_{jrd} \cdot \sum_{n,t} Y_{jrndt} \qquad (8)$$

$$- \sum_{j,d} D_{jd} \cdot Z_{jd} \} \rightarrow \max.$$

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PART V

Models of Labor, Services, and Settlement Structure

FOUR APPROACHES TO COMMUTER ANALYSIS FOR THE SILISTRA REGION

A.E. Andersson
A. La Bella

INTRODUCTION

In an earlier analysis of commuter behavior for the Silistra region, we proposed a rather data-consuming procedure (see Andersson and La Bella 1979).

In this paper we restrict ourselves to a consideration of commuting trips between residences and workplaces. We present four different modeling approaches: the first three are normative, aimed at calculating the macroeconomically optimal commuting pattern; the fourth approach, based on a minimal information procedure, is aimed at forecasting a potential commuting pattern as a consequence of alternative scenarios for distribution of residences and workplaces.

MODEL I

The common framework is constituted by considering n settlements, among which people are allowed to commute, m economic sectors, and the following sets of constraints:

$$\sum_{jh} T_{ijh} = L_i \quad , \qquad \forall i \quad , \tag{1}$$

$$\sum_{i} T_{ijh} \leq D_{jh} \quad , \qquad \forall i,h \quad . \tag{2}$$

We should also include a constraint on transportation capacity:

$$\sum_{ij} c_{ijh} T_{ijh} \leq C_h \quad , \tag{3}$$

where*

D_{jh} is labor demand in region j and sector h;

L_i is labor supply in region i; and

T_{ijh} are commuters from region i to region j in sector h.

We assume here that L_i is a function of the relative housing situation in settlement i with respect to the average housing level. Moreover, D_{jh} will be considered to be dependent on capital stock installed in settlement j and sector h. Therefore, we have

$$L_i = L_i(H_i, H) \quad , \tag{4}$$

$$D_{jh} = D_{jh}(K_{jh}) \quad , \tag{5}$$

where

H_i is an indicator of housing availability and quality in settlement i;

H is an indicator of average housing availability; and

K_{jh} is the capital stock in settlement j and in sector h.

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*Note that (1) and (2) implicitly state that

$$\sum_i L_i \leq \sum_j D_{jh} \quad ;$$

i.e., that integrated over all the regions, the global labor demand is not less that the global labor supply.

The most ambitious procedure would be the following. With a given distribution of population and capital over the n settlements and the m sectors, what would be the macroeconomically optimal pattern of commuting? It should be noted that, in this formulation of the problem, we assume H_i, H, and K_{jh} in equations (4) and (5) as fixed for all i, j, and h.

A solution to the above problem can be obtained using a Cobb-Douglas (or some more general) production function for each sector and settlement:

$$Y_{jh} = A_{jh} K_{jh}^{\alpha_{jh}} \left(\sum_i T_{ijh}\right)^{\beta_{jh}} , \tag{6}$$

where

Y_{jh} is output of sector h in settlement j;
A_{jh} is a constant; and
α_{jh}, β_{jh} are positive parameters.

We can now specify the total production as

$$Y_{**} = \sum_j \sum_h A_{jh} K_{jh}^{\alpha_{jh}} \left(\sum_i T_{ijh}\right)^{\beta_{jh}} , \tag{7}$$

and, therefore, the optimal commuting pattern may be obtained by solving the following problem:

$$\max Y_{**} , \tag{8}$$

subject to (1), (2), and (3).

The model outlined above is very data consuming, and it is uncertain whether we can get all the parameter estimates needed for this model at this stage of the Silistra case study.

Applying the first-order necessary conditions for an optimum of problem (8), we get the following results for any sector:

$$\text{Commuter flow between area i and area j} = \frac{\text{Marginal productivity of labor in area j} \times \text{labor demand in area j}}{\begin{Bmatrix}\text{Shadow price of}\\ \text{labor supply in}\\ \text{area i}\end{Bmatrix} + \begin{Bmatrix}\text{Shadow price of}\\ \text{labor demand}\\ \text{in area j}\end{Bmatrix} + \begin{Bmatrix}\text{Marginal cost of per-}\\ \text{sonal transportation}\\ \text{from area i to area j}\end{Bmatrix}}$$

Attractive as this model may be, it suffers from the major drawback of large, detailed data requirements. We have, therefore, designed a simplified version that can approximate the solutions of the first more general commuter model.

MODEL II

$$\underset{\{T_{ijh}\}}{\text{Maximize}} \sum_{ijh} MP_{L,jh} T_{ijh} , \tag{9}$$

subject to (1), (2), and (3).

The problem of estimating the marginal productivities of labor ($MP_{L,jh}$) can be approached by use of a first-order Taylor approximation of the production function $Q(K,L)$.

$$Q^a_{jh} = Q(K^*,L^*)_{jh} + MP_{K,jh} \Delta K_{jh} + MP_{L,jh} \cdot \Delta L_{jh} + R_{jh} , \tag{10}$$

where

Q^a_{jh} is the estimated production in area j and sector L;

$Q(K^*,L^*)_{jh}$ is production at level K*, L* in area j and sector h;

$MP_{K,jh}$ is marginal productivity of capital in area j and sector h; and

R_{jh} is a residual term.

Define

$$Q^a_{jh} - Q(K^*,L^*)_{jh} \equiv \Delta Q_{jh} \quad ,$$

$$MP_{K,jh} \equiv \gamma_{jh} \quad ,$$

$$MP_{L,jh} \equiv \mu_{jh} \quad , \qquad (11a)$$

$$K_{jh} \equiv I_{jh} = \text{investment} \quad .$$

Thus,

$$\Delta Q_{jh} = \gamma_{jh} I_{jh} + \mu_{jh} \Delta L_{jh} + R_{jh} \quad . \qquad (11b)$$

This equation (11b) can be estimated on time series for any area and sector in order to generate the $MP_{L,jh} \equiv \mu_{jh}$ for the goal function.

If it were impossible to get even the more limited data required for model II, we would be forced to use a cost-minimizing approach as in model III.

MODEL III

$$\underset{\{T_{ijh}\}}{\text{Minimize}} \quad \sum_{ijh} c_{ijh} T_{ijh} \quad , \qquad (12)$$

where

c_{ijh} are the unit transportation costs between each origin-destination pair for any category of workers,

subject to (1) and (2) and $T_{ijh} \geqq 0$ (for all i,j,h).

This macrooptimizing model is the least data-consuming commuter model that can be proposed; it is also easy to implement on computer.

It might be necessary to estimate the parameters for Silistra as a whole under the assumption that there are no productivity differences, with the exception of those reflected by location of production units.

As a further simplification, useful for policy design, we shall assume a linear approximation also for function (4):

$$L_i \simeq \alpha_i H_i \quad . \tag{13}$$

MODEL IV

The fourth approach is aimed at calculating the most probable pattern of commuting resulting from a given distribution of residences and workplaces. It is obvious that the pattern calculated in this way can be significantly different from that obtained from models I to III.

A complete discussion of the use of entropy maximizing and information minimizing techniques can be found in Snickars and Weibull (1977) and Willekens, Por, and Raquillet (1978). For our purpose, it is sufficient to recall that the most probable distribution of commuting trips compatible with constraints (1) and (2) can be found by maximizing the so-called "information function" given as

$$W = \sum_i \sum_j \sum_h T_{ijh} \ln \left[\frac{T_{ijh}}{T^o_{ijh}}\right] \quad , \tag{14}$$

subject to (1), (2), and (3). Where $\{T^o_{ijh}\}$ represents the observed commuter pattern.

We can associate the information minimization problem with the following Lagrangean:

$$\min_{\{T_{ijh}\}} : L = W + \sum_i \sum_h \lambda_{ih} [L_{ih} - \sum_j T_{ijh}]$$

$$+ \sum_j \sum_h \mu_{jh} [D_{jh} - \sum_i T_{ijh}] \tag{15}$$

$$+ \sum_h \nu_h [C_h - \sum_i \sum_j T_{ijh} c_{ijh}] \quad ,$$

with

$$\mu_{jh} \geq 0 \quad \text{and} \quad \mu_{jh}[D_{jh} - \sum_i T_{ijh}] = 0 \quad \forall j,h \ . \tag{16}$$

From the first-order conditions we obtain

$$\frac{\partial L}{\partial T_{ijh}} = + \ln [\frac{T_{ijh}}{T^o_{ijh}}] + 1 - \lambda_{ih} - \mu_{jh} - \beta_h \ c_{ijh} = 0 \ , \tag{17}$$

and, therefore

$$T_{ijh} = T^o_{ijh} \exp \ (1-\lambda_{ih} - \mu_{ih} - \beta_h c_{ijh}) \ . \tag{18}$$

Posing

$$A_{ih} = \frac{e^{1-\lambda_{ih}}}{L_{ih}} \ , \qquad B_{jh} = \frac{e^{-\mu_{ih}}}{D_{jh}} \ , \tag{19}$$

we obtain

$$T_{ijh} = A_{ih} \times B_{jh} \times L_{ih} \times D_{jh} \ e^{-\beta_h C_{ijh}} \ ; \tag{20}$$

substituting in (1), (2) and using (6), we obtain

$$A_{ih} = [\sum_j B_{jh} \ D_{jh} \ e^{-\beta_h c_{ijh}}]^{-1} \ , \tag{21}$$

$$B_{jh} = \begin{cases} 1/D_{jh} & \text{if} \quad D_{jh} > \sum_i T_{ijh} \\ [\sum_i A_{ih} \ L_{ih} \ e^{-\beta_h c_{ijh}}]^{-1} & \text{if} \quad D_{ih} = \sum_i T_{ijh} \end{cases} \ , \tag{22}$$

which can be used for solving the information minimizing problem iteratively (see e.g., Willekens 1977; and Willekens, Por, and Raquillet 1978). The parameter β_h represents the sensitivity of category h of commuters to the commuting cost and can be used for calibrating the model.

DATA REQUIREMENTS

The above models will be run for n areas within the Silistra region and three sectors (industry, services, and agriculture). The number n of the areas will be specified according to the quantity and quality of available information.

The following data will be required:

1. an observed pattern of commuter flows $\{T^o_{ijh}\}$ (n x n x 3 observations);
2. the transportation costs c_{ijh} between each origin-destination pair for any category of workers (n x n x 3/2 observations, if n is even; (n x n x 3/2) + 1 observation otherwise);
3. the labor supply L_i in any area (n observations);
4. the labor demand D_{jh} in any area and sector (n x 3 observations); and
5. the housing situation (measured in total square meters $\{H_i\}$ available in any area: n observations).

The above requirements represent the necessary data basis for all four models previously described and are also sufficient for running models III and IV.

Models I and II require some supplementary information; in particular, model I requires information on:*

6. the capital stock K_{jh} in any area and sector (n x 3 observations); and
7. the labor supply L_{jh} in any area and sector (n x 3 observations).

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* A simplified form of model I could be run making use of the following more aggregated information, if 6 and 7 were not available:

-- the capital stock $\{K_h\}$ by sector for the whole region (3 observations); and
-- the labor supply $\{L_h\}$ by sector for the whole region (3 observations).

Model II requires, instead of 6 and 7, time-series information over a minimum period of 10 years about the following:

8. investment $\{I_{jh}(t)\}$ by area, sector, and year (n x 3 x 10 observations);
9. labor supply $\{L_{jh}(t)\}$ by area, sector, and year (n x 3 x 10 observations); and
10. value of production $\{Q_{jh}(t)\}$ by area, sector, and year (n x 3 x 10 observations).

Particular attention should be given to the consistency of time spans: in fact all the yearly observations for any variable must be relative to the same 10-year interval.

