

1 When is enough? Minimum sample sizes for on-road  
2 measurements of car emissions

3 *Yuche Chen<sup>‡</sup>\*, Yunteng Zhang<sup>‡</sup>, Jens Borken-Kleefeld<sup>†</sup>\**

4 <sup>‡</sup>Department of Civil and Environmental Engineering, University of South Carolina, USA

5 <sup>†</sup>International Institute for Applied System Analysis, Laxenburg, Austria

6 \* Corresponding author: Yuche Chen, [chenyuc@cec.sc.edu](mailto:chenyuc@cec.sc.edu); Jens Borken-Kleefeld,

7 [borken@iiasa.ac.at](mailto:borken@iiasa.ac.at)

8

9 Key words: Monte Carlo simulation, bootstrap, in-use surveillance, diesel car, Europe, Euro 6,  
10 mean, variance, remote emission sensing.

11

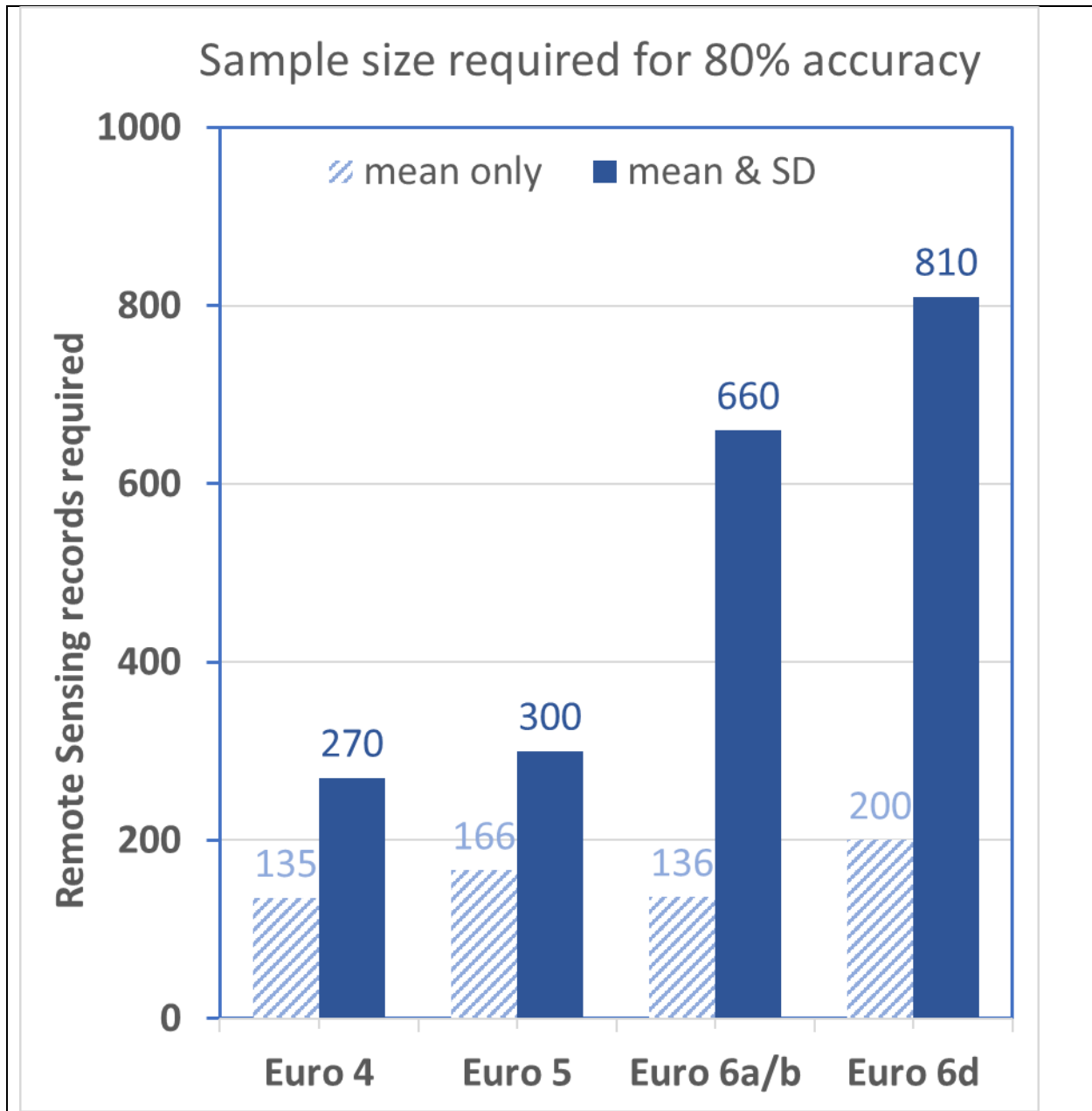
12 **ABSTRACT**

13 The power of remote vehicle emission sensing stems from the big sample size obtained and its  
14 related statistical representativeness for the measured emission rates. But how many records are  
15 needed for a representative measurement and when does the information gain per record become  
16 insignificant? We use Monte Carlo simulations to determine the relationship between the sample  
17 size and the accuracy of the sample mean and variance. We take the example of NO emissions  
18 from diesel cars measured by remote emission monitors between 2011 and 2018 at various  
19 locations in Europe. We find that no more than 200 remote sensing records are sufficient to  
20 approximate the mean emission rate for Euro 4, 5 and 6a,b diesel cars with 80% certainty within  
21 a  $\pm 1$  g NO per kg fuel tolerance margin ( $\sim \pm 50$  mg NO per km). Between 300 and 800 remote  
22 sensing records are needed to approximate also the variance of the mean NO emission rates for  
23 those diesel car technologies. This translates to only 2 and up to 9 measurement days  
24 respectively to characterize the means and their variance for a car fleet typical in Europe.

25

26

### TOC Graphic



## 32 INTRODUCTION

33 Vehicle emission remote sensing has been routinely applied in numerous US states for three  
34 decades now, followed by applications in Europe, in Hong Kong and more recently in mainland  
35 China.<sup>1-5</sup> Typical campaigns last between a couple of days to many weeks, with up to a million  
36 of emission records collected. An obvious assumption in these campaigns is that the larger the  
37 measurement sample the more representative it is for the fleet and driving conditions  
38 investigated. However, every further day of measurement adds to the costs. Too few records on  
39 the other hand mean that the validity of the whole sample can be compromised and hence efforts  
40 were wasted. Despite its fundamental importance we have not found sound guidance in the  
41 literature about the minimum number of emission records needed for a representative  
42 measurement nor on the statistical power of the data. Statistical theories provide tools to  
43 determine the relationship between sample size and confidence of population mean estimation.  
44 However, these formulations usually assume certain distribution characteristics of the data as a  
45 priori, such as normality, independence etc., which are usually not valid for vehicle exhaust  
46 emission rates. In addition, although population variance is an important statistic in assessing  
47 distribution and variability of vehicle emissions<sup>17,27,28</sup>, there is no existing method that constitute  
48 relationship between sample size and confidence of population variance estimation.

