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A FORMAL DESCRIPTION OF CONTRACTUAL COMMITMENT

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ABSTRACT

A formal language for describing concepts of contractual commitment is presented. It is based on the deontic logic of von Wright, extended to include first order individuals and reference to specific times. This requies a somewhat different version of possible worlds than what von Wright uses.

The applications of this language are to the formal representation of financial and commercial contracts as well as systems of contract law and commercial regulation. This is intended as the basis for decision support system applications capable of interpreting and advising on contracts and regulations.

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I. INTRODUCTION

This paper presents a formal language for describing concepts involving contractual commitment. The language is defined using a model theoretic semantics based on "possible worlds," an approach currently popular in the literature of formal logic and linguistics (see e.g., van Fraassen (1971), Thomason (1974), Cresswell (1973)). The notation and form of presentation adopted here is based on (Dowty 1978), which serves as an excellent background tutorial.

The applications of a computer system implementing an axiomatized form of the formal language presented here are manifold. For instance, much of the legislation regarding contracts, exchange and taxation could potentially formalized in a language of this sort. Thus legal retrieval systems such as LEXIS and WESTLAW, which are based on keyword matches, could be superceded by a system performing deductions on theorems expressing the content of the pertinent laws. More than simple retrieval, such a system would be capable certain analyses which presently require the expertise of a professional lawyer. Even more important, the formulation of laws and regulations in a formal language such as proposed here would allow the system of legislation to be mechanically verified for consistency, completeness, redundancy, etc.

Such a facility could therefore help to remedy a problem cited in Lee (1980c), with large governmental bureaucracies which make, interpret or enforce these laws: the system of rules becomes much too complex for a single person to comprehend totally. Hence knowledge of the law tends to be spread between multiple individuals, so that use of the laws must contend with the coordination problems between control procedures, paper work, etc. Modification of the laws becomes all the more difficult since it involves not only the legislation itself but also these organizational coordination problems.

The generation of natural language "legalese" from the formalized versions of laws and regulations does not present difficult computational problems. A system, called AUTOTEXT, written by the author performed a similar function in a different subject domain (see Lee 1980b, appendix). Going the other way, i.e., converting formal language translations of their natural language forms, however, presents a more formidable problem. In a criticism of certain efforts to use formal languages as a tool for analyzing natural languages, Jardine (1975:229), comments

> The illusion that much has been achieved in this field may arise from the relative ease with which NL [natural language] sentences can often be generated from sentences of a formal language. But whilst this *may* be a valuable first step towards the construction of rules which "go the other way," in itself it merely corroborates the uncontroversial claim that NL can capture fragments of many formal languages.
>
> To see the gulf which lies between translation from a formal language into NL and its converse, consider definite pronouns. To generate pleasingly colloquial NL representatives for sentences of a predicate calculus it is fairly easy to write programs which eliminate or reduce repetition of names and definite descriptions by introducing definite pronouns, and which do so without introducing unacceptable ambiguities. But "going the other way" it is exceedingly difficult to write a program which disambiguates the reference of definite pronouns using contextual information to find the admissible substitutions of names and definite descriptions.

The applications we foresee for the type of work here, however, avoid this criticism. We do not claim that this formalized language has all the flexibility and nuances capable in natural language. However, the fact that a formal language does not have this flexibility is, we argue, advantageous for these types of applications. One principle difference between a formal and a natural language is that in the first case the rules of interpretation and inference are fixed, whereas in the second they depend on the consensus of the speakers, which may and often does change, even within the span of a single conversation. For instance, in the page following a particular interpretation is given to the term "explication," which the author and reader will (presumably) agree on throughout this paper, though perhaps neither of us would use that special sense of this term in other situations. In situation of legislation and regulation this is precisely the feature of natural language that one wants to avoid: the interpretation of these pronouncements should be as fixed and uniform as possible. A way of accomplishing this is to formulate these pronouncements in formal terminology that reduce the dimensions of ambiguity to a limited number of primitive terms.

The purpose of this work is thus one of "explication," Carnap's term for the task of "making more exact a vague or not quite exact concept used in everyday life or in an earlier stage of scientific or logical development" (Carnap 1947). In addition, as argued in Lee (1981), definition of a formal descriptive language is viewed as a preliminary step to the logical axiomatization and eventual automation of the concepts captured in the language.

The language described here is a model theoretic formalization of a notation, called CANDID, originally proposed in Lee (1980a). Here we will be primarily concerned with the so-called "deontic"* aspects of that notation. As in Lee (1980a), the approach here builds on the "deontic logic" of von Wright (1968). Section II is therefore a summary of von Wright's formalism and its model theoretic interpretation. Section III adds several extensions to this formalism which adapt von Wrights general concepts of obligation, etc. to specific situations of contractual commitment.

As mentioned, the mode of presentation here uses a so-called "model theoretic semantics" (also called "denotational semantics").

Briefly, the idea behind this is that there is some universe of discourse consisting of sets of *objects*. The symbols of the formal language "stand for" or denote these objects. likewise, combinations of symbols also have an exact denotation. Thus the syntactic rules describe the vocabulary of symbols and their allowable combinations while the semantic rules describe the denotation of these individual symbols and their combinatons. One particularly important set in the universe is the set {True, False}, called the set of truth values. Other sets will be added to the universe as we proceed.

II. SUMMARY OF VON WRIGHT'S DEONTIC LOGIC

A deontic logic is one which formalizes the concepts of obligation permission, obligation and prohibition. It is now generally recognized that these concepts are inter-definable —that obligation and permission are logical duals whereas prohibition is the negation of permission.

Von Wright actually presents two deontic calculi, the second being a generalization of the first. Both of these are based on a logic of action which in turn includes a concept of change.

Our summary will proceed from elementary to complex—i.e., from an ordinary propositional calculus of states, to a calculus of change, and then action, through a modul calculus to the deontic calculi.

* "deontic" refers to concepts of ethical/legal obligation, permission, and prohibition.

A. Propositional Calculus

The various stages of von Wright's deontic logic build on an elementary propositional calculus (PC): By way of introduction, and to help orient the reader to the model theoretic descriptions used throughout this paper, we present this here as the language PC.

1. *Syntax of PC*

a. *Basic expressions*

Propositional constants are denoted as single upper case letters or as an alphanumeric string of characters beginning with a capital letter, e.g., P, Q, Raining.

Metalanguage variables for propositions will be denoted as lower case Greek letters, e.g., α, β, γ, Φ, Ψ.

b. *Formation rules*

The set of *meaningful expressions*, denoted ME, is defined recursively as follows:

Syn_{PC}.1: Every propositional constant is in ME.

Syn_{PC}.2: If $\Phi \in$ ME then $\sim\Phi \in$ ME.

Syn_{PC}.3 If Φ and Ψ are in ME then so is $(\Phi \& \Psi)$.

2. *Semantic Rules*

A model M for PC is any ordered pair <D,F> such that D (the *universe of discourse)* is a non-empty set of propositional constants and F (the *interpretation function)* is any function whose domain is D and whose range is the set {False,True}, representing falsehood and truth, respectively. The semantic rules of PC define recursively for any meaningful expression Φ, the *extension* of Φ with respect to model M, abbreviated Den_M (Φ) as follows:

Sem_{PC}.1.: If Φ is any basic expression, the Den_M $(\Phi) = F(\Phi)$.

Sem_{PC}.2: If $\Phi \in$ ME then Den_M Φ = True iff Den_M $\sim\Phi$ is False, and Den_M $\sim\Phi$ is False otherwise.

Sem_{PC}.3: If Φ and Ψ are in ME, then Den_M $(\Phi \& \Psi)$ is True iff both Den_M Φ and Den_M Ψ are True.

3. Further Definitions

The symbol "::=" is a metalanguage symbol read "is defined as."

For α and β in ME

$$(\alpha \vee \beta) ::= {\sim}({\sim}\alpha \,\&\, {\sim}\beta)$$
$$(\alpha \rightarrow \beta) ::= ({\sim}\alpha \vee \beta)$$
$$(\alpha \leftrightarrow \beta) ::= (\alpha \rightarrow \beta) \,\&\, (\beta \rightarrow \alpha)$$

4. Comment: Logic Proofs in PC

So far, we have described the formal language PC, which gives precise rules for interpreting, i.e., determining the extension or denotation of, any meaningful expression.

As discussed more fully in e.g., van Fraasen (1971), a *logic* is a further specification of a formal language that in addition to the above language description also specifies certain expressions in ME as *axioms* and provides certain *transformation* or *inference* rules which, when applied repeatedly to the axioms are capable of generating any other meaningful expression in the language. The sequence of transformations which lead to a particular expression is called a *proof* and an expression derived in this way is called a *theorem*. The axioms of a logic are therefore theorems by virtue of a null transformation.

A logic for the propositional calculus language described above is as follows.

a. Axioms: (from van Fraasen 1971:78)

$Ax_{PC}.1$: $\alpha \rightarrow (\alpha \,\&\, \alpha)$

$Ax_{PC}.2$: $(\alpha \,\&\, \beta) \rightarrow \alpha$

$Ax_{PC}.3$: ${\sim}(\alpha \,\&\, \beta) \rightarrow {\sim}(\beta \,\&\, \alpha)$

$Ax_{PC}.4$: $(\alpha \rightarrow \beta) \rightarrow ({\sim}(\gamma \,\&\, \beta) \rightarrow {\sim}(\gamma \,\&\, \alpha))$

b. Inference rules:

$IR_{PC}.1$: substitution: any meaningful expression may be substituted for the metalanguage variables.

$IR_{PC}.2$: detachment (modus ponens): if α and $\alpha \rightarrow \beta$, then β

$IR_{PC}.3$: extensionality if $\alpha \leftrightarrow \beta$, then α may be substituted for β and vice versa, without changing the denotation of the expression in which it appears.

B. Formal Description of Change: The T Calculus

Von Wright interprets the meaningful expressions in PC as representing "some arbitrary state of affairs, such as that it is raining or that a certain window is shut" (von Wright 1968:13). That is, they represent some property of the (actual or possible) world, unbound with respect to time. (This interpretation is discussed in more detail later.)

The first step in extending the PC is to introduce a concept of *change* in these states of affairs. Von Wright does this by introducing a connective T, where Φ T Ψ is read Φ "and then" Ψ. For instance, if R is the proposition "it is raining" and S is the proposition "the sun is shining," then R T S indicates that "it is raining and then the sun is shining." The language for the T calculus (TC) is described as follows:

1. Syntax

a. Basic expressions

(as for PC)

b. Formation rules

The set of meaningful expressions, ME, is defined recursively as follows:

$\text{Syn}_{TC}.1$: Every propositional constant is in ME.