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* If information 8-10 were not available, a simplified version of model II could make use of the following more aggregated data:

-- investment $\{I_h(t)\}$ by sector and year for the whole region (3x10 observations);
-- labor supply $\{L_h(t)\}$ by sector and year for the whole region (3x10 observations); and
-- value of production $\{Q_h(t)\}$ by sector and year for the whole region (3x10 observations).

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MODELING SETTLEMENT SYSTEMS AS REGIONAL UNITS

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N. Grigorov
K. Evtimov

INTRODUCTION

In Bulgaria the settlement system is the basic unit for organizing and managing regional socioeconomic activities. This unit consists of a nucleus settlement surrounded by several supporting settlements. In 1979, the country was organized into 291 such systems, some of which have historically close cultural, economic, and other relations between their individual settlements. The systems are grouped into three categories according to the development of their internal links. Sixty-nine settlement systems contain settlements with historical links (type A); for a further 85 systems these links are in the process of being established (type B); and for the remaining 142 systems such links will be formed in the near future (type C). The regional bodies that manage the settlement systems are called Municipal People's Councils.

Although modelling activities are being carried out for all settlement systems in Bulgaria, this paper deals only with that part of the work related to those in the Silistra region. This region contains 10 settlement systems. Settlement system Silistra is of type A, Dulovo and Tutrakan are of type B, and those remaining--Glavinitsa, Zefirovo, Kaynardja, Ishirkovo, Okorsh, Sitovo, and Alfatar--are of type C.

DATA BASE FOR THE SET OF MODELS

The settlement system is characterized by complexity, dynamic changes, and openness. On account of these features,it is necessary to obtain detailed information for the management of the system. This information is organized according to the degree of computer implementation and type of models used for problem solution. In modeling settlement systems,the main idea has been to build a set of interactive models based on an integrated information system, which includes a series of heuristic procedures and two types of statistical and mathematical models: specialized and general models. The specialized models deal with the following groups of problems:

-- land use,
-- environment,
-- population,
-- agriculture,
-- industry,
-- industrial infrastructure,
-- housing,
-- public services,
-- recreation, and
-- management.

The general models are designed to determine the main parameters for all groups of problems in a uniform way, using, for example, input-output analysis, system dynamics analysis.

DESCRIPTION OF THE SET OF MODELS

The model system is organized as shown in Figure 1. All models are connected to the linkage model, which described the structure of the settlement system. The linkage model consists of eleven rows of problems (Figure 2); each problem is specified below.

First Row

1. Collection of data of international relations.
2. Collection of data on public opinion.
3. Collection of data on the effects of technological progress.
4. Collection of data on regional aspects of national economic growth.

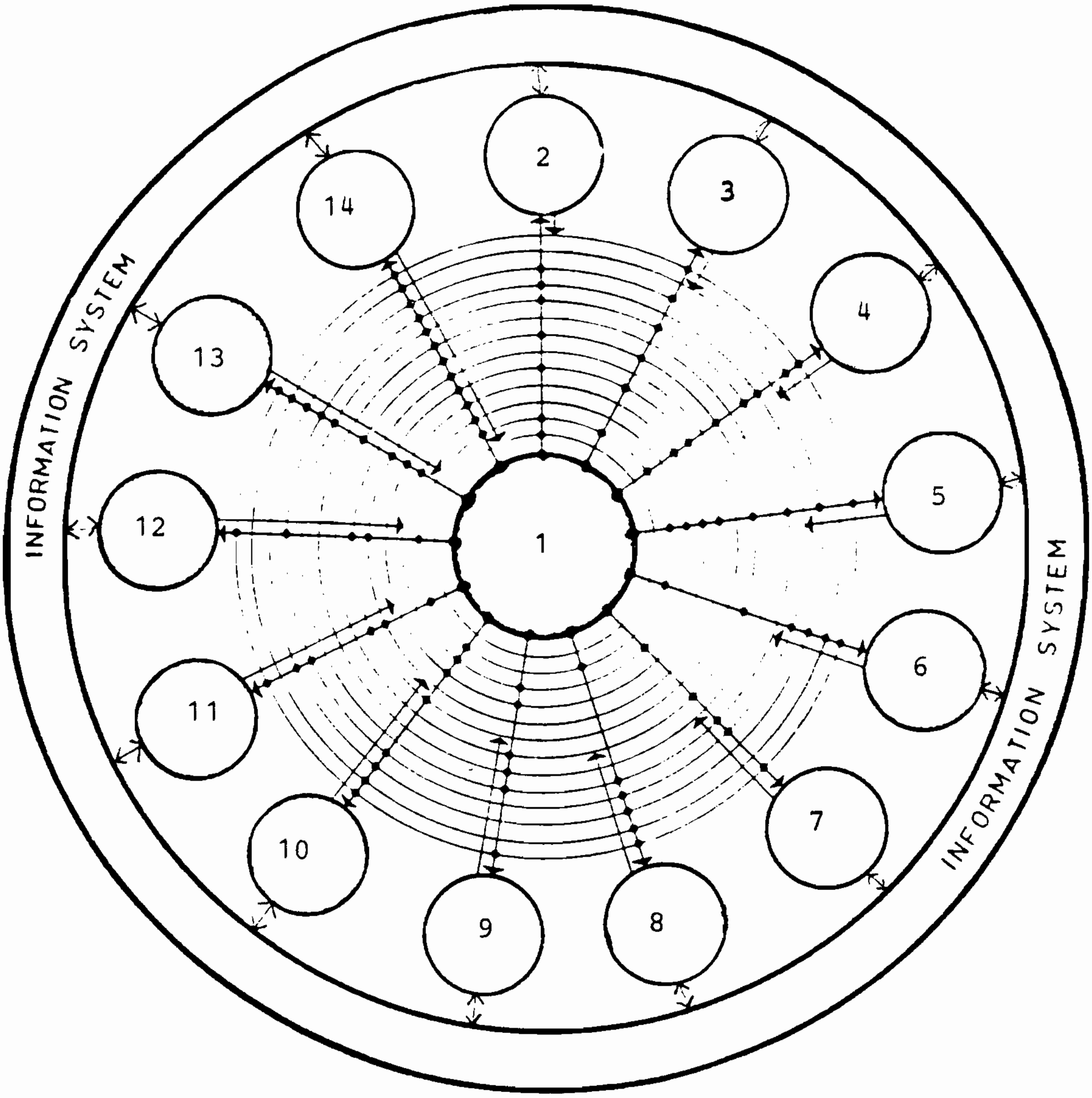

Figure 1. The links within the set of models of the settlement system (1--linkage model; 2--model of settlement system organization; 3--land use model; 4--environment model; 5--population model; 6--agriculture model; 7--industry model; 8--model of other economic activities; 9--industrial infrastructure model; 10--model of housing; 11--model of transport; 12--model of public services; 13--model of recreation; 14--management model).

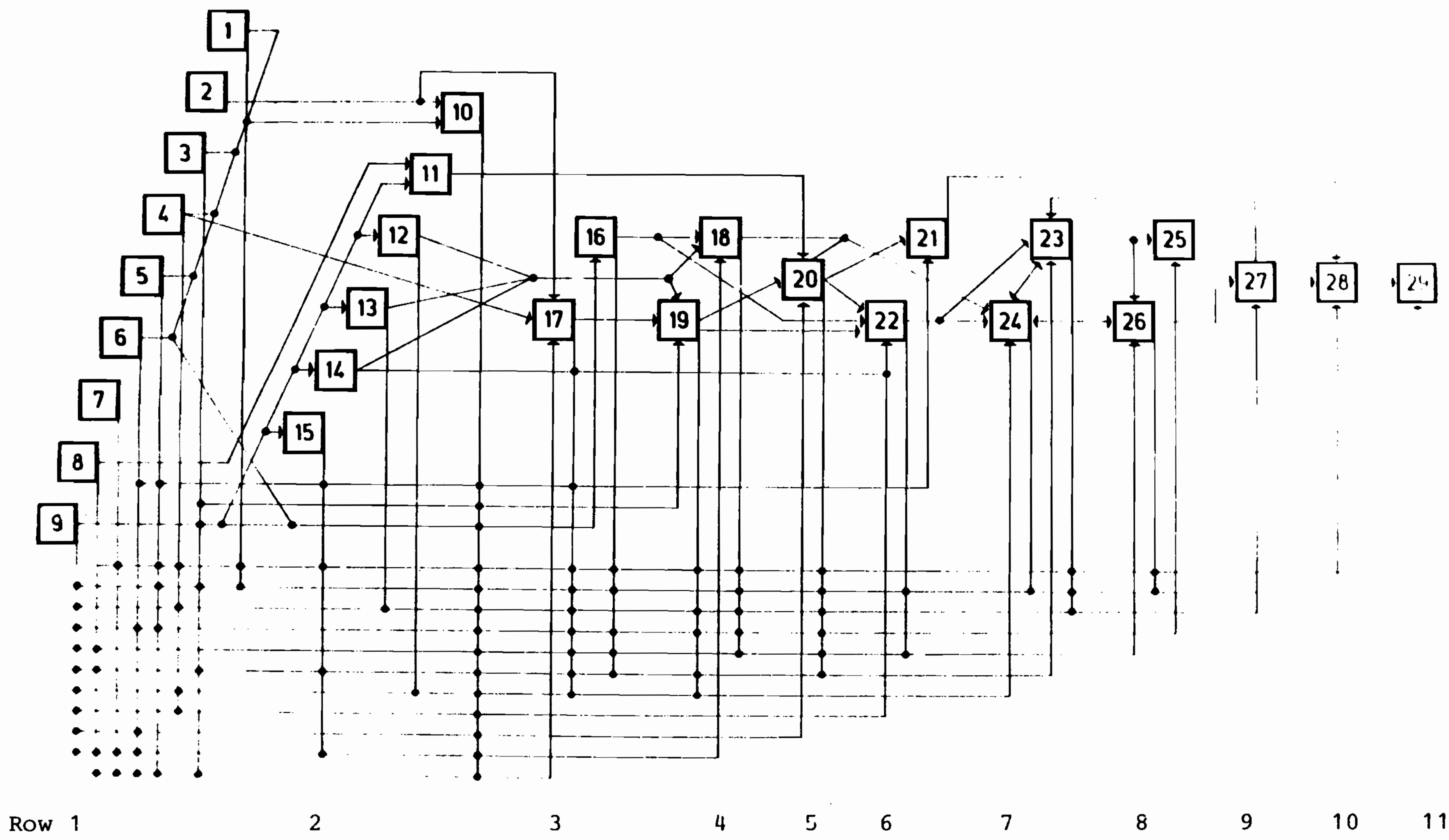

Figure 2. The linkage model describing the structure of the settlement system (the problems--numbers enclosed within squares--are defined in the text).

5. Collection of data on living standards.
6. Collection of data on natural resources and geographic and climatic conditions.
7. Collection of data on demographic processes.
8. Collection of data on past socioeconomic growth.
9. Specification of general social indicators.

Second Row

10. Determination of optimal socioeconomic growth of the settlement system.
11. Selection of a nucleus settlement for the system and organization of the territory around this central settlement.
12. Determination of the maximum water requirements for each settlement.
13. Determination of the maximum energy requirements for each settlement.
14. Determination of the maximum additional area of land that may be included within the boundaries of each settlement.
15. Evaluation of housing and service requirements for each settlement.

Third Row

16. Determination of the optimal number of settlements within the system.
17. Determination of the conditions necessary to achieve optimal economic growth in the settlement system.

Fourth Row

18. Determination of the optimal population size of each settlement.
19. Assessment of future economic stimuli that would encourage the development of the settlement system.

Fifth Row

20. Determination of the role of settlement systems specialization within the national framework.

Sixth Row

21. Determination of the investment required for recreational facilities.
22. Determination of land use within each settlement.

