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A DYNAMIC GAME APPROACH TO ANALYZE BUFFER STOCK ACTIVITIES ON OLIGOPOLISTIC MARKETS

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FOREWORD

Understanding the nature and dimensions of the world food problem and the policies available to alleviate it has been the focal point of the IIASA Food and Agriculture Program since it began in 1977.

National food systems are highly interdependent, and yet the major policy options exist at the national level. Therefore, to explore these options, it is necessary both to develop policy models for national economies and to link them together by trade and capital transfers. For greater realism the models in this scheme are being kept descriptive rather than normative. In the end it is proposed to link models of about twenty-five countries, which together account for nearly 80 per cent of important agricultural attributes such as area, production, population, exports, imports, and so on.

In the national policy models, particularly in the linked system of twenty-five national models, a large number of policy paramters are involved. To reduce the dimensionality of these parameters we need to identify structural relationships among these parameters. Werner Gueth in this paper uses a game theoretic approach to explore some aspects of the interdependence of policies of various agents on international markets. The understanding obtained from such explorations will be useful in reducing the number of independent policy parameters that have to be specified to use the system of linked models.

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My interest in studying buffer stock activities is a result of my lively discussions with Desmond McCarthy (IIASA), who indicated to me the attempts toward forming boards on some special world food markets. Together with Bozena Lopuch (IIASA), I have numerically analyzed the influence of various parameters on the market process. Since this numerical analysis has been greatly generalized by Andreas Thiemer, the paper has been restricted to a purely analytic discussion. Both Bozena Lopuch and Andreas Thiemer indicated to me some errors of an earlier version. It is, furthermore, a pleasure to thank Marianne Spak (IIASA) for her excellent typing of the difficult manuscript.

ABSTRACT

On oligopolistic markets with dynamic production functions one can often observe attempts toward cooperative agreements to "stabilize" prices. On the other hand, it is a well-known fact that cartel agreements are difficult to control, especially when the sellers come from different countries. In such a situation partial agreements - for instance, founding a common marketing board - seem more likely than detailed cartel agreements.

In this paper we analyze the economic institution of a common buffer stock agency which can shift supply from present to future periods. Although the buffer stock agency determines the prices, individual production amounts are chosen independently by the sellers. We study the case of a price buffer stock agency which cannot produce, as well as the situation in which a major seller controls the buffer stock. One interesting result is that it makes quite an important difference whether the buffer stock agency is or is not able to produce. Loosely speaking one can say that only when the buffer stock is controlled by a producer do the other sellers not have to consider the future effects of present supply decisions.

1. INTRODUCTION

On the so-called world markets for various agricultural products (cocoa, coffee, etc.) there seems to be a permanent tendency to form a board (cocoa board, coffee board, etc.). Since most of these markets have a rather inelastic demand at given prices (i.e., a good harvest will usually diminish total revenues on the market), the main purpose of board formation is presumably to "stabilize" supply. Sometimes stabilization of supply might take the form of disposal; i.e., since there is a permanent oversupply on the market, the board simply tries to withhold supply, perhaps by directing it to another market (wheat, for instance, can be used to feed animals or to produce alcohol) or even by destroying the oversupply. Here we do not want to consider situations in which stabilization simply has the form of withholding; some strategic aspects of these situations could easily be analyzed within the standard framework of static oligopoly theory. It will be assumed instead that stabilization of supply is always done via building up so-called buffer stocks which allow the sellers to shift present supply to later periods.

The strategic situation on the market certainly depends on the levels of stocks built up until the present period; i.e., on the previous decision behavior of the sellers. This indicates that market situations in which stabilization is done via stock adjustment should be looked at as dynamic games in which players do not face a static conflict situation but rather interact in time. Stabilization of supply by stock adjustment can, of course, be done individually or collectively. Here it is assumed that sellers want to have some coordinated way of withholding supply. This does not necessarily require that all or some sellers form a cartel assigning quotas to all its members. If the members of such a cartel come from countries with different economic backgrounds, it is usually difficult to keep the group together. Therefore, it is more reasonable to assume that collective decision behavior is limited to some decentralized form of withholding supply via the institution of a buffer stock agency. So one of the countries engaged on the market – probably a major seller like Brazil on the coffee and cocoa market – may, for instance, run the only buffer stock; or some of the sellers may found an independent buffer stock agency in order to "stabilize the market" by shifting supply to future periods. In any case we will restrict our attention to situations in which there is just one buffer stock. This will greatly simplify the computational problems involved in solving the dynamic game. In the terminology of the theory of optimal control this amounts to saying that we restrict ourselves to situations with a one-dimensional state space.

Apart from special situations, solving dynamic games often requires lengthy computations. Here we want to simplify the computational problem as far as possible; i.e., we will look at games which represent the relevant aspects within a rather simple framework. In particular, we will not attempt to develop one model which tries to incorporate all relevant strategic aspects of running a buffer stock.

There is a long tradition in agricultural economics of discussing optimal control of food stocks (see Gardner 1969, and the references mentioned there). But analysis has always been based on non-strategic situations with just one player, the one who controls the buffer stock. So what is new in the present work is that we study the optimal control of food stocks by using the theory of noncooperative dynamic games.

In the following we will first analyze the problem in a deterministic framework, which means that the only uncertainty which the players have to face is that they do not know what their opponents are going to do. Player 1 runs the buffer stock, which may or may not be able to produce. Players $2,\dots,n$ (≥ 2) are pure producers. Since we want mainly to study how to control so-called world markets, we refer to players for the most part as countries. In every period all producers first simultaneously choose their investment amounts, which determine the production levels in the following periods. Afterwards the board has to announce the price for the next period. In order to be believed, the board must always adjust its sales amounts in such a way that total supply equals total demand at the prices it has chosen.

It should be mentioned that we study the so-called closed-loop model, which allows the players to react to what they have observed in the past and which is the only satisfying approach to studying the strategic interaction in time as it arises in the context of running a buffer stock on an oligopolistic market. The game is solved by backward induction; i.e., we first look at the possible situations in the last periods and then, by anticipating these results for the future periods, we determine what is done in the present period. Main emphasis is put on analytic discussion of how the various parameters influence the economic results. This is usually done by assuming symmetry of the countries 2,...,n, which allows us also to study the influence of the market structure on the economic results.

An extensive numerical discussion can be found in Thiemer (1981), where a more general version of our model with deterministic expectations for future demand and supply conditions is analyzed. It allows for quadratic cost functions and includes the effect that past production activities may cause higher production costs in the future. In mathematical terminology the latter is done by introducing two state variables – the buffer stock and the scarcity of natural resources.

After having discussed the solution of deterministic games with a finite number of decision periods, we will indicate how one might study games with infinitely many decision periods. To illustrate how one can include the stochastic nature of the economic relationships, we shall later consider a game which differs from the symmetric case of the deterministic game only by the assumption that the marginal productivities are stochastic variables. Again we will discuss the influence of economic parameters on economic results, concentrating on those parameters which have been used to describe the stochastic nature of the production process. In the final section we summarize our results and indicate possible generalizations to incorporate some additional aspects which might be relevant for a strategic analysis of collective buffer stocks on international food markets.

2. RUNNING A BOARD IN A DETERMINISTIC FRAMEWORK

2.1. Overview

In the following we want to analyze games whose sets of players include a food stock agency – usually referred to as player 1 – and which assume that future productivity and demand conditions are known to all sellers. Since in many situations it is not the individual producers but rather national selling agencies – whose action variables are highly aggregated economic variables – which have to be considered as players, this assumption might not be so unjustified as it is on the level of individual producers.

We will use the following notation: capital letters refer to aggregated variables, lower case letters to individual variables; subindices indicate the period, upper indices the player or country; the vector of individual decision variables is usually indicated by dropping the upper index.

$1,\ldots,n(\geq 2)$	countries in the world, player 1 is the board player
$y_t^i \geq 0$	country i's total production in period t
$Y_t \geq 0$	world production in period t
$X_t \geq 0$	world demand in period t
$B_t \geq 0$	buffer stock level in period t
$s_t^i \geq 0$	country i's total sales amount in period t
$S_t \geq 0$	world sales amount in period t
$p_t \geq 0$	world market price in period t
$c_{t-1}^i \geq 0$	cost level of country i in period t
$t = 1,\ldots,T$	periods considered by all the players

The notation c_{t-1}^i for the cost level in period t should indicate that this variable is determined in period $t - 1$. The following equations follow by definition (summation over i with no further specification refers to summation from $i = 1$ to $i = n$; summation over $i \neq j$ refers to summation over $i = 1$ to $i = n$ with the exception of $i = j$):

$$Y_t = \sum_i y_t^i \qquad \text{total production in period t} \tag{1}$$

$$S_t = \sum_i s_t^i \qquad \text{total sales in period t} \tag{2}$$

$$S_t = X_t \qquad \text{market clearing condition} \tag{3}$$

$$B_{t+1} = B_t + Y_t - S_t \qquad \text{stock development equation} \tag{4}$$

To complete the underlying economic structure we assume $B_2 = 0$ and

$$X_t = \alpha_t - \beta_t p_t \qquad \alpha_t > 0\,, \qquad \beta_t > 0 \qquad \text{for all t} \tag{5}$$

$$y_t^i = \rho_t^i + \delta_t^i c_{t-1}^i \qquad \delta_t^1 \geq 0\,, \qquad \delta_t^j > 0\,, \qquad \sum_i \rho_t^i < \frac{\alpha_t}{2} \qquad \text{for all t and } j \geq 2 \tag{6}$$

$$s_t^j = y_t^j \qquad \text{for } j = 2,\ldots,n \quad \text{and all t} \tag{7}$$

Thus we assume that only country 1 is able to store the product, whereas the others have to sell whatever they produce. According to the restrictions on the parameters, all non-board countries can always produce more by investing more. The condition $\sum_i \rho_t^i < \frac{\alpha_t}{2}$ eliminates only those situations in which there is enough supply without investment and stocks. Although one can also allow for small positive values of B_2, we assume $B_2 = 0$ to simplify our analysis. The reason for not having B_2 too large will become obvious when solving the game. Player 1 must not necessarily be one of the countries in the world. For instance, it can be simply the buffer stock agency, which then, of course, is unable to produce; i.e., we must have $\rho_t^1 = 0$ and $\delta_t^1 = 0$ for all t. In that case the storage capacity would be used only to store some of the other countries' production. In the following we will still refer to player 1 as country 1, although our analysis will be general enough to include the case in which country 1 is just a buffer stock agency. To avoid situations in which storage is essentially disposal, we impose furthermore

$$\frac{\alpha_{t+1}}{2\beta_{t+1}} < \frac{1}{\delta_{t+1}^j} \qquad \text{for } j = 2,\ldots,n \text{ and all t} \tag{8}$$

i.e., marginal production costs $1/\delta^j_{t+1}$ for countries $j \geq 2$ are assumed to be bounded from below by the price $\alpha_{t+1}/2\beta_{t+1}$, which maximizes total revenues in period t + 1.

In every period $t \geq 1$ the market decision process is assumed to have the following two stages:

Stage 1 (production decision stage):

Knowing all the previous decisions – i.e., all decisions in the earlier periods – all countries $i = 1,\ldots,n$ choose simultaneously and independently the amount $c^i_t \geq 0$ which they are going to invest in production and which determines their output y^i_{t+1} in the next period t+1 via the production hypothesis specified above.

