Social evolution leads to persistent corruption

Joung-Hun Lee[a,b], Yoh Iwasa[b,c], Ulf Dieckmann[d,e], and Karl Sigmund[d,f,1]

[a]Institute of Decision Science for a Sustainable Society, Kyushu University, 819-0395 Fukuoka, Japan; [b]Department of Biology, Kyushu University, Nishiku, 819-0395 Fukuoka, Japan; [c]Department of Bioscience, School of Science and Technology, Kwansei-Gakuin University, 669-1337 Sanda-Shi Hyogo, Japan; [d]Evolution and Ecology Program, International Institute for Applied Systems Analysis, 2361 Laxenburg, Austria; [e]Department of Evolutionary Studies of Biosystems, The Graduate University for Advanced Studies (Sokendai), Hayama, Kanagawa 240-0193, Japan; and [f]Faculty for Mathematics, University of Vienna, 1090 Vienna, Austria

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Cooperation can be sustained by institutions that punish free-riders. Such institutions, however, tend to be subverted by corruption if they are not closely watched. Monitoring can uphold the enforcement of binding agreements ensuring cooperation, but this usually comes at a price. The temptation to skip monitoring and take the institution's integrity for granted leads to outbreaks of corruption and the breakdown of cooperation. We model the corresponding mechanism by means of evolutionary game theory, using analytical methods and numerical simulations, and find that it leads to sustained or damped oscillations. The results confirm the view that corruption is endemic and transparency a major factor in reducing it.

cooperation | corruption | evolutionary game theory | social contract | punishment

According to Gallup polls, a majority holds that corruption is the "world's problem number one," ahead of climate change, poverty, or terrorism (1). By threatening the institutional foundations of society, corruption endangers the social contract on which moral behavior and the rule of law are grounded.

In the words of Nobel Laureate Elinor Ostrom, institutions are "tools that offer incentives to enable humans to overcome social dilemmas" (2). Such social dilemmas arise whenever selfish interests threaten the public good. Social contracts offer a way out: By punishing free-riders, institutions curb the behavior of selfish individuals and promote the welfare of the community, ideally enhancing the long-term benefit of all participants (3–6).

However, this principle has always been endangered by corruption. Institutions are implemented by individuals who are not exempt from selfish motives. To pick up Ostrom's phrasing: Corruption provides tools that offer incentives to enable humans to subvert social contracts.

The pervasive influence of corruption has moved into the focus of contemporary social research (1). The time when Nobel laureate Gunnar Myrdal could rightly say that "corruption seems almost taboo as a research topic" (7) is well past. Its study attracts an increasing number of lawyers, political scientists, economists, historians, and sociologists (see, e.g., refs. 8–17). It is well established that the corruption of judicial institutions lowers investments and therefore economic growth (18, 19). In many countries, and in many fields of activity, corruption has a major impact. Tellingly, a state's standing in the Corruption Perceptions Index is closely correlated with its gross domestic product (GDP) per capita, its national income per adult, its Global Competitiveness Index, and its World Happiness Index (*SI Appendix*, Figs. S1–S4).

Corruption is defined as the "illegitimate use of public roles and resources for private benefits" (20). It comes in a multitude of guises, such as favoritism, clientelism (also known as pork-barreling), embezzlement of public money, etc. Here, we consider one aspect only, namely, the bribery of public institutions whose task it is to uphold mutual cooperation by penalizing cheaters. Following Ostrom's approach, we do not restrict the meaning of "public institutions" to agencies run by state officials, but include, for instance, soccer referees, journalists, or executives of nongovernmental organizations (NGOs)—all those wielding power based on social trust and required by their role to punish or otherwise chastise rule-breakers or law-offenders.

Here, we analyze a basic model of this type of corruption by means of evolutionary game theory (21–24). We strive for a minimal model capturing key dynamics relevant for many specific systems. In particular, we show that the adaptation of individual agents to the current social situation leads to sustained or damped oscillations that reflect the waxing and waning of institutional corruption in response to the waning and waxing of cooperation within the society.

If institutions are viewed as "guardians" of the community, then it is up to the community to "guard the guardians." This is usually a costly endeavor. Whenever it is neglected, corruption can spread. In response, cooperation breaks down. Such a crisis reinvigorates efforts by would-be cooperators to closely watch the rule-enforcing institution and only invest when they can trust that the judicial system is reliable. Eventually, this curbs corruption and bolsters economic activity. But when cooperation and honesty prevail again, the efforts to watch the integrity of the institution become superfluous, and therefore slacken. And, thus, another cycle starts. This recurrence of corruption reflects what, in the jargon of social science, is termed a "wicked" problem—one that cannot be solved for good.