49 Here we work on real-world data and explore their inherent relationships. We propose a  
50 bootstrap-sampling based Monte Carlo simulation to determine the relationship between size of  
51 the emission measurement sample and the statistical performance of sample mean and variance.  
52 We carry out the simulations on a set of 130,000 remote sensing emission records of Diesel cars  
53 measured between 2011 and 2018 at 23 locations across Switzerland, Sweden and the United  
54 Kingdom.<sup>25</sup> This unique dataset covers vehicles up to 25 years old, measurement ambient

55 temperature from 0 to 43 Celsius degree and instantaneous vehicle specific power up to 54 kW  
56 per ton. These records comprehensively cover a wide range of real-world driving conditions and  
57 a broad spectrum of Europe's passenger car fleet. We consider this the best available sample of  
58 real emission rates to conduct our analysis on. This allows exploring of methods in deciding  
59 minimum sample size of vehicles with different emission standards under the control of vehicle  
60 age, power and ambient temperature conditions.

## 61 LITERATURE REVIEW

62 Sample size determination is an important component in empirical studies. A minimum number  
63 of measurements is needed to detect statistically significant effects. Traditional methods in  
64 determining sample sizes are dependent on the underlying population distribution. For example,  
65 equation (1) is used to determine a sample size whose sample mean is within E units from the  
66 population mean with  $(1-\alpha) \times 100\%$  confidence.<sup>7</sup>  $\sigma$  is the standard deviation of sample mean and  
67  $z_{\alpha/2}$  is a critical value calculated based on normality assumption of population.

$$68 \quad n = \frac{z_{\alpha/2}^2 \times \sigma^2}{E^2} \quad (1)$$

69 According to Central Limit Theory (CLT), the distribution of the sample means will be  
70 approximately normally distributed and sample mean is an unbiased estimator of population  
71 mean. Therefore, the closed-form solution in Eq. (1) is applicable in determining sample size  
72 whenever mean emission statistics are the focus. However, there is no closed-form equation that  
73 determines sample size based on accuracy of variance statistics. Variance of emission is an  
74 important statistic which determines the spread of on-road vehicle emission and has been used in  
75 various of studies focusing on estimating confidence interval of emission, distribution and  
76 variability of vehicle emissions.<sup>17,27,28</sup> However, limited attention is given to estimate population

77 variance of emission based on sample variance. The variance of sample with size  $n$  is commonly  
78 assumed to follow chi-squared ( $\chi^2$ ) distribution with  $n-1$  degree of freedom. But this requires  
79 sample to be drawn from normal distribution. Instantaneous vehicle emission rates however are  
80 known to be skewed<sup>8</sup> and are not normally distributed. Thus, there is a lack of knowledge in  
81 determining sample size to achieve accuracy in variance estimation statistics.

82 Monte Carlo simulation approach has recently been utilized for sample size determination to  
83 achieve accurate mean estimation. Monte Carlo is a numerical experiment that generates  $T$ -time  
84 sampling simulations each with  $n$  draws with or without replacement from a random sample with  
85 a prescribed probability distribution.<sup>9</sup> Each sample generates one sample mean estimator and one  
86 sample variance estimator. Given a sufficient large simulation time  $T$ , e.g. 1000 times, it is  
87 possible to examine the statistical robustness of using mean and variance of sample size  $n$  to  
88 approximate population mean and variance.

89 Muthén and Muthén is one of the early literatures to use Monte Carlo simulation in determining  
90 sample size.<sup>10</sup> Parameter estimate bias, standard error bias and coverage were reported using  
91 different sample sizes. It was found that non-normality and missing data are major factors of  
92 sample size. Shi and Lee utilized Monte Carlo simulations to calculate sample size needed for  
93 group randomized trials with unequal group sizes in cancer prevention and health promotion  
94 research.<sup>11</sup> They found that the widely used formula for sample size in group randomized trials is  
95 not applicable when group sizes vary, which is commonly observed in empirical research setup.  
96 Qumsiyeh utilized a bootstrap sampling technique in Monte Carlo simulations to find required  
97 sample sizes to achieve various confidence levels in health care related statistical experiments.<sup>12</sup>  
98 The required sample size was proven to be smaller than the one computed based on exact method

99 as shown in equation (1). This has practical consequence because small sample size without  
100 sacrificing predicting power means less labor and cost in conducting research.

101 In on-road vehicle emission measurement studies, researchers install equipment on roadside and  
102 measure vehicle emissions for a certain period in one year or in multiple years to collect enough  
103 data for emissions analyses. Huang et al. compared vehicle emission measurement techniques  
104 under real-world driving conditions and concluded that on-road remote sensing is an effective  
105 and economic tool to monitor and control vehicle emissions.<sup>4</sup> However, they also pointed out  
106 major challenges in applying remote sensing technology, which include robustness of sampling  
107 process. In review of existing real-world vehicle emission studies, it shows that measurement  
108 sample sizes are either determined by researchers' experience or constrained by research  
109 budgets, but have not been derived systematically.<sup>13-15</sup> There is a knowledge gap in  
110 systematically determining necessary sample size to achieve statistical robustness.

## 111 **INPUT DATA AND DATA HANDLING**

112 Three spectroscopic remote sensing (RS) instruments were used to conduct vehicle emission  
113 measurement in this study, including the FEAT instrument developed by the University of  
114 Denver and the Opus AccuScan RSD 4600 and RSD 5000. These instruments have been used  
115 and discussed extensively in previous studies.<sup>1-2,16-18</sup> RS instruments are placed at a roadside; the  
116 concentration of certain pollutants (CO<sub>2</sub>, CO, HC, NO) in the plume of the vehicles passing is  
117 proportional to the attenuation of the light transmitted through the plume. The increment in the  
118 concentration relative to the background measured immediately before is then attributed to the  
119 vehicle. The incremental pollutant concentration is then divided by the incremental concentration  
120 of CO<sub>2</sub>, which in turn is proportional to the fuel burnt in the engine. This ratio presents the  
121 instantaneous fuel specific emission rate of the vehicle. Instantaneous speed and acceleration of

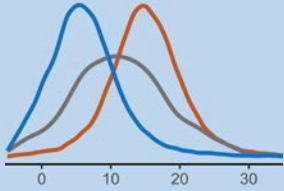
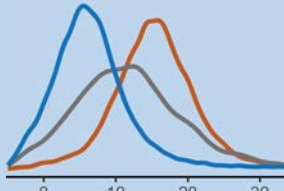
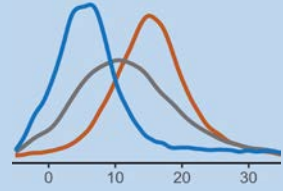
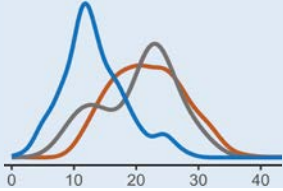
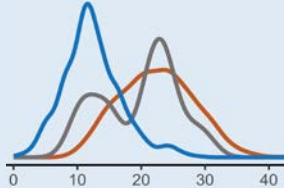
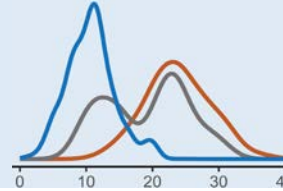
122 passing vehicles are recorded as well. A possible instrument drift is corrected by regular  
123 calibration with an external reference gas. The information on the vehicle technology is retrieved  
124 via the recorded license plate from the vehicle registry. This provides information on the model  
125 year and emission certification standard, the make and model, the engine size and power, the fuel  
126 type and curb weight.