Stage 2 (price determination stage):

Knowing the vector $\mathbf{c}_t = \left(c^1_t, \ldots, c^n_t\right)$ of production decisions, country 1 announces the price p_{t+1} which will be the valid price for all trades within period t + 1. After this decision all countries are informed about p_{t+1}.

To complete the description of the game model we still have to specify the payoff functions for all the countries. In every period $t \geq 1$ country $j = 2,\ldots,n$ tries to maximize the sum of its discounted future profits; i.e., for $j = 2,\ldots,n$ the payoff level H^j_t of country j in period t is given by

$$H^j_t = \sum_{\tau \geq t+1} (d^j_t)^{\tau-(t+1)} \left[p_\tau y^j_\tau - c^j_{\tau-1}\right] \qquad 0 \leq d^j_\tau \leq 1 \qquad \text{for all t} \tag{9}$$

where d^t_τ is the discount factor expressing country j's time preferences in period τ. It should be mentioned that the whole term $p_\tau y^j_\tau - c^j_{\tau-1}$ is completely determined by the decisions in period $\tau-1$, since the variables p_τ and y^j_τ are either decision variables of the decision period $\tau-1$ or functions of them. This clearly reveals that the present decisions influence the future only via the buffer stock equation (4) and not via dynamic production functions. In the terminology of control theory this amounts to saying that equation (4) describes the transition law of the dynamic system. Player 1's profit in period τ is determined by his producer's revenues $p_\tau y^1_\tau - c^1_{\tau-1}$, which will vanish in case of $\rho^1_\tau = 0 = \delta^1_\tau$, and by his way of running the buffer stock. The profit from running the buffer stock is determined by the sum of the stock adjustments $s^1_\tau - y^1_\tau$ evaluated by p_τ and the "storage costs"

$$\varepsilon_\tau + \mu_\tau B_\tau + \xi_\tau B^2_\tau \qquad \xi_\tau > 0, \qquad \mu_\tau < 0 \tag{10}$$

which, as we assume, are a quadratic function of the storage level B_τ in period τ. The parameter ε_τ might include a fixed income of player 1 which results from given periodic transfers by all the other countries to player 1 to pay him for running the buffer stock. The interpretation of (10) as "storage cost" may be misleading: by $\mu_\tau < 0$ we want to exclude the unrealistic event of zero stocks in earlier periods by imposing a kind of penalty for low stocks. So it is essentially due to $\xi_\tau > 0$ that above a certain level higher stocks induce higher "storage costs." Consequently in every period $t \geq 1$ player 1 tries to maximize

$$H^1_t = \sum_{\tau \geq t+1} \left(d^1_\tau\right)^{\tau-(t+1)} \left[p_\tau s^1_\tau - c^1_{\tau-1} - \varepsilon_\tau - \mu_\tau B_\tau - \xi_\tau B^2_\tau\right] \tag{11}$$
$$- \hat{p}_{T+2}(B_{T+2} - \hat{B}_{T+2})^2$$

where $0 \le d_\tau^1 \le 1$ for all τ. $\hat{B}_{T+2}$ is the target stock for the time after the end of the game (or, in the terminology of control theory, the transition level of the state variable), and $\hat{p}_{T+2}$ (≥ 0) is the parameter determining the penalty for not reaching the target stock. Here it will be assumed that $\hat{B}_{T+2} = 0$ and $\hat{p}_{T+2} = \infty$; i.e., that the transition stock level $\hat{B}_{T+2}$ has to be zero. Given $\hat{B}_{T+2} = B_{T+2}$ the penalty term in (11) can obviously be neglected. If player 1 is just a buffer stock agency, we will have $c_{\tau-1}^1 = 0$ and

$$s_\tau^1 = X_\tau - \sum_{i \ne 1} y_\tau^i \qquad (12)$$

because of $y_\tau^1 = 0$.

What remains to be specified is the number $T(\ge 1)$ of periods $t = 1,\dots,T$; i.e., the number of production periods which will take place. Of course, for $T = \infty$ the discount factor d_t^j must always be strictly smaller than 1 (except for a finite number of periods) for all players $j = 1,\dots,n$, in order to assure finite payoff levels. In the beginning it will be assumed that T is finite. Later on we will indicate how to handle the case $T = \infty$.

2.2. The case of finitely many periods

The solution concept for dynamic games with finitely many periods is that of a subgame perfect equilibrium point (this concept was introduced by Selten 1975; for applications to dynamic games see Selten 1965; Gottwald and Gueth 1980; and Boege et al. 1980). Here we will not give an abstract definition of subgame perfect equilibrium points, nor will we try to determine the (unique) subgame perfect equilibrium point of the games described. This would require complicated descriptions of the decision behavior in situations which cannot occur anyway in the case of rational decision makers. What we will do instead is to compute only the actual play, i.e., the decisions actually made according to the solution. This is done by first solving the possible situations in the last period T. With the knowledge of what will be done in period T, the situations in period T - 1 actually become one-period games, as is the case for the situations in period T. Continuing in the same way we can determine the decision behavior in all periods by backward induction. For a more detailed description of the computational procedure, which is often referred to as dynamic programming, see the papers mentioned above.

We derive the decision behavior by induction over $t' = T - t$. First we start with the decision behavior for $t' = 0$. Then it will be shown how to derive the decision behavior for $t' + 1$ under the assumption of known decision behavior for t'. To compute the behavior of the one-period games which have to be considered in the course of backward induction, one usually has to solve a system of n linear nonhomogeneous equations, which under reasonable assumptions will always have a unique solution. For the sake of brevity we do not investigate this problem in great detail. In order to allow for an easy analytic discussion of the economic results, we will always discuss the symmetric case after having derived the equation system for the general case. The symmetric case here refers to the situation in which all countries $j = 2,\dots,n$ always have the same payoff function; i.e., where we have

$$\delta_t^j = \delta_t \qquad (13)$$

$$\rho_t^j = \rho_t \qquad (14)$$

$$d_t^j = d_t \qquad (15)$$

for all periods $t = 1,\dots,T$ and for all countries $j = 2,\dots,n$. Because of the symmetry of the countries $j \geq 2$ it will be possible to study the influence of the market structure on the market results simply by varying the number, n, of countries in the world.

2.2.1. Decision behavior in period T

According to the procedure of backward induction one first has to determine the price p_{T+1} which will be chosen by player 1 after the production decision stage in period T. One easily derives

$$p_{T+1} = \begin{cases} \dfrac{\alpha_{T+1}}{2\beta_{T+1}} & \text{for } \dfrac{\alpha_{T+1}}{2} < B_{T+1} + Y_{T+1} \\ \dfrac{\left[\alpha_{T+1} - B_{T+1} - Y_{T+1}\right]}{\beta_{T+1}} & \text{for } \dfrac{\alpha_{T+1}}{2} \geq B_{T+1} + Y_{T+1} \geq 0 \end{cases} \tag{16}$$

Now let us investigate whether $B_{T+1} + Y_{T+1} > \alpha_{T+1} / 2$ can ever result in the case of rational decision makers. First we observe that because of $\alpha_{T+1} / 2\beta_{T+1} < 1 / \delta^j_{T+1}$ for all $j = 2,\dots,n$ the board player can induce the countries $j > 1$ to choose $c^j_T = 0$. Furthermore, player 1 can always ensure that $B_{T+1} = 0$, since $B_2 = 0$ and $1 / \delta^j_{t+1} > \alpha_{t+1} / 2\beta_{t+1}$ for all previous decision periods, t, and all countries, j; i.e., player 1 can induce every country $j > 1$ to produce only its minimal amount without decreasing total revenues. Thus according to the actual play as determined by the solution of the game, the situation $B_{T+1} + Y_{T+1} > \alpha_{T+1} / 2$ will not occur.

For the symmetric case it will be shown below that the assumption of $\alpha_{T+1} / 2 \geq B_{T+1} + Y_{T+1}$ is – under rather mild restrictions for the parameters ρ^j_{T+1} $(j = 1,\dots,n)$ – a self-fulfilling prophecy in the sense that, starting with $\alpha_{T+1} / 2 \geq B_{T+1} + Y_{T+1}$, the optimal investment decisions always lead to situations satisfying $\alpha_{T+1} / 2 \geq B_{T+1} + Y_{T+1}$ regardless of the value B_{T+1} of the state variable.

Although we eliminated by our assumption the situation $B_{T+1} + Y_{T+1} > \alpha_{T+1} / 2$, which induces $B_{T+2} > 0$, we would like to consider briefly the case in which part of the total production – namely B_{T+2} – is not used to serve demand. There are two main to exclude such situations. First of all it would seem to be rather unreasonable for country 1 to store more than is actually needed and thus to increase its storage costs, although it foresees that the additional stock will never be used. That is why we believe that in our model one should not include situations in which storing activities are, at least partially, disposal.

If it is profitable in certain periods to set aside supply which will not be used later on, one obviously can determine – at least locally – the decisions in those periods without considering future effects. This supports our belief that some aspects of disposal activities can reasonably be investigated within the framework of static oligopoly theory, whereas pure shifts of supply between periods certainly require long-run consideration in a dynamic set up.

Anticipating that

$$p_{T+1} = \frac{\left[\alpha_{T+1} - B_{T+1} - Y_{T+1}\right]}{\beta_{T+1}} \tag{17}$$

country i (≥ 2) wants to maximize

$$\left[\alpha_{T+1} - B_{T+1} - \sum_j (\rho^j_{T+1} + \delta^j_{T+1} c^j_T)\right] \cdot \frac{\rho^i_{T+1} + \delta^i_{T+1} c^i_T}{\beta_{T+1}} - c^i_T \tag{18}$$

From the necessary condition for a local maximum we derive

$$c^i_T = \frac{\alpha_{T+1} - B_{T+1} - \sum_{j \neq i} (\rho^j_{T+1} + \delta^j_{T+1} c^j_T) - \rho^i_{T+1} - \dfrac{\beta_{T+1}}{\delta^i_{T+1}}}{2\delta^i_{T+1}} \tag{19}$$

Country 1 wants to maximize

$$\left[\alpha_{T+1} - B_{T+1} - \sum_j (\rho^j_{T+1} + \delta^j_{T+1} c^j_T)\right] \cdot \frac{\rho^1_{T+1} + \delta^1_{T+1} c^1_T + B_{T+1}}{\beta_{T+1}} - c^1_T \tag{20}$$

which yields accordingly

$$c^1_T = \frac{\alpha_{T+1} - 2B_{T+1} - \sum_{j \neq 1} (\rho^j_{T+1} + \delta^j_{T+1} c^j_T) - \rho^1_{T+1} - \dfrac{\beta_{T+1}}{\delta^1_{T+1}}}{2\delta^1_{T+1}} \tag{21}$$

so the n equations resulting from the necessary conditions for a local payoff maximum yield the following system of n linear equations in the n unknowns c^i_T; i = 1,...,n :

$$2\delta^1_{T+1} c^1_T + \sum_{j \neq 1} \delta^j_{T+1} c^j_T = \alpha_{T+1} - 2B_{T+1} - \sum_j \rho^j_{T+1} - \frac{\beta_{T+1}}{\delta^1_{T+1}} \tag{22}$$

and

$$2\delta^i_{T+1} c^i_T + \sum_{j \neq i} \delta^j_{T+1} c^j_T = \alpha_{T+1} - B_{T+1} - \sum_j \rho^j_{T+1} - \frac{\beta_{T+1}}{\delta^i_{T+1}} \qquad \text{for } i = 2,\dots,n. \tag{23}$$