$\text{Syn}_{TC}.2$: If $\Phi \in$ ME then $\sim\Phi$ is in ME

$\text{Syn}_{TC}.3$: If Φ and Ψ are in ME then so is $(\Phi \& \Psi)$

$\text{Syn}_{TC}.4$: If Φ and Ψ are in ME then so is $(\Phi \text{ T } \Psi)$.

2. Semantic rules

A model M for TC is any ordered quadruple <D, J, <, F>, where D is a non-empty set of propositional constants, J is a set of points in time ordered by the predicate <, and F is any function whose domain is <D, J> and whose range is the set {False,True}.

The semantic rules of TC define recursively for any meaningful expression Φ, the *denotation* of Φ, abbreviated $\text{Den}_{M,j}\ \Phi$, as follows:

$\text{Sem}_{TC}.1$: If Φ os any basic expression, then $\text{Den}_{M,j}(\Phi) = F(\Phi,j)$

$\text{Sem}_{TC}.2$: If $\Phi \in$ ME then $\text{Den}_{M,j} \sim\Phi$ = True iff $\text{Den}_{M,j}\ \Phi$ is False, and $\text{Den}_{M,j} \sim\Phi$ is False otherwise.

$\text{Sem}_{TC}.3$: If Φ and Ψ are in ME, then $\text{Den}_{M,j}(\Phi \& \Psi)$ is True iff both $\text{Den}_{M,j}\ \Phi$ and $\text{Den}_{M,j}\ \Psi$ are True.

$\text{Sem}_{TC}.4$: If Φ and Ψ are in ME, then $\text{Den}_{M,j}(\Phi \text{ T } \Psi)$ is True iff $\text{Den}_{M,j'}\ \Psi$ is True for the unique j' such that for all j'', not $(j < j'' < j')$.

3. *Further definitions*

(same as for PC).

4. *Logic for the T Calculus*

Using the axioms inference rules for the PC logic, von Wright proposes the following additional axioms for the T calculus:

$Ax_{TC}.1$: Distributivity:
$(\alpha \vee \beta)\ T\ (\Phi \vee \Psi) \leftrightarrow (\alpha\ T\ \Phi) \vee (\alpha\ T\ \Psi) \vee (\beta\ T\ \Phi) \vee (\beta\ T\ \Psi)$

$Ax_{TC}.2$: Coordination:
$(\alpha\ T\ \beta)\ \&\ (\alpha\ T\ \Psi) \rightarrow \alpha\ T\ (\beta\ \&\ \Psi)$
earlier (von Wright 1965) this was
$(\alpha\ T\ \beta)\ \&\ (\Phi\ T\ \Psi) \leftrightarrow (\alpha\ \&\ \Phi)\ T\ (\beta\ \&\ \Psi)$

$Ax_{TC}.3$: Redundancy:
$\alpha \leftrightarrow \alpha\ T\ (\beta \vee {\sim}\beta)$

$Ax_{TC}.4$: Impossibility
${\sim}(\alpha\ T\ (\beta\ \&\ {\sim}\beta))$

5. *Additional Theorems, Comments*

$Th_{TC}.1$: $(\alpha\ T\ \beta) \vee (\alpha\ T\ {\sim}\beta) \vee ({\sim}\alpha\ T\ \beta) \vee ({\sim}\alpha\ T\ {\sim}\beta)$

$Th_{TC}.2$: $(\alpha\ T\ \alpha) \vee (\alpha\ T\ {\sim}\alpha) \vee ({\sim}\alpha\ T\ \alpha) \vee ({\sim}\alpha\ T\ {\sim}\alpha)$

This is a corollary of $Th_{TC}.1$. The four disjuncts here are regarded as the four types of elementary changes or state transformations.

$Th_{TC}.3$: ${\sim}(\alpha\ \&\ {\sim}\alpha)\ T\ \beta$
The second Principle of Impossibility.

$Th_{TC}.4$: $(\alpha\ T\ \beta) \rightarrow \alpha$

$Th_{TC}.5$: $\alpha\ \&\ (\beta\ T\ \gamma) \leftrightarrow (\alpha\ \&\ \beta)\ T\ \gamma$

$Th_{TC}.6$: $((\alpha\ T\ \beta)\ T\ \gamma) \leftrightarrow (\alpha\ T\ (\beta\ \&\ \gamma))$

Comment (me): As indicated by $Th_{TC}.4$ and $Th_{TC}.6$, the perspective of the T connective is from the time of the left argument--i.e., the right argument is asserted as a state that ***will follow***, but is yet in the future.

Comment (VW): "The connective T is ***not*** associative. $(\alpha\ T\ \beta)\ T\ \gamma$ is not equivalent to $\alpha\ T\ (\beta\ T\ \gamma)$. The first expression refers, in fact, to ***two*** successive points in time only, the second refers to ***three***."

Comment (me): This is because $(\alpha\ T\ \beta)$ "resolves to" the time-reference of its first argument. The preceding remark points out that T expressions may be iterated, e.g., $\alpha\ T\ \beta\ T\ \Phi\ T\ \Psi$, etc. However, because T is not associative this would be syntactically ambiguous. We therefore adopt the convention of evaluation from right to left, e.g.,

$$\alpha\ T\ \beta\ T\ \Phi\ T\ \Psi ::= (\alpha\ T\ (\beta\ T\ (\Phi\ T\ \Psi)))$$

C. Formal Description of Action: The TI Calculus

Von Wright portrays *action* as a composite concept. This depends on another connective, "I" for "instead of," which behaves similarly to T. Indeed, the axioms he proposes which govern I are exactly analogous to those for T. In (von Wright 1967:124-5), von Wright comments:

> The description to the left of *I* is, in the *I*-expression, asserted to hold true of a world in which there is a certain agent. The description to the right holds true of the world which would be, if from the world which is we remove (in thought) the agent.
>
> This "experiment of thought" calls for some comments. The "removal" of the agent does not mean the removal (in thought) of him *body*. The physical presence of the agent may have a causal influence on the world which is not at all connected with his actions. His physical absence would then make a difference to the world, —but *this* difference does not tell us anything about his actions. The "removal" of the agent is the removal (in thought) of whatever *intentions* he may have. It is, therefore, the removal of his *qua agent*.
>
> One could substitute for this experiment of thought one in which the contrast is between a world in which the agent is present physically and a world from which he is about physically. Then the comparison of the states would tell us for which changes and non-changes the agent, through his presence, is *causally responsible*. This class of changes (and not-changes) includes, but is not necessarily included in, the class of changes (and not-changes) for which he is responsible also *qua agent*.

In von Wright (1968:44-45), he adds:

> Both connectives, "*T*" and "*I*", could be called "co-ordinators of possible worlds." "*T*" coordinates the world which *is* now and the world which *will be* next. "*I*" coordinates the world *as it is* with an agent in it and the world *as it would be*, if the agent remained passive.

An *action*, indicating the effect of some agent to change the world, involves the combination of a T expression and an I expression in what is called a TI expression:

$$\alpha \, \mathrm{T} \, (\beta \, \mathrm{I} \, \gamma)$$

is read "α and next β instead of γ," i.e., that because of the influence of some (unspecified) agent, the world changes from state of affairs α to β instead of γ, as it would have without the agent.

Since the connective I really only has interest when combined with T in TI expressions, we skip over a separate description of the "I calculus," and go directly to a statement of the language for the TI calculus, TIC. We see that a new dimension is introduced at this level, that of the application of a proposition not only to a point in time, but also to one or another "possible worlds." At the moment we will assume this to be understood without further explanation. The concept of a possible world will be examined in more detail later on.

THE LANGUAGE TIC:

1. Syntax

a. Basic expressions

(as for PC)

b. Formation rules

The set of meaningful expressions, ME, is defined recursively as follows:

Syn_{TIC}.1: Every propositional constant is in ME.

Syn_{TIC}.2: If $\Phi \in$ ME then $\sim\Phi$ is in ME.

Syn_{TIC}.3: If Φ and Ψ are in ME then so is:

$$(\Phi \,\&\, \Psi)$$
$$(\Phi \text{ T } \Psi)$$
$$(\Phi \text{ I } \Psi)$$

2. Semantic Rules

A model M for TC is any ordered sextuple, $\langle D, I, Ins, J, <, F\rangle$, where D is a non-empty set of propositional constants, I is a set of possible worlds, Ins is a two place relation coordinating possible worlds, J is a set of times, $<$ is a linear ordering on J, and F is any function whose domain is $\langle D, I, J\rangle$ and whose range is the set {False,True}.

The semantic rules of TIC define recursively for any meaningful expression Φ, the denotation of Φ, abbreviated $Den_{M,i,j}\,\Phi$, as follows:

Sem_{TIC}.1: If Φ is any basic expression, then $Den_{M,i,j}\,\Phi = F(\Phi,i,j)$

Sem_{TIC}.2: If $\Phi \in$ ME then $Den_{M,i,j} \sim\Phi$ = True iff $Den_{M,i,j}\,\Phi$ is False, otherwise $Den_{M,i,j} \sim\Phi$ = False.

Sem_{TIC}.3: If Φ and Ψ are in ME, then $Den_{M,i,j}\,(\Phi \,\&\, \Psi)$ is True iff both $Den_{M,i,j}\,\Phi$ and $Den_{M,i,j}\,\Psi$ are True.

Sem_{TIC}.4: If Φ and Ψ are in ME, then $Den_{M,i,j}\,(\Phi \text{ T } \Psi)$ is True iff $Den_{M,i,j}\,\Phi$ is True and $Den_{M,j,j'}\,\Psi$ is True for the unique j' such that for all j'', not $(j < j'' < j')$.

Sem_{TIC}.5: If Φ and Ψ are in ME, then $Den_{M,i,j}\,(\Phi \text{ I } \Psi)$ is in ME iff $Den_{M,i,j}\,\Phi$ is True and $Den_{M,i',j'}$ is True for some world i', such that $\langle j, j'\rangle \in$ Ins and for all times, j'.

3. *Further Definitions*

(same as PC).

4. *Logic for the TI Calculus*

Using the inference rules and axioms for the PC logic, as well as the axioms for the TC logic, additional axioms are provided here which control the I connective. As can be seen, they parallel those for T.

For all α, β, Φ, and Ψ in ME:

$Ax_{TIC}.1$: $(\alpha \vee \beta)\ I\ (\Phi \vee \Psi) \leftrightarrow (\alpha\ I\ \Phi) \vee (\alpha\ I\ \Psi) \vee (\beta\ I\ \Phi) \vee (\beta\ I\ \Psi)$

$Ax_{TIC}.2$: $(\alpha\ I\ \beta)\ \&\ (\alpha\ I\ \Phi) \rightarrow \alpha I(\beta\ \&\ \Phi)$

$Ax_{TIC}.3$: $\alpha \leftrightarrow \alpha\ I\ (\beta \vee {\sim}\beta)$

$Ax_{TIC}.4$: ${\sim}(\alpha\ I\ (\beta\ \&\ {\sim}\beta))$

D. Modals and the Deontic Calculus

von Wright introduces the formal concepts of permission and obligation by extension from interpretations of modal logic.