127 We use a collection of 130,000 RS records that were measured during eight year across Europe  
128 to serve as population. Table 1 summarizes the different measurement campaigns, testing  
129 conditions and passenger car fleet characteristics. The table groups the data by emission  
130 standards to facilitate comparison within and across country-specific measurements. In this  
131 paper, we focus on NO emissions of Euro 4, Euro 5 and Euro 6a,b diesel cars and demonstrate  
132 our approach for searching a minimum sample size. Euro 4 to Euro 6a,b cars account for 78  
133 percent of all diesel car records. The choose of the NO emission is due to its importance for air  
134 quality, its low measurement error of  $\pm 15\%$  (unlike NO<sub>2</sub>) and because its emission rate is  
135 arguably the least variable among the emissions measured. Therefore, it presents an ideal case  
136 for the analysis and we consider our results as lower bounds for minimum sample sizes for other  
137 pollutants or emission concepts. The Monte Carlo simulation approach proposed here can  
138 however easily serve as a template for sample size determination of other pollutants, vehicle  
139 types and control stages.

140 Vehicle emissions increase with age or mileage, respectively. However, Chen and Borken-  
141 Kleefeld<sup>15</sup> did not find a relevant deterioration on NO and NO<sub>x</sub> emissions for Euro 4 diesel cars.  
142 More recently by Carslaw et al.<sup>19</sup> did likewise not find a relevant change in the NO emission  
143 rate with vehicle mileage for diesel Euro 5 and Euro 6a,b cars. Therefore, we need not  
144 discriminate records by vehicle age or mileage here.



145 **Table 1.** Summary of remote sensing testing conditions and passenger car fleet characteristics in  
 146 UK (blue), Sweden (gray) and Switzerland (orange)

|   |    | Euro 4 Diesel Car   | Euro 5 Diesel Car  | Euro 6a,b Diesel Car  |
|---|----|---|--|---|
| # of measurements                             | UK | 23,825  | 32,071   | 12,136  |
|   | SE | 617   | 5,106  | 3,426   |
|   | CH | 17,257  | 29,725   | 7,072   |
| Measurement Year and instrument               | UK | FEAT: 2012, 2013, 2017, 2018; RSD 4600: 2013, 2015; RSD 5000: 2017, 2018            |  |   |
|   | SE | RSD 5000: 2016  |  |   |
|   | CH | RSD 4600: 2011-2015; RSD 5000: 2016, 2017   |  |   |
| Average age (years)                           | UK | 7.4   | 3.8  | 2.0   |
|   | SE | 8.3   | 5.0  | 1.7   |
|   | CH | 8.3   | 4.3  | 2.5   |
| Average NO emission rates +1 SD (g / kg fuel) | UK | 11.0 (8.0)  | 12.6 (9.4)   | 6.0 (9.2)   |
|   | SE | 10.9 (10.9)   | 11.4 (10.6)  | 5.9 (7.7)   |
|   | CH | 11.0 (9.3)  | 12.6 (10.3)  | 6.0 (8.6)   |
| VSP (kW/ton)                                  |    |    |    |    |
| Ambient Temperature (C)                       |    |  |  |  |

147

148 **METHODOLOGY**

149 Here, we aim to find the smallest sample size of emission records whose mean and standard  
 150 deviation are reasonably close to the ‘true’ mean and standard deviation of the full population.

151 The population is the collection of 130,000 RS records that were measured during eight year  
 152 across Europe as shown in Table 1. We define terminologies as in Table 2.

153 **Table 2.** Terminology Definition

| Terminology       | Definition   |
|-------------------|--|
| <b>Population</b> | The set of all measured emission rates stratified by vehicle emission control technology, denoted as $X$ |

|  |  |
|--|--|
| <b>Population size</b>                         | Total number of measurements in population, $N$  |
| <b>Population mean</b>                         | Mean of all elements in the population, $\mu = \frac{\sum_{i=1}^N X_i}{N}$   |
| <b>Population Standard Deviation</b>           | Mean of all elements in the population, $\sigma = \sqrt{\frac{\sum_{i=1}^N (X_i - \mu)^2}{N}}$                                 |
| <b>Sample mean</b>                             | The mean of a $n$ size sample from the population $\bar{x}_j^n = \frac{\sum_{i=1}^n x_i}{n}$                                   |
| <b>Sample Standard Deviation</b>               | The standard deviation of a $n$ size sample from the population $s_j = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x}_j^n)^2}{n}}$    |
| <b>Error of sample mean</b>                    | The absolute deviation of sample means from population mean, $v_j =  \bar{x}_j^n - \mu $                                       |
| <b>Error of sample standard deviation (SD)</b> | The absolute deviation of sample standard deviation from population standard deviation, $w_j =  s_j - \sigma $                 |
| <b>Tolerated error of estimation</b>           | The maximum error of sample means/standard deviation that counts as an accurate sample, $V/W$                                  |
| <b>Certainty ratio</b>                         | Ratio of accuracy, $A_n = \frac{\sum_{j=1}^T I(v_j \leq V \ \& \ w_j \leq W)}{T}$ , in $T$ simulations with size $n$ each time |

154  
 155 We utilize a bootstrap-based Monte Carlo simulation approach that consists of  $T$ -fold random  
 156 sampling simulations and evaluating the statistical performance of the sample mean to determine  
 157 minimum sample size.  $T$  is a large number, i.e. 1000, as suggested in Monte Carlo simulation  
 158 literature.<sup>9,12</sup> Specifically, in each iteration of the sampling simulation, we conduct a bootstrap  
 159 experiment, i.e. drawing  $n$  samples with replacement from a stratified population  $X$  that has  $N$   
 160 emission records. With replacement means we replace an item once it is drawn from population.  
 161 The purpose of bootstrap is to construct an empirical distribution based on observed data which  
 162 can be used to asymptotically infer statistics of true stratified population. Literature have shown  
 163 that bootstrap sampling can guarantee asymptotic feature of sample mean and variance  
 164 distribution to that of population mean.<sup>20</sup>

165 The  $T$ -time simulations will generate a series of sample means  $\bar{x}_1^n \dots \bar{x}_j^n \dots \bar{x}_T^n$  and sample  
166 variances,  $s_1^2, \dots s_j^2 \dots s_T^2$ . The errors of sample mean and standard deviation for each simulation,  
167  $v_j, w_j$ , defines closeness of  $j^{\text{th}}$  sample mean and standard deviation to the population mean and  
168 standard deviation (SD). We classify an accurate estimation as if  $v_j \leq V$  and  $w_j \leq W$  where  $V$   
169 and  $W$  are our tolerated errors for mean and SD. The certainty ratio of  $T$ -time simulations,  $A_n$ ,  
170 represents the consistency of sample mean and SD estimations when emission measurements are  
171 repeated. Higher  $A_n$  corresponds to greater confidence of observing accurate sample mean and  
172 SD estimations. Specifically, if  $n$  sample are drawn out of population dataset, we can be  $A_n$   
173 confident that the sample mean and SD are within  $\pm V$  and  $\pm W$  of population mean and SD,  
174 respectively. Fixing  $V$  and  $W$ , we expect  $A_n$  to increase as sample size  $n$  enlarges. The advantage  
175 of Monte Carlo simulation is to explicitly explore the relationship between sample size, accuracy  
176 and confidence performance of the sample mean, here based on a large empirical dataset.

