Resources Policy 61 (2019) 311–347

Contents lists available at ScienceDirect

Resources Policy

journal homepage: www.elsevier.com/locate/resourpol

Understanding feedbacks between economic decisions and the phosphorus resource cycle: A general equilibrium model including material flows

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ARTICLE INFO

Keywords:
Phosphorus recycling
General equilibrium
Fertilizer choice
Agricultural economics
Material flow analysis
Socio-economic metabolism

ABSTRACT

By combining an economic two-sector general equilibrium model with a material flow model we study the coupled human-resource-environment feedbacks associated with phosphorus use and recycling, and the economic and environmental effects of implementing phosphorus recovering technologies from waste water. Using recycled phosphorus as fertilizer increases environmental quality and profits in the agricultural sector. Furthermore, the economy does not depend as much on mineral fertilizer imports and is therefore more resilient to a price increase on the global phosphorus market. However, there is a need to improve the quantity and quality of recycled phosphorus products. Overall, reduction of phosphorus in soil and water bodies as result of economic decisions is only possible if phosphorus is recovered from waste water and the prices of imported mineral fertilizer rise. Policy makers can support this technological change by subsidizing recycled phosphorus or introducing taxes or tolls for imported mineral fertilizer to increase its price. Alternatively, societal values would have to change. Such a change may be induced by putting a higher preference on a healthy environment and hence being willing to pay more for food and consequently production inputs like phosphorus fertilizer recycled from sewage sludge.

1. Introduction

Phosphorus (P) is an essential element for plant and animal growth and is necessary to maintain profitable crop and livestock production (Sharpley and Beegle, 2001). Food production depends on nonrenewable mineral phosphorus supplies from a finite P stock (Childers et al., 2011; Dawson and Hilton, 2011). There is a general consensus that the quality and accessibility of remaining reserves are decreasing and costs will increase in the medium and long term, additional to possible price shocks of phosphate fertilizer like in 2008 (Cordell and White, 2011).

Efficient fertilizer use not only increases profits Venezian (1962), also sustainable P use is crucial to preserve food security and this affects households' consumption decisions and consequently the whole economy.

Furthermore, P fertilizer use can negatively influence the environment. Mineral P fertilizer contains heavy metals (Zoboli et al., 2016a) and P is the critical element for eutrophication in most fresh waters besides nitrogen (N). Overfertilization leads to increased P stocks in soil and consequently emissions to surrounding water bodies increase. Generally, anthropogenic activities are the main causes of pollution and environmental problems (Ghazi et al., 2014). Pesticide and fertilizer consumption may reach problematic levels (Saysel et al., 2002) and rural living, livestock, paddy field, and precipitation alternately become the leading source of non-point source (NPS) pollution (Carpenter et al., 1998; Yuan et al., 2017).

Even though measures to ensure P availability are under discussion and the European Commission included phosphate rock into the revised list of Critical Raw Materials (EU-Commission et al., 2014) in 2014, only two countries have introduced new legislation enforcing P recycling from wastewater, namely Switzerland (Der Schweizerische, 2015) and Germany (VVEA, 2017). Moreover, economic incentives like subsidies for P recycling or increased prices for mineral fertilizer, their

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https://doi.org/10.1016/j.resourpol.2019.02.010
Received 7 June 2018; Received in revised form 10 February 2019; Accepted 11 February 2019

consequences and environmental feedbacks still need to be investigated.

Various approaches shed light on different aspects of this complex topic.

Egle et al. (2015) describes technologies for recovering phosphorus from municipal wastewater and Egle et al. (2016) compares them under technological, environmental and economic assessment. Amann et al. (2018) then presents the overall environmental impact assessment of 17 phosphorus recovery technologies considering P recovery potential, heavy metal and organic micropollutant decontamination potential and fertilizer efficiency. Although these studies provide an extensive informative basis, from an economic perspective they are limited to the quantification of the implementation costs and do not account for potential feedbacks in the economy.

Other branches of science model emissions of P into water bodies and address the issue of eutrophication in watersheds. Non-point source pollution from agricultural soils and water quality can be described and assessed with a large variety of models, ranging from physically based to empirical approaches, from spatially lumped to distributed and from deterministic to stochastic (Tsakiris and Alexakis, 2012). In this context, Sharpley et al. (2015) identified the poor knowledge and understanding of the economic dimension as one of the main research gaps regarding the reduction of agricultural emissions of P into water bodies.

Material Flow Analysis (MFA) constitutes the ideal technique to capture the manifold issues around P management, i.e. P availability, P recycling and P emissions, and to investigate them simultaneously with a system perspective. Over the past decade, a very large number of MFAs for P were performed at city, region, country and international level (see review of Chowdhury et al., 2014). By describing and balancing flows and stocks of P across all major anthropogenic and natural compartments, such studies enabled the identification and quantification of problems and lacks of efficiencies in societal P use, but also pointed out where the largest potentials for improvements lie. In Austria, which is used as case-study for the present work, this was done by Egle et al. (2014) and Zoboli et al. (2016a). Based on the knowledge gained through MFA, different attempts were made to understand how regional and national P management could be optimized (Vadenbo et al., 2014; Hanserud et al., 2016; Zoboli et al., 2016b; Kliglmair et al., 2017). Although all these works are based on solid system understanding provided by the detailed mass balances, none of them includes in its scope the economic dimension of the proposed measures.

Economically extended material flow analysis (EE-MFA) is an attempt to study causal relationships between economically motivated human behavior and resource consumption (Kytzia et al., 2004). However, the simplified linear relations of EE-MFA serve not as an appropriate tool for an in-depth analysis of the economic system.

Bouman et al. (2000) compares economic models which incorporate material flows: mathematical methods, data requirements and demarcation of the problem are different for stochastic frontier analysis (SFA), life cycle assessment (LCA), input-output analysis (IOA) and partial equilibrium models. Additionally, goals and assumptions defining the role of materials, the rigidity of economic relations and physical constraints, and economy-environment interactions can differ. E.g. LCA of fertilizers (Skowrońska and Filipek, 2014) may include the environmental impact of systems recycling plant nutrients (Spångberg, 2014) or impacts of fertilizer production (Amann et al., 2018).

Economic models, i.e. partial equilibrium models, analyze fertilizer demand via a cost (Boyle, 1982) or via a profit function (Larson and Vroomen, 1991; Liverpool-Tasie, 2017). Subsequently, (computable) general equilibrium models capture firm and consumer behavior to analyze fertilizer use or agricultural pollution (Diamond, 1967; Joy, 1973; Liu et al., 1996; Hassan and Hallam, 1996; Beghin et al., 1997; Calzadilla et al., 2010, e.g.) in the context of growth, trade or the environment. So far (partial) equilibrium models have looked at fertilizer decisions and their environmental impacts (Boyle, 1982; Larson and Vroomen, 1991; Liverpool-Tasie, 2017), but none of these models has integrated material flows. Bouman et al. (2000) applied MFA and a partial equilibrium model to the same environmental problem to compare the methods and advocates to integrate them.

Other economic studies have used cost-benefit analysis (CBA) to assess the trade-offs between the benefits derived from polluting activity and the environmental losses that derive from pollution Dixon et al. (1994); United Nations Environment (2016); Cazcarro et al. (2016). E.g. Gren et al. (2008); Wustenberghs et al. (2008); Bryhn (2009) and Hautakangas et al. (2014) study P management around the Baltic sea. Lancelot et al. (2011) applied integrated impact assessment methodology to the same issue. Moreover, simulation models explore and illustrate dynamics of socio-ecological systems and can simulate farm decision-making in agricultural systems (e.g. Schreinemachers and Berger, 2011). Yang et al. (2011) provides a very general model for recycling materials. Carpenter et al. (1999) modeled a lake subject to phosphorus pollution and Saysel et al. (2002) is the only Agent-Based Modeling and Simulation (ABMS) and System Dynamics Modeling and Simulation (SDMS) literature published between 2000 and 2015 (Moon, 2017) that also includes agricultural pollution. It yields that its two key environmental factors, pesticide and fertilizer consumption, may reach problematic levels.

None of the mentioned models includes two-way coupled feedbacks between economic activities and environment.

Cools et al. (2011) applies an alternative model approach by coupling a hydrological water quality model and an economic optimization model. They set up a scenario for nitrogen to obtain a cost-effective emission reduction. It includes wastewater treatment plant (WWTP) efficiency, fodder efficiency and fertilizer tuning, but is only build on a linear costs' assumption.

More complex multi-market equilibrium analysis on European agriculture like Moschini et al. (2005) does not consider fertilizer use. Also System Dynamics Modeling and Simulation (SDMS) literature published between 2000 and 2015 (Moon, 2017) is lacking investigation on P fertilizer use.

It is important to analyze coupled human-environmental feedbacks, i.e. to understand households' consumption decisions and their environmental impact as well as farmers fertilizer decisions. To do so, we combine material flow analysis (MFA) methodology and terminology (Baccini and Brunner, 1991, 2012; Brunner and Rechberger, 2004) with an economic model framework. This gives additional value to each model type: on the one hand, we include farmers' and households' decisions and resulting optimized profits and utility in a resource management framework. To our knowledge it is the first attempt to endogenously describe material flows based on economic decisions. On the other hand, we introduce the P cycle into a two-sector general equilibrium model. This integration allows to analyze welfare change and income distribution effects of change in physical system.

The model is calibrated to fit an Austrian P time series described in Zoboli et al. (2016a) and respective economic data from Wien-Statistik (2010). The parameter choice is location specific, but relevant to many watersheds, regions or countries with a similar soil structure and economy.

The dynamic economic general equilibrium model is solved analytically and numerically (using Matlab©) to point out the most important mechanisms that influence the decision makers.

The aim of the presented work is to establish a conceptual model with an economy-wide perspective on sustainable P use and understand coupled human-resource-environment feedbacks embedded in an economic framework. Based on that understanding we can investigate implications of introducing P recovery technologies (see e.g. Egle et al., 2014), of price changes in mineral P fertilizer (see e.g. Kytzia et al., 2004; Zoboli et al., 2016b), or of different societal preferences for a healthy environment (see e.g. Ambec and Coria, 2013). This contributes to better understand the socio-economic metabolism (Ayres, 1989; Fischer-Kowalski and Haberl, 1998) and adds to the fields of bio-economic models (Dellink et al., 2011; Flichman and Allen, 2015), coupled hydro-economic models (Bekchanov et al., 2017) and socio-hydrology (Pande and Sivapalan, 2017).

The basic model is introduced in the first section. In the results section we first present analytical solutions before we continue with numerical results. Within our framework we discuss different scenarios: First, a price shock of imported fertilizer similar to 2008. Second, the use of different recycling technologies to understand economic and environmental impacts. And third, an *environmental friendly society* with relatively higher individual preferences for a healthy environment compared to consumption to point out societal impact. A sensitivity analysis captures uncertainty. The last sections conclude and we discuss model limitations and future directions. Derivations, proofs, and the numerical Matlab©-algorithm are found in the Appendixes.

2. Conceptual model

We develop an analytical general equilibrium model for a two sector economy. The model introduced in this paper is based on a reduced form of the phosphorus (P) cycle (Egle et al., 2014) and adds an economic dimension to explain the demand for phosphorus by various stakeholders. The aim of the model is to capture the mechanisms and interlinks between major stakeholders and decision makers (as represented by households and various economic sectors). An overview of the model including the inflows and outflows between the different sectors of the economy and households is given in Fig. 1. Between crop production and animal husbandry are monetary and non-monetary inter-industry linkages. Households supply labor and demand food. We add P flows as a second layer to the economic model and introduce a waste water treatment plant to recover P. Crop farmers can choose a combination of recycled P fertilizer and imported mineral fertilizer to produce high-quality grain and fodder. Farmers in animal husbandry decide on the amount of fodder. The model agents, i.e. households, farmers and the WWTP, and their interactions are described in details in the following subsections.

2.1. Economic model

We model a closed economy representing one million inhabitants L living in urban and rural areas. Only a small proportion of the population works in the agricultural sector, everyone else works in other sectors (L_x). Farmers can choose to work in crop production (L_g) or animal husbandry (L_a).

2.1.1. Households

We assume that all agents are employed ($L(t) = L_g(t) + L_a(t) + L_x(t)$) and that the labor supply for agriculture is constant. Within the agricultural sector we allow workers to decide to either work for crop production or animal husbandry.

$$L_g(t) + L_a(t) = L - L_x = const. \tag{1}$$

Households demand vegetarian products $c_g^c(t)$ (represented by grain) and meat products $c_a^c(t)$ and derive utility from environmental quality $E(t+1)$ in the next period. The environmental quality reflects the water quality with respect to eutrophication as well as the accumulation of heavy metals and organic micropollutants in soils. Household's consumption choice impacts environmental quality indirectly. Consequently feedbacks between households consumption decisions and environmental quality occur.

Households maximize their utility function $u(t)$ in every period. Other goods $x(t)$ than grain $c_g^c(t)$ and animal products $c_a^c(t)$ are assumed constant and therefore not subject to optimal choice. We choose the form of a log-utility function that yields the optimal consumption decision as a constant budget share. γ_c, α_c and ϵ are positive constants reflecting the importance of the respective consumption goods.

$$\max_{\{c_g^c(t), c_a^c(t)\}} u(c_g^c(t), c_a^c(t), E(t+1)) = \max_{\{c_g^c(t), c_a^c(t)\}} \gamma_c \log(c_g^c(t)) + \alpha_c \log(c_a^c(t)) + \epsilon \log(E(t+1)) \tag{2}$$

Household's consumption cannot exceed the income earned, described in Eq. (3), where $p_g^c(t)$, $p_a^c(t)$, $p_x(t)$ are the prices for grain products, the animal products, and composite other goods, respectively. Households supply labor to the firms in exchange for a wage $w(t)$, and receive profits from grain production $\pi_g(t)$, animal husbandry $\pi_a(t)$, fertilizer trader $\pi_G(t)$, waste water treatment plant $\pi_w(t)$ and other sectors $\pi_x(t)$. Moreover the value of fertilizer imports ($p_m(t)P_m(t)$) is assigned to the households. So the household's budget constraint (Eq. (3)) is aligned with the national budget constraint.

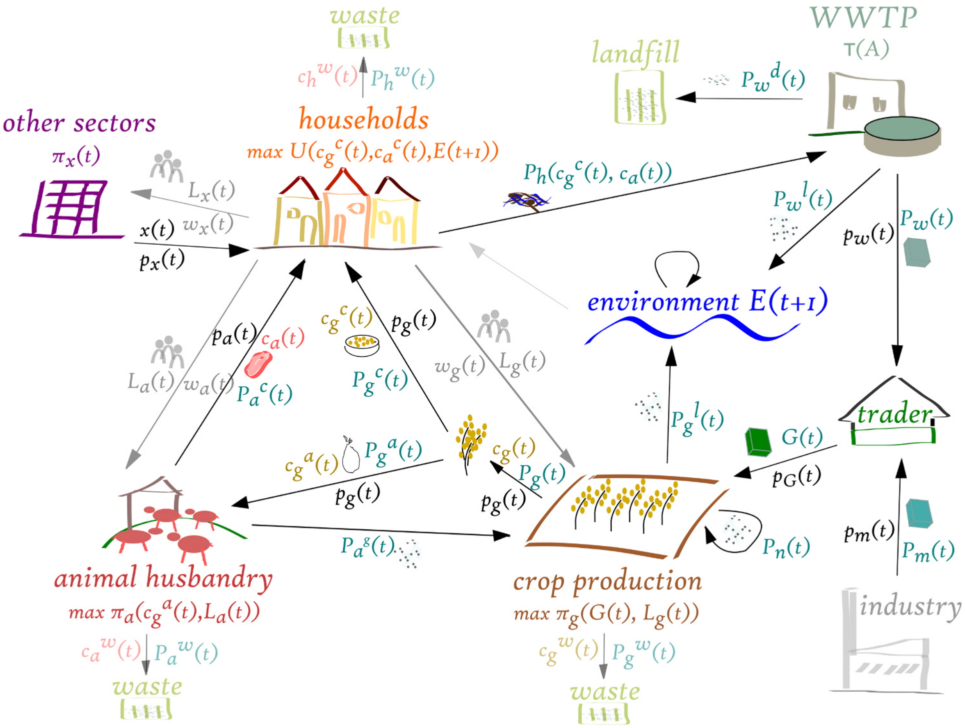

Fig. 1. Outline of the model framework. We denote with the letter g the grain field, a the animal husbandry, h the households, w the waste water treatment plant (WWTP), m the mineral fertilizer from industry and x the other sectors. The subscripts of the consumption c, prices p and phosphorus P variables describe the source of the flow and the superscript describes the destination.

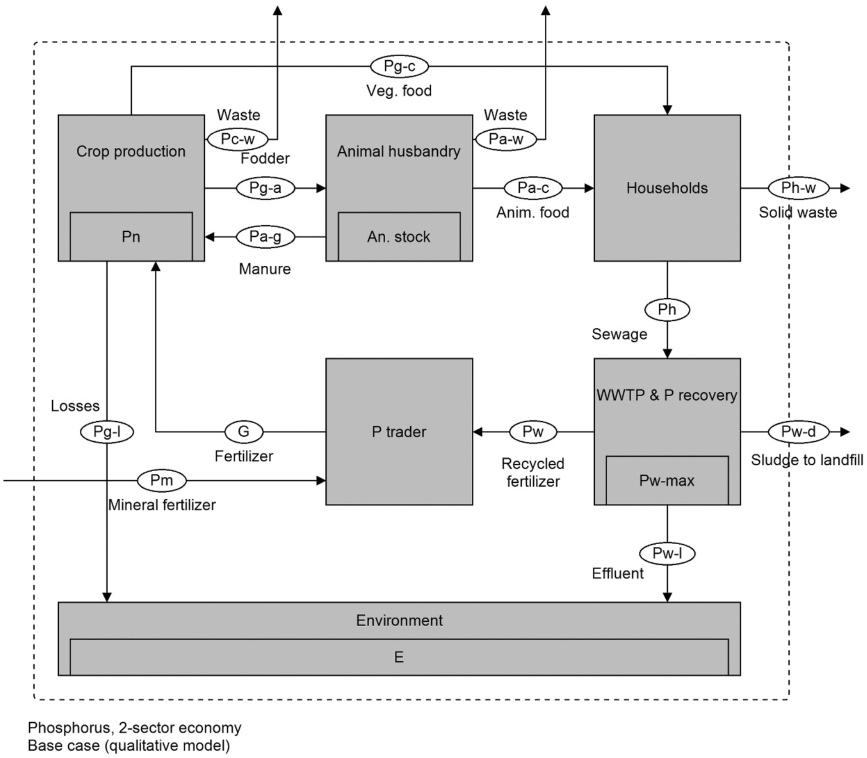

Fig. 2. Schematic illustration of the phosphorus flow model of the 2-sector economy (Fig. 1) generated with the MFA software STAN (http://www.stan2web.net/). Processes (= balance volumes) with or without stocks are represented as boxes, and flows are shown as arrows including variable names.

$$p_g^c(t)c_g^c(t) + p_a^c(t)c_a^c(t) + p_x x \le w(t)L(t) + \pi_g(t) + \pi_a(t) + \pi_G(t) + \pi_w(t) + \pi_x + p_m(t)P_m(t) \qquad (3)$$

2.1.2. Crop production

We assume that phosphorus (P) fertilizer denoted by G and labor inputs L_g are chosen to maximize profits of the crop production c_g. All other production inputs like agricultural land, seeds, alternative fertilizer and irrigation are assumed constant and summarized within the crop production technology ϕ_g. The production elasticities α_g and β_g describe the importance of the production inputs P fertilizer and labor, respectively. In addition to labor $L_g(t)$ and P fertilizer $G(t)$, the production of grain also depends on P in manure $P_a(t-1)$ and P in the soil ($P_n(t-1)$). The production function of grain $c_g(t)$ is therefore given as follows:

$$c_g(t) = \phi_g((\chi_a - \ell(P_a(t-1) + (\chi_n - \ell)P_n(t-1) + G(t))^{\alpha_g}(\phi_L L_g(t))^{\beta_g}, \qquad (4)$$

with χ_a and χ_n denoting the respective efficiency of P in manure and in the soil, ℓ is the P loss via runoff from the field and ϕ_L denotes the labor efficiency. After every period farmers adapt the fixed proportion ϕ_g^c of their yield that is supplied to the households as high-quality products $c_g^c = \phi_g^c c_g(t)$ and the share $\phi_g^a = 1 - \phi_g^c$ that is supplied as fodder $c_g^a = \phi_g^a c_g(t)$ to the animal husbandry sector. The different quality of the products implies different prices on the market. Grain farmers maximize the profit function

$$\max_{\{G(t),L_g(t)\}} \pi_g(G(t), L_g(t)) = \max_{\{G(t),L_g(t)\}} p_g(t)c_g(G(t), L_g(t)) - p_G(t)G(t) - w(t)L_g(t). \qquad (5)$$

with the composite price $p_g(t) = \phi_g^c p_g^c(t) + \phi_g^a p_g^a(t)$.

