TWO ESSAYS ON ALONSO'S THEORY OF MOVEMENT

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FOREWORD

Declining rates of national population growth, continuing differential levels of regional economic activity, and shifts in the migration patterns of people and jobs are characteristic empirical aspects of many developed countries. In some regions they have combined to bring about relative (and in some cases absolute) population decline of highly urbanized areas; in others they have brought about rapid metropolitan growth.

The objective of the Urban Change Task in IIASA's Human Settlements and Services Area was to bring together and synthesize available empirical and theoretical information on the main determinants and consequences of such urban growth and decline. The Task was concluded in 1981, and since then attention has turned to disseminating its principal results.

The two essays republished in this report are part of the Task's dissemination effort. They examine aspects of William Alonso's general theory of movement, developed over a decade ago. In them, Jacques Ledent argues that the Alonso model and one of Alan Wilson's well-known spatial-interaction models are equivalent formulations. A valuable contribution of these two essays is the development of a method for fitting the Alonso model to data on interregional migration.

A list of recent publications in the Urban Change Series appears at the end of this report.

ANDREI ROGERS
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Calibrating Alonso's general theory of movement: the case of interprovincial migration flows in Canada *

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Abstract. First, it is shown that Alonso's general theory of movement relies on a standard doubly-constrained spatial interaction model. Such a finding then suggests the use of a biproportional adjustment method (RAS method) to adequately estimate the systemic variables specified in the underlying model. This eventually leads to the development of a complete and precise methodology for calibrating the Alonso model. This methodology is illustrated with the help of an application to data on interprovincial migration in Canada.

Key words: Alonso's theory of movement, Alonso's model, spatial interaction, calibration, interprovincial migration in Canada.

0. **Introduction**

In the last decade, William Alonso has, in successive efforts, developed a general framework for the analysis of movement (Alonso, 1973, 1975, 1976, 1978). Owing to its logical consistency as well as to its elegant presentation, Alonso's theory rapidly caught the attention of regional scientists, among whom it is nowadays very popular (*).

This theory is generally held to be a common logical and mathematical framework for the large number of models of movement proposed in the past few decades in the social sciences and related areas. However, the view of Alonso's theory as a more general framework for thinking about the diverse existing models overstates somewhat its real worth. As shown in this paper, Alonso's theory simply relies on a standard doubly-constrained spatial interaction model.

Nevertheless, the peculiarity of its formulation, namely, the consideration of place-to-place flows along with that of the total flows out of and into each place in a simultaneous and consistent way, represents a significant contribution to the modeling of mobility phenomena. More specifically, owing to its

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(*) For example, at the 19th European Meeting of the Regional Science Association held in London in the summer of 1979, W. Isard chaired a three-hour panel session which was entirely devoted to a discussion of Alonso's theory. This paper develops reflections and ideas presented by the author at that panel session.

inclusion of variables and parameters reflecting the impact of the system (i.e., the influence of alternative places) on place-to-place flows, Alonso's theory constitutes a potential tool for gaining insights into the interaction of mutually exclusive places (groups) which could not be obtained otherwise. But, in spite of the general discussion generated by this theory (Anselin, Isard, 1980; Dziewonski, 1979), little progress has been made toward its practical use.

The problem here is mainly one of having a reliable methodology for fitting the model underpinning Alonso's theory to actual data sets. To our knowledge, the only calibration attempt to date was carried out by Alonso himself in his first publication (Alonso, 1973). However, the methodology he used was seriously hampered by difficulties originating from the simultaneous measurement of the two mutually dependent systemic variables appearing in the model (Alonso, 1973, Appendix A, p. 100).

Fortunately, the observation made in this paper that Alonso's theory is centered around a well-known spatial interaction model suggests a more precise method for measuring the aforementioned systemic variables. This eventually leads to the development of a satisfactory methodology for estimating the parameters intervening in the specification of the underlying model. An application to the case of interprovincial migration in Canada is provided to illustrate the methodology proposed.

This paper consists of seven sections. Section One, intended as a background section, presents a rapid overview of Alonso's theory. Some of the issues which its formulation raises are briefly discussed in Section Two. Then, Section Three justifies the identification of the underlying model with a standard doubly constrained model of spatial interaction. This leads to the development of appropriate methodologies for calibrating the Alonso model (Section Four) as well as for utilizing it as a forecasting model (Section Five). Finally, Sections Six and Seven report on the application of the proposed calibration method to the case of Canadian interprovincial migration: Section Six focuses on the measurement of Alonso's systemic variables, whereas Section Seven deals with the estimation of the two main parameters involved in Alonso's theory. Note that the latter relies on a regression analysis of migration flows whose results are compared with those of a more classical regression analysis.

1. **Alonso's theory of movement: an overview**

The strength of Alonso's theory which, for a large part, explains its appeal is its circular nature which makes alternative expositions possible. Alonso himself has provided various versions of his theory (Alonso, 1973, 1975, 1976, 1978) whereas Anselin and Isard (1980) have proposed yet another formulation. Our exposition below is broadly similar to the variant proposed by Alonso in his latest effort (Alonso, 1978).

In brief, Alonso's theory of movement pertains to a closed system of mutually exclusive classes (regions in a nation, sectors in an economy, etc.) consisting of units (people, commodities, etc.) which can move to another class.

It is concerned with building a mathematical framework for the flow of the units moving between the various, for example n, classes in the system.

Each class can be regarded as an origin as well as a destination. When considered as an origin, it has unfavorable characteristics-summarized in a general measure v_i ($i = 1, ..., n$) — which induce units to leave. Alternatively, when considered as a destination, it has favorable characteristics-summarized in a general measure w_j ($j = 1, .., n$) — which attract units.

Central to Alonso's theory of movement is the idea that place-to-place flows are not only determined by the unfavorable characteristics of the origin and the favorable characteristics of the destination but are also affected by the characteristics of the alternative origins and destinations. In brief, the impact exerted by the system is assumed to be accounted for by two variables, the pull-in D_i exerted on each origin i and the push-out C_j exerted on the flows terminating in j (*). The former may be thought of as a demand or a draw-hence the notation D — and the latter as a measure of competition, crowding, or congestion-hence the notation C —.

First, let us consider the total flows out of each class. We may, for example, assume that movements out of class i are proportional to the unfavorable characteristics of class i and depend on the pull-in of the system D_i with a rate of response α_i. Thus

$$M_{i.} = v_i \, D_i^{\alpha_i} \qquad \forall i = 1, ..., n. \tag{1}$$

Similarly, we may assume that movements into class j are proportional to the favorable characteristics of class j and depend on the push-out of the system C_j with a rate of response β_j. Thus

$$M_{.j} = w_j \, C_j^{\beta_j} \qquad \forall j = 1, ..., n. \tag{2}$$

Further, departing slightly from Alonso's exposition, let us consider the flow of units M_{ij} moving from a particular class i to a particular class j. In accordance with a widely accepted theory of migration in which place-to-place flows depend on factors associated with the areas of origin and destination as well as with intervening obstacles (Lee, 1966), let us posit that M_{ij} is proportional to:

— the measure v_i of the unattractive properties of class i, weighted by the ease of movement out of i (i.e., $D_i^{\alpha_{i-1}}$);

— the measure w_j of the attractive properties of class j, weighted by the ease of entry into j (i.e., $C_j^{\beta_{j-1}}$);

— a relational term t_{ij} reflecting the effect of distance between i and j.

(*) These two variables remain undefined for the time being: they will be derived later.

Thus

$$M_{ij} = v_i \ w_j \ t_{ij} \ D_i^{\alpha_i-1} \ C_j^{\beta_j-1} \qquad \forall i, j = 1, ..., n. \qquad (3)$$

The next step is the evaluation of the impacts exerted by the system on the alternative classes, i.e., the formal definition of the pull-in D_i and the push-out C_j. This comes naturally from ensuring a certain consistency between the place-to-place flow equation (3) and the total flow equations (1) and (2).

From equation (3), it is easy to derive the total flow of migrants out of and into each class by summing over all possible destinations

$$M_{i.} = \sum_j M_{ij} \qquad \forall i = 1, ..., n \qquad (4)$$

and origins

$$M_{.j} = \sum_i M_{ij} \qquad \forall j = 1, ..., n. \qquad (5)$$

respectively.

We immediately obtain that

$$M_{i.} = v_i \ D_i^{\alpha_i-1} \sum_j w_j \ C_j^{\beta_j-1} \ t_{ij} \qquad \forall i = 1, ..., n$$

and

$$M_{.j} = w_j \ C_j^{\beta_j-1} \sum_i v_i \ D_i^{\alpha_i-1} \ t_{ij} \qquad \forall j = 1, ..., n.$$

Then, comparing the above formulas with (1) and (2), yields

$$D_i = \sum_j w_j \ C_j^{\beta_j-1} \ t_{ij} \qquad \forall i = 1, ..., n \qquad (6)$$

and

$$C_j = \sum_i v_i \ D_i^{\alpha_i-1} \ t_{ij} \qquad \forall j = 1, ..., n \qquad (7)$$

two formulas which indicate that the competition variable is part of the formula defining the draw variable and vice versa; the derivation of the two systemic variables thus requires a simultaneous calculation.

Clearly, formula (6) shows that the draw variable D_i is a weighted sum of the attractive characteristics w_j of all the other classes in the system, where the weights applied to any w_j express the influences of distance (t_{ij}) and competition ($C_j^{\beta_j-1}$).

Similarly, formula (7) shows that the competition variable C_j is a weighted sum of the unfavorable characteristics v_i of all the other classes in the system where the weights applied to any v_i express the influences of both distance (t_{ij}) and ease of exit ($D_i^{\alpha_i-1}$).

Now, summarizing the above presentation of Alonso's theory, it appears that its underlying model consists of five equations, two of which — (4) and (5) — are identities. The remaining equations can be:

Equations (1) and (2) or, alternatively, (6) and (7)
Equation (3) or any equivalent equation describing the place-to-place flows.

This result indeed makes the circularity and involution of Alonso's theory quite clear. For example one could start with the definitions of the systemic variables D_i and C_j [see Anselin and Isard (1980) who provide a good *a priori* justification of equations (6) and (7)]. Then, one would derive (1) and (2) by substituting (6) and (7) into the equations obtained from (3) by summing over all possible destinations and origins respectively.

2. **Issues**

In broad terms, Alonso's general theory of movement raises two types of issues. The first type relates to the specification of the equations underlying the theory, whereas the second type refers to the nature of the flows with which the theory is concerned.

With regard to the specification of the equations underlying the theory, two questions come to mind:

(a) Do equations (1) through (3) require the presence of a constant term as suggested by Anselin and Isard (1980)? The answer to this question is negative, since such constant terms can be incorporated in the definition of the systemic variables D_i and C_j.

