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Sharing Costs Fairly

A Practical Introduction to Cooperative Game Theory

Based on Research by H. Peyton Young, N. Okada, and T. Hashimoto

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The Need for Game Theory

Three neighboring cities have inadequate water supply systems. Their forecasts of water demand in each of the areas ten years ahead tell them that something has to be done. It is clear to decision makers in each city that they must either build a joint system or pay a lot more for separate systems.

Their common objective is to provide sufficient water at the lowest cost, dividing the cost fairly among the three communities. The easiest way is to allocate costs in proportion to population, and let it go at that. But as anyone who has ever been involved in such an allocation knows, the easiest way may be far from fair.

In fact, there may be no completely equitable solution to such problems, which turn up in great numbers of cases involving publicly regulated enterprises. What decision makers need is not the *correct* answer to cost allocation, which does not exist, but rather, a reasonable – *nearly fair* – answer.

To get the best solution to cost allocation problems, decision makers must deal, at least to some extent, with advanced mathematical methods such as game theory. While few officials responsible for resource development will have degrees in mathematics, they nevertheless need at least a rudimentary understanding of what these methods are, why they are used, and how they work in real applications. This Executive Report has been prepared to provide such understanding.

It is based largely on a Research Report published by the International Institute for Applied Systems Analysis (IIASA) in Laxenburg, Austria. In RR-80-32: Cost Allocation in Water Resources Development – A Case Study of Sweden, IIASA scientists H. Peyton Young, N. Okada, and T. Hashimoto compare traditional methods of allocating costs with more recently developed methods involving cooperative game theory. Their purpose is to show how these methods work in practice.

In preparing their report, the authors, who are from the US and Japan, drew upon information and suggestions provided by scientists from several other countries, including IIASA scientists J. Kindler of Poland and L. de Maré of Sweden, and collaborating scientists O. Menshikova and I. Menshikov of the USSR. Such far-ranging international collaboration on problems of mutual concern is typical of the research done at IIASA.

An actual water resource development project in the Skåne region of southern Sweden was chosen for study. Its cost allocation problems are common ones, and they have been well documented over the last decade.

After applying several currently popular game theory methods of cost allocation to the water project in Sweden, the three authors of the IIASA Research Report felt that these methods do indeed have many significant advantages over naive methods. However, they also noted that, in actual applications, it may be unrealistic to expect the kind of information and technical understanding that game theory requires.

They concluded that, as a practical matter, there are cases when using a single numerical criterion for costing, such as apportioning costs by population or by demand, may be preferable to more complicated methods.

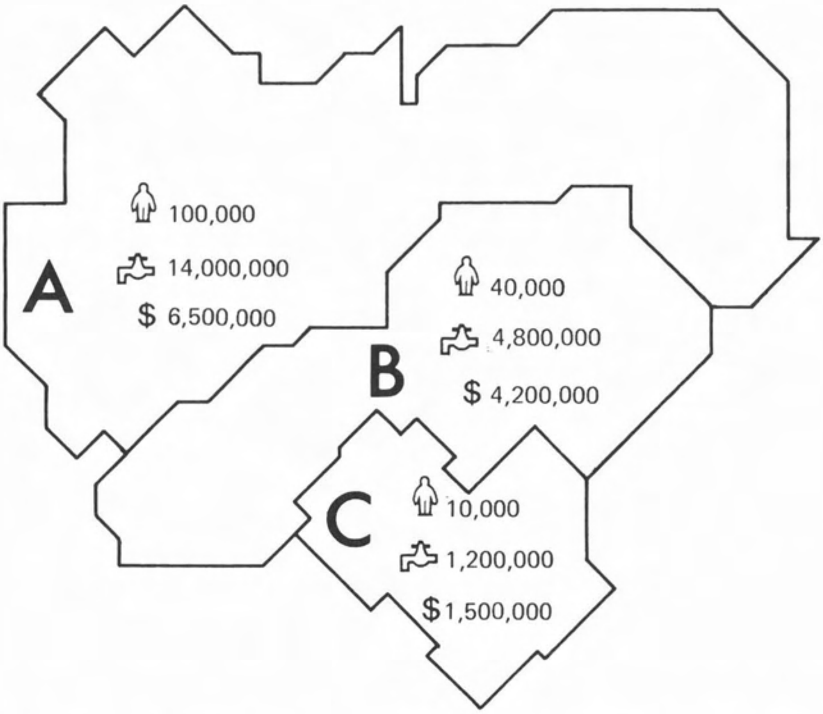
WHY THE PROBLEM IS COMPLEX

The three hypothetical cities mentioned at the outset illustrate the complexity of the problem. City A can build its own separate water facility for \$6.5 million. City B, which is smaller, can build for \$4.2 million, and City C, smaller yet, can build for \$1.5 million. The total cost of building three separate water facilities is \$12.2 million.

The economies of scale are readily apparent from the table at the beginning of this report. A joint waterworks serving all three cities would cost \$10.6 million, or \$1.6 million less than the cost of building three separate facilities. A three-city facility is also cheaper than any combination of a two-city facility and a separate facility.

The decision makers concerned with water resource development in the three cities are in complete agreement. A three-city facility is best. But a fair allocation of the cost turns out to be a serious problem.

WATER COSTS OF THREE CITIES



The population, water use (cubic meters per year), and cost of building separate water facilities are shown for three adjacent cities. In a typical program, the annual water use would be a forecast of needs several years ahead, when new separate or joint facilities would be completed.

One way is to allocate in proportion to population. City A has 100 000 inhabitants, B has 40,000, and C has 10,000. To build a $10.6 million joint waterworks using this method, City A would pay $7.1 million, City B $2.8 million, and City C $700,000.

The solution is simple – but not fair. City A would pay $7.1 million to join with the others, and yet it could build its own separate facility (and have complete control over it) for only $6.5 million. At the same time both B and C would be getting a big break, with City C paying less than half of what a separate facility would cost it.

Allocating costs on the basis of use gives even worse results. When the project is completed, City A will be using 14 million cubic meters of water a year (at a greater rate per capita than the other two cities because of its greater commercial and industrial development). City B will use 4.8 million cubic meters a year, and City C will use 1.2 million cubic meters a year.

