DYNAMIC LINEAR PROGRAMMING MODELS OF ENERGY, RESOURCE, AND ECONOMIC-DEVELOPMENT SYSTEMS

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FOREWORD

Much of the work of IIASA's Energy Systems Program has been devoted to conceiving, building, and using mathematical models in order to arrive at a consistent and globally comprehensive synthesis of separate semiquantitative insights. Such insights relate, for example, to the resources situation and to economic problems, as well as to technology.

This kind of modeling turns out to be complex. For one thing, most of the logical structures are large mathematical models; for another, they are complex in both their meanings and their implications for wider contexts. For this reason, the International Institute for Applied Systems Analysis (IIASA) has evolved a standard set of mathematical models (MEDEE, MESSAGE, IMPACT, and, to an extent, MACRO), which have been described in many IIASA publications, and especially thoroughly in the book *Energy in a Finite World: A Global Systems Analysis* (Ballinger, Cambridge, Massachusetts, USA, 1981).

However, in addition to the development of the standard model set, other efforts have aimed at investigating and understanding the behavior of the energy system – and these have been important in their own right. In particular, Anatoli Propoi and Igor Zimin have developed an approach that unifies aspects of energy supply, energy resources, and the development of an economy. The purpose of this report is to set forth some details of this line of investigation.

WOLF HÄFELE
Leader
Energy Systems Program

CONTENTS

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SUMMARY

This report develops a unified dynamic linear programming approach to studying long-range development alternatives in the energy sector. With the demand for energy and the supply of nonenergy resources needed to develop the energy supply system given exogenously, the report first seeks the optimal mix, phased over time, of different energy technologies. Next, it considers the problem of finding, for primary energy resources, the optimal mix over time of different exploration and extraction technologies. The third part of the report uses an optimization version of a dynamic input–output model to study the macroeconomic impacts of the energy sector. Finally, the report discusses the interactions among these models, presents a general dynamic linear programming framework, and takes up some related methodological issues.

INTRODUCTION

This report is an attempt to review and extend methodological research into the development of complex systems. One very typical, and probably the most urgent, example of this sort of problem is the analysis and planning of the long-term development of energy systems. During the last decade, interest in energy problems has considerably increased all over the world and we have witnessed significant progress in the field (A.A. Makarov and Melentjev 1973; Häfele and Manne 1974; Häfele 1974; Hudson and Jorgenson 1975; Häfele and Sassin 1976; Belyaev et al. 1976; Häfele and A.A. Makarov 1977; Häfele et al. 1977; A.A. Makarov 1977; Kononov 1977; Behling et al. 1977; Hoffman and Jorgenson 1977). However, most of this work has been concerned with the detailed implementation of different energy models. As regards methodological mathematical analysis of the problem, we must of course expect a slight time lag at first, but preliminary attempts have already been made (see, for example, Alta Conference 1975; Tomlin 1976).

Meanwhile, when we analyze the outputs of various energy models implemented in different ways, many methodological questions arise: for example, how should models of energy supply, resources, and the economy be linked into an overall (national) system; what is the most appropriate form of world ("global") energy model – a game-theoretical, optimization, or simulation model; how does our uncertainty concerning future input data influence our degree of certainty about the correctness of present decisions; etc. These questions do not only relate to energy models but are also of concern for any problems involving the long-term development of a complex system (Aganbegyan et al. 1974; Aganbegyan and Valtukh 1975); one example is the analysis of the long-term interaction between manpower and economic development (Propoi 1978).

This report tries to answer some of the questions outlined above. The first three sections describe basic dynamic optimization models – of energy supply, resources, and economic development – all formally presented in a unified dynamic programming framework (Propoi 1973, 1976; Ho 1979). Section 1 considers models of Energy Supply Systems (ESS); the demand for energy and the supply of nonenergy resources needed to develop the ESS are given exogenously, and we seek the optimal mix, phased over a period of time, of different energy technologies. Section 2 examines resource models. Here the problem is to find, for primary energy resources, the optimal mix over time of different extraction and exploration technologies. Section 3 describes dynamic linear programming models of an economy; these are basically optimization versions of dynamic input–output models.

In describing these models, we have tried to concentrate on the most typical features of each, omitting the various details of implementation in order to obtain three basic formalized models which could be useful for subsequent mathematical analysis. The internal structure of the report follows on directly from this: in each of the first three sections we start by considering a basic model and then examine some related real models which can be viewed as modified versions of the basic model.

Sections 1–3 consider each model independently on a national (or regional) level. Methods for linking different models (for example, energy–economy or resources–energy) are discussed in Section 4, while Section 5 suggests a canonical form for the dynamic linear programming problem to which all the models can be reduced. The report closes with a recapitulation of the main conclusions and suggestions for further research.

This report is primarily a review, intended to give the various models a unified presentation, thus providing a basis for further development of methods for the solution and analysis of such models.

1 ENERGY SUPPLY MODELS

We begin with models of Energy Supply Systems (ESS) because ESS play central roles in any study of energy resources. The main purpose of the ESS models is to study major energy options over the next 25–50 years and longer, thus determining the optimal feasible transition from the mix of technologies for energy production currently used (basically fossil fuels), to a more progressive and, in some sense, optimal future mixture of technologies (nuclear, coal, solar, etc.) for a given region (or country). When considering

ESS models, we will basically follow the Häfele–Manne model (Häfele and Manne 1974), and then discuss different versions and modifications of the models.

In formulating Dynamic Linear Programming (DLP) problems, it is useful to identify

(i) the *state equations* of the systems with the *state* and *control* variables clearly separated
(ii) the *constraints* imposed on these variables
(iii) the *planning period* T – the *number* of periods during which the system is considered and the *length* of each period
(iv) the *performance index* (or objective function) gives some quantitative measure of the performance of a program

We will now consider these four stages separately as applied to the ESS models.

1.1 Basic Model

1.1.1 State Equations

The ESS model is broken down into two subsystems: energy production and conversion, and resource consumption. Hence, two sets of state equations are needed.

Energy production and conversion subsystem. The subsystem consists of a certain number of technologies for energy production (fossil, nuclear, solar, etc.). The state of the subsystem during each period t is described by the values of capacities during that period t for all energy-production technologies.

Let

$y_i(t)$ be the capacity of the ith energy-production technology ($i = 1, 2, \ldots, n$) during period t;
n be the total number of different technologies for energy production to be considered in the model; and
$v_i(t)$ be the increase in the capacity of the ith technology over period t ($i = 1, 2, \ldots, n$).

It is assumed that the lifetime of each unit of productive capacity, for example each power plant, is limited: this limited lifetime, characteristic of facilities based on technology i, will be denoted by τ_i.

Thus, the state equations which describe the development of the energy production and conversion subsystem will be as follows

$$y_i(t+1) = y_i(t) + v_i(t) - v_i(t - \tau_i) \qquad (i = 1, 2, \ldots, n;\ t = 0, 1, \ldots, T-1) \tag{1.1}$$

with the given initial conditions

$$y_i(0) = y_i^0 \qquad (i = 1, 2, \ldots, n) \tag{1.2}$$

The increase in the capacity of the ith technology, $v_i(t)$, during the period preceding the time horizon considered ($t < 0$) is also assumed to be known

$$v_i(-\tau_i) = v_i^0(-\tau_i), \ldots, v_i(-1) = v_i^0(-1) \qquad (i = 1, 2, \ldots, n) \tag{1.3}$$

where $\{v_i^0(-\tau_i), \ldots, v_i^0(-1)\}$ are given numbers.

Equations (1.1) and (1.2) can be rewritten in vector form

$$y(t+1) = y(t) + v(t) - v(t - \boldsymbol{\tau}) \tag{1.1a}$$

$$y(0) = y^0 \tag{1.2a}$$

Here

$y(t) = \{y_i(t)\}$ is a *state vector* of the subsystem in period t, describing the state of the energy production and conversion subsystem $i (i = 1, 2, \ldots, n)$ in this period;
$v(t) = \{v_i(t)\}$ is a *control vector*, describing control actions affecting subsystem $i (i = 1, 2, \ldots, n)$ in period t; and
$\boldsymbol{\tau} = \{\tau_i\}$ $(i = 1, 2, \ldots, n)$

Resource consumption subsystem. State equations of this subsystem describe the dynamics of cumulative amounts of extracted primary energy resources.

Let

$z_j(t)$ be the cumulative amount of the jth resource extracted by the beginning of period (sometimes year) t, where $(j = 1, 2, \ldots, m)$;
m be the total number of different primary resources under consideration; and
$q_{ji}(t)$ be the fraction of the jth resource (primary energy input) required for loading the capacity of the ith energy production technology (secondary energy output) in period t $(i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, m)$; $q_{ji}(t)$ represents the conversion process $j \to i$.

Generally, some capacity will not always be completely loaded; therefore we introduce a new variable $u_i(t)$ which represents the degree of utilization of productive capacity based on technology i $(i = 1, 2, \ldots, n)$ in period t. It is evident that

$$u_i(t) \leqslant y_i(t) \qquad (i = 1, 2, \ldots, n) \tag{1.4}$$

or, in vectorial form

$$u(t) \leqslant y(t) \tag{1.4a}$$

If we assume that the primary energy resource extraction during period t is proportional to the degree of utilization of energy-production capacity in this period, we can write the state equations in the form

$$z_j(t+1) = z_j(t) + \sum_{i=1}^{n} q_{ji}(t)\, u_i(t) \tag{1.5}$$

with initial conditions

$$z_j(0) = z_j^0 \qquad (j = 1, 2, \ldots, m) \tag{1.6}$$

or, in matrix form

$$z(t+1) = z(t) + Q(t)u(t) \tag{1.5a}$$

$$z(0) = z^0 \tag{1.6a}$$

Here $z(t)$ is a state vector and $u(t)$ is a control vector. The subsystems (1.1) and (1.5) are linked by means of the inequalities (1.4).

If the conversion process $j \to i$ is denoted by the matrix $\tilde{Q}(t) = \{\tilde{q}_{ij}(t)\}$, then eqn. (1.5a) should be rewritten as

$$z(t+1) = z(t) + \tilde{Q}^T(t)u(t) \tag{1.5b}$$

where $\tilde{Q}^T$ denotes the transpose of the matrix $\tilde{Q}$.

In some cases it is necessary to introduce variables representing stocks of the primary resources extracted (inventory resources). Let $\tilde{z}_j(t)$ be such a variable for the jth resource and $w_j(t)$ the amount of this resource extracted annually. The state equation for the inventory subsystem will then be as follows

$$\tilde{z}(t+1) = \tilde{z}(t) + w(t) - Q(t)u(t)$$

In the above case, $\tilde{z}(t) = 0$ for all t, and $w(t) = Q(t)u(t)$. This is a reasonable assumption because, in the long term, one can neglect the accumulation of stocks of resources.

It should be noted that the real equations of the resource-consumption subsystem are more complex [see Häfele and Manne (1974) and the discussion in Section 1.2].

1.1.2 Constraints

The state equations (1.1) and (1.5) specify the dynamic constraints on variables, but we also have a number of static constraints on variables for each period t.

Nonnegativity. It is evident that no variables introduced into the state equations (1.1) and (1.5) can be negative

$$v(t) \geqslant 0, \; y(t) \geqslant 0, \; u(t) \geqslant 0, \; z(t) \geqslant 0 \tag{1.7}$$

Availability. To begin with, upper limits are imposed on the annual construction rates

$$v_i(t) \leqslant \bar{v}_i(t) \qquad (i = 1, 2, \ldots, n) \tag{1.8}$$

where $\bar{v}_i(t)$ are given numbers. In a more general form, these constraints may be written as

$$F(t)v(t) \leqslant f(t) \tag{1.9}$$

where $f(t)$ is the vector of nonenergy inputs which are needed for the energy production subsystem. The matrix $\mathbf{F}(t)$ denotes the amounts of these resources required for the construction of one unit of capacity using the ith technology in period t. Limits on the rates of introduction of new technology can also be written in the form of eqns. (1.8) or (1.9). More general cases, where the time lags between investment decisions and actual increases in capacity are taken into account, are considered in Section 3.1. In such a situation we can directly link the ESS model with the economic model described in Section 3.

The constraints on the availability of the primary energy resources may be given in the form

$$z(t) \leqslant \bar{z}(t) \tag{1.10}$$

where $\bar{z}(t)$ is the vector of all available energy resources (resources in place) in period t, and $z(t)$ is calculated from eqn. (1.5).

The constraints on the availability of the secondary energy-production capacities are given by inequality (1.4).

Demand. The intermediate and final demands for energy are assumed to be given for all planning periods considered. Hence the demand constraints can be written as

$$\sum_{i=1}^{n} d_{ki}(t) u_i(t) \geqslant d_k(t) \tag{1.11}$$

or

$$\mathbf{D}(t) u(t) \geqslant d(t) \tag{1.11a}$$

where

$d(t) = \{d_k(t)\}$ is the given vector for all t ($t = 0, 1, \ldots, T-1$) of energy demand, both intermediate and final (that is, including both the electrical and nonelectrical components of final demand); and

$\mathbf{D}(t) = \{d_{ki}(t)\}$ is the matrix with components $d_{ki}(t)$, defining either intermediate consumption of secondary energy k per unit of total secondary-energy production, or the conversion efficiency when producing one unit of secondary energy k from energy originally produced using technology i.

1.1.3 Planning Period

The planning period is broken down into T steps, where T is given exogeneously. Each step is of a certain length (e.g., one, three, or five years). Häfele and Manne (1974) chose a planning period of 75 years and each step corresponded to three years, so in that case $T = 25$. Since information on the coefficients of the model becomes more inaccurate with an increasing number of steps it is useful to consider steps which are not all of equal length. For example, Marcuse et al. (1976) decided on a planning period of 100 years, divided into ten steps of varying length (the first five periods of six years each, the next three periods of ten years each, and the last two periods of twenty years each).

1.1.4 Objective Function

The choice of the objective function is one of the more important stages in model building. Full discussion of the economic aspects of ESS modeling objectives is beyond the scope of this report. Here we would like specifically to emphasize only two points: first, in many cases the objective functions can be expressed as linear functions of state and control variables, thus making it possible to use Linear Programming (LP) techniques. Second, the optimization procedure should not be viewed as a final part of the planning process (yielding a "unique" optimal solution), but only as a tool for analyzing the connection between policy alternatives and system performance. Thus in practical applications a policy analysis with various different objective functions is required. For our purpose, however, it is sufficient to limit ourselves to some typical examples of objectives.

Let us consider the objective function which expresses the total capital costs, discounted over time, for both the construction and the operation of units of productive capacity based on technology i

$$J = \sum_{t=0}^{T-1} \beta(t) \left[\sum_{i=1}^{n} c_i^u(t) u_i(t) + \sum_{i=1}^{n} c_i^v(t) v_i(t) \right] \tag{1.12}$$

where

$c_i^u(t)$ are the operating and maintenance costs for units of productive capacity based on technology i in period t;

$c_i^v(t)$ are the investment costs for units of productive capacity based on technology i in period t; and

$\beta(t)$ is the discount rate.

We can express this in vector form as

$$J = \sum_{t=0}^{T-1} \beta(t)\ [(c^u(t), u(t)\ \ +(c^v(t), v(t))] \tag{1.12a}$$

Note that the scalar product $(c^u(t), u(t))$ expresses not only direct operating and maintenance costs during step t but may also indirectly include the cost of primary resources consumed during this step. In a more explicit way, this cost can be written as $(c^u(t), Q(t)u(t))$, where $c^u(t)$ should increase with the cumulative amount of resources consumed. This leads to a nonlinear objective function. A reasonable approximation in this case is a stepwise function for $c^u(t)$. Thus, $c^u(t)$ in eqn. (1.12) can be a stepwise function, with values for each step which depend on the values of cumulative extraction of resources $z(t)$ [or on the difference $\bar{z}(t) - z(t)$].

1.1.5 Statement of the Problem

To begin with, we introduce a number of definitions. A sequence of vectors

$$v = \{v(0), \ldots, v(T-1)\},\ u = \{u(0), \ldots, u(T-1)\}$$

are *controls* of the system; a sequence of vectors

$$y = \{y(0), \ldots, y(T)\}$$

determined by eqns. (1.1) and (1.2) defines a *(capacity) trajectory* of the system; and a sequence of vectors

$$z = \{z(0), \ldots, z(T)\}$$

determined by eqns. (1.5) and (1.6) is a *(cumulative resources) trajectory* of the system.

Sequences of control and state vectors $\{v,u,y,z\}$ which satisfy all the constraints of the problem [for example eqns. (1.1)–(1.11) in this case] are called *feasible*. Having chosen *feasible controls* v and u one can obtain, by using eqns. (1.1)–(1.3), (1.5), and (1.6), *feasible state trajectories* y and z. Thus

$$J = J(y(0), z(0), v, u) = J(v,u)$$

A feasible control $\{v^*, u^*\}$ which minimizes the objective function described in eqn. (1.12) or the equation above will be called an *optimal control*.

We can now formulate the optimization problem for the energy supply system.

Problem 1.1. Given the state equations

$$y(t+1) = y(t) + v(t) - v(t-\tau) \quad (1.1a)$$

$$z(t+1) = z(t) + \mathbf{Q}(t)u(t) \quad (1.5a)$$

with initial conditions

$$y(0) = y^0 \quad (1.2a)$$

$$z(0) = z^0 \quad (1.6a)$$

and known parameters

$$v(-\tau) = v^0(-\tau), \ldots, v(-1) = v^0(-1) \quad (1.3)$$

find controls $\{v,u\}$, and corresponding trajectories $\{y,z\}$, which satisfy the constraints $v(t) \geqslant 0; u(t) \geqslant 0; y(t) \geqslant 0;$ and $z(t) \geqslant 0$

$$\mathbf{F}(t)v(t) \leqslant f(t) \quad (1.9)$$

$$u(t) \leqslant y(t) \quad (1.4a)$$

$$z(t) \leqslant \bar{z}(t) \quad (1.10)$$

$$\mathbf{D}(t)u(t) \geqslant d(t) \quad (1.11a)$$

and minimize the objective function

$$J(v,u) = \sum_{t=0}^{T-1} \beta(t)[(c^u(t), u(t)) + (c^v(t), v(t))] \quad (1.12a)$$

Verbally, the policy analysis in the energy supply system model, which is formalized as Problem 1.1, can be stated as follows.

