

# ***WORKING PAPER***

INTERNATIONAL TRADE LINKAGES IN INTER-  
REGIONAL I/O ECONOMIC MODELING: THE  
MODEL FOR THE TUSCANY CASE STUDY

A. Cavalieri

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## ABSTRACT

This paper deals with the treatment of the international linkages and the relationships between foreign flows and inter-regional ones, in a context of a biregional input-output model, which will be built within the Tuscany case study.

In the first part of the paper, different approaches are surveyed with reference to some existing models including I/O matrices, even if they were not designed at a biregional level.

The second part of the paper aims at describing the approach we have followed in designing the model for the Tuscany region with respect to the international trade components.

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1. INTRODUCTION

Even if the national economic variables of regions within a nation and the linkages between them are the core of the analysis in most interregional and multiregional models, in some regions, the international trade linkages have a specific and relevant local role.

The impacts of the changes in the international division of labor over a nation are not uniform with respect to the single regions. In fact, the national import and export figures are pure statistical entities, because the different export and import specialization and foreign market orientations at regional level.

In addition, some regions have a specific role in acting as trade intermediaries towards foreign countries by distributing import flows and by collecting export flows via interregional trade.

In spite of these considerations, international trade flows are very seldom regarded as affected by regional determinants.

This paper deals with the treatment of the international linkages and relationships between foreign flows and interregional ones in the context of a biregional I/O model which will

be built for the Tuscany region, within the Tuscany Case Study Project.\*

The specific interest in analyzing the international trade from a bottom-up point of view, derives from the fact that the regional export, in Tuscany, has shown a high elasticity of its weight on national export according to the international demand cycles (A. Cavalieri, 1980).

The regional high specialization in some typical "mature" products is the main explanatory factor of such strong fluctuations. The reaction to the international variables affecting the export of the kind of commodity is fairly different in Tuscany and in the rest of Italy. A top-down approach based on fixed share of national export flows does not produce, in this case, significant results at the regional level.

In addition, the model is aimed to take into consideration the interrelations between interregional and international trade which is normally neglected, even if they play an important role in a small, open regional economy.

In the first part of the paper, different approaches are surveyed with reference to some existing models including regional I/O matrices even if they were not designed at a biregional level. In fact, we have faced the nature of such models by reformulating them in order to carry on some possible approaches in treating international and interregional trade in an I/O biregional model.

The choice of the models as well as their reformulation into a biregional context, is a very personal one.

Our selection was made in order to introduce the second part of the paper which aims at describing the approach we have followed in designing the model for the Tuscany region with respect to the international trade components.

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\* This project is being carried out by the Regional Development Group of IIASA in cooperation with IRPET (Regional Institute for Economic Planning of Tuscany) and IASI (Institute for Systems Analysis and Informatics).

## 2. SOME EXAMPLES ON DIFFERENT APPROACHES IN THE TREATMENT OF INTERNATIONAL AND INTERREGIONAL TRADE

As we stated above, in this paragraph, some examples derived from existing models will be shown. It is important to note that we will not deal with the procedures used in estimating the trade coefficients or the gross flows; attention will be given only to the approach used in including the international and interregional trade into an I/O model.

Because the models analyzed were designed in order to focus attention on different aims, the original structures are quite different. The examples we have chosen are nevertheless the closest to the model specified for the Tuscany region, even if this one is a biregional one whereas most surveyed models are multiregional ones.

An extensive comparative study of multiregional economic models is currently performed by the Regional Development Task at IIASA, and the Free University, Amsterdam (see Nijkamp and Rietveld, 1980). Within this study, a preliminary comparison on interregional and international linkages in multiregional economic models was carried out by F. Snickars (1981).

Our limited survey is strictly oriented to the specification of the biregional I/O model for the Tuscany region; for this reason, a common list of relations is used in order to make easier the comparison between the different approaches and the scheme designed for the Tuscany model.

In the rest of the paper, therefore, the following notations are used:

Variables:

$X_T, X_R$  = gross production vector in Tuscany and the rest of Italy;

$F_T, F_R$  = internal final demand vector in Tuscany and the rest of Italy;

$M_T^R, M_R^T$  = interregional import vector in Tuscany and the rest of Italy;

$E_T^W, E_R^W$  = foreign export vector in Tuscany and the rest of Italy;

$M_T^W, M_R^W$  = international import vector in Tuscany and the rest of Italy;

$E_T^R, E_R^T$  = interregional export vector in Tuscany and the rest of Italy;

$E_N^W, E_{NT}^W$  = vector and scalar of national export;

NET, NER = net interregional trade in Tuscany and the rest of Italy.

Parameters:

$A_T, A_R$  = I/O matrix of technological coefficient;

$\mu_T, \mu_R$  = diagonal matrix of foreign import shares of demand components, Tuscany and the rest of Italy;

$\hat{\mu}_T, \hat{\mu}_R$  = diagonal matrix of foreign import shares of gross production, Tuscany and the rest of Italy;

$\mu'_T, \mu'_R$  = diagonal matrix of foreign import shares of intermediate consumption, Tuscany and the rest of Italy;

$\mu''_T, \mu''_R$  = diagonal matrix of foreign import shares of internal final demand, Tuscany and the rest of Italy;

$m_T, m_R$  = diagonal matrix of interregional import shares of demand components, Tuscany and the rest of Italy;

$\hat{m}_T, \hat{m}_R$  = diagonal matrix of interregional import shares of gross production, Tuscany and the rest of Italy;

$m'_T, m'_R$  = diagonal matrix of interregional import shares of intermediate consumption;

$m''_T, m''_R$  = diagonal matrix of interregional import shares of internal final demand, Tuscany and the rest of Italy;

$\varepsilon_T, \varepsilon_R$  = vector of foreign export share of national export (scalar), Tuscany and the rest of Italy;

$\varepsilon'_T, \varepsilon'_R$  = diagonal matrix of foreign export share of national export (vector), Tuscany and the rest of Italy.

## 2.1 The Scheme of the REGAL Model

This multiregional optimization model for the allocation of private and public investment, production, employment and population over economic sectors and regions, (see F. Snickars and A. Granholm, 1981) is not aimed to treat explicitly interregional and international trade.

Nevertheless, the linkages between the regional economic variables and the external trade flows are treated in a way which



is normally followed in many other interregional models based on the top-down approach.