Allocating the \$10.6 million according to anticipated water demand would make City A's water bill \$7.4 million, City B's \$2.6 million, and City C's \$600,000. This way, City A pays an even greater penalty for joining the group, while B and C can pay even less.

TWO ALLOCATION PRINCIPLES

All proportional allocations are as unsatisfactory as the two described above because they ignore lower-cost alternatives available to entities and to coalitions of entities smaller than the whole coalition. To be fair, allocation in the example must provide individual rationality (no city paying more than it would alone) and group rationality (no possible subgrouping of cities paying more than it would as a group). This is the principle of *rationality*.

Individual rationality makes it unfair to charge City A more than \$6.5 million for its share in the joint waterworks because that is how much it would cost A to build a separate water facility. Group rationality makes it unfair to charge City A plus City B more than \$10.3 million, the cost to these two cities of building jointly and leaving out City C.

Note that fairness as applied by the principle of rationality is more than an ideal. Any of the three cities would be tempted to back out of a three-city project costing that city more than it would have to pay alone or in a two-city project.

A second principle of allocation is that no entity in a joint project should pay less than the *marginal cost* of including it. Cities A, B, and C can be served for \$10.6 million, but A and B can be served, excluding C, by a two-city facility for \$10.3 million. Therefore, C should pay at least \$300,000, the marginal (or separable) cost of serving it. Similarly, A should pay not less than its marginal cost of \$5.3 million, B not less than \$2.6 million.

Like rationality, marginal cost extends to all possible combinations. Every group of entities should be charged at least the additional cost of serving it; no group should subsidize any other group. For example, the marginal cost of including the group AB in the group ABC is \$9.1 million (\$10.6 million minus \$1.5 million).

- *Rationality* requires that no entity (such as a city) or group of entities be charged more in the total coalition than it would pay on its own.
- *Marginal cost* requires that no entity or group of entities be charged less than the cost of including it in the total coalition.

In the mathematical formulations of game theory, both of these requirements define the same set of inequalities. When all costs have been allocated, the two objectives are equivalent.

FIVE ALLOCATION METHODS

Five recently developed techniques of cost allocation were evaluated by the IIASA scientists in relation to an actual water development project in Sweden. They are described below in brief, general terms. All of them derive from *cooperative game theory*, which is based on mathematical formulas that are too technical to serve the purposes of this Executive Report.

In game theory, difficult problems are seen as games to be played. To avoid confusion, individuals, groups, and sometimes even coalitions of groups are referred to as "players" when they act together.

The term "game theory" refers to the determination of how a game should or would be played under certain circumstances. (The term "gaming" refers to the design of games and experimentation to see how subjects actually play such games.) In "cooperative" game theory, the players are assumed to be able to form coalitions for their mutual benefit. In other situations, each player may act competitively in his own interest, as an example in Chapter 3 will illustrate.

In game theory, the principles of rationality and marginal cost can be expressed in a set of inequalities. The *core* of the game defines a restricted set of solutions to the set of inequalities. The inequalities incorporate all the limits imposed by both principles, so that unfairness can be identified, manipulated, and reduced.

The core can be seen as a set of guidelines for cost allocation that narrows the choice of acceptable possibilities. As in the three-city waterworks example, the core usually identifies a range of costs, not a unique answer. In many cases there may be no answer at all, because no cost allocations can be made to satisfy rationality or marginal cost. This may happen in projects where the rate of cost savings begins to decrease as the size of the project increases beyond some point.

When there is no unique set of costs, one for each player, but only a range of such costs, or when there is no set of costs at all, adjustments must be made. One of the most common approaches in game theory is to look for a natural way to modify the inequalities that define the core. Such modifications can narrow the range of costs or produce one where none existed.

The first three of the cost allocation methods outlined below modify the core.

The nucleolus. When the core is "empty," so that no cost allocations meet the tests of rationality and marginal cost, it is because the best cost alternatives for some of the players are too good compared with the best alternatives for the whole group of players. In this case, one approach is to tax all combinations of separate and joint projects except a joint project involving the whole group. The idea is to impose the smallest uniform tax that will make a whole-group project advantageous to all players.

When the core is not empty, it is usually too full. This means that it gives a range of answers. In this case, the choice can be narrowed by the opposite technique – subsidizing all combinations other than the whole group by a uniform amount. If this still results in several answers, the choice can be narrowed further to a single answer, called the nucleolus, by an extension of the same reasoning.

The weak nucleolus. This method imposes a minimum uniform tax on any individual if he takes any course of action other than joining the whole group of players. This makes the whole-group project advantageous to all, but under a different set of advantages and drawbacks from those of the nucleolus method. Individual users can also be subsidized to narrow alternatives further, and this too produces results that differ from uniformly subsidizing coalitions.

The proportional nucleolus. This variation on the theme modifies the core by putting a minimum tax or subsidy on players in proportion to how much they save. For example, neither City A nor City C can do any better by building a two-city water facility (AC) than it can by going it alone (because they lack a common border). But both of them can save money in a two-city or three-city coalition involving City B. Subsidizing B will narrow the range of admissible costs for the whole group, ABC.

The Shapley value. This allocation method was suggested by L.S. Shapley in 1953, and it still offers a reasonable and relatively simple answer to some costing problems. Rather than modifying the core, this method assumes that each player has joined the common group in some identifiable order. It also assumes that all possible orders for signing up are equally likely.

If a group of players has already signed up, the additional cost of including the next player to arrive determines his marginal cost contribution. When the marginal cost contributions of each player are determined for every possible sequence of joining and averaged, the result is the Shapley value.

SHAPLEY VALUES
(millions of dollars)

The Shapley value is each city's average marginal contribution to a three-city coalition. First, the cost of adding each city to the coalition is determined for all six possible orders of joining. Then the costs for each city are totaled and averaged. The marginal contribution figures can be derived from the information on the inside front cover of this report.