In modal logic, the notation "$\Diamond\Phi$" commonly used to indicate "it is possible that Φ." In the terms and to describe the semantics of TIC this would have the interpretation: If $\Phi \in$ ME then $Den_{M,i,j}(\Diamond\Phi)$ is True iff $Den_{M,i',j'}(\Phi)$ is True for some $i' \in I$ and some $j' \in J$.

That is, $\Diamond\Phi$ is true if and only if Φ is true in some possible world at some time. The dual concept of possibility, necessity, is denoted $\Box\Phi$ and is defined as follows:

$$\Box\Phi ::= {\sim}\Diamond{\sim}\Phi$$

These two operators refer to *logical* possibility and necessity. That is, $\Box\Phi$ indicates Φ to be tautological, ${\sim}\Diamond\Phi$ indicates that Φ is contradictory. Between these two is the notion of contingent truth, indicated by $\Diamond\Phi$.

Within this area of logically contingent truth, one can apply the prevailing physical theories and designate certain logically contingent truths to be impossible or necessary according to the laws of nature. If we designate the quality of a world being naturally possible by "Nat," we can then define this more restricted concept of natural possibility, $(\Diamond_N \Phi)$ as: If $\Phi \in$ ME then $Den_{M,i,j} \Diamond_N \Phi$ is True iff $Den_{M,i',j'}(\Phi)$ is True for some $j' \in I$ such that Nat(j'), and some $j' \in J$.

The concepts of permission and obligation are developed in analogous fashion. Here, instead of qualifying contingent truth with possibility according to natural laws, it is qualified by its acceptability under some code of ethics or legal system. For the applications we have in mind, this will be the system of laws of some sovereign government (or perhaps a world governing body). The quality of a world being permissible in this system will be designated as "Per." The corresponding concept of deontic possibility might thus be denoted as "$\Diamond_D \Phi$." However, following von

Wright, we will use the more suggestive notation, $P\Phi$, to indicate that "Φ is permitted."

It's semantic interpretation would then be as follows: If $\Phi \in$ ME then $Den_{M,i,j}$ ($P\Phi$) is True iff $Den_{M,i',j'}$ (Φ) is True for some i' $\in$ I and some j' $\in$ J such that Per(i').

The concept of obligation or deontic necessity, abbreviated "O", is defined as the logical dual:

$$O\Phi ::= \sim P \sim \Phi.$$

Following the semantic definition, this says that Φ must be true in all permitted worlds at all times.

Natural possibility, we observed, was a restriction of the concept of logical possibility. Correspondingly, deontic possibility is reasonably viewed as a restriction on natural possibility. Von Wright (1967:133-4), notes (using "M" for "$\Diamond_N$"):

> The concept of possibility within the limits of natural law (including the laws of "human nature") we have denoted by "*M*". The concept of possibility within the limits of a normative order we shall denote by "*P*." It seems plausible to regard "*P*" as the narrower concept in the sense that the expression "*P*(—)" entails the expression "*M*(—)," when the blanks in both expressions are filled by the same description of an action or a life. To accept this relation between 'P' and 'M' is tantamount to accepting a (rather strong) version of the well-known principle which is usually formulated in the words "ought implies can."

The language of the deontic calculus, DC, can now be summarized:

1. Syntax of DC

a. Basic expressions

(same as for PC)

b. Formation rules

Same as for TIC with the addition:

Syn_{DC}.4: If Φ is in ME then $P\Phi$ is in ME.

2. Semantic Rules

A model M for DC is any septtuple <D, I, Ins, Per, J, <, F>, where D is a non-empty set of propositional constants, I is a set of possible worlds, Ins is a two place relation coordinating possible worlds, Per is a subset of I (the permissible worlds), J is a set of times, < is a linear ordering on J, and F is any function whose domain is <D, I, J> and whose range is the set {False,True}.

The semantic rules of DC define recursively for any meaningful expression Φ, the extension of Φ, denoted $Den_{M,i,j}$ Φ as follows:

Sem_{DC}1-5: (Correspond to semantic rules 1-5 for TIC)

Sem_{DC}.6: If Φ is in ME then $Den_{M,i,j}$ PΦ = True iff $Den_{M,i',j'}$ Φ = True for some i' ∈ Per and some j'.

3. *Additional Definitions*

Same as for PC with the addition:

$$O\,\Phi ::= {\sim}P\,{\sim}\Phi.$$

4. *Logic for the Deontic Calculus*

a. *Inference rules*

(Same as for PC).

b. *Axioms*

The axioms of PC.

The 4 axioms for T (presented for TC).

The 4 axioms for I (presented for TIC).

Plus:

Ax_{DC}.1: $P(\Phi \vee \Psi) \leftrightarrow P\Phi \vee P\Psi$

Ax_{DC}.2: $P\Phi \vee P\,{\sim}\Phi$

III. SEMANTIC INTERPRETATION

It is important to note how von Wright intends the variables in his calculi to be interpreted. In von Wright (1965:294): the variables (and, presumably their truth functional compounds) refer to "generic propositions" which "are not true or false 'in themselves.' They have a truth-value only relative to a (point in) time. They may be true of one time, false of another. And they may be repeatedly true and false. Let the generic proposition be, e.g., that it is raining. It may be true of today, false of tomorrow, but true again of the day after tomorrow. (The relativity of generic propositions to a location in space will not be considered.)"

In von Wright (1967) he comments:

> The notion of a state of affairs is thus basic to the notion of change. I shall not attempt to answer here the question what a state (of affairs) is. I shall confine myself to the following observation:

One can distinguish between states of affairs in a *generic* and an *individual* sense. Individually the same state, e.g., that the sun is shining in Pittsburgh on 18 March 1966 at 10 a.m., obtains only once in the history of the world. Genercally the same state, e.g., that the sun is shining, can obtain repeatedly and in different places. Of the two senses, the generic seems to me to be the primary one. An individual state is, so to speak, a generic state instantiated ("incarnated") on a certain occasion in space and time.

In the sequel "state" will always be understood in the generic sense. As schematic descriptions of generic states we shall use the symbols $p,q,r,...$, or such letters with an index-numeral.

Let us assume that the total state of the world on a given occasion can be completely described by indicating for every one of a finite number n of states $p_1, \ldots, p_n$ whether it obtains or does not obtain on that occasion. A description of this kind is called a *state- description*. As is well known, the number of possible total states is 2^n if the number of ("elementary") states is n. We can arrange them in a sequence and refer to them by means of state-descriptions: $s_1, \ldots, s_{2n}$.

A world which satisfies the above assumption could be called a *Wittgenstein- world*. It is the kind of world which Wittgenstein envisaged in the *Tractatus*. I shall not here discuss the (important) ontological question, whether our real world *is* a Wittgenstein-world, or not. The answer is perhaps negative. But nobody would deny, I think, that, as a simplified model of "a world," Wittgenstein's idea is of great theoretical interest—and state-descriptions of great practical importance. Our study of changes and actions will throughout employ this model.

In a reply to a critique of this paper, von Wright adds:

I agree with Robison that the distinction between generic and individual states of affairs is problematic. An individual state is spatio-temporally fully specified. A generic state can be generic in the spatial and individual in the temporal component; or *vice versa;* or it can be generic in both components. A description of the total state of the world must, of course, not contain both p and not-p. Therefore, if we let "the world" embrace the whole of space, any generic state of affairs p, the presence or absence of which may be a characteristic of the world, must be individualized in the spatial component. p could then be, e.g., the state that *it is raining in Pittsburgh.* If, on the other hand, we confine "the world" to a specified location ("point") in space, the states of affairs which characterize it need not be individualized in either component. p could now be, e.g., the state that *it is raining*.

In von Wright (1968:13) he starts with the simple explanation: "Let next 'p' represent some arbitrary state of affairs, such as that it is raining or that a certain window is shut." Later, p. 16, he adds:

> A few words should be said about the reading of the formulae. In my first construction of a system of deontic logic the variables were treated as schematic names of actions. According to this conception, "P_p could be read "It is permitted to p." This conception, however, is connected with difficulties and inconveniences. It is, first of all, not clear whether the use of truth-connectives for forming compound names of action is logically legitimate. It is, furthermore, obvious that, on this view of the variables, higher order expressions become senseless. "P_p" itself cannot be the name of an action; therefore it cannot occur within the scope of another deontic operator either.
>
> It now seems to me better to treat the variables as schematic sentences which express propositions. This agrees with the course "taken by most subsequent authors on deontic logic. Instead of "proposition" we can also say "possible state of affairs."—According to this conception, "P_p" may be read "it is permitted that it is the case that) p."
>
> Against this reading, however, it may be objected that it does not accord very well with ordinary usage. Only seldom do we say of a state of affairs that it is permitted, obligatory, or forbidden. Usually we say this of actions. But it is plausible to think that, when an action is permitted, etc., then a certain state of affairs is, in a "secondary" sense permitted, etc., too. This is the state which, in a technical sense to be explained later, can be called the result of the action in question.
>
> We can take account of this combination of action and resulting state of affairs in our reading of deontic formulae. Instead of saying simply "to p" or "that p" we employ the phrase "see to it that p". "The formulae "P_p" is thus read "it is permitted to see to it that (it is the case that) p" or "one may see to it that p." It should be noted, however, that this reading, though convenient and natural, is somewhat restrictive since it applies *only* to norms which are rules of action.

On p. 18 he adds the additional definitions:

> The single variables will be said to represent *elementary states* within the universe. The 2^n different (order of conjuncts being irrelevant) so-called *state- descriptions* in terms of the n variables represent *total states* of the universe. These total states will also be called *possible worlds* (in the universe of elementary states represented by the propositional variables of the set).

As these excerpts illustrate, von Wright uses two kinds of variables (depending on his purposes), an (elementary) *state* (denoted as p,q etc. as in the preceding syntax), and a composite notion which he variously calls a *state description*, *total state*, *Wittgenstein world*, or *possible world*. We belabor this in order to enunciate a change we propose to make in this interpretation.

Von Wright's notion of a possible world seems similar to one which Cresswell (1973:3-4]) attributes to Carnap:

> Carnap recognizes his debt to Wittgenstein for the notion of a possible world and introduces the notion of a *state- description*. If we assume that there are a set of atomic sentences which may be either true or false without prejudice to the truth or falsity of any other atomic sentences then a state-description is a class which contains for every atomic sentence either that sentence or its negation.