Using the notation

$$\mathbf{A}_{T+1} = \begin{vmatrix} 2\delta^1_{T+1} & \delta^2_{T+1} & \delta^3_{T+1} & \dots & \delta^n_{T+1} \\ \delta^1_{T+1} & 2\delta^2_{T+1} & \delta^3_{T+1} & \dots & \delta^n_{T+1} \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \delta^1_{T+1} & \delta^2_{T+1} & \delta^3_{T+1} & \dots & 2\delta^n_{T+1} \end{vmatrix} \tag{24}$$

$$\mathbf{c}_T = (c^1_T, \cdots, c^n_T)^T \tag{25}$$

$$\mathbf{b}_{T+1} = \begin{vmatrix} \alpha_{T+1} - 2B_{T+1} - \sum_j \rho^j_{T+1} - \frac{\beta_{T+1}}{\delta^1_{T+1}} \\ \alpha_{T+1} - B_{T+1} - \sum_j \rho^j_{T+1} - \frac{\beta_{T+1}}{\delta^2_{T+1}} \\ \cdot \\ \cdot \\ \cdot \\ \alpha_{T+1} - B_{T+1} - \sum_j \rho^j_{T+1} - \frac{\beta_{T+1}}{\delta^n_{T+1}} \end{vmatrix} \quad (26)$$

the equation system can be written as

$$\mathbf{A}_{T+1}\mathbf{c}_T = \mathbf{b}_{t+1} \quad (27)$$

It can be easily seen that det $\mathbf{A}_{T+1} \neq 0$ if $\delta^j_{T+1} > 0$ for $j = 1,\ldots,n$. Consequently, the equation system (27) has a unique solution if $\delta^1_{T+1} > 0$. If $\delta^1_{T+1} = 0$ the solution is unique, too, since then player 1 has to determine only p_{T+1} in period T, since $c^1_T = 0$; i.e., one has only an $(n - 1) \times (n - 1)$ matrix $\mathbf{A}_{T+1}$ instead of an $n \times n$ matrix.

The symmetric case:

We first want to show that the symmetry of the countries $j = 2,\ldots,n$ implies that these countries j choose the same amount c^j_T in equilibrium. Let us therefore consider the decisions c^j_T and c^k_T of two countries $j, k \geq 2$ as determined by the formula above: From

$$|c^j_T - c^k_T| = |-(\rho_{T+1} + \delta_{T+1}c^k_T)/2\delta_{T+1} + (\rho_{T+1} + \delta_{T+1}c^j_T)/2\delta_{T+1}| \quad (28)$$

it immediately follows that one must have

$$c^j_T = c^k_T \qquad \text{for } j,k = 2,\ldots,n. \quad (29)$$

The equation system (27) can therefore be simplified to

$$2\delta^1_{T+1}c^1_T + (n-1)\delta_{T+1}c^2_T = b^1_{T+1} \quad (30)$$

$$\delta^1_{T+1}c^1_T + n\delta_{T+1}c^2_T = b^2_{T+1} \quad (31)$$

where b^j_{T+1} denotes the j-th component of the vector $\mathbf{b}_{T+1}$. The unique solution for the symmetric case is thus

$$c^1_T = \frac{nb^1_{T+1} - (n-1)b^2_{T+1}}{(n+1)\delta^1_{T+1}} \quad (32)$$

$$c^j_T = \frac{2b^2_{T+1} - b^1_{T+1}}{(n+1)\delta_{T+1}} \qquad \text{for } j = 2,\ldots,n \quad (33)$$

With the help of (32) and (33) we are now able to compute Y_{T+1} in order to check whether our initial assumption

$$\frac{\alpha_{T+1}}{2} \geq B_{T+1} + Y_{T+1} \quad (34)$$

is satisfied. We get

$$Y_{T+1} = \frac{\sum_j \rho^j_{T+1}}{n+1} + \frac{n}{n+1}\alpha_{T+1} - B_{T+1} - \frac{\beta_{T+1}}{n+1}\left[\frac{1}{\delta^1_{T+1}} + \frac{n-1}{\delta_{T+1}}\right] \tag{35}$$

Substituting (35) into (34) yields

$$\frac{\beta_{T+1}}{\alpha_{T+1}} \geq \frac{\left[\dfrac{2\sum_j \rho^j_{T+1}}{\alpha_{T+1}} + n - 1\right]\delta_{T+1}}{2\left[\dfrac{\delta_{T+1}}{\delta^1_{T+1}} + n - 1\right]} \tag{36}$$

Assume now that $\sum_j \rho^j_{T+1} = 0$. Condition (36) can then be expressed as

$$\frac{\beta_{T+1}}{\alpha_{T+1}} \geq \frac{\delta_{T+1}}{2\left[1 + \dfrac{\delta_{T+1}}{\delta^1_{T+1}(n-1)}\right]} \tag{37}$$

It is easy to see that the right-hand side of (37) increases if n increases. Thus a sufficient condition for (37) can be obtained by substituting the right-hand side of (37) by its limit value for $n \to \infty$, which yields:

$$\frac{\beta_{T+1}}{\alpha_{T+1}} \geq \frac{\delta_{T+1}}{2} \tag{38}$$

or

$$\frac{\alpha_{T+1}}{2\beta_{T+1}} \leq \frac{1}{\delta_{T+1}} \tag{39}$$

which is always satisfied because of the restrictions on the parameters.

Thus it has been shown that the initial assumption (34) is justified, since it proves to be a self-fulfilling prophecy. The optimal investment decisions given the expectation that (34) holds always lead to situations satisfying (34) regardless of the value B_{T+1} of the state variable in period T and thus of the actual play so far.

Discussion of the behavior in the last decision periods:

In the following we assume the symmetric case and that every period is considered as the last; i.e., that the decision behavior in t prescribed by the solution of the game is always given by formulae (16), (32),and (33) when we substitute t for T. This assumption cannot be simply justified by letting $d^j_{T+1} \to 0$ for all players and periods. In a last period all stock must be sold, which is not true for an earlier period even if the players themselves do not care about the future. Consequently, one has

$$c^1_t = \frac{nb^1_{t+1} - (n-1)b^2_{t+1}}{(n+1)\delta^1_{t+1}} \tag{40}$$

$$c^j_t = \frac{2b^2_{t+1} - b^1_{t+1}}{(n+1)\delta_{t+1}} \qquad \text{for } j = 2,\ldots,n \tag{41}$$

$$p_{t+1} = \frac{\alpha_{t+1} - B_{t+1} - Y_{t+1}}{\beta_{t+1}} \tag{42}$$

The symmetric case allows us to explore the market results for various market structures as indicated by the number n of countries in the world. Here we only investigate the case of competitive markets which results for $n \to \infty$. Since $\rho_{t+1} > 0$ would imply an infinite production amount, we require $\rho_{t+1} = 0$. One easily derives

$$\lim_{n\to\infty} c_t^1 = \frac{\beta_{t+1} \dfrac{\delta_{t+1}^1 - \delta_{t+1}}{\delta_{t+1}^1 \delta_{t+1}} - B_{t+1}}{\delta_{t+1}^1} \qquad \text{if } \delta_{t+1}^1 \geq \delta_{t+1} \tag{43}$$

$$\lim_{n\to\infty} c_t^j = 0 \qquad \text{for } j = 2,\ldots n \tag{44}$$

$$\lim_{n\to\infty} Y_{t+1} = \alpha_{t+1} - B_{t+1} - \frac{\beta_{t+1}}{\delta_{t+1}} \tag{45}$$

$$\lim_{n\to\infty} p_{t+1} = \frac{1}{\delta_{t+1}} \tag{46}$$

Thus the price in equilibrium is determined by the (constant) marginal costs $1/\delta_{t+1}$ of the non-board countries, which corresponds to our intuition about the economic results of competitive markets. It is perhaps surprising that the price level does not depend on the stock level B_{t+1} . The reason is that by our formula for p_{t+1} we implicitly assumed

$$Y_{t+1} + B_{t+1} < \frac{\alpha_{t+1}}{2} \tag{47}$$

i.e., we excluded the case in which the stock would be large enough to serve demand at the equilibrium price. If all countries are rational decision makers, this condition will always be satisfied, since a rational and myopic country 1 will always induce a zero stock level. So what one actually will observe in the case of $n = \infty$ is

$$c_t^1 = \beta_{t+1} \frac{1 - \dfrac{\delta_{t+1}}{\delta_{t+1}^1}}{\delta_{t+1}^1 \delta_{t+1}} \tag{48}$$

$$Y_{t+1} = \alpha_{t+1} - \frac{\beta_{t+1}}{\delta_{t+1}} \tag{49}$$

$$p_{t+1} = \frac{1}{\delta_{t+1}} \tag{50}$$

Thus, even in the case of an atomistic market structure country 1 will produce a positive amount if $\delta_{t+1}^1 > \delta_{t+1}$; i.e., if it uses more effective production techniques.

In the same way one can analyze how changes of the parameters α_{t+1}, β_{t+1}, δ_{t+1}^1, and δ_{t+1} influence the investment amounts c_t^1 and $\sum_{j\geq 2} c_t^j$ as well as the resulting production amounts y_{t+1}^1 and $\sum_{j\geq 2} y_{t+1}^j$. Since these effects are rather straightforward, we do not want to discuss them here in detail. It is obvious that a decrease of δ_{t+1} – i.e., an increase of marginal costs of the non-board countries $j \geq 2$ – will cause a smaller supply and that a decrease of α_{t+1} (or β_{t+1}) will diminish (or increase) total production. But it is not so easy to see how an increase of δ^1_{t+1} – ie., a decrease of marginal costs in country 1 – effects the investment amount c^1_t in country 1. On one hand the lower marginal costs make it profitable in country 1 to produce more; on the other hand country 1 has to invest less because of its greater productivity. Computing the derivative of

$$c^1_t = \frac{\frac{\beta_{t+1}}{\delta_{t+1}}\left[1 - \frac{\delta_{t+1}}{\delta^1_{t+1}}\right]}{\delta^1_{t+1}} \tag{51}$$

with respect to δ^1_{t+1} and considering that this formula is only valid for $c^1_t \geq 0$ shows that c^1_t will be increased by an increase of δ^1_{t+1} if

$$\frac{\delta^1_{t+1}}{2} < \delta_{t+1} < \delta^1_{t+1} \tag{52}$$

and that c^1_t is diminished as soon as δ^1_{t+1} enters the range

$$\frac{\delta^1_{t+1}}{2} > \delta_{t+1} \tag{53}$$

2.2.2. Computation of the behavior in earlier periods by backward induction

In the following we want to show how to compute the decision behavior in period t - 1 given the decision behavior in period t.

It was proved for the last decision period T that the price p_{T+1} and all investment amounts are linear functions of B_{T+1}, the state variable of the decision period T. Since p_{T+1} and c^j_T (j = 1,...,n) are linear functions of B_{T+1}, because of the definition of the payoff functions H^j_T it follows that H^j_T is a quadratic function of B_{T+1}.

It will be shown below that for t = 1,...,T the payoff levels H^j_t (j = 1,...,n) as determined by the solution of the game are quadratic functions of the stock level B_{t+1}, which is the state variable for period t. Since we have proved that this assertion is right for t' = T – T = 0 , it remains to be shown that the assertion is correct for t' + 1 = T – (t + 1) under the induction hypothesis that the assertion is valid for t' = T – t .