If we reformulate the REGAL model in a biregional one, and we specify only the external flows, we get the following structure:

$$X_T + M_T^W + M_T^R = (A_T X_T + F_T) + E_T^W + E_T^R ,$$

$$X_R + M_R^W + M_R^T = (A_R X_R + F_R) + E_R^W + E_R^T ,$$

where

$$M_T^W = \hat{\mu}_T X_T ; E_T^W = \varepsilon_T E_N^{\bar{W}} ; E_T^R = M_T^R = \hat{m}_R X_R ,$$

$$M_R^W = \hat{\mu}_R X_R ; E_R^W = \varepsilon_R E_N^{\bar{W}} ; E_R^T = M_R^T = \hat{m}_T X_T .$$

In terms of the multiplier, we have:

$$X_T = [(I + \hat{\mu}_T + \hat{m}_T) - A_T]^{-1} \cdot (F_T + \varepsilon_T E_N^{\bar{W}} + \hat{m}_R X_R) ,$$

$$X_R = [(I + \hat{\mu}_R + \hat{m}_R) - A_R]^{-1} \cdot (F_R + \varepsilon_R E_N^{\bar{W}} + \hat{m}_T X_T) .$$

Both the imports from the other region and from the rest of the world are determined endogenously as a fixed share of the regional gross production; in the current literature that it means they are supposed to be of a complementary type. The import flows do not affect the matrix A and the final demand components according to the scheme: final demand → I/O matrix → gross production → import.

No specification is given on the import share of the I/O coefficients and of the final demand components as can be shown in Figure 1.

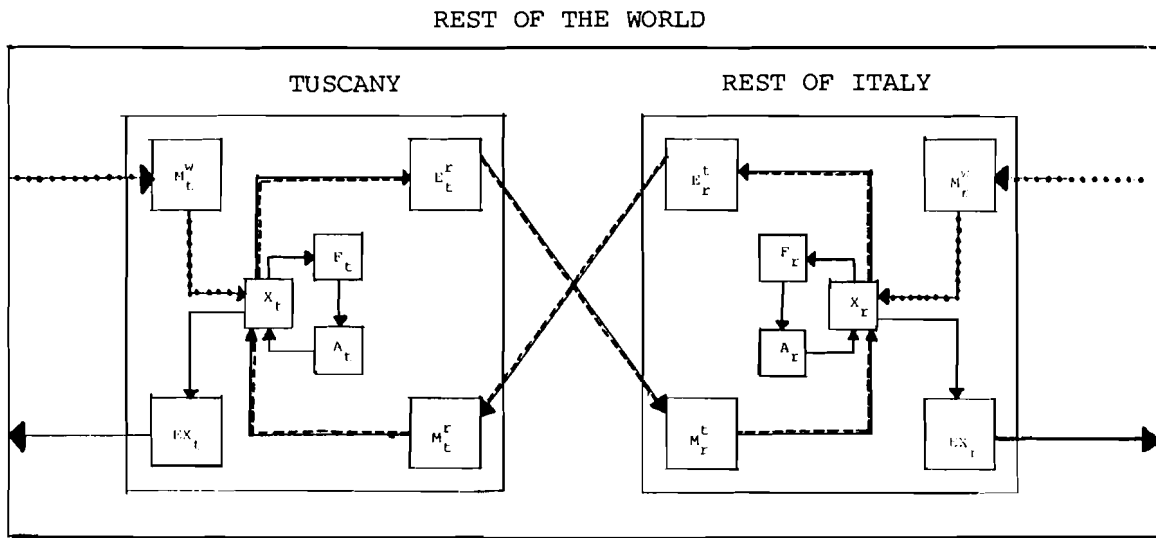


Figure 1. External trade scheme in the REGAL model.

Looking at Figure 1 it is also clear that the interregional trade does not include foreign production, i.e. it is supposed that the regional import meets only complementary needs inside the importer region itself.

As far as the regional export flows to the rest of the world are concerned, they are estimated as a fixed share of the national vector, which is exogenously derived from a national model where constraints by BOP (Balance of Payment) are supposed to exist.

This model is designed in order to focus attention on the interregional impact of some planning policies. All the variables that are not subjected to planning policy (as the international flows are) are treated in a simple way which becomes even simpler when the model is reformulated in a biregional form.

## 2.2 The Scheme of the VERDI Model

This biregional input-output model (P. Costa, L. Malfi, D. Martellato, 1980) was designed in order to analyze the interdependencies between a single region and the rest of the country.

It is an open and static model where the components of the final demand are exogenously determined.

The input-output model is closed only with regard to inter-regional trade, according to the Chenery-Moses approach (Chenery and Cao-Pinna, 1953), applied to Italy more than twenty years ago.

The general structure of the VERDI model is the following:

$$X_T = [(A_T X_T + \bar{F}_T) - M_T^W - M_T^R] + \bar{E}_T^W + E_T^R ,$$

$$X_R = [(A_R X_R + \bar{F}_R) - M_R^W - M_R^T] + \bar{E}_R^W + E_R^T ,$$

where

$$M_T^W = \mu_T (A_T X_T + \bar{F}_T) ; \quad E_T^R = M_R^T = m_R (A_R X_R + \bar{F}_R) ,$$

$$M_R^W = \mu_R (A_R X_R + \bar{F}_R) ; \quad E_R^T = M_T^R = m_T (A_T X_T + \bar{F}_T) ,$$

by substituting, we have:

$$X_T = [I - (1-m_T-\mu_T)A_T]^{-1} \cdot [(1-m_T-\mu_T)\bar{F}_T + m_R(A_R X_R + \bar{F}_R) + \bar{E}_T^W] ,$$

$$X_R = [I - (1-m_R-\mu_R)A_R]^{-1} \cdot [(1-m_R-\mu_R)\bar{F}_R + m_T(A_T X_T + \bar{F}_T) + \bar{E}_R^W] .$$

The imports from the other region as well as from the rest of the world are specified as a fixed share of the demand components, both intermediate and final ones, excluding export flows as shown in Figure 2.

The internal demand is split into domestic and external components; by this way, imports are supposed to be competitive and the multiplier effect is estimated according to the following scheme: final demand (reduced by import propensity) + I/O matrix

(where the coefficients are reduced by the same import propensity)  $\rightarrow$  gross production.

REST OF THE WORLD

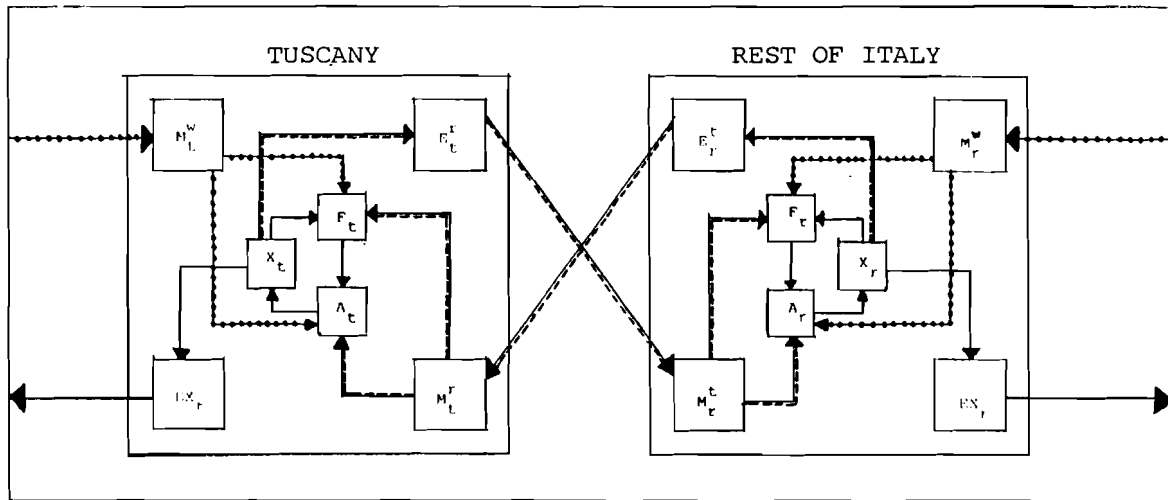


Figure 2. External trade scheme in the VERDI model.