(b) Is the choice of the exponents α_{i-1} and β_{j-1} when applied to the draw and competition terms in equation (3) restrictive? The answer to this is again negative, for the choice of more general exponents α_i-x_i and β_j-y_j does not fundamentally affect the model. This can be seen as follows. Summing the place-to-place flows over all origins and destinations leads to:

$$D_i^{x_i} = \sum_j w_j \, C_j^{\beta_j - y_j} \, t_{ij} \qquad \forall i = 1, \ldots, n$$

and

$$C_j^{y_j} = \sum_i v_i \, D_i^{\alpha_i - x_i} \, t_{ij} \qquad \forall i = 1, \ldots, n.$$

Then, redefining the draw and competition variables as

$$D_i' = D_i^{x_i} \text{ and } C_j' = C_j^{y_j}$$

and defining the following exponents

$$\alpha_i' = \frac{\alpha_i}{x_i} \quad \text{and} \quad \beta_j' = \frac{\beta_j}{y_j}$$

leads to an unchanged model: variables and parameters with a prime sign are simply substituted for the original ones.

Turning now to the issue relating to the nature of the flows with which the theory is concerned, we must first underline that we have purposely avoided the use of the word movements to characterize the flows taking place between the various classes (or groups).

As is well known, changes occurring over a finite period of time in a system of mutually exclusive groups can be observed from two different perspectives. One perspective looks at all the movements made between each pair of groups during the observation period regardless of the group in which the «migrators» were present at the beginning of the observation period. The alternative perspective simply considers the pairwise transitions resulting from the comparison of the groups in which the various units in the system are present at the beginning and end of the observation periods. These two perspectives, known as the movement and transition perspectives (Ledent, 1980), are indeed different in nature and have different implications.

Which conceptualization of intergroup transfers had Alonso in mind when devising his theory? Various indications scattered in his papers — mainly the reference to stayers' flows as well as the consideration of the Markov model of migration as a particular case of the general theory — suggests that Alonso's thoughts are more in line with the transition perspective than with the movement perspective.

However, a focus on transitions is rather limiting because it implies an incomplete view of the exchanges occurring between the alternative classes. Fundamentally, the transition perspective reflects a consolidated view of the movement perspective. Hence it ignores the multiple moves that each individual may have made during the observation period, retaining only the apparent moves out of or within each class observed between the start and end of the observation period. It thus follows that the most complete picture of a spatial interaction system is the one which would be obtained by considering the whole of the movements made between alternative classes. As a corollary, the image which would be drawn from the sole consideration of the transitions made between two points in time would be less meaningful since it would rely on reduced information.

Nevertheless, Alonso's theory subsumes both the movement and transition perspectives because the choice of either approach does not raise any theoretical problem. The only consequence of adopting the transition perspective instead of the movement perspective appears to be the consideration versus the non-consideration of flows M_{ii} of stayers: stayers' flows are indeed irrelevant to the movement perspective.

Note that the inclusion of such flows in most applications based on data consistent with the transition perspective is likely to blur the picture of spatial interaction taking place in the system at hand. The fact that the flows of stayers M_{ii} are generally much higher than the migration flows M_{ij} $(j \neq i)$ heavily influences the values of the systemic variables D_i and C_j: it is thus better to ignore the flows of stayers.

Consequently, the Alonso model will be fitted to actual data — regardless of whether they are data on movements or transitions — without consideration of such elements as M_{ii}. In practice, there is little choice as to whether one will use movement or transition data. Only in the case of migration, is a choice theoretically possible since data in the form of movements can be obtained from population registers and data in the form of transitions can be obtained from population censuses.

3. The Alonso model as a standard doubly-constrained model of spatial interaction

By combining equations (1), (2), and (3), it is readily established that

$$M_{ij} = \frac{1}{D_i\, C_j} M_{i.}\, M_{.j}\, t_{ij} \qquad \forall i,j = 1, \ldots, n \qquad (8)$$

an equation which shows that Alonso's theory of movement relies on a standard doubly-constrained model of spatial interaction. In addition, the draw and competition measures, D_i and C_j, appear to be the reciprocals of the balancing factors of this doubly-constrained model. We have that

$$D_i = \sum_j M_{.j}\, C_j^{-1}\, t_{ij} \qquad \forall i = 1, \ldots, n \qquad (9)$$

and

$$C_j = \sum_i M_{i.}\, D_i^{-1}\, t_{ij} \qquad \forall j = 1, \ldots, n. \qquad (10)$$

The latter result is germane to the observation made by Kirby (1970) that the balancing factors of a doubly constrained spatial interaction model of the multiplicative type as specified in (8) can be interpreted as a measure of the accessibility (in terms of attractiveness and repulsion) of one class with respect to other classes.

Note that the doubly-constrained model of spatial interaction suggested by (8) subsumes some of the classical models used in gravity and entropy theory. [For a review of such models, see Wilson (1974) or Nijkamp (1979)]. For example, assuming that t_{ij} is a simple function of the friction measure d_{ij} relating to origin i and destination j, we see that

if $t_{ij} = d_{ij}^{-h}$ $(h > 0)$, Alonso's theory of movement relies on a generalized formulation of the traditional gravity model constrained at both the origin and destination

$$M_{ij} = \frac{1}{D_i C_j} \frac{M_{i.} M_{.j}}{d_{ij}^{h}}; \qquad \forall i,j = 1, ..., n \qquad (11)$$

if $t_{ij} = \exp(-hd_{ij})$ $(h > 0)$, Alonso's theory of movement relies on a doubly-constrained entropy-derived model

$$M_{ij} = \frac{1}{D_i C_j} M_{i.} M_{.j} \exp(-hd_{ij}). \qquad \forall i,j = 1, ..., n \qquad (12)$$

4. A methodology for calibrating the Alonso model

The observation in Section Three that Alonso's theory of movement relies on a standard doubly-constrained model of spatial interaction immediately suggests a precise methodology for calibrating the underlying model.

Clearly, on the basis of information known about t_{ij}, $M_{i.}$ and $M_{.j}$, the draw and competition measures D_i and C_j can be assessed by solving the system of equations defined by (9) and (10). Actually, finding the solution to this system is nothing else than solving the following biproportional adjustment problem (sometimes referred to as the R.A.S. problem): find the matrix $\mathbf{M} = (M_{ij})$ of place-to-place flows which has row and column totals equal to the observed out- and inmigration flows respectively and which is biproportional to the matrix of relational terms $\mathbf{T} = (t_{ij})$.

As already noted above, D_i and C_j are simply the reciprocals of the balancing factors resulting from this adjustment problem. (Note that they are defined up to a constant multiplicative factor). In practice, they can be obtained in a recursive manner using a method originally proposed by Stone (1962), but alternative algorithms are possible [for a review of these algorithms, see Willekens (1980)].

Note that the feasibility of the above procedure rests on the availability of the **T** matrix. Since the values of the relational terms are generally unknown, only the calibration of particular versions of (8) — such as the gravity model (11) or the entropy-derived model (12) — allows for the estimation of the draw and competition measures. At the same time this yields the value $\hat{h}$ of the h-coefficient appearing in the expression of the relational term t_{ij} in terms of d_{ij}.

Various calibration techniques for these models have been proposed in numerous papers. [For a study in depth of these techniques, see for example Batty and Mackie (1972), or Openshaw (1976)]. Broadly speaking, these methods can be classified into two groups. The first group consists of methods in which the observed parts of the constraints determine the parameters, regardless of the actual fit of the model to the observed pattern of place-to-place flows. They include maximum-likelihood and entropy-maximizing methods which, if it is assumed that the sampling distribution is multivariate normal, are equivalent.

By contrast, the methods of the second group attempt to maximize model performance (i.e., yield a predicted value of the place-to-place flow matrix **M** as close as possible to its observed value). These methods include

(a) a nonlinear, least squares method which seeks to minimize the sum of squares of the differences between the observed and predicted flows (this method does not suppose any assumption about the form of the sampling distribution)

$$\min SS = \sum_i \sum_j (M_{ij} - \hat{M}_{ij})^2$$

where $\hat{M}_{ij}$ is the predicted value of the flow between i and j;

(b) a method which attempts to minimize the following chi-square statistic (this method assumes that the observed flow matrix is subject to sampling errors)

$$\min \chi^2 = \sum_i \sum_j \frac{(M_{ij} - \hat{M}_{ij})^2}{\hat{M}_{ij}}.$$

In practice, if the data available relate only to the total flows out of or into each class, the maximum likelihood/entropy-maximizing method will be used. For its implementation, one will, for example, use the algorithm proposed by Hyman (1969), which Vermot-Desroches (1979) describes as being very efficient.

In case the data available consist of the matrix of place-to-place flows, one can, if time and resources permit, perform the various calibration methods mentioned above and select the one which offers the best model performance on the basis of both the least-squares and chi-square criteria. But, in general, lack of resources may lead to select a single method.

Of course, the calibration of the Alonso model does not stop with the estimation of the draw and competition variables. The next step is the estimation of the exponents α_i and β_j of the draw and competition variables in equations (1) and (2), respectively. First of all, this raises the problem whether these two exponents are identical within the system or whether they may vary from one place to another. Following Dziewonski (1979) who argues that, in countries which are strongly integrated both socially and economically, the same exponents can apply to all places, we thus assume

$$\alpha_i = \alpha \qquad \forall i = 1,\dots, n$$

and

$$\beta_j = \beta \qquad \forall j = 1,\dots, n.$$

Second, the feasibility of estimating α and β requires the knowledge of the internal structure of v_i and w_j. Recalling that the variables v_i and w_j are composite variables which reflect the unfavorable characteristics of place i and the attractive characteristics of place j, we may write:

$$v_i = \prod_{k=1}^{K} X_{k_i}^{u_k} \qquad \forall i = 1,\dots, n$$

and

$$w_j = \prod_{l=1}^{L} Y_{l_j}^{v_l} \qquad \forall j = 1,\dots, n$$

where X_{k_i} is the value of the k-th unfavorable characteristic of place i
u_k is the elasticity or movement response of group i to changes in the value of k-th characteristic (*)
Y_{l_j} is the value of the l-th favorable characteristic of place j
v_l is the elasticity or movement response of group i to changes in the value of the l-th characteristic (*).

Then, the values of the α and β exponents — as well as the values of the various elasticities u_k and v_l — can be found by performing the following regression analyses in double logarithmic form:

$$\ln M_{i.} = u_0 + \sum_{k=1}^{K} u_k \ln X_{k_i} + \alpha \ln D_i \tag{13}$$

and

$$\ln M_{.j} = v_0 + \sum_{l=1}^{L} v_l \ln Y_{l_j} + \beta \ln C_j. \tag{14}$$

(*) The u_k and v_l elasticities are assumed to be independent of the place of reference for the same reason that the α and β elasticities were made independent of the place of reference.