2.1.3. Animal husbandry

To generate meat products, we assume fodder and labor as the variable production inputs. Other production factors like land are assumed constant with the total factor productivity ϕ_a. The production elasticities α_a and β_a reflect the impact of the production inputs fodder and labor on animal husbandry, respectively. Farmers in animal husbandry demand labor $L_a(t)$ and fodder $c_g^a(t)$ to produce animal products $c_a^c(t)$ with the following production technology:

$$c_a^c(t) = \phi_a(\psi c_g^a(t))^{\alpha_a}(\phi_L L_a(t))^{\beta_a} \qquad (6)$$

The relation of fodder to final meat products is given by the inverse feed conversion ratio (FCR) ψ and labor efficiency ϕ_L scales the work force. Farmers maximize the profit function

$$\max_{\{c_g^a(t),L_a(t)\}} \pi_a(c_g^a(t), L_a(t)) = \max_{\{c_g^a(t),L_a(t)\}} p_a(t)c_a^c(c_g^a(t), L_a(t)) - p_g(t)c_g^a(t) - w(t)L_a(t). \qquad (7)$$

2.1.4. Market equilibrium

Every period farms in crop production and animal husbandry decide on the production inputs to maximize their profit $\pi_g(t)$ and $\pi_a(t)$, respectively, and households maximize their utility function $u(t)$ by choosing the consumption goods. The optimal decisions of the agents result in supply and demand functions for every market. The detailed derivation of the market equilibria for vegetarian products $c_g^c(t)$, grain fodder $c_g^a(t)$, animal products $c_a^c(t)$, labor $L(t)$ and P fertilizer $G(t)$, is given in Appendix A. The corresponding prices that result in equilibrium are $p_G(t)$, $p_g^c(t)$, $p_g^a(t)$, $p_a(t)$, and $w(t)$.

2.2. P dynamics

In addition to the economic model framework we construct a mass balance model to capture the P cycle based on Zoboli et al. (2016a). Again, we use Fig. 1 to outline the model dynamics, but focus on the corresponding P flows replicated in Fig. 2.

P can be recovered from the household's sewage sludge $P_h(t-1)$ and is sold as recycled fertilizer product $P_w(t)$ or goes to landfill $P_w^d(t)$. Additional to the P in the soil $P_n(t)$ grain farmers apply manure $P_a^g(t)$,

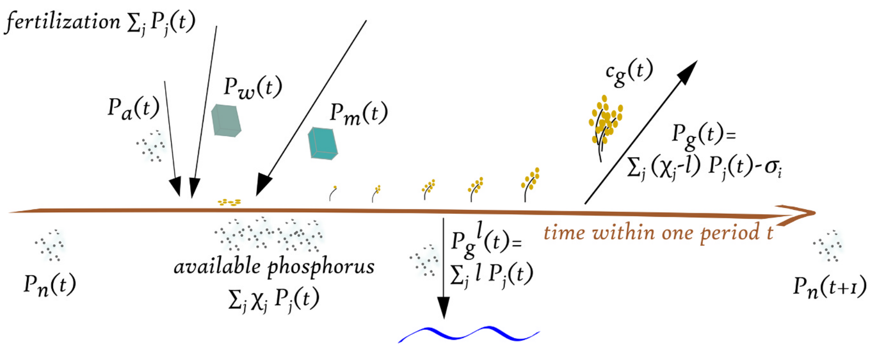

Fig. 3. Phosphorus dynamics within one period.

mineral fertilizer $P_m(t)$ and recycled fertilizer $P_w(t)$ to the field, where mineral and recycled fertilizer are supplied as a composite fertilizer product G by a trader. While plants are growing (Fig. 3) a part ℓ goes as runoff $P_g^\ell(t) = \ell[P_w(t) + P_m(t) + P_a^g(t) + P_n(t)]$ to the receiving water bodies.

Plant harvest contains $P_g(t)$ tons of P. The rest remains in the soil as $P_n(t+1)$ for the next period $t+1$.

$$P_n(t+1) = P_n(t) - P_g^\ell(t) - P_g(t) \tag{8}$$

Plant harvest is processed into a share $\frac{c_g^c(t)}{c_g(t)}$ of qualitative vegetarian food for households containing $P_g^c(t)$ tons of P and a share $\frac{c_g^a(t)}{c_g(t)}$ of fodder for animal husbandry containing $P_g^a(t)$ tons of P, whereas a fraction ω_g^ℓ ends up as food waste ($\omega_g^\ell P_g = P_g^w$).

$$P_g^c(t) + P_g^a(t) = (1 - \omega_g^\ell)P_g(t) \tag{9}$$

A share ζ_a^g of $P_g^a(t)$ in the fodder returns as manure $P_a^g(t+1)$ to the field and the remaining share $\zeta_a^c = 1 - \zeta_a^g$ is processed into animal products $c_a^c(t)$ containing $P_a^c(t)$ tons of P. Again, during processing a fraction ω_a^ℓ ends up as food waste ($\omega_a^\ell P_g^a = P_a^w$).

$$P_a^g(t) + P_a^c(t) = (1 - \omega_a^\ell)P_g^a(t) \tag{10}$$

The P in sewage sludge is a fraction of P in the household's consumption goods $P_h(t) = \zeta_h^g P_g^c(t) + \zeta_h^a P_a^c(t)$. The rest goes into solid wastes, which are not considered further.

2.2.1. P recovery technology from wastewater

The P recovery technology A recovers P from sewage sludge of the households $P_h(t)$ to offer the recycled product $P_w(t+1)$ in the next period to the trader. We choose four different P recovery technologies to represent the various possible recovery routes mentioned in Egle (2014): direct application of sewage sludge without treatment (A1), phosphorus recovery from sludge liquor, i.e. Ostara Pearl Reactor® (A2), from sewage sludge, i.e. Stuttgart process (A3) and from sewage sludge ash, i.e. EcoPhos® (A4).

These technologies A are characterized by the following four criteria: Firstly, the fertilizer efficiency $\chi_w(A) \in [0, 1]$ determines the plant availability of the fertilizer product, i.e the percentage of applied P that the plant can actually absorb considering timing of P application and chemical structure. Secondly, the accumulation of heavy metals and micropollutants negatively influences the environment with a price $p_w^f(A)$ for one ton of applied fertilizer product. Thirdly, the recovery potential of phosphorus treated with technology A is $\tau(A) \in [0, 1]$. Fourthly, the price $p_w(A)$ to sell one ton of recycled fertilizer product is an estimate based on the annualized investment costs $I(A)$ to build technology A, the operating costs $V(A)$ using the technology A and the logistic costs to bring the recycled product to the fields. A technology operates roughly 15 years without large additional investments. Waste water treatment plants (WWTP) with design capacities = 100,000 PE treat more than 55% of municipal wastewater in Austria Egle et al. (2014). Therefore investment costs are calculated for ten representative 100.000-population-equivalent (PE) plants to serve one Mio. inhabitants. We summarize the technologies and their characteristics in Table E.5.

Before P recovery, a proportion ω_ℓ from the waste water is going as effluent $P_w^\ell(t)$ to the water bodies and after applying technology A a certain part $P_w^d(t+1)$ has to go to landfill (Fig. 1 and 2). The rest can be the maximum supply of $P_w(t+1)$ on the P market. Everything that cannot be sold as $P_w(t+1)$ and is not discharged into water bodies as $P_w^\ell(t)$ is landfilled. We summarize the relations in the following equations.

$$P_w^\ell(t) = \omega_\ell P_h(t) \tag{11}$$

$$\begin{aligned} P_w^{maxsupply}(t) &= \tau(A)(P_h(t-1) - P_w^\ell(t-1)) \\ &= \tau(A)(1 - \omega_\ell(A))P_h(t-1) \end{aligned} \tag{12}$$

$$P_w^d(t) = P_h(t-1) - P_w(t) - P_w^\ell(t-1) \tag{13}$$

The profit of the WWTP for selling $P_w(t)$ to the trader is

$$\pi_w(t) = p_w(A)P_w(t). \tag{14}$$

2.2.2. P in plants

Different P products are applied to the crop fields. The ability of a plant to absorb P depends on the type of fertilizer (composition, chemical species, etc.), the time of application, and the soil conditions at the site. We define these properties as efficiency χ_i, $i \in \{a, n, m, w\}$ of the product type i. With respect to soil type, we assume a generic average representing major soil type in Austria (acid or alkaline). P in the soil ($P_n(t)$) is always available but generally difficult to absorb for the plant (χ_n). Manure $P_a(t)$ from animal husbandry is not always applied when the plants are ready to take it and so its efficiency χ_a is affected. Contrary, the timing of application of mineral fertilizer $P_m(t)$ can be well chosen by the farmers and furthermore, the chemical structure allows for high take up rates of the plant (χ_m). The efficiency χ_w of the recycled P product $P_w(t)$ depends on the recycling technology A. The ideal plant demand for P is $\bar{P}$, whereas the actual for the plant available P might differ (Eq. (15)).

$$\begin{aligned} P(t) = {} & (\chi_w - \ell)P_w(t) + (\chi_m - \ell)P_m(t) + (\chi_a - \ell)P_a(t-1) \\ & + (\chi_n - \ell)P_n(t-1). \end{aligned} \tag{15}$$

2.2.3. Trader

The trader supplies the crop farmers with fertilizer $G(t)$ and produces fertilizer by combining recycled phosphorus $P_w(t)$ and imported mineral fertilizer $P_m(t)$. He aims to meet the fertilizer quantity $G(t)$ (Eq. (16)) demanded by the grain farmers considering fertilizer products' efficiency χ and runoff ℓ after application of the fertilizer $G(t)$.

$$G(t) = (\chi_w - \ell)P_w(t) + (\chi_m - \ell)P_m(t) \tag{16}$$

The trader also aims to meet the plant's maximum fertilizer demand $\bar{G}$

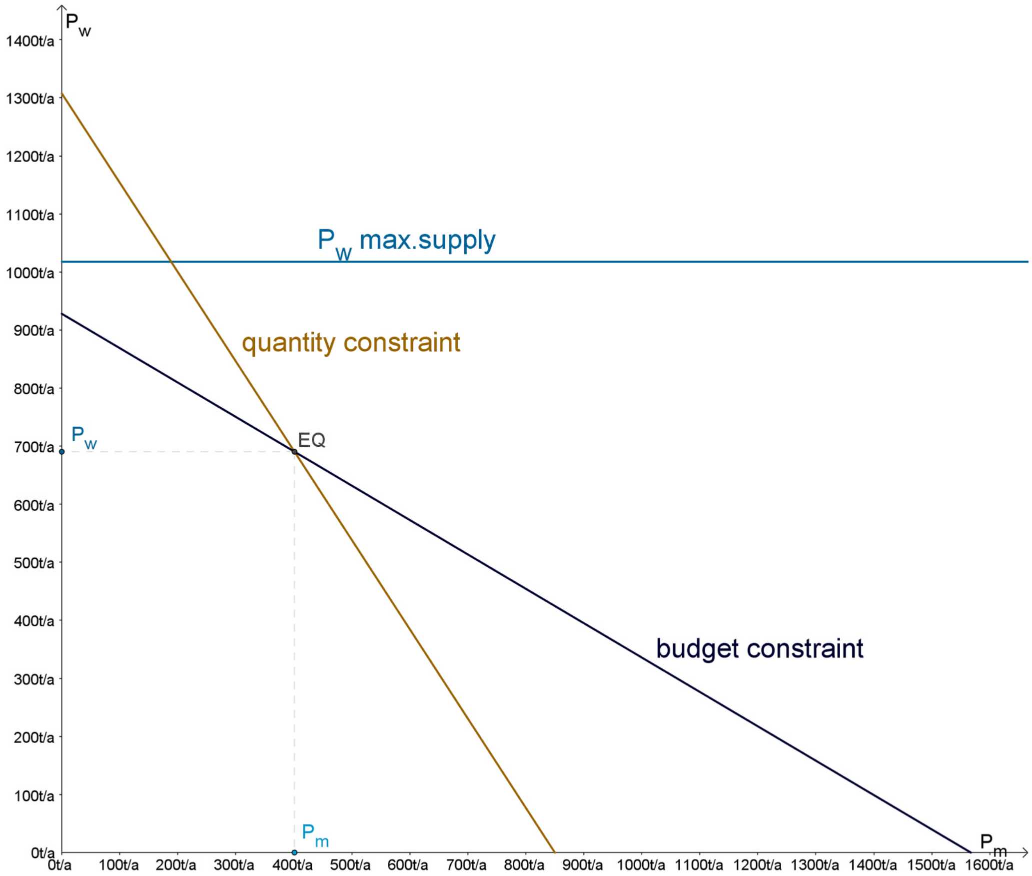

Fig. 4. The constraints for the traders P_m and P_w supply.

according to $\bar{P}$, hence, avoiding overfertilization by fulfilling the quantity constraint $\bar{G} = (\chi_w - \ell)P_w(t) + (\chi_m - \ell)P_m(t)$ in Fig. 4. However, the trader is only a functional intermediary and cannot earn profits. Therefore he is obliged to fulfill his budget constraint (Eq. (17)) in Fig. 4.

$$p_G(t)G(t) = p_m(t)P_m(t) + p_w(A)P_w(t) \tag{17}$$

The trader cannot sell more recycled P products $P_w(t)$ than the WWTP is supplying ($P_w(t)^{maxsupply}$ in Fig. 4).

To sum up, the trader aims for an appropriate fertilizer quantity and is restricted to the zero-profit-condition. This yields the amount of $P_m(t)$ and $P_w(t)$ used for agricultural production. If the plants quantity demand $\bar{G}(t)$ cannot be met with the farmers willingness to pay for fertilizer $p_G(t)$ the trader sells $G(t) = \bar{G}(t) + \sigma(t)$ and the budget constraint reads $p_G(t)[\bar{G}(t) + \sigma] = p_m(t)P_m(t) + p_w(A)P_w(t)$. All cases for under- and overfertilization ($\sigma(t) \neq 0$) are explained in Appendix B.

2.2.4. Environmental quality

Environmental quality in the model is a stylized variable that takes into account the potential damages caused by P in water bodies and the accumulation of contaminants on soils deriving by the application of fertilizers. We conceived it as a positive index between zero (environmental disaster, i.e. algae bloom) and a maximum $\bar{E}$ (well balanced ecosystem). It decreases by P emissions from agricultural fields $P_g^{\ell}(t)$ and from waste water treatment effluents $P_w^{\ell}(t)$, and by accumulation of heavy metals and organic micropollutants via fertilizers $P_w(t)$ or $P_m(t)$ in the fields. Environment can also regenerate with a rate $\delta > 1$. This results in the following dynamics for environmental quality, following standard environmental economic models explained in e.g. Caravaggio and Sodini (2018) and empirical work of Zoboli et al. (2016b).

$$E(t+1) = \delta(E(t)) - P_g^{\ell}(t) - P_w^{\ell}(t) - p_w^f(A)P_w(t) - p_m^f P_m(t) \tag{18}$$

3. Results

The model can be solved explicitly for all variables, which allows extensive analysis of interrelationships. Thus, we first present main analytical results based on the market equilibria derived in Appendix A. Then, we continue with numerical results of the model based on a time frame of 15 years reflecting the operating time of a P recovering technology. The parameter values are calibrated according to Austrian data (Wien-Statistik, 2010; Zoboli et al., 2016a) and listed in Table E.3. An overview of all variables and parameters included in the model is given in Appendix E. For each numerical simulation we assume that the available recycling technology is fixed and solve for general equilibrium prices and quantities in each period.

Furthermore, the numerical results of three different scenarios are discussed. (1) The mineral fertilizer price p_m increases in period $t = 7$ from 2.040 EUR/t to 3.500 EUR/t, similar to the price increase in 2008. (2) The economic and environmental impact of different recycling technologies characterized in Table E.5 is compared. (3) The household preference for a healthy environment ϵ in Eq. (2) is increased from 0.1 to 3 and 5 to reflect an *environmental friendly society*. Finally, a sensitivity analysis for crucial model parameters points out their impact on the results.

3.1. Analytical results

Since a central aim of the paper is to understand the phosphorus cycle and its relation to the specific fertilizers applied, we first consider

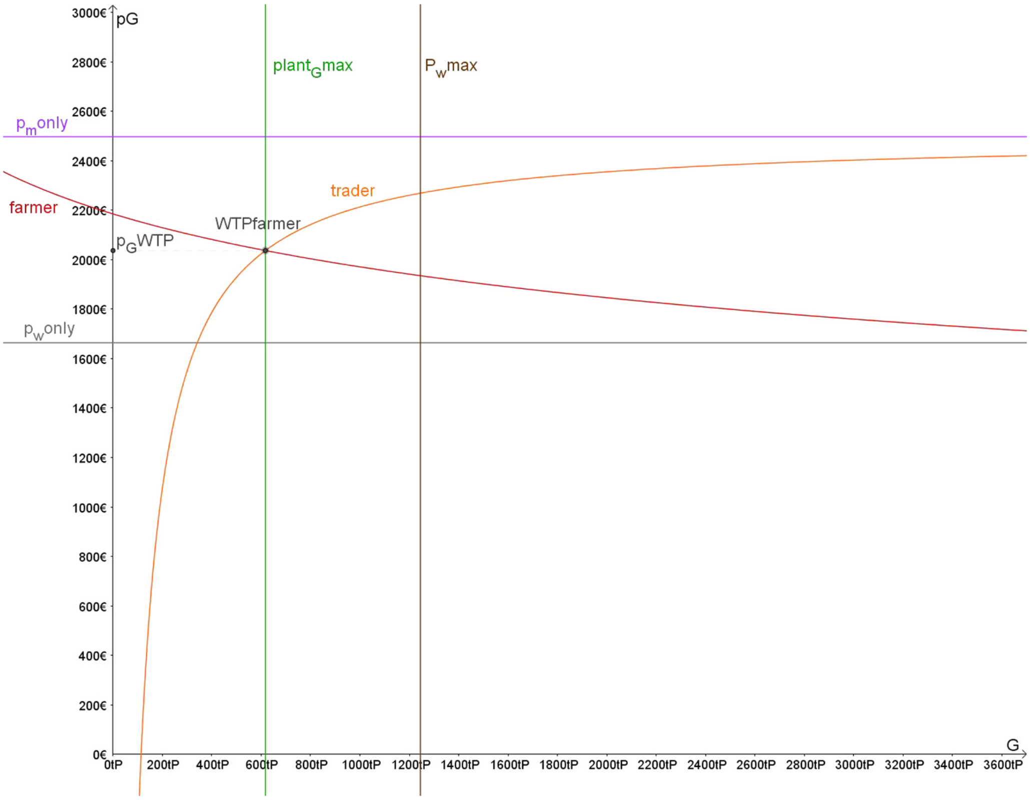

Fig. 5. Shows the fertilizer market for the composite fertilizer G (x-axis) if the effective price level (y-axis) of the mineral fertilizer $\frac{p_m}{\chi_m}$ is above the effective price level of the recycled P product $\frac{p_w}{\chi_w}$. The ascending traders supply curve intersects the decreasing farmers' demand function. The intersection *WTPfarmer* yields the market equilibrium quantity G^{WTP} equivalent to the plants fertilizer demand $plant_G max$ below the maximum G-supply $P_w max$, and the corresponding market equilibrium price saturates $p_G^{WTP} \in [\frac{p_w}{\chi_w}, \frac{p_m}{\chi_m}]$.

analytical results on the market for fertilizers. A further important consideration of our model is to understand the environmental dimension of phosphorus use, considering households demand environmental quality. We therefore present main analytical feedback mechanisms between household demand of grain and animal products and the environment.