By adopting the viewpoint of evolutionary game theory, we implicitly posit that economic agents are guided by self-interest. They preferentially adopt strategies that ensure an advantage in the present state of the population. This choice of a strategy can occur either through social learning—i.e., by imitating agents achieving a higher payoff—or through rationally, but myopically, choosing whatever is currently the most promising option. Such a process of adaptation affects the frequencies of the strategies, which in turn affects the strategies' expected payoffs. The resulting feedback between frequencies and payoffs drives the evolutionary dynamics.

Significance

Corruption is widely perceived as a major problem. Bribery of judicial institutions undermines the trust needed for joint efforts and economic investments. Transparency can reestablish trust, but at the cost of constant supervision of the institutions. Reducing such vigilance is advantageous in the short term. In the long run, it leads to more cheating and less cooperation. This can create cyclic outbursts of corruption or maintain corruption at a stable level.

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[1]To whom correspondence may be addressed. Email: karl.sigmund@univie.ac.at.

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Results

We start with an interaction that is representative of a social dilemma: a "mutual aid game" or "public good game" (24, 25). For simplicity and ease of exposition, we assume that each instance of this game involves only two players and reduces to a donation game: Both players must decide, independently, whether or not to confer a benefit b to the other player, at own cost c. The two alternatives are named C (for "cooperation") and D (for "defection"), respectively. If $c < b$, we obtain a typical one-shot prisoner's dilemma. The matrix describing the resultant payoffs a focal player adopting the strategies in the rows obtains when interacting with another player adopting the strategies in the columns is shown in Table A:

Table A

	C	D
C	$b-c$	$-c$
D	b	0

Table B

	C	D
C	$b-c-f$	$-c-f$
D	$b-A-f$	$-A-f$

Table C

	C	D
C	$b-c-f$	$-c-f$
D	$b-B-f$	$-B-f$

No matter what the other player does, C always yields less than D to the focal player, so that if both players are guided by self-interest, they end up by mutually defecting. Their payoff is 0, instead of the payoff $b - c > 0$ obtained through mutual cooperation. In a population of players meeting randomly, both rational utility-maximization and imitation through social learning lead to a homogenous population consisting of defectors only.

This outcome can be overturned by penalizing defectors. We assume that each game is watched by an umpire, ensuring that any player who defects must pay a fine $A > c$. The fine is not part of the income of the umpire (umpires have to live even if all players cooperate). Rather, we shall assume that both players have to pay a fee f to the umpire before engaging in the donation game, with $f < b - c$, so that mutual cooperation still pays (see payoff matrix in Table B). Clearly, C is now the better move for the focal player, no matter what the other player does. The umpire effectively releases the players from the social trap, at a cost f.

This, however, assumes that the umpire is honest. An umpire who is corrupt, rather than honest, will accept (in addition to the fee) a bribe B from the defector, rather than impose a fine A. If $B < c$, defection becomes the dominant strategy, again (see payoff matrix in Table C).

Thus, all depends on whether the umpire is honest or corrupt. In the absence of specific information, players are reduced to a game of chance with unknown odds. But if we assume that umpires imitate the successful behavior of other umpires, then social learning will corrupt them all, since the bribes offer a supplementary income. The same holds if umpires chose rationally, but myopically, their best reply. (For this reason, our model can easily be adapted to the case that the population of players is faced with a single umpire, as will be discussed later.)

As a consequence of the corruption of the umpires, cooperation breaks down. The challenge is to escape from this social trap, and the investigation of when this is possible is the main aim of the present study.

We assume that players have the opportunity to find out whether their umpire is honest or corrupt, by paying a supplementary cost h for information (an obvious modification of the model works even if that information is incomplete). Players can then react by dropping out of the game if the prospective umpire does not suit their intentions. A player who invests h in inquiring about the umpire's reputation will be said to be *prudent*. A player who trusts luck and skips the cost h will be said to be *optimistic*. Optimistic defectors (ODs) hope that the umpire will refrain from punishing them after accepting a bribe, while optimistic cooperators (OCs) hope that the umpire will punish coplayers who cheat them.

To sum up, there are two populations of size M and N, respectively—namely, the players (of the donation game) and the umpires. The former correspond to economic agents, the latter to judiciary authorities. Umpires can be honest (H) or corrupt (C). Players can be OCs, prudent cooperators (PCs), ODs, or prudent defectors (PDs). The game proceeds as follows. Two players meet randomly and are assigned an umpire (this could be realized by a state-wide law system or, under more anarchic conditions, by hiring a law enforcer, such as a strong-man or sheriff). Prudent players pay h to learn whether their umpire is honest or corrupt; a PD who learns that the umpire is honest drops out of the game, and so does a PC who learns that the umpire is corrupt. If both players are willing to proceed, then both pay a fee to the umpire, each at a cost f. If the umpire is corrupt, defectors pay additionally a bribe B. Now, the usual donation game is played between the two players, with parameters b and c. If the umpire is honest, the defectors have to pay a fine A. And with this, the game is over.