At the beginning of the planning period, energy production capacities broken down into several "homogeneous" technologies (fossil, nuclear, solar, etc.) are known [eqn. (1.2a)]. There are various possible options for developing these initial energy production capacities in the system during the period considered. These options are subject to constraints on the availability of primary energy resources [eqns. (1.5a), (1.6a), (1.10)], and constraints on the availability of nonenergy resources [eqn. (1.9)] required for the construction of new units of energy production capacity. Each of these options has its own advantages and disadvantages, and the problem consists of finding an optimal mix of these options, which, over a given period,

- meets the given demand for secondary energy [eqn. (1.11a)]
- satisfies the constraints on the availability of primary energy resources and nonenergy resources [eqns. (1.9), (1.10)]
- minimizes the total costs (for both construction and operation) [eqn. (1.12a)]

There are two important vector parameters in the model, both of which are given exogenously: the amount of nonenergy resources $f(t)$ available during the planning period, and the demand $d(t)$ for secondary energy. These values mainly affect the interaction of the energy supply system with the economic development system (see Section 4).

1.2 Discussion

The version of an energy supply system (ESS) model considered above is somewhat simplified, but nevertheless it reveals the major features of real systems. The actual implementation of the various ESS models is naturally more detailed and complicated; it depends to a great extent on the general approach selected for the overall ESS model, and on the assumptions about energy and the economy used for building its separate submodels. We will not, however, pay too much attention to the physical peculiarities of different ESS models but will rather try to emphasize the methodological characteristics of the various models and their relationships to Problem 1.1. It should be noted that some of the notation used below is different to that used in the original versions of the models to facilitate analysis and comparison.

1.2.1 Häfele–Manne Model

To illustrate the model described above, we will consider the Häfele–Manne model (Häfele and Manne 1974; Suzuki 1975) in rather more detail. In the model a 75-year planning horizon is subdivided into 25 intervals, each three years in length. Total energy production capacity is divided into two groups: new technologies, for which additional capacity is being constructed during the planning horizon and some "old" technologies. We denote the vectors of new and old capacities by $y(t) = \{y_i(t)\}$ $(i = 1, 2, \ldots, n)$ and $y_0(t) = \{y_{0i}(t)\}$ $(i = 1, 2, \ldots, n_0)$, respectively. The vector $y(t)$ refers to capacity installed or added to during the planning horizon and based on such technologies as coal (COAL), petroleum, gas, etc. (PETG), the light water reactor (LWR), the fast breeder reactor (FBR), electrolytic production of hydrogen (ELHY), etc.; the exogenous vector $y_0(t)$ refers to the amount of capacity based on fossil fuels (coal, petroleum, gas, etc.) already available at the beginning of the planning horizon. It is assumed that all units of new capacity are retired after 30 years of service, and that they are operated at a constant rate throughout the 30-year period. Thus, the state equations for the energy production subsystem can be written in the form of eqn. (1.1), where i = COAL, PETG, LWR, FBR, ELHY, etc; $t = 0, 1, \ldots, 24$; $\tau_i = 10$ for all i; and $v_i(t)$ is the increase in the capacity of the ith technology in the three years included in time period t [by assumption $v_i(t) = 3\tilde{v}_i(\tilde{t})$, where $\tilde{v}_i(\tilde{t})$ is the annual increase in year $\tilde{t} = 0, 3, 6, \ldots$].

Häfele and Manne (1974) assume a total loading of capacities

$$u_i(t) = y_i(t) \tag{1.13}$$

In this case, the state equations for the energy consumption subsystem have the form

$$z_j(t+1) = z_j(t) + a_j[y_j(t) + y_{0j}(t)] \qquad (j = \mathrm{COAL, PETG}) \tag{1.14}$$

for coal and for petroleum and gas; in other words, the cumulative consumption $z_j(t+1)$ of coal or petroleum and gas by the beginning of period $t+1$ is equal to the cumulative consumption $z_j(t)$ of this resource by the beginning of period t plus the consumption by the existing production capacity $y_j(t) + y_{0j}(t)$ during period t.

For natural uranium (NU) we have the equation

$$\begin{aligned} z_{\mathrm{NU}}(t+1) = z_{\mathrm{NU}}(t) &+ [a^1_{\mathrm{NU}} y_{\mathrm{LWR}}(t) - a^2_{\mathrm{NU}} y_{\mathrm{NUHC}}(t)] \\ &+ b^1_{\mathrm{NU}} [v_{\mathrm{LWR}}(t+1) - v_{\mathrm{LWR}}(t-10)] \\ &+ b^2_{\mathrm{NU}} [v_{\mathrm{HTR}}(t+1) - v_{\mathrm{HTR}}(t-10)] \end{aligned} \tag{1.15a}$$

Examining the terms on the right-hand side of eqn. (1.15a), we see first that natural uranium is required in period t for the current refueling of existing light water reactor (LWR) capacity; we note also that part of the total requirement can be met by using *high cost* natural uranium (NUHC), which therefore appears as a negative term. Additional amounts of natural uranium are required for setting up new LWR and HTR (high temperature reactor) capacity three years later (in the next period, $t+1$); because the spent fuel is reprocessed, uranium is effectively released when the LWR and HTR facilities are retired at the end of their service lifetime of ten three-year periods [this accounts for the negative terms $v_{\mathrm{LWR}}(t-10)$ and $v_{\mathrm{HTR}}(t-10)$, respectively].

For natural uranium it is appropriate to speak of cumulative resource *consumption*, but for man-made plutonium we must consider cumulative resource *production*, which alters the sense of the state equation. For plutonium the state equation includes the following elements. The cumulative sum $[z_{\mathrm{PLUT}}(t+1)]$ of plutonium produced by the beginning of period $t+1$ is equal to the cumulative sum $[z_{\mathrm{PLUT}}(t)]$ of the plutonium produced by the beginning of period t, plus production $[y_{\mathrm{LWR}}(t)]$ from LWRs during period t, plus the gain $[y_{\mathrm{FBPL}}(t)]$ from fast breeder reactors (FBRs) during period t, plus amounts

$[v_{FBR}(t-10)]$ "reclaimed" from FBRs retired at the end of their 30-year lifespan, minus consumption $[v_{FBR}(t)]$ for setting up new FBR capacity during period t. Stating this mathematically

$$z_{PLUT}(t+1) = z_{PLUT}(t) + a^1_{PLUT} y_{LWR}(t) + a^2_{PLUT} y_{FBLP}(t) + b_{PLUT}[v_{FBR}(t-10) - v_{FBR}(t)] \tag{1.15b}$$

It should be emphasized that these equations are given here only for illustration: complete explanation of the equations would require a description of the nuclear cycle, which would fall outside the scope of this report [for further details see, for example, Häfele and Manne (1974)]. Here we will merely state that in matrix form these equations may be written as

$$\mathbf{Z}(t+1) = \mathbf{Z}(t) + \mathbf{A}_0 y_0(t) + \mathbf{A}_1 y(t) + \mathbf{A}_2 y(t-1) + \mathbf{B}_1 v(t) + \mathbf{B}_2 v(t+1) - \mathbf{B}_3 v(t-\tau) \tag{1.15c}$$

and over the long term they can in fact be reduced to eqn. (1.5).

Demand constraints in the model (Häfele and Manne 1974) are written in the form

$$\mathbf{D}y(t) + \mathbf{D}_0 y_0(t) \geqslant d(t) \tag{1.16}$$

for final demand and

$$\mathbf{D}_1 y(t) + \mathbf{D}_2[v(t+1) - v(t-\tau)] \geqslant 0 \tag{1.17}$$

for intermediate demand. Only two types of demand are considered, namely, for electrical and nonelectrical energy. Häfele and Manne give the objective function in a linear form similar to eqn. (1.12) for their model societies 1 and 2, and in a nonlinear form

$$J = \sum_{t=0}^{T-1} [a_1(t) d_1^{b_1}(t) + a_2(t) d_2^{b_2}(t)] \tag{1.18}$$

for their model society 3. In the last case it is assumed that demands [$d_1(t)$ for electrical and $d_2(t)$ for nonelectrical energy] are responsive to prices and hence are endogenously determined in the model.

1.2.2 ETA Model

The model for Energy Technology Assessment (ETA) is closely related to the energy supply system model considered above. The model was developed by Manne (1976, 1977) and represents a further development of the nonlinear version (model society 3) of the Häfele–Manne model. ETA is a medium-sized, nonlinear programming model (with linear constraints). It contains, for a 15-stage planning horizon (each stage 5 years long), a total of 300 rows, 700 columns, and 2500 nonzero matrix elements. The model was solved using MINOS – a general-purpose production code developed by Murtagh and Saunders (1978) for solving large-scale nonlinear programs with linear constraints; the code is based on the reduced-gradient algorithm and, on an IBM 370/168, takes 70 seconds to solve the first case and 30 seconds for each subsequent case (Manne 1976, 1977).

Formally, the ETA model constraints have the form of eqns. (1.1)–(1.3) and (1.13–(1.17). The objective function may be viewed in either of two equivalent ways: maximizing the sum of consumers' plus producers' surplus, or minimizing the sum of the costs of conservation measures plus interfuel substitution costs plus the costs of energy supply. In the latter case it is essentially a combination of eqns. (1.12) and (1.18). Because the objective function is formulated in this way, ETA automatically allows for price-induced conservation and also for interfuel substitution.

1.2.3 MESSAGE

The models considered above (Problem 1.1) are formally DLP models of general type (one-index models). By introducing energy flows (from supply points to demand points) we arrive at DLP models of the transportation type (two-index models). The energy models MESSAGE (Agnew et al. 1978a, b) and DESOM [see Marcuse et al. (1976) and Section 1.2.4 below] can both be written in this form. It should be noted that such models cannot be directly handled by transportation or network algorithms, and that therefore conventional LP-packages were used for their solution (Agnew et al. 1978b; Marcuse et al. 1976). The extension of transportation algorithms to handle this particular type of problem was reported recently by Krivonozhko and Propoi (1979).

MESSAGE (Model for Energy Supply Systems Alternatives and their General Environmental impact) was developed by Voss, Agnew, and Schrattenholzer at the International Institute for Applied Systems Analysis (IIASA) as an extension of the Häfele–Manne model. The model differs from its predecessors (Häfele and Manne 1974; Suzuki 1975) in that it includes all allocated secondary energy to end users, incorporates an increased number of supply technologies, makes distinctions between different price categories of natural resources, and adds the costs of resources extracted to the objective function (Agnew et al. 1978a, b).

A simplified diagram of the MESSAGE model is presented in Figure 1. Each conversion process is linked to the other blocks of the system by flows of energy inputs and outputs. Each primary energy resource is either converted into a secondary energy form by a central-station conversion process (e.g., coal converted to electricity) or used directly as a fuel by a decentralized conversion process or end-use technology (e.g., coal used for space heating).

We will now describe a very simplified version of the energy flow model.

Let $x_{jil}(t)$ be the energy flow in period t from supply category j (e.g., primary resource j) to demand category l (e.g., end-use technology l) using conversion process i. Then, following the usual procedure for transportation problems, we can define the supply of energy l which should be greater than or equal to the given demand $d_l(t)$

$$\sum_{j,i} \alpha_{jil} x_{jil}(t) \geqslant d_l(t) \tag{1.19}$$

On the other hand, the total consumption $w_j(t)$ of primary energy resource j in period t is limited by the availability of this resource

$$\sum_{i,l} \beta_{jil} x_{jil}(t) = w_j(t) \tag{1.20}$$

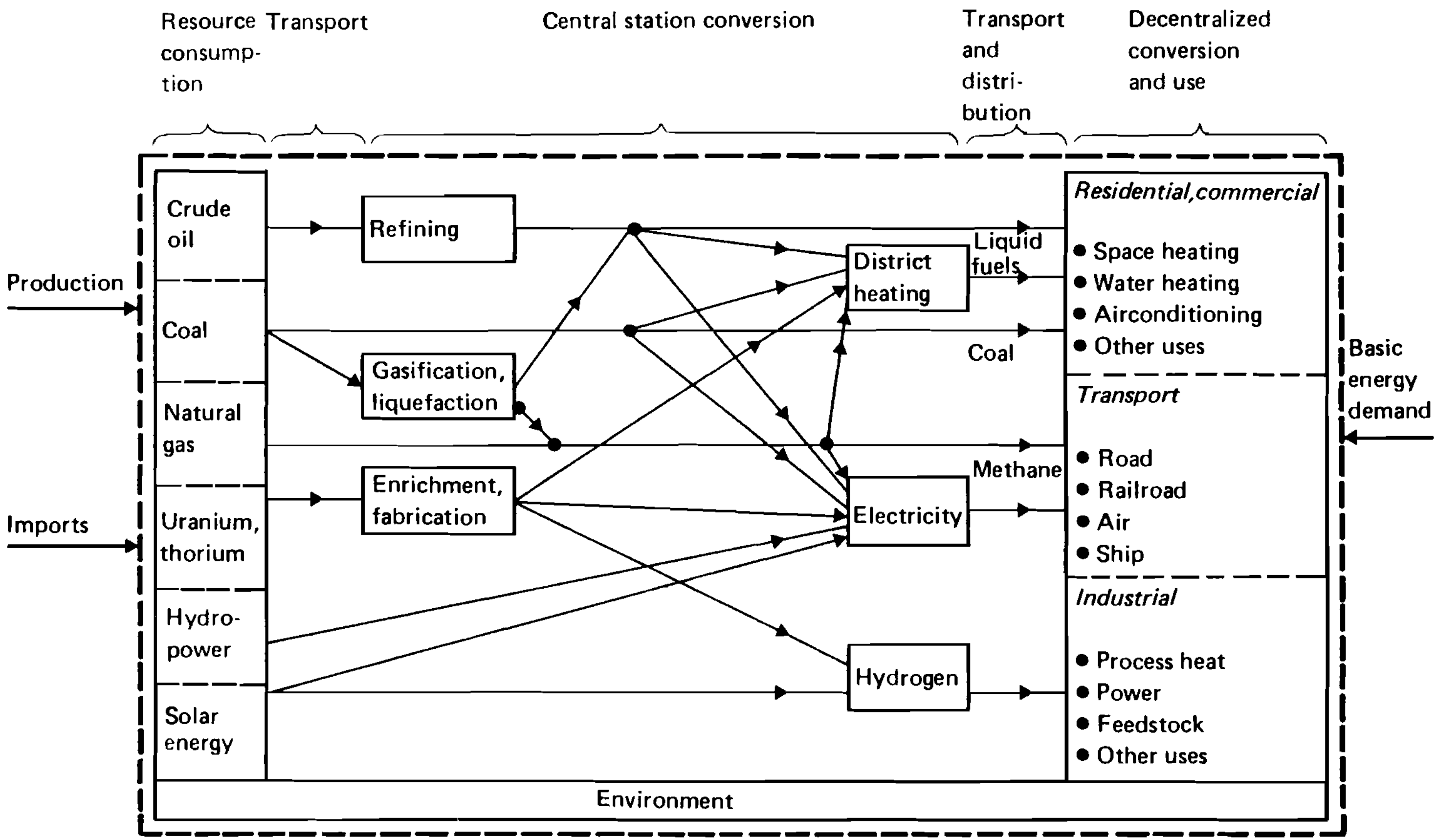

FIGURE 1 Simplified structure of the model MESSAGE.

$$z_j(t+1) = z_j(t) + w_j(t) \tag{1.21}$$

$$z_j(t) \leqslant \bar{z}_j(t) \tag{1.22}$$

Here $z_j(t)$ and $\bar{z}_j(t)$ have the same meaning as in eqns. (1.5) and (1.10). The degree of utilization $u_i(t)$ of process i is also limited by the available production capacity $y_i(t)$

$$\sum_{j,l} \gamma_{jil} x_{jil}(t) = u_i(t) \tag{1.23}$$

$$u_i(t) \leqslant y_i(t) \tag{1.24}$$

In eqns. (1.19), (1.20), and (1.23) α_{jil}, β_{jil}, and γ_{jil} are coefficients of energy-resource conversion efficiency (for examples see the next sections).

The development of the production capacity subsystem is described by state equations similar to eqn. (1.1).

We are now in a position to formulate a DLP model as follows.

Problem 1.2. Given the state equations

$$y_i(t+1) = y_i(t) + v_i(t) - v_i(t-\tau_i) \qquad (i = 1, 2, \ldots, n)$$

$$z_j(t+1) = z_j(t) + w_j(t) \qquad (j = 1, 2, \ldots, m)$$

with the initial conditions

$$y_i(0) = y_i^0; \; z_j(0) = z_j^0; \; v_i(t-\tau_i) = v_i^0(t-\tau_i)$$

$$(i = 1, 2, \ldots, n; \; j = 1, 2, \ldots, m; t < \tau_i)$$

find controls $\{x_{jil}(t)\}, \{v_i(t)\}$ and corresponding state variables $\{y_i(t)\}$, $\{z_j(t)\}$ which satisfy the conditions

$$u_i(t) = \sum_{j,l} \gamma_{jil} x_{jil}(t) \leqslant y_i(t)$$

$$w_j(t) = \sum_{i,l} \beta_{jil}(t) x_{jil}(t)$$

$$d_i(t) \leqslant \sum_{j,l} \alpha_{jil} x_{jil}(t)$$

$$v_i(t) \leqslant \bar{v}_i(t); \; z_j(t) \leqslant \bar{z}_j(t)$$

$$v_i(t) \geqslant 0, \; x_{jil}(t) \geqslant 0; \; y_i(t) \geqslant 0; \; z_j(t) \geqslant 0$$

and minimize the objective function

$$J = \sum_{t=0}^{T-1} \beta(t) \left[\sum_{i=1}^{n} c_i^1 u_i(t) + \sum_{i=1}^{n} c_i^2 v_i(t) + \sum_{j=1}^{m} c_j^3 w_j(t) \right] \tag{1.25}$$

The typical dimensions of the MESSAGE model are as follows. The planning horizon T is 65 years, divided into 13 periods of five years each. The numbers of each type of constraint are: demand, $7 \times T$; resources, $5 \times T$; total availability of resources, 17×1; intensity of resource extraction, $2 \times T$; and capacity loading, $5 \times T$; in addition, there are $35 \times T$ equations for capital stocks. Together with the other constraints this gives us, in terms of conventional LP problems, about 1100 rows and 1200 columns, with some 90 constraints for each period.