3.1.1. Market for fertilizer

The market mechanisms determine the specific quantity of the composite fertilizer G used by the farmers. Farmer's profit maximization (Eq. (5)) yields their fertilizer demand curve (Eq. (A.13)) displayed in Figs. 6–8. Their willingness to pay the price p_G^{WTP} for fertilizer G results from meeting the optimal plant supply $G = \bar{G}$ (curve $plant_G max$ in Figs. 6–8) at market equilibrium point *WTPfarmer*. Note, that the fertilizer traders, and consequently the grain farmers, face price and quantity constraints as illustrated in Fig. 5–8, and explained in Fig. 5 and the following Corollar.

Corollar 1. *The price* $p_G(t)$ *of the composite fertilizer good* $G(t)$ *is constrained by the effective price levels* $\frac{p_m(t)}{\chi_m}$ *(in case only mineral fertilizer is used) and* $\frac{p_w(t)}{\chi_w}$ *(in case only recycled P is applied). Any combination of mineral and recycled fertilizers results in a price combination of these two prices and cannot exceed its interval.*

Proof. See Appendix C.1.☐

In consequence of the Corollar, farmers will over- or underfertilize if their demand for the composite fertilizer good $G(t)$ is above or below both effective price levels, respectively. If for example the demand of farmers is above the more expensive fertilizer price (Fig. 7–8), farmers spend all their available budget $p_G^{WTP}\bar{G}$ (curve *pGcosts*) on recycled P, i.e. $G(t) = (\chi_w - \ell)P_w(t)$ for its price $p_G(t) = \frac{p_w}{\chi_w} = p_w only$. If the trader would sell a mixed fertilizer product $G(t) = (\chi_w - \ell)P_w(t) + (\chi_m - \ell)P_m(t)$, its price would be lower ($p_G(t) < p_w only$) and allow to buy more fertilizer $G(t)$. Consequently, the trader's supply curve would be above the budget constraint curve *pGcosts* (Fig. 7–8).

In case the WWTP supply $P_w max$ cannot meet the demand for recovered phosphorus $(\chi_w - \ell)P_w$ (Fig. 8) the trader would add mineral fertilizer $P_m(t)$ to receive a mixed fertilizer product $Gmix(t) = P_w max(t) + (\chi_m - \ell)P_m(t)$. Hence, farmers spend all available budget *pGcosts* and pay the composite fertilizer price $p_G(t) = p_G sold$.

Depending on whether the price level of mineral fertilizer or recycled phosphorus is cheaper, the supply curve of the trader will be either upward sloping (Fig. 5) or downward sloping (Fig. 6) as we show in Proposition 1. The demand curve of farmers is always downward sloping in the price level of the composite fertilizer good $p_G(t)$.

Proposition 1. *The traders' supply function changes its qualitative behavior if the farmers are willing to pay more for the composite fertilizer than the world market price for P* (p_m), e.g. Fig. 6.

Proof. See Appendix C.2.☐

In other words, if farmers are willing to pay (WTP) more for the composite fertilizer ($p_G^{WTP} > \max(\frac{p_w}{\chi_w}, \frac{p_m}{\chi_m})$) traders will sell more of the expensive product, i.e. recycled fertilizer P_w in Fig. 7.

3.1.2. Environmental quality

Household consumption decisions (c_g^c, c_a^c) influence P flows in water bodies which can cause eutrophication. Since we assume that households care for environmental quality, the consumption decisions will be influenced by their impact on the environment. These interesting feedbacks within the coupled human-water system are described by the

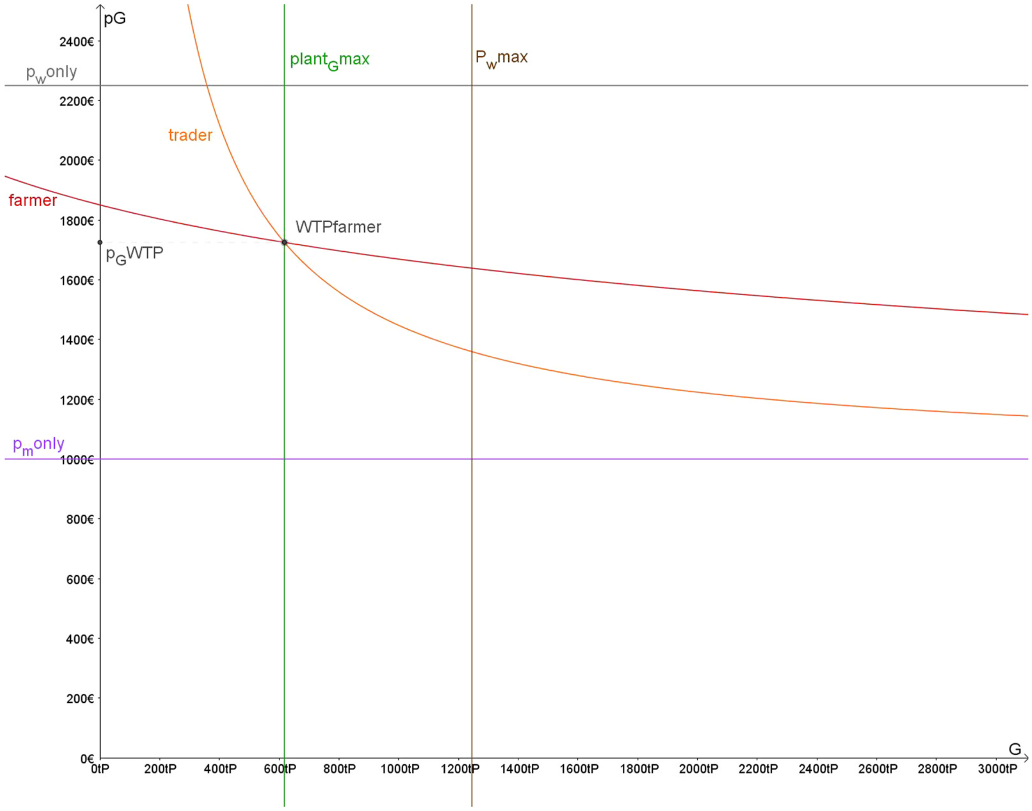

Fig. 6. Shows the fertilizer market for the composite fertilizer *G* if the effective price level of the mineral fertilizer $\frac{p_m}{\chi_m}$ is below the effective price level of the recycled P product $\frac{p_w}{\chi_w}$.

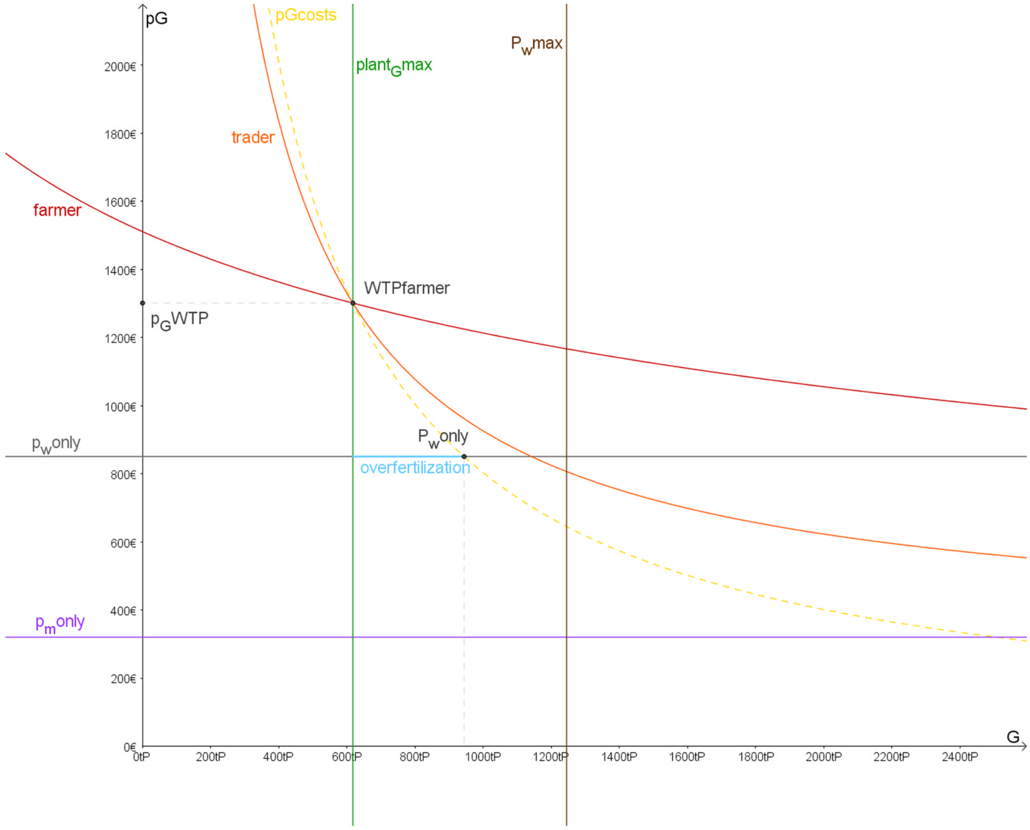

Fig. 7. The fertilizer market for the composite fertilizer *G* if only recycled P, P_w is used and farmers budget allows to overfertilize.

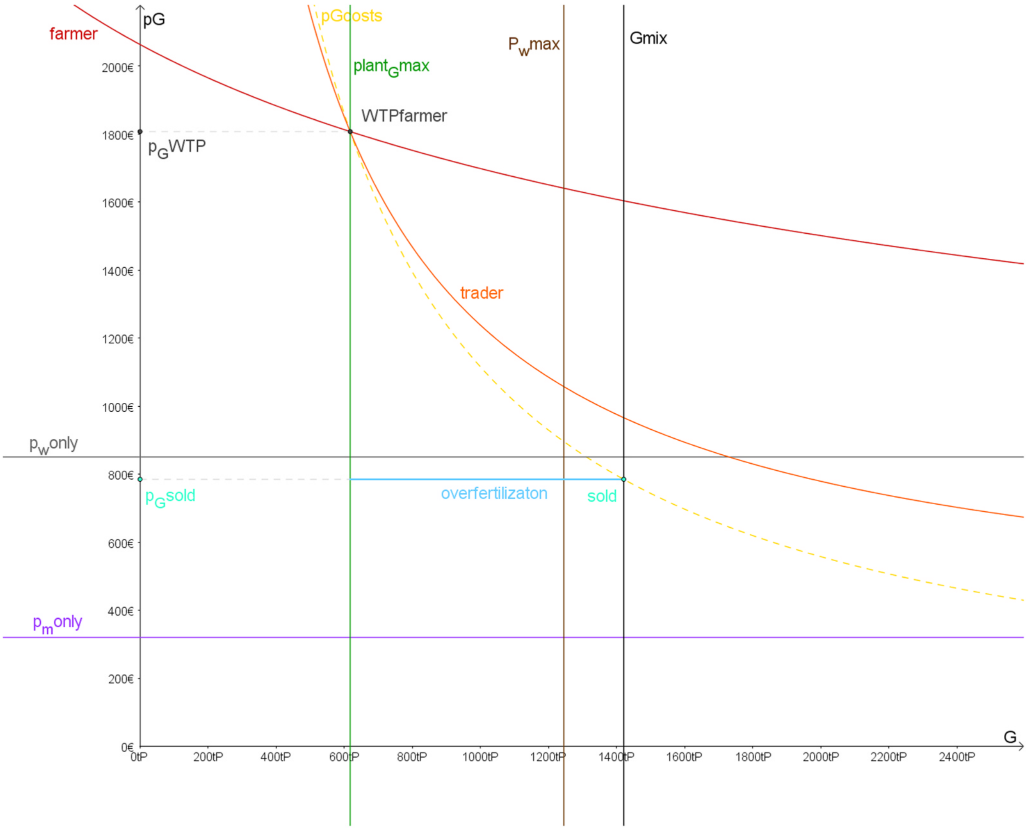

Fig. 8. The fertilizer market for the composite fertilizer G in case of overfertilization. Farmers take the full potential of recycled P and add some mineral fertilizer to buy the maximum amount of fertilizer $G(t) = Gmix$ for a resulting composite price $p_G(t) = p_G sold$ considering their budget constraint $p_G costs$.

dynamics of the environment (Eq. (18)) and by the optimal household demand for grain c_g^c (Eq. (19)) and the analogous demand for the meat consumption good c_a^c Considering Eq. (20) and (21) (see Appendix A) and the fact that $\left[\frac{p_g^c}{p_a^c}\frac{\partial E}{\partial c_a^c} - \frac{\partial E}{\partial c_g^c}\right]$ is negative yield the specific coupled feedbacks described below.

$$(c_g^c)^D = \gamma_c\left[\frac{p_g^c}{p_a^c}\frac{\alpha_c}{c_a^c} + \frac{\epsilon}{E(c_g^c, c_g^a)}\left[\frac{p_g^c}{p_a^c}\frac{\partial E}{\partial c_a^c} - \frac{\partial E}{\partial c_g^c}\right]\right]^{(-1)} \tag{19}$$

$$\frac{\partial E}{\partial c_g^c} = -\omega_\ell\frac{(1-\omega_g^\ell)P_g}{c_g(c_g^c, c_a^c)^2}[\zeta_{hg} - \zeta_{ha}\zeta_a^c]c_g^a(c_a^c) \tag{20}$$

$$\frac{\partial E}{\partial c_a^c} = -\omega_\ell\frac{(1-\omega_g^\ell)P_g}{c_g(c_g^c, c_a^c)^2}\left[\zeta_{ha}\zeta_a^c c_g(c_g^c, c_a^c) - \zeta_{hg}c_g^c + \zeta_{ha}\zeta_a^c\frac{c_g^a(c_a^c)}{\alpha_a c_a^c}\right] \tag{21}$$

Generally, more consumption decreases environmental quality. A decrease in environmental quality would again increase consumption, since individuals derive utility from consumption and environmental quality and these are substitutes. A society acting like that would in the long term deteriorate environment.

Otherwise, if e.g. environmental quality is really bad, the impact of environment is the dominant decision driver, and an increase in environmental quality leads to less consumption and this in turn increases environmental quality further. The utility would increase over time until the environment has reached a maximum. Then the negative loop would start again. The only exception is if the society prefers animal products, i.e. α_c is significantly larger than γ_c. In that case grain consumption would actually increase if environmental quality increases.

3.2. Numerical results

In Austria P in sewage sludge is partly recycled via direct application of sewage sludge on the agricultural fields additional to spreading animal manure. The additional P demand is met by importing mineral fertilizer. In the base case for our analysis we therefore allow the choice of a combination of mineral fertilizer and direct application of sewage sludge. The parameters are summarized in Tables E.3 and E.4 and the initial values in Table E.2. Fig. 9 summarizes the P flows in our model framework and displays the flow values for the first and nineth model period. They represent the most important flows of P in an economy. Such a P flow diagram helps to identify strategies to improve environmental quality with respect to P, to choose a certain P recovery technology and to monitor dynamics of P stock in soil and water bodies. Stocks of P in crop production are P accumulation in soil, whereas stocks of P in animal husbandry and in the P recovery process represent only the transfer of P to the next period. The largest and therefore most important P flows are the fertilizer applications via manure or bought fertilizer on the crop fields. Households consume most P via vegetarian food. Fig. 10 presents a base case simulation of the evolution of the most important P flows and stocks of Fig. 9: P in soil together with the evolution of the mineral fertilizer and recycled phosphorus. In addition we also record the maximum P supply over time and the quantity of overfertilization, which is the change of stock in "Crop production" for each period.

In the presented base case scenario, mineral fertilizer is cheap and direct application of sewage sludge is even cheaper. Farmer's willingness to pay for fertilizer is therefore above the actual market price (cf. Fig. 7 and 8) and consequently they overfertilize (Fig. 10 years 1–7). So the P stock in soil increases. This was the case in Austria in the 1980s and 1990s.

A larger P stock implies that farmers do not have to acquire as much additional fertilizer and can afford to spend more on employing labor and hence wages increase since the demand for labor rises. Higher wages in turn increase the income of households, who can then afford more expensive food and buy more meat products. Agricultural farmers

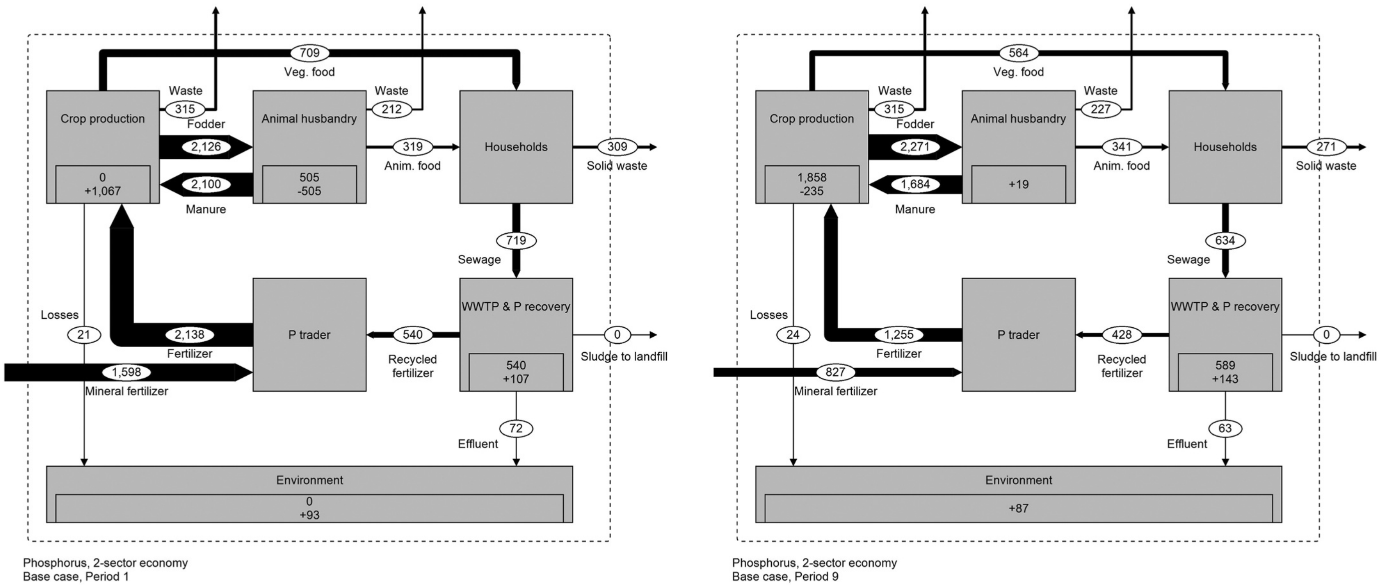

Fig. 9. Sankey-style diagram (i.e. arrow widths are directly proportional to flow quantities) of P flows in period 1 (left) and period 9 (right) of the base case model.

adapt and plant more fodder crops. So the supply of vegetarian products decreases and its price increases stronger than the price of fodder products. The fact that inflation for food was higher than for other consumption goods in Austria in the past years fits to that picture.