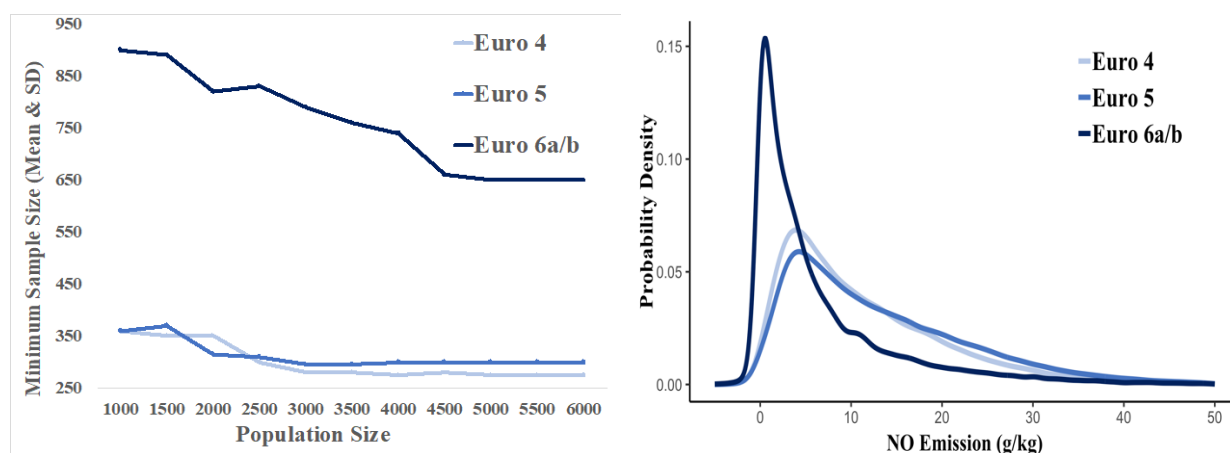
## 177 **INFLUENCE OF POPULATION SIZE ON MINIMUM SAMPLE SIZE**

178 First, we need to make sure that our populations are large enough not to constrain or bias the  
179 subsequent analysis. This is also referred to as finite population correction. It is recommended to  
180 use finite population correction factor to adjust variance/standard deviation estimate when  
181 sample size is greater than 5 percent of a population. Here, we aim to empirically find the  
182 population size that does not need to apply correction factor. The tolerated error of mean  
183 estimation is set to 1 g NO/kg fuel; this corresponds to about 50 mg NO per km, i.e. about 5% to  
184 10% of the average emission rate for Euro 5 diesel cars. The tolerated error of population  
185 standard deviation is set to be 0.5 g NO/kg fuel. In normal distribution, mean plus/minus 2 times  
186 standard deviation covers 95% of data in distribution. We borrow this idea and set the tolerated

187 error for standard deviation at 0.5 g NO/kg fuel so that two times of it equal to 1 g NO/kg fuel,  
188 which is comparable to tolerated error of mean estimation. As certainty rate of estimation, i.e. the  
189 confidence metric, we require 80%, which is based on our engineering knowledge. In the  
190 remainder of this paper, we vary the certainty rate to test sensitivity of our results to certainty  
191 rate.

192 In this analysis, we choose different size of sub-population by randomly draw from the actual  
193 population for Euro 4, 5 and 6a,b diesel car NO emission measurement. We use the Monte Carlo  
194 simulation approach to find minimum sample size for each size of sub-population. Figure 1  
195 shows that the minimum sample sizes are not affected by statistical fluctuations if sub-population  
196 size is greater than 2500 for Euro 4/Euro 5 and 4500 for Euro 6a,b. Obviously, the exact numbers  
197 depend on the required tolerance and certainty, with more stringent requirements leading to  
198 bigger population size thresholds and larger minimum sample sizes. The important observation  
199 here is that we always have records larger than 2500 for Euro 4/Euro 5 or 4500 for Euro 6a,b,  
200 even in the stratified analysis below, so that our minimum sample size results are robust and will  
201 not be influenced by finite population.

202



203

204 **Figure 1.** Minimum sample size (mean and SD) vs population size for NO emission estimation  
205 of Euro 4-6 diesel cars, all data from three countries. Default tolerated error of mean is 1 g NO /  
206 kg fuel, tolerated error of standard deviation at 0.5 g NO/kg fuel, certainty rate of estimation  
207 80%.