1.2.4 DESOM

DESOM (Dynamic Energy System Optimization Model) (Marcuse et al. 1976) was developed at the Brookhaven National Laboratory and is an extension of the Brookhaven Energy System Optimization Model (BESOM) which was a static, single-period LP model. In DESOM the demand sector has been disaggregated into technology-related end uses (22 mutually-exclusive end uses as defined by their energy-conversion processes). The general structure of DESOM is similar to that outlined in Problem 1.2.

Let us consider the state equations for the development of capacity of type i in the form

$$y_i(t+1) = y_i(t) + v_i(t) - v_i(t - \tau_i) - v_{0i}(t) \tag{1.26}$$

where the meaning of the control $v_i(t)$ and state $y_i(t)$ variables is the same as in eqn. (1.1); $v_{0i}(t)$ is the exogenously given decrease in existing (old) capacity of type i during period t.

Marcuse et al. (1976) introduced a scenario variable $\alpha(t)$ which restricts the rate of growth of capacity

$$y_i(t+1) \leqslant \alpha(t) y_i(t) \tag{1.27}$$

Generally $\alpha(t)$ is greater than one, which implies that installed capacity may expand during period t; if $\alpha(t)$ is less than one then capacity will decrease during period t.

Using eqn. (1.26) one can rewrite inequality (1.27) in the following form, which is similar to the inequality given by Marcuse et al.

$$y_{0i}(t+1) + \sum_{g=t-\tau_i}^{t} v_i(g) \leqslant \alpha(t) \left[y_{0i}(t) + \sum_{g=t-1-\tau_i}^{t-1} v_i(g) \right]$$

where

$$y_{0i}(t) = y_i(0) - \sum_{g=0}^{t-1} v_{0i}(g)$$

is the inherited capacity (capital stock of old capacities) for conversion process i at the beginning of period t (given exogenously).

To link the production subsystem with the resource-consumption subsystem, Marcuse et al. introduced demand and other constraints on intermediate energy flows. Each intermediate energy flow has associated with it a demand efficiency and a supply efficiency. The demand efficiency measures the energy loss as the intermediate flow is converted into a final energy product; the supply efficiency measures the energy loss from the point of extraction of the primary energy source to the intermediate energy flow.

If we let $x_{kl}(t)$ be the amount of intermediate energy flow in period t from supply category k to meet final energy demand l, we can define

$$u_i(t) = (1/\Delta) \sum_{(k,l)\in\Omega(i)} [x_{kl}(t)/r_{kl}] \tag{1.28}$$

where

- r_{kl} is the load factor for intermediate energy flow from supply category k to final demand category l;
- Δ is the length of period, generally, $\Delta = \Delta(t)$;
- $\Omega(i)$ is the set of indices (k, l), which defines the path of intermediate energy flow from supply k to final demand l associated with conversion process i; and
- $u_i(t)$ is the amount of installed capacity for conversion process i required in period t to deliver $x_{kl}(t)$, in other words, $u_i(t)$ is the degree of utilization of conversion process i in period t.

Evidently, the amount of installed capacity available in period t must be sufficient to produce intermediate energy flows which utilize the capacity for conversion process i in period t

$$(1/\Delta) \sum_{(k,l)\in\Omega(i)} [x_{kl}(t)/r_{kl}] \leqslant y_i(t) \tag{1.29}$$

which is similar in form to inequality (1.4).

Capacity is required to meet both base-load and peak demands in the electrical sectors. Off-peak electrical intermediate energy flows that use capacity installed for peak requirements are not included in inequality (1.29). For electricity-conversion processes

$$(1/\Delta) \sum_{(k,l)\in\Omega(i)} x_{kl}(t) \leqslant q_i y_i(t) \tag{1.30}$$

where q_i is an overall load factor, applied to all electrical capacity, which states that a conversion facility of type i can only operate for a proportion q_i of the time.

By introducing intermediate energy-flow variables it is possible to write down demand and resource constraints. The total amount of energy from intermediate energy flows $x_{kl}(t)$ must be sufficient to meet the demands $d_l(t)$

$$\sum_k d_{kl} x_{kl}(t) = d_l(t)$$

for each demand category l. Here the d_{kl} are demand coefficients representing the overall technical efficiency of a conversion technology for some intermediate energy flow from supply category k to meet final energy demand l.

On the other hand, intermediate energy flows $x_{kl}(t)$ in period t define a demand for primary energy resource j

$$\sum_{k,l} s_{jkl} x_{kl}(t) = w_j(t) \tag{1.31}$$

where

s_{jkl} are supply coefficients representing the overall technical efficiency of the conversion technology for intermediate energy flow based on resource j from supply k to final demand l; and

$w_j(t)$ is the amount of resource j used in period t.

Introducing the cumulative amount $z_j(t)$ of resource j extracted by the beginning of period t, one can write the state equation for the resource-consumption subsystem in the form

$$z_j(t+1) = z_j(t) + w_j(t); \; z_j(0) = z_j^0 \tag{1.32}$$

which is similar in form to eqn. (1.5). It is also evident that

$$z_j(t+1) = z_j(0) + \sum_{g=0}^{t} w_j(g)$$

Marcuse et al. (1976) built into DESOM upper and lower limits on cumulative resource extraction

$$\underline{z}_j \leqslant z_j(t) \leqslant \bar{z}_j \tag{1.33}$$

$\bar{z}_j$ is associated with the real world availability of resource j, whereas the lower limit $\underline{z}_j$ assures some minimum consumption. In addition to the constraints (1.33), DESOM contains a restriction on the rate of growth of resource extraction, namely that the amount of resource j extracted in period $t + 1$ must be no greater than $\beta_j(t)$ times the amount of resource j extracted in period t

$$w_j(t+1) \leqslant \beta_j(t) w_j(t) \tag{1.34}$$

Generally $\beta_j(t) > 1$; to simulate the phasing out of a resource over time one can set $\beta_j(t) \leqslant 1$ for later periods.

As in other models, DESOM contains environmental constraints, which are written in the form

$$\sum_{k,l} e_{klm} x_{kl}(t) \leqslant E_m(t) \tag{1.35}$$

where

e_{klm} is the amount of emission of type m for intermediate energy flow from k to l; and

$E_m(t)$ is the maximum permissible amount of emission of type m in period t.

The objective of the problem is to minimize the total discounted cost, i.e.

$$J = \sum_{t=0}^{T-1} \gamma(t) \left[\sum_{kl} c_{kl}^1(t) x_{kl}(t) + \sum_i c_i^2(t) v_i(t) + \sum_j c_j^3(t) w_j(t) \right] \tag{1.36}$$

where

$c^1_{kl}(t)$ is the cost for intermediate energy flows (undiscounted);
$c^2_i(t)$ is the annual cost during period t for building capacity for conversion process i; and
$c^3_j(t)$ is the cost for resource j in period t.

Consideration of the variables $v_i(t)$ in the last time period is in fact incorporated in DESOM but is not shown in eqn. (1.36).

Thus the optimization problem for the DESOM model can be formulated as follows.

Problem 1.3. Given the state equations

$$y_i(t+1) = y_i(t) + v_i(t) - v_i(t-\tau_i) - v_{0i}(t)$$

$$z_j(t+1) = z_j(t) + w_j(t)$$

with initial states

$$y_i(0) = y_i^0$$

$$z_j(0) = z_j^0$$

and known parameters

$$v(-\tau_i) = v^0(-\tau_i), \ldots, v(-1) = v^0(-1)$$

$$v_{0i}(t) \qquad (t = 0, 1, \ldots, T-1)$$

find controls $\{v_i(t)\}$, $\{w_j(t)\}$, $\{x_{kl}(t)\}$, and corresponding trajectories $\{y_i(t)\}$, $\{z_j(t)\}$, which satisfy the constraints

$$v_i(t) \geqslant 0; \; x_{kl}(t) \geqslant 0; \; y_i(t) \geqslant 0; \; z_j(t) \geqslant 0$$

$$\sum_k d_{kl} x_{kl}(t) = d_l(t)$$

$$\sum_{k,l} s_{jkl} x_{kl}(t) = w_j(t)$$

$$(1/\Delta) \sum_{k,l} [x_{kl}(t)/r_{kl}] \leqslant y_i(t) \qquad (i \in I^1)$$

$$(1/\Delta) \sum_{k,l} x_{kl}(t) \leqslant q_i y_i(t) \qquad (i \in I^2)$$

$$\underline{z}_j(t) \leqslant z_j(t) \leqslant \bar{z}_j(t)$$

$$y_i(t+1) \leqslant \alpha(t) y_i(t)$$

$$w_j(t+1) \leqslant \beta_j(t) w_j(t)$$

and minimize the objective function

$$J = \sum_{t=0}^{T-1} \gamma(t) \left[\sum_{kl} c_{kl}^1(t) x_{kl}(t) + \sum_i c_i^2(t) v_i(t) + \sum_j c_j^3(t) w_j(t) \right]$$

On examination of Problem 1.3, one can see that it is very similar to those considered earlier [if we exclude the special method of introducing the intermediate flows $x_{kl}(t)$].

As reported by Marcuse et al. (1976), the model without environmental constraints had 130 row constraints and 750 variables per period. The first version of the model contains a four-period optimization problem and it takes about 30 minutes to solve on an IBM 370/155. A standard base case is being developed; this case will cover the 100-year period from 1973 to 2073. It will consist of six five-year periods to provide considerable detail from now until the turn of the century; three ten-year periods to allow for the simulation of large-scale introduction of fusion and solar technologies in the early 21st century, and finally two twenty-year periods to reduce truncation effects.

A new version of DESOM, the MARKet ALlocation Model (MARKAL), has been developed recently at the Brookhaven National Laboratory (Kydes 1978). MARKAL is currently being used by the International Energy Agency in planning strategic energy options.

1.2.5 SPI Model

This model has been developed (A.A. Makarov and Melentjev 1973; Belyaev et al. 1976; A.A. Makarov 1977; Kononov 1977; Häfele and A.A. Makarov 1977) at the Siberian Power Institute (SPI), Siberian Department of the USSR Academy of Sciences, to analyze possible energy development strategies and to compare the trends in different branches of science and technology. The model is part of a system of models for long-term energy development forecasting (for a time horizon of 30–40 years). As this system of models has already been described at length elsewhere, we will discuss here only the more important features of the SPI energy supply systems model.

The SPI model has a specific block structure with detailed descriptions, for each region k and year t, of the production, interconnection, and conversion of energy at all stages ranging from the extraction of primary energy (different kinds of fossil fuel, nuclear fuel, hydro, solar, geothermal energy), via the production and distribution of secondary energy (liquid, solid, and gaseous fuels, secondary nuclear fuel, electrical energy, steam, hot water), to the production of final energy utilized in industry, transport, agriculture, and the municipal and service sectors. For each year t the model consists of oil, coal, gas, nuclear, and electrical energy blocks; for each region k it consists of fuel and electrical energy supply blocks. Each block can be generated, introduced into a computer, and updated independently.

For each region k and year t the balance equations for production and distribution are as follows.

For primary energy α

$$\sum_{j(\alpha)} a_{\alpha j}^k(t) x_{\alpha j}^k(t) + \sum_{k'} x_\alpha^{kk'}(t) = \sum_{j(\beta)} b_{\alpha j}^k(t) x_{\beta j}^k(t) + \sum_{k'} b_\alpha^{kk'}(t) x_\alpha^{kk'}(t) + d_\alpha^k(t)$$

For secondary energy β

$$\sum_{j(\beta)} a^k_{\beta j}(t) x^k_{\beta j}(t) + \sum_k x^{kk'}_\beta(t) = \sum_{j(\gamma)} b^k_{\beta j}(t) x^k_{\gamma j}(t) + \sum_{k'} b^{kk'}_\beta(t) x^{kk'}_\beta(t) + d^k_\beta(t)$$

For final energy γ

$$\sum_{j(\gamma)} a^k_{\gamma j}(t) x^k_{\gamma j}(t) = d^k_\gamma(t)$$

The various terms in the balance equations have the following meanings.

$x^k_{\alpha j}(t), x^k_{\beta j}(t), x^k_{\gamma j}(t)$	are, respectively, the amounts of primary (α), secondary (β), and final (γ) energy produced using technology j in region k and year t;
$x^{kk'}_\alpha(t), x^{kk'}_\beta(t)$	are, respectively, the (unknown) levels of transportation of primary (α) and secondary (β) energy from region k to region k' in year t;
$a^k_{\alpha j}(t), a^k_{\beta j}(t), a^k_{\gamma j}(t)$	are energy conversion coefficients;
$b^k_{\alpha j}(t), b^k_{\beta j}(t)$	are energy conversion coefficients related to intermediate energy consumption;
$b^{kk'}_\alpha(t), b^{kk'}_\beta(t)$	specify energy losses during transportation; and
$d^k_\alpha(t), d^k_\beta(t), d^k_\gamma(t)$	are, respectively, demands for primary (α), secondary (β), and final (γ) energy in region k and year t.

The constraints on nonenergy resources [referred to later in this report as WELMM factors (Grenon and Lapillone 1976); see also the footnote on p. 27], which are similar to inequality (1.9), are written in the form

$$\sum_{\alpha,k,j} f^k_{i\alpha j}(t) x^k_{\alpha j}(t) + \sum_{\beta,k,j} f^k_{i\beta j}(t) x^k_{\beta j}(t) + \sum_{\gamma,k,j} f^k_{i\gamma j}(t) x^k_{\gamma j}(t) \leqslant f_i(t)$$

For each nonrenewable kind of primary energy α we have a constraint

$$\sum_{j,k,t} x^k_{\alpha j}(t) \leqslant \bar{z}_\alpha$$

which is similar to inequalities (1.31)–(1.33).

It can be seen that these conditions, though much more detailed in form, have the same structure as the constraints of the models discussed earlier. The description of the dynamics of system development differs however in some respects. In the SPI model (A.A. Makarov 1977), the equations linking blocks t and $t + 1$ have the following form

$$\sum_{j\in J_0} x_{ij}(t) + \sum_{j\in J_1} x_{ij}(t) = \tilde{y}_i(t+1) = \sum_{j\in J_0} x_{ij}(t+1) + x_i(t+1)$$

where

i denotes a particular energy unit (plant, power station, etc.); and
j denotes the type of conversion process.

The set of indices J_0 is associated with conversion (or production) capacity which exists at the beginning of period t ("old capacity") and the set of indices J_1 is associated with capacity which was built during period t ("new capacity"); thus $y_i(t+1)$ is the production capacity of type i at the end of year t (or at the beginning of year $t+1$); $x_i(t+1)$ is the capacity of type i which is dismantled in year $t+1$.

The above equations can be rewritten in a form closer to that of the state equation (1.1)

$$\sum_{j \in J_0} x_{ij}(t+1) = \sum_{j \in J_0} x_{ij}(t) + \sum_{j \in J_1} x_{ij}(t) - x_i(t+1)$$

By comparison it is evident that the term $\Sigma_{j \in J_0} x_{ij}(t)$ may be associated with the term $y_i(t)$ in eqn. (1.1), whereas the term $\Sigma_{j \in J_1} x_{ij}(t) - x_i(t+1)$ corresponds to the term $v_i(t) - v_i(t - \tau_i)$ in eqn. (1.1).

The other peculiarity of the SPI model is the objective function. The minimization of the total discounted cost was not considered to be altogether adequate in view of the uncertainty in prices. Therefore, the objective function of the model is given in the form of discounted consumption of total expenditures of different material resources and manpower (WELMM factors)

$$J = \sum_{t=0}^{T-1} \sum_i \beta(t) E_i(t) f_i(t)$$

where the coefficient $E_i(t)$ converts the amounts of each resource i into a unified system of units and $\beta(t)$ is a discounting factor.

The dimensions of the SPI model are 500–600 constraints and 4000–5000 variables for the long-range planning variant and 1200–1300 constraints and 6000–7000 variables for the five-year planning problem. To solve these optimization problems a special program package has been developed which gives a three- to four-fold reduction of computation time compared to the conventional simplex method (A.A. Makarov 1977).

2 RESOURCES MODEL

The resources model is designed for the evaluation of long-term resource exploration and extraction strategies. It also provides inputs for the energy supply model (see Section 1), essentially by establishing relations between available quantities of given natural resources and their possible costs of production or extraction (Naill 1972; Brobst and Pratt 1973; Govett and Govett 1974; Kaya and Suzuki 1974; McKelvey 1974; Mesarovic and Pestel 1974; Grenon 1976; Grenon and Lapillone 1976; Grenon and Zimin 1977; Ayres 1978; Kydes 1978).

We will consider the production of natural resources over a given planning horizon at a regional (or national) level. The lengths of each time step and of the whole planning horizon correspond to those in the energy supply model. The availabilities of various resources are expressed in physical units and costs are measured in monetary units.

The model's structure is similar to that of the energy supply model in the sense that it is a DLP model in which the optimal mix of technologies for exploration and extraction of natural energy resources is determined.

2.1 Basic Model

2.1.1 State Equations

The model consists of two subsystems: the resource-accounting subsystem and the capital-stocks subsystem. Using the definitions provided by McKelvey and others (Brobst and Pratt 1973; Govett and Govett 1974; Kaya and Suzuki 1974; McKelvey 1974), the first subsystem describes the movement of resources from the "speculative" to the "hypothetical" category and from the "hypothetical" to the "identified" category. Both renewable and nonrenewable resources may be considered. The second subsystem describes the accumulation and depletion of capacity (capital stocks) for the exploration and extraction of both renewable and nonrenewable resources.