However, this notion is somewhat modified in current uses of possible world semantics. Cresswell observes (p.4):

> The big advance in the semantical study of modal logic after Carnap was *to remove possible worlds from the dependence on language* which they have in Carnap's work and treat them as primitive entities in their own right, in terms of which the semantical notions required by the modal system can be defined.

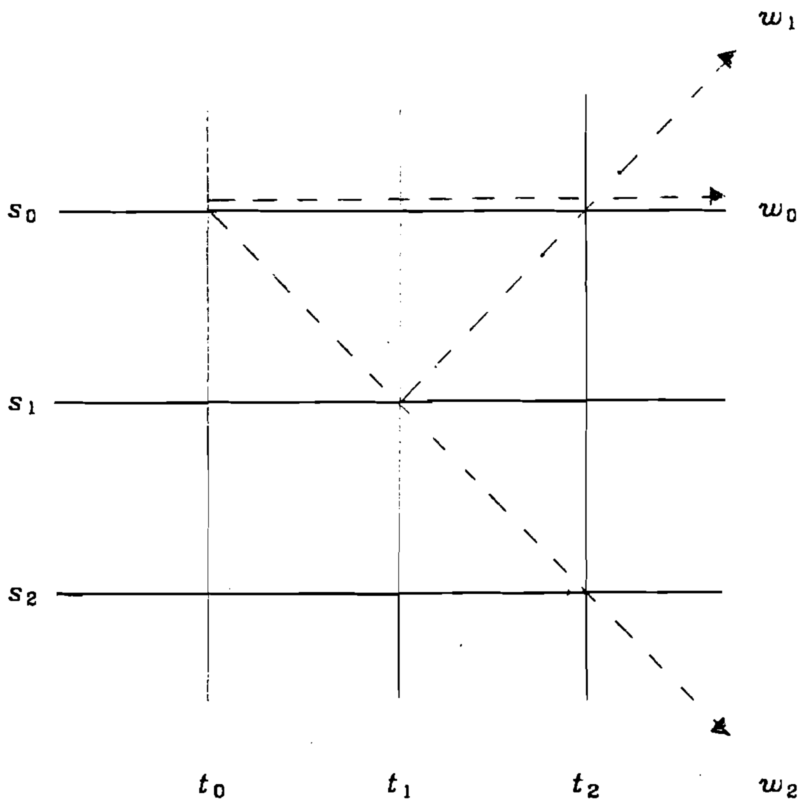
In the remainder of this paper we too adopt the view of a possible world as a primitive concept. This view may be related to that of von Wright by means of an intermediate interpretation. Let us refer to von Wright's concept of a possible world as a "VW world" and the more current view of a possible world, as reported by Cresswell, as a "C-world." Let us call the view of a possible world by a third, intermediate interpretation on "I world."

Recall that a VW world was unbound with respect to time. An I world will be a VW world extended across time. An I world is thus *individuated* by a state description at a particular point in time. An I world is therefore by this interpretation a sequence of state description/time point pairs. This is illustrated in Figure 1. s_0, s_1, and s_2 indicate state descriptions, t_0, t_1, and t_2 indicate time points and w_0, w_1, and w_2 indicate possible worlds.

The possible worlds are therefore the paths through these states across time, e.g.,

$$w_0 = \{\langle s_0, t_0\rangle, \langle s_0, t_1\rangle, \langle s_0, t_2\rangle\}$$
$$w_1 = \{\langle s_0, t_0\rangle, \langle s_1, t_1\rangle, \langle s_0, t_2\rangle\}$$
$$w_2 = \{\langle s_0, t_0\rangle, \langle s_1, t_1\rangle, \langle s_2, t_2\rangle\}$$

etc.



There are in total 27 such paths, hence 27 possible worlds distinguishable from these three state descriptions and three time points. In general, for m state descriptions and n points in time there will be m^n I worlds (i.e., one choses from m possible states at each of n points in time).

If time is considered to be continuous, the set of I worlds obviously becomes infinite over any interval of time.

Under this interpretation, von Wright's state descriptions become predicates of possible worlds, predicates which uniquely identify an I world at a given time.

The difference between an I-world and a C-world is in the linguistic dependence of the former. In an I-world, a state description, a conjunct consisting of each elementary proposition or its negation, serves to uniquely identify the I world at a point in time. A C-world does not have this feature. For a given state description and point in time, there may be many C-worlds which the vocabulary is not refined enough to distinguish.

In the discussion to follow, we will interpret possible worlds to be C-worlds, unless otherwise indicated.

We now proceed to re-interpret von Wright's operators and connectives according to this view.

IV. EXTENSIONS FOR DESCRIBING CONTRACTUAL COMMITMENT

A contractual commitment (as we view it) differs from the general concept of obligation in that it is an obligation for some particular party, say x, to another party, say y, to do some action, e.g., Φ, within some specified time interval, e.g., before time t. This requires that we bring variables and constants for individual entities and times into the object language.

A. First Order Predicate Calculus

Let us consider first the problem of recognizing entities within the object language. This involves, essentially, extending the role played by the propositional calculus, to that of a first order predicate calculus (FOPC), i.e., introducing individual constants and variables as well as quantifiers.

Partly to set the stage for later developments, we will introduce the FOPC as a "type theoretic" language (see e.g., Dowty (1978: 40-55)). Basically, this approach assigns a syntactic category, called a type, to each of the symbols in the language, and then proceeds to describe further characteristics of the language in terms of relationships between in these types. Principally, this allows greater compactness in the language specification.

At this level, there are two basic types, e (for entity) and t (for truth value). Individual constants and variables will have type e, propositions have type t. More complex symbols will be denoted as relations between types. To make effective use of the notation of functional application, these will be confined to two place relations which may however have other relations in either of their places. So, for instance,

<e,t> is a one place predicate (mapping entities to truth values)

<e,<e,t>> is a two place predicate (mapping entities to one place predicates).

<t,t> is an operator (mapping truth values to truth values)

<t,<t,t>> is a connective (mapping a truth value to an operator).

With this brief background, we introduce the language FOPC.

1. Syntax of FOPC

1 The set of ***types***, defined as follows:

a) e is a type

b) t is a type

c) if a and b are any types, then <a,b> is a type.

2. The basic expressions of FOPC consist of:

constants for each type a

-- constants of type e are denoted as a lower case alpha numeric string beginning with a "@", e.g., @a, @ron, @alec

-- constants of type t or <a,t> where a is any type, are denoted by an alphanumeric string beginning with a capital letter, e.g., P, Q, Raining, Married.

-- all other constants will be assigned special notations in the syntactic rules and definitions.

Variables for each type a.

-- variables of type e are denoted as a lower case alpha numeric string beginning with a letter, e.g., x, y, z1, z2.

-- variables for all other types are denoted as an alpha numeric string, beginning with a "?", e.g., ?P, ?Q.

Note: in the metalanguage, the italicized letters u and v will be used to denote variables, and as before, lower case Greek letters denote constants.

a. Formation rules of FOPC

The set of ***meaningful expressions*** of type a, denoted ME_a, for any type a (i.e., the well formed expressions for each type) is defined recursively as follows:

Syn_{FOPC}.1: For each type a, every variable and constant of type a is in ME_a.

Syn_{FOPC}.2: For any types a and b, if $\alpha \in ME_{<a,b>}$ and $\beta \in ME_a$, then $\alpha(\beta) \in ME_b$

Syn_{FOPC}.3: If $\Phi \in ME_t$ and u is a variable (of any type) then $\forall u\ \Phi \in ME_t$

Syn_{FOPC}.4: If $\Phi \in ME_t$ then $\sim\Phi \in ME_t$

Syn_{FOPC}.5: If Φ and Ψ are in ME_t, then $[\Phi \;\&\; \Psi] \in ME_t$

2. Semantics of FOPC

Given a non-empty set D (regarded as the domain of *individuals* or *entities)*, the set of ***possible denotations*** of meaningful expressions of type a, abbreviated D_a, is given by the following recursive definition:

(1) $D_e = D$

(2) $D_t = \{False, True\}$

(3) $D_{<a,b>} = D_b^{D_a}$ for any types a and b, where Y^X stands for "the set of all possible functions from the set X into the set Y."

A *model* for FOPC is an ordered pair <D,F> such that D is as above and F is a function assigning a denotation to each constant of FOPC of type a from the set D_a.

An *assignment of values to variables* (or simply a *variable assignment)*, g is a function assigning to each variable a denotation from the set D_a for each type a.

The denotation of an expression α relative to a model M and variable assignment g, abbreviated $Den_{M,g}(\alpha)$ is defined recursively as follows:

Sem_{FOPC}.1: If x is a constant, then $Den_{M,g}(\alpha) = F(\alpha)$.

Sem_{FOPC}.2: If x is a variable, then $Den_{M,g}(\alpha) = g(\alpha)$.

Sem_{FOPC}.3: If $\alpha \in ME_{<a,b>}$ and $\beta \in ME_a$, then $Den_{M,g}(\alpha(\beta)) = Den_{M,g}(\alpha)(Den_{M,g}(\beta))$ where Y(X) stands for "the value of the function Y when applied to the argument X."

Sem_{FOPC}.4: If $\Phi \in ME_t$, then $Den_{M,g}(\sim\Phi)$ is True iff $Den_{M,g}(\Phi)$ is False, and $Den_{M,g}(\sim\Phi)$ is False otherwise.

Sem_{FOPC}.5: If Φ and Ψ are in ME_t, then $Den_{M,g}[\Phi \& \Psi]$ is True iff both $Den_{M,g}(\Phi)$ and $Den_{M,g}(\Psi)$ are True.

Sem_{FOPC}.6: If $\Phi \in ME_t$ and u is a variable, then $Den_{M,g}(\forall u\ \Phi)$ = True iff for all g' such that g' is exactly like g except possibly for the value assigned to u, $Den_{M,g'}(\Phi)$ = True.

3. *Further Definitions*

For α and β in ME_t:

$$[\alpha \vee \beta] ::= \sim[\sim\alpha \ \& \sim\beta]$$
$$[\alpha \rightarrow \beta] ::= [\sim\alpha \vee \beta]$$
$$[\alpha \leftrightarrow \beta] ::= [\alpha \rightarrow \beta] \ \& \ [\alpha \rightarrow \beta]$$

For $\Phi \in ME_t$ and u a variable

$$[\exists u\ \Phi] ::= [\sim\forall u \sim \Phi]$$

B. Lambda Abstraction

One additional concept will be useful, that of so-called *lambda abstraction*. Dowty (1978:55) introduces this by comparison to the familiar notation for defining a set by means of a predicate, e.g., if Φ is a one place predicate,

$$\{x \mid \Phi(x)\}$$

is the set of individuals in the domain which satisfy this predicate. The operator λ, is used in the object language to the same effect, e.g.,

$$\lambda x\ \Phi x$$

denotes the set of individuals in the domain which satisfy Φ. More specifically, if u is of type e, and e, and $\Phi \in ME_t$, then $\lambda u[\Phi u]$ is the set of <e,t> pairs mapping individuals to truth values.