Induction hypothesis:

For j = 1,...,n country j's payoff level H^j_t can be written as a quadratic function of B_{T+1}:

$$H^j_t = u^j_{t+1} + v^j_{t+1}B_{t+1} + w^j_{t+1}B^2_{t+1} \quad \text{for} \qquad (j = 1,\dots,n) \tag{54}$$

Accordingly the payoff level H^j_{t-1} in country j can be written as

$$H^1_{t-1} = p_t(\rho^j_t + \delta^j_t c^j_{t-1}) - c^j_{t-1} + d^j_{t+1}u^j_{t+1} + d^j_{t+1}v^j_{t+1}B_{t+1} + d^j_{t+1}w^j_{t+1}B^2_{t+1} \tag{55}$$

for j = 2,...,n and in case of country 1 as

$$H^1_{t-1} = p_t s^1_t - c^1_{t-1} - \varepsilon_t - \mu_t B_t - \xi_t B^2_t + d^1_{t+1}u^1_{t+1} + d^1_{t+1}v^1_{t+1}B_{t+1} + d^1_{t+1}w^1_{t+1}B^2_{t+1} \tag{56}$$

In view of

$$B_{t+1} = B_t + Y_t - X_t \tag{57}$$

and considering that Y_t and X_t are functions of the decision variables p_t and c^j_{t-1} (j = 1,...,n) in period t - 1, one can easily see that according to the induction hypothesis all payoff levels H^j_{t-1} are functions of the decision variables in t - 1 and the state variable B_t of decision period t - 1.

Again we first have to compute which price p_t is determined by country 1. Regarding the fact that

$$s^1_t = \alpha_t - \beta_t p_t - \sum_{j \neq 1} (\rho^j_t + \delta^j_t c^j_{t-1}) \tag{58}$$

the necessary condition for (local) maximization of H^1_{t-1} by p_t for given investment decisions in t - 1 yields

$$p_t = P_t + R_t B_t + R_t \delta^1_t c^1_{t-1} - \sum_{j \neq 1} \delta^j_t S_t c^j_{t-1} \tag{59}$$

where we use the following shorthand:

$$P_t = \frac{\rho^1_t + \left[\alpha_t - \sum_k \rho^k_t\right]\left[1 - 2d^1_{t+1}w^1_{t+1}\beta_t\right] + d^1_{t+1}v^1_{t+1}\beta_t}{2\beta_t\left[1 - d^1_{t+1}w^1_{t+1}\beta_t\right]} \tag{60}$$

$$R_t = \frac{d^1_{t+1}w^1_{t+1}}{1 - d^1_{t+1}w^1_{t+1}\beta_t} \tag{61}$$

$$S_t = \frac{1 - 2d^1_{t+1}w^1_{t+1}\beta_t}{2\beta_t\left[1 - 2d^1_{t+1}w^1_{t+1}\beta_t\right]} \tag{62}$$

The price p_t as determined by (59) must, of course, satisfy the condition that $\beta_t p_t \geq \alpha_t - Y_t - B_t$.

Define

$$L = 1 + 2\delta^j_{t+1}w^j_{t+1}(1 - \beta_t S_t)\beta_t \tag{63}$$

$$K = -2\beta_t R_t + 2d^1_{t+1}w^1_{t+1}\beta_t(1 + \beta_t R_t) \tag{64}$$

$$U^j_t = LP_t - \rho^j_t S_t - \frac{1}{\delta^j_t} + d^j_{t+1}v^j_{t+1}\left[1 - \beta_t S_t\right) + 2d^j_{t+1}w^j_{t+1}\left[\sum_k \rho^k_t - \alpha_t\right]\left[1 - \beta_t S_t\right] \tag{65}$$

$$V^j_t = LR_t + 2d^j_{t+1}w^j_{t+1}(1 - \beta_t S_t) \tag{66}$$

$$W^j_{t,i} = L\delta^i_t S_t - 2d^j_{t+1}w^j_{t+1}(1 - \beta_t S_t)\delta^i_t \tag{67}$$

where i ≠ 1, i ≠ j, and for j = 2,...,n

$$W^j_{t,j} = L\delta^j_t S_t + \delta^j_t S_t - 2d^j_{t+1}w^j_{t+1}(1 - \beta_t S_t)\delta^j_t \tag{68}$$

$$W^j_{t,1} = L\delta^1_t R_t - 2d^j_{t+1}w^j_{t+1}\delta^1_t(1 - \beta_t S_t) \tag{69}$$

$$U^1_t = KP_t + \left[\alpha_t - \sum_{j \neq 1} \rho_t j\right]R_t - \frac{1}{\delta^1_t} + d^1_{t+1}v^1_{t+1}(1 + \beta_t R_t) + 2d^1_{t+1}w^1_{t+1}\left[\sum_k \rho^k_t - \alpha_t\right](1 + \beta_t R_t) \tag{70}$$

$$V_t^1 = KR_t + 2d_{t+1}^1 w_{t+1}^1 (1 + \beta_t R_t) \tag{71}$$

$$W_{t,1}^1 = - K\delta_t^1 R_t - 2d_{t+1}^1 w_{t+1}^1 \delta_{t+1}^1 (1 + \beta_t R_t) \tag{72}$$

$$W_{t,j}^1 = K\delta_t^j S_t + \delta_t^j R_t - 2d_{t+1}^1 w_{t+1}^1 \delta_t^j (1 + \beta_t R_t) \tag{73}$$

From the necessary condition for a local maximum of H_{t-1}^j, $j = 2,\ldots,n$ we derive

$$\sum_i W_{t,i}^j c_{t-1}^i = U_t^j + V_t^j B_t \qquad \text{for } j = 2,\ldots,n \tag{74}$$

In the same way local maximization of H_{t-1}^1 implies

$$\sum_i W_{t,i}^1 c_{t-1}^i = U_t^1 + V_t^1 B_t \tag{75}$$

Altogether these equations yield the linear equation system

$$\mathbf{A}_t \mathbf{c}_{t-1} = \mathbf{b}_t \tag{76}$$

where

$$\mathbf{A}_t = \begin{bmatrix} W_{t,1}^1 & & W_{t,n}^1 \\ \vdots & \cdots & \vdots \\ W_{t,1}^n & & W_{t,n}^n \end{bmatrix} \tag{77}$$

$$\mathbf{c}_{t-1} = (c_{t-1}^1, \cdots, c_{t-1}^n)^T \tag{78}$$

$$\mathbf{b}_t = (U_t^1 + V_t^1 B_t, \ldots, U_t^n + V_t^n B_t)^T \tag{79}$$

For $\delta_t^j > 0$ $(j = 1, \ldots, n)$ det $\mathbf{A}_t$ will not vanish, because of the special structure of the matrix $\mathbf{A}_t$ defined by (77). For the case in which country 1 is a pure buffer stock agency, one again has to consider the corresponding $(n-1)\times(n-1)$ matrix instead of $\mathbf{A}_t$ since $c_{t-1}^1 = 0$. Thus there will be a unique solution

$$\mathbf{c}_{t-1} = \mathbf{A}_t^{-1} \mathbf{b}_t \tag{80}$$

of the equation system (76).

Since all investment amounts c_{t-1}^j of the vector $\mathbf{c}_{t-1}$ given by (80) are linear functions of B_t, the same is true for p_t because of (59). According to (57) B_{t+1} is therefore a linear function of B_t. Thus not only the present profits but also the future payoff components of the payoff functions (55) and (56) are quadratic functions of the state variable B_t; i.e., all payoffs H_{t-1}^j $(j = 1,\ldots,n)$ as determined by the solution of the game are quadratic in the state variable B_t of the decision period t - 1, which had to be shown. According to the play as implied by the solution, all countries will determine their action variables according to linear decision functions assigning a unique decision for every stock level, which is the state variable of that period. Furthermore, their payoff levels according to the solution are quadratic functions of the corresponding state variable, i.e., the stock level of the next period.

Our induction proof has two major shortcomings. First of all we assumed that all decision behavior is based on local optimization. This implies a restriction for the initial condition B_2; the stock level B_2 should not exceed a critical value. On the other hand, that is why we can compute only the play as determined by the solution and not the solution as such. The solution as such would require that one also determine the decisions for situations which result because of former mistakes. Such mistakes might cause too high stock levels, which would not otherwise occur. The other shortcoming is that we never investigated the sufficient conditions for local payoff maximization. It is rather obvious because of our economic assumptions that these are satisfied for the decisions in the last period. That this is also the case for the earlier periods is usually proved by backward induction, which often requires lengthy arguments (see Boege et al. 1980). We therefore think that one should check the sufficient conditions when actually computing the play according to the procedure described above, which requires very simple computations. When computing various runs for rather large values of T this was the way in which Gottwald and Gueth (1980) checked the sufficient conditions for local payoff maximization.

The symmetric case

For the symmetric case the induction hypothesis would require

$$u^j_{t+1} = u_{t+1} \tag{81}$$

$$v^j_{t+1} = v_{t+1} \tag{82}$$

$$w^j_{t+1} = w_{t+1} \tag{83}$$

for all $j = 2,\dots,n$. Given the symmetry of the future payoff components in the payoff functions (55) one can show in the same way as for the last decision period T that all sellers $2,\dots,n$ must choose the same investment amount; i.e., $c^j_{t-1} = c_{t-1}$ for $j = 2,\dots,n$. The symmetry of the producers $2,\dots,n$ also implies

$$U^j_t = U_t \tag{84}$$

$$V^j_t = V_t \tag{85}$$

for $j = 2,\dots,n$. We thus have

$$p_t = P_t + R_t B_t + R_t \delta^1_t c^1_{t-1} - (n-1)\delta_t S_t c_{t-1} \tag{86}$$

$$A_t c^1_{t-1} + B_t c_{t-1} = U_t + V_t B_t \tag{87}$$

$$C_t c^1_{t-1} + D_t c_{t-1} = U^1_t + V^1_t B_t \tag{88}$$

where $A_t = W^j_{t,1}$ (for $j \geq 2$) and $C_t = W^1_{t,1}$ and where B_t and D_t are defined according to

$$B_t = (n-1)LS_t\delta_t + \delta_t S_t - 2d_{t+1}w_{t+1}(n-1)\delta_t(1 - \beta_t S_t) \tag{89}$$

and

$$D_t = (n-1)\left[K\delta_t S_t + \delta_t R_t - 2d^1_{t+1}w^1_{t+1}\delta_t(1 + \beta_t R_t)\right] \tag{90}$$