As in the former model, also here, the foreign production goods are not included in the biregional trade which is concerned only with national production consumed inside the country.

The model is designed in order to evaluate the impact effects on the economy of a region from changes in the national economic variables, with particular reference to interdependencies due to the trade patterns between the regions and the rest of the country.

### 2.3 The Scheme of the MORSE Model

The MORSE model, (L. Lundqvist, 1981), is a dynamic multi-regional I/O model aimed to integrate economic, employment and energy planning in a regional perspective.

Theoretically, the model combines traits from input-output analysis, interregional programming and equilibrium growth theory.

As the model is concerned mainly with an objective function representing goals in economic, employment and energy planning, the external trade flows are not the core of the analysis. Nevertheless, some specifications in the treatment of the linkages to the rest of the world within a top-down approach, could be interesting even in a biregional I/O model more oriented to a better specification of the international trade.

By reformulating the MORSE model's structure into a bi-regional model, we get the following structure:

$$X_T + M_T^W = [(A_T X_T + F_T) - M_T^R] + E_T^W + E_T^R ,$$

$$X_R + M_R^W = [(A_R X_R + F_R) - M_R^T] + E_R^W + E_R^T ,$$

where

$$M_T^W = \hat{\mu}_T X_T ; \quad E_T^R = M_T^R = m_R^A X_R + m_R^F F_R ; \quad E_T^W = \epsilon_T^E E_{TN}^W ,$$

$$M_R^W = \hat{\mu}_R X_R ; \quad E_R^T = M_R^T = m_T^A X_T + m_T^F F_T ; \quad E_R^W = \epsilon_R^E E_{TN}^W .$$

In terms of the multipliers, we have:

$$X_T = [(I + \hat{\mu}_T) - (1 - m_T^A) A_T]^{-1} \cdot [F_T (1 - m_T^F) + m_R^A X_R + m_R^F F_R + \epsilon_T^E E_{TN}^W] ,$$

$$X_R = [(I + \hat{\mu}_R) - (1 - m_R^A) A_R]^{-1} \cdot [F_R (1 - m_R^F) + m_T^A X_T + m_T^F F_T + \epsilon_R^E E_{TN}^W] .$$

As far as the scalar of national export is concerned,  $E_{TN}^W$  is constrained at the national level by BOP goals:

$$E_{TN}^W = \overline{BOP} - M_N^W ,$$

$$E_{TN}^W = \overline{BOP} - (\hat{\mu}_T X_T + \hat{\mu}_R X_R) .$$

A specific feature of this model is the different treatment of import between interregional and international flows (Figure 3). Whereas the first ones are considered as competitive and therefore are linked to demand components, the second ones are supposed to be complementary and are linked to regional gross production.

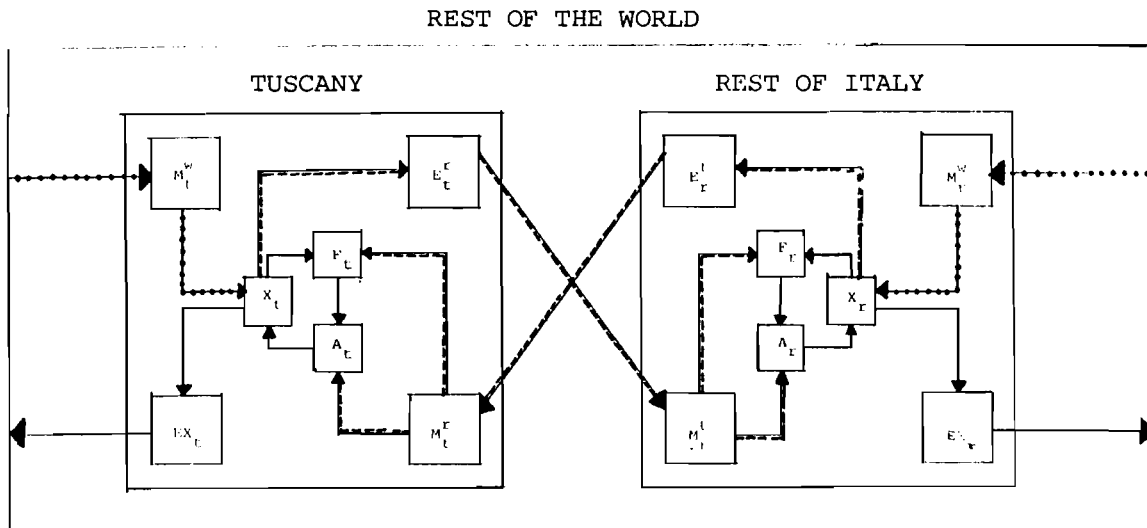


Figure 3. External trade scheme in the MORSE model.

In addition, by permitting different rates of self-sufficiency for intermediary goods and for final demand, the Chenery-Moses framework is somewhat extended.

As in the other models surveyed until now, the interregional trade is concerned only with domestic production demanded inside the country.

As far as the regional export is concerned, a matrix share is used to breakdown at the regional and sectoral level the total national export which is derived from a national econometric model.

In order to avoid the strong assumption on the fixed coefficients of that matrix the parameters  $\varepsilon_T^I$ ,  $\varepsilon_R^I$  are subjected to change over time, according to the trends in the sectoral mixing of the national export.

The same procedure is adopted for parameters  $\hat{\mu}_T$ ,  $\hat{\mu}_R$  which link regional import to gross production in the region.