Seventh Row

23. Determination of the structure of investment in productive activities within each settlement.
24. Determination of the structure of water use for productive activities within each settlement.

Eighth Row

25. Determination of the structure of investment for environmental protection in each settlement.
26. Determination of the structure of investment in housing within each settlement.

Ninth Row

27. Determination of the structure of energy use for productive activities within each settlement.

Tenth Row

28. Determination of the structure of investment in transport and communication networks within each settlement.

Eleventh Row

29. Determination of the links required between settlements within a given settlement system.

At present, models of population, housing, public services, and recreation have been developed. The remaining models, necessary for the functioning of the model system, will be constructed in accordance with the description of the general model system outlined in these Proceedings in the paper by Christov and Panov.

DEVELOPMENT OF THE SYSTEM OF MODELS

The model system to be used for analysis of settlement systems is interactive and includes a linkage model, a data base, a package of computer programs for mathematical and statistical analysis and a bank of models formed as a result of this analysis. The interactive system is operated through the linkage model (Figure 3), and this process is demonstrated below taking problem 20 "determination of the role of settlement systems specialization within the national framework" as an example.

The set of all settlement systems in the country is given $C_i (i = 1,2,...,291)$. Each settlement system is characterized by N parameters $X_j^i (i = 1,2,...,291;\ j = 1,2,...,N)$. The set C must be divided into subsets; we have decided on a division according to the function of the settlement systems. The solution may be obtained using the following taxonomic method.

In the multidimensional parameter space a distance between two points is introduced satisfying the constraints

$$R(a,b) > 0 \text{ if } a \neq b \quad , \tag{1}$$

$$R(a,a) = 0 \quad , \tag{2}$$

$$R(a,b) \neq R(b,a) \quad . \tag{3}$$

The triangle axiom is not obligatory and that is why the space may not be metric. The smaller the distance between two points, the greater their similarity.

One subset includes points whose similarity exceeds a certain boundary value. The average similarity of the points within the subset significantly exceeds the similarity of points from different subsets.

The above algorithm "Forel I", developed by Elkino, Elkin, and Zagoronyko, has been modified to account for certain features peculiar to each settlement.

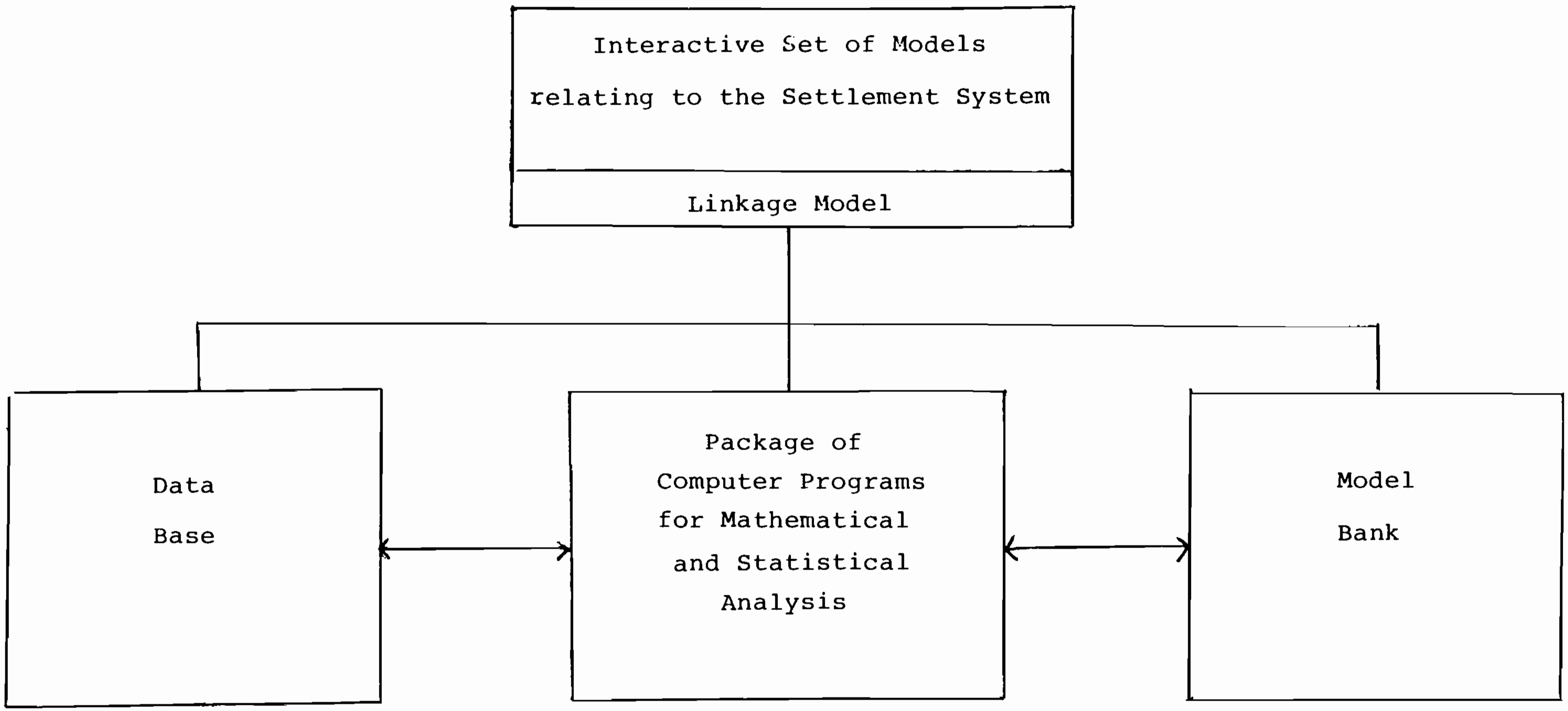

Figure 3. Operation of the interactive set of models.

CONCLUDING REMARKS

To classify the settlement systems of all regions in Bulgaria according to the state of their economic development, the radius of different subsets was calculated. Thus, five types of settlement systems were determined: ranging from type 1--highly developed--to type 5--underdeveloped. Within the Silístra region, settlement system Silistra is of type 2, Tutrakan and Dulovo are of type 3, Alfatar is of type 4, and Sitovo, Okorsh, Zefirovo, Glavinitza. Ishirkovo and Kaynardja are of type 5.

A MATHEMATICAL PROGRAMMING APPROACH TO LAND ALLOCATION IN REGIONAL PLANNING

A.E. Andersson
M. Kallio

1. INTRODUCTION

Many disciplines (e.g., theoretical geography, economics, operations research) have attempted to tackle the problem of finding an efficient allocation of land. Numerous approaches are used and their basic features regarding the treatment of space and time dimensions vary. The alternatives are illustrated in Table 1.

Table 1. A classification of approaches to the spatial allocation problem.

	Discrete time	Continuous time
Discrete Space	Most mathematical programming approaches (Andersson and La Bella 1979)	Most optimal control models (Isard et al. 1979)
Continuous Space	New urban economics (Mills 1972) Weber models (Cooper 1967, Nijkamp and Paelinck 1975)	Isard's dynamic transportation-location models (Isard et al. 1979) Beckmann-Puu transportation-location models (Beckmann 1953 and Puu forthcoming)

Approaches involving continuous time and/or continuous space have (up until now) proved to be of limited practical value, except for some qualitative analysis. We have chosen a discrete space and time model, which is able to cope with practical complications in generating policy alternatives for two case studies at IIASA: a long-term development study of the Malmö area in southern Sweden and long-range planning of the Silistra region in Bulgaria. We intend to formulate a regional development program as a (dynamic) mathematical programming problem and provide a procedure for finding an optimal solution for such a problem under various criteria.

We first provide an introductory discussion of regional planning. Thereafter, the problem is formulated as a dynamic (nonconvex) quadratic programming problem with integer variables. We develop a solution procedure for our programming problem, based on the theory of optimization over networks, and illustrate this procedure using a numerical example.

2. CHARACTERISTICS OF REGIONAL DEVELOPMENT

In short, the physical aspect of the regional development planning problem may be stated as a problem of finding a suitable trajectory of the locational pattern of various activities in a region. To elaborate on this statement, the following three considerations are taken into account: (i) the current or initial locational pattern of resources within the region, (ii) future expectations (or plans) for the total volume of different types of activities over time, and (iii) the criteria used to evaluate alternative development of locational patterns for these activities. We shall now discuss each of these considerations in some detail.

The current situation may be described by a map indicating the distribution of resources within different subregions. As an example, the regional subdivisions and the main road networks for Skåne (including the Malmö area) and Silistra are given in Figures 1 and 2. Resources here are understood in general terms: they include both natural resources and various types of

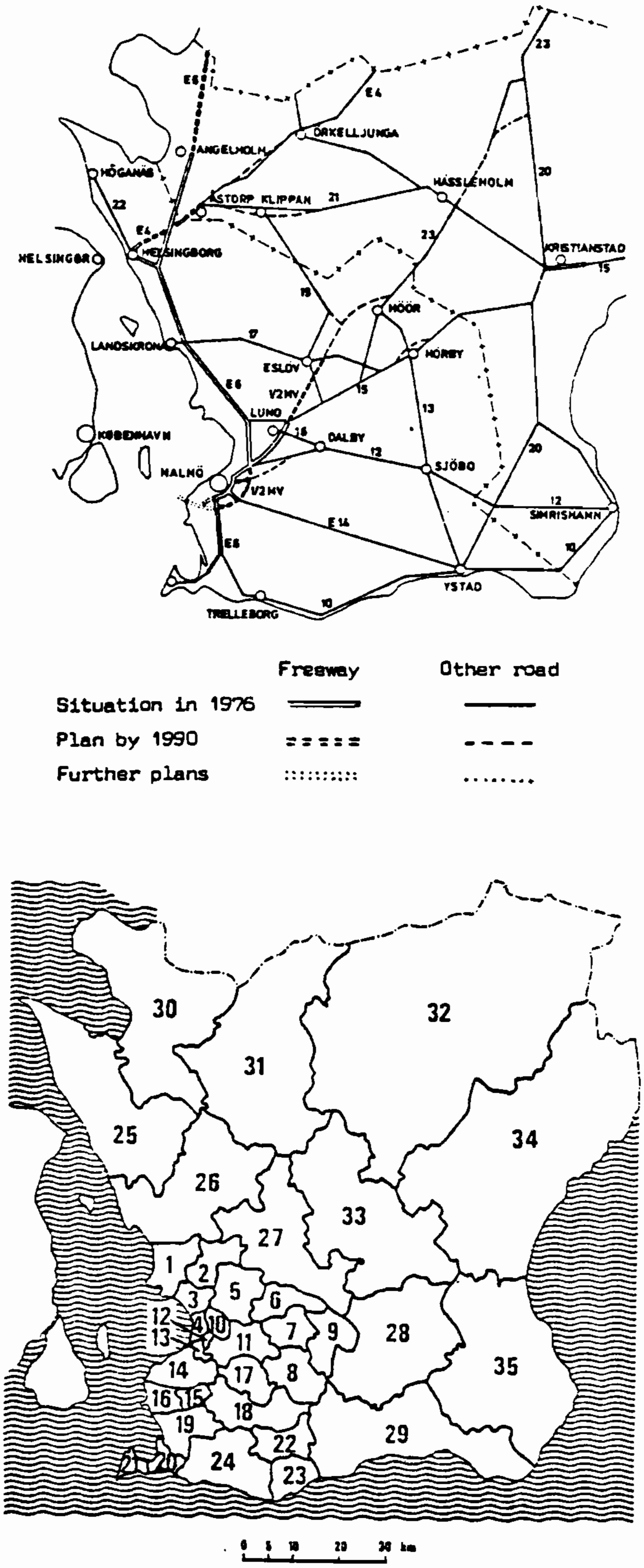

Figure 1. Regional subdivisions and the network of main roads in Skåne.