With the help of

$$\det_t = B_t C_t - A_t D_t \tag{91}$$

$$E_t = \left[B_t U^1_t - D_t U_t\right] / \det_t \tag{92}$$

$$F_t = \left[B_t V^1_t - D_t V_t\right] / \det_t \tag{93}$$

$$G_t = \left[C_t U_t - A_t U^1_t\right] / \det_t \tag{94}$$

$$H_t = \left[C_t V_t - A_t V^1_t\right] / \det_t \tag{95}$$

the investment decision functions for period t - 1 can be written as

$$c^1_{t-1} = E_t + F_t B_t \tag{96}$$

$$c_{t-1} = G_t + H_t B_t \tag{97}$$

Inserting (96) and (97) into (84) yields

$$p_t = I_t + J_t B_t \tag{98}$$

where

$$I_t = P_t + R_t \delta_t^1 E_t - (n-1)\delta_t S_t G_t \tag{99}$$

$$J_t = R_t + R_t \delta_t^1 F_t - (n-1)\delta_t S_t H_t \tag{100}$$

Substituting the decision functions (96), (97), and (98) into the payoff functions (55) yields

$$H_{t-1}^j = u_t + v_t B_t + w_t B_t^2 \qquad \text{for} \quad j = 2,\ldots,n \tag{101}$$

where

$$K_t = 1 + (n-1)\delta_t H_t + \delta_t^1 F_t + \beta_t J_t \tag{102}$$

$$L_t = \sum_k \rho_t^k - \alpha_t + \beta_t I_t + (n-1)\delta_t G_t + \delta_t^1 E_t \tag{103}$$

$$v_t = I_t \delta_t H_t + J_t(\rho_t + \delta_t G_t) - H_t + d_{t+1} v_{t+1} K_t + 2 d_{t+1} w_{t+1} K_t L_t \tag{104}$$

$$w_t = J_t \delta_t H_t + d_{t+1} w_{t+1} K_t^2 \tag{105}$$

In the same way one derives

$$H_{t-1}^1 = u_t^1 + v_t^1 B_t + w_t^1 B_t^2 \tag{106}$$

where

$$v_t^1 = -I_t\left[\beta_t I_t + (n-1)\delta_t H_t\right] + J_t\left[\alpha_t - \beta_t I_t - (n-1)\rho_t - (n-1)\delta_t G_t\right] + d_{t+1}^1 v_{t+1}^1 K_t + 2 d_{t+1}^1 w_{t+1}^1 K_t L_t \tag{107}$$

$$w_t^1 = -J_t\left[\beta_t J_t + (n-1)\delta_t H_t\right] - \varepsilon_t + d_{t+1}^1 w_{t+1} K_t^2 \tag{108}$$

Thus we have shown how to derive the payoffs as functions of the state variable B_t for period $t-1$ if these functions are known for the next period t. The formulae for v_t and w_t as well as v_t^1 and w_t^1, together with the decision functions (96), (97), and (98), yield a straightforward procedure to compute the play determined by the solution and the initial condition B_2. One first derives by backward induction the decision functions

$$p_{t+1} = I_{t+1} + J_{t+1} B_{t+1} \tag{109}$$

$$c_t^1 = E_{t+1} + F_{t+1} B_{t+1} \tag{110}$$

$$c_t^j = G_{t+1} + H_{t+1} B_{t+1} \qquad \text{for } j = 2,\ldots,n \tag{111}$$

for all periods $t = 1,\ldots,T$ as shown above. The initial value B_2, i.e. the value of the state variable in the starting period 1, together with the decision functions

$$p_2 = I_2 + J_2 B_2 \tag{112}$$

$$c_1^1 = E_2 + F_2 B_2 \tag{113}$$

$$c_1 = G_2 + H_2 B_2 \tag{114}$$

then determines the actual decisions c_1^1, c_1 ($= c_1^j$ for $j = 2,\ldots,n$) and p_2 in period 1 and thus the stock level B_3, which is the state variable for the next period 2. One therefore can proceed in period 2 as in period 1 and so forth until one has determined the actual decisions c_T^1, $c_T = (c_T^j$ for $j = 2,\dots,n)$ and p_{t+1} for the last decision period T.

Besides the fact that the non-board countries 2,...,n will not have identical decision and payoff functions, one can compute the play for the non-symmetric case according to the same procedure. There the backward induction procedure requires, of course, the solution of T linear equation systems.

Discussion of the solution

Although computation of the economic results is relatively easy for the symmetric case, the formulae for the decision variables will be very complex in general and too complicated to allow a detailed analytic investigation. In the following we therefore restrict our discussion to the symmetric case with two decision periods, i.e. T = 2. This case still captures the essential dynamic aspect of the game situation, since in decision period 1, called the present, country 1 has to decide whether it wants to build up a positive stock B_3 in order to shift supply from the present to the next period T = 2, called the future. To simplify the formulae we assume $\rho_t^1 = \rho_t = 0$ for $t = 2,3$; i.e., production amounts are always zero if there is no investment. To make possible a more interesting discussion, we allow the initial state variable B_{t+1} to be positive, although we still exclude those cases where B_{t+1} is so large that B_{T+2} would be positive according to the solution.

From the equation for p_{T+1} and c_T^1 and c_T^2 we derive

$$p_{T+1} = \frac{\alpha_{T+1}}{\beta_{T+1}} - \frac{B_{T+1}}{\beta_{T+1}} - \frac{(n-1)b_{T+1}^2 + b_{T+1}^1}{(n+1)\beta_{T+1}} \tag{115}$$

or

$$p_{T+1} = \frac{\alpha_{T+1}}{(n+1)\beta_{T+1}} + \frac{n-1}{(n+1)\delta_{T+1}} + \frac{1}{(n+1)\delta_{T+1}^1} \tag{116}$$

This shows that (again under the implicit assumption that $B_{T+1} + Y_{T+1} < \alpha/2$, which will be always satisfied if the players are rational) the price determined in the last decision period T depends only on the parameters α_{T+1} and β_{T+1} of the demand function, on the productivity coefficients, and on the market structure as indicated by the number n of countries in the world. As a consequence we have

$$H_T^2 = \left[\delta_{T+1}\left[\frac{\alpha_{T+1}}{(n+1)\beta_{T+1}} + \frac{n-1}{(n+1)\delta_{T+1}} + \frac{1}{(n+1)\delta_{T+1}^1}\right] - 1\right] \tag{117}$$

$$\frac{\alpha_{T+1} - 2\dfrac{\beta_{T+1}}{\delta_{T+1}} + \dfrac{\beta_{T+1}}{\delta_{T+1}^1}}{(n+1)\delta_{T+1}} \qquad \text{for } j = 2,\dots,n$$

and thus

$$v_{T+1} = w_{T+1} = 0 \tag{118}$$

For country 1 we get

$$H_T^1 = p_{T+1}\left[\alpha_{T+1} - \beta_{T+1}p_{T+1} - \frac{n-1}{n+1}\left[\alpha_{T+1} + \frac{\beta_{T+1}}{\delta_{T+1}^1} - 2\frac{\beta_{T+1}}{\delta_{T+1}}\right]\right] \tag{119}$$

$$-\frac{\alpha_{T+1} - (n+1)B_{T+1} - n\dfrac{\beta_{T+1}}{\delta^1_{T+1}} + (n-1)\dfrac{\beta_{T+1}}{\delta_{T+1}}}{(n+1)\delta^1_{T+1}}$$

$$-\varepsilon_{T+1} - \mu_{T+1}B_{T+1} - \xi_{T+1}B^2_{T+1}$$

and thus

$$v^1_{T+1} = \frac{1}{\delta^1_{T+1}} - \mu_{T+1} \tag{120}$$

$$w^1_{T+1} = -\xi_{T+1} \tag{121}$$

Since only the future payoffs have to be discounted, we can neglect the time dependency of the discount factors, which therefore will be written as d^1 and d, respectively. The decision variables of the present period 1 will be denoted by t, whereas those of the future period 2 have been denoted by T. By applying the recursive formulae we get

$$P_T = \frac{\alpha_T(1 + 2d^1\xi_{T+1}\beta_T) + d^1\beta_T\left[\dfrac{1}{\delta^1_{T+1}} - \mu_{T+1}\right]}{2\beta_T(1 + \beta_T d^1\xi_{T+1})} \tag{122}$$

$$R_T = -\frac{d^1\xi_{T+1}}{1 + \beta_T d^1\xi_{T+1}} \tag{123}$$

$$S_T = \frac{1 + 2d^1\xi_{T+1}\beta_T}{2\beta_T(1 + \beta_T d^1\xi_{T+1})} \tag{124}$$

$$A_T = R_T\delta^1_T \tag{125}$$

$$B_t = -n\delta_T S_T \tag{126}$$

$$C_T = -2\beta_T R_T\delta^1_T[R_T + (1 + \beta_T R_T)d^1\xi_{T+1}] - 2(1 + \beta_T R_T)d^1\xi_{T+1}\delta^1_T \tag{127}$$

$$D_T = (n-1)\Big\{2\beta_T S_T\delta_T[R_T + (1 + \beta_T R_T)d^1\xi_{T+1}] - [R_T\delta_T + 2(1 + \beta_T R_T)d^1\xi_{T+1}\delta_T]\Big\} \tag{128}$$

$$U^1_T = -2\beta_T P_T[R_T + (1 + \beta_T R_T)d^1\xi_{T+1}] + \alpha_T R_T + (1 + \beta_T R_T)d^1\left[\frac{1}{\delta^1_{T+1}} - \mu_{T+1} + 2\xi_{T+1}\alpha_T\right] \tag{129}$$

$$U_T = -P_T + \frac{1}{\delta_T} \tag{130}$$

$$V^1_T = -2\beta_T R_T[R_T + (1 + \beta_T R_T)d^1\xi_{T+1}] - 2(1 + \beta_T R_T)d^1\xi_{T+1} \tag{131}$$

$$V_T = -R_T \tag{132}$$

which completely determines the decisions c^1_t, c^j_t (j = 2,...,n) and p_{t+1} in the present period t = T − 1 = 1 according to

$$p_{t+1} = I_{t+1} + J_{t+1}B_{t+1} \quad (133)$$

$$c_t^1 = E_{t+1} + F_{t+1}B_{t+1} \quad (134)$$

$$c_t^j = G_{t+1} + H_{t+1}B_{t+1} \qquad \text{for } j = 2,\dots,n \quad (135)$$

Since the explicit formulae for the decision functions p_{t+1}, c_t^1, and c_t^j are still too complex, our discussion of how some important economic parameters influence the present decision behavior will proceed by our first investigating their influence on the terms used to define the coefficients of the decision functions in t and then on the decision functions themselves.

The discount factor d of the non-board countries

The first interesting fact to observe is that, because $w_{T+1} = 0$ and thus $d_{T+1}w_{T+1} = 0$, the discount factor, d, of the non-board countries does not affect the decision behavior in the present period at all. Although the non-board countries cannot store supply, this is not an obvious consequence (Gottwald and Gueth 1980, have examined a dynamic game in which the discount factor of oil traders strongly influences the decision behavior, although the traders were not able to store oil). One can imagine that the non-board countries could influence the stock level simply by changing their investment amounts, to which country 1 will possibly react. The fact that the discount factor d has no impact on the decision behavior might be due to the specific information conditions prevailing in our model (country 1 knows the production amounts of the non-board countries when determining the price). This indicates that a slight change in the model structure might strongly affect the economic results.

Influence of the number n

It can be seen that only B_T and D_T – and, of course, those coefficients depending on them – are influenced by the number n of producers. Furthermore, one can see that the coefficients of the decision functions depend only via the ratio $(n - 1)/n$ on the number of producers. This shows that the decision functions converge rather rapidly to the decision functions of the competitive situation.

The productivity of the pure producer countries

Besides the effect on U_T we can observe that an increase of δ_T has essentially the same consequences as an increase of n (B_T and D_T depend on n and δ_T only via $n\delta_T$ or $(n - 1)\delta_T$). We thus confine ourselves to the question of how δ_T influences the results via U_T. Since U_T is increased by a decrease of $\delta_T(>0)$, the coefficient E_T is negatively and the coefficient G_T positively influenced by this effect on U_T . For a given value of the stock level B_T the effect on c_{T-1}^1 is thus negative and the effect on c_{T-1} positive. In spite of the positive effect on c_{T-1}, the production amount of the countries 2,...,n will generally decrease because of their reduced marginal productivity.