The consumption behavior and employment structure resulting from the model replicates the current situation in Austria. Households eat slightly more vegetarian products than animal products. And more people are employed in the grain sector (62%) than in animal husbandry.

3.3. Price level increase of imported mineral fertilizer

In 2008 the price of imported mineral fertilizer increased from 2040EUR/tP to 3800EUR/tP. Such a price rise is introduced for the years 7–15 (Fig. 10) to allow us to study how such a price increase in our model will affect the fertilizer market and consequently all the other markets. We can identify decreasing wages and increasing prices of alternative fertilizer and consumption goods, decreasing mineral fertilizer imports and a decreasing P stock.

As soon as the price of the mineral fertilizer rises, the grain farmer's willingness to pay increases and equals the even higher market price of P fertilizer. So farmers cannot afford overfertilization any longer (Fig. 10 years 8–15) and apply less fertilizer. In the base case farmers can choose between mineral fertilizer and direct application of sewage sludge. The quality of sewage sludge, i.e. the P availability for the plants, is too low to guarantee that the plant receives enough P to grow to its full extent. Farmers are not willing to buy 100% of the available recycled P (max P supply in Fig. 10) and have to also decrease the amount of direct application of sewage sludge to spend the money rather on the more efficient mineral fertilizer (Fig. 10 years 8–11).

Applying less fertilizer lowers the P stock in soil over time. Consequently, demand of P fertilizer increases to compensate P in soil. Farmers cannot afford the increased demand of P fertilizer and underfertilize. This quantity effect is intensified by a price effect: Lower grain prices decrease revenues and fertilizer costs increase. Sensitivity analysis has shown that even lowering the number of employees and their wages cannot compensate these price changes.

Most important, farmers do not build up P stock after a fertilizer price increase. The degradation of environmental quality is reduced.

The severe overall fertilizer price change affects all other prices. In the short term, the fertilizer price increase in the grain production sector increases production input costs. Consequently, fodder supply decreases (Fig. 11(a)). Since fertilizer requires more monetary resources for production, farmers can pay less for the complementary production input labor. Wages decrease because less labor is demanded in the grain sector. Consequently the second agricultural production sector, animal husbandry, can spend more on its other production input, fodder, and fodder demand increases in the short term (Fig. 11(b)).

In the long term workers move from the more labor intensive grain sector to the animal husbandry sector. Still, 58% of agricultural labor remain in the grain sector in the base case and, overall, wages decline. Hence, lower income leads to reduced food demand. This results in two

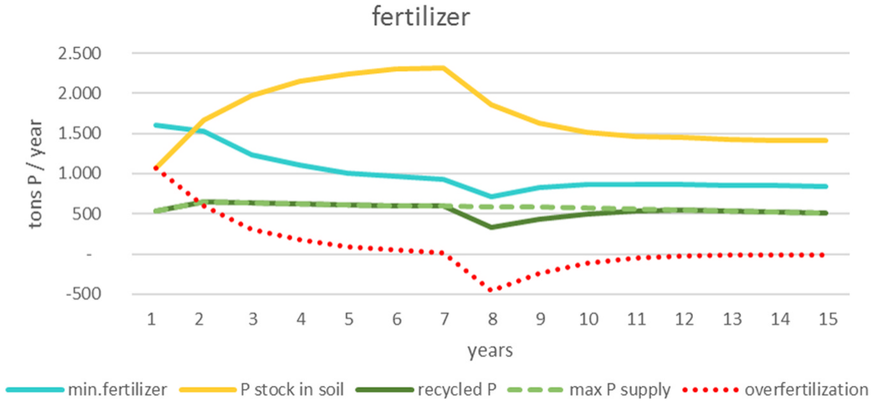

Fig. 10. Fertilizer use over time given the option of direct application of sewage sludge, i.e. recycled P fertilizer. The price increase of mineral fertilizer in year 7 decreases the use of P fertilizer in the short term and the level of mineral fertilizer in the long term, and consequently lowers the P stock in soil. Max P supply is the maximum available amount of recycled P that the WWTP offers to the farmers and overfertilization show if farmers apply more or less P than the plants can absorb.

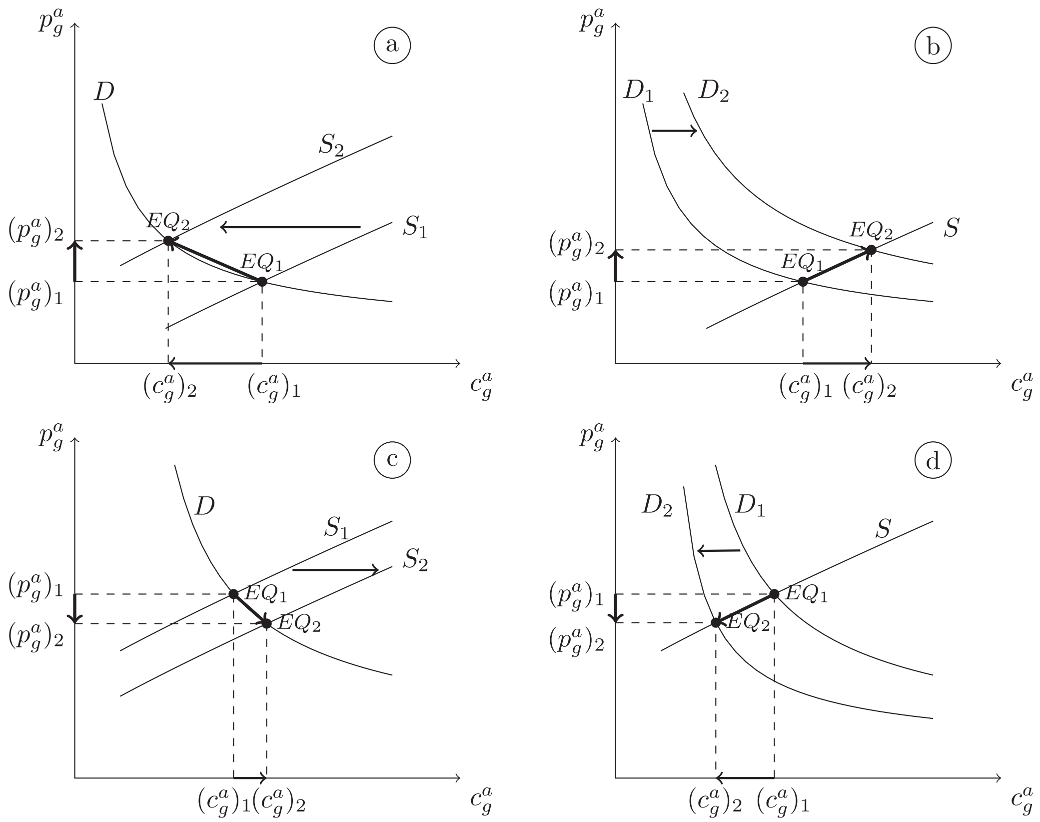

Fig. 11. Direct (a,b) and indirect (c,d) effects of mineral fertilizer price p_m changes for fodder c_g^a supply S and demand D.

opposite effects. Reduced household demand for high quality grain products combined with the short term fodder price increase forces agricultural farmers to shift to a crop variety that produces more lower quality fodder products. Consequently fodder supply increases (Fig. 11(c)) and supply for high quality grain products decrease. Therefore prices for vegetarian products increase. This shifts the household's demand to a more dairy or meat based diet. To serve the resulting increased demand of animal products, fodder demand increases (Fig. 11(b)). Contrary, overall reduced food demand also reduces animal husbandry's demand of the intermediate good fodder (Fig. 11(d)).

The resulting fodder price p_g^a dynamics of the first long term effect characterize the final price change, but they are twofold: The increased fodder demand leads to a fodder price increase (Fig. 11(b)), whereas the increased fodder supply yields a fodder price decrease (Fig. 11(c)). In the base case, where farmers use mineral fertilizer and direct application of sewage sludge, the fodder price increases 3% (Fig. 12 (A1)). Whenever a P recovering technology is used to treat sewage sludge, the second effect is dominant, because fertilizer efficiency is higher and therefore it is easier to increase supply. In case of Ostara Pearl Reactor® the fodder price decreases 5%, for applying the Stuttgart process fodder price decreases 8% and for the most expensive technology EcoPhos® the price decreases even 13% (Fig. 12 (A4)). The lower production input prices in the animal husbandry allow for lower prices for animal products.

The increased share of fodder products in the grain farmer's product portfolio leads to a lower average price per produced ton of grain, even though the price for household's grain consumption goods increased. In the end households pay higher prices with lower income.

If the price of recycled P is low, the effects of the price increase for mineral fertilizer are much stronger. If e.g. recovered P from EcoPhos® treatment can be sold for less than 2000EUR/tP, the reduction of

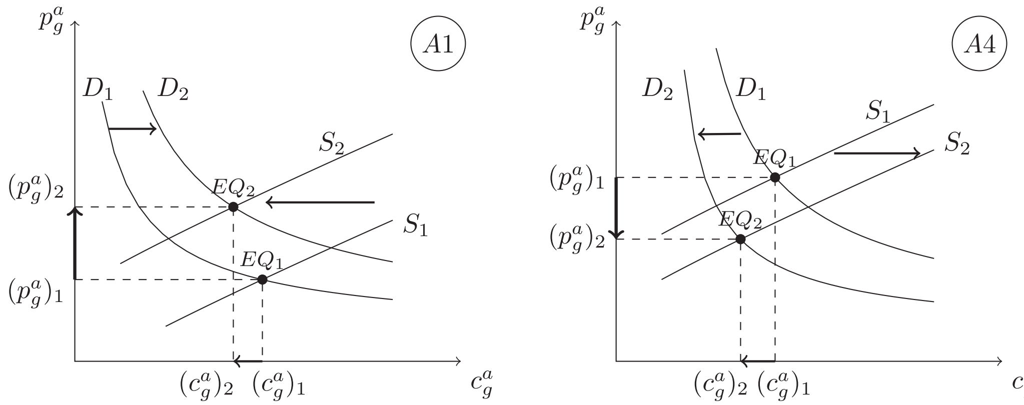

Fig. 12. Long term effects on the fodder c_g^a market after a mineral fertilizer price p_m change.The fodder price p_g^a increases given the possibility of direct application $A1$ or decreases by using P recycling with EcoPhos® technology $A4$.

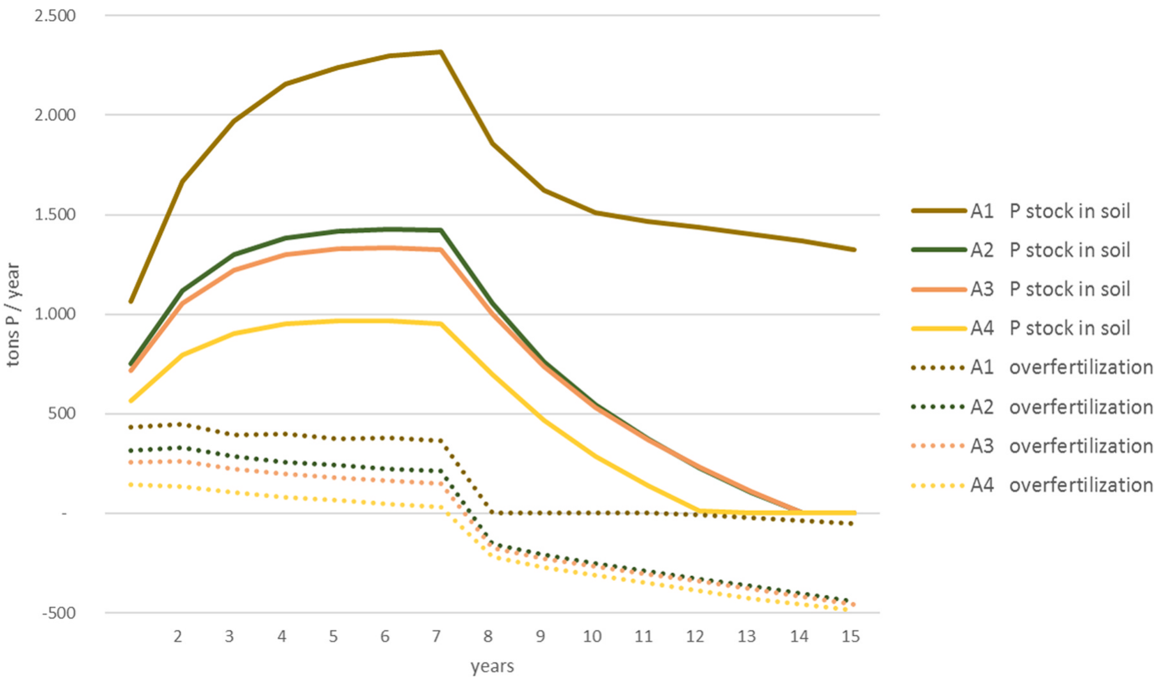

Fig. 13. P accumulation in soil and corresponding overfertilization (stock and stock changes in "crop production" in Fig. 2) over time, when grain farmers choose, additional to mineral fertilizer, also fertilizer from different recycling technologies: Direct application of sewage sludge (A1), Ostara Pearl Reactor® (A2), Stuttgart process (A3), EcoPhos® (A4). Note that negative overfertilization is underfertilization and forces the plant to take up P from the soil.

overfertilization due to increased prices for mineral fertilizer would more than double.

3.4. Different P recycling technologies

Next, we study the market changes for the four different recovery methods (A1-A4) introduced in Section 2.2.

Offering P recovery technologies (A2, A3, A4) keeps the P stock in the soil significantly lower compared to no recycling or allowing only direct application of sewage sludge (A1) additional to the application of mineral fertilizer (Fig. 13).

However, for grain farmers efficient grain production is more important than environment. Given any P recycling method (A1-A4) and stable fertilizer prices grain farmers overfertilize and the P stock in the soil increases every year (Fig. 13, year 1–7). This reflects the current situation in Austria.

Even after the fertilizer price increase in year 7, the combination of direct application of sewage sludge (A1) and mineral fertilizer is cheap enough to ensure plants will receive enough P fertilizer. As indicated in Fig. 13, for year 8–15, overfertilization is close to zero, because enough P fertilizer is applied, and plants absorb less P from the P stock in the soil. This changes when we introduce P recovery technologies (A2, A3, A4). After a price increase of mineral fertilizer the price of the fertilizer composition increases sufficiently and therefore marginal costs of fertilization become larger than the marginal revenue for applying as much fertilizer as necessary for the plant to grow to its full potential. As a result, farmers underfertilize slightly and force the plants to absorb P from the soil. Consequently, environment recovers faster.

After the mineral fertilizer price increase the prices farmers have to pay for different recycled P fertilizer products also change significantly. If recycled P is already expensive the actual price change is not as strong as for cheap recycled fertilizer, but still increases at least 50%.

However, Fig. 14 displays, that a price increase of imported mineral fertilizer has only short term effects on the use of imported mineral fertilizer if a P recovery technology (A2, A3, A4) is installed. A few years after the price increase firms on the market adjust all other quantities and prices to the higher price of the mineral fertilizer and apply as much or even more mineral fertilizer as before the price increase. Only in case of direct application of sewage sludge (A1) its high P recovery potential and its low price allow a reduction of imported mineral fertilizer.

Generally, the total level of applied fertilizer is decreasing more if, additional to a more expensive mineral fertilizer, the recovered P fertilizer product is more expensive. This leads to a stronger underfertilization (Fig. 14) and therefore a more significant reduction of P stock in soil.

Implementing any P recovery technology (A2, A3, A4) and offering the recycled product as P fertilizer would lead to reusing 100% of the recovered P (Fig. 14) independent of the price level of mineral fertilizer. Since P in treated recycled products is more easily available for the plants, less overfertilization happens. An important observation is that farmers would always optimally choose the maximum available amount of recovered P. Nevertheless, this is not yet enough to meet the plant's P demand and farmers have to add imported mineral fertilizer or exploit the P stock in the soil.

Treated recycling products cause a higher fertilizer price. This affects the grain production more than animal husbandry and we can observe a slight increase in meat consumption. This fertilizer price effect due to a more costly recovering technology implies the same mechanisms as in the base case after the price shock. Generally, implementing costly P recovery technologies does not significantly change consumption levels compared to the base case with the possibility of cheap direct application of sewage sludge, but prices change.

Households spend significantly more on food consumption if no P recovering technology is implemented. However, quality has its price. If we only compare the scenarios with implemented P recovering technologies, the most expensive recovering technology leads to the highest prices for vegetarian products. Nevertheless, prices are still lower than in the scenario (A1) with no P recovering technology.

Economies benefit from investing in P recycling. Better recovering technologies, i.e. a greater recovering potential and better plant availability of the recycled material, enable farmers to buy less imported mineral fertilizer. This is important for two reasons: First, the economy depends less on the P world market and its prices. Second,

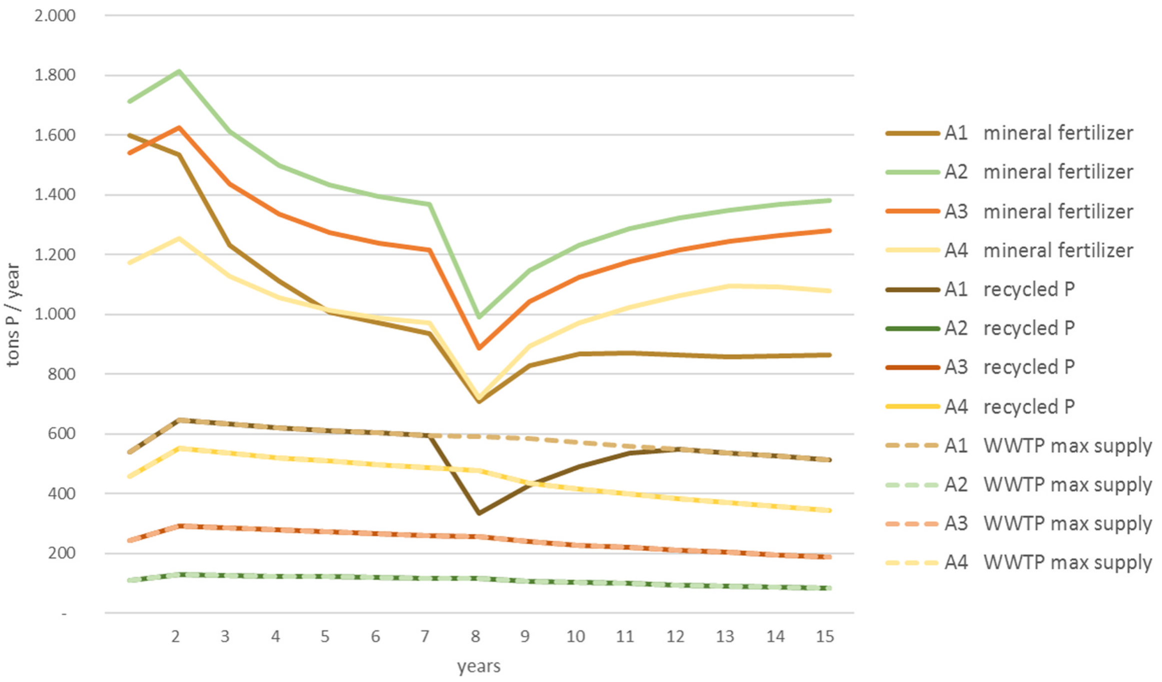

Fig. 14. Farmer's choice of optimal fertilizer composition given the four different P recovery technologies. Time series of mineral fertilizer and recycled P in Fig. 2 and the respective maximum amount of available P for the plant.

environmental quality is higher since P recovered by a better technology includes less heavy metals and organic micropollutants, and less P ends up in water bodies where P can cause eutrophication.