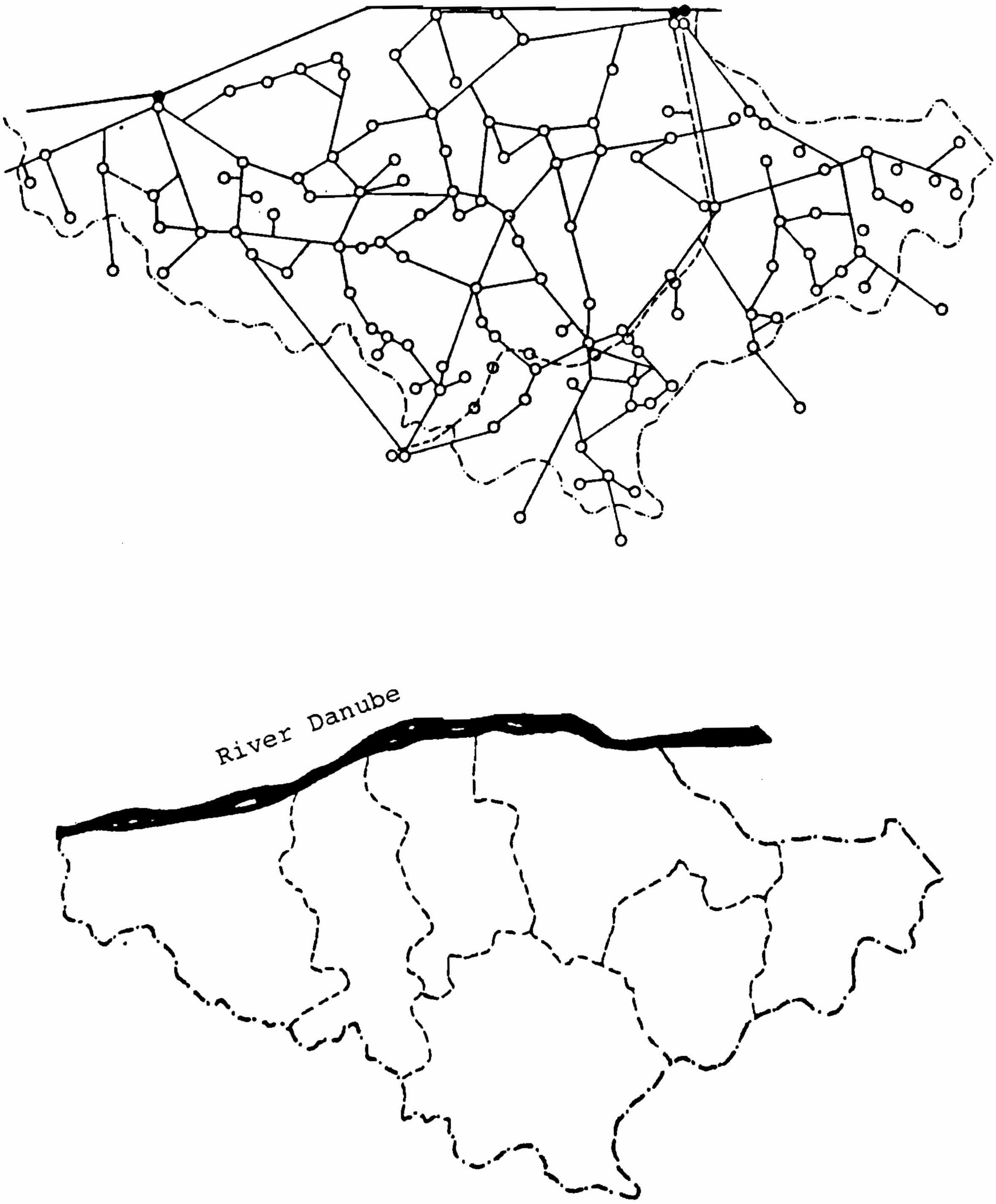

Figure 2. Regional subdivisions and the network of main roads in the Silistra region.

capacity. Examples of resources are capacity for industrial, transportation, farming activities, water resources, renewable resources (such as forests), and nonrenewable resources (such as mineral deposits).

Resources are used over time for various activities, such as industrial or agricultural production. We shall assume the total volume of such activities is known over the time period under study. Such forecasts may be available in the framework of central planning (as in Bulgaria) or they may be estimated using econometric techniques (as in Sweden), for instance. A feasible locational pattern at a given point of time is one that provides sufficient resources to achieve the estimated activity levels within the region at that time. This may require an increase in some of the resources (such as housing or industrial units) or it may allow a decrease over time (such as use of mineral resources).

In general, there is much freedom in designing feasible patterns: there are alternative locations for most of the activities and, furthermore, certain sites may change activity over time. Then, the following question arises: what are the criteria that we should take into account while comparing alternative feasible locational patterns over time? Clearly, a single criterion is insufficient. Over the whole planning horizon, we must simultaneously account for investment costs for changing the capacity for various activities in various subregions over time, the operating cost of the production activities, communication within the region, and environmental problems created by a certain locational pattern. Economies of scale are assumed to play an important role in determining the production costs. Furthermore, the location of a production unit relative to the location of natural resources and other production units may, of course, represent a significant share in operating costs.

3. A PLANNING MODEL

3.1. Feasible Allocation of Land

Our next task is to formulate the planning problem into a mathematical programming model. We shall first describe the set of alternative location patterns in terms of mathematical relations. Thereafter, we formulate a precise statement of the proposed criteria for evaluating alternative plans.

As indicated above, we adopt a discrete time and discrete space formulation; i.e., we consider the planning horizon to be partitioned into T periods ($t = 0,1,2,\ldots, T-1$) and the region partitioned into R subregions ($r = 1,2,\ldots, R$). For instance, each time period t may be five years in length, in which case the planning horizon may consist of three to five such periods. Each region r is associated with a land area L_r and an initial capacity x^r_{i0} for activity i, $i = 1,2,\ldots, I$. Thus, there is an area L_r available for these activities in subregion r during each period t.

We shall denote by x_0 the vector whose components are x^r_{i0}. The allocation of land for different activities i for the first period $t = 0$ is determined by the initial state x_0. For other periods, the land use may be altered through investment decisions. Let $y^r_i(t)$ be the increase of capacity i (for activity i) in subregion r during t, let $d^r_i(t)$ be the decrease (demolition), and let $x^r_i(t)$ be the total capacity i in region r at the beginning of period t, for all i, r, and t. In this notation we have, for all t,

$$x(t+1) = x(t) + y(t) - d(t) \quad , \tag{1}$$

where

$x(t) \equiv (x^r_i(t))$, $y(t) \equiv (y^r_i(t))$, and $d(t) \equiv (d^r_i(t))$ are nonnegative vectors with I x R components; and

$x(0) = x_0$.

One way of handling economies of scale is to consider a set of indivisible production units only. Assuming that these units correspond to real alternatives, the production cost estimates can be given relatively easily. This approach leads to an integer programming formulation. In particular, for our purposes, it is sufficient to consider only one plant size that yields an average production cost close to the minimum possible and yet is a relatively small unit compared with the total capacity increase required. Thus, the vector y(t) indicates that the capacity increases have to be expressed by a nonnegative integer vector.

Notice in equation (1) that no physical depreciation is assumed. Thus, the operating cost is assumed to cover the reinvestment cost that is needed to maintain the capacity over period t. For the amount d(t) to be demolished, we may have a lower and an upper bound denoted by L(t) and U(t), respectively:

$$L(t) \leq d(t) \leq U(t) \quad . \tag{2}$$

This may be due to initially existing capacity, which ought to be closed down during period t. We shall assume d(t) and x_0 to be integer vectors as well, so that x(t) is an integer vector.

Let $z_i(t)$ be the total amount of capacity i at the beginning of period t, and denote $z(t) \equiv (z_i(t))$. Thus, we have

$$z_i(t) = \sum_r x_i^r(t) \quad . \tag{3}$$

A minimum requirement for capacities is given by a vector $Z(t) \equiv (Z_i(t))$, corresponding to an estimate of the total volume of activities within the region:

$$z(t) \geq Z(t) \quad . \tag{4}$$

The land availability constraint can approximately be taken into account through the following inequality:

$$\sum_i x_i^r(t) \leq L_r \quad , \qquad \text{for all r and t} \quad . \tag{5}$$

Although this may seem quite restrictive, it is reasonable to assume in our study that the same amount of land is needed for each unit of various industrial activities. Such a unit is roughly determined by the chosen scale of the production units. For other activities, for which economies of scale are less important, the unit of capacity is determined so that its land requirement is about the same as that for an industrial unit. The purpose of this slightly restrictive assumption is to obtain a network flow formulation, which then greatly simplifies the analysis of our model.

An example of the network structure of our model for a 2-period, 2-region and 2-activity case is given in Figure 3. The nodes on the left refer to the land available and those on the right to the installed capacity. The vertically directed arcs on the left describe unoccupied land and those on the right the capacity carried over from one period to the next (for which we have a lower bound given by equation (4)). The other arcs, which are horizontal and may not be directed, refer to land allocations (for the two activities) or land made available (through demolition). The conservation equations for each node together with the lower bounds, given by equation (4) (for the vertical flows on the right), and the integrality requirements constitute the constraints for a possible land allocation.

3.2. Evaluation Criteria

We consider the following decision criteria: (a) investment and demolition costs, operating costs (including transportation of raw material and industrial products), (b) private communication costs (such as commuting, recreation, and leisure time), and environmental considerations including (c) congestion and (d) environmental synergisms. We intend to quantify these considerations as follows.

3.2.1. Investment, demolition, and operating costs

Let $B_i^r(t)$ and $D_i^r(t)$ be the unit cost of investment and demolition, respectively, for capacity i in subregion r during period t. Define $B(t) \equiv (B_i^r(t))$ and $D(t) = (D_i^r(t))$. Then, the total investment and demolition cost for period t is given as $B(t)y(t) + D(t)d(t)$.

We divide the operating costs into the interaction costs between the activities to be located (such as transportation of goods, communication) and other operating costs (which may also be dependent on the locations of the production units). The costs of interactions between activity i and j located in areas r and s, respectively, will be written as $x_i^r(t)c_{ij}^{rs}(t)x_j^s(t)$, where $x_j^r(t)$ and $x_j^s(t)$ define (as above) the number of units located, and $c_{ij}^{rs}(t)$ is the cost of interaction per unit of activity i on zone r and per unit of activity j on zone s.