Note that the presence of constant terms u_0 and v_0 in (13) and (14) respectively — which normally are not called for by the specification of equations (1) and (2) — is necessitated by the fact that D_i and C_j are measured up to a constant multiplicative factor.

In practice, the feasibility of performing the regression analyses based on equations (13) and (14) might be hampered by a low value of the number of degrees of freedom due, for example, to the consideration of a limited number of places and of a large number of independent variables. Alternatively, if the matrix of place-to-place flows is available, one can reasonably expect to find the values of the α and β parameters by performing a regression analysis based on the sole equation (3) rather than equations (1) and (2). In such circumstances, one would fit to the available set of data the following equation:

$$\ln M_{ij} = w_0 + \sum_{k=1}^{K} u_k \ln X_{k_i} + \sum_{l=1}^{L} v_l \ln Y_{l_j} + \alpha' \ln D_i + \beta' \ln C_j - h' \ln d_{ij} \qquad (^*)\ (15)$$

where w_0 is the constant term and h' the distance elasticity of the place-to-place migration flows. The α and β coefficients would be then obtained by adding one unit to the estimated values of α' and β'.

Note here that the estimate h' can be different from the value $\hat{h}$ obtained when estimating the values of the systemic variables C_i and D_j. It can be shown that the measures of the systemic variables obtained as indicated earlier are consistent with (a) a certain value of the parameter h entering the function expressing the relational terms t_{ij} in terms of the distance d_{ij}, and also with (b) the estimated values $\hat{M}_{ij}$ of the place-to-place migration flows. Thus, the simultaneous estimation of the α and β coefficients from the place-to-place flows will not be obtained by fitting (15) to the available data but by fitting the following regression equation:

$$\ln \frac{\hat{M}_{ij}}{d_{ij}^{-\hat{h}}} = w_0 + \sum_{k=1}^{K} u_k \ln X_{k_i} + \sum_{l=1}^{L} v_l \ln Y_{l_j} + a' \ln D_i + \beta' \ln C_j \qquad (16)$$

where $\hat{M}_{ij}$ is the matrix of the adjusted (estimated) place-to-place flows coming out of the measurement of C_i and D_j

$\hat{h}$ is the estimated value of h coming out of the same measurement.

(*) This assumes that the relational term t_{ij} is given by a negative power of the distance term d_{ij}. Alternatively, if t_{ij} is given by a negative exponential function of the distance d_{ij}, the last term of (15) will simply be minus $h'd_{ij}$.

Note that, since (16) requires the knowledge of the estimated matrix $\hat{M}_{ij}$ rather than the observed M_{ij}, this allows for an estimation of the α and β parameters in case the available migration data are limited to the total migration flows $M_{i.}$ and $M_{.j}$. (In these circumstances, a maximum likelihood or an entropy-maximizing method allows for the derivation of $\hat{M}_{ij}$).

To summarize, it appears that the calibration of Alonso's model requires two successive stages:

in a first stage, the systemic variables are estimated using one of the classical calibration methods generally applied to gravity and entropy models;

in a second stage, the parameters reflecting the response of migration to changes in the systemic variables are estimated from an appropriate regression analysis.

Finally, note that the interest of this calibration method goes beyond the estimation of the systemic variables and their corresponding elasticities. In effect, this calibration method attempts to explain place-to-place flows not only in terms of the characteristics of the origin and destination — as is traditionally done — but also in terms of the characteristics of the rest of the system. Thus, with reference to the case of interregional migration, the methodology developed above provides a way to test the influence of the rest of the system (i.e., the places other than the places of origin and destination) on place-to-place migration flows. In particular, this methodology allows one to answer the problem that some researchers have tried to tackle with relatively moderate success (see, for example, Alperovich et *al.*, 1977; Wadycki, 1979); namely, the importance of intervening opportunities in the determination of place-to-place migration flows.

5. **Using the Alonso model as a forecasting model**

In their review of Alonso's theory of movement, Anselin and Isard (1980) claim that a critical deficiency of this theory is its non-dynamic character. However, such a criticism appears to be improperly addressed unless there is a divergence on the notion of what constitutes a dynamic model and what does not.

For us, Alonso's model is dynamic in the sense that it allows one to calculate the population of each class in successive time periods t, t+1, etc.. This property was illustrated by Alonso himself in his first paper (Alonso, 1973).

In effect the size P_i (t+1) of group i at time (t+1) is linked with the same group at time t by the relation

$$P_i(t+1) = P_i(t) + B_i(t) - D_i(t) + M_{.i}(t) - M_{i.}(t) \tag{17}$$

where $B_i(t)$ is the number of units added to the system in group i: in a multiregional demographic system, they would be the number of babies born in region i between times t and t+1;

$D_i(t)$ is the number units in group i disappearing from the system: in a multiregional demographic system, they would be the number of deaths occurring in region i between times t and t+1;

$M_{.i}$ (t) and $M_{i.}$ (t) are the total number of units entering or leaving group i between times t and t+1.

From equation (17), it is clear that the population of each class i in successive times t, t+1, etc. can be obtained once the various flows on the right-hand side of (17) are determined for the series of the corresponding time intervals. In fact, the determination of these flows does not raise any problem. On the one hand, it is a simple matter to relate the "births" and "deaths" occurring in group i with the size of the same group. On the other hand, the total flows of units entering and leaving group i can be easily calculated on the basis of the equations shown in the first part of the paper.

Indeed, if the parameters α and β (as well as the u_k and v_l elasticities) are known, the availability of the values taken by the variables X_{k_i} and Y_{l_j} entering the composite variables v_i and w_j allows one to calculate D_i and C_j by solving iteratively the system of equations (6) and (7). Then, inserting the systemic variables thus obtained into (1) and (2) yields the requested estimates of the total number of units entering and leaving each group i.

Thus, Alonso's model appears to be a dynamic forecasting too (*), capable of producing alternative simulations of the future based on various assumptions regarding the values of the independent variables entering the composite variables v_i and w_j.

6. An application to interprovincial migration flows in Canada (I): Measurement of the systemic variables

The methodology proposed in Section Four for calibrating the Alonso model will now be illustrated with an application to the case of interprovincial

(*) The following digression might be of interest to mathematical demographers. As we will see later on, the population sizes at the origin and destination are likely to be the most significant variables entering the composite variables v_i and w_j. This suggests the specification of a general model of place-to-place migration flows.

$$M_{ij} = P_i^u P_j^v t_{ij} D_i^{-1} C_j^{-1} \quad (3')$$

$$\text{where } D_i = \sum_j P_j^v C_j^{-1} t_{ij} \quad (6')$$

$$C_j = \sum_i P_i^u D_i^{-1} t_{ij} \quad (7')$$

which encompasses virtually all of the migration models proposed by mathematical demographers. [This model can be easily operationalized since (6') and (7') can be easily solved for values of D_i and C_j in an iterative manner].

migration flows in Canada. The present section reports on the measurement of the systemic variables, while the next one deals with the estimation of the model parameters.

The migration data used for the purpose of this illustration are the data on the total number of families leaving a given province for another province which are published annually by Statistics Canada (see Statistics Canada 1977 for a chronological series of such annual migration figures). Observe that these data are counts of moves rather than of transitions: if a family makes several moves across provincial boundaries during a given year, it appears in the data as many times as the family moves.

For the purpose of this paper, the annual data were consolidated into three sets covering the periods 1961-66, 1966-71, and 1971-76. The corresponding matrices of interprovincial flows, each reflecting an annual average over these three periods, are shown in Table A1 of the Appendix.

Both the gravity model (11) and the entropy-derived model (12) were fitted to these three data sets using a nonlinear least-squares method (*). The problem here was one of finding the value of h minimizing the sum of squares of the difference between the estimated and observed values of M_{ij}. This was solved in an iterative manner as follows. First, we picked an initial value h_0 of h ($h_0 = 0.1$ in the case of the gravity model), and, using Stone's (1962) algorithm already mentioned, we calculated the matrix **M** biproportional to the corresponding matrix of relational terms such that its row and column sums were equal to the observed total out- and in-migration flows. Then we increased h_0 by a quantity Δh ($\Delta h = 0.1$ in the case of the gravity model) and, using again Stone's algorithm, obtained a new estimate of the matrix **M**. Generally, the sum of squares of the residuals relating to the rew estimate of **M** was smaller than in the first iteration. So, we simply increased the previous estimate of h by Δh and repeated the previous operation until we obtained a sum of squares value greater than in the preceding iteration. If h_1 is the value of h corresponding to the iteration which sees an increase in the sum of squares value, this means that the optimal value of h is located somewhere in between $h_1 - 2\Delta h$ and h_1. Thus, we repeated the procedure described above, starting with $h = h_1 - 2\Delta h$ and proceeding with increases equal to a tenth of Δh until we obtained an increase of the sum of squares value (for say $h = h_2$). Next, the above procedure was repeated starting with

$h = h_2 - 2\frac{\Delta h}{10}$ and proceeding with increases equal to a hundredth of Δh.

This was pursued until we obtained the desired number of significant digits for the value of h. Table 1 shows the successive values of h and the square root of the corresponding sum of squares values obtained in the calibration of the

(*) The matrix of interprovincial distances (road distances between the provinces' principal cities) appears in Table A3 of the Appendix.

Table 1 — Calibration of the gravity model for the period 1971-76: evolution of the square root of the sum of squares of the residuals with the distance friction coefficient h

h	$\sqrt{SS}$	h	$\sqrt{SS}$
0.1	5232.5	0.81	1300.2
0.2	4636.6	⋮	
0.3	4029.9	0.90	1130.7
0.4	3421.5	0.91	1125.7
0.5	2823.4	0.92	1123.7
0.6	2252.5	0.93	1124.5
0.7	1737.1		
0.8	1331.4	0.911	1125.5
0.9	1130.7	⋮	
1.0	1204.6	0.921	1123.658
		0.922	1123.641
		0.923	1123.653

model for the period 1971-76. Finally, a value of h equal to 0.922 and an index of model performance equal to 1123.6 were obtained. The corresponding matrix of adjusted (predicted) flows as well as the matrix of the ratios of the adjusted to the actual flows are shown in Tables A4 and A5 of the Appendix.

Note that the fit of the gravity model was better than that of the entropy-derived model: model performance was poorer in the latter case with an index equal to 1656.8. The figures in Table 2 summarizing the results of the calibration of the two alternative models for the three periods available indicate that

(a) the fit of the gravity model improves with time while no such conclusion can be drawn for the entropy derived model;

(b) the value of the distance friction coefficient is practically the same for the three periods in the case of the gravity model — its lowest values is 0.916 in period 1961-66 and its highest 0.924 in period 1966-71 — while it tends to decrease with time in case of the entropy-derived model — from 0.865×10^{-3} in 1961-66 to 0.721×10^{-3} in 1971-76 —.