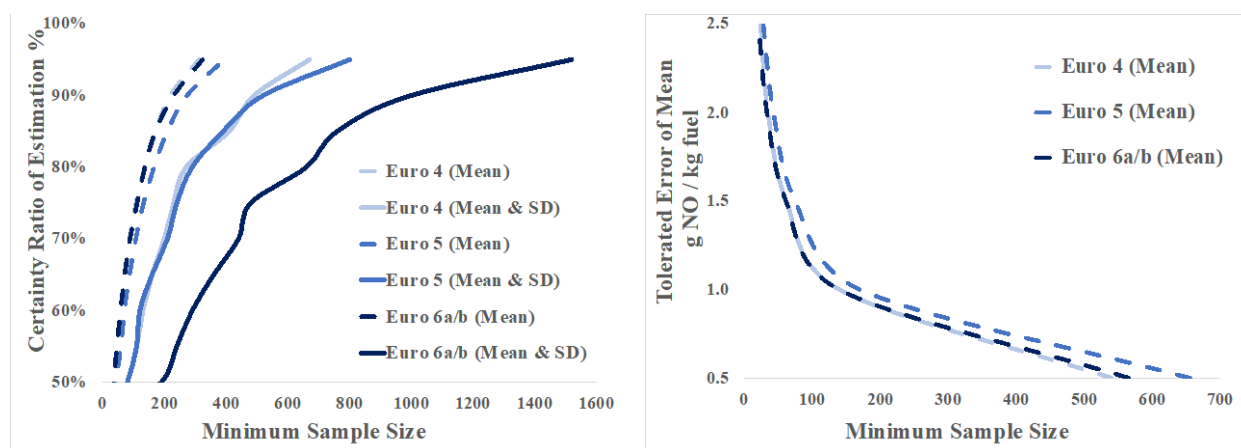
208

## 209 **SAMPLE SIZE AS FUNCTION OF TOLERATED ERROR AND ESTIMATION**

### 210 **CERTAINTY RATE**

211 The minimum sample size is clearly a function of the tolerated error for the mean and standard  
212 deviation  $V, W$  and the certainty ratio of estimation  $A_n$ . Figure 2a shows the minimum sample  
213 size for the NO emission rate of diesel cars first as a function of the certainty ratio. A larger  
214 sample size leads to higher confidence in using the sample mean and sample's standard deviation  
215 to estimate population mean and standard deviation, as expected. However, the increase in  
216 certainty diminishes as the sample size becomes bigger. To achieve 70%, 80% or even 90%  
217 certainty in both, mean and standard deviation estimate, a sample size of about 200, 300 or 500  
218 records is needed for Euro 4 and Euro 5 diesel cars, and significantly large size of 440, 660 and  
219 1010 records for Euro 6a,b cars, respectively. If only the mean is of interest then about half that  
220 number would be sufficient, with the notable exception of Euro 6a,b cars: The mean estimate  
221 requires only about 90, 140 and 230 records respectively for the chosen certainties. This is a  
222 consequence of the very different shape of distribution of the Euro 6a,b records compared to the  
223 earlier diesel generations. The stark difference in sample size between the mean and the standard  
224 deviation estimate is a result of the Monte Carlo simulation further discussed below. Next we  
225 explore the relationship between the tolerated error and the minimum sample size (Figure 2b).  
226 For illustration, we choose to vary the tolerated error for the mean estimation from 0.5 to 2.5 g  
227 NO / kg fuel and keep the certainty ratio fixed at 80%. As expected, the minimum sample size  
228 increases when the tolerated error is reduced, with the exact form established here from

229 observations. We find that a tolerated error for mean of 1 g NO / kg fuel is actually a tipping  
230 point for the reduction rate for all three Euro classes investigated here: Below 1 g NO / kg fuel, a  
231 further decrease of the tolerated error results in a strong increase in the minimum sample size.  
232 For instance the minimum sample size increases from 50 to 100 to then about 600, when the  
233 tolerated error is reduced from 2 to 1 to finally 0.5 g NO / kg.



234  
235 **Figure 2.** Minimum sample size vs certainty rate of estimation based on tolerated error of mean  
236 at 1 g NO/kg fuel and tolerated error of standard deviation at 0.5 g NO/kg fuel (left, 2a);  
237 minimum sample size vs tolerance error of mean based on 80% certainty ratio of mean  
238 estimation (right, 2b).

239  
240 Table 3 compares the minimum sample sizes derived from the traditional closed form solution of  
241 equation (1) with the results from our Monte Carlo simulation. We confirm with empirical data  
242 the sample size results when the interest is only in the mean values of the population. However,  
243 we show at the same time that more than two times that number of records is needed to estimate  
244 the standard deviation of the distribution with the same accuracy. The distributions for Euro 4  
245 and Euro 6a,b cars are quite different e.g. in terms of skewness, peak, possession of symmetry  
246 (Figure 1b). The Monte Carlo simulation approach utilizes both population variance and shape of  
247 distribution to determine sample sizes that can guarantee robustness estimation of both

248 population mean and variance. These sample sizes are consistently larger than sample sizes  
249 obtained based on closed form solution as shown in Equation (1).

250 **Table 3.** Minimum sample size as a function of required certainty in standard deviation  
251 estimation for diesel cars Euro 4, 5, 6a,b, numbers, in parenthesis are sample size calculated  
252 using Equation (1).

| Certainty Ratio  | 70%       |           | 80%       |           | 90%       |           |
|------------------|-----------|-----------|-----------|-----------|-----------|-----------|
|                  | Mean      | Mean & SD | Mean      | Mean & SD | Mean      | Mean & SD |
| <b>Euro 4</b>    | 87 (87)   | 200       | 135 (133) | 270       | 218 (219) | 490       |
| <b>Euro 5</b>    | 107 (110) | 210       | 166 (169) | 300       | 273 (278) | 520       |
| <b>Euro 6a,b</b> | 88 (85)   | 440       | 136 (131) | 660       | 229 (215) | 1010      |

253

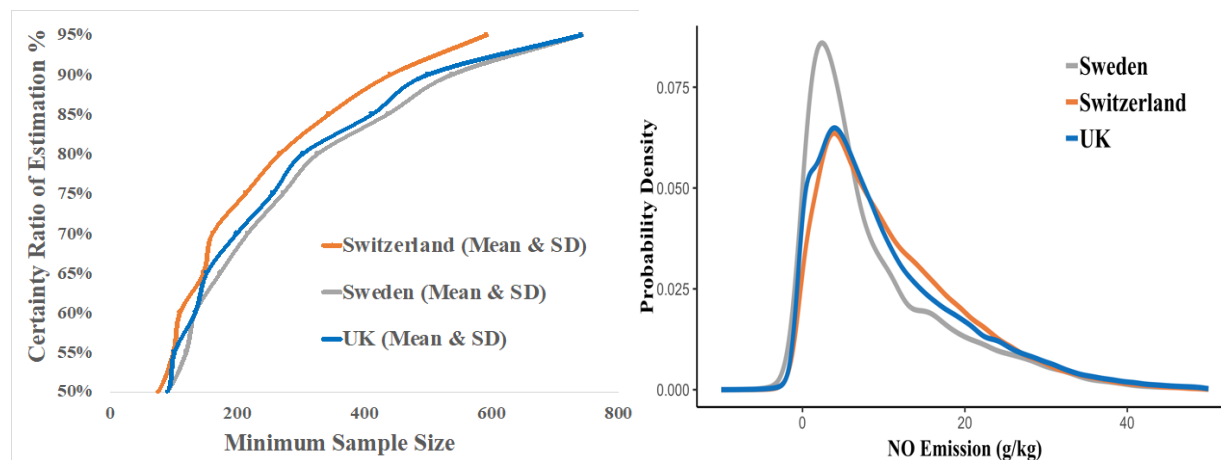
254

255

## 256 **INFLUENCE OF LOCATION ON MINIMUM SAMPLE SIZE**

257 Previous studies of remote sensing have demonstrated heterogeneity of emissions behavior at  
258 different measurement locations.<sup>21</sup> As shown in Table 1, our data were collected from three  
259 countries and contain heterogeneous vehicle specific power and ambient temperature  
260 distributions. Thus, we differentiate the data by country and explore the relationship between  
261 minimum sample size and certainty ratio of estimation for each location specifically (Figure 3a).  
262 We observe that to achieve the same level of confidence in estimation, i.e. certainty ratio, Swiss  
263 data require the smallest sample size, followed by UK and Sweden. For example, to achieve 80%  
264 certainty ratio in mean and standard deviation estimation for Euro 5 cars, a sample of around 300  
265 records are required for any location (exactly 277, 302, 326 in Switzerland, UK and Sweden  
266 respectively). This is a remarkably consistent result despite different fleets, different instruments,  
267 driving conditions and ambient temperatures in the different locations. This means that records  
268 from different sites, i.e. from different instruments, vehicle fleets and driving conditions can be

269 collectively analysed together, at least for NO emissions from diesel cars Euro 4, Euro 5 and  
270 Euro 6a,b.