In matrix terms, the vector  $\varepsilon_i^I$  (in region  $T$  and region  $R$ ) as well as the diagonal matrix  $\hat{\mu}$  (in region  $T$  and region  $R$ ), is split into two components in the following way:

$$\begin{bmatrix} \epsilon_1^t \\ \cdot \\ \cdot \\ \cdot \\ \epsilon_i^t \\ \cdot \\ \cdot \\ \cdot \\ \epsilon_n^t \end{bmatrix} = \begin{bmatrix} \epsilon_1^* & & & \emptyset \\ & \cdot & & \cdot \\ & & \cdot & \cdot \\ & & & \cdot \\ & & & \cdot \\ & & \epsilon_i^* & \\ & & & \cdot \\ & & & \cdot \\ & & & \cdot \\ \emptyset & & & \epsilon_n^* \end{bmatrix} \begin{bmatrix} \epsilon_1^t \\ \cdot \\ \cdot \\ \cdot \\ \epsilon_i^t \\ \cdot \\ \cdot \\ \cdot \\ \epsilon_n^t \end{bmatrix}$$

where

$$E_i^t = \frac{E_{Ni}}{E_{TN}^W} \text{ at time } t ,$$

and

$$\begin{bmatrix} \hat{\mu}_1 & & & \emptyset \\ \cdot & & & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & \hat{\mu}_1 & \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \emptyset & & & \hat{\mu}_n \end{bmatrix} = \begin{bmatrix} \mu_1^* & & & \emptyset \\ \cdot & & & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & \mu_i^* & \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \emptyset & & & \mu_n^* \end{bmatrix} \cdot \begin{bmatrix} \mu_1^t & & & \emptyset \\ \cdot & & & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & \mu_i^t & \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \emptyset & & & \mu_n^t \end{bmatrix}$$

where

$$\mu_i^t = \frac{M_{Ni}^W}{M_{TN}^W} \text{ at time } t .$$

While both the diagonal matrix  $\epsilon^*$  and  $\mu^*$  are taken as fixed over time, the diagonal matrix  $\mu^t$  and the vector  $\epsilon^t$  are subjected to change over time, according to historical trends or short-term forecasts.

In this way, the consistency between the sectoral mixing at national and regional levels is achieved for each time  $t$ .

The model contains a typical national policy variable in order to constrain the foreign flows: the balance of payment (BOP). Because of the high dependence of the export from exogenous international variables, by determining BOP, the model (closed with respect to the other final demand components) indicates the level of the gross production which is consistent with the export flows.

## 2.4 The Scheme of the REGIS Model

The REGIS model (R. Courbis and G. Cornilleau, 1978) is a reduced form of the national REGINA model. Both models are of the multiregional, interdependent kind, build around a core constituted by a production model of input-output type.

The sectoral disaggregation, as well as the territorial one, is highly detailed; the model takes into account the interregional mobility of many economic and social factors.

Even if the REGIS is one of the most comprehensive and sophisticated multiregional models, the interregional trade is not specified in terms of gross flows. On the other hand, the REGIS model is a good example of a detailed and theoretically justified treatment of foreign trade at a regional level.

By reducing the REGIS model to a biregional one, we lose the main specific characteristics of this model; in addition, by focussing our attention on the external trade flows at a regional level, we analyze a part of the model which is not specified as the multiregional trade is concerned.

By bearing in mind these strong limitations, the biregional structure of the REGIS model is a very simple one:

$$X_T = (A_T X_T + F_T) - M_T^W + E_T^W + NET \quad ,$$

$$X_R = (A_R X_R + F_R) - M_R^W + E_R^W + NET \quad ,$$

where

$$NET = -NER \quad .$$

The multiplier of the model is the traditional Leontieff inverse matrix:

$$X_T = [I - A_T]^{-1} \cdot (F_T - M_T^W + E_T^W + NET) \quad ,$$

$$X_R = [I - A_R]^{-1} \cdot (F_R - M_R^W + E_R^W - NET) \quad .$$



In this structure no direct impact of the interregional trade flows is supposed to operate in the production and demand formation (see Figure 4), whereas many other factors (not taken into consideration here) have many indirect effects via interregional mobility or interdependencies.

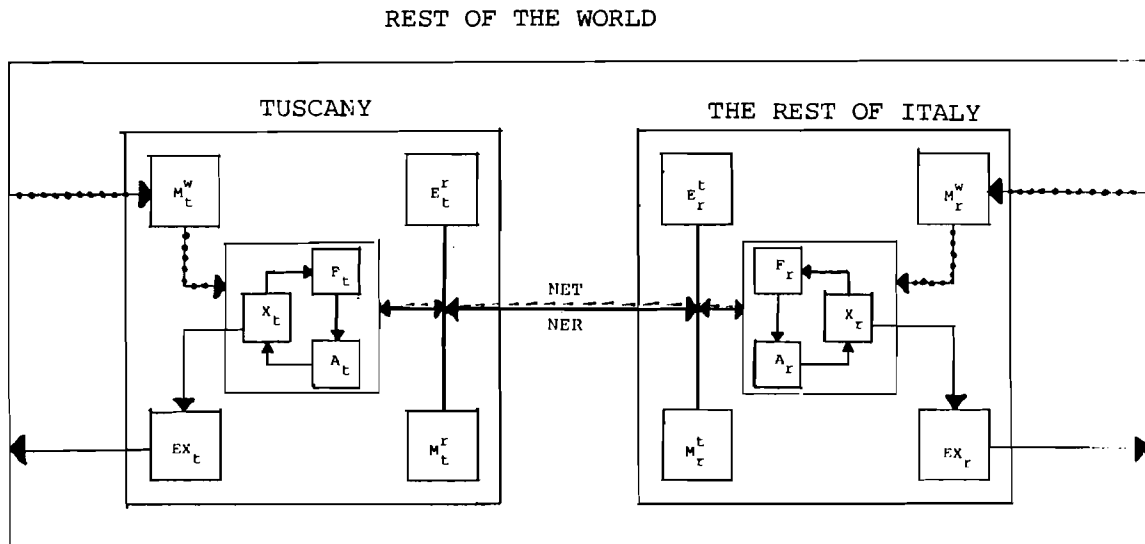


Figure 4. External trade scheme in the REGIS model.

The REGIS model employs import and export functions econometrically specified at the regional level; there is a different treatment of import with respect to competitive or complementary products. Here the difference between competitiveness and complementarity is more theoretically based than in most models. Competitiveness is assumed to be determined only by national variables. In this case, the regional import is estimated as a fixed share of the national one:

$$M_T^W = K_T M_N^W, \quad M_R^W = K_R M_N^W.$$

For complementary goods, instead, it is assumed that import flows are linked to the growth of regional demand and to the production prices related to import prices, via an econometric function at the regional level:

$$M_T^W = \alpha_T D_T^{bT} \cdot (P/PM)^{cT},$$

$$M_R^W = \alpha_R DT_R^b \cdot (P/PM)^{CR} ,$$

where

DT = vector of regional total demand (index numbers);  
P = vector of production price at the national level;  
PM = vector of import at the national level;  
b,c = elasticity parameters (demand, relative price).