The converse concept to lambda abstraction is called *lambda conversion* which is essentially only functional application. E.g., for a variable v, of type e,

$$\lambda u\ [\Phi\ u]\ (v)$$

applies the variable v to the function $\lambda u[\Phi u]$, resulting in $\Phi(v)$. This seems to bring us back where we started from in the first place. The advantage however, as Dowty points out, is to make the syntax of the language "flexible." More to the point, it allows reference to predicates and other functions as extensional sets, independent of the variables to which they are applied. (More extensive explanation is given in Dowty, (1978:Section 1.8), and Cresswell, (1973:chapter 6).)

The use of lambda abstraction is not limited to variables of type e, but in fact may be used with variables of any type. Syntactically, it behaves just like the quantifiers, serving to bind the variables.

Recognition of lambda abstraction and conversion in the calculus requires the following additional syntactic and semantic rules:

Syn_λ.1: If $\alpha \in ME_a$ and u is a variable of type b, then $\lambda u\ \alpha \in ME_{<b,a>}$.

Syn_λ.2: If $\alpha \in ME_{<a,b>}$ and $\beta \in ME_a$, then $\alpha\ (\beta) \in ME_b$.

Sem_λ.1: If $\alpha \in ME_{<a,b>}$ and u is a variable of type b, then $Den_{M,i,j,g}\ (\lambda\ u\ \alpha)$ is that function h with domain D_b such that for any object x in that domain $h(x) = Den_{M,i,j,g'}\ (\alpha)$, where g' is that value assignment exactly like g with the possible difference that g'(u) is the object x.

Sem_λ.2: If $\alpha \in ME_{<a,b>}$ and $\beta \in ME_a$, then $Den_{M,i,j,g}\ (\alpha\ (\beta))$ is $Den_{M,i,j,g}\ (\alpha)\ (Den_{M,i,j,g}\ (\beta))$ (i.e., the result of applying the function $Den_{M,i,j,g}\ (\alpha)$ to the argument $Den_{M,i,j,g}\ (\beta)$).

We should note that the introduction of lambda abstraction by comparison to definition of sets by some critical predicate can be slightly misleading. For u a variable of type a, and Φ a predicate,

$$\{u \mid \Phi\ u\}$$

is a set of individuals of type a, i.e., the subset of all individuals of type a which satisfy Φ.

$$\lambda\ u\ \Phi$$

on the other hand is a set of ordered pairs, <a,t> one for each element of type a in the domain, and whose second place is True if this individual satisfies Φ, False otherwise.

On the other hand, it is seen that the basic information contained in these two concepts is equivalent. Correspondingly, the predicate of elementhood,

$$u \in \alpha$$

has its analog in lambda conversion (functional application):

$$\alpha(u).$$

C. First Order Deontic Calculus

If we now combine this definition of the FOPC language with the extensions von Wright added to the PC, we arrive at a first order deontic calculus, FODC. Its description would be as follows:

1. *Syntax of FODC*

a. *Basic expressions*

(same as for FOPC)

b. *Formation rules*

Syn_{FODC}.1-5: Same as Syn_{PC}.1-Syn_{PC}.5.

Syn_{FODC}.6;7: Same as Syn_{λ}.1, Syn_{λ}2.

Syn_{FODC}.8-9: If Φ and Ψ are in ME_t, then so are

Syn_{FODC}.8: $[\Phi \ T \ \Psi]$

Syn_{FODC}.9: $[\Phi \ I \ \Psi]$

Syn_{FODC}.10: If $\Phi \in ME_t$, then so is $[P \ \Phi]$.

2. *Semantics of FODC*

Given a non-empty set D (the domain of entities), the set of possible denotations of meaningful expressions of type a, abbreviated D_a, is given by the following recursive definition:

(1) $D_e = D$

(2) $D_t = \{False, True\}$

(3) $D_{<a,b>} = D_b^{D_a}$ for any types a and b.

A *model* for FODC is an ordered septuple <D, I, Ins, Per, J, <, F> where D is as above, I is a set of possible worlds, Ins is a two place relation on I coordinating possible worlds (those with and those without the influence, Per is a subset of I (the permissible worlds), J is a set of times, < is a linear ordering on J and F is a function that assigns an appropriate denotation to each constant of FOPC relative to each pair <i,j> for $i \in I$ and $j \in J$. (Thus "$F(\alpha, <i,j>) = \beta$" is to be interpreted as that the extension (denotation) of α in possible world i at time j is the object β.)

The set of *possible denotations* of type a is defined as follows:

$$D_e = D$$

$$D_j = J$$

$$D_t = \{False, True\}$$

$$D_{<a,b>} = D_b^{D_a} \text{ for any types a and b.}$$

A variable assignment, g, is a function assigning to each variable a denotation from the set D_a for each type a.

The denotation of an expression α relative to a model M, a possible world i, time j and value assignment g, abbreviated $Den_{M,i,j,g}(\alpha)$, is defined recursively as follows:

Sem_{FODC}-1: If α is a constant, then $Den_{M,i,j,g}(\alpha) = F(\alpha)$.

Sem_{FODC}-2: If α is a variable, then $Den_{M,i,j,g}(\alpha) = g(\alpha)$.

Sem_{FODC}-3: If $\alpha \in ME_{<a,b>}$ and $\beta \in ME_a$, then $Den_{M,i,j,g}(\alpha(\beta)) = Den_{M,i,j,g}(\alpha)(Den_{M,i,j,g}(\beta))$.

Sem_{FODC}-4: If $\Phi \in ME_t$, then $Den_{M,i,j,g}(\sim\Phi)$ is True iff $Den_{M,i,j,g}(\Phi)$ is False and $Den_{M,i,j,g}(\sim\Phi)$ is False otherwise.

Sem_{FODC}-5: If Φ and Ψ are in ME_t, then $Den_{M,i,j,g}[\Phi \& \Psi]$ is True iff both $Den_{M,i,j,g}(\Phi)$ and $Den_{M,i,j,g}(\Psi)$ are True.

Sem_{FODC}-6: If $\Phi \in ME_t$ and *u* is a variable, then $Den_{M,i,j,g}(\forall u\, \Phi)$ = True iff for all g' such that g' is exactly like g except possibly for the value assigned to u, $Den_{M,i,j,g'}(\Phi) = 1$.

Sem_{FODC}-7: If Φ and Ψ are in ME_t, then $Den_{M,i,j,g}[\Phi\ T\ \Psi]$ is True iff $Den_{M,i,j,g}(\Phi)$ is True and $Den_{M,i,j',g}(\Psi)$ is True for the unique j' such that for all j'', not ($j < j'' < j'$).

Sem_{FODC}-8: If Φ and Ψ are in ME_t, then $Den_{M,i,j,g}[\Phi\ I\ \Psi]$ is True iff $Den_{M,i,j,g}(\Phi)$ is True and $Den_{M,i',j',g}(\Psi)$ is True for some world i', such that $<i,i'> \in Ins$, and for all times, j'.

Sem_{FODC}-9: If $\Phi \in ME_t$ then $Den_{M,i,j,g}[P\ \Phi]$ = True iff $Den_{M,i',j',g}(\Phi)$ = True for some $i' \in Per$ and some j'.

3. *Further Definitions*

For α and β in ME_t

$$[\alpha \vee \beta] ::= \sim[\sim\alpha \,\&\, \sim\beta]$$
$$[\alpha \rightarrow \beta] ::= [\sim\alpha \vee \beta]$$
$$[\alpha \leftrightarrow \beta] ::= [\alpha \rightarrow \beta] \,\&\, [\beta \rightarrow \alpha]$$

For $\Phi \in ME_t$ and u a variable,

$$[\exists u\ \Phi] ::= [\sim\forall u \sim\Phi]$$

For $\Phi \in ME_t$

$$[O\ \Phi] ::= [\sim P \sim\Phi]$$

The next problem to be considered is the recognition of times *within* the object language. This can be done relatively easily. Adopting a notation suggested by Rescher and Urguhart (1971), the expression

$$(R\ u)\Phi$$

is read that the formula Φ is "realized" at time u. This can be assimilated into the preceding FODC language by means of the following additions.

Consistent with our earlier megtalanguage notation using J as a set of times, with j used to indicate elements of J, we revise the specification of types as follows:

e is a type
j is a type
t is a type
if a and b are types, <a,b> is a type.

Variables and constants of type j and type <j,t> will be denoted in the same fashion as variables and constants of type e.

To the formation rules we add the following:

If $\Phi \in ME_t$ and u is a variable of type j, then $[(R\ u)\ \Phi] \in ME_t$.

The denotations of each type are correspondingly as follows:

$$D_e = D$$

$$D_j = J$$

$$D_t = \{False, True\}$$

$$D_{<a,b>} = D_a^{D_b} \text{ for any types a and b.}$$

The following is added to the semantic rules:

Sem_{FODC}.10: If $\Phi \in ME_t$ and u is a variable of type j, then $Den_{M,i,j,g}$ $[(R\ u)\ \Phi]$ is True iff $Den_{M,i,j',g}$ (Φ) = True for all j' = g(u).

Several additional definitions will prove useful.

$$u <= v ::= [u < v] \vee [u = v]$$
$$u > v ::= \sim[u <= v]$$
$$u >= v ::= [u > v] \vee [u = v]$$

D. Time Spans

The variables and constants of type <j,t> denote sets of times. Of special interest are sets of contiguous points in time, i.e., *time spans.* To designate this, we introduce an additional function, span, defined as follows.

For variables u,v and w of type j,

$$\text{span} ::= \lambda u\ \lambda v\ \lambda w\ [(u <= w)\ \&\ (w <= v)]$$

Note that the variables must be of type j, since the definition depends on "<," a relation only defined over the set J.

Span is thus a function of type <j, <j, <j,t>>>. By applying two (time point) arguments to it, e.g., span (u)(v), the result will be of type <j,t>, i.e., the set of points between u and v (or, strictly, the set of pairs <j,t>, indicating by a 1 in the right hand place which points on the time line are between u and v, inclusive.)

Note that by the application of a third argument, e.g., span (u)(v)(w) the result is of type t, i.e., true iff w is between or equal to u and v.

Further realization operators can be defined as convenient. For instance, for u a variable of type <j,t>, and $\Phi \in ME_t$

$$(RT\ u)\ \Phi ::= \forall v\ u(v) \rightarrow (R\ v)\ \Phi$$

Reading: Φ is "realized throughout" time span u.

$$(RD\ u)\ \Phi ::= \exists v\ u(v)\ \&\ (R\ v)\ \Phi$$

Reading: Φ is "realized during" time span u.