Before continuing with the description of the resource model, let us consider a simple example, which illustrates how the dynamics of the process will be described. Let $x(t)$ be the total amount of nonrenewable resource in place at the beginning of period t. By applying given extraction technologies it is only possible to extract a certain proportion of the total amount of this resource in place. We will denote the extractable (or recoverable) amount of the resource by $\hat{x}(t)$: it is convenient to refer to $\hat{x}(t)$ as a net value and to $x(t)$ as a gross value. The relationship between the gross and net values of the resource may be described by

$$x(t) = \hat{x}(t)/\delta$$

where $\delta (0 < \delta < 1)$ is the recoverability factor of the resource (for a fixed technology) during period t.

Bearing this in mind, we can describe the process in three ways: in terms of gross values, net values, or a mixture of both. Let $u(t)$ be the (gross) amount of the resource extracted in period t, and $\tilde{u}(t)$ be the (gross) amount of the resource moved during the same period from the hypothetical to the identified category. Then the balance equation is

$$x(t+1) = x(t) - u(t) + \tilde{u}(t) \qquad (t = 0, 1, \ldots, T-1)$$

It is evident that

$$x(t) \geqslant 0 \qquad (\text{for all } t)$$

which is equivalent to

$$\sum_{g=0}^{t} u(g) \leqslant x(0) + \sum_{g=0}^{t} \tilde{u}(g) \qquad (t = 1, 2, \ldots, T)$$

To obtain a description in "net" units, all the variables must be multiplied by δ. Due to the linearity of the relationships

$$\hat{x}(t+1) = \hat{x}(t) - \hat{u}(t) + \hat{\tilde{u}}(t)$$

In practice, a mixed description is generally used

$$x(t+1) = x(t) - \hat{u}(t)/\delta(t) + \tilde{u}(t)$$

In this case, the condition

$$x(t) \geqslant 0$$

is equivalent to

$$\sum_{g=0}^{t} u(g) \leqslant \delta \left[x(0) + \sum_{g=0}^{t} \tilde{u}(g) \right]$$

The value

$$x(0) + \sum_{g=0}^{t} \left[\tilde{u}(g) - \hat{u}(g) \right] \geqslant (1-\delta) \left[x(0) + \sum_{g=0}^{t} \tilde{u}(g) \right]$$

denotes the (gross) amount of the resource remaining in place after t periods of extraction.

From this point onwards we will use the mixed description but, for simplicity, we will omit the "hat" sign on variable $\hat{u}(t)$ (Grenon and Zimin 1977).

Nonrenewable resources. Let

- $x_i^1(t)$ be the (gross) amount (or stock) of an identified nonrenewable resource i at period t;
- $u_{mi}^1(t)$ be the (net) amount of resource i extracted by technology m during period t (extraction intensity);
- M_i^1 be the total number of extraction technologies which can be applied to nonrenewable resource i;
- $u_{ki}^2(t)$ be the (gross) amount of resource i moved from the hypothetical to the identified category by exploration technology k during period t; and
- K_i^1 be the total number of exploration technologies which can be applied to nonrenewable resource i.

Then the dynamics (in total amounts) of identified nonrenewable resources will be as follows

$$x_i^1(t+1) = x_i^1(t) - \sum_{m \in M_i^1} u_{mi}^1(t)/\delta_{mi}^1(t) + \sum_{k \in K_i^1} u_{ki}^2(t) \qquad (2.1)$$

Here $\delta_{mi}^1(t)$ is the recoverability of resource i by technology m during period t.

For hypothetical resources (all variables are "gross" values) we introduce, in a similar way

- $x_i^2(t)$ as the total amount of resource i in the hypothetical category in period t; and
- $u_i^3(t)$ as the total amount of resource i moved from the speculative to the hypothetical category as a result of exploration activity during period t.

Note that in this case we do not single out different exploration technologies, in contrast to the case of moving resources from the hypothetical to the identified category.

The state equations for this group of hypothetical nonrenewable resources will be as follows

$$x_i^2(t+1) = x_i^2(t) - \sum_{k \in K_i^1} u_{ki}^2(t) + u_i^3(t) \tag{2.2}$$

Similarly, for the speculative category of nonrenewable resources

$$x_i^3(t+1) = x_i^3(t) - u_i^3(t) + u_i^4(t) \tag{2.3}$$

where

- $x_i^3(t)$ is the total estimate of resource i in the speculative category during period t; and
- $u_i^4(t)$ is the change in the estimate of resource i in the speculative category during period t as a result of improved scientific knowledge.

In the state equations (2.1)–(2.3), $\{x_i^1(t), x_i^2(t), x_i^3(t)\}$ $(i = 1, 2, \ldots, N_1)$ are state variables for the nonrenewable resources subsystem, $\{u_{mi}^1(t), u_{ki}^2(t), u_i^3(t), u_i^4(t)\}$ $(m \in M_i^1, k \in K_i^1, i = 1, \ldots, N_1)$ are control variables, and $i = 1, \ldots, N_1$, where N_1 is the total number of categories of nonrenewable resources considered.

Renewable resources. In a similar way we can write the state equations for renewable resources such as solar, geothermal, etc., as follows

$$y_i^1(t+1) = y_i^1(t) + \sum_{k \in K_i^2} v_{ki}^2(t) \tag{2.4}$$

$$y_i^2(t+1) = y_i^2(t) - \sum_{k \in K_i^2} v_{ki}^2(t) + v_i^3(t) \tag{2.5}$$

$$y_i^3(t+1) = y_i^3(t) - v_i^3(t) + v_i^4(t) \qquad (i = 1, 2, \ldots, N_2) \tag{2.6}$$

where

- $y_i^1(t)$ is the total available flow of renewable resource i in period t;
- $y_i^2(t)$ is the total hypothetical flow of resource i in period t;
- $y_i^3(t)$ is the total speculative flow of resource i in period t;
- $v_{ki}^2(t)$ is the intensity of exploration technology k applied to resource i in period t;
- $v_i^3(t)$ is the total flow of renewable resource i moved from the speculative to the hypothetical category as a result of exploration activity during period t;
- $v_i^4(t)$ is the change in the estimated flow of renewable resource i in the speculative category during period t as a result of improved scientific knowledge;
- K_i^2 is the total number of exploration technologies for resource i; and
- N_2 is the total number of categories of renewable resources considered.

In the renewable-resources subsystem (2.4)–(2.6), $\{y_i^1(t), y_i^2(t), y_i^3(t)\}$ $(i = 1, 2, \ldots, N_2)$ are the state variables, and $\{v_{ki}^2(t), v_i^3(t), v_i^4(t)\}$ $(k = 1, 2, \ldots, K_i^2;\ i = 1, 2, \ldots, N_2)$ are the control variables.

Initial conditions are assumed to be given for all resource categories

$$\left.\begin{array}{ll} x_i^1(0) = x_i^{1,0};\ x_i^2(0) = x_i^{2,0};\ x_i^3(0) = x_i^{3,0} & (i = 1, 2, \ldots, N_1) \\ y_i^1(0) = y_i^{1,0};\ y_i^2(0) = y_i^{2,0};\ y_i^3(0) = y_i^{3,0} & (i = 1, 2, \ldots, N_2) \end{array}\right\} \tag{2.7}$$

Dynamics of extraction and exploration capacity. Alongside the subsystems which describe resource extraction and exploration themselves, it is necessary to introduce a subsystem describing the *development* of resource extraction and exploration capacity. This can be done by using equations similar to eqn. (1.1). For the extraction part of the subsystem, let

$z_m(t)$ be the extraction capacity of type m in period t;
$w_m(t)$ the increase of the mth extraction capacity during period t; and
τ_m the service lifetime of units of capacity of type m.

Then the state equations for this submodel will be as follows

$$z_m(t+1) = z_m(t) + w_m(t) - w_m(t - \tau_m) \tag{2.8}$$

where, in the general case, $m \in M_1 \cup M_2$, the union of two sets

M_1 (the total set of technologies for extracting nonrenewable resources); and
M_2 (the total set of technologies for extracting renewable resources.)

Initial conditions are given as follows

$$z_m(0) = z_m^0 \tag{2.9a}$$

$$w_m(t - \tau_m) = w_m^0(t - \tau_m) \qquad (0 \leqslant t \leqslant \tau_m - 1) \tag{2.9b}$$

The dynamics of the development of exploration capacity can be described in a similar way, but for simplicity these equations are omitted here.

2.1.2 Constraints

The activities of exploration and extraction of natural resources are subject to a number of constraints. In the sections which follow we will examine how the model deals with physical, recoverability, availability, and demand constraints.

Physical sense. By virtue of their physical meaning, all the variables in the model are nonnegative

$$\left.\begin{array}{l} x_i^1(t) \geqslant 0;\ x_i^2(t) \geqslant 0;\ x_i^3(t) \geqslant 0 \\ u_{mi}^1(t) \geqslant 0;\ u_{ki}^2(t) \geqslant 0;\ u_i^3(t) \geqslant 0;\ u_i^4(t) \geqslant 0 \\ \qquad (i = 1, 2, \ldots, N_1;\ m = 1, 2, \ldots, M_1;\ k = 1, 2, \ldots, K_1) \end{array}\right\} \quad (2.10)$$

$$\left.\begin{array}{l} y_i^1(t) \geqslant 0;\ y_i^2(t) \geqslant 0;\ y_i^3(t) \geqslant 0 \\ v_{mi}^1(t) \geqslant 0;\ v_{ki}^2(t) \geqslant 0;\ v_i^3(t) \geqslant 0;\ v_i^4(t) \geqslant 0 \\ z_m(t) \geqslant 0;\ w_m(t) \geqslant 0;\ m \in M_1 \cup M_2 \\ \qquad (i = 1, 2, \ldots, N_2;\ m = 1, 2, \ldots, M_2;\ k = 1, 2, \ldots, K_2) \end{array}\right\} \quad (2.11)$$

Recoverability. The recoverability of a resource is assumed to be associated with the type of resource and the technology used for its extraction. As mentioned previously, the non-negativity condition for nonrenewable resources may be stated as

$$x_i^1(t) \geqslant 0 \qquad (2.12)$$

which [from eqns. (2.1) and (2.7)] is equivalent to

$$\sum_{g=0}^{t} \sum_{m \in M_i^1} u_{mi}^1(g)/\delta_{mi}^1(g) \leqslant x_i^{1,0} + \sum_{g=0}^{t} \sum_{k \in K_i^1} u_{ki}^2(g) \qquad (i = 1, 2, \ldots, N_1) \qquad (2.12a)$$

For renewable resources the corresponding constraints may be written as

$$\sum_{m \in M_i^2} v_{mi}^1(t)/\delta_{mi}^2(t) \leqslant y_i^1(t) \qquad (i = 1, 2, \ldots, N_2) \qquad (2.13)$$

Here $v_{mi}^1(t)$ is the amount of the renewable resource i utilized by technology $m \in M_i^2$ during period t (the "extraction" intensity). In contrast to eqn. (2.1), this variable does not enter eqn. (2.4) for renewable resources, because utilization of such resources (solar, geothermal, etc.) does not influence their source.

From eqns. (2.4) and (2.7), condition (2.13) is equivalent to

$$\sum_{m \in M_i^2} v_{mi}^1(t)/\delta_{mi}^2(t) \leqslant y_i^{1,0} + \sum_{g=0}^{t-1} \sum_{k \in K_i^2} v_{ki}^2(g) \qquad (2.14)$$

Availability. In their simplest form, these constraints can be expressed as upper bounds on control variables

$$u_{mi}^1(t) \leqslant \bar{u}_{mi}^1(t);\ u_{ki}^2(t) \leqslant \bar{u}_{ki}^2(t);\ u_i^3(t) \leqslant \bar{u}_i^3(t);\ u_i^4(t) \leqslant \bar{u}_i^4(t) \qquad (2.15)$$

and

$$v_{mi}^1(t) \leqslant \bar{v}_{mi}^1(t);\ v_{ki}^2(t) \leqslant \bar{v}_{ki}^2(t);\ v_i^3(t) \leqslant \bar{v}_i^3(t);\ v_i^4(t) \leqslant \bar{v}_i^4(t) \qquad (2.16)$$

These constraints are similar to those of inequality (1.8), and express very approximately the availability over time of various technologies for exploration and extraction.

The development of a given resource system may often require the input of other resources (such as land, manpower, etc.) which are external to the system itself (referred to here as WELMM* factors). These constraints can be written in a form similar to that of inequality (1.9)

$$\sum_{s,i} r_{si}^{vlu}(t) u_{si}^{v}(t) \leqslant R^{vlu}(t) \tag{2.17}$$

$$\sum_{q,i} r_{qi}^{vlv}(t) v_{qi}^{v}(t) \leqslant R^{vlv}(t) \qquad (l = 1, 2, \ldots, L; v = 1,2,3,4) \tag{2.18}$$

where

$R^{vlu}(t), R^{vlv}(t)$ are, respectively, the amounts of nonrenewable and renewable external resource l (or WELMM factor l), available in period t for each group of exploration activities v;

L is the total number of WELMM factors considered as external to the model; and

$r_{si}^{vlu}(t), r_{qi}^{vlv}$ are, respectively, the (normative) consumptions of nonrenewable and renewable WELMM factor l per unit of productive output; and

$$s \in M_i^1, \text{ if } v = 1; \; s \in K_i^1, \text{ if } v = 2$$

$$q \in M_i^2, \text{ if } v = 1; \; q \in K_i^2, \text{ if } v = 2$$

The subscripts s and q on the left-hand sides of inequalities (2.17) and (2.18) should be dropped if $v = 3$ or 4. In practical terms, coefficients $r_{si}^{vlu}(t)$ and $r_{qi}^{vlv}(t)$ are negligibly small for $v = 2, 3$, or 4.

The other important type of availability constraint is connected with the linkage of resource-extraction and production capacity: the extraction of resources during each period is limited by the production capacity available

$$\sum_i u_{mi}^1(t) \leqslant z_m(t) \qquad (m \in M_1) \tag{2.19}$$

$$\sum_i v_{mi}^1(t) \leqslant z_m(t) \qquad (m \in M_2) \tag{2.20}$$

where $z_m(t)$, $m \in M_1$, and $m \in M_2$ are defined from eqn. (2.8).

In its turn, the development of the extraction-capacity subsystem (2.8) may itself be limited by the amount of resources available for construction of new capacity. In this case, the control variables $w_m(t)$ in eqn. (2.8) are subject to constraints which are similar to those described in inequalities (2.17) and (2.18).

* Grenon and Lapillone (1976) originally used WELMM as an abbreviation for Water, Energy, Land, Materials, and Manpower; however in this report we use the term "WELMM factor" to mean any arbitrary resource which is external to the system in question.

Demand. Demands are exogeneous for the resource model. These constraints can be written in the form

$$\sum_{m \in M_i^1} u_{mi}^1(t) \geqslant d_i^u(t) \qquad (i = 1, 2, \ldots, N_1) \tag{2.21}$$

for nonrenewable resources, and in the form

$$\sum_{m \in M_i^2} v_{mi}^1(t) \geqslant d_i^v(t) \qquad (i = 1, 2, \ldots, N_2) \tag{2.22}$$

for renewable resources, where $d_i^u(t)$ and $d_i^v(t)$ are, respectively, the demands for nonrenewable and renewable resource i in period t.

It should be noted that accurate estimation of the demands $d_i^u(t)$ and $d_i^v(t)$ is very important in the resource model: this is because these parameters exert a strong influence on the timing and corresponding costs of putting into operation new extraction technologies and on the intensity of exploration activities, and therefore, finally, on the optimal solution itself.

2.1.3 Objective Function

A variety of different objective functions is possible for the resource system development. Following the ESS model procedure, we define the objective function so as to minimize the total discounted costs required to implement a given resource-development strategy

$$
\begin{aligned}
&J(\boldsymbol{u}^1, \boldsymbol{u}^2, \boldsymbol{u}^3, \boldsymbol{u}^4, \boldsymbol{v}^1, \boldsymbol{v}^2, \boldsymbol{v}^3, \boldsymbol{v}^4, \boldsymbol{w}) = \\
&\sum_{t=0}^{T-1} \beta(t) \Bigg\{ \Bigg[\sum_{m,i} c_{mi}^{1u} u_{mi}^1(t) + \sum_{k,i} c_{ki}^{2u} u_{ki}^2(t) + \sum_i c_i^{3u} u_i^3(t) + \sum_i c_i^{4u} u_i^4(t) \Bigg] \\
&+ \Bigg[\sum_{m,i} c_{mi}^{1v} v_{mi}^1(t) + \sum_{k,i} c_{ki}^{2v} v_{ki}^2(t) + \sum_i c_i^{3v} v_i^3(t) + \sum_i c_i^{4v} v_i^4(t) \Bigg] + \sum_m c_m^z z_m(t) + \sum_m c_m^w w_m(t) \\
&+ \Bigg[\sum_{l,m,i} c_{mi}^{1lu} r_{mi}^{1lu} u_{mi}^1(t) + \sum_{l,k,i} c_{ki}^{2lu} r_{ki}^{2lu} u_{ki}^2(t) + \sum_{l,i} c_i^{3lu} r_i^{3lu} u_i^3(t) + \sum_{l,i} c_i^{4lu} r_i^{4lu} u_i^4(t) \Bigg] \\
&+ \Bigg[\sum_{l,m,i} c_{mi}^{1lv} r_{mi}^{1lv} v_{mi}^1(t) + \sum_{l,k,i} c_{ki}^{2lv} r_{ki}^{2lv} v_{ki}^2(t) + \sum_{l,i} c_i^{3lv} r_i^{3lv} v_i^3(t) + \sum_{l,i} c_i^{4lv} r_i^{4lv} v_i^4(t) \Bigg] \Bigg\}
\end{aligned}
$$

Here

$c_{mi}^{1u}, c_{ki}^{2u}, c_i^{3u}, c_i^{4u}$	are exploration costs for nonrenewable resources;
$c_{mi}^{1v}, c_{ki}^{2v}, c_i^{3v}, c_i^{4v}$	are exploration costs for renewable resources;
c_m^z	are operational costs;
c_m^w	are capital investment costs; and
$c_{mi}^{1lu}, c_{mi}^{1lv}$, etc.	are costs of WELMM factors (external resources).