We shall always assume that h, the cost of information, is so small that the following inequalities hold:

$$f + h < b - c, \quad [1]$$

$$c + h < A, \quad [2]$$

$$B + h < c, \quad [3]$$

$$h < \sqrt{f(f+B)}. \quad [4]$$

The first three inequalities are modifications of $f < b - c$, $c < A$, and $B < c$, conditions obviously needed to make economic sense of the game, the penalty, and the bribe, respectively. Inequality **4** is less immediate: The cost of information must be lower than the geometric mean of the incomes that an honest and a corrupt umpire can expect from a client. This condition, as we shall see, provides an escape from the economic stalemate caused by the social trap, if only for a while.

The average payoffs for the four types of players (OC, PC, OD, and PD) and the two types of umpires (H and C) can easily be computed as functions of their frequencies (*Methods*), assuming that the populations are well-mixed and players and umpires meet randomly. The average payoff values depend in an obvious way on the numbers of players M_1, M_2, M_3, and M_4 of types OC, PC, OD, and PD, respectively, and on the numbers of umpires N_1 and N_2 of types H and C. These numbers sum up to M and N, respectively.

We now examine what happens when all individuals, players and umpires, can update their strategies from time to time, in between playing many games. The updating of strategies by social learning is due to two mechanisms. With a small probability, an individual switches at random to another strategy. This corresponds to random exploration. Otherwise, an individual X imitates another individual Y's strategy with a certain likelihood. This can be modeled in various ways: All that matters is that the likelihood that X imitates Y increases in P_Y and/or decreases in P_X, the average payoffs of Y and X, respectively (*Methods*).

The resulting imitation–exploration dynamics depends, of course, on various parameters. What is robust is a cycling tendency (Fig. 1 *A* and *B* and *SI Appendix*, Fig. S7), possibly damped by large populations, high exploration rates, weak selection, or high information costs (Fig. 1 *C* and *D* and *SI Appendix*, Figs. S8 and S9). The oscillations are essentially driven by the fact that when umpires are honest, OCs prevail. Corrupt umpires then sneak in, benefitting from the lack of control. Once they are frequent enough, PCs spread, in alliance with ODs, and pave the way for the resurgence of honest umpires.