Influence of the storage cost parameters

It should first be indicated that the storage cost parameter ε_{T+1} does not affect the decision behavior at all. Furthermore, μ_{T+1} and ξ_{T+1} can influence the decisions only if $d^1 > 0$. If ξ_{T+1} tends to zero, the same will be true for R_T, V_T^1 and V_T. This implies that F_T, H_T, and J_T also converge to zero. Thus for smaller values of ξ_{T+1} the present decision behavior becomes less dependent on the level B_T of the state variable. It should be mentioned that the case $\xi_{T+1} = 0$

cannot be analyzed by letting ξ_{T+1} approach zero, since $|E_T|$ will converge to infinity. This indicates that the solution for $\xi_{T+1} = 0$ probably does not rely on local payoff maximization. The storage cost parameter μ_{T+1} influences only the terms U_T^1 and U_T and thus only the constants E_T, G_T, and I_T of the respective decision functions. Consequently, a change of μ_{T+1} will influence the decisions but not the reactivity of the decisions with respect to changes of the state variable. Since P_T is decreased by an increase of μ_{T+1}, the coefficient U_T is greater if μ_{T+1} is greater. For U_T^1 this result is not so clear, since the effect via P_T will be at least partially compensated by a direct effect of μ_{T+1}. Thus it seems reasonable to expect a negative effect on E_T, whereas the effects on G_T and I_T are more difficult to estimate. In general one would expect a reduction of present production Y_T and of B_{T+1}.

Country 1 as a pure buffer stock agency

In the case of $\delta_T^1 = 0$ one clearly must have $c_{T-1}^1 = 0$, which also can be derived with the help of our formulae. In case $\delta_T^1 = 0$ we cannot rely on our formulae to discuss the present decision behavior. The reason is that $\delta_T^1 = 0$ implies

$$A_T = C_T = 0 \tag{136}$$

and therefore a situation in which the present decision functions are not well defined. That is why one has to go to the original system of equations (87) with $c_T^1 = 0$, which yields

$$c_{T-1} = \frac{U_T}{B_t} + \frac{V_T}{B_T} B_T \tag{137}$$

and

$$p_T = P_T + R_T B_T - (n-1)\delta_T S_T c_{T-1} \tag{138}$$

If we want to use the values for U_T, V_T, B_T, P_T, R_T, and S_T as listed above, we have to assume that δ_{T+1}^1 is positive. Thus the present decision functions for the case of $\delta_T^1 = 0$ and $\delta_{T+1}^1 > 0$ are

$$c_{T-1} = \frac{P_T - \frac{1}{\delta_T}}{n\delta_T S_T} + \frac{R_T}{n\delta_T S_T} B_T \tag{139}$$

$$p_T = \frac{n-1}{n} \frac{P_T\left(\frac{n}{n-1} S_T - 1\right) - \frac{1}{\delta_T}}{S_T} + \frac{n-1}{n} \frac{R_T}{S_T}\left(\frac{n}{n-1} S_T - 1\right) B_T \tag{140}$$

If we now approximate the situation of an extremely myopic buffer stock agency by $d^1 \to 0$, we get $R_T \to 0$, and thus the present decision behavior becomes less dependent on the stock level B_T (again it is implicitly assumed that B_T is not too large). The price will be

$$p_T = \frac{n-1}{2n\beta_T} \cdot \left[\frac{\alpha_T}{2\beta_T}\left(\frac{n}{n-1} \cdot \frac{1}{2\beta_T} - 1\right) - \frac{1}{\delta_T}\right] \tag{141}$$

and the investment amount

$$c_{T-1} = \frac{\frac{\alpha_T}{2\beta_T} - \frac{1}{\delta_T}}{n\delta_T \cdot \frac{1}{2\beta_T}} \tag{142}$$

If one wants to investigate the case in which player 1 is a pure buffer stock agency in the present and in the future – i.e., $\delta^1_T = \delta^1_{T+1} = 0$ – one has to go even further back in order to derive the decision functions for this case. Since $\delta^1_{T+1} = 0$ implies

$$c_T = \frac{\alpha_{T+1} - B_{T+1}}{n\delta_{T+1}} - \frac{\beta_{T+1}}{n(\delta_{T+1})^2} \tag{143}$$

$$p_T = \frac{\alpha_{T+1} - B_{T+1}}{n\,\beta_{T+1}} + \frac{n-1}{n\,\delta_{T+1}} \tag{144}$$

we no longer have the result that the coefficients v_{T+1} and w_{T+1} are equal to zero; i.e., the pure producer countries will now take into account the future effects of their present supply decisions. This indicates that the opposite result for $\delta^1_{T+1} > 0$ was due to the fact that the pure producers expect player 1 to adjust his supply amount in order to achieve the right relation between present and future supply. If this is impossible for player 1, the pure producer countries themselves have to consider the future effects. Again the time structure of the decision process is probably essential to an explanation of the different results for $\delta^1_{T+1} > 0$ and $\delta^1_{T+1} = 0$, especially that player 1 can adjust the price to the production amounts previously determined.

Computation of the actual play for the symmetric case

In order to compute the actual play for the symmetric case, one has to know v^j_{T+1} and w^j_{T+1} (j = 1,...,n) – see equations (118), (120), and (121) – as well as the formulae which enable us to derive v^j_t and w^j_t with the help of the coefficients v^j_{t+1} and w^j_{t+1} of the next period – see equations (104), (105), and (107), (108). Defining

$$F_{T+1} = J_{T+1} = 0 \tag{145}$$

$$E_{T+1} = \frac{\alpha_{T+1} - \rho^1_{T+1} - (n-1)\rho_{T+1} - 2\frac{\beta_{T+1}}{\delta_{T+1}} + \frac{\beta_{T+1}}{\delta^1_{T+1}}}{(n+1)\delta_{T+1}} \tag{146}$$

$$G_{T+1} = \frac{\alpha_{T+1} - \rho^1_{T+1} - (n-1)\rho_{T+1} - n\frac{\beta_{T+1}}{\delta^1_{T+1}} + (n-1)\frac{\beta_{T+1}}{\delta_{T+1}}}{(n+1)\delta_{T+1}} \tag{147}$$

$$H_{T+1} = -\frac{1}{\delta^1_{T+1}} \tag{148}$$

$$I_{T+1} = p_{T+1} \tag{149}$$

all decision functions can be expressed by

$$c^j_t = E_{t+1} + F_{t+1}B_{t+1} \tag{150}$$

$$c^1_t = G_{t+1} + H_{t+1}B_{t+1} \tag{151}$$

$$p_{t+1} = I_{t+1} + J_{t+1}B_{t+1} \tag{152}$$

for j = 2,...,n and t = 1,...T. Starting with B_2 one can first compute the actual decisions for t = 1 and then for t = 2, etc., where B_{t+1} is determined by B_t according to

$$B_{t+1} = B_t + \rho_t^1 + (n-1)\rho_t + \delta_t^1 c_{t-1}^1 + (n-1)\delta_t c_{t-1}^2 - \alpha_t + \beta_t p_t \tag{153}$$

It has been mentioned above that it will be easy to check the sufficient conditions for local payoff maximization when computing the actual play for numerical examples. It is easy to see that for the last decisions these conditions are always satisfied. So what actually has to be checked in the process of backward induction is whether

$$\frac{\partial^2}{\partial p_t \partial p_t} H_{t-1}^1 = -2\beta_t\left[1 - d_{t+1}^1 w_{t+1}^1 \beta_t\right] < 0 \tag{154}$$

$$\frac{\partial^2}{\partial c_{t-1}^j \partial c_{t-1}^j} H_{t-1}^j = -W_{t,j}^j < 0 \tag{155}$$

$$\frac{\partial^2}{\partial c_{t-1}^1 \partial c_{t-1}^1} H_t^1 = -W_{t,1}^1 < 0 \tag{156}$$

It has already been indicated that the case of a pure buffer stock agency cannot simply be investigated by setting $\delta_t^1 = 0$ for all periods t. Due to

$$p_{T+1} = \frac{(\alpha_{T+1} - B_{T+1} - Y_{T+1})}{\beta_{T+1}} \tag{157}$$

and

$$c_T = \frac{\left[\alpha_{T+1} - B_{T+1} - (n-1)\rho_{T+1}^1 - \dfrac{\beta_{T+1}}{\delta_{T+1}}\right]}{n\delta_{T+1}} \tag{158}$$

one has the following initial conditions:

$$G_{T+1} = \left[\alpha_{T+1} - (n-1)\rho_{T+1} - \frac{\beta_{T+1}}{\delta_{T+1}}\right] \tag{159}$$

$$H_{T+1} = -\frac{1}{n\delta_{T+1}} \tag{160}$$

$$I_{T+1} = \frac{[\alpha_{T+1} - (n-1)(\rho_{T+1} + \delta_{T+1}G_{T+1})]}{\beta_{T+1}} \tag{161}$$

$$J_{T+1} = -\frac{[1 + (n-1)\delta_{T+1}H_{T+1}]}{\beta_{T+1}} \tag{162}$$

$$w_{T+1} = \delta_{T+1}J_{T+1}H_{T+1} \tag{163}$$

$$v_{T+1} = (\delta_{T+1}J_{T+1} - 1)H_{T+1} + G_{T+1}\delta_{T+1}J_{T+1} + \rho_{T+1}J_{T+1} \tag{164}$$

$$w_{T+1}^1 = J_{T+1}(-\beta_{T+1}J_{T+1} - (n-1)\delta_{T+1}H_{T+1}) - \xi_{T+1} \tag{165}$$

$$v_{T+1}^1 = J_{T+1}[\alpha_{T+1} - \beta_{T+1}J_{T+1} - (n-1)(\rho_{T+1} + \delta_{T+1}G_{T+1})] - \mu_{T+1} - I_{T+1}[\beta_{T+1}J_{T+1} + (n-1)\delta_{T+1}H_{T+1}] \tag{166}$$

Given these initial conditions one can easily compute the decisions in the earlier periods by using just equation (88) to determine c_t.

2.3. How to study the infinite game?

In case of $T = \infty$ (we will simply speak of the infinite game) there is no last period T which can serve as the starting period for solving the game recursively. This already indicates that the games with $T=\infty$ are strategically very different from those with a finite number of decision periods. As a matter of fact, the set of equilibrium points usually explodes when switching from $T < \infty$ to $T = \infty$. In the literature this is usually illustrated with the help of the repeated prisoners' dilemma game.

In order to define a game with $T = \infty$, one would first have to define the parameters $\alpha_t, \beta_t, \rho_t^i, \delta_t^i, d_t^i$ $(i = 1,\dots,n)$ for all the infinitely many decision periods t, which indicates that there is no unique game for the case of $T = \infty$. Instead of giving an infinite list of parameter vectors, one usually will assume a certain trend for every one of these parameters, which should be bounded in order to assure finite payoffs in combination with the discount factors d_t^i , which should always be strictly smaller than 1 except for a finite number of periods. Once a trend has been specified for every parameter – which will always be assumed in this section – a game is uniquely defined just by the number T of decision periods, and accordingly there is a unique game for the case $T = \infty$.