We have at this point extended the deontic calculus to recognize individual entities as well as temporal reference. However, several further problems remain in order to adequately describe contractual commitment.

E. Identifying the Agents of Actions

One issue is that we need to particularize actions to identify the agent involved. This entails adding an additional place to the I connective, i.e., of the form $(\alpha\ \text{I}u\ \beta)$. This will lead to a corresponding revision of the predicate Ins, call it Ins', where

$$\text{Ins}'\ (u,i',i'')$$

indicates that world i' is the case rather than i'' due to the influence of agent u.

This requires replacing the former syntactic and semantic rules for I as follows:

Syn'_{FODC}.9:If α and β are in ME_t and u is a variable or constant of type e, then $(\alpha \text{ Iu } \beta) \in ME_t$.

Sem'_{FODC}.9:If Φ and Ψ are in ME_t, and u is a variable or constant of type e, $Den_{M,i,j,g}$ $[\Phi \text{ Iu } \Psi]$ is True iff $Den_{M,i,j,g}$ (Φ) is True and $Den_{M,i'',j',g}$ (Ψ) is True for some world i' such that $\langle g(u), \langle i', i'' \rangle\rangle \in Ins'$, for all times j'.

When substituted in a TI expression this provides an explication for the sense that x *does* some action Φ.

We still however need to account for the sense that x is obligated *to y* to do Φ. Before addressing that, however, we need to introduce a notation for contingent permission and obligation.

F. Contingent Permission and Obligation

As discussed in more detail in the appendix, von Wright goes beyond the deontic definitions described so far to what he calls a "dyadic" version of the deontic logic. For various reasons (noted in the appendix), we are unable to incorporate that here. However, we do have need of an analogous concept to his contingent permission and obligation. Using a notation analogous to his, we write

$$P\ \alpha/\beta$$

to indicate that in some permissible world, both β and α are true. Contingent obligation is defined as

$$O\ \alpha/\beta ::= {\sim}P {\sim}\alpha/\beta$$

which may be interpreted that in any world, if β is true then if the world is permissible, then α is true.

The scoping and quantification may be a bit hard to follow in these explanations. To help clarify, we will temporarily make use of formal notation in the megtalanguage, distinguishing this from the object language by enclosing it in double brackets, e.g., [[ ]].

In this notation, w will be a variable for possible worlds.

Thus,

$$\begin{gathered} P\ \alpha/\beta ::= [[\exists w\ \beta(w)\ \&\ Per(w)\ \&\ \alpha(w)]] \\ O\ \alpha/\beta ::= [[{\sim}\exists w\ \beta(w)\ \&\ Per(w)\ \&\ {\sim}\alpha(w)]] \\ \leftrightarrow [[\forall w\ {\sim}\beta(w)\ V\ {\sim}Per(w)\ V\ \alpha(w)]] \\ \leftrightarrow [[\forall w\ \beta(w) \rightarrow (Per(w) \rightarrow \alpha(w))]] \end{gathered}$$

We find it useful to generalize these concepts of conditional permission and obligation to arbitrary many levels.

We therefore define

$$\begin{gathered} P(\alpha/\beta_1/\beta_2/\ldots/\beta_n) ::= \\ [[\exists w\ \beta_n(w)\ \&\ \ldots\ \&\ \beta_2(a)\ \&\ \beta_1\ (w)\ \&\ Per(w)\ \&\ \alpha(w)]] \end{gathered}$$

Analogously, we define the generalized form of conditional obligation as:

$$D(\alpha / \beta_1 / \beta_2 / \ldots / \beta_n) ::= {\sim}P({\sim}\alpha / \beta_1 / \beta_2 / \ldots / \beta_n)$$
$$\leftrightarrow [[{\sim}\exists w(\beta_n(w) \,\&\, \ldots \,\&\, \beta_2(w) \,\&\, \beta_1(w) \,\&\, Per(w) \,\&\, {\sim}\alpha(w))]]$$
$$\leftrightarrow [[\forall w\, {\sim}\beta_n(w) \vee \ldots \vee {\sim}\beta_2(w) \vee {\sim}\beta_1(w) \vee Per(w) \vee {\sim}\alpha(w)]]$$
$$\leftrightarrow [[\forall w\, \beta_n(w) \rightarrow (\ldots \rightarrow (\beta_2(w) \rightarrow (\beta_1(w) \rightarrow (Per(w) \rightarrow \alpha(w)]]]$$

(Here the additional square right bracket is meant to close all open left hand parentheses.)

To incorporate these concepts of conditional permission and obligation in the formal language, the following additions are needed:

Syn. If α, β_2, ..., β_n are all in ME_t, then $P(\alpha/\beta_2/\ldots/\beta_n)$ is in ME_t.

Sem. If α, β_1,..., β_n are all in ME_t, then $Den_{M,i,j,g}\ P(\alpha/\beta_1/\ldots/\beta_n) = 1$ iff for some i', i' $\in$ Per, and $Den_{M,i',j,g}\ (\alpha)$ is True and $Den_{M,i',j,g}\ (\beta_k)$ is True for k = 1, ..., n.

Def. If α, β_2, ..., β_n are all in ME_t, then $O(\alpha/\beta_1/\ldots/\beta_n) ::= {\sim}P({\sim}\alpha/\beta_1/\ldots/\beta_n)$.

G. The Benefactors of Contractual Commitments

As mentioned above, while the formal language is now refined to distinguish the agent of actions in contractual commitments, we yet lack a way of identifying the other party, what we might call the "benefactor" of the obligation or permission.

The commitment to this party might at first examination be considered as a sort of local obligation separate from the overall legal system represented by O and the other deontic operators. However, if when we deal with contractual, as opposed to say informal, obligation between two parties, we are nonetheless referring to obligations allowed and enforced within a broad system of contract law. There are therefore certain circumstances prescribed in law which allow x to become (legally) obligated to g to do Φ.

For instance, x's obligation to give y some object, say z, may only come in force if y pays x some sum of money (perhaps only a partial or token payment). Contracts are thus often stated as pairs of obligations, with opposite roles of the same two parties. However, neither obligation may in fact become effective until all or part of the other has been executed. These conditions for creating a contractual obligation, however, depend on the specifications of the legal system governing the parties.

(International contracts, involving perhaps several legal systems, entail further complications which we ignore here.)

By this view x becomes generally obligated to do Φ. That is however not quite the case in a contractual obligation. In a contract, if y defaults and does not do Φ, y has recourse to certain *legal actions* against x. But these do not come automatically; y must initiate them in the form of a lawsuit, or some similar type of appeal to the governing body for enforcement of his/her claims against x.

This leads us to the view that contractual obligation is not a general obligation for x to do Φ, but rather a permission on the part of y to take legal action against x if x does not do Φ. This notion of "legal action" can obviously be very complex and as well varies depending on the government having jurisdiction. I do believe though that the possibility of taking legal action is a necessary element to explicate obligation. It is therefore adopted as a primitive predicate, viz

$$LA(x,y)$$

indicates a "legal action of x against y."

With this assumption, we are now able to define a concept of contractual obligation:

$$O(x,y)\ \Phi ::= P\ LA\ (y,x)/{\sim}\Phi(x).$$

$O(x,y)\ \Phi$ has the reading that "x is obligated to y to Φ," and is defined as the permission of y to take legal action against x if x does not Φ.

Note that "O" here for contractual obligation is not the same as the O for general obligation. The two are distinguished by the presence of the two arguments in the case of contractual obligation.

As was the case with general obligation and permission, we take contractual obligation and permission to be dual concepts:

$$\begin{aligned} P(y,x)\ \Phi &::= {\sim}O(x,y)\ {\sim}\Phi \\ &::= {\sim}[P\ LA(y,x)/{\sim}({\sim}\Phi(x))] \\ &::= {\sim}P\ LA(y,x)/\Phi(x). \end{aligned}$$

Note that the places are reversed in contractual permission and its dual obligatory form. The definition says that if y permits x to Φ, then y is not permitted to take legal action against x if x does Φ.

This conforms with usual intuitions. A contractual permission of y to x allows x to do something he/she would normally be forbidden (not permitted) to do, i.e.,

$$\begin{aligned} {\sim}P(y,x)\ \Phi &::= O(x,y)\ {\sim}\Phi \\ &::= {\sim}P\ LA(y,x)/\Phi(x) \end{aligned}$$

i.e., normally, y would be allowed legal action against x if x did Φ. A permission to do Φ is thus a suspension of this right to take legal action.

The concepts of conditional obligation and permission can be extended to the contractual case:

$$\begin{aligned} O(x,y)\ \Phi/\Psi &::= P[LA(y,x)/{\sim}\Phi(x)/\Psi] \\ &::= [[\exists w\ \Psi(w)\ \&\ {\sim}\Phi(x,w)\ \&\ Per(w)\ \&\ LA(y,x,w)]] \end{aligned}$$

Reading: x is obligated to y to do Φ given Ψ is defined that it is permitted for y to take legal action against x given that x does not do Φ given Ψ which, in the symbolic metalanguage form, is in turn defined that in some permitted world, x has not done Φ, Ψ is true and y takes legal action.

Correspondingly

$$\begin{aligned} P(x,y)\ \Phi/\Psi &::= {\sim}O(y,x)\ {\sim}\Phi/\Psi \\ &::= {\sim}P\ LA(x,y)/{\sim}\Phi(y)/\Psi \\ &::= [[\forall w\ \Psi(w) \rightarrow (\Phi(y,w) \rightarrow (Per(w) \rightarrow LA(x,y,w)))]] \end{aligned}$$

Reading: the permission of x to y to do Φ given Ψ is defined (last line) that in any possible world, if Ψ is true then if y does Φ then if the world is permitted there is no legal action taken by x against y.

In all the above cases, the enforcement of the contractual obligation (or permission) has been the application (or suspension) of some "legal action" which we have adopted as a primitive concept. However, in many contracts, the enforcement is a specific action which we would want to explicate in the calculus, e.g., the right to claim ownership of some particular asset serving as collateral for a loan in the case of default.

We will indicate the relationship to an enforcement action by the connective OE read "or else."

In the case of contractual obligation this is defined:

$$O(x,y)\ \Phi\ OE\ \gamma ::= P(x,y)\ \gamma/{\sim}\Phi(x)$$

Reading: the obligation of x to y to do Φ or else γ is defined as the permission of x to y given that x does not do Φ.

This has a natural extension to cases of conditional contractual obligation:

$$O(x,y)\ \alpha/\beta\ OE\ \gamma ::= P(x,y)\ \gamma/{\sim}\alpha(x)/\beta$$

Reading: the oblitation of x to y to do x given β or else γ is defined as the permission of x to y to do γ given that x does not do α given β.