Order of joining	Marginal contribution A	B	C
ABC	6.5	3.8	0.3
ACB	6.5	2.6	1.5
BAC	6.1	4.2	0.3
BCA	5.3	4.2	1.1
CAB	6.5	2.6	1.5
CBA	5.3	3.8	1.5
Total	36.2	21.2	6.2
Shapley value	6.033	3.533	1.033

For example, if City A were the first to contemplate building a waterworks, its marginal cost contribution would come to $6.5 million. If City B then joined in, the resulting two-city facility would cost $10.3 million. City B's marginal cost contribution would then be $3.8 million ($10.3 million minus $6.5 million). Adding City C would provide a three-city facility costing $10.6 million, so in this case C's marginal cost contribution would be a modest $300,000 ($10.6 million minus $10.3 million). The method may still be useful even though two-city and three-city facilities cannot actually be built as marginal increments to smaller facilities.

As can be seen from the example, the last of the three cities to join the group fares best. But all orders of joining are considered in calculating the Shapley value. The marginal cost contributions of each of the three cities for the six possible orders (ABC, ACB, BAC, BCA, CAB, and CBA) are averaged. This allocation method assigns the following costs for a $10.6 million three-city facility: City A, $6,033,000; City B, $3,533,000; City C, $1,033,000.

The separable costs–remaining benefits (SCRB) method. This method is commonly used to allocate the costs of water development projects. It is based on the simple and therefore highly appealing idea that joint costs should be allocated, more or less, in proportion to the willingness of the user to pay, which in turn is a reflection of the benefit to the user of the proposed facilities.

First, each player is assigned the *marginal or separable cost* of including him in the project. (The computation was illustrated in the discussion of the marginal cost principle on page 4.) This means that City A will be charged $5.3 million, B $2.6 million, and C $0.3 million. However, the sum of these separable costs, $8.2 million, is not enough to cover the cost of the joint $10.6 million project; it is still necessary to find another $2.4 million. This is known as the *nonseparable cost* of the scheme.

The *justifiable cost* for a player is either the cost of going alone or the amount that he is willing to pay, whichever is lower. In this example, we assume that for each player the cost of going alone is smaller, that is, each player could be justified in building alone if necessary.

The *remaining benefit* is the justifiable cost less the marginal cost. The SCRB method proposes that the nonseparable cost be divided among the participants in proportion to their remaining benefits. For example, City A would have to pay $6.5 million for a separate facility; at the moment it is only being charged its marginal cost, $5.3 million. Thus, if only the separable costs are charged, A would benefit by $1.2 million, B would have a remaining benefit of $1.6 million, and C a remaining benefit of $1.2 million. The total remaining benefit is therefore $4 million. This means that A, which takes 1.2/4 of the total remaining benefit, should pay 1.2/4 of the nonseparable cost, or $0.72 million. This is added to its separable cost contribution of $5.3 million to give a total of $6.02 million. Similarly, B would pay $3.56 million (2.6 + 0.96) and C $1.02 million (0.3 + 0.72).

Variations of the SCRB method first allocate the separable costs and then assign the nonseparable cost in proportion to population, use, or some other criterion. The shortcoming of the

SCRB method is that the simple process of allocating costs in proportion to benefits can sometimes become unrealistic because of the need to start by allocating the separable costs. This can lead to some strange results, as will be shown in the next chapter.

Few cost allocations, in fact, are as simple and free of complicating considerations as the one undertaken by hypothetical cities A, B, and C. The next chapter suggests how the methods outlined here might apply to an actual problem of cost allocation.

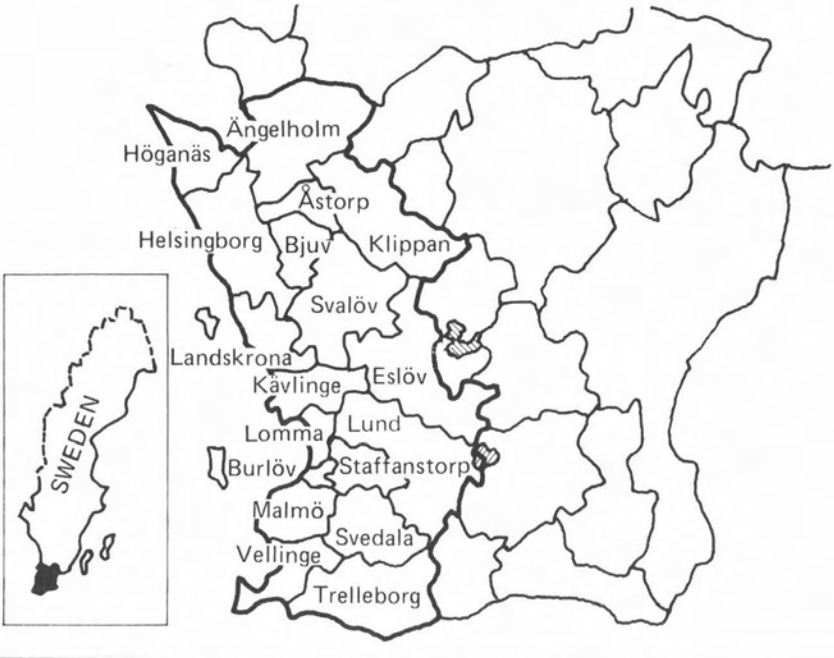
2

Testing the Methods

The basic cost allocation principles and methods described in the preceding chapter were applied to a water development project involving 18 municipalities in the Skåne region of southern Sweden. The study dealt with data for the period 1970–80 that were readily available in detail from the administrators of the project.