One way to determine the solution of the infinite game would be to look at the limit of the solutions of the finite games for $T \to \infty$; i.e., one would first try to prove that the limit exists and then show that it is an equilibrium point of the game with $T = \infty$. The solution of the infinite game thus derived could be called the asymptotically convergent equilibrium solution of the infinite game. We believe that the asymptotically convergent equilibrium solution is the most reasonable concept to approach the infinite game, since the infinite game as such is only understandable as the limit of the finite games. Here we will not specify certain trends for the parameters nor will we investigate the asymptotically convergent equilibrium solution; for examples see Selten (1965) and Boege et al. (1980).

According to another approach one has to consider the infinite game as such and not as a limit of finite games. In this case one would probably specify a set of axioms – possibly related to our solution concept for finite games but altogether somewhat stronger – which uniquely determine the decision behavior in the given game situation. Possible axioms would be, for instance, the equilibrium property and subgame consistency in the sense that the decision behavior must be the same in two periods t and t' if $B_{t+1} = B_{t'+1}$, since those games can be considered as strategically equivalent (see Selten and Gueth 1978).

4. INCLUDING STOCHASTIC PRODUCTION FUNCTIONS

The deterministic game model assumes that every player knows exactly how the economic variables react to the decisions of the various players. In other words, there is no other uncertainty involved but the one about what one's opponents are going to do. As was indicated at the beginning, buffer stocks are often used on markets whose products have a rather inelastic demand elasticity at given prices. Since these are mostly agricultural products and since the output of agricultural production processes is strongly influenced by events like rain, storms, etc., and their respective distribution in time, it seems very important to show how the analysis of managing a buffer stock can be extended to stochastic economic relationships. In doing so we want to stay as much as possible within the economic framework underlying the deterministic game model. For the sake of simplicity we will investigate only the symmetric case.

Consider the symmetric case of the deterministic game model. Most typically it will be the production functions

$$y_t^j = \rho_t + \delta_t c_{t-1}^j \qquad \text{for } j = 2,\ldots,n \tag{167}$$

and

$$y_t^1 = \rho_t^1 + \delta_t^1 c_{t-1}^1 \tag{168}$$

which are stochastic in nature. Of course, total demand might also be a stochastic variable, but at least for agricultural products highly aggregated demand levels seem to be more predictable than the results of the production process.

Within the framework of linear production functions the uncertainty might be due to stochastically determined parameters ρ_t^i and to stochastically determined productivity coefficients δ_t^i $(i = 1,\ldots,n)$. Here we will assume that $\rho_t^i = 0$ for all i and t and that both the productivity coefficient δ_t^1 and the productivity coefficient δ_t (which is the same for all countries 2,...n), are stochastic variables whose actual values are determined according to the uniform distribution over the interval $[a^1,b^1]$ and $[a,b]$, respectively, where

$$0 < a^1 \leq b^1 ; \qquad 0 < a \leq b , \qquad \frac{\beta_t}{\alpha_t} > \frac{a+b}{4} \qquad \text{for all } t \tag{169}$$

Since the productive effect of investment in agricultural production is uncertain, we have to specify how this affects the problem of determining the stock level. It was our idea that the board determines a price and is willing to adjust its sales amount in such a way that demand at the chosen price is equal to supply. This was essential since, in order to be believed, the buffer stock agency must be willing to enforce the price which it had previously chosen. In the stochastic framework the problem of choosing a price which can be believed by all agents becomes more complicated. Consider the situation of given investment amounts c_{t-1}^j, $j = 1,\ldots,n$. The board, i.e. player 1, has to decide about the price p_t which determines demand $X_t = \alpha_t - \beta_t p_t$ in period t. If everyone were to trust that this price p_t will actually be the prevailing price in period t, one obviously would have

$$B_t + a^1 c_{t-1}^1 + a \sum_{j \neq 1} c_{t-1}^j \geq X_t = \alpha_t - \beta_t p_t \tag{170}$$

i.e., country 1 can choose only a price $p_t(\geq 0)$ which satisfies

$$p_t \geq \frac{\alpha_t - B_t - a \sum_{j \neq 1} c_{t-1}^j - a^1 c_{t-1}^1}{\beta_t} \tag{171}$$

Given this restriction for the set of possible prices for all periods $t=2,\ldots,T+1$, we can proceed to solve the dynamic stochastic game with $T < \infty$ as for the finite deterministic game. It should be mentioned that the payoffs for the stochastic games are uniquely defined because of their unique definition for all actual economic developments and because we specified the probability distributions according to which the stochastic variables δ_t^1 and δ_t $(t = 2,\ldots,T+1)$ are determined.

Decision behavior in period T

In the last decision period player 1 will obviously choose

$$p_{T+1} = \begin{cases} \dfrac{\alpha_{T+1}}{2\beta_{T+1}} & \text{for} \quad \dfrac{\alpha_{T+1}}{2} \leq B_{T+1} + a^1 c_T^1 + a \sum\limits_{j \neq 1} c_T^j \\ \dfrac{\alpha_{T+1} - B_{T+1} - a^1 c_T^1 - a \sum\limits_{j \neq 1} c_T^j}{\beta_{T+1}} & \text{otherwise} \end{cases} \tag{172}$$

It should be mentioned that this price setting behavior relies on the expectation of minimal productivity coefficients, which implies that the restriction for the set of possible prices p_{T+1} is satisfied. Thus the only difference from the strategic choice of investment amounts in the last period T consists of the fact that the linear decision function (172) is defined by other coefficients than in the deterministic case. It will be shown below that we can restrict our attention to the price decision function

$$p_{T+1} = \frac{\alpha_{T+1} - B_{T+1} - a^1 c_T^1 - a \sum\limits_{j \neq 1} c_T^j}{\beta_{T+1}} \tag{173}$$

Let E denote the expectation operator. Country i = 2,...,n wants to maximize

$$E\left[p_{T+1} \cdot y_{T+1}^i\right] - c_T^i = \left[p_{T+1} \cdot \frac{a+b}{2} - 1\right] c_T^i \tag{174}$$

As for the deterministic case, one can show that in a subgame perfect equilibrium point one must have

$$c_T^j = \frac{\alpha_{T+1} - B_{T+1} - a^1 c^1 - \dfrac{2\beta_{T+1}}{a+b}}{na} \quad \text{for} \qquad j = 2,\ldots,n \tag{175}$$

From the local maximization of country 1's expected payoff

$$E\left[p_{T+1} \cdot s_{T+1}^1\right] - c_T^1 = p_{T+1}\left[\alpha_{T+1} - \beta_{T+1} p_{T+1} - \frac{a+b}{2} \sum_{j \neq 1} c_T^j\right] - c_T^1 \tag{176}$$

we derive

$$c_T^1 = \frac{\alpha_{T+1} - 2B_{T+1} - \dfrac{\beta_{T+1}}{a^1} - \dfrac{3a-b}{2} c_T^2 (n-1)}{2a^1} \tag{177}$$

The two equations for c_T^1 and c_T^j ($j \geq 2$) imply

$$c_T^1 = \frac{n b_{T+1}^1 - (n-1) \dfrac{3a-b}{2a} b_{T+1}^2}{\left[n \dfrac{a+b}{2a} + \dfrac{3a-b}{2a}\right] a^1} \tag{178}$$

$$c_T^j = \frac{2 b_{T+1}^2 - b_{T+1}^1}{n \dfrac{a+b}{2} + \dfrac{3a-b}{2}} \qquad \text{for } j = 2,\ldots,n \tag{179}$$

where

$$b^1_{T+1} = \alpha_{T+1} - 2B_{T+1} - \frac{\beta_{T+1}}{a^1} \tag{180}$$

$$b^2_{T+1} = \alpha_{T+1} - B_{T+1} - \frac{2\beta_{T+1}}{a+b} \tag{181}$$

The minimal production in period T+1 is thus

$$\frac{b^1_{T+1} + (n-1)\frac{a+b}{2a}b^2_{T+1}}{n\frac{a+b}{2a} + \frac{3a-b}{2a}} . \tag{182}$$

The condition that $\frac{\alpha_{T+1}}{2} > B_{T+1} + a^1 c^1_T + (n-1)ac^2_T$ is therefore equivalent to

$$\frac{\beta_{T+1}}{\alpha_{T+1}} > \frac{(n-1)(a+b)}{4\left[\frac{a}{a^1} + n - 1\right]} = \frac{a+b}{4}\,\frac{1}{1 + \frac{a}{a^1(n-1)}} \tag{183}$$

The right hand side of (183) increases if n is increased. Furthermore, it converges to (a + b) / 4 for $n \to \infty$. Because of (169) it is justified to use the price decision function (173) instead of (172). It is important to observe that condition (183) does not depend at all on the value of the state variable B_{T+1} . Thus, whether condition (183) is satisfied or not is not determined by the actual play so far but only by the parameters of the game situation.

Discussion of the solution for the case of myopic countries

As for the deterministic game, we would like to discuss briefly the behavior for $d^1_t = d_t = 0$ for all $t \geq 2$ where our analysis concentrates on the question of how the uncertainty about the result of the production process affects the economic development. In the case of extremely myopic players the decisions in all periods $t \geq 1$ are given by

$$c^1_t = \frac{nb^1_{t+1} - (n-1)\frac{3a-b}{2a}b^2_{t+1}}{\left[n\frac{a+b}{2a} + \frac{3a-b}{2a}\right]a^1} \tag{184}$$

$$c^j_t = \frac{2b^2_{t+1} - b^1_{t+1}}{n\frac{a+b}{2} + \frac{3a-b}{2}} \qquad \text{for } j = 2,\dots,n \tag{185}$$

$$p_{t+1} = \frac{\frac{\alpha_{t+1}}{\beta_{t+1}} + \frac{1}{a^1} + \frac{n-1}{a}}{n\frac{a+b}{2a} + \frac{3a-b}{2a}} \tag{186}$$

where b^1_{t+1} and b^2_{t+1} are given by b^1_{T+1} and b^2_{T+1} by substituting t for T. For $n \to \infty$ we get

$$p_{t+1} \to \frac{2}{a + b} \tag{187}$$

i.e., the competitive price is determined by the expected marginal costs – which are the inverse of the expected marginal productivity – of the non-board countries.

The formula for the price p_{t+1} shows that only the lower bound a^1 for country 1's marginal productivity is important for the price setting behavior, whereas for the non-board countries both the lower bound, a, for the marginal productivity and the upper bound, b, enter the formula. The special case in which there is no uncertainty about the productivity in the non-board countries, i.e. $a = b$, implies

$$p_{t+1} = \frac{\alpha_{t+1}}{(n + 1)\beta_{t+1}} + \frac{1}{(n + 1)a^1} + \frac{n - 1}{(n + 1)a} \tag{188}$$

This indicates that the buffer stock agency reacts to the event of worst productivity in order to exclude situations where it cannot satisfy all the demand at its chosen price.