Foreign export is also estimated by econometric functions with regional specifications based on different elasticity parameters, where the independent variables are the export prices at the national level (PX):

$$E_T^W = \gamma_T + \beta_T PX^{K_T} ,$$

$$E_R^W = \gamma_R + \beta_R PX^{K_R} .$$

In addition to the equations there are national sums corresponding to regional export and import, both endogenous. Thus, foreign trade is altogether regionally specified in this model, which is a very unusual property in the current modeling practice where the top-down approach is normally used.

### 3. A BIREGIONAL I/O MODEL DESIGNED FOR THE TUSCANY REGION

This model is designed in order to have a core to which are linked some submodels oriented to analyzing specific aspects of the regional reality and the interdependencies with the rest of the nation and the rest of the world.

The core of the model is constituted by the I/O matrix for the Tuscany region which is at present available for the years 1975 and 1977 (built by an indirect procedure) and for the year 1978 (built by a direct survey procedure).

The linkages between Tuscany and the rest of Italy are based on the trade coefficients matrix which links the Tuscany I/O matrix

and the rest of Italy, and on an interregional population model which will be developed into a demo-economic interregional model.

The linkages between Tuscany and the rest of the world via international trade are provided both by a regional model of export (bottom-up approach) and by a breakdown from the INFORUM project (top-down approach) on which IRPET is working in order to link Italy to the INFORUM model (C. Almon and D. Nyhus, 1977).

Particular emphasis is given in the model to the treatment of public components of the final demand in order to achieve a detailed specification of the linkages between the public authority expenditures (especially at the local level) and the productive sectors.

The general scheme of the models system that is developed within the Tuscany case study is shown in Figure 5.

This paper deals only with the parts of the model, as well as their linkages, that are concerned with external trade. These parts are shown in the scheme by doubled-lined squares; they are concerned with regional export and tourism model (box A), international export and import linkages - INFORUM (box B), regional linkages demand for consumption (only tourist consumption) and exports (box C-1), and interregional trade linkages (box C-2).

### 3.1 General Structure of the Model with Reference to Interregional, International Trade Flows and Their Linkages

The interregional trade interdependencies are treated in this model, to some extent, as in the VERDI model; in addition, a linkage between interregional flows and international ones is modeled in order to take into account also the re-exports from the importer region, according to the following two schemes:

foreign production → foreign regional import → inter-regional export

foreign regional export ← interregional import ← regional production

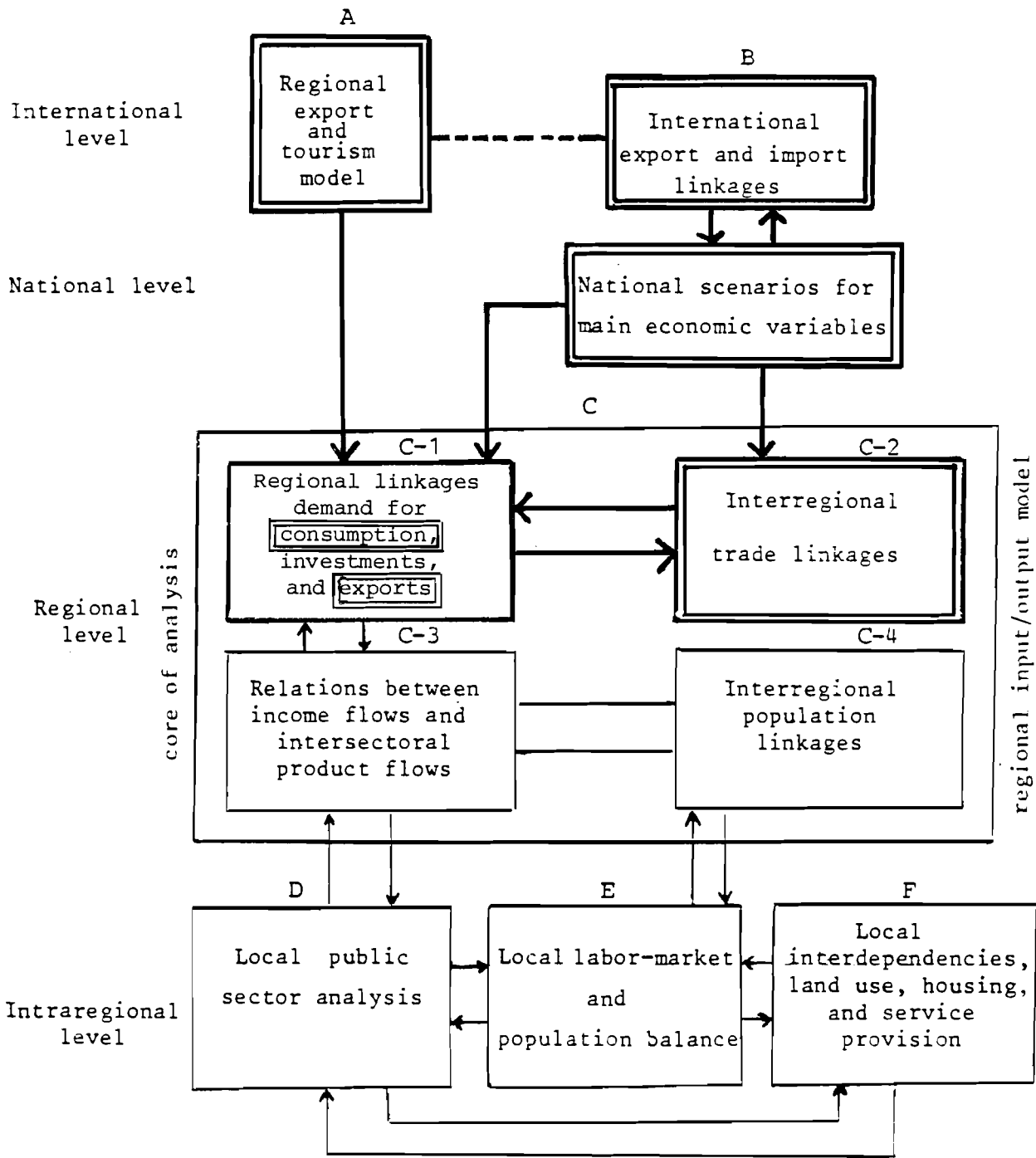


Figure 5. Tuscany case study: general model scheme.

Foreign regional import is not supposed to be re-exported (via interregional trade) to foreign markets.