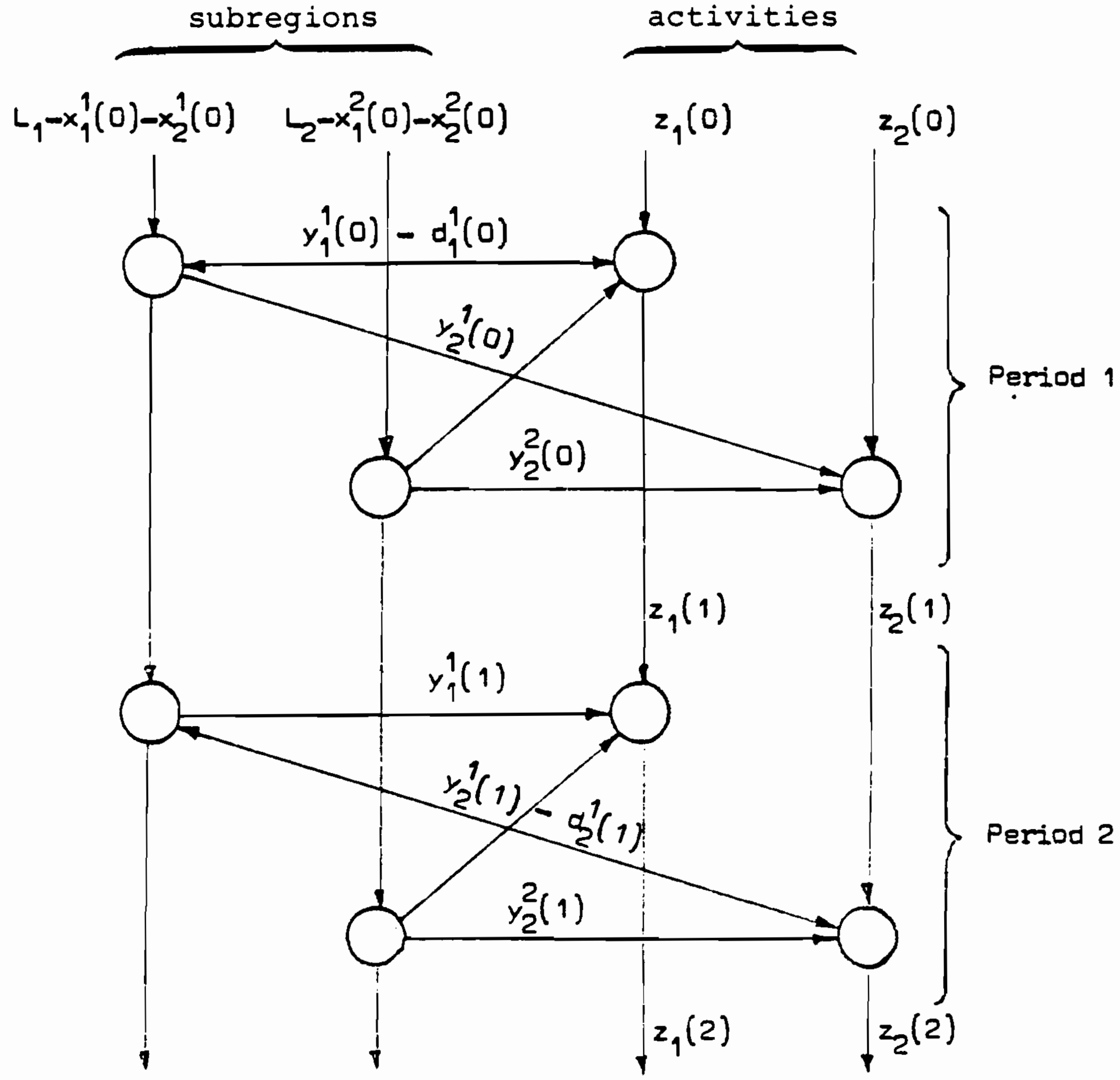

Figure 3. The network structure of a model with T = I = R = 2.

<u>Remark</u>: In order to take advantage of the network structure of the model, the subregional capacity levels $x_i^r(t)$ should be suppressed; i.e., one solves $x_i^r(t)$ from (1) and substitutes elsewhere. While doing so, one has to pay special attention to restrict the demolition activities in order to maintain non-negativity for the $x_i^r(t)$ variables. For instance, one may allow demolition only for certain time periods and for some initially existing capacity.

Such a formulation of interaction costs was first proposed by Koopmans and Beckmann (1957). Similar formulations have later been developed by Lundqvist and Karlqvist (1972), Andersson (1974), Snickars (1972), and Los (1978).

We may interpret the interaction cost $c_{ij}^{rs}(t)$ as a <u>potential transportation (communication) cost</u>. The cost $c_{ij}^{rs}(t)$ is then given as a product of the following factors: the frequency of interaction of one unit of activity i with activity j divided by $Z_j(t)$ (the estimated total number of units j), a nondecreasing function of the distance between subregions r and s, and the unit cost of interaction. The product of the first two factors yields an estimate for the number of interactions for one unit of activity i in subregion r with one unit of j in subregion s.

Let $F_i^r(t)$ be the other operating costs for one unit of activity i in subregion r during t. Such a term may, for instance, include interaction costs between the industrial unit and some prelocated sites of interaction (such as a mineral deposit, water supplying area, port of export). If we define a square matrix $C(t) \equiv (c_{ij}^{rs}(t))$ and a vector $F(t) = (F_i^r(t))$, then for period t, the total investment, demolition, and interaction costs, denoted by $I_1(t)$, can be written as

$$I_1(t) = B(t)y(t) + D(t)d(t) + F(t)x(t) + x(t)C(t)x(t) \quad . \tag{6}$$

Alternatively, the interaction costs may be taken through as an <u>accessibility</u> concept, which will now be defined. The accessibility A_{ij}^{rs} of a unit j in subregion s for a unit i in subregion r is defined as a product of frequency of interaction of one unit of i with j, and a nonincreasing function of the distance between subregions r and s. Defining a square matrix A as (A_{ij}^{rs}), the total system accessibility is given as $x(t)Ax(t)$. Because a high level of accessibility is desirable, we may replace the interaction cost $x(t)C(t)x(t)$ in equation (6) by the negative of the total system accessibility (possibly multiplied by a positive scalar, since accessibility may not be measured in monetary units).

Both potential transportation (communication) costs and accessibility are of fundamental importance in spatial planning problems. Accessibility has been a dominating concept in the recent development of regional theory. It has been given an axiomatic foundation by Weibull (1976), and our definition above is consistent with his assumptions. Because accessibility adds to the dimensionality of our decision criteria, we shall consider potential transportation costs as a measure of the communication costs.

3.2.2. Private communication costs

We account for private communication costs in a way similar to that of the above. However, a distinction between private and other communication costs is made because these constitute two separate criteria for evaluation in our planning problem.

Private communication costs, denoted by $I_2(t)$, may then be given as

$$I_2(t) = F_p(t)x(t) + x(t)C_p(t)x(t) \quad , \qquad (7)$$

where

$F_p(t)$ is a vector of unit communication costs between housing and prelocated sites (such as recreation areas, i.e., lakes, rivers, forests, etc.); and

$C_p(t)$ is a matrix of potential communication costs of connecting the housing units to other activities to be located.

Thus, components of $F_p(t)$, which do not correspond to the housing activities, are defined as equal to zero. Similarly, components of $C_p(t)$ are equal to zero if they do not correspond to interaction with a housing unit.

3.2.3. Congestion

Excessive congestion of activities is the most obvious kind of environmental problem. We measure congestion by capital density allocated by subregions (i.e., congestion at zone r is defined as $\sum_i K_i^r(t)x_i^r(t)/L_r$, where $K_i(t)$ is the capital stock per unit of activity i). Average congestion in a regional system is defined as the weighted sum of the congestion of each subregion. If the ratio of capital stock in subregion r and total capital stock within the region is taken as such weights, then the average congestion, denoted by $I_3(t)$, is written as

$$I_3(t) = \sum_r [\sum_i K_i^r(t) x_i^r(t)]^2 / (K(t) L_r) \qquad (8)$$

$$x(t)G(t)x(t) \quad ,$$

where

G(t) is an appropriately defined square matrix; and

K(t) is the total capital stock.

3.2.4. Environmental synergisms

Environmental problems are normally of a much more complicated and synergistic nature than those described in our congestion cost measure. A unit of heavy industry is, for instance, of little environmental consequence if located, together with other heavy industries, a considerable distance from housing. On the other hand, if it has to be located close to housing or outdoor recreation, the disturbance can be enormous. Because of the public good nature of pollution, one has to take into account the number of persons affected. An environmental interaction matrix $E = (E_{ij}^{rs})$ would consequently measure the disturbance between different activities i and j located in region r and s, respectively. Naturally, numerical values for the parameters E_{ij}^{rs} may be very difficult to assess. In order to account for the environmental effects at least qualitatively, one might use powers of ten as values for these parameters (e.g., 0.1, 1, 10, 100, etc.). A measure $I_4(t)$ for environmental synergism effects may then be given as

$$I_4(t) = x(t)Ex(t) \quad . \qquad (9)$$

For each of the four criteria c and for each time period t, we define a weighting factor $\beta_c(t)$ that accounts for the time preference. Thus, our planning problem becomes a 4-criteria optimization problem, where the criteria I_c are given by

$$I_c = \sum_t \beta_c(t) I_c(t) \quad , \quad \text{for } c = 1,2,3,4 \quad . \tag{10}$$

We do not propose that a particular multicriteria optimization technique should be used. Rather, we suggest that simply nonnegative weights λ_c should be used for the criteria in order to form a linear scalarizing function that would be minimized. In this way, a linear approximation for the (negative) utility function g is given as

$$g = \sum_c \lambda_c I_c \quad . \tag{11}$$

Of course, different values for the parameters λ_c may be used in order to generate a set of interesting development alternatives for the region.

3.3. Summary of the Model

In summary, the planning problem (P) is to find nonnegative integer vectors x(t), y(t), d(t), and z(t), for all t, to

minimize g in (11)

(P) subject to (1) - (5) and

with the initial state $x(0) = x_0$.

The objective function of this problem is a quadratic form. However, in general this function is not convex. It is easy to see that, for instance, the potential transportation cost matrix C(t) normally is not positive semidefinite. If we have a static 1-activity and 2-zone problem, and the transportation costs are equal to the distances d_{rs} (between subregions r and s), then the potential transportation cost is given as

$$xCx = (x^1, x^2) \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} x^1 \\ x^2 \end{bmatrix} , \tag{12}$$

where the diagonal elements d_{rr} are equal to zero. Clearly, if $d_{rs} > 0$ for $r \neq s$, our matrix C is not positive semidefinite, since for $(x^1, x^2) = (1,-1)$ we have $xCx < 0$. It can be shown that this result holds for multiactivity multizone problems in general (see Snickars 1972). Our planning problem will thus not necessarily have a unique optimum. Instead it is reasonable to expect a number of locational patterns to correspond to local optima, one or more of which are also global optima. This phenomenon is illustrated by a numerical example in section 5.

4. A SOLUTION TECHNIQUE

In this section we consider the network formulation of the problem (P); i.e., we assume that variables $x_i^r(t)$ have been solved from (1), substituted elsewhere, and that their nonnegativity is guaranteed without an explicit consideration. Let x be a vector whose components are our decision variables $y_i^r(t)$, $d_i^r(t)$, and $z_i(t)$, for all i, r and t. Let us denote our objective function in equation (11) by $g = g(x)$ and the set of all nonnegative vectors x satisfying our constraints (1) - (5) by S. In this notation our problem (P) may be restated as finding an integer vector x to

$$\text{(P)} \qquad \min_{x \in S} g(x) \quad .$$

Formally, the set S can be described as the set of feasible solutions to a transshipment network as illustrated in Figure 3.

We exploit the fact that every linearized problem (P) (a problem where the objective function of (P) is replaced by a linear function) is a transshipment problem for which very efficient solution techniques exist (see, e.g., Bradley et al. 1977). This is due to the fact that every extreme point of S is an integer solution provided that L_r, $D_j(t)$, and $Z_j(t)$ are integers for all r, j, and t (see, e.g., Dantzig 1963). Thus, while solving the linearized problem, the integrality requirement can be relaxed.

We shall propose the following approach for solving (P):

1. Choose an initial solution $x^0 \in S$, and set the iteration count k to 0.
2. Solve the linearized problem (L):
   $$\min_{x \in S} \nabla g(x^k)x$$
   for an optimal solution $z^k \in S$ (here $\nabla g(x^k)$ denotes the gradient of $g(x)$ at $x = x^k$).