Specific enforcements may likewise be considered for contractual permission, though this is much less natural—(indeed I can think of no practical example).

The definition would go as follows:

$$P(x,y)\ \Phi_y\ OE\ \gamma ::= {\sim}O(x,y)\ {\sim}\Phi_y\ OE\ \gamma ::= {\sim}P(y,x)\ \gamma_x/\Phi(y)$$

Reading: the permission of x to y to Φ or else γ is to say that y is not obligated to x not to Φ or else γ which is to say that y does not permit x to γ given that y does Φ.

I. Formal Summary: Language CC (Contractual Commitment)

1. *Syntax of CC*

a. *Types*

Let t, e and j be any fixed objects. Then the set of *types* is defined recursively as follows:

i. t is a type

ii. e is a type

iii. j is a type

iv. If a and b are types, then <a,b> is a type.

b. *Basic expressions*

i. For each type a, CC contains a denumerably infinite set of *non-logical constants* (or simply *constants*), $C_{n,a}$, for each natural number n. The set of all constants of type a is denoted Con_a.

ii. For each type a, CC contains a denumerably infinite set of *variables* $V_{n,a}$ for each natural number n. The set of all variables of type a is denoted Var_a.

c. *Syntactic rules of CC*

The set of *meaningful expressions* of type a, denoted ME_a, is defined recursively as follows:

Syn_{CC}.1:	Every variable of type a is in ME_a
Syn_{CC}.2:	Every constant of type a is in ME_a
Syn_{CC}.3:	If $\alpha \in ME_a$ and u is a variable of type b, then $\lambda\, u\, \alpha \in ME_{<b,a>}$.
Syn_{CC}.4:	If $\alpha \in ME_{<a,b>}$ and $\beta \in ME_a$, then $\alpha(\beta) \in ME_b$.
Syn_{CC}.5:	If α and β are both in ME_a, then $\alpha = \beta \in ME_t$.
Syn_{CC}.6-7:	If Φ and Ψ are in ME_t, then the following are also in ME_t:
	Syn_{CC}.6: $\sim\Phi$
	Syn_{CC}.7: $\Phi \,\&\, \Psi$
Syn_{CC}.8:	If $\Phi \in ME_t$ and u is a variable of any type, then $\forall u\, \Phi \in ME_t$
Syn_{CC}.9:	If Φ and Ψ are in ME_t, then $\Phi\ T\ \Psi \in ME_t$
Syn_{CC}.10:	If Φ and Ψ are in ME_t and u is of type e then $\Phi\ Iu\ \Psi \in ME_t$.
Syn_{CC}.11:	If $\Phi \in ME_t$, then $P\Phi \in ME_t$
Syn_{CC}.12:	If $\alpha, \beta_1, \ldots, \beta_n$ are all in ME_t, then $P(\alpha / \beta_1 / \ldots / \beta_n) \in ME_t$
Syn_{CC}.13:	If $\Phi \in ME_t$ and u is a variable of type j, then $[R\ u\ \Phi] \in ME_t$

2. Semantics of CC

A *model* for CC is an ordered octuple $<D, I, Ins', Per, LA, J, <, F>$ such that D, I and J are non-empty sets, Ins' is a relation on D X I X J, (where one world is a counter factual alternative to another because of the influence of some agent in D), Per is a subset of I (the permitted worlds), LA is a relation on D X D X I (the predicate for legal action), $<$ is a linear ordering on the set J, and F is a function that assigns an appropriate denotation to each constant of CC relative to each pair $<i,j>$ for $i \in I$ and $J \in J$. The set of *possible denotations* of type a is defined as follows:

i. $D_e = D$

ii. $D_j = J$

iii. $D_t = \{False, True\}$

iv. $D_{<a,b>} = D_b^{D_a}$ for any types a and b.

An assignment of values to variables, g, is a function having as domain the set of all variables and giving as value for each variable of type a a member of D_a.

The denotation of an expression α relative to a model M, a possible world i, time j, and value assignment g, abbreviated $Den_{M,i,j,g}(\alpha)$, is defined recursively as follows:

$Sem_{CC}.1$: If α is a constant, then $Den_{M,i,j,g}(\alpha) = F(\alpha)$

$Sem_{CC}.2$: If α is a variable, then $Den_{M,i,j,g}(\alpha) = g(\alpha)$.

$Sem_{CC}.3$: If $\alpha \in ME_{<a,b>}$ and u is a variable of type b, then $Den_{M,i,j,g}(\lambda u\ \alpha)$ is that function h with domain D_b such that for any object x in that domain, $h(x) = Den_{M,i,j,g'}(\alpha)$, where g' is that value assignment exactly like g with the possible difference that g'(u) is the object x.

$Sem_{CC}.4$ If $\alpha \in ME_{<a,b>}$ and $\beta \in ME_a$, then $Den_{M,i,j,g}(\alpha(\beta))$ is $Den_{M,i,j,g}(\alpha)(Den_{M,i,j,g}(\beta))$ (i.e., the result of applying the function $Den_{M,i,j,g}(\alpha)$ to the argument $Den_{M,i,j,g}(\beta)$).

$Sem_{CC}.5$: If α and β are in ME_a, then $Den_{M,i,j,g}(\alpha = \beta)$ is True if and only if $Den_{M,i,j,g}(\alpha)$ is the same as $Den_{M,i,j,g}(\beta)$.

$Sem_{CC}.6$: If $\Phi \in ME_t$, then $Den_{M,i,j,g}({\sim}\Phi)$ is True if and only if $Den_{M,i,j,g}(\Phi)$ is False, and $Den_{M,i,j,g}({\sim}\Phi)$ is False otherwise.

$Sem_{CC}.7$: If Φ and Ψ are in ME_t, then $Den_{M,i,j,g}[\Phi \ \& \ \Psi]$ is True if and only if both $Den_{M,i,j,g}(\Phi)$ and $Den_{M,i,j,g}(\Psi)$ are True.

$Sem_{CC}.8$: If $\Phi \in ME_t$ and u is a variable of type e, then $Den_{M,i,j,g}(\forall u\ \Phi)$ is True if and only if $Den_{M,i,j,g}(\Phi)$ is True for all g' exactly like g except possibly for the value assigned to u.

$Sem_{CC}.9$:. If Φ and Ψ are in ME_t, then $Den_{M,i,j,g}(\Phi\ T\ \Psi)$ is True iff $Den_{M,i,j,g}(\Phi)$ is True and $Den_{M,i,j',g}(\Psi)$ is True for the unique j' such that $j < j'$ and for all j'', either not $j < j'' < j'$ or $j'' = j'$.

$Sem_{CC}.10$: If Φ and Ψ are in ME_t and u is of type e then $Den_{M,i,j,g}[\Phi\ Iu\ \Psi]$ is True iff $Den_{M,i,j,g}\ \Phi$ is True and $Den_{M,i',j,g}(\Psi)$ is True for some i' such that $<g(u),i,i'> \in Ins'$.

Sem_{CC}.11: If $\Phi \in ME_t$, then $Den_{M,i,j,g}$ $P\Phi$ is True iff $Den_{M,i',j',g}$ Φ is True for some i' such that i' $\in$ Per and some j'.

Sem_{CC}.12: If $\alpha, \beta_1, \ldots, \beta_n$ are all in ME_t, then $Den_{M,i,j,g}$ $P(\alpha/\beta_1/\ldots/\beta_n)$ is True iff for some i', such that i' $\in$ Per and $Den_{M,i,j,g}(\alpha)$ is True and $Den_{M,i',j,g}(\beta_k)$ is True for $\beta_k = \beta_1/\ldots/\beta_n$.

Sem_{CC}.13: If $\Phi \in ME_t$ and u is a variable of type j, then $Den_{M,i,j,g}$ [R u Φ] is True iff $Den_{M,i',j',g}(\Phi)$ = True for all j' = g(u).

3. Additional Definitions

i.-iii. For α and β in ME_t

i. $[\alpha \vee \beta] ::= {\sim}[{\sim}\alpha \,\&\, {\sim}\beta]$

ii. $[\alpha \rightarrow \beta] ::= [{\sim}\alpha \vee \beta]$

iii. $[\alpha \leftrightarrow \beta] ::= [\alpha \rightarrow \beta] \,\&\, [\beta \rightarrow \alpha]$

iv. For $\Phi \in ME_t$ and u and v variables of type e,
$\exists u\, \Phi ::= {\sim}\forall u {\sim}\, \Phi$

v. For $\Phi \in ME_t$ $O\Phi ::= {\sim}P\, {\sim}\Phi$

vi. If $\alpha, \beta_1, \ldots, \beta_n$ are all in ME_t, then $O(\alpha/\beta_2/\ldots/\beta_n)$
$::= {\sim}P\, {\sim}\alpha/\beta_1/\ldots/\beta_n$.

For u, v and w variables of type j,

vii. $[u \geq v] ::= [u < v] \vee [u = v]$

viii. $[u > v] ::= {\sim}[u \leq v]$

ix. $u \geq v ::= [u > v] \vee [u = v]$

x. span $::= \lambda u\, \lambda v\, \lambda w\, [(u \leq w) \,\&\, (w \leq v)]$

For u a variable of type t, and $\Phi \in ME_t$

xi. $RT\, u\, \Phi ::= \forall v\, u(v) \rightarrow [R\, v\, \Phi]$

xii. $RD\, u\, \Phi ::= \exists v\, u(v) \rightarrow [R\, v\, \Phi]$

If $\Phi \in ME_t$ and u and v are of type e, then

xiii. $O(u,v)\, \Phi ::= P\, LA(v,u)/{\sim}\Phi(u)$

xiv. $P(u,v)\Phi ::= {\sim}O(v,u)\, {\sim}\Phi$

If α, β and γ are in ME_t and u and v are variables of type e, then

xv. $O(x,y)\, \alpha/\beta ::= P(LA(v,u)/{\sim}\alpha(x)/\beta)$

xvi. $P(u,v)\, \alpha/\beta ::= {\sim}O(v,u)\, {\sim}\Phi/\Psi$

xvii. $O(u,v)\, \alpha\, OE\, \gamma ::= P(u,v)\, \gamma/{\sim}\alpha(x)$

xviii. $O(u,v)\, \alpha/\beta\, OE\, \gamma ::= P(u,v)\, \gamma/{\sim}\alpha(u)/\beta$

APPENDIX: COMMENTS ON VON WRIGHT'S DYADIC DEONTIC CALCULUS

Von Wright goes beyond the deontic definitions described here to introduce what he calls a "dyadic" (as opposed to the above "modadic") deontic logic. In this form, permission is denoted

$$P\Phi/\Psi$$

read as "Φ is permitted *given* Ψ," where Ψ is thus a condition on the permissability of Φ. The monadic form, $P\Phi$, is thus a degenerate case where Ψ is a tautology. Dyadic obligation is defined:

$$O\,\Phi/\Psi ::= {\sim}P\,{\sim}\Phi/\Psi$$

This dyadic logic is motivated to avoid certain paradoxes which arise in the monadic form. (We point out that these are paradoxes in the *logic* of DC, not in the semantic interpretation of the language. That is to say, the (monadic) axioms provided seem to be inadequate in that one is able to generate from them certain theorems that are semantically unacceptable. The principle one is Ross's paradox: that $O\Phi \rightarrow (\Phi \vee \Psi)$ is derivable from the axioms. This says e.g., that if some state of affairs is obligatory, then that state or any other obligatory—e.g., if one ought to mail a letter then one ought to mail or burn it.