The year 1970 was chosen as a vantage point. A water supply

THE SKÅNE REGION OF SWEDEN
(18 municipalities)



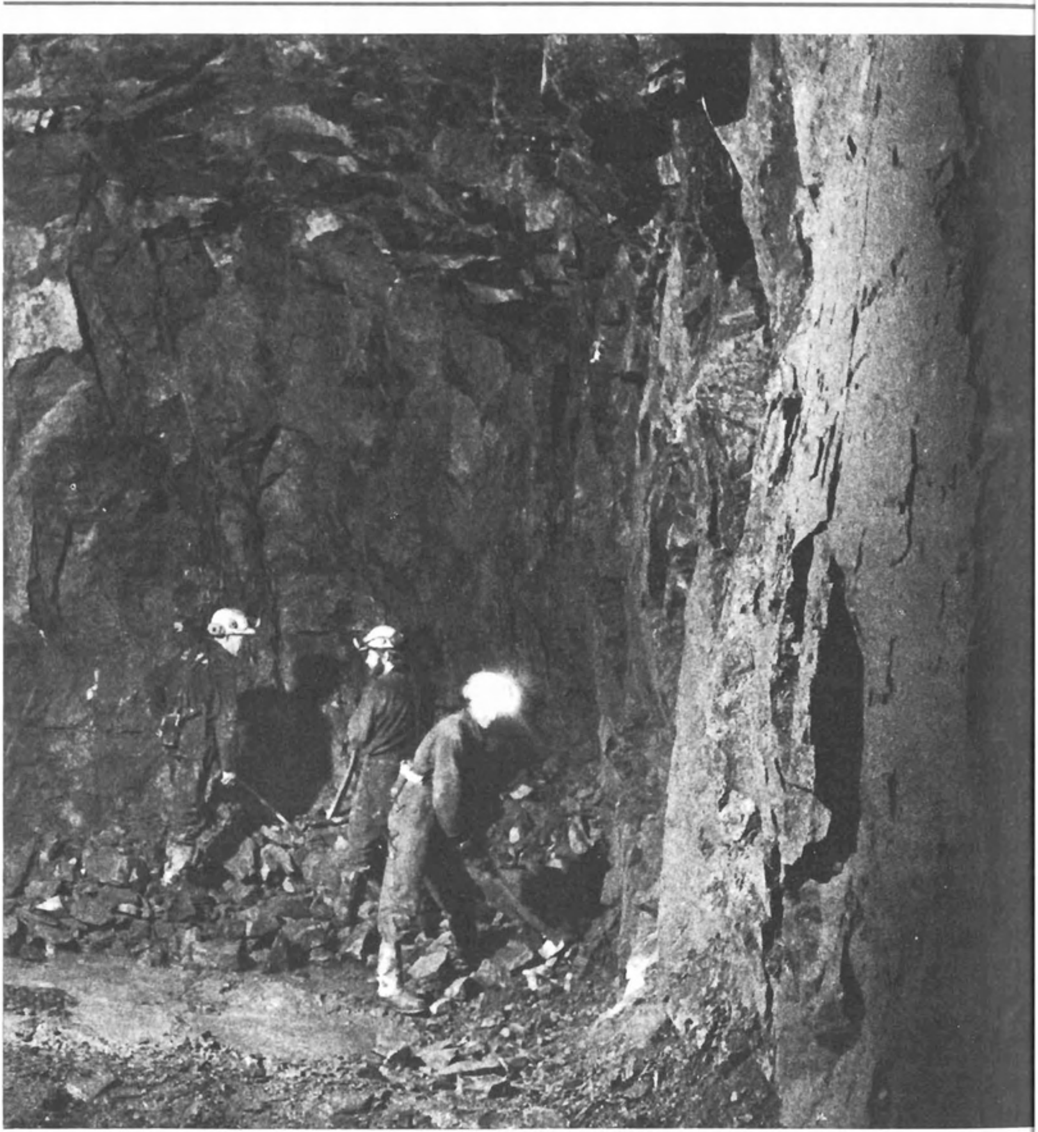
system was designed and its costs allocated by various methods to fill the needs perceived and forecast from 1970 to 1980. This made it possible to see how the municipalities would have fared under each of the allocation methods in a real-world setting. The approach also served to show how the methods work and to demonstrate their shortcomings when applied to a complex and changing cost allocation problem.

ESTABLISHING GROUPS

Most of the water now supplied to the region comes from three sources – local groundwater and two separate pipeline systems from different lakes. By the late 1940s, some of the municipalities were aware that local water resources might not be sufficient for future needs, so they began looking for off-site sources. In the 1960s several of them formed the Sydvatten (South Water) Company, an association for planning the long-term water supply of the region.

Working out a waterworks. Sydvatten Company executives charged with planning the long-term water supply of the Skåne region include (left to right) Economist Carl Erik Davidson, Managing Director Lars Reingardt, Project Manager Nils Mårtenson, and Planning Engineer Ingvar Bornmyr.

By the late 1960s, Sydvatten had begun to design a major project for obtaining water from Lake Bolmen, a source outside the region that would require a tunnel 80 kilometers long. The project has been undertaken, but it is not expected to have any impact on the area's water supply until the late 1980s.



More costs to come. By the late 1980s, a tunnel 80 kilometers long will bring water from Lake Bolmen to the Skåne region. The photograph (by courtesy of Scandia Photopress) shows workers excavating a section of the tunnel.

From the outset it was clear to the Swedish planners that the success of the Lake Bolmen project would depend on the number of municipalities joining in. That, in turn, would depend on the cost to each community compared with the cost of developing its own on-site water resources.

Cost allocation for the project was originally based on population, and it remains so today. But unforeseen changes have occurred since 1970, such as greatly escalated costs, improved estimates of local resources that are more optimistic, and rates of growth in demand that are lower than expected. As a result, the population-based cost allocation has been brought into question. The decision makers involved have been open to the possibility of trying a different approach.

To deal with cost allocation in tangible terms, the IIASA team confined its study to local supply possibilities during a 10-year period – excluding new off-site sources such as Lake Bolmen. The supply options include expanding the two pipeline systems and making greater use of local groundwater sources.