Table 2 — Calibration of the gravity and entropy models for alternative periods: values of the distance friction coefficient (h) and the model performance index (square root of the sum of squares of the residuals)

	Period	1961-66	1966-71	1971-76
Gravity	h	0.916	0.924	0.922
Model	$\sqrt{SS'}$	1825.9	1352.5	1123.6
Entropy	h	0.000865	0.000778	0.000721
Model	$\sqrt{SS'}$	1734.4	1521.4	1656.8

As for the systemic variables resulting from the above calibration method, their normalized values appear in Table 3 (only the values obtained with the gravity model are shown). Interestingly enough, the draw and competition variables take on remarkably similar values, thus suggesting that they are highly correlated. As a matter of fact, their correlation coefficient varies between 0.982 for the period 1966-71 and 0.991 for the period 1971-76. Thus, the rankings of the ten provinces according to increasing values of both C_i and D_J are roughly similar, and even identical for the period 1971-76. In this latter period, the typical ranking is as follows:

1. British Columbia
2. Newfoundland
3. Ontario
4. Alberta
5. Nova Scotia
6. Manitoba
7. New Brunswick
8. Saskatchewan
9. Prince Edward Island
10. Quebec.

Another interesting result suggested by Table 3 is the relative invariance of the normalized values of the draw and competition variables over time, so that the rankings of the ten provinces according to increasing values of D_i and C_j for the other periods look roughly the same. For example, the only differences that the 1966-71 rankings present with respect to the one above concern Nova Scotia and Prince Edward Island — which gain one rank in the D-ranking — Newfoundland, Manitoba and Saskatchewan — which gain one rank in the C-ranking—.

Following the above observations, two comments are here in order. First of all, the similar values taken in each province by the systemic variables may appear to contradict the interpretation of Alonso's systemic variables as accessibility measures in terms of attractiveness and repulsion. In effect, as a consequence of such an interpretation, we would have expected these variables to be correlated negatively rather than positively. Our result can be contrasted with the evidence provided by Vermot-Desroches (1979) whose calibration of Cesario's versions of models (11) and (12) (see Cesario, 1974, 1975).(*) to the interregional flows by rail of petroleum products in France leads to values of the systemic variables conforming with our *a priori* expectations. Actually, the different results obtained by Vermot-Desroches and ourselves can be simply attributed to the apparent differences existing in the spatial interaction patterns observed. On the one hand, the negative correlation obtained by Vermot-Desroches follows from the asymmetric interaction pattern he deals with. On the other hand, the positive correlation here is simply the consequence of the highly symmetric pattern of interregional migration (**), a

(*) The Cesario versions of models (11) and (12) group the terms concerning both the production (v_i and D_i) and the attraction (w_j and C_j) zones.
(**) Indeed, if the interprovincial flow matrix $\mathbf{M} = (M_{ij})$ is symmetric, the C and D indices are identical in each province.

Table 3 — Calibration of the gravity model for alternative periods: normalized values of the systemic variables C_i and D_j

	1961-66			1966-71			1971-76		
Province	D_i (1)	C_j (2)	(1)/(2)	D_i (1)	C_j (2)	(1)/(2)	D_i (1)	C_j (2)	(1)/(2)
Newfoundland	0.05668	0.05674	0.9990	0.05614	0.05633	0.9966	0.06608	0.06695	0.9807
Prince Edward Island	0.12659	0.13853	0.9138	0.12536	0.13566	0.9241	0.12083	0.12225	0.9875
Nova Scotia	0.09759	0.09967	0.9792	0.09679	0.10072	0.9610	0.09909	0.10046	0.9847
New Brunswick	0.11737	0.12300	0.9543	0.11623	0.12273	0.9470	0.11236	0.11654	0.9574
Quebec	0.15554	0.14927	1.0419	0.16236	0.15307	1.0606	0.13860	0.14421	0.9346
Ontario	0.06945	0.07066	0.9830	0.06367	0.07216	0.8824	0.07071	0.07114	0.9772
Manitoba	0.10042	0.09565	1.0499	0.09915	0.09368	1.0584	0.10470	0.10135	1.0426
Saskatchewan	0.11628	0.10982	1.0589	0.12763	0.11530	1.1069	0.12782	0.11870	1.1035
Alberta	0.10214	0.09678	1.0554	0.09713	0.09285	1.0461	0.09424	0.09179	1.0522
British Columbia	0.05793	0.05990	0.9675	0.05555	0.05749	0.9662	0.06559	0.06661	1.0034
Coefficient of correlation between D_i and C_j		0.9891			0.9821			0.9906	

fact well-known to students of migration since Ravenstein (1885) observed that "each main current of migration produces a compensating counter-current".

The second comment we would like to make here concerns a possible interpretation of the ranking order of the ten Canadian provinces according to the values of the systemic variables. Actually, it turns out that this ranking is akin to the ranking of the provinces according to increasing values of their total out- and in-migration rates, or of the sum of these (*) (see Table 4).

The only major difference concerns the position of Quebec which is at the bottom in the former list and at the top in the latter. The other but minor differences appear to be the reverse order in which Ontario, Newfoundland and British Columbia are to be found and the two-rank gain of Manitoba.

Table 4 — Migration propensity index during the 1971-76 period

Province	Index Value	Ranking
Newfoundland	6.96	3
Prince Edward Island	11.21	10
Nova Scotia	9.06	5
New Brunswick	9.18	6
Quebec	2.75	1
Ontario	3.75	2
Manitoba	9.54	8
Saskatchewan	10.95	9
Alberta	9.25	7
British Columbia	8.34	4

Are there any obvious reasons accounting for those differences? A positive answer to this question will in fact be obtained in the second stage of the calibration of the Alonso model, which is the object of the next section.

7. An application to interprovincial migration flows in Canada (II): Estimation of the model parameters

Once the systemic variables have been measured, the next stage in our calibration of the Alonso model to the case of interprovincial migration in Canada consists of estimating the model parameters, i.e., the elasticities α and β of the systemic variables D_i and C_j respectively.

(*) Because the average number of families present in each province during each period was unavailable, we proxied the aforementioned rates by migration indices in which the denominators were taken as the arithmetic average of the provincial populations at the beginning and end of the period. The migration index referred to in Table 4 is the sum of the total out- and in-migration proxies.

In accordance with the principles set fourth in Section Four, this estimation is based on a regression analysis in which the independent variables include the variables traditionally used in regression analyses of migration flows (for an extensive survey of the determinants of migration, see Greenwood, 1975) as well as the two systemic variables proposed by Alonso in this theory of movement.

In a first step however, we perform a classical regression analysis — i.e., in which we ignore Alonso's systemic variables — whose objective is to allow, by comparing its results with those obtained from the more complete regression analysis, a better assessment of the importance of the systemic variables in the migration decision in Canada. The explanatory variables used in the course of this analysis are for each province:

The size of its population P
Its unemployment rate U
Its weekly wage rate (in real terms) W(*).

In each of the three periods studied, the population size variable is taken as the arithmetic average of the observed populations at the beginning and end of the period whereas the socio-economic variables are taken equal to their values observed in the starting year of each period (see Table A3 of the Appendix for the actual values of the variables considered).

The first observation made is that, regardless of the independent variable chosen (total outmigration flow, total inmigration flow, or place-to-place migration flow) or the observation period examined, large residuals are consistently obtained in the case of migration flows originating from or ending in Quebec. Thus, we add to the set of explanatory variables a dummy variable X, normally equal to zero but taking the value 1 if the corresponding migration flow originates from or ends in Quebec.

When attempting to explain the total migration flows entering and leaving each province in terms of the four explanatory variables P, U, W and X, we note in both cases that the regression coefficients of the population variable as well as of the dummy variable are highly significant. In particular, the negative sign of the dummy variable coefficients indicates the existence of a lower propensity to move into and out of Quebec: it can be attributed to its linguistic peculiarity (see Termote, Frechette, 1979). By contrast, the two socioeconomic variables perform poorly. For example, for the period 1971-76, they appear to have the wrong sign in both the out and immigration equation: however, the coefficient of the latter variable is not statistically significant. Can we explain the poor performance of these socio-economic variables? On the one hand, the result concerning the wage rate variable can be attributed to a

(*) Initially, we also used real per capital income (INC), which we discarded after discovering the high correlation existing between this variable and the wage rate variable. (Their correlation coefficient is equal to 0.81).

high colinearity with the population variable (in effect the coefficient of correlation between the population and the wage rate variables is equal to 0.83). On the other hand, the result concerning the unemployment rate might just be another manifestation of the evidence that high unemployment rates do not necessarily imply higher outmigration and smaller immigration (see Greenwood, 1975).

The first two equations of Table 5 show the "best" regression equations obtained in the case of the 1971-76 out- and in-migration flows. (The socio-economic variables behaving unexpectedly have been removed from these equations except for the unemployment rate in the first equation. We will see later on why).

Of course, another good reason for the poor performance of the unemployment variables might be that the above analysis of the total migration flows is based on too few observations (ten in each case). Thus, we might expect to obtain better results if we analyze the place-to-place flows for which we have 90 observations.

For each of the three periods studied, it turns out that the main factors explaining those flows are distance and size of the population at both the origin and the destination. In all cases these variables have t-statistics having an absolute value no less than 9. As for the coefficient of the distance variable, it is found to be in close agreement with the value h of the distance elasticity obtained in the first phase of this calibration effort. For the period 1971-76, the "best" regression equation yields a distance elasticity equal to 0.978, whereas the biproportional adjustment method of the preceding section produced a value of 0.922. Also highly significant are the dummy variables at the origin and destination. The somewhat higher values of their coefficients with respect to their values in the total migration equations indicates that the aforementioned peculiarity of Quebec does not solely reflect a comparatively smaller impact of Quebec's characteristics on the migration process, but also a comparatively higher effect of distance in the case of a move originating or ending in Quebec.

An immediate consequence of the latter result is that the estimation — in Section Six — of the systemic variables D_i and C_j was probably incorrectly performed owing to a relative underestimation of the relational terms concerning the migration flows originating or ending in Quebec; whence the extreme positions taken by Quebec in the two ranking orders of the provinces considered in the previous section.

As for the socio-economic variables, they do not appear to perform much better than in the case of the total out- and inmigration flows even though, in the case of the 1971-76 period, the coefficient of the unemployment variable relating to the origin now appears to have the correct sign and a relatively high t-value (see third equation of Table 5). The same result is not obtained in the case of the other two periods.