Transportation costs can also be included in the model.

2.1.4 Statement of the Problem

Finally we can formulate the problem of optimal development of the resource system as follows.

Problem 2.1. Given the state equations for the nonrenewable resources subsystem ($i = 1, 2, \ldots, N_1$)

$$x_i^1(t+1) = x_i^1(t) - \sum_{m \in M_i^1} u_{mi}^1(t)/\delta_{mi}^1(t) + \sum_{k \in K_i^1} u_{ki}^2(t);\ x_i^1(0) = x_i^{1,0}$$

$$x_i^2(t+1) = x_i^2(t) - \sum_{k \in K_i^1} u_{ki}^2(t) + u_i^3(t);\ x_i^2(0) = x_i^{2,0}$$

$$x_i^3(t+1) = x_i^3(t) - u_i^3(t) + u_i^4(t);\ x_i^3(0) = x_i^{3,0}$$

for the renewable resources subsystem ($i = 1, 2, \ldots, N_2$)

$$y_i^1(t+1) = y_i^1(t) + \sum_{k \in K_i^2} v_{ki}^2(t);\ y_i^1(0) = y_i^{1,0}$$

$$y_i^2(t+1) = y_i^2(t) - \sum_{k \in K_i^2} v_{ki}^2(t) + v_i^3(t);\ y_i^2(0) = y_i^{2,0}$$

$$y_i^3(t+1) = y_i^3(t) - v_i^3(t) + v_i^4(t);\ y_i^3(0) = y_i^{3,0}$$

and for the extraction capacity subsystem ($m \in \{1, \ldots, M_1\}$ and $m \in \{1, \ldots, M_2\}$)

$$z_m(t+1) = z_m(t) + w_m(t) - w_m(t - \tau_m);\ z_m(0) = z_m^0$$

$$w_m(t - \tau_m) = w_m^0(t - \tau_m) \qquad (0 \leqslant t \leqslant \tau_m - 1)$$

find controls $\{u_{mi}^1(t), u_{ki}^2(t), u_i^3(t), u_i^4(t)\}$, $\{v_{mi}^1(t), v_{ki}^2(t), v_i^3(t), v_i^4(t)\}$, and $\{w_m(t)\}$, and corresponding trajectories $\{x_i^1(t), x_i^2(t), x_i^3(t)\}$, $\{y_i^1(t), y_i^2(t), y_i^3(t)\}$, and $\{z_m(t)\}$, which satisfy the following constraints

(a) nonnegativity

$$u_{mi}^1(t) \geqslant 0;\ u_{ki}^2(t) \geqslant 0;\ u_i^3(t) \geqslant 0;\ u_i^4(t) \geqslant 0$$

$$v_{mi}^1(t) \geqslant 0;\ v_{ki}^2(t) \geqslant 0;\ v_i^3(t) \geqslant 0;\ v_i^4(t) \geqslant 0$$

$$x_i^1(t) \geqslant 0;\ x_i^2(t) \geqslant 0;\ x_i^3(t) \geqslant 0$$

$$y_i^1(t) \geqslant 0;\ y_i^2(t) \geqslant 0;\ y_i^3(t) \geqslant 0$$

(b) recoverability

$$x_i^1(t) \geqslant 0 \qquad (i = 1, 2, \ldots, N_1)$$

$$\sum_m v_{mi}^1(t)/\delta_{mi}^2(t) \leqslant y_i^1(t) \qquad (i = 1, 2, \ldots, N_2)$$

(c) external-resource availability

$$\sum_{s,i} r_{si}^{vlu}(t)u_{si}^{v}(t) \leqslant R^{vlu}(t)$$

$$(l = 1, 2, \ldots, L;\ v = 1, 2, 3, 4)$$

$$\sum_{q,i} r_{qi}^{vlv}(t)v_{qi}^{v}(t) \leqslant R^{vlv}(t)$$

(d) production-capacity availability

$$\sum_{i} u_{mi}^{1}(t) \leqslant z_m(t) \qquad (m \in M_1)$$

$$\sum_{i} v_{mi}^{1}(t) \leqslant z_m(t) \qquad (m \in M_2)$$

(e) demand

$$\sum_{m} u_{mi}^{1}(t) \geqslant d_i^{u}(t) \qquad (i = 1, 2, \ldots, N_1)$$

$$\sum_{m} v_{mi}^{1}(t) \geqslant d_i^{v}(t) \qquad (i = 1, 2, \ldots, N_2)$$

and minimize the objective function ($v = 1, 2, 3, 4$)

$$J(u, v, w) = \sum_{t=0}^{T-1} \beta(t)\Bigg[\sum_{v,s,i} c_{si}^{vu}u_{si}^{v}(t) + \sum_{v,s,i} c_{si}^{vv}v_{si}^{v}(t) + \sum_{m} c_m^{z} z_m(t) + \sum_{m} c_m^{w} w_m(t)$$
$$+ \sum_{v,l,s,i} c_{si}^{vlu} r_{si}^{vlu} u_{si}^{v}(t) + \sum_{v,l,s,i} c_{si}^{vlv} r_{si}^{vlv} v_{si}^{v}(t)\Bigg]$$

This particular objective function is given here only for illustration. Many other objectives, for instance, the minimization of the total production costs of primary energy resources and effect of their use in the energy sector, are of practical interest, and some examples of such modifications of the model are given in the next section.

2.2 Discussion

The formulation of Problem 2.1 is general enough to allow different modifications to the basic problem. These modifications make it possible to carry out policy analyses for extraction and/or exploration activities, for a single resource or for a group of resources, for a region or a country; it is also possible to determine optimal balances of these activities for nonrenewable and renewable resources. We will now consider some examples of these modifications and particular cases of Problem 2.1.

2.2.1 Extraction and Exploration Model

First we consider the analysis of the interrelationships between extraction and exploration activities for a given nonrenewable energy resource (e.g., coal, oil, etc.).

The problem is as follows. For a given region (or country) there are known initial values for identified and hypothetical stocks of the resource, classified in n different categories (e.g., onshore crude oil, natural gas, and offshore crude oil). There are also M different extraction and K different exploration technologies. The degree of utilization of these technologies depends, during a given period, on the extraction and exploration capacity available during the same period. The problem is to determine the optimal mix of extraction and exploration activities over a given planning horizon which is, at the same time, balanced with the development of the exploration-, extraction-, and production-capacity subsystems and yields the maximum output over the same horizon.

Using the conditions of Problem 2.1, this problem can be formalized as follows.

Problem 2.2 Let the initial stocks of identified and hypothetical resources be given, respectively, as

$$x_i^1(0) = x_i^{1,0} \text{ and } x_i^2(0) = x_i^{2,0} \tag{2.23}$$

with state equations for extraction activities

$$x_i^1(t+1) = x_i^1(t) - \sum_{m \in M_i} u_{mi}^1(t)/\delta_{mi}^1(t) + \sum_{k \in K_i} u_{ki}^2(t) \tag{2.24}$$

and for exploration activities

$$x_i^2(t+1) = x_i^2(t) - \sum_{k \in K_i} u_{ki}^2(t) + \tilde{u}_i^2(t) \tag{2.25}$$

where $\tilde{u}_i^2(t)$ is the increase in the hypothetical stocks of resource i during period t (the discovery rate). In addition, let the initial values of the extraction and exploration capacities be given, respectively, by

$$z_m^1(0) = z_m^{1,0} \text{ and } z_k^2(0) = z_k^{2,0} \tag{2.26}$$

with the state equations

$$z_m^1(t+1) = z_m^1(t) + w_m^1(t) - w_m^1(t - \tau_m^1) \tag{2.27}$$

$$z_k^2(t+1) = z_k^2(t) + w_k^2(t) - w_k^2(t - \tau_k^2) \tag{2.28}$$

The intensities of extraction and exploration activities, $u_{mi}^1(t)$ and $u_{ki}^2(t)$, as well as the intensities of construction of new extraction and exploration capacity, $w_i^1(t)$ and $w_i^2(t)$, are subject to budgetary and other resource constraints

$$\sum_{m,i} r_{mil}^{1u}(t)u_{mi}^1(t) + \sum_{k,i} r_{kil}^{2u}(t)u_{ki}^2(t) + \sum_i r_{il}^{1w}(t)w_i^1(t) + \sum_i r_{il}^{2w}(t)w_i^2(t) \leqslant R_l(t) \tag{2.29}$$

$$\sum_i u_{mi}^1(t) \leqslant z_m^1(t); \ \sum_i u_{ki}^2(t) \leqslant z_k^2(t) \tag{2.30}$$

$$x_i^1(t) \geqslant 0 \tag{2.31}$$

Find nonnegative control sequences $\{u_{mi}^1(t)\}$, $\{u_{ki}^2(t)\}$, and $\{w_i^1(t)\}$, $\{w_i^2(t)\}$, and corresponding nonnegative state variables $\{x_i^1(t)\}$, $\{x_i^2(t)\}$, and $\{z_i^1(t)\}$, $\{z_i^2(t)\}$, which maximize the total output of resource i

$$J = \sum_{t=0}^{T-1} \sum_{m,i} \kappa_i u^1_{mi}(t) \tag{2.32}$$

where κ_i is the energy conversion factor for resource i. Here $\tilde{u}_i^2(t)$ (the discovery rate) is considered as a scenario variable.

2.2.2 Extraction Model

If the increase $\{\tilde{u}_i(t)\}$ of the identified resource is considered as a scenario variable (but not as a result of controllable exploration activities), then the state equations for the extraction system are simplified

$$x_i(t+1) = x_i(t) - u_i(t)/\delta_i(t) + \tilde{u}_i(t);\ x_i(0) = x_i^0 \tag{2.33}$$

where $\tilde{u}_i(t)$ is the amount of resource i moved from the hypothetical to the identified category during period t, and $u_i(t)$ is the total amount of resource i extracted during period t (in this example, different extraction technologies are not singled out).

The development of the extraction capacity subsystem is described by a state equation similar to eqn. (2.27)

$$z_i(t+1) = z_i(t) + w_i(t) - w_i(t-\tau_i);\ z_i(0) = z_i^0 \tag{2.34}$$

with the constraints

$$u_i(t) \leqslant z_i(t);\ u_i(t) \geqslant 0;\ w_i(t) \geqslant 0;\ z_i(t) \geqslant 0 \tag{2.35}$$

$$\sum_i r_{il}^w(t) w_i(t) + \sum_i r_{il}^u(t) u_i(t) \leqslant R_l(t);\ w_i(t) \geqslant 0 \tag{2.36}$$

$$x_i(t) \geqslant 0 \tag{2.36a}$$

The problem is to determine the extraction policy for a given identified resource, subject to constraints on extraction capacity (2.35), availability of external resources (2.36), and recoverability of the given resource (2.36a), which gives the maximum total output during the planning period.

The objective function may be written again as (2.32), or, if we introduce $\xi(t)$ as the cumulative amount of the resource extracted

$$\xi(t+1) = \xi(t) + \sum_i \kappa_i u_i(t);\ \xi(0) = 0 \tag{2.37}$$

as the maximization of $\xi(T)$.

2.2.3 Exploration Model

This model allows us to determine those exploration policies which will move the maximum amount of resources from the hypothetical to the identified category. The subsystem is a counterpart of the extraction subsystem and is described by the equations

$$x_i(t+1) = x_i(t) - u_i(t) + \tilde{u}_i(t);\ x_i(0) = x_i^0 \tag{2.38}$$

$$z_i(t+1) = z_i(t) + w_i(t) - w_i(t-\tau_i);\ z_i(0) = z_i^0 \tag{2.39}$$

$$u_i(t) \leqslant z_i(t);\ u_i(t) \geqslant 0,\ z_i(t) \geqslant 0 \tag{2.40}$$

$$\sum_i r_{il}(t) w_i(t) \leqslant R_l(t);\ w_i(t) \geqslant 0 \tag{2.41}$$

$$x_i(t) \geqslant 0 \tag{2.42}$$

$$J = \sum_{t=0}^{T-1} \sum_i u_i(t) \to \max \tag{2.43}$$

2.2.4 Cost Minimization

In the examples above the objective was to maximize the output from the extraction and/or the exploration subsystems. For many practical purposes it is also necessary to calculate the relationship between the optimal cost J^* and the cumulative availability of a given resource [for example, for calculating cost coefficients in the objective function (1.12) of the energy supply system model]. This can be done by using a simple optimization model

$$\left.\begin{aligned}
&x_i(t+1) = x_i(t) - u_i(t)/\delta_i(t) + \tilde{u}_i(t);\ x_i(0) = x_i^0\\
&z_i(t+1) = z_i(t) + w_i(t) - w_i(t-\tau_i);\ z_i(0) = z_i^0\\
&\xi(t+1) = \xi(t) + \sum_i \kappa_i(t) u_i(t);\ \xi(0) = 0\\
&\sum_i \kappa_i u_i(t) \geqslant d(t);\ u_i(t) \geqslant 0\\
&u_i(t) \leqslant z_i(t);\ z_i(t) \geqslant 0\\
&x_i(t) \geqslant 0
\end{aligned}\right\} \tag{2.44}$$

$$J = \sum_t \sum_i [c_i^u(t) u_i(t) + c_i^w(t) w_i(t)] \to \min \tag{2.45}$$

This model differs from the extraction model in two ways: demand constraints are included (2.44), and the objective function (2.45) is formulated differently. Resource constraints (2.36) are omitted here because they are implicitly accounted for by cost coefficients $c_i^u(t)$ and $c_i^w(t)$ in objective function (2.45).

Clearly, in this simple model

$$\sum_i \kappa_i u_i^*(t) = d(t)$$

for optimal $u_i^*(t)$. Hence

$$\xi(t+1) = \xi(t) + d(t);\ \xi(0) = 0 \tag{2.46}$$

and

$$\xi(T) = \sum_{t=0}^{T-1} d(t) \tag{2.47}$$

The problem is, therefore, to calculate cost–supply curves

$$J^* = J[u^*, z(T)] = \varphi[z(T)]$$

It should be noted that the behavior of these curves is strongly dependent on the behavior of the demand curve $d(t)$.

2.2.5 Dimensions of the Models

Finally, we will calculate the typical dimensions of the resources model. Let

- M be the total number of different countries in a region;
- L be the number of resource provinces within a country;
- K be the number of basins within a province;
- T be the length of the planning horizon;
- l be the number of different resource categories in a basin;
- m be the number of different technologies which can be used in exploration and extraction; and
- k be the number of WELMM factors limiting extraction.

One can see that the model will have a total of $(3l + m)KLM$ state equations, $(2l + k + m)$ KLM constraints (nonnegativity constraints are not included here), and $3lmKLM$ control variables for each period.

For example, consider a region consisting of only one country with two resource provinces. Assuming that the average number of basins in a province is three, the average number of different resource categories is two (for instance, crude oil and natural gas), the number of different technologies is two, and the number of limiting WELMM factors is two, we calculate that, for each period, the model would have 48 state equations, 48 constraints, and 72 control variables. Thus, for a problem of quite realistic size, the resources model is manageable and can be handled even by standard LP-solving programs.

2.2.6 Resource Modeling under Conditions of Uncertainty

One of the intrinsic features of the resources model is uncertainty in the values of various parameters, particularly for the speculative and hypothetical resource categories. The conventional method for handling this difficulty is to consider these parameters as scenario variables [e.g., $\tilde{u}_i^2(t)$ in eqn. (2.25), or $\tilde{u}_i(t)$ in eqn. (2.33)], carrying out numerous computer runs for different hypothetical values of the variables.

A more sophisticated approach is to consider "maxmin" problems associated with the given model. The maxmin approach allows us to evaluate upper and lower limits of the objective function for optimization problems under conditions of uncertainty, and to elaborate extraction and exploration policies which guarantee the required results within a given range of uncertain parameters. Methods for solving maxmin DLP problems have been considered by Propoi and Yadykin (1974).

Yet another approach to the treatment of uncertainty conditions in resource models is the statement of the problem in a multistage stochastic programming framework (Ermoliev 1978).

3 ECONOMIC DEVELOPMENT MODELS

In this section we present a model which simulates optimal behavior of the entire economy of a given region for various different objectives. Interest in such models has been increasing in recent years because they allow us to calculate various "optimal" mixes of the dynamics of such important economic indicators as production levels, capital investment, and levels of intermediate and final consumption of goods produced. A number of different optimization models of economic development have been described previously (see, for example, Kantorovich 1965; V.V. Makarov 1966; Ivanilov and Petrov 1970a, b; Aganbegyan et al. 1974; Aganbegyan and Valtukh 1975). However, we will not analyze all these models here, but will restrict ourselves to describing a multibranch industrial model named INTERLINK (Zimin 1976a, b, 1977, 1980), which is conceptually based on its predecessor, the π-model developed at the Computer Center of the USSR Academy of Sciences (Ivanilov and Petrov 1970a, b). The model presented below may be viewed as a simplified version of the original π-model.

3.1 Basic Model

3.1.1 State Equations

The system under consideration is broken down into two subsystems, describing production and the development of capacity (or capital stock accumulation).

Production subsystem. The operation of industry is described in terms of n producing sectors. Let

- $x_i(t)$ be the cumulative production in sector i ($i = 1,2, \ldots, n$) up to period t;
- $u_i(t)$ be the gross output (production level) of sector i during period t;
- $v_i(t)$ be the additional capital stock (plant, equipment, etc.) constructed in period t; and
- $a_{ij}(t)$ be the input–output coefficients (i.e. the number of units of i required to produce one unit of j).

In addition, we assume that

- τ_j is the time (number of periods) required to construct and put into operation additional capacity in sector j;
- $b_{ij}(\tau)$ are capital coefficients representing the amount of sector i products required to build unit capacity in sector j, to be available for production τ periods later;
- $w_i(t)$ is the final consumption of sector i products during period t; and
- $s_i(t)$ is the net amount of sector i products exported during period t.