Any P recovery technology (A2, A3, A4) leads to better environmental quality and profits of grain farmers are generally higher than in the base case scenario. In any case, profits of grain farmers are higher than profits for animal husbandry. However, there are also some economic trade-offs. Wages and consumption levels decrease.

To sum up, the most important benefit of implementing a recovery technology is the increase of environmental quality. Even though direct application of sewage sludge has advantages for plant growth and farmers profits, it yields the worst environmental quality in comparison to recovering technologies A2-A4.

3.5. *Environmental friendly society*

A different mind set causes a different optimal choice of P recovering technologies. In the base case household utility decreases by implementing a better and therefore more costly P recovering technology. This is why there are still no P-recycling technologies implemented e.g. in Austria. If environmental quality plays a crucial role in the societies objective, it is optimal to invest in better technologies (i.e. EcoPhos®) and adjust consumption accordingly.

If people value environment, they also care about it. By slightly adapting the consumption behavior, less fertilizer is needed to meet the demand and consequently less P can run off into surrounding water bodies.

A positive mind set towards environment even leads to higher wages and higher profits in agriculture and animal husbandry. The reason is a higher willingness to pay for food.

3.6. *Sensitivity analysis*

The choice of the parameter values describing the different recovery technologies (Table E.5) is crucial for the model output. The main results of a sensitivity analysis for fertilizer efficiency $\chi_w(A)$ and recovery potential $\tau(A)$ are discussed below, followed by an outline of model limitations.

A better P availability $\chi_w(A)$ in the recycling product for the plant would obviously lead to a more efficient use of the recycled P product and consequently less application of the product. Still, the qualitative model outcome would only change for expensive P products. If we assume a plant availability of 80% for recovered P from the expensive EcoPhos process instead of 100%, 20% less recycled P from the WWTP would be consumed. Still, P stock in the soil would increase since plants absorb less.

For any other recovering technology reduction in $\chi_w(A)$ would only lead to an increasing use of mineral fertilizer, if imported mineral fertilizer is expensive. The resulting increase of fertilizer prices yields that the farmers are only willing to pay more for more efficient fertilizer, i.e. mineral fertilizer. If e.g. P recovered with the Stuttgart process performs with an efficiency of only 50% P plant availability instead of 85%, farmers would not use any of the recycled P after a price increase for mineral fertilizer. The same would apply to P recovered by the EcoPhos process if efficiency decreases from 100% to 80%.

The recovery potential $\tau(A)$ directly impacts the amount of available P after the recycling process. If more recycled P is available farmers buy adequately less mineral fertilizer. If the plant availability of recycled P is lower than of mineral fertilizer ($\chi_w < \chi_m$), the P stock in soil increases. This is the case for any recycling technology besides EcoPhos, because it would be too expensive to overfertilize.

4. Discussion and conclusion

In this paper we have developed a two sector general equilibrium model and studied how households' consumption decisions and farmers' profit maximization affect phosphorus (P) flows and resulting P stocks in soil and water bodies. Different P recovery technologies treat waste water and provide recycled P fertilizer products. Farmers can choose recycled P products and alternatively imported P mineral fertilizer. Environmental quality reflects contaminants in soil and P in water bodies, and is best if P is recycled from waste water and then used as fertilizer. This has even positive consequences on the economy. Profits in crop production increase, wages are higher and food prices remain stable.

A reduction of P stock in the soil can be achieved by implementing a recovering technology, whereas at the same time prices of imported mineral fertilizer have to be high. However, Koppelaar and Weikard (2013) points out that costs for recycling P from wastewater are above any foreseeable phosphate rock fertilizer price. Cheap imported mineral fertilizers provide no incentive to lower fertilizer application as argued in Withers et al. (2014). If policy makers help to lower prices for recovering P or even to increase mineral P prices by taxes or tolls, a technological change would occur with more effective P use and lower P losses, resulting in better environmental quality.

Economically, a price increase of imported mineral fertilizer would increase food prices and reduce household income for employees in agriculture.

To avoid such economic impacts of world market fertilizer prices the economy has to be independent. Thus, it is important to enhance recycling to close the loop of P resource use. Childers et al. (2011); Koppelaar and Weikard (2013); Zoboli et al. (2016b) also argue for a similar strategy to secure P access for farmers. Our model does not include additional strategies like reducing P loss from soils by erosion abatement (Zoboli et al., 2016b), or prospect and explore new sources of mineral P (Childers et al., 2011).

A more advanced recovery technology (i.e. recovering P from ashes) allows a stronger decrease of P in soil and water bodies. Whenever recycled P is offered, farmers use 100% of the available recovery products independent of the price of mineral fertilizer. To cover the total fertilizer demand there is a need, first, to increase the collection of sewage sludge and waste products containing P and, second, to increase the recovery potential of P recovery technologies.

These conclusions are based on the modeling choices: Besides direct application of sewage sludge, only recovery technologies that ensure the production of fertilizers with relatively high plant availability and low levels of contaminants are taken into account. This explains their superior performance in comparison to direct application of sewage sludge and also the positive feedbacks they trigger both in the economy and in the environment. If technological options offering products with a poorer quality (Egle et al., 2016) were included in the model, the outcomes would likely be very different.

Without any incentives to change the current consumption and fertilization behavior, overfertilization and high P stocks in soil remain a threat for the environment. Furthermore, food prices would further increase as this is the current case in Austria (Wien-Statistik, 2010).

A different mindset can crucially change economic decisions and consequently the environment. A green society - i.e. having higher preferences for environmental quality - would choose to implement the most advanced and expensive P recovery technology. This would not only improve environmental quality, but also increase households' utility, wages and profits. Withers et al. (2014) also relates greater public awareness of the environmental consequences to significant economic, environmental, and resource-protection gains. To sum up, agents with a green mindset are willing to pay higher food prices and consequently higher production input costs to ensure high environmental quality. Similar results are found in e.g. Hughner et al. (2007).

An important outcome of this work is the combination of an economic general equilibrium model with material flow analysis. This helps experts in resource management as well as economists to relate their work to a broader context. We propose a first framework to endogenously describe and explain material flows depending on economic decisions. Through our approach we can understand how economic decisions influence the environment based on mapping pollutant flows in a particular case and vice versa. These insights can be applied to any developed region or country, not only to Austria.

5. Model shortcomings and directions for further research

Like any conceptual model it cannot perfectly reflect reality. It has not been econometrically estimated and, therefore, it does not serve as a forecasting model. However, considering plausible assumptions the results will shed light on what might actually happen if conditions resembling the scenario settings occur. Generally, the interdisciplinary modeling approach includes limitations from a perspective of every single discipline. Bouman et al. (2000) also mentions challenges of combining MFA modeling techniques and a partial equilibrium model. In this paper the nonlinear production functions slightly loosen the animal physiological and biological relationships of fodder and respective meat products. On the other side, linear production functions, as typically applied in material flow models by using transfer coefficients, would not be coherent with standard economic literature where labor plays a crucial role as factor input.

The standard economic assumption of well informed and rational farmers would lead to no overfertilization, hence, abandon the core of this study. So, we introduced the concept of a trader, serving as intermediary and regulatory agency. Aiming for appropriate fertilizer quantity and the zero-profit-condition distinguishes the modeled trader from a commercial fertilizer seller, who would only aim to maximize profits. Finding an alternative model approach to capture the phenomenon of overfertilization can be interesting for future research.

Further future work can include a more detailed modeling of the agricultural sector by including different crop applications like biofuel, more cost types or economic regulators. Some specific regulation policies can be consumption tax or direct investments into environment.

Moreover, capturing the feedbacks of households' decisions and the environment over many periods by aggregating the utility and deriving optimal long term strategies is interesting work for future research.

Last, but not least, bioslurry uses, biogas nutrients, nutrient supply through recycling food waste, human health effects of environmental pollution and transportation aspects (Kok et al., 2018) could be additionally considered in further modeling analysis.

Acknowledgments

We would like acknowledge financial support from the Austrian Science Fund (FWF) as part of the Vienna Doctoral Programme on Water Resource Systems (DK-plus W1219-N28) and our colleagues within the DK resource management cluster for helpful feedbacks. We want to thank Stefani Rivic and Anna Dugan for helping with proofs and Michael Freiberger for fruitful discussions. Furthermore we would like to acknowledge Prof. Roy Brouwer for his support, Prof. Horatio Rus for very useful inputs and Prof. Bryan Tolson for sharing his Dynamically Dimensioned Search (DDS) Algorithm.

Appendix A. Market equilibrium

A.1. Demand and supply functions

We derive the demand and supply function of the households and the firms to obtain the market equilibria.

Households. maximize

$$\begin{aligned}\mathcal{L}_h(c_g^c, c_a^c, \mu_h) &= \gamma_c \log(c_g^c(t)) + \alpha_c \log(c_a^c(t)) + \epsilon \log(E(t+1))\\ &+ \mu_h[p_g(t)c_g^c(t) + p_a^c(t)c_a^c(t) + p_x x - (w(t)L(t) + \pi_g(t) + \pi_a(t) + \pi_w + \pi_x + p_m(t)P_m(t))]\end{aligned} \tag{A.1}$$

considering

$$\begin{aligned}E(t+1) &= E(c_g^c(t),\, c_g^a(t))\\ &= \delta E(t) - \ell[P_w(t) + P_m(t) + P_a^g(c_g^c(t-1),\, c_a^c(t-1)) + P_n(t)] - P_w^\ell(c_g^c(t),\, c_a^c(t)) - p_w^f P_w(t) - p_m^f P_m(t).\end{aligned} \tag{A.2}$$

The first order conditions are the following. For easier reading we suppress the time argument t.

$$\frac{\partial \mathcal{L}_h}{\partial c_g^c} = \frac{\gamma_c}{c_g^c} + \frac{\epsilon}{E(c_g^c,\, c_a^c)} \frac{\partial E(c_g^c,\, c_a^c)}{\partial c_g^c} + \mu_h p_g^c = 0 \tag{A.3}$$

$$\frac{\partial \mathcal{L}_h}{\partial c_a^c} = \frac{\alpha_c}{c_a^c} + \frac{\epsilon}{E(c_g^c,\, c_a^c)} \frac{\partial E(c_g^c,\, c_a^c)}{\partial c_a^c} + \mu_h p_a^c = 0 \tag{A.4}$$

$$\frac{\partial \mathcal{L}_h}{\partial \mu_h} = p_g^c c_g^c + p_a^c c_a^c + p_x x - (wL + \pi_g + \pi_a + \pi_w + \pi_x + p_m P_m) = 0 \tag{A.5}$$

We define

$$\begin{aligned}E'(c_g^c,\, c_a^c) :=\; & \frac{p_g^c}{p_a^c}\frac{\partial E}{\partial c_a^c} - \frac{\partial E}{\partial c_g^c}\\ =\; & \frac{p_g^c}{p_a^c}\frac{\partial(-\ell P_a^g(c_g^c,\, c_a^c) - P_w^\ell(c_g^c,\, c_a^c))}{\partial c_a^c} - \frac{\partial(-\ell P_a^g(c_g^c,\, c_a^c) - P_w^\ell(c_g^c,\, c_a^c))}{\partial c_g^c}\\ =\; & -w_\ell \frac{(1 - w_g^\ell(P_g}{c_g(c_g^c,\, c_a^c(^2}[(\frac{p_g^c}{p_a^c}\zeta_{ha}\zeta_a^c - \zeta_{hg})c_g(c_g^c,\, c_a^c) + (1 - \frac{p_g^c c_g^a(c_a^c)}{\alpha_a p_a^c c_a^c})(\zeta_{hg} c_g^c + \zeta_h a \zeta_a^c c_g^a(c_a^c))].\end{aligned} \tag{A.6}$$

Expressing μ_h from Eq. (A.3) and inserting in Eq. (A.4) yields an implicite function for the grain demand

$$c_g^c = \gamma_c[\frac{p_g^c}{p_a^c}\frac{\alpha_c}{c_g^c} + \frac{\epsilon}{E(c_g^c,\, c_g^a)}E'(c_g^c,\, c_g^a)]^{(-1)} \tag{A.7}$$

and we can reformulate the budget constraint to obtain the animal product demand as function of the grain demand

$$(c_a^c)^D = \frac{\left(wL + \pi_g + \pi_a + \pi_w + \pi_x + p_m P_m\right) - \left(p_g^c c_g^c + p_x x\right)}{p_g^c}. \tag{A.8}$$

Using $c_g^a(c_a^c) = \frac{1}{\psi}(\frac{c_a^c}{\phi_a}L_a^{1-\beta_a})^{\frac{1}{\alpha_a}}$ inserting Eq. (A.8) in Eq. (A.7) we obtain an implicite function for $(c_g^c)^D$ that only depends on prices and L_a, which will be a result of the labor market equilibrium.

The supply for labor is

$$L^S = 1000000. \tag{A.9}$$

Grain farmers. demand labor and fertilizers and maximize

$$\mathcal{L}_g(G,\, L_g,\, \mu_g) = p_g(t)\phi_g[\chi_a P_a + \chi_n P_n + G]^{\alpha_g}[\phi_L L_g]^{\beta_g} - p_G G - wL_g. \tag{A.10}$$

Note, that P_a and P_n are taken from the previous period $t - 1$. From the first order conditions

$$\frac{\partial \mathcal{L}_g}{\partial G} = \alpha_g p_g \phi_g[(\chi_a - \ell)P_a + (\chi_n - \ell)P_n + G]^{\alpha_g - 1}[\phi_L L_g]^{\beta_g} - p_G = 0 \tag{A.11}$$

$$\frac{\partial \mathcal{L}_g}{\partial L_g} = \beta_g p_g \phi_g[(\chi_a - \ell)P_a + (\chi_n - \ell)P_n + G]^{\alpha_g}\phi_L^{\beta_g} L_g^{\beta_g - 1} - w = 0 \tag{A.12}$$

we can derive the demands for fertilizer and labor.

$$G^D(t) = \left(\frac{\phi_g \phi_L^{\beta_g}\alpha_g^{1-\beta_g}\beta_g^{\beta_g}p_g(t)}{p_G(t)^{1-\beta_g}w(t)^{\beta_g}}\right)^{\frac{1}{1-\alpha_g-\beta_g}} - ((\chi_a - \ell)P_a(t-1) + (\chi_n - \ell)P_n(t-1)) \tag{A.13}$$

$$L_g^D(t) = \left(\frac{\phi_g \phi_L^{\beta_g}\alpha_g^{\alpha_g}\beta_g^{1-\alpha_g}p_g(t)}{p_G(t)^{\alpha_g}w(t)^{1-\alpha_g}}\right)^{\frac{1}{1-\alpha_g-\beta_g}} \tag{A.14}$$

Note, the fertilizer price is constraint with the world market price p_m and the price for recycled P p_w (proof see Appendix C.1).

The crop supplies (limited with the crop intake of P to $\bar{P}$) are

$$c_g^S(t) = \left(\frac{\phi_g \phi_L^{\beta_g}\alpha_g^{\alpha_g}\beta_g^{\beta_g}p_g(t)^{\alpha_g+\beta_g}}{p_G(t)^{\alpha_g}w(t)^{\beta_g}}\right)^{\frac{1}{1-\alpha_g-\beta_g}} \tag{A.15}$$

$$(c_g^c)^S(t) = \phi_g^c c_g^S(t) \tag{A.16}$$

$$(c_g^a)^S(t) = \phi_g^a c_g^S(t) \tag{A.17}$$

The average price for grain is $p_g(t) = \phi_g^c p_g^c(t) + \phi_g^a p_g^a(t)$.

Animal husbandry. maximizes by choosing the amount of fodder and labor.

$$\mathcal{L}_a(c_g^a, L_a) = p_a^c \phi_a (\psi c_g^a)^{\alpha_a} [\phi_L L_a]^{\beta_a} - p_g^a c_g^a - wL_a \tag{A.18}$$

yields

$$\frac{\partial \mathcal{L}_a}{\partial c_g^a} = \alpha_a p_a^c \phi_a \psi^{\alpha_a} (c_g^a)^{\alpha_a - 1} [\phi_L L_a]^{\beta_a} - p_g^a = 0 \tag{A.19}$$

$$\frac{\partial \mathcal{L}_a}{\partial L_a} = \beta_a p_a^c \phi_a (\psi c_g^a)^{\alpha_a} \phi_L^{\beta_a} L_a^{\beta_a - 1} - w = 0 \tag{A.20}$$

the optimal demands and supply are

$$(c_g^a)^D(t) = \left(\frac{\phi_a \phi_L^{\beta_a} \psi^{\alpha_a} \alpha_a^{1-\beta_a} \beta_a^{\beta_a} p_a^c(t)}{\left(p_g^a\right)^{1-\beta_a} w^{\beta_a}}\right)^{\frac{1}{1-\alpha_a-\beta_a}} \tag{A.21}$$

$$L_a^D(t) = \left(\frac{\phi_a \phi_L^{\beta_a} \psi^{\alpha_a} \alpha_a^{\alpha_a} \beta_a^{1-\alpha_a} p_a^c(t)}{\left(p_g^a\right)^{\alpha_a} w^{1-\alpha_a}}\right)^{\frac{1}{1-\alpha_a-\beta_a}} \tag{A.22}$$

$$(c_a^c)^S(t) = \left(\frac{\phi_a \phi_L^{\beta_a} \psi^{\alpha_a} \alpha_a^{\alpha_a} \beta_a^{\beta_a} (p_a^c)^{\alpha_a+\beta_a}}{\left(p_g^a\right)^{\alpha_a} w^{\beta_a}}\right)^{\frac{1}{1-\alpha_a-\beta_a}} \tag{A.23}$$

Trader. The zero-profit condition is the budget constraint of the trader.

$$\pi_G = p_G(\bar{G} + \sigma) - p_w(A)P_w - p_m P_m = 0 \tag{A.24}$$

The production function is the combination of the fertilizer types.

$$\bar{G} + \sigma = (\chi_w(A) - \ell)P_w + (\chi_m - \ell)P_m \tag{A.25}$$

For no overfertilization $\sigma = 0$ the quantity constraint for an ideal fertilizer supply $\bar{P}$ of the plant is fulfilled.

$$G(t) = \bar{G} = \bar{P} - (\chi_a - \ell)p_a^c(t-1) - (\chi_n - \ell)P_n(t-1) \tag{A.26}$$

Furthermore the trader is constraint to a maximum supply from the WWTP.

$$P_w \leq P_w^{maxsupply} \tag{A.27}$$

In case budget, quantity and WWTP constraint can be fulfilled we derive the following demand for fertilizer on the world markets (see also Fig. 4).

$$P_w^D = \frac{(1 - (\chi_m - \ell)\frac{p_G}{p_m})G}{(\chi_w - \ell) - (\chi_m - \ell)\frac{p_w}{p_m}} \tag{A.28}$$

$$P_m^D = \frac{p_G G - p_w P_w^D}{p_m} \tag{A.29}$$

A.2. Market clearing

We obtain the prices p_g^c, p_g^a, p_a^c, w, p_G from the competitive markets for c_g^c, c_g^a, c_a^c, L, G, respectively. The open markets for P_m and P_w face infinitely elastic supply and exogenous prices p_m and p_w, respectively.