3. Solve the line search problem (Q):
   minimize $g(\alpha x^k + (1 - \alpha) z^k)$
   for an optimal solution $\alpha^k \in [0,1]$.
4. Stop (i) if $\alpha = 0$, or (ii) if $\min_{i \leq k} g(z^i) - g(\alpha^k x^k + (1-\alpha^k) z^k) < \delta$, where δ is an appropriate tolerance, or (iii) if another appropriate criterion is satisfied (such as computing time); otherwise replace x^k by $\alpha^k x^k + (1 - \alpha^k) z^k$, k by k + 1, and return to step 2 .

As mentioned above, the linearized problem (L) is a transshipment problem and it can be solved extremely efficiently. For computations, we shall use the code reported in Kallio et al. (1979). The optimal (basic) solution for (L) satisfies the integrality requirements for all variables $x_i^r(t)$. Thus, z^k is feasible for (P). We approximate the optimal solution of (P) by the best of the solutions z^k generated by the above procedure. This, of course, may not be an exact solution for (P).

Problem (Q) is a quadratic problem with one variable α and one constraint $0 \leq \alpha \leq 1$. Thus, (Q) is extremely simple. Let (R) be the problem that is obtained by relaxing the integrality requirement on x in (P). Solution x^{k+1} is the best for problem (R) and can be found when moving from x^k in the direction z^k. Thus, the sequence $\{x^k\}$ generated by this procedure is exactly the same as that generated by the Frank-Wolfe method (1956) when applied to problem (R). If x^k converges to an optimal solution x^* for (R) and $\bar{x}$ is optimal for (P), then

$$g(x^*) \leq g(\bar{x}) \leq \min_{i \leq k} g(z^i) \quad , \tag{13}$$

(i.e., $g(x^*)$ is a lower bound on the optimal value of (P)). We may never know $g(x^*)$ but may still be motivated to use the difference of $\min_{i<k} g(z^i)$ and $g(x^k)$ as a stopping criterion, the best feasible values for (P) and (R) found so far. This is illustrated in Figure 4 .

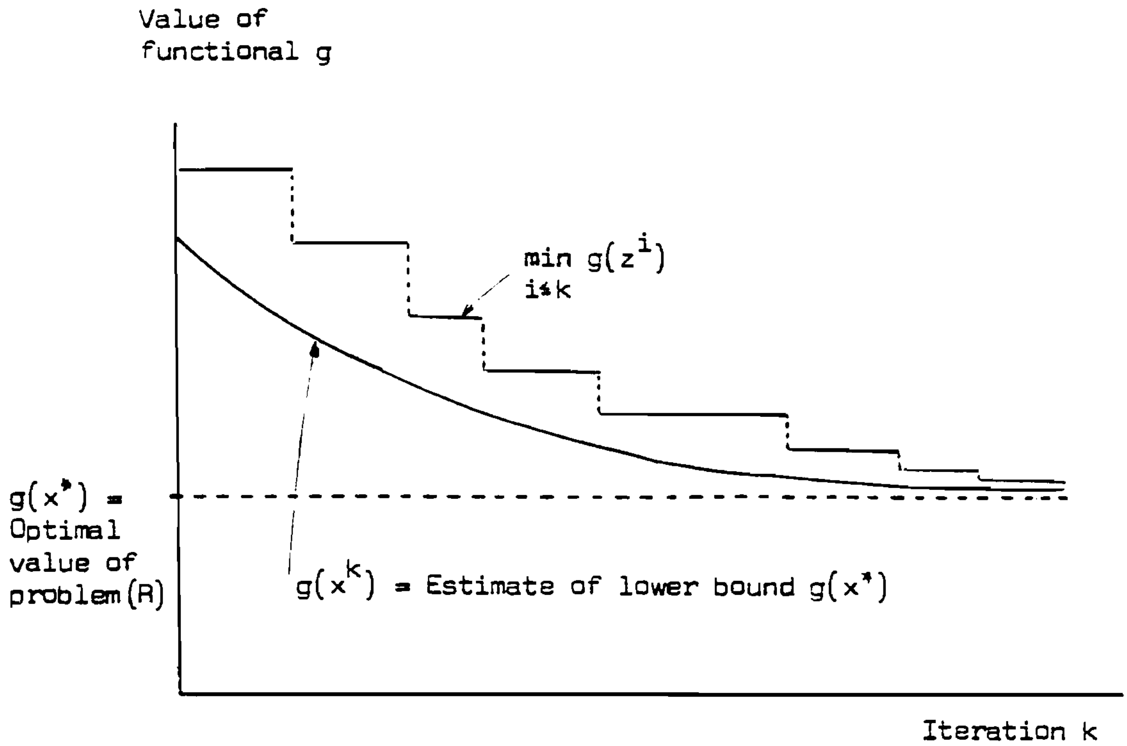

Figure 4. Functional value for problem (R) (the best one found so far) and for problem (P) as a function of iteration count k.

If g is a convex function, x^k converges to x^*. Otherwise we may apply the method several times starting with different solutions x^0.

5. A NUMERICAL EXAMPLE

As a simple example, we consider a static problem with four regions and four activities. The activities j, their building requirements Z, the regions r, and their land availability L_r are described in Table 2.

Let x_i^r be the number of units i to be located to region r, and denote $x = (x_1^1, x_2^1, x_3^1, \ldots, x_1^4, x_2^4, x_3^4, x_4^4)$. If the investment costs are assumed to be independent of region, they can then be considered as a constant term and thus omitted from further consideration. Our linear term in the objective function shall then consist only of the communication cost between housing and recreation facilities. The linear term is then given as $cx = (0,0,0,54900,0,0,0,45500,0,0,0,32800,0,0,0,39400)x$.

Table 2. An example of land requirements and availability.

j	Activity	Z_j	Region r	L_r
1	agriculture	5	A	1
2	industry	4	B	2
3	service	3	C	5
4	housing	6	D	10

The quadratic term xQx consists of congestion and communication costs, where the matrix $Q = (Q_{ij}^{rs})$ is given as

$$Q_{ij}^{rs} = \begin{cases} \alpha_{ij} d_{rs} + 1/L_r & \text{if } r = s \text{ and } i = j = 4 \; , \\ \alpha_{ij} d_{rs} & \text{otherwise} \; , \end{cases}$$

and

$$(a_{ij}) = \begin{bmatrix} 2 & 3 & 1 & 0 \\ 1 & 5 & 3 & 1 \\ 1 & 4 & 3 & 10 \\ 1 & 4 & 6 & 8 \end{bmatrix} , \qquad (d_{rs}) = \begin{bmatrix} 20 & 30 & 50 & 100 \\ 30 & 30 & 40 & 80 \\ 50 & 40 & 40 & 50 \\ 100 & 80 & 50 & 50 \end{bmatrix} .$$

The objective function appears to be nonconvex. Thus, we ran our solution procedure starting from randomly generated solutions x^0. The procedure was repeated tens of times, each one taking a few seconds in PDP 11 of IIASA. Two local optima were found. Both of these solutions appeared to be equally good, thus yielding to our location problem alternative global optima as conjectured above. The nonzero components of these solutions are given in Table 3.

Table 3. Two local optima of the example.

j\r	A	B	C	D	total
1	1			4	5
2				4	4
3		2		1	3
4			5	1	6
total	1	2	5	10	

j\r	A	B	C	D	total
1	1	2		2	5
2				4	4
3				3	3
4			5	1	6
total	1	2	5	10	

6. IMPLEMENTATION OF A PLAN

A plan generated, for instance, with the aid of our model is of little value if it cannot be implemented. There are essentially four ways of implementing a plan:

- -- to leave implementation to the market system without constraints but with charges (rent) for the land use;
- -- to use direct central decisions to implement complete investment strategies;
- -- to use the planning system to generate zoning constraints for activities and leave the detailed implementation to the market; or
- -- to use a scheme of negotiations between the allocators of land and the allocators of investment.

Each approach will now be briefly discussed.

6.1. Market Implementation

The market implementation method consists of determining rental values for land in different zones. Subsequently the decision makers of the sectors would be given a possibility to choose their own preferred location which under the sectoral criteria would yield the desired location pattern. In the following we shall provide some theoretical background on the existence of such rental values.

Consider first a simple case where n activities i are to be located on n available subregions r. Let b_i^r be the net benefit of activity i (excluding the rent for land) given that it has been located on subregion r. Suppose that according to the plan, the locations are determined so that the total net benefit is as large as possible. An optimal plan then results as a solution to the following assignment problem (Dantzig 1963):

$$\text{maximize} \sum_{ir} b_i^r x_i^r$$

$$\text{subject to} \sum_i x_i^r \leq 1 \quad , \tag{14}$$

$$\sum_r x_i^r = 1 \quad , \tag{15}$$

$$x_i \geq 0 \quad .$$

For an optimal basic solution, x_i^r is equal to 1 if activity i is to be located on subregion r and it is zero otherwise.

Let p_r and π_i denote the optimal dual multipliers for constraints (14) and (15), respectively. If, according to the (optimal) plan, subregion r is assigned to activity i, then $b_i^r - p_r - \pi_i = 0$. We shall interpret p_r as the rent for subregion r. Thus, $\pi_i = b_i^r - p_r$ is the profit for activity i. Given the rental values p_r, another location k for i would yield a profit of $\pi_i^k \equiv b_i^k - p_k$. By the optimality condition, $b_i^k - p_k - \pi_i \leq 0$, or $\pi_i^k = b_i^k - p_k \leq \pi_i$; i.e., any other location k for i would yield a profit π_i^k that is no higher than π_i. Thus, profit maximization of each activity separately yields an optimal location pattern under these rental prices.

It is often believed that a decentralized pricing system cannot be used to allocate a resource if there are economies of scale leading to indivisibilities. In fact, it has been shown by Koopmans and Beckmann (1957) for the above example, that decentralized implementation of the optimal solution cannot be achieved in general if the goal function is nonlinear, for example, quadratic. The same is usually true when integrality constraints are superimposed on the system, i.e., when capacity for some activities has to be built in given units of size. In our case, both nonconvexity and the integrality requirement (due to economies of scale) are likely to prevent a market implementation of the plan. The pure market solution to the implementation problem may then be ruled out.

6.2. Centralized Implementation

Another extreme procedure for implementation is the central decision principle where the plan is enforced by the regional authority. However, this procedure is extremely information-demanding at the level of central planning. A planning model, for instance,the one described in this paper, is by numerical necessity of a highly aggregated nature. Such aggregation may rule out a centralized implementation scheme with its requirements of detailed information, i.e., with fine disaggregation into fairly homogeneous branches of industry. One might also argue that it is impossible, or at least uneconomical, to generate very disaggregated technological and administrative data at the central level.

6.3. Zoning

One way of using a model for regional planning is as a constraint-generator for more detailed decision making. A compromise between centralized and market implementation is the "zoning principle" according to which central authorities constrain land use for each subregion to fall within an aggregate category of activities leaving all detailed decisions to the market. It is obvious that a planning model can be used to generate such constraints on land use.

6.4. Negotiation

Another implementation procedure is a negotiation scheme that also may be seen as a compromise between the pure planning and market approaches. This procedure, however, comes closer to the market implementation. The allocation model may be used to generate a reasonably representative set of pareto optimal locational patterns. These solutions may then be used as reference points in the negotiation between the land-allocating authorities and the sectoral decision makers (on investments in new units of production and other activities).

The choice between different implementation approaches cannot be determined objectively but must be decided in an institutional analysis relevant to the region and country.

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PART VI

Data Problems and Computer Use

A DATA BANK FOR AGRICULTURE AND WATER PROJECTS IN THE UPPER NOTEC REGION

A. Kałuszko
M. Kurowski

INTRODUCTION

A pilot project has been established to solve the economic and water resource problems of the Upper Notec region in Poland. This project is the first in Poland in which methods of systems analysis are used for the solution of regional problems and the experience gained will be used for similar agriculture and water projects in other regions.