Then the state equations describing the production subsystem can be written as follows

$$x_i(t+1) = x_i(t) + u_i(t) - \sum_{j=1}^{n} a_{ij}(t)u_j(t) - \sum_{j=1}^{n}\sum_{\tau=0}^{\tau_j} b_{ij}(\tau)v_j(t-\tau) - w_i(t) - s_i(t)$$

$$(i = 1, 2, \ldots, n;\ t = 0, 1, \ldots, T-1) \tag{3.1}$$

Initial inventories and preplanning controls are assumed to be given by

$$x_i(0) = x_i^0 \qquad (i = 1, 2, \ldots, n;\ t = 0, 1, \ldots, \tau_i - 1) \tag{3.2}$$

$$v_i(t-\tau_i) = v_i^0(t-\tau_i) \qquad (i = 1, 2, \ldots, n;\ t = 0, 1, \ldots, \tau_i - 1) \tag{3.3}$$

Assuming that $\tau_j = \bar{\tau}$ for all sectors $j (j = 1, 2, \ldots, n)$ eqn. (3.1) can be rewritten in matrix form

$$x(t+1) = x(t) + (\mathbf{I} - \mathbf{A}(t))u(t) - \sum_{\tau=0}^{\bar{\tau}} \mathbf{B}(\tau)v(t-\tau) - w(t) - s(t) \tag{3.1a}$$

where

$x(t) = \{x_i(t)\}$ is a state vector, $u(t) = \{u_i(t)\}$;
$v(t) = \{v_i(t)\}, w(t) = \{w_i(t)\}$ are control vectors; and
$s(t) = \{s_i(t)\}$ is considered here as an exogenous vector.

For some particular problems, the export/import variables must be considered as control (or decision) variables. In these cases the net export $s(t)$ is better represented as follows

$$s(t) = s^+(t) - s^-(t) \qquad (s^+(t) \geqslant 0, s^-(t) \geqslant 0)$$

where $s^+(t)$ is the import vector and $s^-(t)$ is the export vector.

Development of capacity subsystem. Let

$y_i(t)$ be the production capacity in sector i $(i = 1, 2, \ldots, n)$ at time t; and
$d_i(t)$ be the depreciation factor in sector i during period t.

Then the dynamics of production capacity may be written as follows

$$y_i(t+1) = (1 - d_i(t))y_i(t) + v_i(t-\tau_i) \qquad (i = 1, 2, \ldots, n) \tag{3.4}$$

The initial capital stocks (plant, equipment, etc.) are given as

$$y_i(0) = y_i^0 \tag{3.5}$$

Assuming again for simplicity that

$$\tau_i = \bar{\tau} \qquad \text{(for all } i)$$

we can rewrite eqn. (3.4) in matrix form

$$y(t+1) = [\mathbf{I} - \mathbf{D}(t)]\, y(t) + v(t - \bar{\tau}) \tag{3.4a}$$

where $\mathbf{D}(t)$ is a diagonal matrix with $d_i(t)$ on the main diagonal, and $y(t) = \{y_i(t)\}$ $(i = 1, 2, \ldots, n)$ is a state vector for the production capacity subsystem.

3.1.2 Constraints

It is evident that any economic system operates within certain constraints; this implies a range of physical, economic, institutional, and other limits to our choice of the control variables which we will use in the model.

Physical Sense. All state and control variables are nonnegative

$$\left.\begin{array}{c} u_i(t) \geqslant 0;\ v_i(t) \geqslant 0;\ w_i(t) \geqslant 0;\ x_i(t) \geqslant 0;\ y_i(t) \geqslant 0 \\ (i = 1, 2, \ldots, n;\ t = 0, 1, \ldots, T-1) \end{array}\right\} \tag{3.6}$$

Resource availability. The production system requires certain external resource inputs for its operation. At their most basic, these are inputs of labor and primary resources. Both constraints can be written in a similar way

(a) for labor resources

$$\sum_{j=1}^{n} l_{kj}(t) u_j(t) \leqslant l_k(t) \qquad (k = 1, 2, \ldots, K) \tag{3.7}$$

where

$l_k(t)$ is the total labor of category $k (k = 1, 2, \ldots, K)$ available in period t; and
$l_{kj}(t)$ are the labor output ratios for sector j.

(b) for other primary resources (described here as WELMM factors)

$$\sum_{j=1}^{n} r_{mj}(t) u_j(t) \leqslant r_m(t) \qquad (m = 1, 2, \ldots, M) \tag{3.8}$$

where

$r_m(t)$ is the total amount of resource category m (WELMM factor m) available during period t; and
$r_{mj}(t)$ are specific resource requirements per unit of sector j production (resource–output ratios) during period t.

In matrix form, inequalities (3.7) and (3.8) become

$$\mathbf{L}(t) u(t) \leqslant l(t) \tag{3.7a}$$

$$\mathbf{R}(t)u(t) \leqslant r(t) \tag{3.8a}$$

Production capacity. The gross output of each sector is limited by the available production capacity in that sector

$$u_i(t) \leqslant y_i(t) \qquad (i = 1, 2, \ldots, n) \tag{3.9}$$

or, in vector form

$$u(t) \leqslant y(t) \tag{3.9a}$$

Inventory. These constraints relate to the possibility of accumulating limited stocks of a given commodity*. For storable goods

$$0 \leqslant x_i(t) \leqslant \bar{x}_i(t) \tag{3.10}$$

where

$\bar{x}_i(t)$ are the given stock capacities; and
$x_i(t)$ are calculated from eqn. (3.1).

For nonstorable goods we write, instead of inequality (3.10)

$$u_i(t) - \sum_{j=1}^{n} a_{ij}(t)u_j(t) - \sum_{j=1}^{n} \sum_{\tau=0}^{\tau_j} b_{ij}(\tau)v_j(t-\tau) - w_i(t) - s_i(t) \geqslant 0 \tag{3.11}$$

or, in matrix form

$$[\mathbf{I} - \mathbf{A}(t)]\, u(t) - \sum_{\tau=0}^{\bar{\tau}} \mathbf{B}(\tau)v(t-\tau) - w(t) - s(t) \geqslant 0 \tag{3.12}$$

It should be stressed that, in many practical cases, the accumulation of large stocks of goods is either physically unreasonable or prohibitively expensive. Hence, $\{x_i(t)\}$ values are small in comparison to the outputs of the system. Therefore we can consider the balance equation (or bill of goods) in the form of an inequality [equivalent to inequality (3.12)]

$$[\mathbf{I} - \mathbf{A}(t)]\, u(t) \geqslant \sum_{\tau=0}^{\bar{\tau}} \mathbf{B}(\tau)v(t-\tau) + w(t) + s(t) \tag{3.13}$$

or as an equation

$$[\mathbf{I} - \mathbf{A}(t)]\, u(t) = \sum_{\tau=0}^{\bar{\tau}} \mathbf{B}(\tau)v(t-\tau) + w(t) + s(t) \tag{3.13a}$$

for both storable and nonstorable goods.

Consumption. Final consumption usually has limits for each sector i. In many cases it can be represented by an inequality of the form

*In addition, note that here we regard such resources as manpower and electricity as nonstorable goods.

$$w_i(t) \geqslant g_i(t)\,\omega(t) \tag{3.14}$$

where

$\omega(t)$ is the total final consumption of all goods; and

$g_i(t)$ is the share of total consumption provided by sector i.

The exogenously-given vector $\boldsymbol{g}(t) = \{g_i(t)\}$ $(i = 1, 2, \ldots, n)$ predefines the profile of final consumption over time. The introduction of a consumption profile allows one to use a scalar control $\omega(t)$ instead of the control vector $\boldsymbol{w}(t)$

$$\boldsymbol{w}(t) \geqslant \boldsymbol{g}(t)\,\omega(t) \tag{3.14a}$$

3.1.3 Objective Function

In the sections above, $\{\boldsymbol{u},\boldsymbol{v},\boldsymbol{w}\} = \{u_i(t), v_i(t), w_i(t)\}$ are control variables, and $\{\boldsymbol{x},\boldsymbol{y}\} = \{x_i(t), y_i(t)\}$ are state variables. The choice of optimal controls depends on the choice of the objective function for a particular problem. We will now consider typical examples of the objective function.

Maximization of the cumulative discounted-goods supply. In this case, the objective function (in monetary terms) is

$$J = \sum_{t=0}^{T-1} \beta(t)\,\omega(t) \tag{3.15}$$

where $\beta(t)$ is the discounting factor. If we consider only the last step of the planning horizon then the objective function (in terms of products) will be

$$J = \sum_{i=1}^{n} h_i^w(T) w_i(T) \tag{3.16}$$

where the $h_i^w(T)$ are weighting coefficients for different products.

Maximization of the final stock of goods

$$J = \sum_{i=1}^{n} h_i^x(T) x_i(T) \tag{3.17}$$

where the $h_i^x(T)$ are weighting coefficients ("costs") for $x_i(T)$.

Maximization of the terminal values of production capacity

$$J = \sum_{i=1}^{n} h_i^y(T) y_i(T) \tag{3.18}$$

where the $h_i^y(T)$ are weighting coefficients for $y_i(T)$.

Minimization of total expenses. This criterion is similar to the objective functions considered in Sections 1 and 2

$$J = \sum_{t=0}^{T-1} \beta(t)[(c^u(t), u(t)) + (c^\nu(t), \nu(t)) + (c^y(t), y(t))] \tag{3.19}$$

where

$c^u(t), c^y(t)$ are, respectively, operating and maintenance costs;
$c^\nu(t)$ is the investment cost; and
$\beta(t)$ is the discounting factor.

For storable goods [see inequalities (3.10)] it is desirable in some cases to extend eqn. (3.19) by including storage costs.

Other objective functions are of course also possible (Kantorovich 1965; V.V. Makarov 1966; Ivanilov and Petrov 1970a, b; Zimin 1976a, b, 1977, 1980). In addition, it should be noted that control targets can also be expressed by additional constraints, such as

$$\omega(T) \geqslant \bar{\omega}(T) \tag{3.20}$$

$$x(T) \geqslant \bar{x}(T) \tag{3.21}$$

$$y(T) \geqslant \bar{y}(T) \tag{3.22}$$

For example, one may wish to minimize the total costs [eqn. (3.19)] under a given level of final consumption as specified by inequality (3.20).

3.1.4 Statement of the Problem

For reference purposes we will now write down a typical optimization problem that frequently occurs in economic models.

Problem 3.1. Given the state equations of the production subsystem

$$\mathbf{x}(t+1) = \mathbf{x}(t) + [\mathbf{I} - \mathbf{A}(t)]\,\mathbf{u}(t) - \sum_{\tau=0}^{\bar{\tau}} \mathbf{B}(\tau)\,\boldsymbol{\nu}(t-\tau) - \mathbf{w}(t) - \mathbf{s}(t) \tag{3.1a}$$

and of the production-capacity subsystem

$$\mathbf{y}(t+1) = [\mathbf{I} - \mathbf{D}(t)]\,\mathbf{y}(t) + \boldsymbol{\nu}(t-\bar{\tau}) \tag{3.4a}$$

with initial conditions

$$\mathbf{x}(0) = \mathbf{x}^0 \tag{3.2a}$$

$$\boldsymbol{\nu}(t-\bar{\tau}) = \boldsymbol{\nu}^0(t-\bar{\tau}) \qquad (0 \leqslant t \leqslant \bar{\tau}-1) \tag{3.3a}$$

$$\mathbf{y}(0) = \mathbf{y}^0 \tag{3.5a}$$

find controls $\mathbf{u} = \{u(0), \ldots, u(T-1)\}$, $\boldsymbol{\nu} = \{\nu(0), \ldots, \nu(T-\bar{\tau}-1)\}$, and $\mathbf{w} = \{w(0), \ldots, w(T-1)\}$, and corresponding trajectories $\mathbf{x} = \{x(0), \ldots, x(T)\}$ and $\mathbf{y} = \{y(0), \ldots, y(T)\}$, which satisfy the following constraints

(a) nonnegativity

$$u(t) \geqslant 0;\; v(t) \geqslant 0;\; w(t) \geqslant 0;\; x(t) \geqslant 0;\; y(t) \geqslant 0 \tag{3.6a}$$

(b) labor availability

$$\mathbf{L}(t)u(t) \leqslant l(t) \tag{3.7a}$$

(c) resource availability

$$\mathbf{R}(t)u(t) \leqslant r(t) \tag{3.8a}$$

(d) production capacity

$$u(t) \leqslant y(t) \tag{3.9a}$$

(e) storable goods inventory

$$x(t) \leqslant \bar{x}(t) \tag{3.10a}$$

(f) nonstorable goods inventory

$$[\mathbf{I} - \mathbf{A}(t)]\, u(t) \geqslant \sum_{\tau=0}^{\bar{T}} \mathbf{B}(\tau)v(t-\tau) + w(t) + s(t) \tag{3.13a}$$

(g) consumption

$$w(t) \geqslant g(t)\omega(t) \tag{3.14a}$$

and maximize the objective function

$$J = \sum_{t=0}^{T-1} \beta(t)\omega(t) \tag{3.15}$$

3.2 Discussion

We will now consider some modifications and extensions of Problem 3.1.

3.2.1 Conversion Model

In many practical cases it is necessary to take into account the process of reconstruction (or conversion) of productive capacity (Ivanilov and Petrov 1970a, b). In this case three of the conditions given above should be replaced.

State equation (3.1) should be replaced by

$$x_i(t+1) = x_i(t) + u_i(t) - \sum_{j=1}^{n} a_{ij}(t)u_j(t) - \sum_{j=1}^{n}\sum_{\tau=0}^{T_j} b_{ij}(\tau)v_j(t-\tau)$$

$$- \sum_{j,s=1}^{n}\sum_{\tau=0}^{T_j^s} b_{ij}^s(\tau)v_j^s(t-\tau) - w_i(t) - s_i(t) \tag{3.23}$$

Here

$v_j^s(t)$ is the additional productive capacity in sector j obtained from conversion of some sector-s capacity started during period t;

$b_{ij}^s(t)$ are the capital coefficients of the conversion $s \rightarrow j$; and

τ_j^s is the number of steps required for the conversion $s \rightarrow j$.

The state equation (3.4) is replaced by

$$y_i(t+1) = [1 - d_i(t)]\, y_i(t) + v_i(t - \tau_i) - \sum_{s=1}^{n} \sum_{\tau=0}^{\tau_i^s - 1} v_i^s(t - \tau) + \sum_{s=1}^{n} k_i^s(\tau_i^s) v_i^s(t - \tau_i^s) \tag{3.24}$$

where $k_i^s(t)$ is the conversion coefficient, which shows the increase in the productive capacity in sector i per unit of conversion activity $s \rightarrow i$.

3.2.2 Capital Stock Subsystem

In some cases it is more convenient to describe the development of the production subsystem in terms of capital stock rather than in terms of productive capacity. In these cases, instead of state equations (3.4) or (3.24) we must introduce state equations

$$c_i(t+1) = [1 - \tilde{d}_i(t)]\, c_i(t) + v_i(t - \tau_i) - \sum_{s=1}^{n} \sum_{\tau=0}^{\tau_i^s - 1} v_s^i(t - \tau) + \sum_{s=1}^{n} v_i^s(t - \tau_i^s) \tag{3.25}$$

where

$c_i(t)$ is the capital stock in sector i during period t; and

$\tilde{d}_i(t)$ is the depreciation factor.

In addition, the production capacity constraints (3.9) are replaced by

$$\gamma_i(t) u_i(t) \leqslant c_i(t) \qquad (i = 1, 2, \ldots, n) \tag{3.26}$$

where $\gamma_i(t)$ is the capital–output ratio. Finally, if no conversion activities are taking place in the system, then the last term on the right-hand side of eqn. (3.25) should be omitted.

3.2.3 Simplified Model

We will now describe a simplified version of Problem 3.1, which may be of interest for more long-range planning and more aggregated systems, such as the case of linking energy and economy submodels. To simplify the model we assume that the period is such that time lags can be ignored and we rule out the possibility of building up stocks of goods; furthermore, we do not consider conversion or reconstruction processes. With these assumptions, the problem can be formulated as follows.

Problem 3.1a. Given the state equations for the capital stock subsystem in the form

$$c(t+1) = [\mathbf{I} - \mathbf{D}(t)]\, c(t) + v(t)$$

with an initial state

$$c(0) = c^0$$

and subject to the following constraints

(a) balance equations

$$[\mathbf{I} - \mathbf{A}(t)]\, u(t) = \mathbf{B}(t)\nu(t) + w(t) + s(t)$$

(b) resource availability

$$\mathbf{L}(t)u(t) \leqslant l(t)$$

$$\mathbf{R}(t)u(t) \leqslant r(t)$$

(c) production

$$\Gamma u(t) \leqslant c(t)$$

(d) consumption

$$w(t) \geqslant g(t)\omega(t)$$

find controls $\{\nu(t),\ u(t),\ \omega(t)\}$, and a corresponding trajectory $\{c(t)\}$, which maximize the objective function

$$J = \sum_{t=0}^{T-1} \beta(t)\omega(t)$$

3.2.4 INTERLINK Model

The INTERLINK model was developed at the International Institute for Applied Systems Analysis (IIASA) by Zimin for modeling the economic development of a region (or country) in the IIASA system of energy development models. It represents a version of the dynamic multisector π-model (Ivanilov and Petrov 1970a, b); its structure is close to that outlined in Problem 3.1 and it is described in detail elsewhere (Zimin 1976a, b, 1977, 1980).

The typical dimensions of the INTERLINK model are as follows: there are 17 state equations (representing sectors of the economy) and 41 constraints for each period. Each period is five years long and there are ten such periods, giving a total planning horizon of 50 years. The corresponding linear programming problem has approximately 600 rows and 600 columns.

4 LINKING THE MODELS

In earlier sections of this report we considered three different models – of the energy supply system, of the primary resources system, and of the economic development system; the most important features of each model were formally presented in Sections 1, 2, and 3, respectively. Each of these models can be used individually for the assessment of energy, resources, and the development of various technologies.