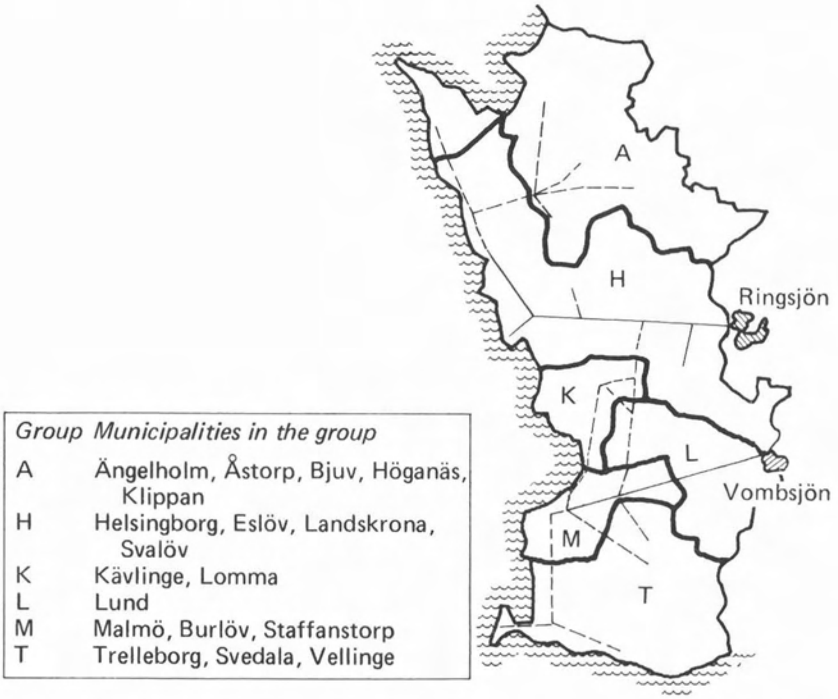
The first problem they encountered in establishing a more equitable cost allocation method was to identify the groups of users (players) in the total water demand system. It would be impractical to develop costs for each of the 262,143 possible groupings of the 18 municipalities.

The municipalities were found to form natural groups based on past associations, geographical locations, existing water transmission systems, and hydrological and geographical features determining the natural routes for water transmission networks.

After careful study, the municipalities were grouped into six independent units. Group H, for example, consists of three municipalities already connected by the Ringsjön water supply system in 1970, together with Svalöv, which would be in the middle of the transmission route serving the other three municipalities in a joint project.

The six groups were then treated as players in an analysis of alternative cost allocations. After an allocation based on the six groups has been determined, an allocation among the municipalities within each group can be made using a similar method. However, this would not necessarily give the same results as would treating all 18 municipalities separately.

SIX GROUPS OF MUNICIPALITIES



Group	*Municipalities in the group*
A	Ängelholm, Åstorp, Bjuv, Höganäs, Klippan
H	Helsingborg, Eslöv, Landskrona, Svalöv
K	Kävlinge, Lomma
L	Lund
M	Malmö, Burlöv, Staffanstorp
T	Trelleborg, Svedala, Vellinge

To test cost allocation methods in an actual water development project, the 18 municipalities of the Skåne region were combined in six groups. Full lines indicate the existing water supply network; broken lines show where the network will be extended when a 10-year water development project is finished. Shaded areas indicate water; the two lakes serving as major water sources for the region are to the east of the region.

DIRECT AND INDIRECT COSTS

Combining municipalities into groups poses a cost allocation problem that does not appear in the simplified example used in Chapter 1. Such groupings have *direct* costs, incurred by each municipality in the group regardless of the water supply system chosen by the group. Costs that depend on whether the group builds a separate facility or joins various coalitions are *indirect* costs to each municipality in the group. The problem is that the distinction between direct and indirect costs can be difficult to make.

For example, water delivery costs within each municipality are direct costs, and theoretically they should be allocated to each municipality independently of the six-group allocation. But in practice such costs are hard to distinguish.

Water delivered by the joint supply network must be pumped to a reservoir for distribution within the municipality. The higher the pressure when the water reaches the municipality, the less the cost of the pumping equipment. Therefore, the costs of distribution within the six groups, which have from one to five municipalities in them, depend on the source chosen within the water supply system.

The water supply system includes two lakes (Ringsjön and Vombsjön), one major groundwater aquifer (Alnarp), and minor on-site sources. The possible routes of a water transmission network (based on a preliminary analysis) are shown on the map on page 15.

The definition of costs is always somewhat arbitrary, regardless of the allocation method, so a method that ignores direct costs is best. A weakness of the SCRB method is that, in basing the calculation on willingness to pay, it must establish direct costs that are hard to define, such as local pumping costs.

To make costs consistent for each method tested, so that SCRB could be compared with the other methods, water pressure was assumed to be the same at each demand point regardless of source. In this way, the cost of distributing water within each community does not depend on source, so it is a direct cost, excluded from the calculation of costs for each of the six groups.

Water delivered to each municipality was assumed to be of uniform quality. To attain this quality, water is treated at the source, so costs of treatment were considered indirect costs and included in the calculation of group costs.

The calculation of total cost for each group was based on mathematical formulas. This provided estimates of the least-cost combination of alternative supply sources for each group separately and for all possible coalitions, including a coalition of all six groups.

Comparing the total cost estimates shows the relative strength of each group, which depends on the cost and availability of local resources and access to the resources of others. For example, Group L finds the cost of going it alone high, despite being close to the major regional sources, Ringsjön and Vombsjön, neither of which it owns. So L has a strong incentive to join with H and M.

Groups H and M have the lowest unit costs because they own the two major supply systems. And they can make their unit costs still lower by joining with other groups, thanks to the economies

THE COSTS OF COALITIONS
(millions of kronor)

The figures were determined by mathematical formulas that expressed costs in terms of availability of local water resources and least-cost alternative access to the resources of others in the region. Commas signify that the least-cost option of a coalition is to break up into the subcoalitions indicated.