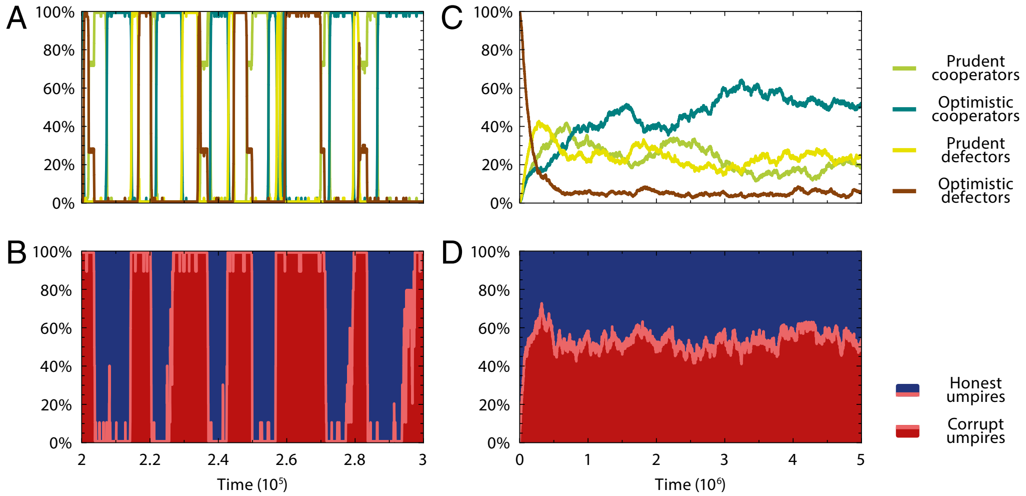

Fig. 1. Strategy dynamics resulting from the social-learning process in the player population and the umpire population. (*A* and *B*) Sustained oscillations. Corrupt umpires can invade and take over when players are OCs. Cooperation then gives way to defection, and investments drop drastically. PCs, however, turn the tide. They can coexist with ODs in sufficient amount to make honesty advantageous for the umpires. At this stage, OCs can spread, and the cycle resumes. The shown social-learning process is close to the exploration-limited case (with vanishing exploration rates; see text). The time-averaged ratio of honest-to-corrupt umpires is 2.15, whereas the theoretical value in the exploration-limited case is 2.54. Similar cycles of corruption prevail for a broad range of other exploration rates and imitation rules, see *SI Appendix*. (*C* and *D*) Damped oscillations. For larger populations, higher exploration rates, weaker selection, or higher information costs, the oscillations can be damped, leading to stable levels of persistent corruption (*SI Appendix*, Figs. S8 and S9). Parameters: $b = 1$, $c = 0.5$, $f = B = 0.2$, $h = 0.1$, $A = 2$, and $M = 50$, $N = 10$, $\mu = 0.001$, $\nu = 0.005$, and $s = 10^{10}$ (*A* and *B*), or $M = 5{,}000$, $N = 1{,}000$, $\mu = 0.01$, $\nu = 0.05$, and $s = 0.3$ (*C* and *D*); thus, in *A* and *B* as in *C* and *D*, new strategies enter both populations through exploration at the same rate.

This waxing and waning of corruption is clearly displayed by individual-based simulations of the social-learning process (Fig. 1). It can be checked analytically in two limiting cases, for large populations and rare exploration, respectively.

Large Populations. When M and N tend to infinity, well-known replicator–mutator equations govern the relative frequencies $x_i = M_i/M$ and $y_j = N_j/N$ of the four types of players and the two types of umpires (*Methods*). When social learning is entirely driven by imitation and unaffected by random exploration, we obtain replicator equations (26) whose analysis proceeds along the usual lines: The eight homogeneous states (with all players and all umpires being of the same type) are clearly fixed points. The behavior on the edges connecting them is shown in Fig. 2. If all umpires are honest, the frequency of OC converges to 1. If all umpires are corrupt, the frequency of OD converges to $h/(f + B)$ and that of PC to $(f + B - h)/(f + B)$, while OC and PD vanish. The condition $h < \sqrt{f(f+B)}$ guarantees that honest umpires can invade at this state and take over. Next, PCs give way to OCs.

If all players are OCs, corrupt and honest umpires do equally well. This is why corruption will drift in again as soon as a small exploration rate is considered. A limit cycle emerges near the boundary of the state space. It follows orbits along some of the edges which, in the replicator equations, connect fixed points of saddle type, as well as along the edge that connects (OC, H) with (OC, C). In the vicinity of this edge, which consists of fixed points of the replicator equations, the flow leads toward corruption

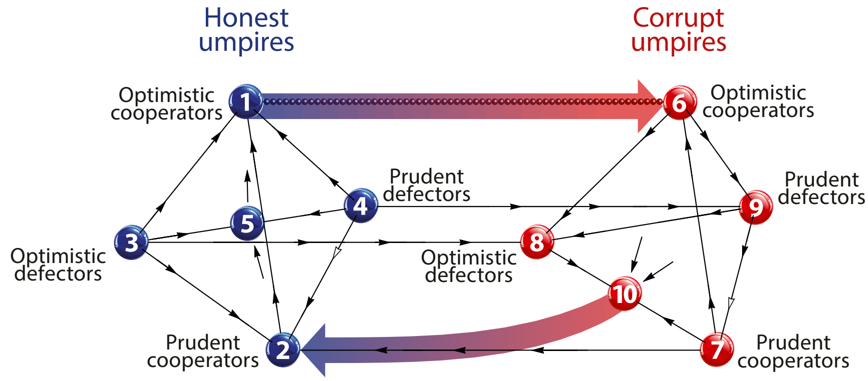

Fig. 2. Backbone of the model. Shown are the replicator dynamics (*Methods*) on a projection of the four-dimensional state space $S = \Delta_4 \times [0,1]$. The simplex to the left is the state space of the player population when all umpires are honest, while that on the right applies when all umpires are corrupt. The colored balls depict fixed points. Apart from the fixed-point edge with $x = (1, 0, 0, 0)$ and $y_1 \in [0, 1]$ (shown by the line of small colored balls at the top), the fixed points are isolated; the absorbing state 5 is given by $x = (0, 0, h/(f + A), (f + A - h)/(f + A))$ and $y_1 = 1$, whereas the absorbing state 10 is given by $x = (0, (f + B - h)/(f + B), h/(f + B), 0)$ and $y_1 = 0$. The small arrowheads indicate whether the linearized flow near the corresponding fixed point leads toward that fixed point or away from it, or, technically speaking, whether the corresponding "transversal eigenvalue" is positive or negative (6). If the arrowhead is open, that eigenvalue is 0: The direction of the flow then follows from its nonlinear components. The two block arrows along the top edge and within the bottom plane indicate the locally prevailing trends from honesty to corruption (top) and back (bottom).