However, this approach of separate, "piecemeal" analysis is limited in its possibilities because many important features of the systems which derive from their interactions with one another are missing. To overcome these deficiencies we need to build models of the whole interacting energy–resources–economy system; we must therefore investigate ways of linking individual models into a coherent whole. This new stage of energy-policy modeling has started relatively recently (A.A. Makarov and Melentjev 1973; Dantzig 1975a; Dantzig and Parikh 1975; Belyaev et al. 1976; Behling et al. 1977; Häfele and A.A. Makarov 1977; Hitch 1977; Hoffman and Jorgenson 1977; Kononov 1977; A.A. Makarov 1977; Manne 1977). Two basic approaches* can be singled out here. In the first approach separate models are integrated into a single optimization problem with one corresponding objective function (Dantzig 1975a, b; Dantzig and Parikh 1975; Dantzig 1976). The second approach is to investigate manually linking a number of independent submodels, each with its own objective function (Behling et al. 1977; Häfele and A.A. Makarov 1977; Hoffman and Jorgenson 1977; A.A. Makarov 1977; Manne 1977).

Both approaches naturally have their own advantages and drawbacks. The major advantage of the first, "machine" approach is that it allows us to take into account all the constraints and interactions between the many factors which influence a given decision and to combine them in some "optimal" way. However, building an integrated model obviously leads to a very large optimization problem which, although sometimes possible to solve, is always very difficult to interpret.

The second, "manual" approach – in which information obtained from one submodel is interpreted by an analyst and provided as input to another submodel – is more attractive but is much more time consuming and may sometimes lead to uncertainty as to whether the "truly optimal" solution for the whole system has been obtained. Later in the report we will refer to this as the "iterative" approach.

It seems sensible to combine the best features of each approach and we will now consider each in turn, starting with the integrated model.

4.1 Integrated Model

Considering the ESS and the economy models, we can see (Figure 2) that there are two main links between them: the final demand for energy, which is an output of the economy model, and the demands for nonenergy resources, which are outputs of the ESS model. We will combine the ESS model (Problem 1.1) and the economy model (Problem 3.1) into one overall system, using the subscripts E for the energy sector and NE for the nonenergy sectors.

For uniformity of presentation we assume that the industrial processes of both the economic and the energy sectors may be described in terms of physical flows. Furthermore, in the model developed below we omit, for simplicity, time lags in the construction and putting into operation of production capacity; in other words, we will use simplified versions of the ESS and economy models.

* "Non-optimization" approaches fall outside the scope of this report and are therefore not considered here (see Hitch 1977).

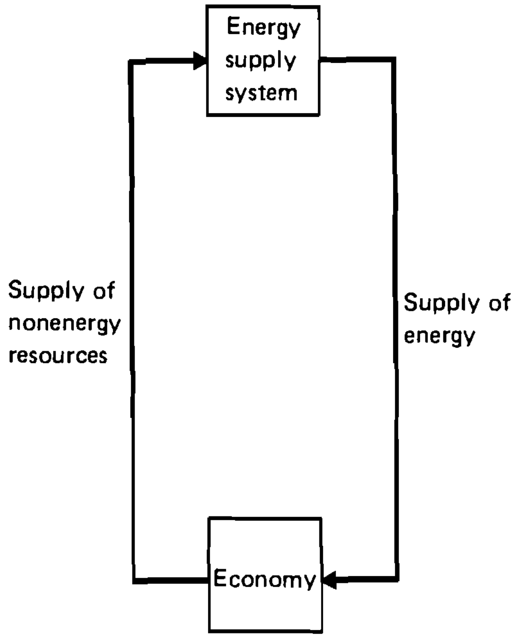

FIGURE 2 Linkage of energy supply and economy models.

4.1.1 State Equations

Production subsystem. This is a combination of state equations (1.1a) and (3.4a), for the energy and nonenergy sectors, respectively, in their simplified form (we describe the depreciation of capacity in the same way for both equations)

$$y_E(t+1) = [\mathbf{I} - \Delta_E(t)]\, y_E(t) + \nu_E(t) \tag{4.1}$$

$$y_{NE}(t+1) = [\mathbf{I} - \Delta_{NE}(t)]\, y_{NE}(t) + \nu_{NE}(t) \tag{4.2}$$

with initial states

$$y_E(0) = y_E^0 \tag{4.3}$$

$$y_{NE}(0) = y_{NE}^0 \tag{4.4}$$

Here $y_E(t)$ and $y_{NE}(t)$ are vectors of production capacity for the energy and nonenergy sectors, and $\nu_E(t)$ and $\nu_{NE}(t)$ are the increases of capacity in these sectors during period t. $\Delta_E(t)$ and $\Delta_{NE}(t)$ are diagonal matrices whose elements are the corresponding depreciation factors.

Energy resource consumption subsystem. To describe the cumulative consumption of primary energy resources we will first use eqn. (1.5a) (instead of the more-detailed version given in Problem 2.1)

$$z_E(t+1) = z_E(t) + Q_E(t)u_E(t) \tag{4.5}$$

$$z_E(0) = z_E^0 \tag{4.6}$$

$$0 \leqslant z_E(t) \leqslant \bar{z}_E(t) \tag{4.7}$$

Here

$z_E(t)$ is the vector of cumulative amounts of primary energy resources extracted at the beginning of period t

$u_E(t)$ is the vector of activities in the energy sector.

The upper limits $\bar{z}_E(t)$ may be estimated from the resource model (see Section 2).

4.1.2 Constraints

The most important constraint in the model is the balance between the production of goods and their consumption. As in the simplified version of the economy model (Problem 3a), we rule out the possibility of building up stocks of goods, and therefore consider the static form of these conditions. For energy output

$$-\mathbf{A}_{NE}^{E}(t)u_{NE}(t) + [\mathbf{I} - \mathbf{A}_{E}^{E}(t)]u_E(t) = \mathbf{B}_{NE}^{E}(t)v_{NE}(t) + \mathbf{B}_{E}^{E}(t)v_E(t) + w_E(t) + s_E(t) \tag{4.8}$$

and for nonenergy products

$$[\mathbf{I} - \mathbf{A}_{NE}^{NE}(t)]u_{NE}(t) - \mathbf{A}_{E}^{NE}u_E(t) = \mathbf{B}_{NE}^{NE}(t)v_{NE}(t) + \mathbf{B}_{E}^{NE}(t)v_E(t) + w_{NE}(t) + s_{NE}(t) \tag{4.9}$$

We also have production-capacity constraints for energy sectors

$$u_E(t) \leqslant y_E(t) \tag{4.10}$$

and for nonenergy sectors

$$u_{NE}(t) \leqslant y_{NE}(t) \tag{4.11}$$

[essentially similar in form to inequalities (1.4) and (3.9), respectively].

Labor-availability constraints (3.7) are written in the form

$$\mathbf{L}_{NE}(t)u_{NE}(t) + \mathbf{L}_E(t)u_E(t) \leqslant l(t) \tag{4.12}$$

and the constraints on WELMM factors [cf. inequality (3.8)] as

$$\mathbf{R}_{NE}(t)u_{NE}(t) + \mathbf{R}_E(t)u_E(t) \leqslant r(t) \tag{4.13}$$

Final consumption constraints (3.14) can be written as

$$w_E(t) \geq g_E(t)\,\omega(t) \tag{4.14}$$

$$w_{NE}(t) \geq g_{NE}(t)\,\omega(t) \tag{4.15}$$

where the given vectors $g_{NE}(t)$ and $g_E(t)$ specify profiles of final consumption for non-energy and energy products, respectively.

Finally, all the variables are obviously nonnegative

$$\begin{aligned} &u_{NE}(t) \geq 0;\; u_E(t) \geq 0;\; v_{NE}(t) \geq 0;\; v_E(t) \geq 0; \\ &y_{NE}(t) \geq 0;\; y_E(t) \geq 0;\; z_E(t) \geq 0;\; \omega(t) \geq 0 \end{aligned} \tag{4.16}$$

4.1.3 Statement of the Problem

We therefore obtain the following optimization problem.

Problem 4.1. Given the state equations

$$y_E(t+1) = [\mathbf{I} - \Delta_E(t)]y_E(t) + v_E(t) \tag{4.1}$$

$$y_{NE}(t+1) = [\mathbf{I} - \Delta_{NE}(t)]y_{NE}(t) + v_{NE}(t) \tag{4.2}$$

with initial states

$$y_E(0) = y_E^0 \tag{4.3}$$

$$y_{NE}(0) = y_{NE}^0 \tag{4.4}$$

find controls $\{v_E(t)\}$, $\{v_{NE}(t)\}$ and $\{u_E(t)\}$, $\{u_{NE}(t)\}$, and corresponding trajectories $\{y_E(t), y_{NE}(t)\}$, which satisfy the following constraints

(a) balance equations

$$[\mathbf{I} - \mathbf{A}_E^E(t)]u_E(t) - \mathbf{A}_{NE}^E(t)u_{NE}(t) = \mathbf{B}_E^E(t)v_E(t) + \mathbf{B}_{NE}^E(t)v_{NE}(t) + w_E(t) + s_E(t) \tag{4.8}$$

$$\begin{aligned} -\mathbf{A}_E^{NE}(t)u_E(t) + [\mathbf{I} - \mathbf{A}_{NE}^{NE}(t)]\,u_{NE}(t) = \mathbf{B}_E^{NE}(t)v_E(t) + \mathbf{B}_{NE}^{NE}(t)v_{NE}(t) \\ + w_{NE}(t) + s_{NE}(t) \end{aligned} \tag{4.9}$$

(b) production capacity

$$u_E(t) \leq y_E(t) \tag{4.10}$$

$$u_{NE}(t) \leq y_{NE}(t) \tag{4.11}$$

(c) primary energy resource availability

$$z_E(t+1) = z_E(t) + \mathbf{Q}_E(t)u_E(t) \tag{4.5}$$

$$z_E(0) = z_E^0 \tag{4.6}$$

$$z_E(t) \leqslant \bar{z}(t) \tag{4.7}$$

(d) labor availability

$$\mathbf{L}_E(t)u_E(t) + \mathbf{L}_{NE}(t)u_{NE}(t) \leqslant l(t) \tag{4.12}$$

(e) WELMM factor availability

$$\mathbf{R}_E(t)u_E(t) + \mathbf{R}_{NE}(t)u_{NE}(t) \leqslant r(t) \tag{4.13}$$

(f) final consumption

$$w_E(t) \geqslant g_E(t)\omega(t) \tag{4.14}$$

$$w_{NE}(t) \geqslant g_{NE}(t)\omega(t) \tag{4.15}$$

(g) nonnegativity

$$\begin{aligned} &u_E(t) \geqslant 0;\; u_{NE}(t) \geqslant 0;\; v_E(t) \geqslant 0;\; v_{NE}(t) \geqslant 0; \\ &y_E(t) \geqslant 0;\; y_{NE}(t) \geqslant 0;\; z_E(t) \geqslant 0;\; \omega(t) \geqslant 0 \end{aligned} \tag{4.16}$$

and which maximize the objective function*

$$J = \sum_{t=0}^{T-1} \beta(t)\omega(t) \tag{4.17}$$

Problem 4.1 is, once again, a DLP model. Its solution , in principle, permits us to investigate the interactions between a (more-detailed) energy sector and the nonenergy sectors of an economy. As mentioned above, we can solve Problem 4.1 as one overall DLP problem, or we can solve it by an iterative procedure, paying special attention to the links between the ESS and the economy parts of the integrated model.

Clearly, in much the same way, the more-detailed statement of the resources model (Problem 2.1) may be included in the integrated model instead of using the simplified eqns. (4.5)–(4.7). We will not, however, develop this possibility here.

In the integrated model there is one important feature which, although clearly visible in the scalar representation, cannot be seen explicitly from the matrix formulation of Problem 4.1. In practise, each of the individual models which are to be integrated into a

* This particular objective function is chosen only for illustrative purposes. Many other objectives are of course of interest for this integrated model.

system may have different levels of aggregation. Moreover, if we are investigating the influence of ESS on economic development, the ESS model should be presented in much more detail than the economy model. For this particular case, a special model has been developed (see below) which determines the influence (or impact) of energy developments upon the economy as a whole.

Therefore, when attempting the linkage of energy, resources, and economy models, one must take into account first, the means of linkage (machine or man–machine), and second, the level of aggregation and specific features of each individual model.

4.2 Iterative Approach

We now consider the iterative interaction between ESS and economy model. The general scheme is as follows.

On examining the integrated model described earlier (Problem 4.1), we see that it is basically the economy model (Problem 3.1) partitioned into energy (E) and nonenergy (NE) sectors. On the other hand, the ESS model is embedded in the integrated model. In fact, eqns. (4.1), (4.3), (4.5)–(4.7), (4.11), and (4.14) are the same as in the Problem 1.1 formulation.

If we define the demand $d_E(t)$ for secondary energy by

$$d_E(t) = \mathbf{A}_{NE}^{E}(t)u_{NE}(t) + \mathbf{B}_{NE}^{E}(t)v_{NE}(t) + w_E(t) + s_E(t) \tag{4.18}$$

and let

$$\mathbf{D}_E(t) = [\mathbf{I} - \mathbf{A}_E^E(t)] \tag{4.19}$$

then we can rewrite eqn. (4.8) as

$$\mathbf{D}_E(t)u_E(t) = d_E(t) + \mathbf{B}_E^E(t)v_E(t) \tag{4.19}$$

which, because of the smallness of the last term on the right-hand side, is similar to the demand constraints (1.11) of the ESS model.

Let us further write down the requirements of the ESS for nonenergy products as follows

$$f_E^{NE}(t) = \mathbf{B}_E^{NE}(t)v_E(t) + \mathbf{A}_E^{NE}(t)u_E(t) \tag{4.20}$$

Taking into account that the amounts of nonenergy products required for the operation and maintenance of energy production systems [the second term on the right-hand side of eqn. (4.20)] are small in comparison with the requirements for construction [the first term on the right-hand side of eqn. (4.20)], it can be seen from eqn. (4.20) and inequality (1.9), that

$$\mathbf{F}(t) = \mathbf{B}_E^{NE}(t)$$

Therefore, we can rewrite eqn. (4.9) as

$$[\mathbf{I} - \mathbf{A}_{\mathrm{NE}}^{\mathrm{NE}}(t)]\, \boldsymbol{u}_{\mathrm{NE}}(t) = f_{\mathrm{NE}}^{\mathrm{NE}}(t) + f_{\mathrm{E}}^{\mathrm{NE}}(t) \tag{4.21}$$

where

$$f_{\mathrm{NE}}^{\mathrm{NE}}(t) = \mathbf{B}_{\mathrm{NE}}^{\mathrm{NE}}(t) v_{\mathrm{NE}}(t) + w_{\mathrm{NE}}(t) + s_{\mathrm{NE}}(t) \tag{4.22}$$

and $f_{\mathrm{E}}^{\mathrm{NE}}(t)$ is defined from eqn. (4.20).

Thus, eqn. (4.19) represents the supply of energy required for the energy sector and, as was mentioned above, is equivalent to the demand constraint (1.11) with $d_{\mathrm{E}}(t)$ fixed; and constraint (4.20) represents the amounts of nonenergy products required by the ESS for a fixed value of $f_{\mathrm{E}}^{\mathrm{NE}}(t)$.

On the other hand, eqns. (4.18) and (4.22), respectively, represent the demands for energy and nonenergy products in the rest of the economy, while eqn. (4.21) shows the supply of goods from the nonenergy sectors.

In addition, we can rewrite constraints (4.12) and (4.13) in the following form

$$\mathbf{L}_{\mathrm{E}}(t)\boldsymbol{u}_{\mathrm{E}}(t) = l_{\mathrm{E}}(t) \tag{4.23}$$

$$\mathbf{L}_{\mathrm{NE}}(t)\boldsymbol{u}_{\mathrm{NE}}(t) = l_{\mathrm{NE}}(t) \tag{4.24}$$

$$l_{\mathrm{E}}(t) + l_{\mathrm{NE}}(t) \leqslant l(t) \tag{4.25}$$

$$\mathbf{R}_{\mathrm{E}}(t)\boldsymbol{u}_{\mathrm{E}}(t) = r_{\mathrm{E}}(t) \tag{4.26}$$

$$\mathbf{R}_{\mathrm{NE}}(t)\boldsymbol{u}_{\mathrm{NE}}(t) = r_{\mathrm{NE}}(t) \tag{4.27}$$

$$r_{\mathrm{E}}(t) + r_{\mathrm{NE}}(t) \leqslant r(t) \tag{4.28}$$

Finally, we find that eqns. (4.1), (4.3), (4.5)–(4.7), (4.10), (4.14), (4.19), (4.20), (4.23), and (4.26), with variables $d_{\mathrm{E}}(t)$, $f_{\mathrm{E}}^{\mathrm{NE}}(t)$, $l_{\mathrm{E}}(t)$, and $r_{\mathrm{NE}}(t)$ given exogenously, give a complete description of the ESS model; similarly, eqns. (4.2), (4.4), (4.11), (4.15), (4.18), (4.21), (4.22), (4.24), and (4.27), with variables $d_{\mathrm{E}}(t)$, $f_{\mathrm{E}}^{\mathrm{NE}}(t)$, $l_{\mathrm{NE}}(t)$, and $r_{\mathrm{NE}}(t)$ given exogenously, describe the rest of the economy.

In the integrated model (Problem 4.1), variables $d_{\mathrm{E}}(t)$, $f_{\mathrm{E}}^{\mathrm{NE}}(t)$, $f_{\mathrm{NE}}^{\mathrm{NE}}(t)$, $l_{\mathrm{E}}(t)$, $l_{\mathrm{NE}}(t)$, $r_{\mathrm{E}}(t)$, and $r_{\mathrm{NE}}(t)$, should be considered as endogenous; in this case constraints (4.21), (4.25), and (4.28) are coupling constraints and the variables just mentioned [$d_{\mathrm{E}}(t)$, etc.] are coupling variables.

Let us assume that we have some initial estimate of the energy demand $\bar{d}_{\mathrm{E}}(t)$ for a given planning period $0 \leqslant t \leqslant T-1$. Solving the ESS model (Problem 1.1) for this demand, we can calculate the required increases in capacity $\bar{v}_{\mathrm{E}}(t)$ of the ESS during the period, and the corresponding values for the production capacity $\bar{y}_{\mathrm{E}}(t)$ and output (degrees of utilization) $\bar{u}_{\mathrm{E}}(t) \leqslant \bar{y}_{\mathrm{E}}(t)$.