Group	Total cost	Group	Total cost	Group	Total cost
A	21.95	AHK	40.74	AHKL	48.95
H	17.08	AHL	43.22	AHKM	60.25
K	10.91	AH,M	55.50	AHK,T	62.72
L	15.88	AH,T	56.67	AHL,M	64.03
M	20.81	A,K,L	48.74	AHL,T	65.20
T	21.98	A,KM	53.40	AH,MT	74.10
		A,K,T	54.84	A,K,LM	63.96
AH	34.69	A,LM	53.05	A,K,L,T	70.72
A,K	32.86	A,L,T	59.81	A,K,MT	72.27
A,L	37.83	A,MT	61.36	A,LMT	73.41
A,M	42.76	HKL	27.26	HKL,M	48.07
A,T	43.93	HKM	42.55	HKL,T	49.24
HK	22.96	HK,T	44.94	HKMT	59.35
HL	25.00	HL,M	45.81	HLMT	64.41
H,M	37.89	HL,T	46.98	KLMT	56.61
H,T	39.06	H,MT	56.49	AHKL,T	70.93
K,L	26.79	K,LM	42.01	AHKLM	69.76
KM	31.45	K,L,T	48.77	AHKMT	77.42
K,T	32.89	K,MT	50.32	AHLMT	83.00
LM	31.10	LMT	51.46	AKLMT	73.97
L,T	37.86			HKLMT	66.46
MT	39.41				
				AHKLMT	83.82

of scale. But H can offer potential partners lower costs than M can, because its source, Ringsjön, has greater excess capacity than Vombsjön, which belongs to M. The higher incremental cost of joining with M must be reflected in the final cost allocation.

COMPARING THE METHODS

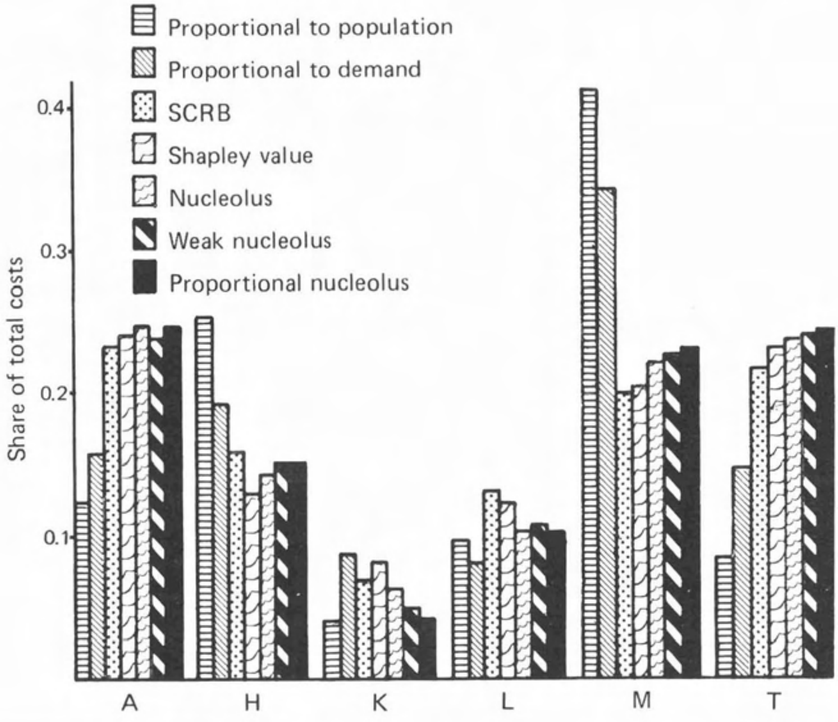
Costs for the six groups and their various combinations were allocated by population, use, and the five methods outlined in Chapter 1. The study assumes that all groups participate in a project at a total cost of Skr 83.82 million. The costs for each group according to each allocation method are shown on page 18.

SEVEN ALLOCATIONS AT A GLANCE
(millions of kronor)

The figures show how costs would be allocated to the six groups of Swedish municipalities by each of the allocation methods outlined in Chapter 1. For further comparison, the individual rationality costs of building separate facilities and the marginal costs are shown below the allocations.

The bar chart illustrates the extent to which the various methods of allocation produce contrasting results.

Allocation method	A	H	K	L	M	T	Total
Proportional to population	10.13	21.00	3.19	8.22	34.22	7.07	83.82
Proportional to demand	13.07	16.01	7.30	6.87	28.48	12.08	83.82
SCRB	19.54	13.28	5.62	10.90	16.66	17.82	83.82
Shapley value	20.01	10.71	6.61	10.37	16.94	19.18	83.82
Nucleolus	20.35	12.06	5.00	8.61	18.32	19.49	83.82
Weak nucleolus	20.03	12.52	3.94	9.07	18.54	19.71	83.82
Proportional nucleolus	20.36	12.46	3.52	8.67	18.82	19.99	83.82
Individual alternative costs	21.95	17.08	10.91	15.88	20.81	21.98	
Marginal costs	17.36	9.85	0.82	6.40	12.89	14.06	



The most obvious fact revealed by the comparison is that the two naive methods of allocation – by proportion of population and of use – produce the greatest contrasts in costs. The results differ markedly from those of all other methods tested. As was true of the hypothetical case in Chapter 1, apportioning costs by population or use charges some groups more than what they would have to pay in pursuing the venture alone. Allocation by population penalizes H and M, while allocation by use penalizes M.

The two proportional methods fail the test of individual rationality by not taking into account differences of access to sources. Charged by population or use, Groups H and M bear the brunt of total costs because they have large populations. The methods also fail the test of marginal cost. Groups A and T are outlying, and their proportional charges by population or use would be far less than what it would cost to include them in the six-group coalition.

Seemingly more reasonable than proportional methods, but actually almost as ill-behaved, is the SCRB method. It meets the test of individual rationality because it is cheaper for each group to join the six-group project than to go it alone. But it fails group rationality because membership of some coalitions smaller than the whole AHKLMT project would be cheaper for the players involved. For example, using separable costs–remaining benefits, the three most centrally located groups, H, K, and L, can build a joint waterworks for Skr 27.26 million. In a six-group project, SCRB would cost them Skr 29.80 million.

The defect of the SCRB method is that it considers only the marginal costs of including players, not the marginal costs of including coalitions of players. The marginal cost of including both M and T is much higher than the sum of their individual marginal costs because when one of them is being served the added expense of serving the other is low. The SCRB method avoids estimating all cost elements, but this shortcut can end in an inequitable final cost distribution.

By contrast. the Shapley value requires alternative costs for all possible subsets. But this method of allocation fails the tests of group rationality and marginal cost. For instance, it charges coalition HKL Skr 430,000 more than its alternative of building a facility as an independent group.