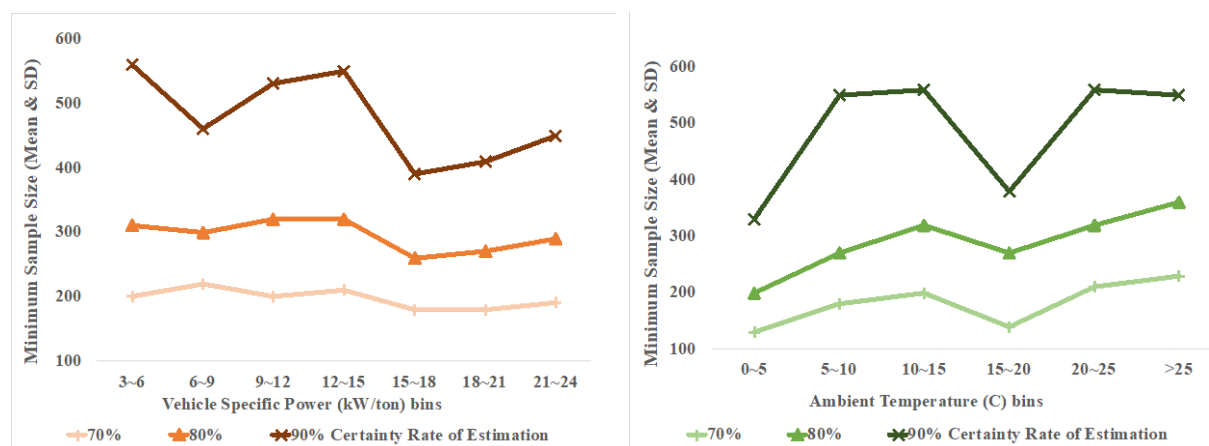
271  
272 **Figure 3.** Minimum sample size (mean and SD) vs certainty ratio of estimation based on Euro 5  
273 diesel cars location-specific NO emission data, default tolerated error is 1 g NO/kg fuel and  
274 standard deviation tolerated error is 0.5 g NO/kg fuel.

### 275 276 **INFLUENCE OF POWER HOMOGENIZATION ON MINIMUM SAMPLE SIZE**

277 Vehicle specific power (VSP) is a metric for estimating engine power demand of a vehicle and  
278 has been extensively used in emission models and remote sensing analysis.<sup>17,22-23</sup> Particularly,  
279 Carslaw et al. found a clear increase of the NO<sub>x</sub> emission rate with increasing VSP.<sup>13</sup> In the  
280 NEDC type-approval driving cycle in Europe VSP ranges from 3 to 22 kW/ton, which is consist  
281 of normal urban driving and extra-urban driving cycles. To assess impacts of vehicle power on  
282 sample size determination, in Figure 4a, we present minimum sample size under various VSP  
283 bins and various accuracy performance metrics, i.e. 70%, 80% and 90% certainty ratio of  
284 estimation,  $A_n$ . The tolerated error of mean estimation  $V$  is fixed at 1 g NO / kg fuel and the  
285 tolerated error of standard deviation  $W$  is fixed at 0.5 g NO / kg fuel. We restrict the data to Euro  
286 5 diesel cars measured in United Kingdom, i.e. the most abundant sample, to identify the  
287 influence of engine load as clearly as possible. Given the certainty ratio of 80%, the minimum



288 sample size is relatively stable at 300 records up to a VSP of 18 kW/ton. When the certainty is  
289 reduced from 80% to 70%, meaning that roughly one third of records is allowed to be outside the  
290 tolerance margin, only 100 records are required. Vice versa, to increase the certainty to 90%,  
291 meaning that only 10% of the sample is allowed outside the tolerance error, then at least 450  
292 records are needed to approximate the population mean.



293  
294 **Figure 4.** Minimum sample size (mean and SD) based on vehicle specific power bins (left, 4a),  
295 temperature bins (middle, 4b) with different certainty ratio of estimation, default tolerated error  
296 of mean 1 g NO/kg fuel and tolerated error of standard deviation 0.5 g NO/kg fuel, UK Euro 5  
297 diesel car NO emission.

298

### 299 INFLUENCE OF TEMPERATURE ON MINIMUM SAMPLE SIZE

300 It has been shown before, that the NO emission rate increases significantly when the ambient  
301 temperature decreases from 20°C to 5°C.<sup>24,26</sup> While this affects the mean rate, it does actually not  
302 affect the sample size to achieve accurate mean and/or SD estimations, as we find empirically.

303 The minimum sample size required is stable at about 300 records across temperatures from 5°C  
304 to 25°C (Figure 4b). The same stable behavior is observed when a higher or lower certainty is  
305 requested: Then the necessary sample size is either about 500 or 200 records to approximate the  
306 population mean. These stable relations are good news for experimentalists and analysts alike:

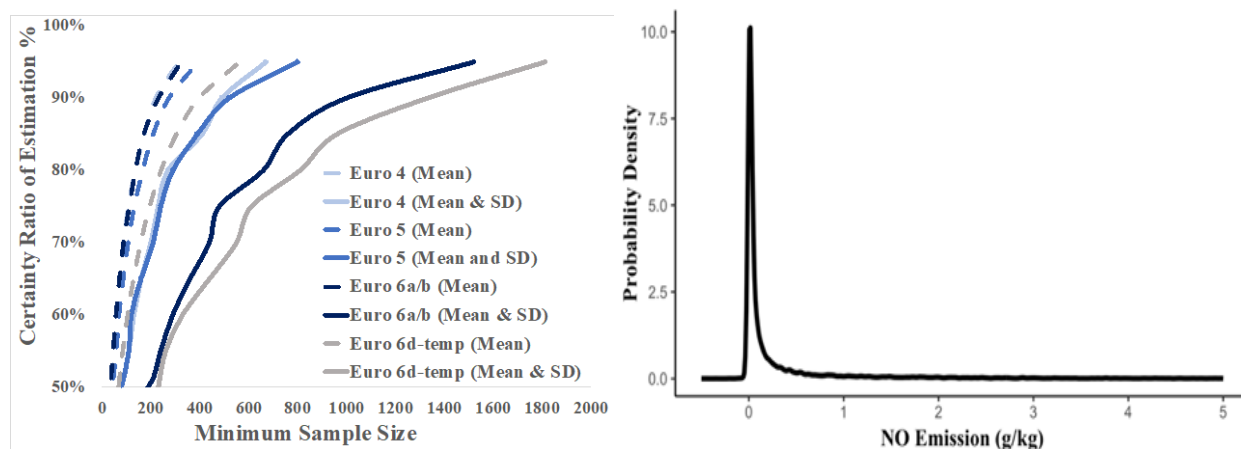
307 The former can be assured that they do not need to spend time on finely controlling driving

308 conditions and ambient temperatures; the latter can justify combining data from different  
309 external conditions in their analysis.