Now including the systemic variables among the independent variables of the total migration equations, we note (see Table 6) few changes in the performance of the various variables. As far as the period 1971-76 is

Table 5 — "Best" regression equations explaining the variations of the total and place-to-place migration flows *without* inclusion of the systemic variables, 1971-76

$$\log M_{i.} = 4.109 + 0.754 \log P_i - 0.545 \log U_i - 0.679\ X_i$$
$$(9.88) \qquad (-1.84) \qquad (-2.41)$$
$$R^2 = 0.982 \quad F = 54.3 \quad m = 8.37 \quad S.E = 0.217$$

$$\log M_{.j} = -2.860 + 0.615 \log P_j + 1.444 \log W_j - 0.953\ X_j$$
$$(3.76) \qquad (1.19) \qquad (-2.62)$$
$$R^2 = 0.966 \quad F = 27.7 \quad m = 8.38 \quad S.E = 0.283$$

$$\log M_{ij} = -6.755 - 0.978 \log d_{ij} + 1.071 \log P_i + 0.882 \log P_j$$
$$(-16.29) \qquad (19.42) \qquad (9.79)$$
$$+ 0.447 \log U_i + 0.674 \log W_j - 1.882\ X_i - 1.842\ X_j$$
$$(2.10) \qquad (1.53) \qquad (-9.21) \qquad (-9.19)$$
$$R^2 = 0.959 \quad F = 132.9 \quad m = 5.46 \quad S.E = 0.465$$

Table 6 — "Best" regression equations explaining the variations of the total and place-to-place migration flows *with* inclusion of the systemic variables, 1971-76

$$\log M_{i.} = -3.736 + 0.702 \log P_i + 1.753 \log U_i + 0.524 \log D_i$$
$$(7.55) \qquad (2.63) \qquad (2.71)$$
$$- 1.114\ X_i$$
$$(-4.43)$$
$$R^2 = 0.992 \quad F = 80.7 \quad m = 8.37 \quad S.E = 0.156$$

$$\log M_{.j} = -3.116 + 0.643 \log P_j + 1.608 \log W_j + 0.302 \log C_j$$
$$(3.71) \qquad (1.26) \qquad (0.76)$$
$$- 1.206\ X_j$$
$$(-2.39)$$
$$R^2 = 0.969 \quad F = 19.4 \quad m = 8.38 \quad S.E = 0.294$$

$$\log \frac{\hat{M}_{ij}}{d_{ij}^{-\hat{h}}} = -3.062 + 0.897 \log P_i + 0.665 \log P_j + 1.413 \log W_j$$
$$(27.13) \qquad (12.33) \qquad (3.58)$$
$$- 0.514 \log D_i - 0.700 \log C_j - 1.280\ X_i - 1.226\ X_j$$
$$(-4.52) \qquad (-5.65) \qquad (-8.93) \qquad (-7.81)$$
$$R^2 = 0.982 \quad F = 308.2 \quad m = 12.28 \quad S.E = 0.273$$

concerned, the comparison of the first two equations in Table 5 and 6 simply reveals the following.

The coefficient of the unemployment rate variable now has the right sign and is significant in the outmigration equation.
The coefficients of the dummy variables are much higher in size.

As for the systemic variables, their coefficients are positive as one would normally expect. But, whereas the draw variable coefficient is significant in the outmigration equation, the competition variable coefficient is not significant in the immigration equation.

With regard to the place-to-place migration flows, let us recall that the appropriate dependent variable is $\hat{M}_{ij}/d_{ij}^{-\hat{h}}$ i.e., the ratio of the corrected (estimated) migration flow from province i to province j to the value of the relational term t_{ij} for the estimated value $\hat{h}$ of the friction coefficient. The results obtained (see for example the third equation of Table 6) confirm some of the results found earlier.

The two population variables and the two dummy variables are highly significant.
The socio-economic variables perform poorly: in the case of the 1971-76 period, the unemployment variable of the origin is not significant any longer regardless of whether or not it is used in the presence of the wage rate variable of the destination.

More interesting however is the finding that the coefficients of *both* systemic variables are positive and highly significant. In the case of the 1971-76 period, their values lead to estimates of the α and β elasticities equal to 0.486 and 0.300, respectively which are broadly similar to those suggested by the total migration equations (0.524 and 0.302, respectively).

As a digression, note that the value of the population size coefficient at the origin appears to be less than 1, i.e., the migration flows out of each province are not proportional to the size of that province's population but rather to a power function of this size where the exponent takes a value less than one. The interest of this observation is to suggest that the minor differences observed in the ranking orders of the Canadian provinces according to their migration propensities and according to their systemic values (differences which concern provinces varying as much in size as British Columbia, Newfoundland and Ontario) were simply due to a size effect.

At this stage, let us again point out the poor performance of the socio-economic variables in the regression analysis of the total and place-to-place migration flows in Canada. We suggest that the main reason for such a finding is the small number of spatial units considered, with the consequence being the small variation of the socio-economic variables across provinces: for example, the ratio of the standard deviation to the mean in the case of log W is less than 3 percent.

In view of the above evidence concerning the socio-economic variables, we may wonder whether we would obtain significantly different results if we ignored these variables when performing a regression analysis of the total and place-to-place migration flows. Indeed, we have performed such an analysis for each of the three periods, 1961-66, 1966-71, and 1971-76, the results of which are reported in Tables 7 through 9. It turns out that we obtain for the remaining variables results similar to those derived in the presence of the socio-economic variables. For example, compare Table 6 and 9 which display

Table 7 — Regression equations explaining the variations of the total and place-to-place migration flows when ignoring economic variables, 1961-66

$$\log M_{i.} = 4.591 + \underset{(10.30)}{0.896} \log P_i + \underset{(3.27)}{1.017} \log D_i - \underset{(-4.34)}{1.641} X_i$$

$$R^2 = 0.975 \quad F = 38.1 \quad m = 8.28 \quad S.E = 0.239$$

$$\log M_{.j} = 3.363 + \underset{(8.37)}{0.987} \log P_j + \underset{(1.92)}{0.801} \log C_j - \underset{(-3.11)}{1.540} X_j$$

$$R^2 = 0.966 \quad F = 27.8 \quad m = 8.20 \quad S.E = 0.311$$

$$\log \frac{\hat{M}_{ij}}{d_{ij}^{-\hat{h}}} = -1.090 + \underset{(24.26)}{0.896} \log P_i + \underset{(25.66)}{0.988} \log P_j + \underset{(0.10)}{0.013} \log D_i$$

$$\underset{(-1.39)}{0.190} \log C_j - \underset{(-10.19)}{1.637} X_i - \underset{(-9.56)}{1.546} \log X_j$$

$$R^2 = 0.975 \quad F = 261.0 \quad m = 12.14 \quad S.E = 0.303$$

Table 8 — Regression equations explaining the variations of the total and place-to-place migration flows when ignoring economic variables, 1966-71

$$\log M_{i.} = 4.025 + \underset{(10.18)}{0.867} \log P_i + \underset{(2.39)}{0.688} \log D_i - \underset{(-3.28)}{1.241} X_i$$

$$R^2 = 0.977 \quad F = 41.7 \quad m = 8.37 \quad S.E = 0.232$$

$$\log M_{.j} = 2.589 + \underset{(8.04)}{0.934} \log P_j + \underset{(0.77)}{0.318} \log C_j - \underset{(-2.84)}{1.432} X_j$$

$$R^2 = 0.966 \quad F = 27.7 \quad m = 8.26 \quad S.E = 0.316$$

$$\log \frac{M_{ij}}{d_{ij}^{-\hat{h}}} = -2.482 + \underset{(23.06)}{0.866} \log P_i + \underset{(24.83)}{0.936} \log P_j - \underset{(-2.48)}{0.316} \log D_i$$

$$-\underset{(-4.97)}{0.666} \log C_j - \underset{(-7.41)}{1.237} X_i - \underset{(-8.85)}{1.445} X_j$$

$$R^2 = 0.977 \quad F = 284.5 \quad m = 12.24 \quad S.E = 0.306$$

Table 9 — Regression equations explaining the variations of the total and place-to-place migration flows when ignoring economic variables, 1971-76

$$\log M_{i.} = 3.284 + \underset{(11.32)}{0.897 \log P_i} + \underset{(1.78)}{0.486 \log D_i} - \underset{(-3.72)}{1.279\, X_i}$$

$R^2 = 0.982$ F = 53.0 m = 8.37 S.E = 0.220

$$\log M_{.j} = 3.255 + \underset{(7.12)}{0.814 \log P_j} + \underset{(0.52)}{0.217 \log C_j} - \underset{(-2.51)}{1.310\, X_j}$$

$R^2 = 0.959$ F = 23.1 m = 8.38 S.E = 0.308

$$\log \frac{\hat{M}_{ij}}{d_{ij}^{\hat{h}}} = 2.741 + \underset{(25.36)}{0.896 \log P_i} + \underset{(22.44)}{0.815 \log P_j} - \underset{(-4.26)}{0.518 \log D_i}$$
$$\underset{(-5.94)}{0.775 \log C_j} - \underset{(-8.33)}{1.276\, X_i} - \underset{(-7.95)}{1.317\, X_j}$$

$R^2 = 0.979$ F = 312.9 m = 12.28 S.E = 0.292

corresponding regression equations for the period 1971-76 with and without inclusion of the socio-economic variables (*).

As a consequence, the equations shown in Tables 7 through 9 can be used to assess the importance of the systemic variables during each of the three observation periods considered.

First, observe that, from one period to the next, the coefficient of the systemic variables in the out- and in-migration flow equations tend to become less significant at the same time that their values decrease toward zero: the impact of the system on the total migration flows out of and into each province tends to diminish over time.

By contrast, the coefficients of the systemic variables in the place-to-place migration flow equation tends, from one period to the next, to become more significant as their values move away from zero toward one. The impact of the system on the spatial pattern of migration flows tends to increase over time.

In fact, the two above observations are in total agreement — they reflect the consistency existing in Alonso's theory of movement between place-to-place and total migration flows — and lead to consistent estimates of the α and β

(*) This shows that in the case of the Canadian interprovincial system, place-to-place migration flows can be adequately represented by a gravity model such as (3') — in which the masses refer only to the populations at the origin and the destination — with the single addition of a dummy variable to account for the specificity of Quebec (a single dummy variable is sufficient since X_i and X_j in each of the place-to-place migration equations have similar coefficients).

parameters which are shown in Table 10 (*). Note the α parameter is consistently higher than the β parameter by 0.2 to 0.3.

Table 10 - Estimated values of the α and β elasticities for alternative periods

Period	α	β
1961-66	1.0	0.8
1966-71	0.7	0.3
1971-76	0.5	0.2

On the basis of the above evidence, it thus follows that, since the early sixties, the total migration out of or into Canadian provinces tends to be influenced less and less by the conditions prevailing in the rest of the country. (Note that the concomitant result concerning the place-to-place migration flows is to be accepted with more caution since, as mentioned above, it is the reflection of the internal consistency of Alonso's theory).

Conclusion

In this paper, we have indicated how the observation that Alonso's theory of movement relies on a standard model of spatial interaction allows for the estimation of the systemic variables entering in its specification.

The application of the method to the case of Canadian interprovincial migration has revealed that the two systemic variables were taking on similar values owing to the rather symmetric pattern of the observed matrix of interprovincial flows. The relative values of these systemic variables across provinces were shown to be primarily affected by the migration propensities of each region — up to a size effect — at the same time that changes in socio-economic conditions were found to have a relatively small role in the migration decision.