The requirements of the ESS in nonenergy resources, $\bar{f}_{\mathrm{E}}^{\mathrm{NE}}(t)$, are calculated from eqn. (4.21). Now we can solve the economy model (Problem 3.1) or the integrated model

(Problem 4.1) with fixed $\bar{u}_E(t)$, $\bar{v}_E(t)$, $\bar{y}_E(t)$, subject to a certain set of assumptions about the future development of the overall economy.

This solution yields degrees of utilization (gross outputs) $\bar{u}_{NE}(t)$ and the additional capital investments $\bar{v}_{NE}(t)$ required in the nonenergy sectors as well as a new value $\bar{d}^*_E(t)$ for the corresponding demand for energy [calculated from eqn. (4.18)]. If the old $\bar{d}_E(t)$ and new $\bar{d}^*_E(t)$ values for energy demand coincide, the procedure terminates; if the values do not coincide, then we must repeat the iteration with a recalculated demand.

Generally speaking, the solution obtained in such a way (if the process converges) is not an optimal solution for Problem 4.1, but is often acceptable because it satisfies all the constraints of the problem and optimizes (separately) two objectives [for example (1.12) and (3.15)] for the energy and nonenergy sectors.

To obtain an optimal solution for the whole of Problem 4.1 by an iterative procedure, one may use different methods of decomposition. In this case the dual variables (marginal estimates), obtained from the solution of the economy model, define the corresponding objective function for the ESS model [instead of using eqn. (1.12)]. The actual convergence behavior depends on the procedure used and the method of implementation. It should also be noted that for this procedure to be implemented the economy model should be sufficiently disaggregated in order to provide the ESS model with shadow prices in sufficient detail.

But, in practice, a single "optimal" solution of Problem 4.1 is not very valuable – regardless of whether it has been obtained "automatically" by applying the simplex method to Problem 4.1, or in some iterative way. Clearly, such a complex system requires a man–machine iterative procedure with a detailed energy–economy analysis composed of separate iterations. Let us now examine the points where human intervention is appropriate. These are as follows

- Changing the objective function for the overall Problem 4.1 and for the ESS model (Problem 1.1). [In fact, this is a vector-optimization problem (Alta Conference 1975)].
- Determining the energy demand $d_E(t)$ not from eqn. (4.18), but rather from a special energy-demand model (see for example Beaujean et al. 1977).
- Determining the nonenergy resource requirements $f_E^{NE}(t)$ for the ESS by using a special model (see Kononov and Tkachenko 1975).
- Changing the parameters of the model (especially those associated with assumptions on technological innovation and profiles of consumption).

Many of these points of human intervention may be considered as attempts to take into account nonlinearities of the system.

It should be noted finally that the methodological problems of linking separate models into coherent overall systems are of great practical importance and have not yet been sufficiently investigated. Some of these questions are discussed at greater length by Kallio et al. (1979).

4.3 Discussion

4.3.1 Pilot Model

This model (Dantzig 1975a, b; Dantzig and Parikh 1975; Dantzig 1976) has been developed by Dantzig and Parikh at Stanford University. It is a DLP model on a pilot scale that describes, in physical terms, various technological interactions within the sectors of the US economy, including a detailed energy sector.

The basic structure of the model is quite similar to that described by Problem 4.1. Dynamic equations include capacity-balance constraints, retraining of labor force constraints. and constraints on raw energy reserves, cumulative discoveries, amounts produced, and intermediate energy stocks.

The capacity-balance constraints are equivalent to eqns. (4.1) and (4.2). The retraining of labor force constraints specify educational and training capacities of the country modeled and are written in the form [compare inequalities (1.27) and (1.34) in the DESOM model described in Section 1.2.4]

$$\boldsymbol{p}(t+1) \leqslant \beta \boldsymbol{p}(t)$$

where the manpower vector $\boldsymbol{p}(t)$ is partitioned into skill groups.

The resource constraints are similar to constraints (2.24) and (2.25) and are intended to allow the inclusion of accurate values for the energy reserves, cumulative discoveries (and amounts produced), and stocks.

The various static constraints represent energy-demand requirements, energy-processing and operating-capacity limitations, and environmental aspects. The energy and non-energy sectors are linked by the balance equation constraints (4.8) and (4.9).

The objective function of the model maximizes the discounted vector of goods received per person, summed over time. It can be expressed as

$$J = \sum_{t=1}^{T} \lambda(t)[\mathbf{M}(t), \boldsymbol{p}(t)]$$

where the matrix $\mathbf{M}(t)$ represents the consumption levels and the vector $\boldsymbol{p}(t)$ is the distribution of the population over different income levels.

When finally completed, the detailed model will include an 87-sector input–output matrix, and the possibility of modeling the energy sector using approximately 150 equations per period. Thus, the number of constraints for each period in an integrated model with a reasonable level of detail may be of the order of 400: 87 for industrial activity, 2 X 87 for capacity constraints, and about 150 for a detailed energy sector. A 20–25-period model (for example, one covering a 75-year planning horizon in 3-year periods) would therefore have between 8,000 and 10,000 constraints.

As noted by Dantzig (1976), such LP models would be among the largest built to date. Therefore as a first step, a much smaller model which (Dantzig 1976) "incorporates many, if not all, of the essential features of its larger counterpart" has been attempted. This pilot model is expected to have about 130 equations per period. For a 30-year model (ten periods of three years each), there will be between 1,250 and 1,400 equations. Initially, the model will be solved using the straightforward simplex method.

4.3.2 IMPACT Model

This is an extension of the model developed by Kononov and Tkachenko at the Siberian Power Institute (Kononov and Tkachenko 1975; Kononov and Por 1979). The model is designed to investigate the influence upon other branches of the national economy of long-term changes in technology and the structure and rate of energy development.

The model is described by the following equations [for more details, see Kononov and Por (1979)].

The direct requirements of the ESS for nonenergy products are given by

$$f_{\mathrm{E}}^{\mathrm{NE}}(t) = \mathbf{A}_{\mathrm{E}}^{\mathrm{NE}}(t) \boldsymbol{u}_{\mathrm{E}}(t) + \sum_{\tau=0}^{\bar{\tau}} \mathbf{B}_{\mathrm{E}}^{\mathrm{NE}}(t-\tau) \boldsymbol{v}_{\mathrm{E}}(t-\tau) \tag{4.29}$$

If we neglect the time lags $\bar{\tau}$ in construction, then eqn. (4.20) is obtained. In the original version of the IMPACT model (Kononov and Por 1979), a "carried forward" presentation is used; in other words

$$f_{\mathrm{E}}^{\mathrm{NE}}(t) = \mathbf{A}_{\mathrm{E}}^{\mathrm{NE}}(t) \boldsymbol{u}_{\mathrm{E}}(t) + \sum_{\tau=t}^{t+\bar{\tau}} \tilde{\mathbf{B}}_{\mathrm{E}}^{\mathrm{NE}}(\tau-t) \boldsymbol{v}_{\mathrm{E}}(\tau) \tag{4.29a}$$

where the matrix $\tilde{\mathbf{B}}_{\mathrm{E}}^{\mathrm{NE}}(\tau - t)$ denotes the contribution for the construction of additional capacity to be put into operation during period $\bar{\tau}$, where $t \leqslant \tau \leqslant t + \bar{\tau}$.

Total (direct and indirect) product (material, equipment, etc.) requirements are derived from eqn. (4.9) [or from eqn. (4.21), where $f_{\mathrm{E}}^{\mathrm{NE}}(t)$ and $f_{\mathrm{NE}}^{\mathrm{NE}}(t)$ are obtained from eqns. (4.22) and (4.29), respectively]

$$[\mathbf{I} - \mathbf{A}_{\mathrm{NE}}^{\mathrm{NE}}(t)] \boldsymbol{u}_{\mathrm{NE}}(t) = \mathbf{B}_{\mathrm{NE}}^{\mathrm{NE}}(t) \boldsymbol{v}_{\mathrm{NE}}(t) + f_{\mathrm{E}}^{\mathrm{NE}}(t) + w_{\mathrm{NE}}(t) + s_{\mathrm{NE}}(t) \tag{4.30}$$

Using $\boldsymbol{v}_{\mathrm{NE}}(t)$ and $\boldsymbol{v}_{\mathrm{E}}(t)$, one can also calculate the total direct and indirect capital investments. In addition, the model includes several equations for evaluating direct and indirect expenditures of WELMM resources.

The model operates in the following way. Problem 1.1 for the given demand $\bar{d}_{\mathrm{E}}(t)$ for secondary energy is solved. Initially, the nonenergy resource constraints (1.9) are not taken into account. The solution of the problem gives the values $\bar{u}_{\mathrm{E}}(t)$ and $\bar{v}_{\mathrm{E}}(t)$, which are inputs for the IMPACT model. Using eqn. (4.29), one can calculate $\bar{f}_{\mathrm{E}}^{\mathrm{NE}}(t)$ for given $\bar{u}_{\mathrm{E}}(t)$ and $\bar{v}_{\mathrm{E}}(t)$. Substituting $\bar{f}_{\mathrm{E}}^{\mathrm{NE}}(t)$ into eqn. (4.30) and solving the linear equations (4.24) with certain additional conditions (Kononov and Por 1979)*

$$\boldsymbol{v}_{\mathrm{NE}}(t) = \max\{\min_{\tau < t} [\boldsymbol{u}_{\mathrm{NE}}(t) - \boldsymbol{u}_{\mathrm{NE}}(\tau)]\,;\ 0\}$$

one can find the indirect investment $\boldsymbol{v}_{\mathrm{NE}}(t)$ in the economy which the ESS needs to meet the given demand $\bar{d}_{\mathrm{E}}(t)$.

Note that we have only described here the general scheme of the IMPACT model. The particular implementation of this model depends greatly on the specific details of the ESS and economy models to be linked.

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* It is assumed here that capital stock is not dismantled and does not wear out.

4.3.3 SPI Model

The interactions between the energy and nonenergy sectors of the national economy have also been analyzed at the Siberian Power Institute, part of the Siberian Branch of the USSR Academy of Sciences. For this analysis a special multisector model has been developed (A.A. Makarov 1977). The model describes the interactions of the energy (E) sector with those nonenergy (NE) sectors which directly or indirectly influence the energy sector. There are eight such nonenergy sectors producing a total of 31 types of product.

The mathematical formulation of the model is close to that described by Problem 4.1 [note that we use here a somewhat different notation from that in the original version of the model (A.A. Makarov 1977)].

The development of the production subsystem is described by state equations which are similar to eqns. (4.1) and (4.2)

$$\sum_e y_{ie\mathrm{E}}(t+1) = \sum_e \delta_i(t) y_{ie\mathrm{E}}(t) + \sum_e v_{ie\mathrm{E}}(t)$$

$$y_{i\mathrm{NE}}(t+1) = \delta_i(t) y_{i\mathrm{NE}}(t) + v_{i\mathrm{NE}}(t)$$

where $\delta_i(t)$ is a depreciation factor. Note that the subscript e used in equations for the energy sector denotes e different technologies for energy production. Thus the energy sector is represented in a more disaggregated form in comparison to the nonenergy sectors of the model.

The balance equations are written in the dynamic form [compare eqns. (3.1), (4.8), and (4.9)]

for the nonenergy sectors

$$z_{i\mathrm{NE}}(t+1) = z_{i\mathrm{NE}}(t) + a_{i\mathrm{NE}}(t) y_{i\mathrm{NE}}(t) - \sum_j a_{ij\mathrm{NE}}(t) y_{j\mathrm{NE}}(t)$$
$$- \sum_j \sum_\tau b_{ij\mathrm{NE}}(t+\tau) v_{j\mathrm{NE}}(t+\tau) - w_{i\mathrm{NE}}(t) - s_{i\mathrm{NE}}(t)$$

for the energy sector

$$z_{i\mathrm{E}}(t+1) = z_{i\mathrm{E}}(t) + \sum_e a_{ie\mathrm{E}}(t) y_{ie\mathrm{E}}(t) - \sum_j a_{ij\mathrm{E}}(t) y_{j\mathrm{NE}}(t) - w_{i\mathrm{E}}(t) - s_{i\mathrm{E}}(t)$$

(For the energy sector the stocks are fuels.)

Here $z_{i\mathrm{NE}}(t)$ and $z_{i\mathrm{E}}(t)$ are the production inventories for the nonenergy and energy sectors, respectively, at the beginning of period t; $a_{i\mathrm{NE}}(t)$ and $a_{ie\mathrm{E}}(t)$ are loading coefficients of production capacity, hence

$$u_{i\mathrm{NE}}(t) = a_{i\mathrm{NE}}(t) y_{i\mathrm{NE}}(t)$$

$$u_{ie\mathrm{E}}(t) = a_{ie\mathrm{E}}(t) y_{ie\mathrm{E}}(t)$$

where $u_{i\mathrm{NE}}(t)$ and $u_{ie\mathrm{E}}(t)$ are the production levels (gross outputs) during period t.

As in the IMPACT model, a "carried forward" ($\tau > 0$) presentation of the requirements for construction is used.

Constraints on manpower and other limited resources are given in a similar way to inequalities (4.12) and (4.13)

$$\sum_i \sum_e r^{\mathrm{E}}_{vie}(t) y_{ie\mathrm{E}}(t) + \sum_e \sum_\tau r^{\mathrm{E}}_{vie}(t,\tau) v_{ie\mathrm{E}}(\tau) + r^{\mathrm{NE}}_{vi}(t) y_{i\mathrm{NE}}(t)$$

$$+ \sum_\tau r^{\mathrm{NE}}_{vi}(t,\tau) v_{i\mathrm{NE}}(\tau) \leqslant r_v(t)$$

The model is solved using an iterative mode.

5 DLP CANONICAL FORM

On considering the models described above, we can see that all of them can be reduced to a single canonical form (Propoi 1973, 1976).

Problem 5.1. Given the state equations

$$x(t+1) = \mathrm{A}(t)x(t) + \sum_{\tau=0}^{\bar{\tau}} \mathrm{B}(\tau)u(t-\tau) \tag{5.1}$$

with initial conditions

$$x(0) = x^0; \ u(t-\tau) = u^0(t-\tau) \qquad (0 \leqslant \tau \leqslant t-1) \tag{5.2}$$

and constraints

$$\mathrm{G}(t)x(t) + \mathrm{D}(t)u(t) \leqslant f(t) \tag{5.3}$$

$$x(t) \geqslant 0; \ u(t) \geqslant 0 \tag{5.4}$$

find the control $u = \{u(0), \ldots, u(T-1-\tau)\}$ and the corresponding trajectory $x = \{x(0), \ldots, x(T)\}$, which maximize the objective function

$$J(u) = [a(T), x(T)] + \sum_{t=0}^{T-1} [(a(t), x(t)) + (b(t), u(t))] \tag{5.5}$$

Here the $\{u(t)\}$ are control variables and the $\{x(t)\}$ are state variables.

One can see that either all the models considered in the previous sections can be reduced to this canonical DLP problem, or that the methods developed for the canonical problem can be directly applied to the models. Problem 5.1 represents a DLP problem in a canonical form and can be viewed either as a "staircase" linear-programming problem or as an optimal control-theory problem. Hence, both methods – linear programming and control theory – can be applied to the solution of Problem 5.1. These methods have been surveyed by Propoi (1973, 1976, 1979).

6 CONCLUSION

Different individual energy–resource–economy models, and their linkage into a coherent overall system, have been discussed in the preceding sections. It has been shown that all these models may be reduced to a canonical form of the DLP problem. Therefore, a unified methodological approach can be developed to analyze and solve the models. Very briefly, several further possible directions for the methodological analysis of energy models may be outlined.

a. Energo-economic analysis. In this report we have concentrated on analyzing the common mathematical features of the models. The analysis of the physical structure of each model – objective functions, constraints, level of aggregation, uniformity of data bank, etc. – from the economic and energy-technology points of view is also of great interest.

b. Vector-optimization methods. Clearly, a single objective function is not a realistic way of modeling energy systems. This problem has been discussed, for example, by Ho (1979).

c. Duality theory. The shadow prices which are the solutions of the dual problem provide a valuable tool for a marginal analysis of the model. The relevant duality theory for the canonical DLP Problem 5.1 has been described by Propoi (1977). The further application of this theory to energy models, as discussed in this report would be useful in many respects.

d. Numerical-solution methods. As mentioned above, Problem 5.1 is an LP problem. Hence, standard LP programs can be (and already have been) applied for the solution of energy models. Special methods which take into account the specific features of DLP problems have also been developed (Ho and Manne 1974; Propoi and Yadykin 1975/1976; Ho 1977; Ho and Loute 1977; Propoi and Krivonozhko 1977, 1978); see also the references given by Propoi (1976, 1979). Preliminary versions of these algorithms show results which are acceptable when compared to the standard simplex methods (Ho 1977; Ho and Loute 1977).

e. Post-optimal analysis. Methods for analysis of solutions, including parametric DLP methods, and sensitivity and stability analysis, are of great practical interest. A general theory of linear and quadratic parametric programming has recently been developed (Propoi and Yadykin 1978).

f. Implementation of the solution. The implementation of the optimal solution is just as important as finding the solution. We must mention here the questions of realization of the optimal solution as a program (that is, as a time sequence of controlling actions) or as a feedback control (that is, as a current control action determined by the current state of the system).

g. Linking the models. The development of methods for linking individual models is, at present, probably the most important issue. Three main areas for investigation are

- Relations between long-, medium-, and short-term energy models (for example, how the optimal solution of an aggregated long-term model relates to the solution of a more detailed short-term model);
- Methods of linking individual models of energy, resources, and the economy into an integrated energy model for a nation or region (some of these methodological questions have been discussed in Section 4 of this report); and
- methods of linking national energy models into a world model.

Various discussions, both of methodology and of actual methods for the computer implementation of linked models, may be found in the literature (see, for example, Moiseev 1975; Behling et al. 1977; Häfele and A.A. Makarov 1977; Hoffman and Jorgenson 1977; Kononov 1977; A.A. Makarov 1977; Manne 1977; Moiseev 1977; Orchard-Hays 1977; Kallio et al. 1979).

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