310 **OUTLOOK: REQUIRED SAMPLE SIZE WHEN THE EMISSION RATE IS MUCH**  
311 **LOWER**

312 The RS measurement data contain emission records from diesel cars certified up to Euro 6a,b,  
313 which are vehicles manufactured before September 2018. Their NO emission rate is  
314 approximately 400-500 mg NO per km, and thus much higher than for gasoline cars that emit  
315 less than 2 g NO per kg fuel or less than 80 mg NO per km on the road. Diesel cars certified to  
316 the current Euro 6d-temp emission standard are also measured in this range. Therefore, the  
317 question is whether many more RS records are needed for a reliable sampling at much lower  
318 average emission rates? For lack of data we cannot answer this from the existing set of RS  
319 records.

320 As a proxy we use modal PEMS data (courtesy S. Hausberger, TU Graz) from six Euro 6d-temp  
321 diesel cars all having an emission rate of no more than 40 mg NO<sub>x</sub> per km over 20 RDE  
322 compliant trips. This constitutes some 116,000 second-by-second emission records. We convert  
323 them into a format compatible with RSD as follows: To alleviate possible issues with time  
324 alignment notably between the NO and the CO<sub>2</sub> sensor we take the running average over  
325 consecutive three second intervals. Next, we calculate the ratio of the three second NO to CO<sub>2</sub>  
326 and fuel consumption respectively. Finally, we filter out all records for negative acceleration or  
327 VSP above 22 kW per ton. This way we generate a set of 39,000 instantaneous emission ratios  
328 comparable to RS measurement conditions. The average NO emission rate of these Euro 6d-temp  
329 cars is 0.8 g / kg fuel, about a factor eight lower than the earlier Euro 6a,b cars. We perform the  
330 sample size analysis on this set.



331  
332 **Figure 5.** Minimum sample size vs average NO emission of Euro 4-6 diesel cars in RS data and  
333 Euro 6d in PEMS data (left, 5a), default tolerated error of mean at 1 g NO / kg fuel, tolerated  
334 error of standard deviation at 0.5 g NO/kg fuel, certainty rate of estimation 80%; Probability  
335 density function of NO emission rates of Euro 6d cars based PEMS experiment (right, 5b).

336 Figure 5a presents minimum sample size versus certainty ratio for Euro 4, 5 and 6a,b using  
337 remote sensing data and Euro 6d-temp using PEMS data. As before the tolerated error of mean  
338 estimation  $V$  is fixed at 1 g NO / kg fuel and the tolerated error of standard deviation  $W$  is fixed  
339 at 0.5 g NO / kg fuel. The certainty rate of estimation is set at 80%. As suspected, that the  
340 minimum sample size for Euro 6d is larger than those of Euro 4, 5 and 6a,b. For example, at 80%  
341 certainty ratio, the minimum sample size for Euro 6d-temp diesel cars is 810 records which is  
342 larger than that of Euro 6a,b (660), and about 2.5 times of those of Euro 4 and Euro 5. However,  
343 that number is within the range found before, meaning that also for vehicles with very clean  
344 exhaust emissions our results indicate the range. One could assume that this then also holds true  
345 for gasoline cars as well.

346 As we see that increase in the required sample size is determined by the shape and in particular  
347 the variance of the underlying emission distribution: A small portion of higher emission records  
348 in Euro 6d-temp diesel cars leads to a high variance in the emission measurements and thus  
349 higher minimum sample size for estimating the average emission level. Figure 5b presents the

350 probability density function of NO emission from Euro 6d, which shows majority of NO  
351 emission of Euro 6d cars are small and the average emission of Euro 6d-temp is well controlled.

## 352 **DISCUSSION**

353 In summary, we propose a bootstrap-based Monte Carlo simulation approach to determine the  
354 minimum sample size in remote sensing measurements of vehicle emissions. The sample size is  
355 given explicitly here as a function of required accuracy and robustness for both the population  
356 mean and its standard deviation. The minimum number depends on vehicle technology, fuel type  
357 and pollutant. Here we explore the empirical relationship for the NO emission rate of European  
358 diesel cars certified to Euro 4, 5 or 6 emission standards. We believe this pollutant presents a  
359 good opportunity to develop the method that is suitable for other vehicle concepts and pollutants.  
360 Because of their bigger variance we expect that the minimum sample will be higher for the other  
361 pollutants. Our results are important for planning measurement campaigns, for appropriately  
362 budgeting resources and for assessing the robustness of the records obtained. This is illustrated  
363 by a simplified example in Table 4: Suppose, RS measurements are conducted at a road with an  
364 average 2000 passenger cars passing during daytime, which is typical of many sites used so far  
365 in Europe. Assume for simplicity an even share of diesel and gasoline cars, meaning that there  
366 are about 1000 diesel cars passing, of which typically 90% have valid records. This fleet might  
367 be composed of around 40% Euro 5 cars (mandatory between 2009 and 2014), 40% newer  
368 Euro 6a,b and 10% Euro 6d cars, the rest being older. Then between 90 and 360 diesel cars of the  
369 respective certification standards could be measured in a single day (and about the same  
370 distribution for gasoline cars). Within half a day the mean values of Euro 5 and Euro 6a,b cars  
371 could be determined with more than 80% certainty; in less than two measurement days also their  
372 variance could be measured representatively. The same campaign would lend similar data for the

373 other technology layers, and the more records come in the more accurate the sample mean and  
374 variance become for those and any other vehicle category and technology layer. These averages  
375 per technology layer are crucial input e.g. to traffic emission and air quality models.

376 If the objective of the RS campaign is market surveillance e.g. of individual engine families, then  
377 more measurement time is needed, depending on the frequency of occurrence of the respective  
378 engines. Assume the ten top selling engine families have at least 2% share in the Euro 6a,b cars.  
379 To determine the mean NO emission rates for these top ten engine families (i.e. needing at least  
380 136 records each) about 19 measurement days would be needed, so roughly two days per month.  
381 Most days would be needed for Euro 6d engine families because they are (so far) less abundant  
382 and need most records for an accurate determination: For a Euro 6 engine family with only 1%  
383 share more than 200 measurement days would be needed to determine its mean emission rate.  
384 This would represent nearly continuous measurements or calls for a change in the measurement  
385 strategy: Either more RS units could be deployed to multiply the data capture, or they should be  
386 deployed to road with higher traffic volume, or to sites where a higher occurrence of the target  
387 engine families is known. The numbers can be easily adopted to a different local situation and  
388 different vehicle categories. Whatever the target, the campaign will always capture very useful  
389 data for the whole fleet and all vehicle categories and technologies occurring at once. How  
390 robust and accurate the values are is essentially ‘only’ a matter of the statistical sample. This can  
391 be boosted by cooperation between different RS campaigns, as illustrated by the CONOx  
392 project<sup>24</sup> that provided the initial sample for this analysis.