It is suspected, however, that the above results are essentially due to the relatively small number of spatial units considered in our Canadian application. It is thus recommended that the methodology proposed above be applied to a more disaggregated spatial level before drawing some general conclusions on (a) the role of Alonso's systemic variables and (b) the role of changes in socio-economic variables in the decision to migrate.

(*) In his empirical work, however based on a less precise calibration method, Alonso (1973) estimated $\alpha = 0.3$ and $\beta = 0.1$ for the United States in the 1955-60 period.

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APPENDIX

Table A1 — Annual average number of family moves, 1961-66; 1966-71; 1971-76

A — 1961-66

from \ to	1	2	3	4	5	6	7	8	9	10	Total out-migration
1. Newfoundland		24	231	95	171	669	28	15	39	45	1317
2. Prince Edward Island	17		143	100	39	238	17	7	26	24	611
3. Nova Scotia	214	167		628	460	1952	112	48	137	341	4059
4. New Brunswick	90	91	579		750	1329	99	24	99	118	3179
5. Quebec	183	35	378	603		4307	228	97	263	407	6501
6. Ontario	366	165	1244	959	4225		1429	559	1255	1697	11899
7. Manitoba	31	16	107	79	289	1604		861	849	959	4795
8. Saskatchewan	12	6	36	24	100	633	903		1823	1038	4575
9. Alberta	37	19	111	72	290	1378	663	1259		3175	7104
10. British Columbia	28	21	253	74	333	1321	492	579	2178		5279
Total inmigration	978	544	3082	2634	6657	13431	3971	3449	6669	7804	

B — 1966-71

to / from	1	2	3	4	5	6	7	8	9	10	Total out-migration
1. Newfoundland		15	226	115	165	1053	30	19	52	61	1736
2. Prince Edward Island	15		137	84	42	231	22	8	30	37	606
3. Nova Scotia	221	151		542	382	1965	126	50	182	368	3987
4. New Brunswick	103	76	566		594	1424	98	34	131	172	3198
5. Quebec	178	43	373	629		5826	281	110	414	768	8622
6. Ontario	662	209	1525	1131	3596		1266	541	1515	2479	12924
7. Manitoba	24	21	120	99	310	1629		798	1051	1346	5398
8. Saskatchewan	13	5	49	33	103	717	1025		2155	1429	5529
9. Alberta	33	21	136	79	255	1454	628	1179		3777	7562
10. British Columbia	39	16	252	99	385	1941	652	650	2681		6715
Total inmigration	1288	557	3384	2811	5832	16240	4128	3389	8211	10437	

C — 1971-76

to / from	1	2	3	4	5	6	7	8	9	10	Total out-migration
1. Newfoundland		24	283	157	158	945	81	15	87	96	1846
2. Prince Edward Island	22		135	92	26	198	21	7	48	28	577
3. Nova Scotia	258	160		602	276	1488	136	53	253	349	3575
4. New Brunswick	138	106	556		496	989	86	33	157	187	2748
5. Quebec	151	46	351	651		4803	239	68	456	699	7464
6. Ontario	1138	282	1749	1366	3804		1488	529	2352	3190	15898
7. Manitoba	58	28	129	99	218	1449		829	1229	1204	5243
8. Saskatchewan	17	8	64	39	65	540	774		2055	1176	4738
9. Alberta	63	35	221	121	269	1587	754	1433		4282	8765
10. British Columbia	69	23	270	150	423	2059	768	844	3569		8175
Total inmigration	1914	712	3758	3277	5735	14058	4347	3811	10206	11209	

SOURCE: Statistics Canada (1977).

Table A2 — Values of the regional socio-economic variables used

	New-found-land	Prince Edward Island	Nova Scotia	New Bruns-wick	Quebec	Ontario	Manitoba	Saskat-chewan	Alberta	British Columbia
1. Population - in thousands										
1961	458	105	737	598	5259	6236	922	925	1332	1629
1966	493	109	756	617	5781	6961	963	955	1463	1874
1971	522	112	789	635	6028	7703	988	926	1628	2185
1976	558	118	829	677	6234	8265	1022	922	1838	2467
2. Unemployment Rate — in percentage										
1961			- - - - -		NOT AVAILABLE		- - - - -			
1966	6.1	7.5	4.8	5.1	4.1	2.6	2.8	1.5	2.6	4.6
1971	8.8	9.5	6.9	6.2	7.3	5.4	5.7	3.5	5.7	7.2
3. Weekly Wage (in real terms) - in dollars										
1961	70	53	63	67	79	80	80	79	82	80
1966	84	64	77	85	98	95	91	94	99	102
1971	125	87	112	121	137	139	133	131	142	142
4. Per Capita Disposable Income (in real terms) - in dollars										
1961	951	942	1271	1189	2137	1934	1697	1666	1701	1783
1966	1365	1357	1722	1698	2117	2573	2286	2272	2398	2442
1971	2212	2105	2635	2656	3169	3899	3457	2980	3467	3483

SOURCE: 1. 1961-1966-1971: Canada Dominion Bureau of Statistics (1968), Statistics Canada (1973)
1976 - Figures from the 1976 Census of Canada quoted in Termote (1978)
2. Figures from Statistics Canada (1976) quoted in Termote and Frechette (1979)
3. Termote, Frechette (1979)
4. Termote, Frechette (1979).

Table A3 — Interprovincial distances (in miles)

	1	2	3	4	5	6	7	8	9	10
1. Newfoundland		900	934	1073	1617	1952	3113	3628	3956	4600
2. Prince Edward Island	900		174	201	745	1080	2241	2756	3084	3728
3. Nova Scotia	934	174		192	776	1111	2272	2787	3115	3759
4. New Brunswick	1073	201	192		584	919	2080	2595	2923	3567
5. Quebec	1617	745	776	584		335	1496	2011	2339	2983
6. Ontario	1952	1080	1111	919	335		1304	1819	2147	2791
7. Manitoba	3113	2241	2272	2080	1496	1304		515	843	1387
8. Saskatchewan	3628	2756	2787	2595	2011	1819	515		328	1042
9. Alberta	3956	3084	3115	2923	2339	2147	843	328		773
10. British Columbia	4600	3728	3759	3567	2983	2791	1387	1042	773	

SOURCE: Termote, Frechette (1979).

Table A4 — Number of family moves predicted by the gravity model: annual average for the 1971-76 period

	1	2	3	4	5	6	7	8	9	10
1. Newfoundland		36	237	150	132	676	93	56	187	278
2. Prince Edward Island	30		152	96	36	158	17	10	32	46
3. Nova Scotia	233	179		804	283	1242	136	78	254	366
4. New Brunswick	134	102	729		240	966	97	54	176	250
5. Quebec	186	62	408	382		4981	266	140	439	600
6. Ontario	796	223	1488	1276	4139		1536	781	2416	3243
7. Manitoba	110	24	163	128	221	1535		532	1217	1314
8. Saskatchewan	64	13	91	70	113	760	518		1957	1152
9. Alberta	155	32	215	164	257	1703	858	1418		3963
10. British Columbia	205	41	275	208	313	2036	825	743	3529	

Table A5 — Ratios of the predicted (by the gravity model) to actual numbers of family moves, 1971-76

	1	2	3	4	5	6	7	8	9	10
1. Newfoundland		1.497	0.839	0.958	0.833	0.716	1.154	3.740	2.147	2.895
2. Prince Edward Island	1.363		1.125	1.040	1.403	0.800	0.818	1.401	0.664	1.634
3. Nova Scotia	0.904	1.117		1.335	1.025	0.835	1.004	1.474	1.005	1.047
4. New Brunswick	0.970	0.963	1.310		0.484	0.977	1.124	1.650	1.121	1.339
5. Quebec	1.234	1.346	1.162	0.586		1.037	1.114	2.058	0.963	0.858
6. Ontario	0.699	0.792	0.851	0.934	1.088		1.032	1.475	1.027	1.017
7. Manitoba	1.896	0.864	1.267	1.289	1.014	1.059		0.641	0.990	1.091
8. Saskatchewan	3.780	1.682	1.423	1.796	1.742	1.407	0.669		0.952	0.979
9. Alberta	2.460	0.906	0.972	1.355	0.957	1.073	1.138	0.989		0.926
10. British Columbia	2.976	1.762	1.018	1.385	0.740	0.989	1.075	0.880	0.989	

Résumé. Cet essai montre d'abord comment la théorie générale des déplacements d'Alonso peut être ramenée à un modèle standard d'interaction spatiale doublement contraint. Cette découverte suggère, ensuite, l'utilisation d'une méthode d'adjustement biproportionelle (la méthode RAS) pour estimer de manière adéquate les variables systémiques specifiées dans le modèle sous-jacent. Ceci conduit, enfin, au développement d'une méthodologie complète et précise pour calibrer le modèle d'Alonso. Cette méthodologie est illustrée par une application à des données sur la migration interprovinciale au Canada.

Riassunto. In questo articolo si mostra, in primo luogo, come la teoria generale degli spostamenti di Alonso si possa ricondurre ad un modello standard di interazione spaziale doppiamente vincolato. Tale riconoscimento suggerisce, dunque, di utilizzare un metodo di calibrazione biproporzionale (il metodo RAS) per stimare adeguatamente le variabili del sistema specificate nel modello implicito. Ciò porta, alla fine, allo sviluppo di una completa e rigorosa metodologia per la calibrazione del modello di Alonso. Questa metodologia è illustrata attraverso un'applicazione ai dati sulla migrazione interprovinciale in Canada.

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On the relationship between Alonso's theory of movement and Wilson's family of spatial-interaction models†

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Abstract. This paper compares the system of equations underlying Alonso's theory of movement with that of Wilson's standard family of spatial-interaction models. It is shown that the Alonso model is equivalent to one of Wilson's four standard models depending on the assumption at the outset about which of the total outflows and/or inflows are known. This result turns out to supersede earlier findings—inconsistent only in appearance—which were derived independently by Wilson and Ledent. In addition to this, an original contribution of this paper—obtained as a by-product of the process leading to the aforementioned result—is to provide an exact methodology permitting one to solve the Alonso model for each possible choice of the input data.

Introduction

Very recently, Wilson (1979) and Ledent (1980) have attracted the attention of regional scientists to the similarities presented by the general theory of movement proposed by Alonso (1973; 1975; 1976; 1978) and the spatial-interaction models developed by Wilson (1971; 1974). However, most unfortunately, their papers arrive at inconsistent results.

On the one hand, Ledent (1980) argues that the Alonso model is identical to a doubly-constrained model of spatial interaction. On the other hand, Wilson (1979) claims that each of the four models constituting his family of standard interaction models (Wilson, 1971) can be seen as a special case of Alonso's underlying model for an appropriate choice of the model parameters. In particular, "Setting one of the parameters α and β to zero essentially makes the Alonso model production or attraction constrained respectively. Setting both to zero makes it doubly constrained." (Wilson, 1979, page 3).