The market clearing conditions are as follows.

$$c_g^c\colon (c_g^c)^D(p_g^c; p_a^c, E, E') = \phi_g^c \left(\frac{\phi_g \phi_L^{\beta_g} \alpha_g^{\alpha_g} \beta_g^{\beta_g} p_g(t)^{\alpha_g+\beta_g}}{p_G(t)^{\alpha_g} w(t)^{\beta_g}}\right)^{\frac{1}{1-\alpha_g-\beta_g}} \tag{A.30}$$

$$c_g^a\colon \left(\frac{\phi_a \phi_L^{\beta_a} \psi^{\alpha_a} \alpha_a^{1-\beta_a} \beta_a^{\beta_a} p_a^c(t)}{\left(p_g^{)} a^{1-\beta_a} w^{\beta_a}}\right)^{\frac{1}{1-\alpha_a-\beta_a}} = \phi_g^a \left(\frac{\phi_g \phi_L^{\beta_g} \alpha_g^{\alpha_g} \beta_g^{\beta_g} p_g(t)^{\alpha_g+\beta_g}}{p_G(t)^{\alpha_g} w(t)^{\beta_g}}\right)^{\frac{1}{1-\alpha_g-\beta_g}} \tag{A.31}$$

$$c_a^c\colon \frac{\left(wL + \pi_g + \pi_a + \pi_w + \pi_x + p_m P_m\right) - \left(p_g^c c_g^c + p_x x\right)}{p_g^c} = \left(\frac{\phi_a \phi_L^{\beta_a} \psi^{\alpha_a} \alpha_a^{\alpha_a} \beta_a^{\beta_a} (p_a^c)^{\alpha_a+\beta_a}}{\left(p_g^a\right)^{\alpha_a} w^{\beta_a}}\right)^{\frac{1}{1-\alpha_a-\beta_a}} \tag{A.32}$$

$$L\colon \left(\frac{\phi_g \phi_L^{\beta_g} \alpha_g^{\alpha_g} \beta_g^{1-\alpha_g} p_g(t)}{p_G(t)^{\alpha_g} w(t)^{1-\alpha_g}}\right)^{\frac{1}{1-\alpha_g-\beta_g}} + \left(\frac{\phi_a \phi_L^{\beta_a} \psi^{\alpha_a} \alpha_a^{\alpha_a} \beta_a^{1-\alpha_a} p_a^c(t)}{\left(p_g^a\right)^{\alpha_a} w^{1-\alpha_a}}\right)^{\frac{1}{1-\alpha_a-\beta_a}} + L_x = L \tag{A.33}$$

$$G\colon \left(\frac{\phi_g \phi_L^{\beta_g} \alpha_g^{1-\beta_g} \beta_g^{\beta_g} p_g(t)}{p_G(t)^{1-\beta_g} w(t)^{\beta_g}}\right)^{\frac{1}{1-\alpha_g-\beta_g}} = \bar{P} \tag{A.34}$$

Note, $(c_a^c)^D$ cannot be explicitly expressed with prices and is still a function of $(c_g^c)^D$. So we cannot derive the prices explicitly.

A.3. Tatonnement algorithm

We eliminate some equations and apply a tatonnement algorithm for the c_g^c and c_a^c markets for every period. The full Matlab code can be found in Appendix D, the outline is found in the following.

1. $p_m(t)$, $p_w(A)$, L given; $E(t)$, $P_n(t)$, $P_h(t-1)$ from previous period
2. identify cheaper and more efficient fertilizer by testing $\frac{p_m \chi_m}{p_w \chi_w}$ smaller or larger than one, derive $\bar{G} = G$ from plan constraint (A.26)
3. initialize p_g^c, p_g^a, p_a, p_G
4. from L market derive w using G from plant constraint (A.26) in the first iteration of the algorithm

$$\left(\phi_g \phi_l^{\beta_g} \beta_g p_g \bar{P}^{\alpha_g}\right)^{\frac{1}{1-\beta_g}} w(t)^{\frac{-1}{1-\beta_g}} + \left(\frac{\phi_a \phi_L^{\beta_a} \psi^{\alpha_a} \alpha_a^{\alpha_a} \beta_a^{1-\alpha_a} p_a(t)}{p_g^a(t)^{\alpha_a}}\right)^{\frac{1}{1-\alpha_a-\beta_a}} w(t)^{\frac{1-\alpha_a}{1-\alpha_a-\beta_a}} + L_x - L = 0 \tag{A.35}$$

5. from fertilizer G market derive p_G^{WTP} using farmers demand and the plant constraint (A.26)

$$p_G^{WTP} = \left(\frac{\phi_g \phi_L^{\beta_g} \alpha_g^{1-\beta_g} \beta_g^{\beta_g} \left(\phi_g^c p_g^c + \phi_g^a p_g^a\right)}{\bar{P}^{1-\alpha_g-\beta_g} w^{\beta_g}}\right)^{\frac{1}{1-\beta_g}} \tag{A.36}$$

6. derive P_m and P_w, p_G^{sold} and G^{sold}
   (a) check if p_G^{WTP} in [$\frac{p_m}{\chi_m}$, $\frac{p_w}{\chi_w}$] without loss of generality or if we have to use a corner solution
   (b) buy only $P_m = p_G^{WTP}\bar{G}/p_m$ if farmer's willingness to pay (WTP) is below the market prices for fertilizers and P_m is more efficient than P_w
   (c) buy only $P_w = p_G^{WTP}\bar{G}/p_w$ or mixed if $P_w > P_w^{maxsupply}$ if farmer's WTP is outside the fertilizer price interval
   (d) if the WTP is within the market fertilizer price range choose fertilizer according to Eq. (A.28) and (A.29), except $P_w > P_w^{maxsupply}$
   (e) derive overfertilization $\sigma = G - \bar{G}$ and the new P stock in the soil $P_n = (1-\ell)(P_m + P_w + P_a^g + P_n) - \bar{P}$
   (f) derive the P in plants $P_g = \min(G, \bar{G}) + (\chi_a - \ell)P_a + (\chi_n - \ell)P_n$
7. derive p_g^a from c_g^a market using $\gamma_g^a = \frac{1}{(\alpha_g+\beta_g)(1-\alpha_a-\beta_a)+(1-\beta_a)(1-\alpha_g-\beta_g)}$

$$p_g^a = \frac{1}{\phi_g^a}\left(\left(\frac{\phi_a \phi_L^{\beta_a} \psi^{\alpha_a} \alpha_a^{1-\beta_a} \beta_a p_a}{w^{\beta_a}}\right)^{1-\alpha_g-\beta_g} \left(\frac{w^{\beta_g}}{\phi_g \phi_L^{\beta_g} \alpha_g^{\alpha_g} \beta_g^{\beta_g} \left(p_g^c\right)^{\alpha_g+\beta_g}}\right)^{1-\alpha_a-\beta_a}\right)^{\gamma_g^a} \tag{A.37}$$

8. prepare tatonnement

$$(\mathbf{c}_g^c)^S = \phi_g^c\left(\frac{\phi_g \phi_L^{\beta_g} \alpha_g^{\alpha_g} \beta_g^{\beta_g} p_g(t)^{\alpha_g+\beta_g}}{p_G(t)^{\alpha_g} w(t)^{\beta_g}}\right)^{\frac{1}{1-\alpha_g-\beta_g}} \tag{A.38}$$

$$(\mathbf{c}_a^c)^S = \left(\frac{\phi_a \phi_L^{\beta_a} \psi^{\alpha_a} \alpha_a^{\alpha_a} \beta_a^{\beta_a} p_a^{\alpha_a+\beta_a}}{\left(p_g^a\right)^{\alpha_a} w^{\beta_a}}\right)^{\frac{1}{1-\alpha_a-\beta_a}} \tag{A.39}$$

$$L_g^D = \left(\frac{\phi_g \phi_L^{\beta_g} \alpha_g^{\alpha_g} \beta_g^{1-\alpha_g} p_g(t)}{p_G(t)^{\alpha_g} w(t)^{1-\alpha_g}}\right)^{\frac{1}{1-\alpha_g-\beta_g}} \tag{A.40}$$

$$L_a^D = \left(\frac{\phi_a \phi_L^{\beta_a} \psi^{\alpha_a} \alpha_a^{\alpha_a} \beta_a^{1-\alpha_a} p_a(t)}{p_g^a(t)^{\alpha_a} w(t)^{1-\alpha_a}}\right)^{\frac{1}{1-\alpha_a-\beta_a}} \tag{A.41}$$

$$\pi_g = \left(p_g^c \phi_g^c + p_g^a \phi_g^a\right)(c_g^c)^S - p_G G - wL_g^D \tag{A.42}$$

$$\pi_a = p_a(c_a^c)^S - p_g(c_g^a)^D - wL_a^D \tag{A.43}$$

$$\pi_w = p_w P_w \tag{A.44}$$

$$P_g^c = \frac{c_g^c}{c_g^c + c_g^a}\zeta_g P_g \tag{A.45}$$

$$P_g^a = \frac{c_g^a}{c_g^c + c_g^a}\zeta_g P_g \tag{A.46}$$

$$P_h = \left(c_g^c + \zeta_a^c c_g^a\right)\frac{\zeta_g P_g}{c_g^c + c_g^a} \tag{A.47}$$

$$P_w^\ell = \omega_\ell(A) P_h \tag{A.48}$$

$$P_g^\ell = \ell(P_w + P_m + P_a^g + P_n) \tag{A.49}$$

$$E(t+1) = \delta E - P_g^\ell - P_w^\ell \tag{A.50}$$

$$(\mathbf{c}_g^c)^D = (c_g^c)^D\left(p_g^c; p_a, E, E'\right) \tag{A.51}$$

$$(\mathbf{c}_a^c)^D = \frac{\left(wL + \pi_g + \pi_a + \pi_w + \pi_x + p_m P_m\right) - \left(p_g^c c_g^c + p_x x\right)}{p_g^c} \tag{A.52}$$

9. tatonnement with dumping factor d:

$$p_a = p_a^{old} - d((c_a^c)^D - (c_a^c)^S) \tag{A.53}$$

$$p_g^c = \left(p_g^c\right)^{old} - d\left((c_g^c)^D - (c_g^c)^S\right) \tag{A.54}$$

10. repeat from step 4 and stop if $|((c_a^c)^D - (c_a^c)^S)| + |((c_g^c)^D - (c_g^c)^S)| < tol$

Appendix B. Overfertilization

The fertilizer market is one of the core elements of the proposed model framework. Farmers demand the composite fertilizer good G for a market price p_G (Eq. (A.13)). A trader combines mineral fertilizer P_m and recycled phosphorus P_w to the fertilizer composition G (Eq. (16)) and sells it for the resulting price p_G (Eq. (17)) under his zero-profit-condition and the price level constraints (see Appendix C.1). We can picture the traders fertilizer supply (Eq. (C.1)), the farmers fertilizer demand, the plant demand $\bar{G}$ and the maximum WWTP supply $P_w^{maxsupply}$ in one graph (e.g. Fig. B.15). If the quantity demand $G(t)$ from the farmers (Eq. (A.13)) cannot be met with the farmers willingness to pay for fertilizer $p_G(t)$ the trader sells $G(t) + \sigma(t) = (\chi_w - \ell)P_w(t) + (\chi_m - \ell)P_m(t)$. All cases for under- and overfertilization $\sigma(t)$ are explained below.

Farmers are willing to pay more than the necessary fertilizer price and the demand of recycled P can be met. The trader sells only recycled P_w and considers the budget constraint. The sold amount G is displayed in Fig. B.15 and the price p_G is exactly the price p_w for $\chi_w P_w$.

Farmers fertilizer demand exceeds the amount of available recycled P fertilizer. Even if the trader sells all the available P_w for the price $p_G = \frac{p_w}{\chi_w}$ farmers are still willing to pay more for fertilizer. So the trader also adds imported mineral fertilizer to the composite fertilizer G_{mix} (Fig. B.16). Consequently, the price for amount $G = \bar{G} + \sigma$ changes to $p_G < \frac{p_w}{\chi_w}$ (Eq. (17)).

After a price shock of mineral fertilizer price relations change to $p_m > p_w$. If farmers WTP is above p_w traders compose the fertilizer mix G analogous to the cases in the previous paragraphs: The trader would only add P_m if the demand for P_w exceeds $P_w^{maxsupply}$. Contrary to the above case the new price p_G would then increase ($p_G > p_w$).

Farmers are willing to pay less than the cheapest available fertilizer. If $p_G^{WTP} < \frac{p_m}{\chi_m} < \frac{p_w}{\chi_w}$ or $p_G^{WTP} < \frac{p_w}{\chi_w} < \frac{p_m}{\chi_m}$, the trader sells as much of the cheapest

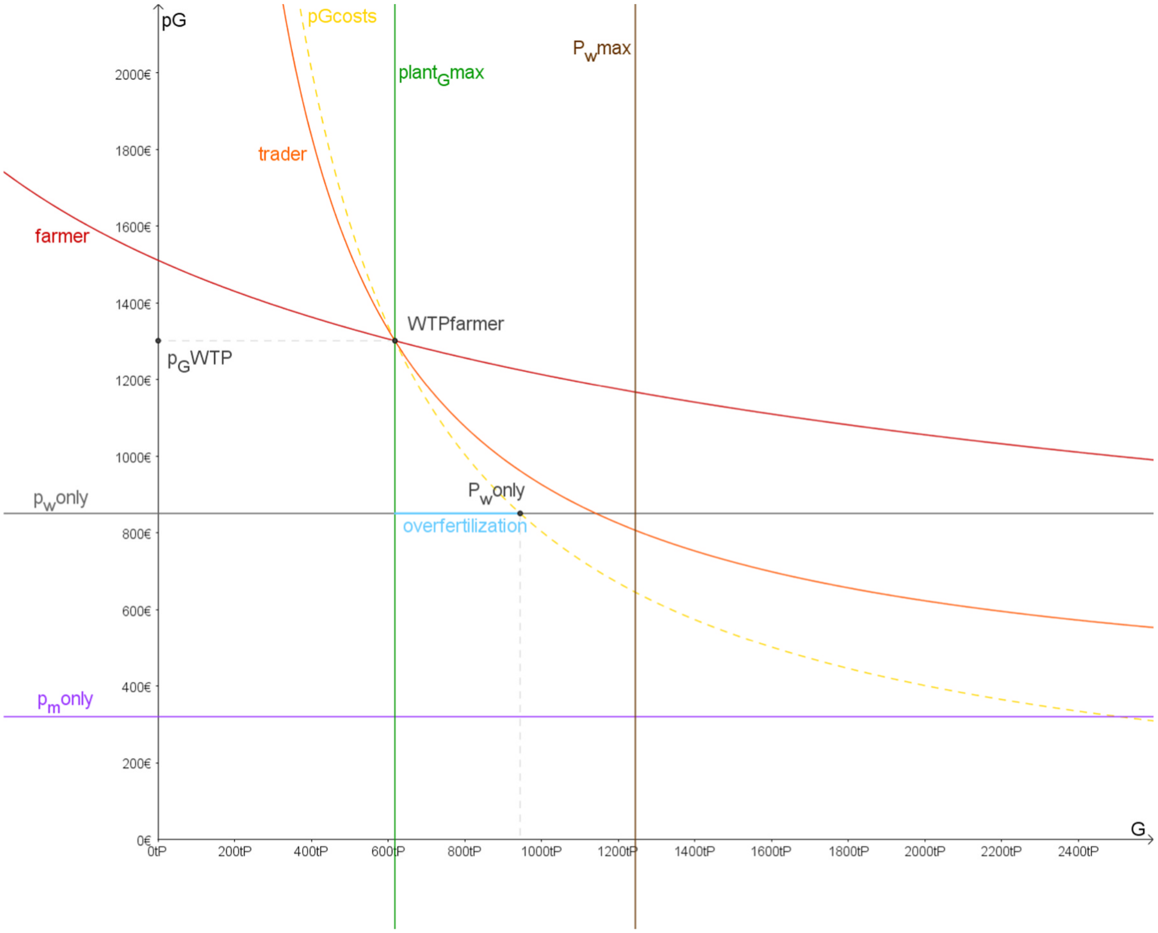

Fig. B.15. Even if farmers pay the highest possible price $p_G = \frac{p_w}{\chi_w}$ the are willing to buy more fertilizer G than the plants actually demand ($plant_G max$) and overfertilize.

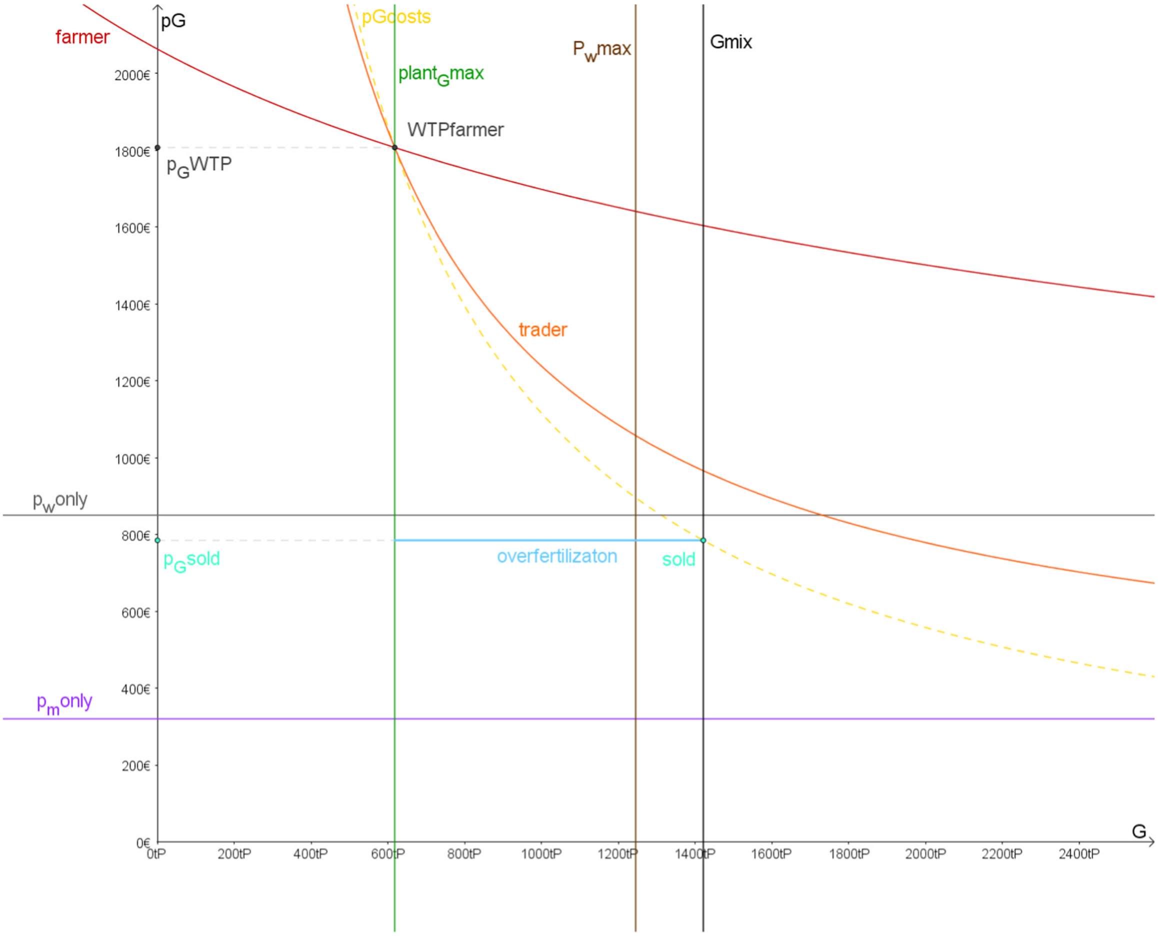

Fig. B.16. The willingness to pay for farmers (G^{WTP}, p_G^{WTP}) allows to sell the fertilizer mix (G^{sold}, p_G^{sold}).

fertilizer as the budget constraint (Eq. (17)) allows. The fertilizer supply is less than the plants demand ($G = \bar{G} - \sigma$) and we are in a case of underfertilization.