(Fig. 3). Indeed, the exploration rate introduces small amounts of OD, PC, and PD players. Most pairs of players contain at least one OC player. Corrupt and honest umpires do equally well for pairs (OC, OC) and (OC, OD). For (OC, PC) pairs, honest umpires have an advantage f; but for (OC, PD) pairs, the advantage of corrupt umpires is even greater, namely, $f + B$. Thus, the bribe B drives the dynamics toward corruption.

If the exploration rate increases further, the limit cycle shrinks. For large values, it turns into a stable equilibrium with all types present. The orbits of the imitation–exploration dynamics spiral toward it, and the frequencies undergo damped oscillations (Fig. 3). Both in the case of a limit cycle and of a stable equilibrium, corruption prevails in the long run.

Rare Exploration. When population sizes are finite and exploration rates are very small, the fate of an invading dissident—i.e., its invasion or fixation—is settled (in the sense that imitation has led to an absorbing state of the pure imitation process) before the next exploration step introduces a new strategy. The dynamics, then, is reduced to transitions between the absorbing states of the imitation process (Fig. 1 *A* and *B*).

It is easy to compute the stationary frequencies of the absorbing states if imitation is "hard"—i.e., always in the direction of the better payoff. We note that in this case, two absorbing states are mixed—one is a mixture of OD and PD (when all umpires are honest) and the other a mixture of OD and PC (when all umpires are corrupt). In particular, if the exploration steps occur with the same (small) frequency in the player population as in the umpire population, the ratio of honest to corrupt umpires is $19N + 87{:}109$, where N is the number of umpires. This result does not depend on the size M of the player population, nor on the other parameters A, B, f, h, b, and c. Even for very small values of N, the majority of umpires will be honest in the long run. But it can always be subverted, for a time, by episodes of corruption.

To sum up, the shift from honesty to corruption occurs when OCs prevail and monitoring is reduced. The shift from corruption to honesty occurs when there are enough PCs, which requires that the information cost h is sufficiently small to make transparency sufficiently high. What corrupt umpires then gain through bribes is more than offset by what they lose in honest investors. This result also holds if we assume, instead of a population of umpires, a single umpire. In this case, the concept of social learning is inappropriate. But if the umpire is "rational" in the sense of adopting the best option (corruption or honesty) given the current state of the player population, analogous cycles emerge.

Discussion

Cyclic behavior has been found in several other models of corruption. In ref. 27, cycles are driven exogenously by the periodic recurrence of elections. In refs. 28 and 29, cycles occur through an interplay of the popularity of politicians and their "hidden assets." Our approach, which is different, centers instead on the effect of judiciary corruption upon economic investment. It is grounded in a long tradition of game-theoretical work on cooperation (23, 24, 30–34).

Experience and experimental games show that players are often willing to engage in the punishment of cheaters, although it is costly to themselves (35). They may do this even if they will never meet again with the defector, and even if they were not personally victimized, but a third party (36). Such behavior has been termed "altruistic punishment" or "strong reciprocation" (37, 38). This form of direct or indirect revenge is frequently efficient, and theoretical models have shown that it can emerge through social learning (39, 40). However, since it consists in "taking the law into one's own hands," the punishment of cheaters