Since logical axiomatization is the next step after semantic formalization in making the concepts described herein computable, I too would like an axiomatization that does not lead to paradoxes.

However, I have not yet been successful at incorporating von Wright's dyadic forms into the model theory interpretation used here. The purpose of this appendix is to explain the difficulty.

The basic problem, as I'll show, stems from the different views of possible worlds in his work and this. The obvious question, then, is why don't I adopt his view of a possible world? One reason is that his worlds have no time dimension, a necessary requirement for describing particular contracts. One other reason is that further extensions of this work make use of Montague's intensional logic which takes the view we have adopted here.

Let me begin by summarizing von Wright's comments. The notation here, $P(\Phi / \Psi)$, is for him $P(p/q)$, which he introduces as follows:

> For the symbol "*P*(*p* / *q*)" we suggest the following reading: "it is permitted that ***p***, ***given that q***." Instead of "given that" we can also say "on condition that" or "relative to that" or "in the circumstances when."
>
> A special comment will be made about the case when the blanks in "*P*(—/ —)" are filled by state descriptions in the terms of the state-descriptions. They describe possible worlds in the universe of elementary states represented by the propositional variables of the set. For an expression of the form "*P*(*s* / *s*')" we suggest the following reading, too: "in the possible world *s*' the possible world *s* is permitted as an alternative world to *s*'." (Strictly: in the possible world described by "s'," etc.)
>
> The suggested piece of terminology may sound a little artificial. But in fact it comes very close to the "meaning" of norms. Norms are (usually) concerned with actions. Action, broadly speaking, is interference with an existing (given) state of affairs (a situation, a world) and consists in substituting for this another (an alternative) state of affairs. The expression "*P*(s / *s*')" thus says that a world of the description "s'" ***may*** become changed to a world of the description "s." (von Wright 1968:23)

Von Wright uses this dyadic concept to differentiate six different types of conditional permission, as follows.

Since these are metalanguage explanations, he gives them in English. However, as the quantification and scoping is a rather hard to follow in this form, we provide a symbolic translation. This symbolism, however, is also meant as metalanguage, not object language. Let $Per(w_1, w_2)$ be a relation indicating that world w_2 is permitted in world w_1. Then the six permission concepts are:

$P_1(\Phi / \Psi)$ ::=: "in ***some possible world in which it is true that*** Ψ ***some*** possible world is permitted in which it is true that Φ."
::= $\exists w_1 \exists w_2\ \Psi(w_1)\ \&\ Per(w_1, w_2)\ \&\ (\Phi(w_2)$

$P_2(\Phi/\Psi)$ ::= "in *all* possible worlds in which it is true that Ψ *some possible world is permitted in which it is true that* Φ"
::= $\forall w_1 \exists w_2\, \Psi(w_1) \rightarrow (Per(w_1,w_2)\ \&\ \Phi(w_2))$

$P_3(\Phi/\Psi)$ ::= "*some* possible world in which it is true that p is such that it (this world) is permitted in *every* possible world in which it is true that Ψ"
::= $\exists w_2 \forall w_1\, \Phi(w_2)\ \&\ (\Psi(w_1) \rightarrow Per(w_1,w_2))$

$P_4(\Phi/\Psi)$ ::= "every possible world in which it is true that Φ is such that it is permitted in some possible world in which it is true that Ψ"
::= $\forall w_2 \exists w_1\, \Phi(w_2) \rightarrow (\Psi(w_1)\ \&\ Per(w_1,w_2))$

$P_5(\Phi/\Psi)$ ::= "in *some* possible world in which it is true that Ψ *all* possible worlds are permiteed in which it is true that Φ"
::= $\exists w_1 \forall w_2\, \Psi(w_1)\ \&\ (\Phi(w_2) \rightarrow Per(w_1,w_2))$

$P_6(\Phi/\Psi)$ ::= "in *all* possible worlds in which it is true that Ψ all possible worlds are permitted in which it is true that Φ."
::= $\forall w_1 \forall w_2\, \Psi(w_1) \rightarrow (\Phi(w_2) \rightarrow Per(w_1,w_2))$

Using the definition

$$O(\Phi/\Psi) ::= {\sim}P({\sim}\Phi/\Psi)$$

there are six corresponding concepts of obligation:

$O_1(\Phi/\Psi)$ ::= "in *all* possible worlds in which it is true that Ψ, *no* possible world is permitted in which it is *not* true that Φ"
::= $\forall w_1 \forall w_2\, \Psi(w_1) \rightarrow (Per(w_1,w_2) \rightarrow \Phi(w_2))$

$O_2(\Phi/\Psi)$ ::= "in *some* possible world in which it is true that Ψ, *no* possible world is permitted in which it is *not* true that Φ"
::= $\exists w_1 \forall w_2\, \Psi(w_1)\ \&\ (Per(w_1,w_2) \rightarrow \Phi(w_2))$

$O_3(\Phi/\Psi)$ ::= "every possible world in which it is *not* true that Φ is such that it (this world) is not permitted in *some* possible world in which it is true that Ψ"
::= $\forall w_2 \exists w_1\, {\sim}\Phi(w_2) \rightarrow (\Psi(w_1)\ \&\ {\sim} Per(w_1,w_2))$

$O_4(\Phi/\Psi)$ ::= "*some* possible world in which it is *not* true that Φ is such that it is not permitted in *any* possible world in which it is true that Ψ"
::= $\exists w_2 \forall w_1\, {\sim}\Phi(w_2)\ \&\ (\Psi(w_1) \rightarrow {\sim}Per(w_1,w_2))$

$O_5(\Phi/\Psi)$ ::= "in *all* possible worlds in which it is true that Ψ *some* possible world is not permitted in which it is *not* true that Φ.
::= $\forall w_1 \exists w_2\, \Psi(w_1) \rightarrow ({\sim}Per(w_1,w_2)\ \&\ {\sim}\Phi(w_2))$

$O_6(\Phi/\Psi)$ ::= "in *some* possible world in which it is true that Ψ *some* possible world is not permitted in which it is *not* true that Φ"
::= $\exists w_1 \exists w_2\, \Psi(w_1)\ \&\ ({\sim}Per(w_1,w_2)\ \&\ \Phi(w_2))$

In von Wright's concept of a possible world, what we called a VW world, a world was uniquely specified by a state description. By adding the time dimension, leading to our intermediate interpretation called an I world, a world became a sequence of state description/time pairs.

It would seem, then, that in this view VW worlds become states of an I world. Thus, the world variables in von Wright's six interpretations of dyadic permission and obligation would become (second order) variables ranging over state descriptions.

For instance, the first definition would then read:

$$P_1(\Phi/\Psi) ::= \exists s_1 \exists s_2 \Psi(s_1) \& Per(s_1,s_2) \& \Phi(w_2)$$

which we would interpret as:

$$P_1(\Phi/\Psi) ::= \exists s_1 \exists s_2 \exists w\, s_1(w) \& \Psi(w) \& Per(w) \& s_2(w) \& \Phi(w).$$

What now becomes unclear is the intended temporal relationship between the two states. If we suppose that the state where q occurs immediately precedes the state where Φ occurs this might be expressed as follows.

Let us define a function "prev" for previous, which returns the time preceding a given time t:

$$prev(t) := \iota t' \forall t'' \sim(t' < t'' < t)$$

Then the definition of P_1 might read:

$$P_1(\Phi/\Psi) ::= \exists s_1 \exists s_2 \exists w \exists t$$
$$s_1(w_1,prev(t)) \& \Psi(w,prev(t)) \& Per(w) \& s_2(w,t) \& \Phi(w,t)$$

Here, however, we are speculating since von Wright did not indicate the temporal relationship of the two (VW) worlds involved in his dyadic definitions.

Nonetheless, making some sort of assumptions about the temporal relationship, one could still distinguish the various types of conditional permission and obligation based on various combinations of quantifiers on the two state variables.

However, as we move to the concept of a C-world, the concept of a state description loses its importance since these no longer distinguish unique possible worlds.

Under this interpretation, which we use here, the dyadic view of conditional probability becomes monadic by dropping reference to these states, i.e.,

$$P(\Phi/\Psi) ::= \exists w\, \Psi(w) \& Per(w) \& \Phi(w)$$

using the definition

$$O(\Phi/\Psi) ::= \sim P(\sim\Phi/\Psi)$$

we have

$$O(\Phi/\Psi) ::= \sim\exists w\, \Psi(w) \& Per(w) \& \sim\Phi(w)$$
$$::= \forall w \sim[\Psi(w) \& Per(w) \& \sim\Phi(w)]$$
$$\forall w \sim\Psi(w) \vee \sim(Per(w) \& \Phi(w))$$

$$\forall w \sim\Psi(w) \vee (Per(w) \rightarrow \Phi(w)$$
$$\forall w \Psi(w) \rightarrow (Per(w) \rightarrow \Phi(w)$$

One further comment. Later (p.77), von Wright addresses the concept of commitment (obligation from one party to another), and shows the inadequacy of two views of conditional obligation, namely

$$O(\Psi \rightarrow \Phi)$$

and

$$\Psi \rightarrow O\Phi$$

and proposes (but does not develop) that a concept of dyadic obligation $O(\Phi/\Psi)$, would be more suitable.

We simply want to note here that our re-interpretation of $O(\Phi/\Psi)$, while monadic in that it only involves one possible world, is nonetheless distinct from either of the preceding two monadic concepts. Using definitions introduced earlier we have:

$$O(\Psi \rightarrow \Phi) ::= \forall w\ Per(w) \rightarrow (\Phi(w) \rightarrow \Psi(w))$$
$$\Psi \rightarrow O\Phi ::= \Psi \rightarrow \forall w\ Per(w) \rightarrow \Phi(w)$$
$$O(\Phi/\Psi) ::= \forall w\ \Psi(w) \rightarrow (Per(w) \rightarrow \Phi(w))$$

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