Actually, a careful examination of the papers by Wilson and Ledent reveals that both of the above results are correct with regard to what they respectively take for granted at the outset. This observation then suggests that the aforementioned inconsistency is only one of appearance. In other words, there should exist a relationship of a more general nature between Alonso's theory of movement and Wilson's spatial-interaction models, a relationship which moreover boils down to the similarities observed by Wilson or Ledent in case some special conditions are met.

In view of this, the purpose of this paper is precisely to uncover the nature of this general relationship and to indicate in what circumstances such a relationship becomes equivalent to those proposed by Wilson and Ledent.

Alonso's theory of movement: an overview

First, in order to facilitate understanding by the reader a brief summary of Alonso's theory is presented. Fundamentally, this theory is concerned with building a mathematical model which describes the flows of elements moving between the mutually exclusive classes of a closed system. Its main feature which makes it

† The reader may care to note a recent issue of *Environment and Planning A* (volume 12, number 6)—released after this paper was accepted for publication—in which were published the working paper by Wilson (1979) alluded to in this paper as well as a paper by Hua and a letter by Alonso relating to the same subject.

particularly worthwhile is that the formulation of these flows reflects not only the influence of the factors which are usually considered to affect movement (that is factors associated with the classes of origin and destination as well as with intervening obstacles), but also the impacts exerted by the system upon alternative classes.

Note that, because Alonso's theory is circular in nature, various equivalent formulations are possible. The specification which we use below differs significantly from Alonso's final exposition (Alonso, 1978): it is a modified version of the formulation proposed by Anselin and Isard (1980), one which represents a more logical démarche in view of the objective of this note.

Let us focus on the flow M_{ij} of elements from class (origin) i to class (destination) j. First, in accordance with the usual explanations of movement, one can reasonably hypothesize that this flow is proportional to:
(a) a global measure v_i of the unattractive characteristics of class i,
(b) a global measure w_j of the attractive characteristics of class j, and
(c) a relational term t_{ij} reflecting the effect of distance between classes i and j.

Second, suppose that the impacts that the system exerts upon the alternative classes can be accounted for by two variables, the pull-in D_i it exerts on each origin i and the push-out C_j it exerts on each destination j. Then, it can reasonably be posited that M_{ij} is proportional to:
(d) D_i to which is applied an exponent $\alpha_i - 1$ reflecting the movement of response from class i to its relation to the other classes in the system, and to
(e) C_j to which is applied an exponent $\beta_j - 1$ reflecting the movement of response of class j to its relation to the other classes in the system.

Thus

$$M_{ij} = v_i w_j D_i^{\alpha_i - 1} C_j^{\beta_j - 1} t_{ij} \,. \tag{1}$$

The problem now is one of defining the systemic variables D_i and C_j. On the one hand, D_i can be regarded as a draw effect representing a weighted sum of the attractive characteristics w_j of all the other classes in the system where the weights applied to any w_j expresses the influences both of distance and of ease of entry into j. Recalling the variables introduced above, these influences can be taken care of by use of t_{ij} and $C_j^{\beta_j - 1}$, respectively. Thus

$$D_i = \sum_j w_j C_j^{\beta_j - 1} t_{ij} \,. \tag{2}$$

On the other hand, C_j can be regarded as a competition effect representing a weighted sum of the unattractive characteristics v_i of all the other classes in the system where the weights applied to any v_i expresses the influences both of distance and of ease of movement out of j. These influences can indeed be taken care of by use of t_{ij} and $D_i^{\alpha_i - 1}$, respectively, and thus

$$C_j = \sum_i v_i D_i^{\alpha_i - 1} t_{ij} \,. \tag{3}$$

From there, the specification of the Alonso model can be completed by deriving from equation (1) the total flow of movements out of and into each class by summing over all possible destinations:

$$M_{i\cdot} = \sum_j M_{ij} \tag{4}$$

and origins:

$$M_{\cdot j} = \sum_i M_{ij} \,, \tag{5}$$

respectively.

One immediately obtains

$$M_{i.} = v_i D_i^{\alpha_i - 1}\left(\sum_j w_j C_j^{\beta_j - 1} t_{ij}\right) ,$$

and

$$M_{.j} = w_j C_j^{\beta_j - 1}\left(\sum_i v_i D_i^{\alpha_i - 1} t_{ij}\right) ,$$

or, after substitution of equations (2) and (3),

$$M_{i.} = v_i D_i^{\alpha_i} , \tag{6}$$

and

$$M_{.j} = w_j C_j^{\beta_j} . \tag{7}$$

The Alonso model: the four cases

Now that the model underlying Alonso's theory of movement in its entirety has been spelled out the respective numbers of equations and variables in contains are counted. As for the former, it is noted that the model consists of seven equation types, type (1)—which contains scalar equations in a number equal to the number of communications existing between alternative classes— and types (2) through (7)— each one of which contains as many scalar equations as there are classes. However, two of the equation types (2) through (7) are redundant as equation types (6) and (7) were obtained by combining equation types (1) through (5). It follows that the core of the Alonso model consists of five equation types, of which (1) or any equivalent equation type describing class-to-class flows is a necessary one.

Attention is now turned to the count of the number of variables. It is first pointed out that the values of the parameters α_i and β_j as well as of the relational term t_{ij} are inputs to the Alonso model. Thus, the model contains seven variable types. One relates to the class-to-class flows M_{ij} which take as many values as there are communications between alternative states, that is, as many as there are scalar equations of type (1). The other variable types are D_i, C_j, v_i, w_j, $M_{i.}$, and $M_{.j}$, all of which take as many values as there are classes.

Therefore, in view of the fact that Alonso's system of equations is identified only if the number of equations and variables are identical, it follows that variable types which are unknown include the class-to-class flows M_{ij} (necessarily) plus four among the six other variable types. Since the systemic variables D_i and C_j are indeed unknowns, this means that two variable types among v_i, w_j, $M_{i.}$, and $M_{.j}$ are endogenously determined (that is, unknowns) whereas the other two are necessarily inputs to the model.

Observing that there are six possible ways of choosing two variables among four, it is concluded that the Alonso model comprises six alternative cases, depending on the composition of the inputs. In practice however, the possibility of knowing both variable types relating to the origin (that is, v_i and $M_{i.}$) or to the destination (that is, w_j and $M_{.j}$) is to be ruled out because, by virtue of equation types (6) and (7), respectively, D_i and C_j would be immediately known.

We are thus left with the result that *the Alonso model comprises four alternative cases depending on which of the total flows out of ($M_{i.}$) and into ($M_{.j}$) each class are known at the outset.* The four cases are, respectively:

case 1 neither outflows ($M_{i.}$) nor inflows ($M_{.j}$) are known,
case 2 only outflows ($M_{i.}$) are known,
case 3 only inflows ($M_{.j}$) are known, and
case 4 both outflows ($M_{i.}$) and inflows ($M_{.j}$) are known.

Let us now examine the model core of these four alternative cases. We already know that, in any case, equation type (1) belongs to the model core. Also, since D_i and C_j are unknowns in all the four cases, one can safely include types (2) and (3) in the model core. Thus, the problem is one of choosing two equation types from (4) through (7), but one should avoid taking the two equation types relating to outflows or to inflows, hence there are four possibilities. Logically, when outflows and/or inflows are known, one can omit from the model core the corresponding equation type(s) (4) and/or (5) and keep within the core the equation type(s) (6) and/or (7) corresponding to the eventual outflows and/or inflows. Thus, the model core of the four alternatives is composed of equations (1) through (3) plus the following two:
case 1 equation types (6) and (7),
case 2 equation types (4) and (7),
case 3 equation types (5) and (6), and
case 4 equation types (4) and (5).
The redundant equations are, in each case, those from types (4) through (7) which are not listed above.

Clearly, when the outflows and/or inflows are known, the corresponding definitional equations—(4) and/or (5)—which are included within the model core act as constraints. This indeed suggests that the Alonso model bears some resemblance to the spatial interaction models developed by Wilson (1971): there seems to exist a close connection between each of the four cases of the Alonso model arrived at above and one of the four standard interaction models proposed by Wilson. More precisely:
case 1 appears to correspond to the unconstrained model,
case 2 to the production constrained model,
case 3 to the attraction constrained model, and
case 4 to the doubly-constrained case.

The remainder of this paper is devoted to revealing such a connection.

Equivalence of the Alonso model and Wilson's standard family of spatial-interaction models

This connection is carried out by examining, in the four cases taken in succession, how one can solve the Alonso model from the available data.

Case 1: neither the outflows $M_{i.}$ nor the inflows $M_{.j}$ are known

By recalling that two among the four variable types $M_{i.}$, $M_{.j}$, v_i, and w_j are known, the assumption underlying this first case implies that v_i and w_j are known inputs to the model. [Note that in this case, the parameters α and β are necessarily different from zero because, if they were not, the total outflows and/or inflows would be known at the same time. This is by virtue of equations (14) and/or (15) to be derived.]

Under these circumstances, the model is solved as follows. First, the system of equations (2) and (3) is solved iteratively to yield values of D_i and C_j. This allows one to derive the values of the following composite variables:

$$V_i = v_i D_i^{\alpha_i - 1} , \qquad (8)$$

and

$$W_j = w_j C_j^{\beta_j - 1} . \qquad (9)$$

Finally, the class-to-class flows can be obtained from equation (1) which can be rewritten as

$$M_{ij} = V_i W_j t_{ij} , \qquad (10)$$

an equation which permits one to conclude that, if neither $M_{i.}$ nor $M_{.j}$ are known, the Alonso model boils down to Wilson's unconstrained spatial-interaction model.

Case 2: only the outflows $M_{i.}$ are known
In this case the inputs to the Alonso model appear to be $M_{i.}$ and w_j (with $\beta_j \neq 0$, otherwise the inflows would also be known). From equation (6), one can then substitute $M_{i.}D_i^{-1}$ for $v_i D_i^{\alpha_i - 1}$ in equation (3). This leads to the system of equations comprised of equation (2) and

$$C_j = \sum_i M_{i.} D_i^{-1} t_{ij} , \tag{11}$$

which can be solved iteratively to yield values of D_i and C_j (as well as values of composite variables W_j defined exactly as before). Then, the class-to-class flows can be obtained from equation (1) rewritten as

$$M_{ij} = D_i^{-1} M_{i.} W_j t_{ij} , \tag{12}$$

in which one has from equations (2) and (9)

$$D_i^{-1} = \left(\sum_j W_j t_{ij} \right)^{-1} . \tag{13}$$

Thus, if the outflows $M_{i.}$ solely are known, the Alonso model boils down to Wilson's production-constrained interaction model in which the balancing factor is the reciprocal of the systemic variable D_i.