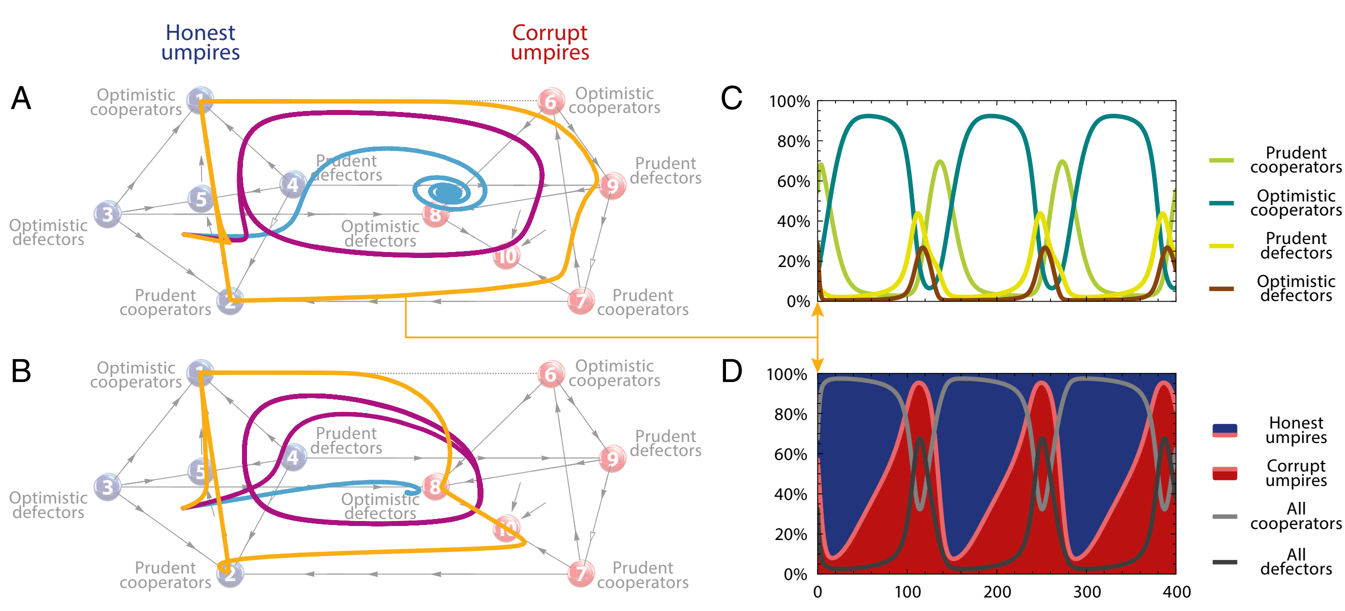

Fig. 3. Strategy dynamics in the large-population limit. Replicator-mutator equations (*Methods*) describe social learning in large populations of players and umpires. The orbits in *A* and *B* have the same starting point ($y_1 = 1$, $x_1 = x_4 = 0.1$, $x_2 = x_3 = 0.4$), but differ in the exploration rates $\mu = \nu$ of players and umpires (orange, 0.0001; violet, 0.02; blue, 0.07). With increasing exploration rates, the limit cycle shrinks and turns into a stable fixed point reached by damped oscillations. The limit cycle for small exploration rates follows orbits along the edges of the replicator equations (Fig. 2): Depending on whether $(c - B)/(A - B) < (c - B - h)/(b - B - f)$ or not, the limit cycle leaves the edge where all players are OCs to visit the absorbing state 8 or 9, respectively, before moving toward the absorbing state 10 and from there again toward more honest umpires. This is shown for large penalties $A = 2$ in *A* and small penalties $A = 0.7$ in *B*. The strategy cycles of the frequencies of the four types in the player population (*C*) and of the two types in the umpire population (*D*) are shown for the orange limit cycle in *A*, together with the frequencies of cooperators and defectors in the player population. Other parameters are as in Fig. 1 *A* and *B*.

by their peers can easily lead to counterpunishment, vendettas, "asocial punishment" (i.e., the punishment of cooperators), and a "war of all against all" (41, 42).

This dark side of peer punishment can be overcome by so-called pool punishment (43). In this case, players are offered, before each round of the public goods game, to contribute to a punishment pool (essentially, the equivalent of a police force). The better the punishment pool is filled, the harsher the fines imposed on the free-riders. Pool punishment is less efficient than peer punishment (since the police must be paid even if nobody has to be punished), but it is more stable (44).

Pool punishment is a rudimentary form of the institutionalized punishing of free-riders (and of other forms of asocial behavior) that underlies the "social contract": Punishment is meted out not by the players, but by an institution strong enough to suppress retaliation. Philosophers are familiar with the concept. The institution to which the agents submit, willingly or not, was called "the sovereign" by Hobbes, Rousseau, and Kant, who lived in an age of absolute monarchs. More recently, Ostrom (2, 45) and others (46) have shown that similar social contracts to enforce cooperation hold in many other types of societies and often emerge spontaneously, bottom-up, without state interference, through voluntary commitment. People frequently agree on rules that are enforced by specialized agents. These persons or grass-roots organizations can be farmers' or fishers' councils, village elders, janitors, sheriffs, judges, local lords, mafia bosses, guild masters, magistrates, ombudsmen, notaries, referees, etc.

While institutionalized punishment avoids the war of all against all looming whenever players take the law into their own hands, it is endangered by corruption. This is the situation we have modeled here. In a previous paper (47), we used a related model to analyze situations in which players decide whether to use a sanctioning institution or not. In contrast, here, players are obliged to use it, if they wish to participate in the game at all. For instance, an international company can chose whether to open a store in a given country or not, but when it does, it must obey that country's legislation—i.e., submit to its "umpires." If the company decides to avoid a specific country with bad corruption status, it acts like a PC.