The remaining three methods – nucleolus, weak nucleolus, and proportional nucleolus – are potentially more desirable than the SCRB and Shapley methods because they start by limiting what each player is charged to individual and group rationality (to provide maximum incentive) and marginal cost (to be fair to all other groups and coalitions). As noted in Chapter 1, these methods further narrow the limits by means of taxes and subsidies.

ANOTHER ALLOCATION PRINCIPLE

To make the best choice from all the allocation methods available, the principle of *monotonicity* should be considered. A minimum requirement of fair allocation, according to this principle, is that, if total costs increase, no player will be charged less, and if total costs decrease, no player will be charged more.

Typically, project costs are not known precisely until after the project has been completed. But the allocation method is usually agreed upon before the project is started. Since total costs are not then known, the allocation may be made for several total cost estimates. Then when all players agree to begin a joint project, a single total cost is taken to be the best estimate and the alternatives are abandoned. This cost is likely to be off the mark by the time the project is finished. So, to evaluate the method of cost allocation, different levels of total cost should be allocated under the assumption of fixed levels of alternative costs.

Several of the methods discussed in this report do not meet the test of monotonicity. For example, suppose the 18-municipality project in southern Sweden ends with a cost overrun of Skr 4 million. That raises the total cost to Skr 87.82 million, which would change costs for all six groups under all the methods, as shown on page 21.

The figures show that by the nucleolus and SCRB methods, group K would pay less, even though the total cost of the joint project had increased. This suggests that neither of these methods is a reasonable way to allocate costs.

Methods that do meet the test of monotonicity are those allocating costs according to some single criterion like population or use, the Shapley value, the weak nucleolus, and the proportional nucleolus. For the Shapley value and weak nucleolus, any change in total costs is distributed equally among the groups.

Even here, though, questions of fairness arise. Why should all the players have to share added costs equally when their shares in the project differ greatly? For example, in the weak nucleolus method, it is possible that a small player contributing almost no costs (or savings) to the coalition would have to pay as much for

WHAT HAPPENS WHEN COSTS RISE
(millions of kronor)

The principle of monotonicity states that, as a minimum requirement, a cost allocation method should not charge any group less if costs unexpectedly rise, or more if they fall. The costs allocated to each group by each method for a facility costing Skr 83.82 million are compared below with the allocations that would be made for an Skr 87.82 million project (assuming a cost overrun of Skr 4 million). The figures in color show that the SCRB and nucleolus methods lack monotonicity.

Allocation method	A	H	K	L	M	T	Total
Proportional to population	10.61	22.00	3.35	8.62	35.86	7.40	87.82
	10.13	21.00	3.19	8.22	34.22	7.07	83.82
Proportional to demand	13.70	16.78	7.64	7.20	29.84	12.66	87.82
	13.07	16.01	7.30	6.87	28.48	12.08	83.82
SCRB	21.42	14.19	5.46	10.97	17.31	18.47	87.82
	19.54	13.28	5.62	10.90	16.66	17.82	83.82
Shapley value	20.67	11.38	7.29	11.03	17.60	19.84	87.82
	20.01	10.71	6.61	10.37	16.94	19.18	83.82
Nucleolus	20.76	13.25	4.51	9.80	19.16	20.33	87.82
	20.35	12.06	5.00	8.61	18.32	19.49	83.82
Weak nucleolus	20.70	13.19	4.61	9.74	19.21	20.38	87.82
	20.03	12.52	3.94	9.07	18.54	19.71	83.82
Proportional nucleolus	20.61	13.20	4.72	9.84	19.14	20.31	87.82
	20.36	12.46	3.52	8.67	18.82	19.99	83.82

the cost overrun as its giant neighbors. This too constitutes an unreasonable allocation.

The fairest way of measuring the shares of each participant in the project may not be by cost. Most of the costs would be incurred in any case by building separate facilities. A reasonable method would divide unforeseen costs, such as overruns, in proportion to the benefits enjoyed. This is what the proportional nucleolus method does. As noted in Chapter 1, it puts a tax on players in proportion to their savings.

Of all the game theory methods considered, the proportional nucleolus appears to be the most attractive. But the application of the various methods to an actual cost allocation problem shows

Putting it together. A new water conduit traverses the Skåne region of southern Sweden, from a lake, Vombsjön, in the east directly to the city of Malmö in group M in the west of the region. Group M suffers most by an allocation of water costs proportional to population because it has the largest and most dense population. It would have to pay Skr 34.22 million for a joint water supply when it could build a separate system for Skr 20.81 million. (Photograph by courtesy of Sydvästra Skånes Kommunalförbund) that merely comparing the results of using the methods does not provide a full evaluation. There remain the questions of acquiring the needed information, of getting the players to produce the information and agree on it, and of assuring that the players will remain committed to the project after they see how much it will cost them.

What if a player cannot afford to join the project? What if a player has not indicated his true willingness to pay? Who decides whether or not outlying municipalities will be included in the project? Some of these questions are considered briefly in the last chapter of this report.

3

Practical Solutions

The true test is how the method works in practice, and by simulating such a test the IIASA scientists have shown that some commonly used cost allocation methods are ineffective.

As noted in Chapters 1 and 2 of this Executive Report, the method should conform to some basic principles of fairness. *Rationality* prohibits charging any player more in the whole coalition than he would pay to build a water supply system alone or in a smaller coalition. *Marginal cost* is what the whole coalition would have to pay to include a new player, and therefore it is the least that the player should be charged.

The method chosen should be adaptable, and in particular it should provide *monotonicity*, so that if the cost of the project rises, no player benefits by paying less than before, and if the cost falls, no player will pay more. It should be insensitive to *direct costs*, so that it does not depend on the identification of costs incurred entirely within each group, as these costs may be difficult or impossible to isolate.

In addition to meeting these needs, the method should be simple, so that it can be presented readily to all the decision makers of the municipalities involved. Similarly, it should be based on information that can be acquired without undue effort.

One of the methods tested, the separable costs–remaining benefits method, is currently used for many types of public projects where costs must be allocated. In fact, SCRB has a number of serious shortcomings. It fails the tests of rationality and marginal cost for coalitions smaller than the whole, and of monotonicity. The method's underlying objective of allocating costs in proportion to benefits is attractive, but difficulties arise from the *ad hoc* introduction of marginal costs.