At present, two main directions of research form the first stage of the project:

-- the General Study of water resource management and land reclamation in the Upper Notec region; and
-- the development of specialized models for solving particular problems.

The General Study forms the basis for the investment program.

The models already developed deal with the following problems:

-- regional agriculture (Albegov et al.);
-- water resource distribution (Gutenbaum et al.);
-- regional development policies (Kulikowski and Krus);
-- water system development (Makowski); and
-- agriculture production (Podkaminer et al.)

Papers describing the models in detail are included in these Proceedings.

The greatest difficulties faced by the researchers involved in the General Study and in model development are related to the collection and organization of the data. Although collected in the same region, the data are from many sources and come in different degrees of aggregation. While some models require them to be organized on a geographical basis, others need arrangement of the data on an administrative basis. Additional difficulties occur because the Upper Notec region lies in three voivodships (provinces); this is inconvenient, especially with regard to the data collected on an administrative basis. Aggregated data exists for individual provinces only and not for the Upper Notec region as a whole. Thus, it is necessary to work out a new basis for aggregation of the data.

Because of the difficulties mentioned above and the fact that most of the data are necessary for more than one model, it was decided that a uniform data base allowing rapid access to selected information should be established. Thus, we have created a computerized data base.

SCOPE OF INFORMATION FOR THE MODELS

We will now briefly indicate the most important data required.

The General Study (GS) required information on

- -- precipitation,
- -- air temperature and humidity,
- -- solar radiation,
- -- winds,
- -- natural and balanced water flows,
- -- soil types and their distribution,
- -- characteristics and organization of local industry,
- -- planned economic development,
- -- population structure,
- -- structure of farms,

-- crops and crop structure,
-- corn equivalent units,
-- number of animals in the region,
-- water supply and storage facilities and water
-- consumption,
-- sewage systems,
-- water quality,
-- land reclamation requirements, and
-- reservoir construction and maintenance costs.

The Generalized Regional Agriculture Model (GRAM) requires information on

-- soil types, their properties and distribution,
-- population structure,
-- structure of farms,
-- crop yields,
-- corn equivalent units,
-- consumption of fertilizers,
-- number of agricultural machines,
-- number of animals,
-- average and maximum water flows,
-- average and maximum water resources, and
-- area of land to be irrigated.

This information should be organized on an administrative basis.

The Water Resource Distribution Model (DWRM) requires information on

-- precipitation,
-- water flows,
-- reservoir water levels,
-- water consumption,
-- area of land to be irrigated,
-- crop structure, and
-- soil types.

This information should be organized on a partial watershed basis.

The Regional Development Policy Model (RDPM) requires information on

- -- costs and effects of different investment policies,
- -- population structure,
- -- number of sectors in the regional economy,
- -- value of production,
- -- investment,
- -- capital,
- -- employment,
- -- water consumption, and
- -- sewage systems.

(value of production through sewage systems: for each sector)

The Water System Development Model (WSDM) requires information on

- -- precipitation,
- -- air temperature,
- -- crop yields,
- -- crop structure,
- -- water consumption,
- -- maximum area of land to be irrigated,
- -- corn equivalent units,
- -- irrigation costs, and
- -- maintenance cost of water supply and storage facilities.

This information should be organized on a subregional basis.

The Agricultural Production Model (APM) requires information on

- -- precipitation,
- -- air temperature,
- -- soil types and their suitability for irrigation,
- -- corn equivalent units, and
- -- area of land to be irrigated.

PARTITIONING OF DATA INTO FILES

From the information presented in the previous section it can be observed that in many cases the same type of data is used in different models. In such data sets there are data consisting of a large number of items, e.g., soil data requires more than 2.5 million words of computer memory. Thus, it is necessary to put all this data into one data base.

All data are grouped into files, with every file composed of a number of subfiles, containing information of the same type. The structure of the files is not hierarchical. The most important information stored in each file is briefly described below.

Soil Data

The basic subfile of the soil data file, designated squares as defined below, is also the basic subfile of the whole data bank. The Upper Notec region is covered by a rectangular grid consisting of squares, with one side representing 500 m and the whole square an area of 25 ha; the squares correspond to those on a 1:25000 map, in which one side is 2 cm in length. For greater precision, every square of 25 ha is divided into four squares of 6.25 ha each. The code for each square is composed of the number of the quarter. The subfile of designated squares contains over 70,000 records.

Every record in this subfile contains information on soil characteristics and soil usage as well as on links connecting given squares to the data in other files, e.g., the codes of the administrative unit, partial watershed, and meteorological stations. Such a subfile structure and means of surface mapping allow information stored in the other files to be used during processing of the soil data.

Climatic Data

The climatic data file contains the data collected since 1951 at 56 climatic stations in the Upper Notec region and its surrounding area. There are seven types of data in this file: precipitation, air temperature, air humidity, low air humidity, solar radiation, wind speed and direction, and wind rose.

There are four subfiles for each type of data: 24-hour data, 10-day data, monthly data, and periodic data. The last three subfiles are computed at the input stage from the first subfile and are stored separately because they are often used by user's programs. The climatic data file is updated once annually.

Hydrological Data

The internal organization of the hydrological data file is the same as for the climatic data file. The subfiles contain the average values of 24-hour flows, 10-day flows, and periodic flows measured at 10 water-level measurement points. The 10-day, monthly, and periodic values of corrected flows for 10 measurement points and 78 balance points are stored in another subfile. The file is updated once annually.

Land Reclamation Data

The land reclamation data file consists of two basic subfiles. The first contains characteristics of 65 streams with a total length of 877 km. Parameters of the sequential stream sections are stored together with water quality indicators. The subfile contains data from 400 land reclamation installations.

Water Resource Data

Characteristics of the reservoirs of the Upper Notec region and their technical installations are stored in two subfiles. The third subfile contains the parameters of canals in the region. The remaining subfiles contain information on the water resources assigned to agriculture, industry, and households and include data on irrigation, water intake, water supply, and the sewage system.

Economic Data

The basic subfile contains economic data on agriculture in the smallest administrative units--gminas--relating to agricultural land use, crops, livestock, and crop and livestock structure. Other subfiles contain data on demographic processes and characteristics of local industry. Some of the agricultural data are organized on a subregional and partial watershed basis.

Financial Data

This file has a number of subfiles on costs. For example, there are prices of land and forest and urban areas, construction and maintenance costs of hydrotechnical installations, etc. These costs are mostly given as matrices representing the cost versus some other parameters.

The seven files presented above contain most of the data described in the previous section. The use of the files by the different models is shown in Table 1.

Table 1. Use of files by the models.

Model or study	Soil data	Climatic data	Hydrological data	Land reclamation data	Water resource data	Economic data	Financial data
GS	+	+	+	+	+	+	+
GRAM	+		+	+	+	+	+
DWRM		+	+	+	+	+	
RDPM					+	+	+
WSDM		+	+	+	+	+	+
APM	+	+		+		+	

ORGANIZATION OF THE DATA BASE

The data bank contains a set of interconnected files, whose organization allows rapid and efficient information retrieval. Data loading and retrieval are accomplished by service routines belonging to the data bank software. The service routines are also used for the management of data stored in files and for their preliminary processing.

In order to reduce the time taken for information retrieval, the data gathered in the data bank are stored in a direct access backing storage (magnetic disc units). In the direct access storage, contrary to the sequential storage, information can be found quickly only if the proper address can be determined. Hence, the files must be organized so that the address of specified data can be calculated with the least possible number of readings from the backing storage. Some typical methods of file organization that can be used directly, or with some modifications according to the amount and the type of data, can be found in Martin (1976) and Sprowls (1976).

The file organization used for the data bank described in this paper is a modification of the indexed sequential file technique. For every file there is an address array stored in the backing storage. This address array is rather small and can be taken as a whole into the computer memory. Its structure can differ according to the type of file. In addition to the address array, the file description array is inserted into the computer memory. For each subfile, the address of the beginning subfile and information on how to use the address array is stored in the file description array. When these two arrays for a given file are in the computer memory, the routines can compute, without further access to the backing storage, the address of any given record within this file. For a few subfiles that are used rather rarely, the computation of the record address can necessitate the reading of the specified record of another subfile.

The organization of the files described above could be used in our data bank under the assumptions of a constant record length for any given subfile and small variability of information stored in files. For many of the files, the updating of data consists only in changing specified fields inside the records. The new records are loaded only into those files in which data are stored for many consecutive years, e.g., meteorological data. Even in this case, updating is made only once annually. Thus, time-consuming reorganization requiring the modification of the file structure, which is performed by the special data bank service routines, is rare.

SERVICE ROUTINES AND USER'S PROGRAMS

The information stored in the data bank is used mainly by Fortran programs. All data bank service routines are also Fortran programs and subroutines, of which the five main service routines are

- the mapping subroutines,
- the input and update program,
- the print program,
- the file description program, and
- the administrative program.

The structure of physical files, stored on magnetic discs, allows direct access to any physical record. However, user's programs do not communicate directly with the physical data base and physical records. They can only gain access to suitably defined logical records. The logical record often consists of the same data as the corresponding physical record, but can also be composed of data from several physical records belonging to different physical files. The mapping of the physical files onto the logical files, and vice versa, is performed by special data bank service routines--the mapping subroutines (Figure 1).

The mapping subroutines use the file description and address arrays in order to find the position of any given physical record. For a given logical record structure, if the relations between the logical and physical records are known, a backing storage address for every field of logical record can be found. The mapping subroutines can be called by any user's program and used for reading or writing any given record.

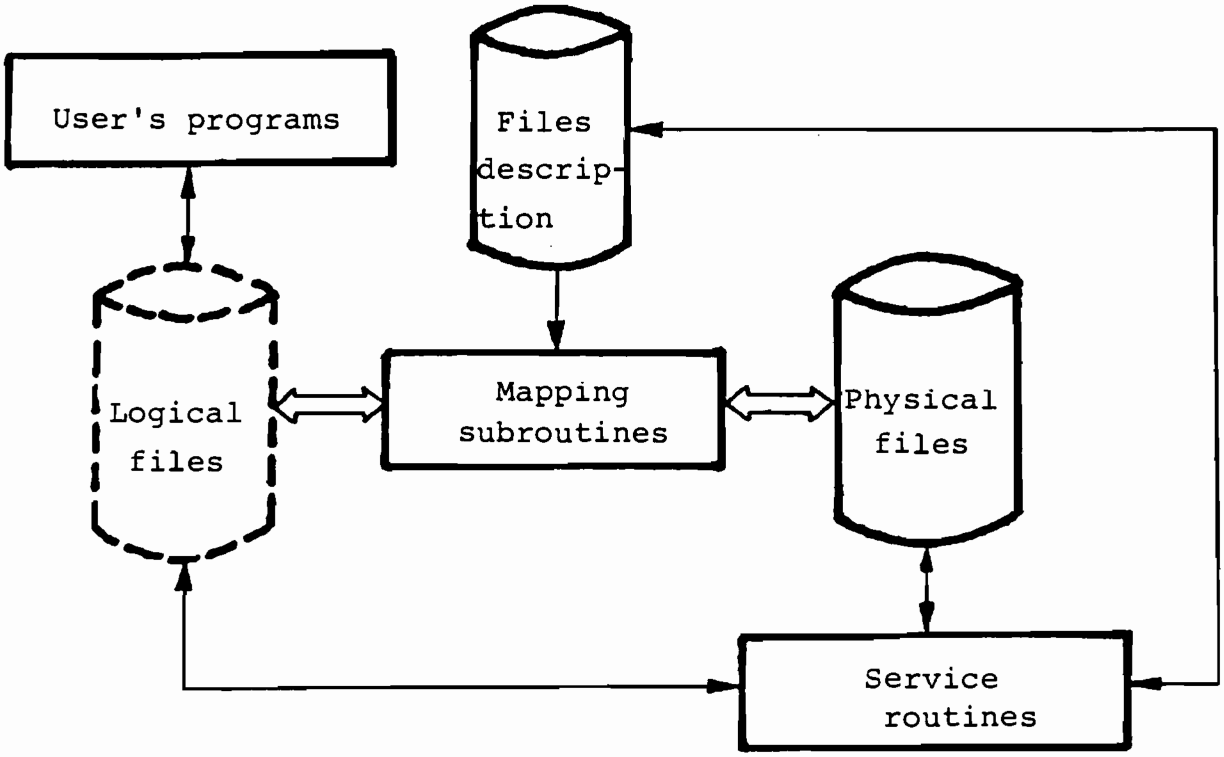

Figure 1. The organization of the data bank.