Our model could and should be extended. More than two players may engage in the joint enterprise. They may have information about each other and react accordingly. Information about umpires can be partial, erroneous, even manipulated. The umpires' propensity to corruption, and the players' propensity to defection, can be heterogeneous. Umpires can take a bribe and punish nevertheless. The offer of a bribe can be a risky move, entailing complex signaling. Umpires often come in hierarchies and can themselves be penalized. Thus, one could investigate an extended model with players, umpires, and a government. If umpires are corrupt and players invest less, the government suffers (less investments, less growth, less taxes, and less votes) and may start an anticorruption drive to promote honesty among umpires. We expect the direct incentives to umpires modeled here and the indirect incentives to umpires occurring in such an extended model to have analogous game-theoretical consequences and have therefore opted for a minimalistic approach that neglects all these issues.

Endemic corruption and fluctuating levels of corruption are widespread all over the world (8, 9). Documented instances of periodic cycles seem less easy to find. This could mean, in line with our model, that individual exploration rates are so high that oscillations are damped. It is even more likely, however, that periodicity is blurred by stochastic effects, time delays, spatial diffusion, and the layers of complexity added over time, by every society, to its governance structure. Nevertheless, the underlying cyclicity seems to manifest itself in a growing trend, all over world, to resort to periodic elections. Rather than driving cycles of corruption, as in the model used in ref. 27, such elections could thus be a response to the recurring need for reestablishing institutional integrity. Real-life models of such cleanup operations could possibly involve a hierarchy of umpires, but the basic strategic driver of honesty is the same as described by our model: More trust leads to more investment in joint enterprises.

Information is needed to uphold social trust and economic activity. If distrust in institutions grows, people are less willing to engage in cooperative interactions, and the social dilemma returns with a vengeance: Players and umpires both suffer. To promote economic life, it must be possible and affordable to check the reliability of the legal machinery provided by an institution. The institution must have a good reputation, whose integrity can be trusted because it can be checked. Transparency is the main tool in the fight against corruption.

Such transparency comes in many forms: as freedom of the press, as "glasnost" (to use a historical Soviet term), as a "sunlight test" (19), as "naming and shaming" (48), as incentives for journalists and "whistleblowers" (49), etc. It is no accident that the world's leading anticorruption NGO is named Transparency International.

Corruption displays a stunning diversity. Our approach has focused on one part only. It is, however, a core part—the corruption of those institutions whose task it is to overcome social dilemmas by enforcing cooperation. The striking negative correlation between corruption and economic performance is well established (18, 50–52); see also the positive correlation between the Corruption Perceptions Index and multiple indicators of social welfare (*SI Appendix*, Figs. S1–S4).

The corruption of judiciary institutions can effectively strangle economic life. Since agents unwilling to invest in joint enterprises no longer need institutions to uphold their agreements, corruption subverts its own basis in the long run. This feedback allows the comeback of cooperation and leads to a (possibly damped) cycle not unrelated to the familiar predator–prey oscillations in ecology.

Methods

Payoffs. If the umpires are all honest, the payoff matrix for OCs, PCs, ODs, and PDs is

$$Q_H = \begin{pmatrix} b-c-f & b-c-f & -c-f & 0 \\ b-c-f-h & b-c-f-h & -c-f-h & -h \\ b-f-A & b-f-A & -f-A & 0 \\ -h & -h & -h & -h \end{pmatrix}.$$

If the umpires are all corrupt, the corresponding payoff matrix for the four types of players is

$$Q_C = \begin{pmatrix} b-c-f & 0 & -c-f & -c-f \\ -h & -h & -h & -h \\ b-f-B & 0 & -f-B & -f-B \\ b-f-B-h & -h & -f-B-h & -f-B-h \end{pmatrix}.$$

An honest umpire receives a payoff of $2f$, except if there is at least one PD among the two players, in which case the umpire's payoff is 0. A corrupt umpire receives a payoff of 0 if at least one among the two players is a PC. In all other cases, the umpire obtains a payoff of $2f$ and, in addition, one or two bribes B, depending on whether there are one or two defectors among the two players, respectively.

Dynamics. Individuals update their strategies by either picking another strategy at random (exploration), with a small probability μ (players) or ν (umpires), or by copying another individual's strategy with a certain likelihood (imitation). For our numerical simulations, we assume that the likelihood that X imitates a randomly chosen role model Y is given by $\Pr(X \to Y) = [1 + \exp(s(P_X - P_Y))]^{-1}$, where $s \geq 0$ is the so-called selection strength. For $s = 0$, we obtain neutral drift. Imitation occurs at random. The larger the value of s, the more relevant the payoff difference becomes. This imitation process is often called the Fermi process. For large values of s, we are close to hard imitation: $\Pr(X \to Y)$ is 1 if $P_Y > P_X$, 0 if $P_Y < P_X$, and 1/2 if $P_Y = P_X$. We stress that there exist many other models of social learning leading to essentially the same results.