Appendix C. Proofs for the fertilizer market

C.1. Proof: price p_G is constraint

The price $p_G(t)$ of the composite fertilizer good $G(t)$ has to be in the interval $[\frac{p_m(t)}{\chi_m}, \frac{p_w(t)}{\chi_w}]$ if $\frac{p_m(t)}{\chi_m} < \frac{p_w(t)}{\chi_w}$ w.l.o.g.

Proof: We can rewrite the production function (Eq. (16)) and budget constraint (Eq. (17)) of the trader into $p_G = \frac{p_m P_m + p_w P_w}{\chi_m P_m + \chi_w P_w}$.. Note, $G = \bar{G} + \sigma$ in case of overfertilization.

First, assume $p_G > \frac{p_w}{\chi_w}$. Then $p_w \frac{\frac{p_m}{p_w} P_m + P_w}{\chi_m P_m + \chi_w P_w} > \frac{p_w}{\chi_w}$. We can rewrite that into $\frac{p_m(t)}{\chi_m} > \frac{p_w(t)}{\chi_w}$, which disproves the assumption and we conclude $p_G < \frac{p_w}{\chi_w}$.

Second, assume $p_G < \frac{p_m}{\chi_m}$. Then $p_m \frac{P_m + \frac{p_w}{p_m} P_w}{\chi_m P_m + \chi_w P_w} < \frac{p_m}{\chi_m}$. We can rewrite that again into $\frac{p_w(t)}{\chi_w} < \frac{p_m(t)}{\chi_m}$, which disproves the assumption and we conclude $p_G < \frac{p_w}{\chi_w}$.

Consequently, $p_G \in [\frac{p_m(t)}{\chi_m}, \frac{p_w(t)}{\chi_w}]$ if $\frac{p_m(t)}{\chi_m} < \frac{p_w(t)}{\chi_w}$. Analogous we can show $p_G \in [\frac{p_w(t)}{\chi_w}, \frac{p_m(t)}{\chi_m}]$ if $\frac{p_m(t)}{\chi_m} > \frac{p_w(t)}{\chi_w}$.

C.2. Proof: trader supply switches at a price level $\frac{p_m(t)}{\chi_m}$

The traders supply function switches from downward (Fig. 5) to upward (Fig. 6) sloping at point $p_G = \frac{p_m(t)}{\chi_m}$.

Proof: From the production function (Eq. (16)) and budget constraint (Eq. (17)) we can derive the traders supply curve

$$p_G(G(t); P_w(t)) = \frac{p_m(t)}{G(t)}\left[\frac{G(t) - \chi_w P_w}{\chi_m} + p_w(A)P_w(t)\right]. \tag{C.1}$$

The partial derivative with respect to G is

$$\frac{\partial p_G}{\partial G} = \frac{p_m(t)}{G(t)^2}\left[\frac{\chi_w P_w}{\chi_m} - p_w(A)P_w(t)\right]. \tag{C.2}$$

So we can express

$$p_G(G(t); P_w(t)) = \frac{p_m(t)}{\chi_m(t)} - \frac{p_m(t)}{G(t)}\left[\frac{\chi_w P_w}{\chi_m} - p_w(A)P_w(t)\right] \tag{C.3}$$

$$= \frac{p_m(t)}{\chi_m(t)} - G(t)\frac{\partial p_G}{\partial G}. \tag{C.4}$$

If $p_G > \frac{p_m(t)}{\chi_m}$, then $\frac{p_m(t)}{\chi_m(t)} - G(t)\frac{\partial p_G}{\partial G} > \frac{p_m(t)}{\chi_m}$ and consequently $\frac{\partial p_G}{\partial G} < 0$. So the trader supply is downward sloping.

Analogous, if $p_G < \frac{p_m(t)}{\chi_m}$, we can derive $\frac{\partial p_G}{\partial G} > 0$ meaning the trader supply is upward sloping.

In the special case $p_G = \frac{p_m(t)}{\chi_m}$ the price of the fertilizer is constant, because only P_m is sold as $G(t)$.

Appendix D. Matlab code

The following is the main Matlab-file `run_Pmodel`.

```
% P—model
% run file
% set parameters and choose model

global alpha_c gamma_c k cabar cgbar alpha_g beta_g alpha_a beta_a psi phi_g ...
    phi_gc phi_ga pw chi_w chi_m chi_n chi_a Pibar l Tend epsilon P1bar
global zeta_hg zeta_ha zeta_a zeta_g zeta_ac Δ_e stepsLg LGepsilon dumpingfactor n ...
    tol %E P_m P_w P_n P_a
global omega F V A pwf tau wl sozPl0

model = 3;
% 0 = simplemodel
% 1 = Extension 1: Including P and trader
% 2 = Extension 2: Including Environment in utility function
% 3 = Final Model: differentiating pg

sozPl0 = 1;
% 0 = social planner does not take P_a into account
% 1 = social planner considers P_a for consumption decision

A = 2; %Technology
% 1 = default for models without WWTP
% 2 = sludge liquor
% 3 = sewage sludge
% 4 = sludge ash

pmchange = 5; %time period when price level changes from cheap to expensive
pmexpensive = 0.9;
pmcheap = 0.7;
pm = [ones(1,pmchange)*pmexpensive,ones(1,Tend—pmchange)*pmcheap];

alpha_c = 0.4;
gamma_c = 0.6;
k = 0;
```

```
cabar = 0;
cgbar = 0;
Gmax = 0.1;
alpha_g =  0.1;
beta_g = 0.7;
phi_g = 1.2; % total factor productivity >1 for grain sector
phi_gc = 1.2; % total factor productivity >1 for grain sector
phi_ga = 1.2; % total factor productivity >1 for grain sector
alpha_a = 0.2;
beta_a = 0.6;
psi = 0.25;   % psi^alpha_a=total factor productivity >1 for meat sector
n = 1;       % labor supply % originally = 1
tol = 0.01; % tolerance for Qsupply-Qdemand in eq1period to derive market equilibrium
dumpingfactor = 0.01; %rather small e.g.0.01

Tend = 20; % time horizon
stepsLg = 100; % number of steps when calculating roots of LG fct
LGepsilon = 0.0001; % tolerance for root search of LG fct

%initialize
price0(1,1)=1;      % pg %$/t       % 150 euro for 1 ton grain
price0(2,1)=1;      % w  % $/a/cap

% introducing P
E=0.7;
P_n=0;
P_a=0;
P_m=0;
P_w=0;
P=zeros(4,Tend);
P_wMaxSupply=1;

P1bar = 0.1;
chi_m = 1;
chi_w = [0.9, 0.6, 0.6, 0.9]; % chi_w(A)
chi_n = 0.4;
chi_a = 0.8;
l = 0.0005;
pw = [0.3,0.01,0.01,0.3]; %pw(A)
```

```
pwD = pw;
zeta_P = 0.75;
zeta_g = 0.6;
zeta_a = 0.1;
zeta_hg = 0.7;
zeta_ha = 0.7;
zeta_ac = 1-zeta_a;
Δ_e = 1.01;
sigma=0.05;
sigma_i=[ones(1,pmchange)*0,ones(1,Tend-pmchange)*sigma];
Pibar = ones(1,Tend)*P1bar +sigma_i;
epsilon = 0.01; %??
P_g = zeta_g*P1bar;

omega = 1/100000;
I = [1000, 1000, 1000, 1000]; %I(A)
F(A) = I(A)/Tend;
V = [0.01, 0.01, 0.01, 0.01]; %V(A)
pwf = [0.01, 0.005, 0.005, 0.006]; %pwf(A)
tau = [1, 0.9, 1, 0.9]; %tau(A)
wl = [1, 0.2, 0.3, 0.1]; %wl(A)

%[cg,cgc,cga,ca,Lg,La]=quantities
%[pg,pa,w]=prices

%-simple model
if model == 0
    [quantity,price,error]=marketEQsimple(price0,Gmax)
    [ u,pi_g,pi_a ] = objectivesimple(quantity,price)

%-Ext1: Including P in farmers decision
elseif model == 1
    for t=1:Tend
        Gmax = Pibar(t)-(chi_a-1)*P_a(t)-(chi_n-1)*P_n(t);
        [quantities(:,t),prices(:,t),diff]=marketEQsimple(price0,Gmax);
        [P_m(t),P_w(t),pwD(t)]= ...
            fertilizer(quantities,prices,P_a,P_n,P_wMaxSupply(t),pm,pw,A,t);
        [u(t),pi_g(t),pi_a(t)] = ...
            objectiveExt1(quantities(:,t),prices(:,t),P_m(t),P_w(t),pm(t));
```

```
        P_a(t+1)=zeta_a*quantities(3,t)/quantities(1,t)*zeta_g*P1bar;
        P_n(t+1)=(1-chi_m)*P_m(t)+(1-chi_w(1))*P_w(t)+(1-chi_n)*P_n(t)
        +(1-chi_a)*P_a(t)+sigma_i(t);
        E(t+1)= Δ_e*E(t)-l*(P_m(t)+P_w(t)+P_n(t)+P_a(t)) - wl(A)*(zeta_hg ...
            *quantities(2,t) / (quantities(8,t) *quantities(5,t)^beta_g)* P_g + ...
            zeta_ha*zeta_a* quantities(3,t) / (quantities(8,t)* ...
            quantities(5,t)^beta_g) *P_g);
        P_wMaxSupply(t+1) = zeta_hg*quantities(2,t)+zeta_ha*quantities(4,t); ...
            %Supply from households for next period = basically P_h
        P(t)=Pflows(quantities(:,t),P_m(t),P_w(t),P_a(t),P_n(t),P_wMaxSupply(t));
    end

%-Ext2: Including Environment in utility function
elseif model == 2
    %figure
    for t=1:Tend
        Gmax = Pibar(t)-(chi_a-l)*P(3,t)-(chi_n-l)*P(4,t);
        [quantities(:,t),prices(:,t),P(:,t),diff]=
        marketEQEnv(price0,Gmax,pw(A),pm(t),P_wMaxSupply,P_g,P(4,t),P(3,t),E(t),t);
        [u(t),pi_g(t),pi_a(t),pi_G(t)] = ...
            objectiveExt2(quantities(:,t),prices(:,t),P(2,t),P(1,t),E(t));
        P(3,t+1)=zeta_a*quantities(3,t)/quantities(1,t)*zeta_g*P1bar;
        P(4,t+1)=(1-chi_m)*P(2,t)+(1-chi_w(1))*P(1,t)+(1-chi_n)*P(4,t)
        +(1-chi_a)*P(3,t)+sigma_i(t);
        E(t+1)= Δ_e*E(t)-l*(P(2,t)+P(1,t)+P(4,t)+P(3,t)) - wl(A)*(zeta_hg ...
            *quantities(2,t) / (quantities(8,t) *quantities(5,t)^beta_g)* P_g + ...
            zeta_ha*zeta_a* quantities(3,t) / (quantities(8,t)* ...
            quantities(5,t)^beta_g) *P_g);
        P_wMaxSupply(t+1) = zeta_hg*quantities(2,t)+zeta_ha*quantities(4,t); ...
            %Supply from households for next period = basically P_h
    end

%-final model: differentiating pg
elseif model == 3
%     quantities=transpose([cgc,cga,ca,G,Lg,La,L]);
%     prices=transpose([pgc,pga,pa,pG,w,pm,pw]);
%     P=transpose([P_m,P_w,P_a,P_n]);
for t=1:Tend
       G = Pibar(t)-(chi_a-l)*P(3,t)-(chi_n-l)*P(4,t);
```

```
        price0=[prices(1,t-1),prices(3,t-1)]; %pgc,pa from last period as initial ...
            prices
        [quantities(:,t),prices(:,t),P(:,t),objectives(:,t),E(t+1),iter]=
        marketEQfinal(price0,G,Pibar,pw(A),pm(t),P_wMaxSupply,P_g,P(4,t),E(t),t);
        P(4,t+1)=(1-chi_m)*P(2,t)+(1-chi_w(1))*P(1,t)+(1-chi_n)*P(4,t)+(1-chi_a)*P(3,t)
        +sigma_i(t); %P_n stock in next period
        P_wMaxSupply(t+1) = zeta_hg*quantities(2,t)+zeta_ha*quantities(4,t); ...
            %Supply from households for next period = basically P_h
    end

end

if model>0
t=linspace(1,Tend,Tend);
P_n=P(4,:);
figure
% plot(t,u,'color','b');hold on
% plot(t,pi_g,'color','r');hold on
% plot(t,pi_a,'color','m');hold on
% plotyy(t,0,[t,Tend+1],E,'color','g');hold on
% legend('u','pi_g','pi_a','E')

[Ax, H1, H2] = ...
    plotyy(t,[u;pi_g;pi_a;quantities(2,:);quantities(4,:);quantities(5,:)],[t,Tend+1], ...
    [E;P_n]);
set(H1(1),'Color','b');
set(H1(2),'Color','r');
set(H1(3),'Color','m');
set(H1(4),'Color',[0.1,0.5,0.1]); %cgc
set(H1(5),'Color',[0.2,0.7,0.3]); %ca
set(H1(6),'Color',[0.9,0.5,0.1]);
set(H2(1),'Color','g');
set(H2(2),'Color',[0.5,0.9,0.5]);
set(get(Ax(1),'Ylabel'),'String','quantities')
set(get(Ax(2),'Ylabel'),'String','Environment, P')
legend([H1;H2],'u','pi_g','pi_a','c_g^c','c_a','L_g','E','P_n','Location',
'northeastoutside');
end
```

The following is the Matlab-file `objectivesimple` for the respective function.

```
function [ u,pi_g,pi_a ] = objectivesimple(quantities,prices)
% objective functions simple model
%transpose[cg,cgc,cga,ca,Lg,La,L,G]=quantities
%transpose[pg,pa,w,pG]=prices

global alpha_c gamma_c k cabar cgbar pG alpha_g beta_g alpha_a beta_a psi phi_g G

u = gamma_c*log(quantities(2))+alpha_c*log(quantities(4))+1;
pi_g = prices(1)*quantities(1)-prices(4)*quantities(8)-prices(3)*quantities(5);
pi_a = prices(2)*quantities(4)-prices(1)*quantities(3)-prices(3)*quantities(6);

end
```

The following is the Matlab-file `objectiveExt1` for the respective function.

```
function [ u,pi_g,pi_a,pi_G] = objectiveExt1(quantities,prices,P_m,P_w,pm)
% objective functions for Ext1 including environment
% environment not in utility function
%transpose[cg,cgc,cga,ca,Lg,La,L,G]=quantities
%transpose[pg,pa,w,pG]=prices

global alpha_c gamma_c pw A

u = gamma_c*log(quantities(2))+alpha_c*log(quantities(4))+1;
pi_g = prices(1)*quantities(1)-prices(4)*quantities(8)-prices(3)*quantities(5);
pi_a = prices(2)*quantities(4)-prices(1)*quantities(3)-prices(3)*quantities(6);
pi_G = prices(4)*quantities(8)-pw(A)*P_w-pm*P_m;

end
```

The following is the Matlab-file `objectiveExt2` for the respective function.

```
function [ u,pi_g,pi_a,pi_G] = objectiveExt2(quantities,prices,P_m,P_w,E)
% objective functions for Ext1 including environment
% environment not in utility function
%transpose[cg,cgc,cga,ca,Lg,La,L,G]=quantities
%transpose[pg,pa,w,pG]=prices

global alpha_c gamma_c epsilon

u = gamma_c*log(quantities(2))+alpha_c*log(quantities(4))+epsilon*log(E)+1;
pi_g = prices(1)*quantities(1)-prices(4)*quantities(8)-prices(3)*quantities(5);
pi_a = prices(2)*quantities(4)-prices(1)*quantities(3)-prices(3)*quantities(6);
pi_G = prices(4)*quantities(8)-prices(5)*P_w          -prices(6)*P_m;

end
```

The following is the Matlab-file `fertilizer` for the respective function.

```matlab
function [P_m,P_w,pwD]= fertilizer(quantities,prices,P_a,P_n,P_wMaxSupply,pm,pw,A,t)
% subproblem for the farmers
% how much P_m and P_w is chosen
% total fertilizer G = Pibar =
% (chi_w-l)*P_w+(chi_m-l)*P_m+(chi_a-l)*P_a+(chi_n-l)*P_n
% prices pG*G=pw*P_w+pm*Pm+0*P_a+0*P_n

global chi_w chi_m chi_n chi_a l

P_w = (1-(chi_m-l)*prices(4,t)/pm(t))*quantities(8,t)/(chi_w(A)-
l-(chi_m-l)*pm(t)/pw(A));
%check parameter constraints:
if P_w < 0
    P_w = 0;
    fprintf('Pw = 0 in period %2.0f ;',t); %--> change parameters for P_w >0
end
if P_w > P_wMaxSupply   %household supply can not meet the demand
    P_w = P_wMaxSupply;
    fprintf('Pw demand > supply in period %2.0f ;',t); %--> change parameters for ...
        P_w even larger
end

P_m = (prices(4,t)*quantities(8,t)-pw(A)*P_w)/pm(t);
pwD = (prices(4,t)*quantities(8,t)-pm(t)*P_m)/P_w; % = pw if household supply can ...
    meet the demand, > pw if PwD>PwS, <pw if PwD=0

end
```

The following is the Matlab-file `Pflows` for the respective function.

```matlab
function [P]=Pflows(quantities,P_m,P_w,P_a,P_n,P_wMaxSupply)
% P flows for period t
% P = transpose([P_m,P_w,P_a,P_n,G,Pg,Pgc,Pga,Pac,Ph,Pgl,Pwl,Pwd,P_wMaxSupply])
% quantities=transpose([cgD,cgc,cga,ca,Lg,La,LD,GD]);

global P1bar zeta_g zeta_ac wl A

Pg = zeta_g*P1bar;
Pgc=quantities(2)/quantities(1)*Pg;
Pga=quantities(3)/quantities(1)*Pg;
Pac=zeta_ac*Pga;
Ph=zeta_hg*quantities(2)+zeta_ha*quantities(4); %P_wMaxSupply
Pgl=l*(P_m+P_w+P_a+P_n);
Pwl=wl(A)*Ph;
```

The following is the Matlab-file `marketEQfinal` for the respective function.

```matlab
function [quantities,prices,P,objectives,E,iter]=
marketEQfinal(price0,G,Pibar,pw,pm,P_wMaxSupply,P_g,P_n,Eold,t,parametersLit,
parametersCalibration)
% solve for market equilibria prices and quantities in period t
% iter is the number of iterations
%     quantities=transpose([cgc,cga,ca,G,Lg,La,L]);
%     prices=transpose([pgc,pga,pa,pG,w,pm,pw]);
%     P=transpose([P_m,P_w,P_a,P_n,P_ga,P_gc,P_gl,P_ac,P_h,P_wl,P_wd]);
% global alpha_c gamma_c k alpha_g beta_g alpha_a beta_a psi phi_gc phi_ga epsilon ...
    dumpingfactor %tol
% global chi_w chi_m  l wl Δ_e sozPl0 A %chi_n chi_a
% global zeta_hg zeta_ha zeta_a  zeta_ac % zeta_g