According to these assumptions, the general structure of the model can be designed in the following terms:

$$X_T = [(A_T X_T + F_T) - M_T^W - M_T^R] + E_T^W + E_T^R ,$$

$$X_R = [(A_R X_R + F_R) - M_R^W - M_R^T] + E_R^W + E_R^T ,$$

where

$$M_T^W = \mu_T (A_T X_T + F_T) + \mu_T^{**} m_R (A_R X_R + F_R) ,$$

$$M_R^W = \mu_R (A_R X_R + F_R) + \mu_R^{**} m_T (A_T X_T + F_T) ,$$

and

$$M_T^R = m_T^* [(A_T X_T + F_T) + m_T^{**} E_T^W] ;$$

$$M_R^T = m_R^* [(A_R X_R + F_R) + m_R^{**} E_R^W] ,$$

by taking into account that

$$E_T^R = M_R^T ; \quad E_R^T = M_T^R ,$$

we have

$$X_T = (1 - \mu_T^* - m_T^*) (A_T X_T + F_T) + (1 - \mu_T^{**}) m_R^* (A_R X_R + F_R) + m_R^{**} E_R^W + (1 - m_T^{**}) E_T^W ,$$

$$X_R = (1 - \mu_R^* - m_R^*) (A_R X_R + F_R) + (1 - \mu_R^{**}) m_T^* (A_T X_T + F_T) + m_T^{**} E_T^W + (1 - m_R^{**}) E_R^W .$$

The final structure of the model in terms of input-output inverse matrix is given by the following equations:

$$\begin{aligned}
 X_T = & [I - (1-\mu_T^* - m_T^*) A_T]^{-1} \cdot [(1-\mu_T^* - m_T^*) F_T \\
 & + (1-\mu_T^{**}) m_R^* (A_R X_R + F_R) \\
 & + m_R^{**} E_R^W + (1-m_T^{**}) E_T^W] \quad ,
 \end{aligned}$$

$$\begin{aligned}
 X_R = & [I - (1-\mu_R^* - m_R^*) A_R]^{-1} \cdot [(1-\mu_R^* - m_R^*) F_R \\
 & + (1-\mu_R^{**}) m_T^* (A_T X_T + F_T) \\
 & + m_T^{**} E_T^W + (1-m_R^{**}) E_R^W] \quad .
 \end{aligned}$$

The interregional and international trade flows are connected as can be illustrated by Figure 6.

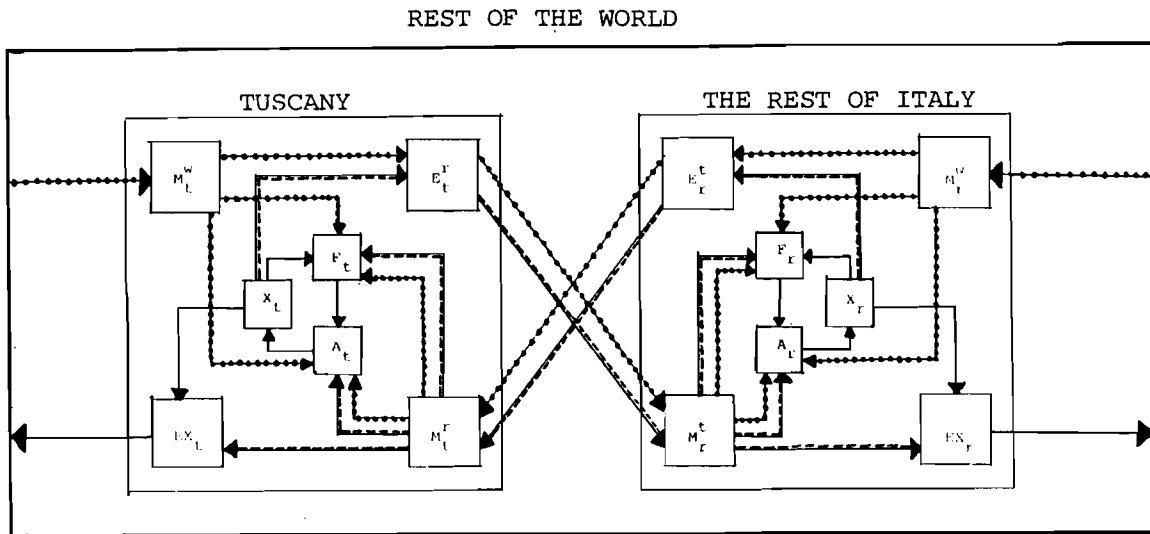


Figure 6. External trade scheme in the Tuscany model.

As it is shown in this figure, the interregional trade is concerned also with imported foreign production and with exported domestic production.

The split of the national import into two parts (used inside the importer region and traded outside the input region) is clear when we aggregate the two regions.

By defining  $A_T X_T + F_T = D_T$  and  $A_R X_R + F_R = D_R$  we obtain

$$\begin{aligned}
 (X_T + X_R) &= (D_T + D_R) - (\mu_T D_T + \mu_R D_R) - (\mu_R^m D_T + \mu_T^m D_R) \\
 X_N &= D_N - M_N^{W*} - M_N^{W**} \\
 &\quad + (E_T^W + E_R^W) , \\
 &\quad + E_N^W ,
 \end{aligned}$$

where

$M_N^{W*}$  = national import used inside the importer regions (Tuscany and the rest of Italy);  
 $M_N^{W**}$  = national import trade outside the importer regions (re-exported import).

### 3.2 The Treatment of Import Flows and Interregional Trade

As far as the import flows are concerned, the main distinction which is usually made is between competitive and complementary imports. Whereas in the economic theory such a distinction is clear (complementary when no domestic substitute is available in the region, competitive when it is), in reality, as well as in modeling work, it is not so. Firstly, by working on a sectoral level (branches of the input-output matrix), each sector includes both competitive and complementary products. Secondly, even if goods are produced in the region, the level of the regional production could not be high enough to meet the needs of the internal demand.

In the regional modeling work it is often assumed that all the import flows are either competitive, or complementary; in addition, by applied fixed coefficients also in the treatment of competitive imports, the distinction with the complementary import is smooth and it has no real economic basis. Taking into account these problems, the imports are often considered either as competitive or complementary according to a very crude assumption. If the import does not affect the production formation, the production is determined by the following equation.

$$X = (I - A)^{-1} (D + E) ,$$

whereas the imports are determined by a second equation:

$$M = m_X \quad \text{or} \quad M = m(D + E) .$$

In this case, the import flows are supposed to be complementary.

In an opposite way, the imports are treated as competitive when they are included in the determination of the production via the following equation:

$$X = (I - mA)^{-1} (D + E) ,$$

if they are competitive with respect to the intermediate products demand, and via the following equation:

$$X = (I - A)^{-1} (D + E - M) ,$$

if they are competitive with respect to the final demand components.

In this last case, the imports are modeled in a similar way to that used in the Keynesian models within an open economy. Taking into account these crude definitions, in the biregional I/O model for the Tuscany region, the imports are considered competitive both for interregional and international flows.

In fact, this approach implies only the assumption that a proportional linkage is supposed to link import and internal demand components via fixed coefficients, i.e. no substitution effect between internal and foreign production is taken into consideration.