Large-Population Limit. In the limiting case of M and N tending to infinity, we can use the replicator–mutator equations governing the relative frequencies $x_i = M_i/M$ and $y_j = N_j/N$,

$$\dot{x}_i = x_i\left[\sum_k Q_{ik}(y)x_k - \bar{Q}\right] - \mu x_i + \mu(1-x_i)/3, \text{ for } i = 1,2,3,4,$$
$$\dot{y}_1 = y_1(1-y_1)[P_{\mathrm{H}}(x) - P_{\mathrm{C}}(x)] - \nu y_1 + \nu(1-y_1),$$

in the state space $S = \Delta_4 \times [0,1]$, where Δ_4 is the (three-dimensional) unit simplex in four-dimensional space. For umpire frequencies y, the matrix $Q(y) = y_1 Q_{\mathrm{C}} + (1 - y_1)Q_{\mathrm{H}}$ is the average payoff matrix for the players, and $\bar{Q} = \sum_{ik} x_i Q_{ik}(y)x_k$ is the average payoff in the player population. For player frequencies x, $P_{\mathrm{H}}(x)$ and $P_{\mathrm{C}}(x)$ are the payoffs of honest and corrupt umpires, respectively; these are independent of y. It is easy to check that $P_{\mathrm{H}}(x) - P_{\mathrm{C}}(x) = 2f(x_2 - x_4)(1 + x_1 + x_3) - 2B(x_3 + x_4)(1 - x_2)$. For a detailed analysis of the replicator dynamics, see *SI Appendix*.

Rare-Exploration Limit. For small exploration rates and hard imitation (i.e., a high selection strength s), the fate of an exploration step is settled before the next exploration step occurs. We assume that the cost h of information is positive, $h > 0$. The case $h = 0$ yields very similar results, see *SI Appendix*.

When all umpires are honest, we see from the payoff matrix Q_{H} that there are five absorbing states: allOC, allPC, allOD, allPD, and OD+PD (a mixture of OD and PC). We number these states from 1 to 5, as shown in the simplex on the left side of Fig. 2.

When all umpires are corrupt, we see from the payoff matrix Q_{C} that the absorbing states are allOC, allPC, allOD, allPD, and OD+PC (a mixture of OD and PC). We number these states from 6 to 10, as shown in the simplex on the right side of Fig. 2.

As shown in the *SI Appendix*, the matrix describing the transition rates among the 10 states is

$$T = \begin{pmatrix} * & 0 & 0 & 0 & 0 & \lambda\mu/N & 0 & 0 & 0 & 0 \\ \mu/3 & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mu/3 & \mu/3 & * & 0 & \mu/3 & 0 & 0 & 0 & 0 & 0 \\ \mu/3 & \mu/6 & 0 & * & \mu/3 & 0 & 0 & 0 & \lambda\mu & 0 \\ \mu/3 & 0 & 0 & 0 & * & 0 & 0 & \lambda\mu & 0 & 0 \\ \lambda\mu/N & 0 & 0 & 0 & 0 & * & 0 & \mu/3 & \mu/3 & 0 \\ 0 & \lambda\mu & 0 & 0 & 0 & \mu/3 & * & 0 & 0 & \mu/3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & 0 & \mu/3 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mu/3 & \mu/3 & * & 0 \\ 0 & \lambda\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * \end{pmatrix},$$

where the asterisks (*) are such that each row sum equals 1. Here, μ is the (very small) probability that an exploratory step occurs in the player population (i.e., μ is the M-fold of the individual exploration rate), and $\nu = \lambda\mu$ is the corresponding probability in the umpire population. For the stationary distribution $x = (x_1,\ldots, x_{10})$, i.e., the left eigenvector of T for the eigenvalue 1, we obtain (up to normalization)

$$x = \left(\frac{N(7+12\lambda)}{3\lambda(2+3\lambda)} + 2, \frac{7+12\lambda}{2+3\lambda}, 0,0,0,2, \frac{1}{2+3\lambda}, 3,1, \frac{7+9\lambda}{3\lambda(2+3\lambda)}\right).$$

The ratio of H:C in the umpire population is $(7 + 12\lambda)N + 33\lambda + 54\lambda^2 : 7 + 48\lambda + 54\lambda^2$. If $\lambda = \nu/\mu = 1$, then the frequencies are (1/15)(19N + 30, 57, 0, 0, 0, 30, 3, 45, 15, 16), and the H:C ratio thus equals 19N + 87:109.

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