The mapping subroutines are also used by the input and update program and the print program. The former allows new records to be inserted into the data bank and obsolete information to be updated. In the input mode, the program reads data in the source document, checks them, and inserts them in the physical data base. The print program prints out or writes computer media selected records or whole subfiles. The two remaining service programs are used for data base administration. The file description program can initiate new files and subfiles. The administrative program is used for copying, regenerating, and reorganizing the files, and the magnetic tape copy of the data base protects against a disc files failure.

The data processing subroutines that are used by more than one user's program, e.g., the subroutine aggregating soil data into various geographical configurations, can also be included in the data bank software.

CONCLUSION

The climatic file is already stored on computer and soil data are being loaded. The service routines are ready and users are now able to utilize this part of the data bank.

The data for the files come from the Central Research and Design Bureau for Irrigation, Drainage, and Water Supply for Agriculture (BIPROMEL). Some of the data are in the form of documents, the rest are in the form of magnetic tapes. The data bank was designed such that new files and subfiles can be added, if necessary.

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SUMMARY OF DISCUSSIONS

SUMMARY OF DISCUSSIONS

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J. Kacprzyk

GENERAL PROBLEMS

The pilot character of both the Notec and Silistra regional development programs was stressed. Thus, the choice of activities to be modeled and the modeling techniques to be used should not be governed solely by conditions prevalent in a particular region, but should provide a sound basis for other applications in future regional programs. From this perspective, even if the approach applied does not give expected results or proves to be, for example, inefficient, the experience should be considered useful.

The discussion focused on the system of regional development models. Evidently, such a system cannot be purely general in concept, but should be capable of addressing issues of importance in the region analyzed. However, it should be easily expandable.

The idea of building a system of regional models around an aggregated macrolevel core model was accepted to be rational for most regions. Such a core model should consider some basic regional factors (e.g., capital, labor, land, etc.) in aggregated form, its results would be used by other, more detailed lower-level models. In turn, the results obtained by lower-level models should be fed into the core model. In building a core model, a careful analysis of, for example, the objective (utility) function, rural and urban components, basic cause-effect relations, etc. is crucial. The equity or efficiency criterion should also be such that it can easily be defined. Since the nature of decision making at the level of the core model usually involves some bargaining, conflict resolution, etc., an appropriate tool should be chosen and a gaming approach would seem to be very promising.

The next important question closely associated with the above discussion is the problem of model linkage. How should specific models be connected to form a system? From the methodological point of view, the idea of linkage is very rational but is not easy to achieve. How should such models be constructed and how should they be operated? The models have different data requirements, involve different planning horizons, etc. Moreover, modelers usually determine the general assumptions for their model within a rather narrow framework, without considering the relations with other models of the system. In this case, the assumptions on which the model is based can often change considerably in the course of development and, as a result, much additional work may be required to link the models.

MODELS OF AGRICULTURE

The discussion of agricultural problems opened with the important question of time horizon. The agriculture models presented were more or less explicitly concerned with regional specialization. The choice of specialization is difficult and requires careful analysis over longer planning horizons, say, 20 years. However, agriculture is an economic activity facing relatively rapid structural changes (e.g., with respect to costs, benefits, labor), which indicates that a shorter planning horizon may be appropriate. Moreover, because of the complexity of regional agriculture, the problems tend to be multidimensional; linear programming is therefore the only practical method of solution.

The discussion then turned to the choice of a static or dynamic model. This is also not a trivial problem. Theoretically, a long-term plan or policy may be devised using a static model period by period (e.g., year by year), but a long-term plan obtained by means of a dynamic model is qualitatively different. Dynamic models, although advantageous from this point of view, are however, more difficult to operate. Thus, there is no universal answer to the type of model that should be used. In many cases, it may be more appropriate to devise a simpler, more aggregated dynamic model. Moreover, different models should be used for different purposes and final proof of the model's validity should always be its usefulness and implementability.

The question of whether to use a small or large model was raised. In many cases, the use of a small model may give many valuable results that can be easily interpreted. However, any attempt to reflect the complex structure of the modeled system with sufficient accuracy (e.g., a complicated policy-making structure, administrative divisions, property structure, production structure) inevitably leads to the need for a large-scale model, which usually takes the form of a large-scale linear programming model. This is unwieldy and difficult to operate. Many modelers are therefore skeptical about the use of large models: such an attitude originates to a considerable extent from their experience of using unsophisticated, conventional mathematical programming systems for this purpose. These systems may work quite well for smaller problems, but to solve a large-scale problem efficiently, use of an advanced system (e.g., SESAME DATAMAT) is essential. Moreover, the solution must be obtained in an interactive mode. Only in this way can large models be used effectively and can valuable results, understandable not only to modelers but also to decision makers, be obtained in disaggregated form. Moreover, use of the systems mentioned above makes sensitivity analysis or parametric solution, the importance of which was emphasized, possible.

As for some specific questions, the use of some natural, physical units (e.g., corn equivalent units) for expressing the volume of agricultural production was appreciated. In most countries agriculture is subsidized to some degree, prices are therefore not always economically justified. This may lead to some difficulties in the modeling activities, giving an inaccurate representation of reality. Hence, the use of a physical measure of agricultural output may often be more appropriate.

MODELS OF WATER SYSTEMS AND THE ENVIRONMENT

The first part of the discussion covered some general topics on the philosophy of modeling water resource management. Water demand cannot usually be considered only in terms of the amount needed, since the time dimension (i.e., when the water is needed) is also extremely important. Dynamic models provide a more adequate description of the problem and more useful results. Uncertainty about, for example, future changes in technology, available resources, demand, should also be properly reflected in the model (e.g., by the introduction of some random elements).

In the study of a water system, the proper price of water is crucial. If it is too low, this could lead to irrational water use, but if it is too high, the transition to the use of a more advanced and efficient technology requiring greater water use could be delayed or even prevented. Cost sharing in a water project among users also plays an important role with regard to the successful construction of a water system and its future use. However, there is no generally accepted rule. The principle predominantly used (i.e., that costs are shared proportional to the planned future use) does not always give the best possible results. Thus, the problem should be carefully analyzed.

The operational control aspects of water systems were then discussed. The construction of an efficient water system may often be practically impossible without specifying some rational strategy of water withdrawal. Hence, an operational control model of a water withdrawal system may be needed. In such a system there are usually some elements of uncertainty that play a crucial role. Thus, to model them adequately, an often complicated stochastic formulation is usually obtained.

However, the collection of reliable data and the solution of the model may be difficult. Moreover, the results can rely heavily on the performance index (i.e., the strategy of water withdrawal). In turn, since, for example, agricultural production activities depend on water withdrawal capacities and the planned scale of water withdrawal depends on the planned agricultural activities, an important feedback emerges. Thus, analysis and solution become very difficult, but the crucial importance of the problem to the development of the regional economy fully justifies such a modeling activity.

MODELS OF INDUSTRY, TRANSPORTATION, AND ENERGY

The discussion opened with some general remarks on possible approaches to industrial development modeling. Usually two approaches are possible. The first consists in using the model to evaluate all the possibilities for development, from which the best one is selected. In the second approach, initially many possible alternatives of industrial development for a region are analyzed by the model. After having run the model, numerous variants are discarded, but many still remain. These are presented for further analysis to the appropriate authorities, who provide feedback in the form of additional comments, proposals, etc. The alternatives are rerun on the model in the light of the feedback received and the process is repeated until the optimal variant is chosen. In most cases, the second procedure is considerably better. However, long delays in the analysis may result from communication with the authorities.

The spatial aspect of industrial development is important and should be reflected in the problem formulation. It should also be considered in the choice of appropriate tools. In general, the spatially oriented and nonspatially oriented elements may have a different time-variability. For instance, the structure of an industry may be changed in, say, 5 years, but the change of transportation network in a region would probably require approximately 20-30 years. Thus, to facilitate the solution, the low-varying elements (e.g., transportation) could be fixed first and afterwards the models concerning aspects of higher variability (e.g., location of activities) could be solved.

The problem with low variability could then be solved for a fixed (e.g., obtained in a previous iteration) pattern of highly varying aspects. Hence, the model would in fact be split into submodels, links between which would be organized according to the time variability of the issues analyzed. Such an approach would probably offer the best possible results. Moreover, it would be the most computationally efficient.

With respect to the time horizon, it was mentioned that, generally speaking, long-range models behave rather better for the case of industry than for agriculture. However, this does not mean that the model builder should not be aware of common problems resulting from the use of long time horizons.

With regard to energy systems, one serious problem is to determine when some new energy resource should be introduced. The time factor is sensitive to fuel prices, material costs, and energy resource structure and availability, etc. Thus, the model depends heavily on accurate forecasting, which is a very difficult task, especially given that unpredictable disturbances (e.g., oil shocks) may qualitatively change the situation.

MODELS OF LABOR, SERVICES, AND SETTLEMENT STRUCTURE

Labor is a very important production factor playing a key role in the economic activities of both the Silistra and Notec regions and its importance was fully acknowledged.

The discussion focused mainly on migration, which was considered to be the most important issue of labor-oriented problems. First, it was stated that the point of departure for any serious analysis should be a thorough investigation of the factors underlying migration. The most common assumption that migration is a function of wage differences is only a rough approximation of reality. This is particularly true in the case of agricultural regions, where the rural population tend to hold traditional attitudes and are generally resistant to change. The results from the labor models should form the basis for policy making or at least provide some hints in this respect. However, policy should not be understood in a narrow sense, as, for example, the location of industry, but should incorporate some auxiliary elements (such as allocation of investment for infrastructure), which often play a decisive role in stimulating migration. In policy evaluation, the problem should also be interpreted in a broad context as multidimensional, involving many issues such as agglomeration and scale effects.

The policies are generally of two types: equity or efficiency oriented (or perhaps a compromise between both). Policy choice, however, depends mainly on political factors. Certain migration trends may to some extent influence policy, in which case some feedback emerges, which may complicate the analysis.

The first step in an analysis of the transportation system should be a thorough study of the nature of the regional transportation problem (i.e., a commuting or freight transportation problem).

In most cases, transportation systems must be planned for longer time horizons, hence some difficulties will occur in obtaining reliable predictions of future tendencies.

One of the more specific problems discussed related to potential versus actual transportation costs. In spite of the intrinsic difficulty in obtaining the realized transportation costs, the importance of their availability was stressed.

DATA PROBLEMS AND COMPUTER USE

Since it is intended that the models for both the Silistra and Noteç regions should be run as a more or less integrated system, the problem of establishing a relevant and efficient data base was considered to be of particular importance.

The data should be sufficiently detailed to be of use to each model in the system. The fact that the data base does not always provide the models with data in the required format was not considered to be a drawback, as long as some of the user's subroutines are able to process the data relatively easily and quickly. Similarly, a moderate overlap of some data in the data base should not be viewed as inefficient, since it is convenient for the user.

The necessity of devising a clear organizational structure (e.g., who is responsible for which changes) to operate the data base efficiently is crucial.

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