As far as the interregional trade coefficients are concerned, the traditional Chenery-Moses procedure is modified in order to include in the interregional trade also the foreign imports and exports. The trade coefficients equations are reformulated in the following way if  $m_T^* = m_T^{**} = m_T$ ; and  $m_R^* = m_R^{**} = m_R$  .



$$X_T + M_T^W = (1-m_T)(D_T+E_T^W) + m_R(D_R+E_R^W) \quad ,$$

$$X_R + M_R^W = (1-m_R)(D_R+E_R^W) + m_T(D_T+E_T^W) \quad ,$$

where  $(1-m_T)$ , as well as  $(1-m_R)$  are the self-sufficiency rate with regard to interregional trade.

$M_T^W$  and  $M_R^W$  are linearly dependent both from the regional demand and from the interregional export (i.e. import of the other regions) demanded for internal utilization through the parameters  $\mu_T^*$ ,  $\mu_R^*$  and  $\mu_T^{**}$ ,  $\mu_R^{**}$ . If  $\mu_T^* = \mu_T^{**} = \mu_T$  and  $\mu_R^* = \mu_R^{**} = \mu_R$ , we get:

$$\mu_T = \frac{M_T^W}{D_T + m_R D_R} \quad \text{and} \quad \mu_R = \frac{M_R^W}{D_R + m_T D_T} \quad ,$$

As in the MORSE model, the parameters  $\mu_T$  and  $\mu_R$  are subjected to change over time according to the expected changes in the sectoral mixing of the import at the national level derived via the INFORUM model.

Future steps in the modeling work will take into account the possibility of treating import flows (or some of them) as real competitive ones. A possible approach could be the one used in the REGIS model, where an economic function links the import flows to some independent variables such as regional demand, price levels, etc., via elasticity parameters. Even in this case the INFORUM model could provide consistent scenarios at the national level.

### 3.3 The Treatment of Export Flows and Tourism

Because of the important economic role of the linkages to the foreign markets, the model for the Tuscany region devotes great attention to a regional determination of the export flows, both of commodities and services (tourists).

A bottom-up approach is used for those commodities which have a high weight in the export mixing of the region and, at

the same time, are supposed to have specific behavioral functions as far as the regional level is concerned.

On the other hand, for commodities whose export functions are not supposed to have regional specifications, we assume that a top-down approach is the most appropriate one.

Whereas in the first case (bottom-up approach), econometric functions at the regional level are used, in the second case (top-down approach), the regional vectors are derived from national forecasts, achieved via INFORUM model to which Italy will be linked in the near future.

In both cases transition matrices are used in order to convert commodities into I/O sectoral branches (31 in Tuscany I/O matrix).

In addition to a highly detailed commodity disaggregation each commodity export function is separately estimated for some groups of foreign commodities, between Tuscany and the rest of Italy.

Figure 7 describes the submodel for the export flows of commodities as it is designed in the Tuscany biregional I/O model.

The following notations are used:

$E_{t,g,c}^W, E_{r,g,c}^W$  = export of Tuscany and the rest of Italy by commodity  $g$  (UICC classification\*) and country  $c$ ;  
 $E_{n,g,c}^W$  = export of Italy by commodity  $g$  (SITC classification\*\*) and country  $c$ ;  
 $T_{i,g}^{UICC}, T_{i,g}^{SITC}$  = transition matrices converting commodities in SITC and UICC classification into I/O sectoral branches;

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\* UICC (Unione Italiana Camere di Commercio) classification is adopted in Italy for export and import data at the provincial level, derived from foreign payment flows.

\*\* SITC (Standard International Trade Classification) classification is adopted by the United Nations and by OECD for the international trade.

$E_{t,i(o,m)}^W, E_{r,i(o,m)}^W$  = vectors of regional export by sectors (o-m), derived from the bottom-up approach;

$E_{t,i(m,n)}^W, E_{r,i(m,n)}^W$  = vectors of regional export by sectors (m-n), derived from the top-down approach;

$$E_{n,g}^W; E_{t,g}^W; E_{r,g}^W = \sum_c E_{n,g,c}^W; \sum_c E_{t,g,c}^W; \sum_c E_{r,g,c}^W ,$$

$$E_{n,i(o,m)}^W; E_{n,i(m,n)}^W = E_{t,i(o,m)}^W + E_{r,i(o,m)}^W ;$$

$$E_{t,i(m,n)}^W + E_{r,i(m,n)}^W .$$

The econometric regional export functions are estimated by techniques applied to independent variables at the international and national level.

Both time series and cross-section data are used, the latter by grouping the single countries into market areas. All the variables are expressed in constant prices.

The regional specification is achieved by estimating different elasticity parameters between Tuscany and the rest of Italy. In addition, by getting each set of parameters for each country and commodity combination, the country share composition, by itself, implies different estimations of the export trends between Tuscany and the rest of Italy, even if the elasticity was the same.

Regional exports are treated by an additive demand and price relationship:

$$E_{T,g,c}^W = K_{T,g,c} + a_{T,g,c} (D_c \cdot \frac{D_c}{D_N})^{b_{T,g,c}} \cdot RP_{g,c}^{c_{T,g,c}} ,$$

$$E_{R,g,c}^W = K_{R,g,c} + a_{R,g,c} (D_c \cdot \frac{D_c}{D_N})^{b_{R,g,c}} \cdot RP_{g,c}^{c_{R,g,c}} .$$

These functions assume that only a part of the exports are price and/or demand dependent due to the constants  $K_{T,g,c}$  and  $K_{R,g,c}$ .

The variable  $RP_{g,c}$  is the index of the relative export prices as they are taken into account by the foreign customers.  $RP_{g,c}$  is equal to the index of the national export price, divided by the index of price in country C; finally, the result is multiplied by the index of exchange between Italy and country C.

The second independent variable is a composite one. The index of the domestic demand in country  $c$  ( $D_c$ ) is, in fact, corrected by the ratio between the same index and that of domestic demand in Italy.

Here it is assumed that a partial substitution effect between foreign market and the national one can influence the business cycle of regional export.

The role of the foreign demand is reduced or amplified according to the differentials in the growth rate between the foreign demand itself and the national one.

By adopting this composite variable, a linkage, even if a very simple one, is established between the regional foreign export and the interregional trade which is dependent, via trade coefficients, by the domestic demand in both regions.

As far as the part of the regional export vector derived from the national forecast (via the INFORUM model) is concerned, the following algorithm is used:

$$E_T^W = \epsilon_T E_N^W ; \quad E_R^W = \epsilon_R E_N^W ,$$

for sectors  $i$  from  $m$  to  $n$  (see Figure 7).

Due to the role of external tourism in Tuscany, the tourist flows are treated as an additional commodity. The procedure is similar to that applied for the export flows, i.e. an econometric function is estimated for each country of origin (including the rest of Italy and Tuscany).