```

```
% Parameter values
% — Calibration
alpha_c = parametersCalibration(1);
gamma_c = parametersCalibration(2);
epsilon = parametersCalibration(3);
alpha_g = parametersCalibration(4);
beta_g = parametersCalibration(5);
alpha_a = parametersCalibration(6);
beta_a = parametersCalibration(7);
Δ_e = parametersCalibration(8);
phi_gc = parametersCalibration(9);
phi_ga = parametersCalibration(10);
% Literature/ model based
psi = parametersLit(1);
chi_w = parametersLit(2);
chi_m  = parametersLit(3);
l = parametersLit(4);
wl = parametersLit(5);
zeta_hg = parametersLit(6);
zeta_ha = parametersLit(7);
zeta_a  = parametersLit(8);
zeta_ac= parametersLit(9);
sozPl0 = parametersLit(10);
k = parametersLit(11);
A = parametersLit(12);
dumpingfactor= parametersLit(13);
P1bar = parametersLit(14);

% 0) algorithm values
N=1000; eps = 1.e—5; %define max no. iterations and error
maxval = 50; %define max value of divergence

% 1) Initialize
pgc= price0(1); % 4.03 not 1.2
pga = price0(2); % 0.29 not 0.1;
pa= price0(3); % 4.87 not 2.2
w = price0(5); % 2.78 not 1;
L = 1;

```

```
while N>0

% 2) from cga and L market derive pga and w (2 Var, 2 Eq.)
[pga,w,¬,¬,¬] = secant2D(pga,w,pa,pgc,Pibar,t,parametersLit, ...
    parametersCalibration); %secant2D(pga0,w0,pa,pgc,Pibar)

% 3) derive pG from GD=G
Lg = (beta_g*Pibar^alpha_g*(pgc*phi_gc+pga*phi_ga)/w)^(1/(1—beta_g));
pG = alpha_g*(pgc*phi_gc+pga*phi_ga)^(1/(1—beta_g))
*(beta_g*Pibar^alpha_g/w)^(beta_g/(1—beta_g))*Pibar^(alpha_g—1);
% old wrong version: Pibar^(alpha_g—1)*alpha_g*(pgc*phi_gc+pga*phi_ga)*Lg^beta_g;

% 4) derive Pm, Pw from GS=G
P_w = (1—(chi_m—1)*pG/pm)*G/(chi_w—1—(chi_m—1)*pw/pm);
   % check parameter constraints
if P_w < 0
    P_w = 0;
    fprintf('Pw = 0 in period %2.0f ;',t); %——> change parameters for P_w >0
end
if P_w > P_wMaxSupply   %household supply can not meet the demand
    P_w = P_wMaxSupply;
    fprintf('Pw demand > supply in period %2.0f ;',t);
    %——> change parameters for P_w even larger
end
   % derive Pm
P_m = (pG*G—pw*P_w)/pm;

% 5) tatonnement algorithm for pa and pgc
cgcS = phi_gc*Pibar^alpha_g*Lg^beta_g;
cgaS = phi_ga*Pibar^alpha_g*Lg^beta_g;
cgaD = cgaS; %market equilibrium was ensured in step 2
La = (psi^alpha_a*alpha_a^alpha_a*beta_a^(1—alpha_a)
*pa/pga^alpha_a/w^(1—alpha_a))^(1/(1—alpha_a—beta_a));
caS = (psi*cgaD)^alpha_a*La^beta_a;
pi_g = pgc*cgcS+pga*cgaS—w*Lg—pG*G;
pi_a = pa*caS—pga*cgaD—w*La;
cgcD = calc_cgcD(pgc,pG,w,pi_g,pi_a,cgaD,G,Eold,P_m,P_w,P_n,parametersLit, ...
    parametersCalibration);
% formerly (w*L+pG*G+pi_g+pi_a—caS*(pa+k))/(pgc+k);
```

```
P_gc = cgcS/(cgaS+cgcS)*P_g;
P_ga = cgaS/(cgaS+cgcS)*P_g;
%P_a(t) = zeta_a*P_ga(t-1); %P_a is actually P_ag, same for zeta_a % from previous ...
    period
P_ac = zeta_ac*P_ga;
P_h = zeta_hg*P_gc+zeta_ha*P_ac;
P_wl = wl*P_h;
P_a = zeta_a*P_ga; %P_a = zeta_a*P_ga; %P_a is actually P_ag, same for zeta_a % ...
    from same period
P_gl = l*(P_w+P_m+sozPl0*P_a+P_n);
E = Δ_e*Eold - P_gl - P_wl;
Estrich = l*zeta_a*cgaS/(cgaS+cgcS)^2*P_g-wl*zeta_hg*cgaS/(cgaS+cgcS)^2
*P_g+wl*zeta_ha*zeta_a*cgaS/(cgcS+cgaS)*P_g;
caD1 = alpha_c*(pgc+k)/(pa+k)/(gamma_c/cgcS+epsilon*Estrich/E);
caD = (w*L+pG*G+pi_g+pi_a-cgcD*(pgc+k))/(pa+k);

pa = pa- dumpingfactor * (caD-caS);
pgc = pgc - dumpingfactor * (cgcD-cgcS);

% 6) stopping criteria
 if abs(caD-caS)+abs(cgcD-cgcS)<eps
      iter=100-N; return;
 end
 if abs(caD-caS)+abs(cgcD-cgcS)>maxval
      iter=100-N; disp(['iterations = ',num2str(iter)]);
      error('Market EQ diverges in period %2.0f ;',t);abort;
 end
 N=N-1;
 iter = 100-N;
end

% 7) rest of calculations
cgc = cgcS;
cga = cgaS;
ca = caS;
u = gamma_c*log(cgc)+alpha_c*log(ca)+epsilon*log(E);
P_wd = P_h-P_w-P_wl;

% 8) output
quantities=transpose([cgc,cga,ca,G,Lg,La,L]);
prices=transpose([pgc,pga,pa,pG,w,pm,pw]);
P=transpose([P_m,P_w,P_a,P_n,P_ga,P_gc,P_gl,P_ac,P_h,P_wl,P_wd]);
objectives=transpose([pi_g,pi_a,u]);
%E;

end
```

The following is the Matlab-file `marketEQEnv` for the respective function.

```
function [quantities,prices,P,diff]=
marketEQEnv(price0,Gmax,pw,pm,P_wMaxSupply,P_g,P_n,P_a,E,t)
% solve for market equilibria prices and quantities in period t
% iter is the number of iterations
global alpha_c gamma_c k alpha_g beta_g alpha_a beta_a psi phi_g tol epsilon ...
    dumpingfactor
global chi_w chi_m  l wl Δ_e sozPl0 A %chi_n chi_a
global zeta_hg zeta_ha zeta_a %zeta_g zeta_ac

% 1) Initialize
pg=price0(1);
w=price0(2);
Pi=0;
diff=100;
iter=0;

%while diff>tol
for iter=1:500
% 2) pG from the Fertilizers market clearing condition with GS = Gmax;
pG  = (phi_g.*beta_g^beta_g.*pg./Gmax.^(1-alpha_g-beta_g)./w^beta_g).^(1/(1-beta_g))
.*alpha_g;
GD  = (beta_g./w).^(beta_g/(1-alpha_g-beta_g)).*(alpha_g./pG).^((1-beta_g)
./(1-alpha_g-beta_g)).*(pg.*phi_g).^(1/(1-alpha_g-beta_g));
%=Gmax after market clearing

% 3) P_w from P_w market
P_w = (1-(chi_m-1)*pG/pm)*GD/(chi_w(A)-1-(chi_m-1)*pw/pm);
   % 3a) check parameter constraints:
```

```
if P_w < 0
    P_w = 0;
    fprintf('Pw = 0 in period %2.0f ;',t); %--> change parameters for P_w >0
end
if P_w > P_wMaxSupply   %household supply can not meet the demand
    P_w = P_wMaxSupply;
    fprintf('Pw demand > supply in period %2.0f ;',t);
    %--> change parameters for P_w even larger
end

% 4) P_m from P_m market
P_m = (pG*GD-pw*P_w)/pm;
% pwD = (pG*GD-pm*P_m)/P_w;
% = pw if household supply can meet the demand, > pw if PwD>PwS, <pw if PwD=0
% if pw ≠ pwD
%     fprintf('pw changed in period %2.0f ;',t);
% end
% pw=pwD;

% 5) pa from the Animal sector market
% Secant method to solve nonlinear equation with implicit pa
paIter=[1.2*pg; pg]; Nullfct=100;
while Nullfct > tol
    % all variables ...Iter are 2x1 vectors
   LaIter = (alpha_a^alpha_a.*beta_a.^(1-alpha_a).*psi^alpha_a.*paIter./pg.^alpha_a
   ./w.^(1-alpha_a)).^(1./(1-alpha_a-beta_a));
   cgaIter = alpha_a./beta_a.*w./pg.*LaIter;
   caSIter = (psi.*cgaIter).^alpha_a.*LaIter.^beta_a; % caS

   cgcIter = (ones(2,1).*(w+pG*Gmax+Pi)-(paIter+k).*caSIter)./(pg+k);
   %expressed in terms of ca from the household budget constraint to avoid E
   E_nexttIter = ones(2,1).*(Δ_e*E-l*(P_m+P_w+P_n+P_a)) - wl(A)
   .*(zeta_hg .*cgcIter ./(cgcIter+cgaIter).* P_g + zeta_ha.*zeta_a.
   * cgaIter ./ (cgcIter+cgaIter) .*P_g);
   E_strichIter = sozPl0.*l.*P_a./(cgcIter+cgaIter)-wl(A).*zeta_hg
   .*cgaIter./(cgaIter+cgcIter).^2.*P_g+wl(A).*zeta_ha.*zeta_a.*cgaIter
   ./(cgaIter+cgcIter).*P_g;
   caDIter  = alpha_c.*(pg+k)./(paIter+k)./(gamma_c./cgcIter+epsilon
   .*E_strichIter./E_nexttIter);
```

```

   pa = paIter(1)- (caDIter(1)-caSIter(1))./((caDIter(1)-caSIter(1))
   -(caDIter(2)-caSIter(2))).*(paIter(1)-paIter(2));
   Nullfct = abs(caDIter(1)-caSIter(1));
   paIter(2)=paIter(1); paIter(1)=pa;
end
ca = (caSIter(1)+caDIter(1))/2;

% Tatonnement algorithm
% - Labor market
Lg = (alpha_g^alpha_g*beta_g^(1-alpha_g)*pg
*phi_g/pG^alpha_g/w^(1-alpha_g))^(1/(1-alpha_g-beta_g));
La = (alpha_a^alpha_a*beta_a^(1-alpha_a)*psi^alpha_a
*pa/pg^alpha_a/w^(1-alpha_a))^(1/(1-alpha_a-beta_a));
LD = Lg+La;
LS = 1;

% - Grain market
cgc = (w+pG*Gmax+Pi-(pa+k)*ca)/(pg+k);
cga = alpha_a/beta_a*w/pg*La;
% only dependent on prices: cga = (beta_a^beta_a*alpha_a^(1-beta_a)*psi^alpha_a
*pa/pg^(1-beta_a)/w^(beta_a))^(1/(1-alpha_a-beta_a));
cgD = cgc+cga;
cgS = (alpha_g^alpha_g*beta_g^beta_g*pg^(alpha_g+beta_g)
*phi_g/pG^alpha_g/w^beta_g)^(1/(1-alpha_g-beta_g));

pg = pg+dumpingfactor*(cgD-cgS);
w = w+dumpingfactor*(LD-LS);

% total error
diff=abs(LD-LS)+abs(cgD-cgS);

% resulting quantities and profits for the next iteration
pi_g = pg*cgS - pG*Gmax - w*Lg;
pi_a = pa*ca  - pg*cga - w*La;
pi_G = pG*Gmax - pw*P_w-pm*P_m;
Pi = pi_g + pi_a + pi_G;

%iter=iter+1;
end

quantities=transpose([cgD,cgc,cga,ca,Lg,La,LD,GD]);
prices=transpose([pg,pa,w,pG,pw,pm]);
P=transpose([P_w,P_m,P_a,P_n]);

end
```

Appendix E. Variables and parameters

The agents in the model (agricultural farmers, animal husbandry, households) will optimally choose every year what we call *decision variable*. Based on that decisions we obtain *endogenous variables*. Parameters and initial values are chosen according to existing literature or calibrated. For the calibration we use data describing the *decision variables* and *endogenous variables* based on Austrian data (scaled to 1.000.000 inhabitants to be nourished) from the years around 2010 stated in Wien-Statistik (2010) and Zoboli et al. (2016a). Most parameters could be derived explicitly from the first order conditions described in Appendix A, the rest has been derived with a calibration algorithm, i.e. minimum mean square error, to reflect the data.

An overview of variables and parameters is given in Tables E.1 and E.2, and Tables E.3, E.4 and E.5, respectively.

Table E.1
Decision variables of the model.

Symbol	Description	Unit
$c_g(t)$	total production of grain	[t/a/cap]
$c_g^c(t)$	household consumption of grain	[t/a/cap]
$c_g^a(t)$	fodder consumption	[t/a/cap]
$c_a^c(t)$	consumption of animal products	[t/a/cap]
$G(t)$	fertilizer mix	[t/a/cap]
$L_g(t)$	labor in grain production	cap
$L_a(t)$	labor in animal husbandry	cap
$P_w(t)$	recycled phosphorus-fertilizer from the waste water treatment plant	[t/a/cap]
$P_m(t)$	mineral fertilizer	[t/a/cap]

Table E.2
Endogenous variables of the model.

Symbol	Description	Unit	Initial value	Source
$E(t)$	quality of environment (amount of phosphorus in the water bodies)	[]	100,000	
$x(t)$	other goods	[t/a/cap]		
$p_g(t)$	price for grain	EUR/t		
$p_g^c(t)$	price for grain household products	EUR/t		
$p_g^a(t)$	price for grain fodder	EUR/t		
$p_a(t)$	price for animal products	EUR/t		
$p_G(t)$	price for fertilizer mix	EUR/t		
$p_x(t)$	price for other goods	EUR/t		
$w(t)$	wages	EUR/cap		
$\pi_g(t)$	profit in grain production	EUR/a		
$\pi_a(t)$	profit in animal husbandry	EUR/a		
$\pi_G(t)$	profit of trader	EUR/a		
$\pi_w(t)$	profit of WWTP	EUR/a		
$\pi_x(t)$	profit in other sectors	EUR/a		
$u(t)$	household utility	[]		
$P_n(t)$	"natural" phosphorus stock in the soil	[t/a/cap]	0	Zoboli et al. (2016a)
$P_g^c(t)$	phosphorus in the consumed grain products	[t/a/cap]		
$P_g^a(t)$	phosphorus in the grain fodder for the animals	[t/a/cap]		
$P_g^\ell(t)$	losses of phosphorus (runoff from the grain field)	[t/a/cap]		
$P_a^g(t)$	phosphorus in the animal manure applied to grain fields	[t/a/cap]	2100	Zoboli et al. (2016a)
$P_a^c(t)$	phosphorus in the consumed animal products	[t/a/cap]		
$P_h(t)$	phosphorus within the waste water from the households	[t/a/cap]	600	Zoboli et al. (2016a)
$P_w^\ell(t)$	losses of phosphorus from the WWTP	[t/a/cap]		
$P_w^\ell(t)$	losses of phosphorus from the WWTP	[t/a/cap]		
$P_w^{maxsupply}(t)$	maximum supply of P from WWTP	[t/a/cap]		
σ	amount of overfertilization	[t/a/cap]		
ϕ_g^c	grain yield fraction for household products	[]	0.75	Wien-Statistik (2010)
ϕ_g^a	grain yield fraction for fodder products	[]	0.25	Wien-Statistik (2010)

Table E.3
Parameters of the model. Note, sources provide direct data or are basis for calibration*.

Symbol	Description	Unit	Value	Source
t	time	a		
T	time horizon	a	15	
L_x	labor in other sectors	cap	999,000	Wien-Statistik (2010)
L	total labor	cap	1000,000	
p_m	price for mineral fertilizer	EUR/t	2040	Egle et al. (2016)
p_m	price for mineral fertilizer after price increase	EUR/t	3500	Egle et al. (2016)
γ_c	grain consumption preference in the utility function $u(t)$	[0,1]	1	
α_c	meat consumption preference in the utility function $u(t)$	[0,1]	1	
ϵ	environmental quality preference in the utility function $u(t)$	[0,1]	0.1	
α_g	output elasticity of fertilizer in grain production	[0,1]	0.25	Wien-Statistik (2010)*
α_a	output elasticity of fodder in animal husbandry	[0,1]	0.25	Wien-Statistik (2010)*
β_g	output elasticity of labor in grain production	[0,1]	0.60	Wien-Statistik (2010)*
β_a	output elasticity of labor in animal husbandry	[0,1]	0.55	Wien-Statistik (2010)*
ϕ_g	total factor productivity for grain	[]	7	Wien-Statistik (2010)*
ϕ_a	total factor productivity for animal husbandry	[]	2.9	Wien-Statistik (2010)*
ϕ_L	labor efficiency	[]	29	Wien-Statistik (2010)*

Table E.4
Parameters of the model.

Symbol	Description	Unit	Value	Source
$\bar{P}$	maximum phosphorus intake of crops	[t/a/cap]	3150	Zoboli et al. (2016a)
j	phosphorus source	$\{w, m, a^g, n\}$		
χ_a	plant availability of phosphorus from manure	[0,1]	0.8	Zoboli et al. (2016a)
χ_n	plant availability of phosphorus from the natural stock in the soil	[0,1]	0.4	Zoboli et al. (2016a)
$\chi_w(A)$	plant availability of recycled phosphorus (see Table E.5)	[0,1]	[0.6, 0.85, 0.85, 1]	Egle et al. (2014)
χ_m	plant availability of mineral fertilizer	[0,1]	1	Egle et al. (2014)
ψ	tons of animal product from one ton grain (inverse FCR)		0.25	Zoboli et al. (2016a)
ζ_a^g	P in the manure of animals as proportion of P in fodder	[0,1]	0.75	Zoboli et al. (2016a)
ζ_a^c	P in the animal products as proportion of P in fodder	[0,1]	0.15	Zoboli et al. (2016a)
ζ_{hg}	P in the households waste water as proportion of P in consumed grain	[0,1]	0.7	Zoboli et al. (2016a)
ζ_{ha}	P in the households waste water as proportion P in consumed animal products	[0,1]	0.7	Zoboli et al. (2016a)
ℓ	proportion of phosphorus run off from the field	[0,1]	0.005	Zoboli et al. (2016a)
δ	regeneration rate of the environment >1		1.001	
w_g^ℓ	loss fraction of grain food waste	[0,1]	0.1	Zoboli et al. (2016a)
w_a^ℓ	loss fraction of animal food waste	[0,1]	0.1	Zoboli et al. (2016a)
w_ℓ	P run-off from waste water into water bodies	[0,1]	0.1	Zoboli et al. (2016a)

Table E.5
Characteristics of the P recovery technologies based on Egle et al. (2014) and expert judgment.

A	technology	$\chi_w(A)$	$p_w^f(A)$	$\tau(A)$	$p_w(A)$	$I(A)$	$V(A)$
		[0,1]	[]	[0,1]	EUR/t	EUR/a	EUR/a
1	direct application	0.60	3	1.00	10	0	0
2	Ostara Pearl Reactor®	0.85	1	0.20	100	95,411	35,446
3	Stuttgart process	0.85	1	0.45	2000	38,164	543,566
4	EcoPhos®	1.00	1	0.85	3300	38,238	392,686
	mineral fertilizer*	1.00	2	1.00	2040	–	–

* Note that mineral fertilizer is not recovered from waste water but imported from abroad.

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