We now want to investigate how the result is influenced by the range [a,b] for the productivity in the non-board countries. Let us therefore consider an initial state $a^o \le b^o$ in which the parameters a and b are changed according to

$$a = \frac{a^o + b^o - k}{2} \quad \text{and} \quad b = \frac{a^o + b^o + k}{2} \quad \text{for } k \ge 0 \tag{189}$$

which obviously implies $a + b = a^o + b^o$. Increasing k obviously means increasing the variance of productivity in the non-board countries, whereas expected productivity stays constant. We get

$$p_{t+1} = \frac{\dfrac{\alpha_{t+1}}{\beta_{t+1}} + \dfrac{1}{a^1} + 2\dfrac{n - 1}{a^o + b^o - k}}{n\dfrac{a^o + b^o}{a^o + b^o - k} + \dfrac{a^o + b^o - 2k}{a^o + b^o - k}} \tag{190}$$

$$= \frac{\left(\dfrac{\alpha_{t+1}}{\beta_{t+1}} + \dfrac{1}{a^1}\right)\left(a^o + b^o\right) + 2(n - 1) - \left(\dfrac{\alpha_{t+1}}{\beta_{t+1}} + \dfrac{1}{a^1}\right)k}{(n + 1)(a^o + b^o) - 2k}$$

i.e., by an increase of k both the numerator and the denominator on the right hand side are increased, which shows that there is no obvious answer as to how an increase of k affects the price. The decrease of a which is implied by a higher value of k forces country 1 either to increase the price or to increase its investment amount c^1_t. Observe that b^1_{t+1} and b^2_{t+1} do not depend on k. To investigate how c^1_t depends on k one therefore can use the following expression for c^1_t

$$c^1_t = \frac{nb^1_{t+1}(a^o + b^o) - (n - 1)b_{t+1}(a^o + b^o) + [2(n - 1)b_{t+1} - nb^1_{t+1}]k}{a^1\left(a^o + b^o\right)(n + 1) - 2a^1k} \tag{191}$$

The denominator in this formula for c_t^1 decreases if k increases. Thus, a sufficient condition to assure that c_t^1 increases with k is given by

$$2(n-1)b_{t+1}^2 - nb_{t+1}^1 \geq 0 \tag{192}$$

or by

$$2b_{t+1}^2 - \frac{n}{n-1} b_{t+1}^1 \geq 0 \tag{193}$$

Since $\frac{n}{n-1} \to 1$ for $n \to \infty$, condition (193) will be satisfied according to (185) when n is large and $c_t^j \geq 0$ for $j \geq 2$. This shows that c_t^1 will increase with k for large values of n whenever the solution determined above will satisfy the non-negativity assumption that $c_t^j \geq 0$ for $j = 2,\ldots,n$.

One should mention that instead of studying the real game situation in which the investment amounts c_t^i $(i = 1,\ldots,n)$ must be non-negative, we have studied the so-called pseudogame which results by neglecting these non-negativity assumptions. Our approach here is to study only the pseudogame whose solution will generally satisfy the non-negativity assumptions and thus coincide with the solution of the game in which the investment amounts must be non-negative.

Since according to our assumptions country 1 has to set the price p_{t+1} in such a way that it can satisfy demand even in the case of worst productivity in all countries, the expected value $E(B_{t+2})$ of stock in period t + 2, which is the state variable for decision period t + 1, must satisfy

$$\begin{aligned} E(B_{t+2}) &\geq \left[\frac{a^1 + b^1}{2} - a^1\right] c_t^1 + (n-1)\left[\frac{a+b}{2} - a\right] c_t^2 \\ &= \frac{b^1 - a^1}{2} c_t^1 + (n-1)\frac{b-a}{2} c_t^2 \end{aligned} \tag{194}$$

i.e., for given investment amounts the expected stock level $E(B_{t+2})$ is a positive linear function of the distances between greatest and lowest marginal productivity in the various countries. Now for $\alpha_t / \beta_t > b^1 > a^1$ and $\alpha_t / \beta_t > b > a$ and B_{t+1} not too large c_t^1 and c_t^2 are positive. Thus inequality (194) implies that, contrary to the deterministic case, the buffer stock level is positive with probability 1 even if the (expected) productivity in the future periods is worse than in the present period.

Computation of the decision behavior in earlier periods

For the last decision period T it was shown that the investment amounts c_T^1, c_T^j $(j = 2,\ldots,n)$ and the price p_{T+1} are linear functions of B_{T+1} . Accordingly, the expected payoffs $E(H_T^j)$ $(j = 1,\ldots,n)$ are quadratic functions of B_{T+1} which, furthermore, are identical for all players $j = 2,\ldots,n$. One can therefore formulate the induction hypothesis in a manner similar to that of the deterministic case:

For $j = 1,\ldots,n$ player j's expected payoff $E(H_t^j)$ can be written as a quadratic function of B_{t+1} where

$$E(H_t^1) = u_{t+1}^1 + v_{t+1}^1 B_{t+1} + w_{t+1}^1 B_{t+1}^2 \tag{195}$$

and

$$E(H_t^j) = u_{t+1} + v_{t+1} B_{t+1} + w_{t+1} B_{t+1}^2 \quad \text{for} \quad j = 2,\ldots,n. \tag{196}$$

The expected payoff levels $E(H_{t-1}^j)$ can thus be written as

$$E(H_{t-1}^1) = E(p_t s_t^1) - c_{t-1}^1 - \varepsilon_t - \mu_t B_t - \xi_t B_t^2 + E(d_{t+1}^1 u_{t+1}^1 + d_{t+1}^1 v_{t+1}^1 B_{t+1} + d_{t+1}^1 w_{t+1}^1 B_{t+1}^2) \tag{197}$$

and

$$E(H_{t-1}^j) = E(p_t y_t^j) - c_{t-1}^j + E(d_{t+1} u_{t+1} + d_{t+1} v_{t+1} B_{t+1} + d_{t+1} w_{t+1} B_{t+1}^2) \tag{198}$$

for $j = 2,\ldots,n$ where

$$E(B_{t+1}) = B_t + E(Y_t) - X_t \tag{199}$$

If one can always rely on local payoff maximization, it is possible to show that this induction hypothesis implies that the decisions p_t, c_t^1 and c_t^j ($j = 2,\ldots,n$) are linear functions of B_t which, furthermore, are identical for all players $j = 2,\ldots,n$. As a consequence all expected payoffs $E(H_{t-1}^j)$ for $j = 1,\ldots,n$ will be quadratic functions of B_t which are also identical for all players $j = 2,\ldots,n$.

Let p_t^* denote the price which locally maximizes player 1's expected payoff $E\left[H_{t-1}^1\right]$ as determined by (197). The price setting of player 1 is determined by p_t^* as long as

$$p_t^* \geq \frac{\alpha_t - a^1 c_{t-1}^1 - a \sum_{j \neq 1} c_{t-1}^j - B_t}{\beta_t} \tag{200}$$

Now one cannot ignore the event of stock levels B_t with

$$B_t \geq (b^1 - a^1) c_{t-2}^1 + (b - a) \sum_{j \neq 1} c_{t-2}^j \tag{201}$$

because of the stochastic nature of productivity. Thus to make sure that inequality (200) is generally satisfied, one will usually need much stronger assumptions than for the deterministic case, where one also had to assure that the local payoff-maximizing price p_t determined by (59) is in the range of

$$p_t \geq \frac{\alpha_t - Y_t - B_t}{\beta_t} \tag{202}$$

If the local payoff-maximizing price p_t^* does not satisfy restriction (200), the price will be given by the right-hand side of (200) and thus also be a linear function of of the stock level B_t . In general the price setting behavior of player 1 will thus be determined by an only piecewise linear function of B_t . We do not want to describe here how to determine the decision behavior if this is the case. In many cases one will have to face some conceptual problems in determining which of the linear functions describing the decision behavior has to be applied in each of the previous periods. These problems result since there is usually no unique self-fulfilling prophecy in that respect and therefore no unique play implied by our solution requirements applied so far (see Gottwald and Gueth 1980, and Boege et al. 1980, where such situations have been studied.

4. FINAL REMARKS

Obviously there are many ways in which one can design dynamic game models with a buffer stock to shift present supply to future periods. One could use prices or production levels instead of investment amounts as strategic variables. If production amounts are the strategic parameters, it is – at least for the deterministic games – easy to have quadratic cost functions instead of linear ones (see Thiemer 1981).

Of course, it is also possible to design game models with more than one buffer stock. In this case the state space would be multi-dimensional instead of one-dimensional. This would imply more complicated decision formulae in the sense that now all buffer stock levels enter into the decisions. But in spite of that their general mathematical structure, in which the decisions are at most linear functions of all state variables, will still be valid if we otherwise stay within our economic framework.

That only one buffer stock is used to store supply will often be due to the fact that a group of producers agreed to found a buffer stock agency or somehow accepted that one of them take over the role of such an agency in addition to being a producer himself. If this is true, one might want to explain how such an agreement can result from independent decisions of many producers or how it can happen that one producer takes over the position of a buffer stock agency. To do this one can include a time period $t = 0$ where it is decided by a non-cooperative bargaining procedure whether a buffer stock agency is to be founded or not or whether one of the producers – and which one of them – should take over the role of such an agency.

Another special feature of our game model is that the number of producers on the market is considered as given. The reason for this was that we had in mind rather well-established world markets in which the number of producers will be determined by the number of countries. Nevertheless it might be that some of the countries have not started production of the specific commodity but are considering whether they should engage on that market at the present time. To determine the group of actual producers on the market endogenously, one might include a market entry/exit stage (before the production decision stage) in which all potential producers can independently determine whether they want to enter or leave that market or not. Here one should, of course, impose market entry and/or market exit costs which the players have to face.

If the buffer stock agency results from the cooperative efforts of many producers (i.e., player 1 is a pure buffer stock agency), the profits of player 1 have to be distributed among those producers. Consider, for instance, the case in which the cooperating producers have agreed on a vector of shares specifying the proportion of player 1's profit that goes to any of them. When making his investment decision a player $j(\geq 2)$, engaged in the foundation of the buffer stock agency, will now consider not only how his decision affects his own future profits but also the payments to him by player 1. Our assumption that player 1 is supposed to maximize H_t^1 might be justified in such a case by requiring an independent buffer stock management whose salary is determined according to a strictly positive function of H_t^1.

This shows that our analysis can be considered only as a starting point for studying the economic institution of cooperative buffer stocks in dynamic systems. Once one gives up the idea of total cooperation, various possibilities of partial cooperation arise which do not require the countries to give up all their independence. With respect to cooperative buffer stocks, one can especially vary the degree of control which the member countries have on the decision behavior of the buffer stock agency. Here it was assumed that the agency is completely independent once it has been founded and that it is interested in increasing its revenues. In the case of a producing buffer stock agency this is, of course, limited by the fact that revenues from production have to be taken into account as well. The reason for introducing an independent agency is that usually an economic institution will develop its own intentions after its establishment. Since this will usually require financial resources, a major motivation will be to earn more.

It is an interesting and important result of our analysis that in the case of a producing buffer stock agency the pure producers do not have to consider the effects their present decisions have for the future via the transition law (4). Whereas a non-producing agency only takes over the responsibility of balancing prices and supply in time, a producing buffer stock agency enables the pure producers to neglect future effects entirely.

The model structure as it was introduced in section 2.1 defines only a class of noncooperative dynamic games. To select a game out of this class, one has to specify the number of countries, the parameters determining the payoff function of each country, and the aggregate demand function, as well as the initial stock level. An attempt has been made to study in great detail how the market results will be influenced if one of these parameters defining a specific game in the class of games is varied. Thus we have learned how the market reacts to changes in the market structure, productivity coefficients, time preferences, demand parameters, etc. There is reason to expect that the sensitivity of the market process to changes in the parameters will be similar for other classes of games by which the workings of cooperative buffer stocks can be investigated. Therefore, in spite of the specific assumptions about the market decision process, our model should provide some general insights into how sensitive oligopolistic world markets with fluctuating supply in time are to are to changes in these variables.

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