The dependent variable is the number of the tourist-days ( $T^C$ ), whereas the independent variables, are the matrix of the demand in country  $c$  ( $D_c$ ) and the national price index, deflated

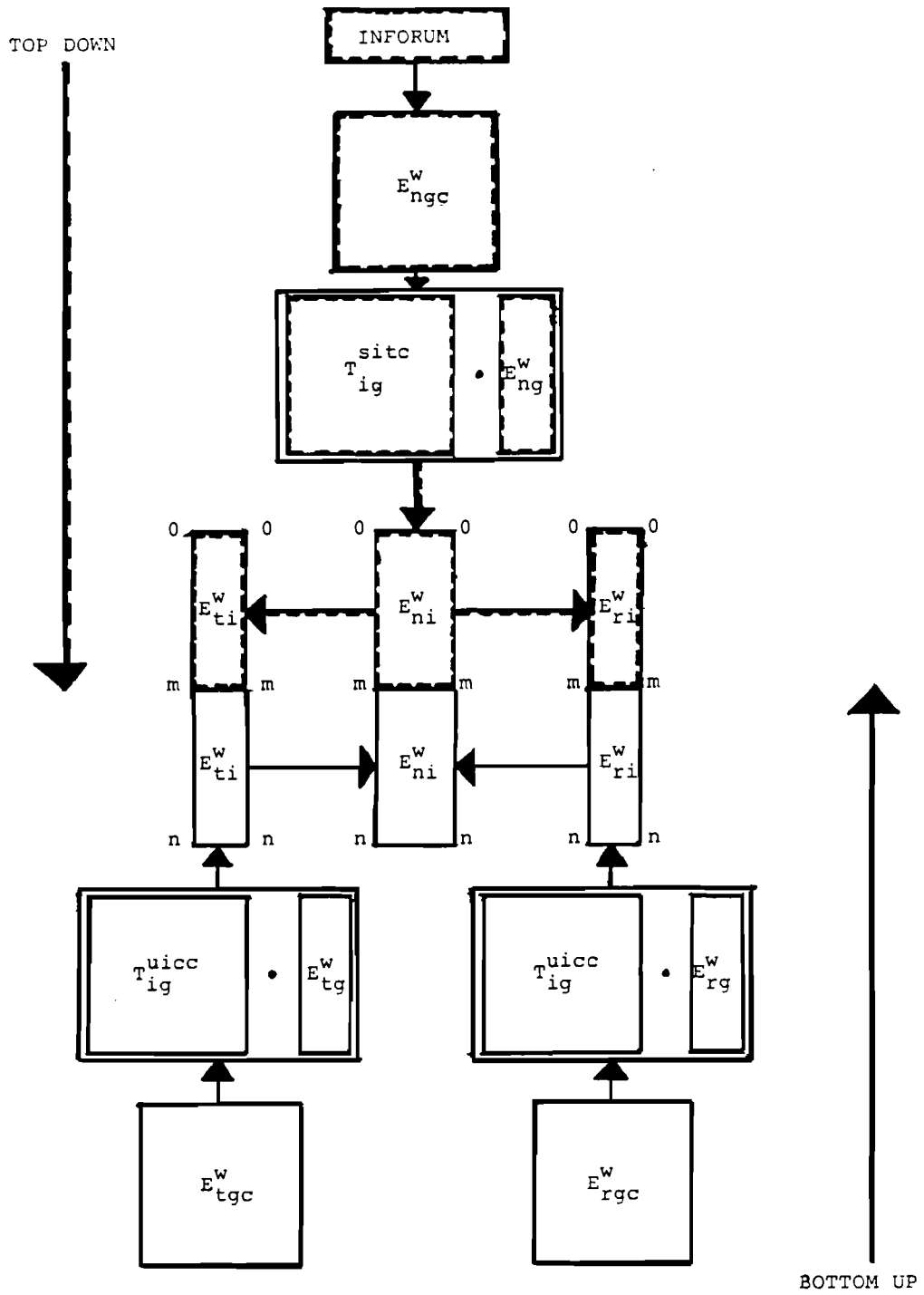


Figure 7. Export flows scheme.

by the price index and exchange index in country  $c$  ( $RP_c$ ); this last variable is assumed to be equal to 1 for the Italian tourists.

The function used has the following form:

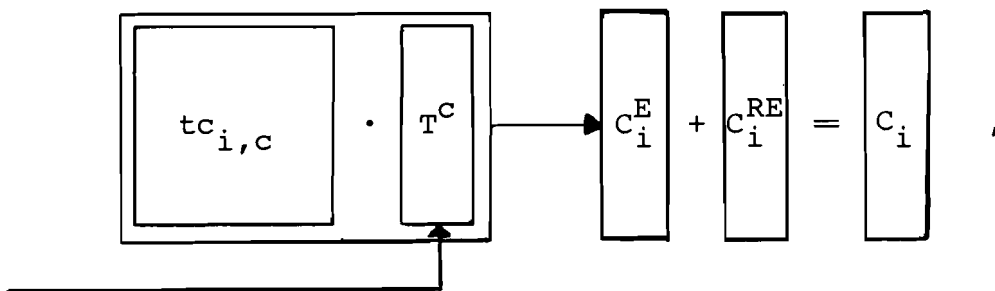
$$T_T^C = K_{T,c} + a_{T,c} \cdot D_C^{b_{T,c}} \cdot RP_C^{c_{T,c}} ;$$

$$T_T^R = K_{T,R} + a_{T,R} \cdot D_N^{b_{T,R}} ;$$

$$T_R^C = K_{R,c} + a_{R,c} \cdot D_C^{b_{R,c}} \cdot RP_C^{c_{R,c}} ;$$

$$T_R^T = K_{R,T} + a_{R,T} \cdot D_N^{b_{R,T}} ;$$

The estimations of the tourist-days are converted into the tourist-consumption through a transition matrix of the sectoral tourist consumption by sector  $i$  and country  $c$  ( $t_{ci,c}$ ) according to the following scheme:



where

$C_i^E$  = consumption of non-residents;

$C_i^{RE}$  = consumption of residents;

$C_i$  = total private consumption in the region.

In the biregional model, the total private consumption, as a component of the final demand, assumes the following form in Tuscany and in the rest of Italy.

$$C^T = C_T^W + C_T^R + C_T^T ,$$

$$C_R = C_R^W + C_R^T + C_R^R .$$

It should be noted that a regional specification of the tourism functions, as well as those for export, is made possible by the availability of regional data over a long period of time.

#### 4. CLOSING REMARKS

The international trade is a weak point in many multiregional economic models, especially as far as the treatment of export is concerned.

The most common treatment of the export flows is to regionalize results of a national economic model. This methodology is appropriate for commodities where export functions are not supposed to have regional specifications. On the other hand, for those commodities supposed to have specific behavioral functions at the regional level, a bottom-up approach is preferred.

The model for the Tuscany region devotes great attention to a regional determination of the export flows both of commodities and services (foreign tourism), as well as the interdependencies between international and interregional trade. Even if the methodology adopted in modeling such linkages is a very simple one, the external trade relationships at the regional level is, to some extent, more comprehensive.

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