The Shapley value is monotonic, but it only satisfies rationality for separate players, not for coalitions of players smaller than the whole coalition.

The three methods that modify the core of the game – the nucleolus, weak nucleolus, and proportional nucleolus – all satisfy rationality and marginal cost. However, the nucleolus is not monotonic. Both the weak nucleolus and proportional nucleolus are monotonic, but the proportional nucleolus makes a better allocation of changes in costs, in a way more consistent with the benefits to each group.

None of the five methods passes the tests of simplicity and ease of information gathering. They are all fairly complicated, and require detailed information on costs. Information on demand and on the optimal scale of development may be unreliable or nonexistent, yet these methods rely on it. This shortcoming is particularly apparent for a project such as the Swedish water development, which was based on estimates of demand and costs 10 years ahead.

The IIASA study concluded that a simple scheme based on allocating costs in proportion to population may be the best practicable solution for the six-group, 18-municipality case in southern Sweden. Insufficient information was available for developing a more equitable approach. Accordingly, the decision makers in the project chose to allocate by population. Its greatest inequity falls on group M, which has the largest and most dense population. It could build its own separate water supply facility for Skr 20.81 million, and yet it would pay Skr 34.22 million for the dubious privilege of sharing a facility – and control over it – with five other groups that are less urban and so have different needs.

A WAY TO FIND DEMAND

The case study by an international group of scientists at IIASA illustrates how costs are allocated in practice and evaluates some common allocation methods. However, another IIASA paper takes the problem one step further and suggests an approach that can overcome the greatest difficulty encountered in the Swedish study, the lack of basic information on willingness to pay.

In Cost Allocation and Demand Revelation in Public Enterprises, an IIASA Working Paper prepared in 1980, H. Peyton Young, who was also instrumental in the Swedish study, describes a simple method for allocating joint costs in cooperative and public enterprises. The method is based on bidding, which is noncooperative. This puts the burden of information gathering on the players themselves.

The approach differs from those applied in the Swedish case study in that it assumes no knowledge of either demands or the optimal level of production. They are discovered by a competitive bidding mechanism. Each group of municipalities in the Swedish project would play this game by first determining what it would be willing to pay to have a water facility under any circumstances. In doing so, it would not consult with any of the other players.

The bidding process begins by each player submitting a sealed bid declaring how much he is willing to pay to join a proposed facility. This amount may be less than what he is truly willing to pay, reflecting the inclination to get a good bargain. Any group of municipalities (player) can bid, including those in outlying areas. When the bids are in, the regulator or auctioneer chooses the coalition of players that maximizes net surplus. In this coalition, the total bids exceed the total costs by the highest amount.

All the players can then bid again, and a new optimum facility is determined. Players left out of the project in the first round will naturally want to raise their bids. Those included in the first round will try to lower their bids and still remain in the project. In this way, the cost allocation process approaches equilibrium.

For example, suppose the six players bid to be included in a joint project as follows: A, 19 (million kronor); H, 12; K, 8; L, 11; M, 17; and T, 19. A comparison of the costs of serving each possible coalition reveals that the sum of bids less costs is highest for the coalition HKL. This excludes groups (or players) A, M, and T.

In the next round, players A, M, and T can be expected to raise their bids slightly, while H, K, and L will keep their bids the same or even decrease them. Round two might produce these bids: A, 20 (million kronor); H, 12; K, 5; L, 9; M, 18; and T, 20. In this case, the six-group coalition maximizes bids less costs. The surplus is only about 18 million kronor, and the result is actually very close to equilibrium.

The regulator terminates the process after a predetermined but undisclosed number of rounds. Or, the process may end when the bids do not change appreciably from one round to the next. Players excluded at this point have no recourse, and the project is undertaken, the players being charged the prices they last bid.

The outcome may not reveal the true demands of the players, but it comes close enough to insure that an efficient level of service is provided. If the players bid more than the total cost of serving them, a surplus may accumulate. Ideally, this surplus is taken as a lump-sum tax on all players in the whole coalition. It could also be put into an escrow account to protect against unfore-

Urban—rural contrast. Group M, the most densely populated of the six groups, has three municipalities, Malmö, Burlöv, and Staffanstorp. But it also has much farmland, which stands in sharp contrast to the urban development, as this view of Burlöv shows. With costs allocated by population or by use, farmers pay considerably more for their drinking water than do their competitors just to the north in groups K and L. (Photograph by courtesy of Sydvästra Skånes Kommunalförbund) seen cost increases, or it could be returned to the bidders in proportion to their bids.

In theory, redistributing a surplus could distort the bidding process if players overbid in anticipation of actually paying less. However, such a strategy is unlikely because the players do not know the other bids. In some cases careful bidding will result in a surplus that is negligible. Where the surplus is considered too large, extra rounds of bidding may be arranged.

A cost allocation based on competitive bidding has several advantages. It is simple and easy to implement, and it leads to an efficient and reasonably fair allocation in the absence of information on demand.

As a test, the competitive bidding method was applied to the Swedish water project. Decision makers from the six groups discussed in Chapter 2 were given information only about their own demand. Rounds of bidding were conducted. Only the winning set of bids was announced after each round. Although the players did not know their costs, as outlined in Chapter 2, within 10 rounds the bidding had converged on a solution within a fraction of 1 percent of a solution in the core, and the resulting surplus was negligible.

Competitive bidding is not presented here as the final word on the problems of allocating costs in public projects. It is most effective when there are few players in the game and the advantages of a coalition are evident and free of qualifying circumstances.

The discussion is presented merely as a way to illustrate the problems of allocation and the ways of solving them. For a more detailed explanation, including the mathematical formulations on which much of the material in this report is based, two publications available from IIASA are recommended:

RR-80-32, Cost Allocation in Water Resources Development – A Case Study of Sweden, by H.P. Young, N. Okada, and T. Hashimoto.
WP-80-130, Cost Allocation and Demand Revelation in Public Enterprises, by H.P. Young.