Case 3: only the inflows $M_{.j}$ are known
This case is the mirror image of case 2 above. The inputs to the Alonso model are $M_{.j}$ and v_i (with $\alpha_i \neq 0$, otherwise the outflows would also be known). By means of equation (7), one can then substitute $M_{.j} C_j^{-1}$ for $w_j C_j^{\beta_j - 1}$ in equation (2). This leads to the system of equations comprised of equation (3) and

$$D_i = \sum_j M_{.j} C_j^{-1} t_{ij} , \tag{14}$$

which can be solved iteratively for values of C_j and D_i (as well as values of composite variables V_i defined exactly as before). Then, the class-to-class flows can be obtained from equation (1) rewritten as

$$M_{ij} = C_j^{-1} V_i M_{.j} t_{ij} , \tag{15}$$

in which one has from equations (3) and (8)

$$C_j^{-1} = \left(\sum_i V_i t_{ij} \right)^{-1} . \tag{16}$$

Thus, if only the inflows $M_{.j}$ are known, the Alonso model boils down to Wilson's attraction-constrained interaction model in which the balancing factor is the reciprocal of the systemic variable C_j.

Case 4: both the outflows $M_{i.}$ and the inflows $M_{.j}$ are known
In this case, by means of the redundant equations (6) and (7), one is led to the system of equations comprised of equations (11) and (14), which can be solved iteratively for values of D_i and C_j. Then, the class-to-class flows can be obtained from equation (1) rewritten as

$$M_{ij} = D_i^{-1} C_j^{-1} M_{i.} M_{.j} t_{ij} , \tag{17}$$

where one indeed has

$$D_i^{-1} = \left(\sum_j M_{.j} C_j^{-1} t_{ij} \right)^{-1} \tag{18}$$

and

$$C_j^{-1} = \left(\sum_i M_{i.} D_i^{-1} t_{ij}\right)^{-1} . \tag{19}$$

Thus, if both outflows and inflows are known at the outset, the Alonso model boils down to Wilson's doubly-constrained interaction model in which the balancing factors are the reciprocals of the systemic variables D_i and C_j.

The conclusion here is that the Alonso model and the standard family of spatial-interaction models developed by Wilson are equivalent: as shown above, the Alonso models boils down to one among the four members of the family depending upon which of the total outflows $M_{i.}$ and inflows $M_{.j}$ are known at the outset.

On looking back at the treatment of the four cases, the central role played by the following system of equations is noted:

$$D_i = \sum_j b_j C_j^{\delta_j} t_{ij} , \tag{20}$$

$$C_j = \sum_i a_i D_i^{\gamma_i} t_{ij} , \tag{21}$$

where

a_i is equal either to v_i or to $M_{i.}$,
b_j is equal either to w_j or to $M_{.j}$,
γ_i is equal either to $\alpha_i - 1$ or to -1, and
δ_j is equal either to $\beta_j - 1$ or to -1.

The fact is that, for any value of γ_i and δ_j[(1)], the above system of equations has a unique solution which in general can be reached by iteration (if $\gamma_i = 0$ and $\delta_j = 0$, the solution is immediately obtained). No attempt is made, in the context of this paper, to provide a formal demonstration of this result which actually is a generalization of the result which is well known to hold in the case $\gamma_i = -1$ and $\delta_j = -1$. It is recalled that this particular choice of the parameters γ_i and δ_j corresponds to the possibility of finding D_i and C_j as the inverse of the balancing factors of the following biproportional adjustment problem—sometimes referred to as the RAS problem: find the matrix **M** which has row and column totals equal to a_i and b_j, respectively, and which is biproportional to the matrix of relational terms **T**.

Summary and conclusions

This paper has sought to uncover the general nature of the connection between Alonso's theory of movement and Wilson's standard family of spatial-interaction models. It was shown that the two frameworks are formally identical: the eventual knowledge of the total outflows and/or inflows considered determines which one among the members of Wilson's family the Alonso model is equivalent to.

How does this result compare with the earlier findings obtained by Wilson (1979) and Ledent (1980)? Clearly, it supersedes them as Wilson's and Ledent's findings can be taken as particular cases.

On the one hand, assuming the total outflows and inflows to be known—which is precisely what Ledent (1980) does—leads to the recovery of his result that the Alonso model can be identified with a doubly-constrained spatial-interaction model.

(1) An exception to this—which Oscar Fisch pointed out to the author—arises in the case $\gamma_i = 1$ and $\delta_j = 1$. Under these circumstances, the system of equations (20)-(21) becomes an homogeneous linear equation system which generally has no other solution than $D_i = 0$ and $C_j = 0$. It is readily established that there would be a nontrivial solution only if the determinant of the matrix $\mathbf{T}^{\mathrm{T}}\mathbf{ATB}-\mathbf{I}$ (where $\mathbf{T}^{\mathrm{T}}$ is the transpose of the matrix **T** of the relational terms, **A** and **B** two diagonal matrices having a_i and b_j, respectively, as typical elements, and **I** an identity matrix) would be zero.

On the other hand, in case the values of u_i and w_j are known, setting α and/or β to zero causes total outflows and/or inflows to be known and allows one to derive each of Wilson's family of spatial-interaction models as special cases of the Alonso model.

Note the possibility of characterizing further, from a different angle, the difference between Wilson's (1979) result and the one derived in this paper. Wilson (1979) appears to have stated a *sufficient* condition for the Alonso model to be identical with this standard family of spatial-interaction models, but his condition (u_i and w_j are known, α and β are equal and/or different from zero, respectively) is not necessary. By contrast, this paper has shown that a *necessary and sufficient* condition for the identity of the two frameworks rested on the eventual knowledge of the outflows and/or inflows, which indeed does not necessitate the inclusion of the variables u_i and/or w_j, among the input data: as has been shown, the values of these variables can be endogenously determined by solving the Alonso model from the iterative procedure which was suggested.

Table 1. The relationship between Alonso's theory of movement and Wilson's family of standard interaction models: a tabular summary.

Case 1: neither $M_{i.}$ nor $M_{.j}$ are known	Case 2: only $M_{i.}$ is known	Case 3: only $M_{.j}$ is known	Case 4: both $M_{i.}$ and $M_{.j}$ are known
Inputs to the model			
v_i, w_j (α_i and $\beta_j \neq 0$)	$M_{i.}, w_j$ ($\beta_j \neq 0$)	$v_i, M_{.j}$ ($\alpha_i \neq 0$)	$M_{i.}, M_{.j}$
Alonso's model			
Model core—equations common to the four alternatives			
$M_{ij} = v_i w_j D_i^{\alpha_i-1} C_j^{\beta_j-1} t_{ij}$ (1) $D_i = \sum_j w_j C_j^{\beta_j-1} t_{ij}$ (2) $C_j = \sum_i v_i D_i^{\alpha_i-1} t_{ij}$ (3)	as for case 1	as for case 1	as for case 1
Model core—additional equations			
$M_{i.} = v_i D_i^{\alpha_i}$ (6) $M_{.j} = w_j C_j^{\beta_j}$ (7)	$M_{i.} = \sum_j M_{ij}$ (4) $M_{.j} = w_j C_j^{\beta_j}$ (7)	$M_{i.} = v_i D_i^{\alpha_i}$ (6) $M_{.j} = \sum_i M_{ij}$ (5)	$M_{i.} = \sum_j M_{ij}$ (4) $M_{.j} = \sum_i M_{ij}$ (5)
Redundant equations			
$M_{i.} = \sum_j M_{ij}$ (4) $M_{.j} = \sum_i M_{ij}$ (5)	$M_{i.} = v_i D_i^{\alpha_i}$ (6) $M_{.j} = \sum_i M_{ij}$ (5)	$M_{i.} = \sum_j M_{ij}$ (4) $M_{.j} = w_j C_j^{\beta_j}$ (7)	$M_{i.} = v_i D_i^{\alpha_i}$ (6) $M_{.j} = w_j C_j^{\beta_j}$ (7)
Corresponding spatial-interaction model			
Type			
Unconstrained	Production-constrained	Attraction-constrained	Doubly-constrained
Equations			
$M_{ij} = V_i W_j t_{ij}$ (10) where $V_i = v_i D_i^{\alpha_i-1}$ (8) $W_j = w_j C_j^{\beta_j-1}$ (9)	$M_{ij} = D_i^{-1} M_{i.} W_j t_{ij}$ (12) where $W_j = w_j C_j^{\beta_j-1}$ (9)	$M_{ij} = C_j^{-1} V_i M_{.j} t_{ij}$ (15) where $V_i = v_i D_i^{\alpha_i-1}$ (8)	$M_{ij} = D_i^{-1} C_j^{-1} M_{i.} M_{.j} t_{ij}$ (17)
Balancing factors			
	$D_i^{-1} = \left(\sum_j W_j t_{ij}\right)^{-1}$ (13)	$C_j^{-1} = \left(\sum_i V_i t_{ij}\right)^{-1}$ (16)	$D_i^{-1} = \left(\sum_j M_{.j} C_j^{-1} t_{ij}\right)^{-1}$ (18) $C_j^{-1} = \left(\sum_i M_{i.} D_i^{-1} t_{ij}\right)^{-1}$ (19)
Fundamental equations			
$D_i = \sum_j w_j C_j^{\beta_j-1} t_{ij}$ (2) $C_j = \sum_i v_i D_i^{\alpha_i-1} t_{ij}$ (3)	$D_i = \sum_j w_j C_j^{\beta_j-1} t_{ij}$ (2) $C_j = \sum_i M_{i.} D_i^{-1} t_{ij}$ (11)	$D_i = \sum_j M_{.j} C_j^{-1} t_{ij}$ (14) $C_j = \sum_i v_i D_i^{\alpha_i-1}$ (3)	$D_i = \sum_j M_{.j} C_j^{-1} t_{ij}$ (14) $C_j = \sum_i M_{i.} D_i^{-1} t_{ij}$ (11)

In addition, the difference observed in the nature of the conditions considered in this paper—sufficient in the case of Wilson (1979), necessary and sufficient in the case of this paper—leads to the following corollary: Wilson has not formally proved the equivalence of Alonso's framework and his—a fact which was demonstrated in this paper—but has simply shown that his framework was a subset of Alonso's system of equations.

Finally, note that, besides uncovering the general nature of Alonso's and Wilson's frameworks—of which a tabular summary is shown in table 1—an original contribution of this paper is to set forth an exact methodology for solving the Alonso model for each possible choice of the input data. The main interest of this is that, in contrast to the general belief that the Alonso model is only a theoretical framework, this model constitutes a tool that can be used for applied analysis: for example, as illustrated by Ledent (1980, section V), the Alonso model is capable of producing alternative simulations of the regional populations of a nation based on various assumptions regarding the values of the variables v_i and w_j.

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