393 A campaign of 20 days would yield about 36,000 valid car records, which would very accurately  
394 provide mean NO emission rates of the top ten engine families for all light duty vehicles but the  
395 latest technology layer (Euro 6d). At indicative costs of 0.5 to 2 €/per record this translates to

396 about 18,000 to 72,000 Euros for the whole campaign for top engine families for light duty  
 397 vehicles, diesel and gasoline alike. This is significantly cheaper than a series of PEMS  
 398 measurement of dozens of individual vehicles and illustrates the important cost saving potential  
 399 when RS is used for market surveillance and pre-screening before detailed emission testing.  
 400 Flexible, small scale measurement campaigns allow capture of different driving conditions and  
 401 fleets, longer term, stationary campaigns allow more detailed analysis down to individual  
 402 vehicles when they are measured repeatedly.

403 **Table 4.** Example for planning the duration of a RS campaign for either fleet or family emission  
 404 rate, mean only (with  $\pm 1$  g NO/kg accuracy) or including standard deviation (as  $\pm 0.5$  g NO/kg).

|                                  | Assumptions on traffic |                           | Required sample size @80% certainty |                  | Measurement days |                  |
|----------------------------------|------------------------|---------------------------|-------------------------------------|------------------|------------------|------------------|
|                                  |                        | Records per day [6-18hrs] | ...for mean only                    | ...for mean & SD | ...for mean only | ...for mean & SD |
| <b>Volume of passenger cars</b>  | 2000                   |                           |                                     |                  |                  |                  |
| <b>Valid records</b>             | 90%                    | 1800                      |                                     |                  |                  |                  |
| <b>Share diesel cars</b>         | 50%                    | 900                       |                                     |                  |                  |                  |
| <b>Share: Euro 4 and older</b>   | 10%                    | 90                        | 135                                 | 270              | 1.5              | 3                |
| <b>Share Euro 5</b>              | 40%                    | 360                       | 166                                 | 300              | 0.5              | <1               |
| <b>Share Euro 6a,b</b>           | 40%                    | 360                       | 136                                 | 660              | 0.4              | <2               |
| <b>Share Euro 6d</b>             | 10%                    | 90                        | 200                                 | 810              | >2               | 9                |
|                                  |                        |                           |                                     |                  |                  |                  |
| <b>a Euro 5 engine family</b>    | 2%                     | 7.2                       | 136                                 | 660              | 19               | 92               |
| <b>a Euro 6a,b engine family</b> | 1%                     | <1                        | 200                                 | 810              | 222              | 900              |

405

## 406 AUTHOR INFORMATION

407 Corresponding Author: Yuche Chen  
 408 Phone: +1-803 777 9105; Fax: +1-8037770670; E-mail: [chenyuc@cec.sc.edu](mailto:chenyuc@cec.sc.edu)

409 Jens Borken-Kleefeld  
 410 Phone: +43-2236 807 570; Fax: +43-2236 807 530; E-mail: [borken@iiasa.ac.at](mailto:borken@iiasa.ac.at)

411

412 **ACKNOWLEDGEMENTS**

413 The authors would like to thank the CONOx group for providing remote sensing data<sup>24</sup> and Prof.  
414 S. Hausberger of Graz University of Technology for providing detailed PEMS data.

415

416 **REFERENCES**

- 417 (1) Carslaw, D.C.; Rhys-Tyler, G. New Insights from Comprehensive On-Road Measurements of  
418 NO<sub>x</sub>, NO<sub>2</sub> and NH<sub>3</sub> from Vehicle Emission Remote Sensing in London, UK. *Atmospheric*  
419 *Environment*, 2013, 81 (December): 339–47. <https://doi.org/10.1016/j.atmosenv.2013.09.026>.
- 420 (2) Chen, Y.;Borken-Kleefeld, J. Real-driving emissions from cars and light commercial  
421 vehicles–Results from 13 years remote sensing at Zurich/CH. *Atmospheric Environment*, 2014,  
422 88, pp.157-164. <https://doi.org/10.1016/j.atmosenv.2014.01.040>
- 423 (3) Bishop, G.A.; Haugen, M.J. The story of ever diminishing vehicle tailpipe emissions as  
424 observed in the Chicago, Illinois area. *Environmental science & technology*, 2018, 52(13),  
425 pp.7587-7593. doi: 10.1021/acs.est.8b00926.
- 426 (4) Huang, Y.; Organ, B.; Zhou, J.L.; Surawski, N.C.; Hong, G.; Chan, E.F.; Yam, Y.S. Remote  
427 sensing of on-road vehicle emissions: Mechanism, applications and a case study from Hong  
428 Kong. *Atmospheric Environment*, 2018, 182, pp.58-74. Doi: [10.1016/j.atmosenv.2018.03.035](https://doi.org/10.1016/j.atmosenv.2018.03.035)
- 429 (5) Guo, H., Zhang, Q., Shi, Y. and Wang, D. On-road remote sensing measurements and fuel-  
430 based motor vehicle emission inventory in Hangzhou, China. *Atmospheric Environment*, 2007,  
431 41(14), pp.3095-3107. <https://doi.org/10.1016/j.atmosenv.2006.11.045>
- 432 (6) Borken-Kleefeld, J., and Dallmann, T.R. “Remote Sensing of Motor Vehicle Exhaust  
433 Emissions.” White Paper. Washington DC, USA: The International Council on Clean  
434 Transportation (ICCT), 2018.
- 435 (7) Kutner, M.H., Nachtsheim, C.J., Neter, J., Li, W. *Applied Linear Statistical Models*, 5th ed.  
436 McGraw-Hill, New York, 2005.
- 437 (8) Zhang, Y., Bishop, G.A., Stedman, D.H. Automobile Emissions Are Statistically Gamma  
438 Distributed. *Environmental Science & Technology*, 1994, 28 (7): 1370–1374.  
439 <https://doi.org/10.1021/es00056a029>
- 440 (9) Beasley, W.H. and Rodgers, J.L. Bootstrapping and Monte Carlo methods. *APA handbook of*  
441 *research methods in psychology*, 2012, 2, pp.407-425.
- 442 (10) Muthén, L.K. and Muthén, B.O. How to use a Monte Carlo study to decide on sample size  
443 and determine power. *Structural equation modeling*, 2002, 9(4), pp.599-620.  
444 [https://doi.org/10.1207/S15328007SEM0904\\_8](https://doi.org/10.1207/S15328007SEM0904_8)

445

446



- 447 (11) Shi, Y. and Lee, J.H. Sample size calculations for group randomized trials with unequal  
448 group sizes through Monte Carlo simulations. *Statistical Methods in Medical Research*, 2018,  
449 27(9): 2569-2580, doi: 10.1177/0962280216682775.
- 450 (12) Qumsiyeh, M., 2013. Using the bootstrap for estimating the sample size in statistical  
451 experiments. *Journal of Modern Applied Statistical Methods*, 12(1), p.9.
- 452 (13) Carslaw, D.C., Williams, M.L., Tate, J.E. and Beevers, S.D. The importance of high vehicle  
453 power for passenger car emissions. *Atmospheric environment*, 2013, 68, pp.8-16.  
454 <https://doi.org/10.1016/j.atmosenv.2012.11.033>
- 455 (14) Pujadas, M., Domínguez-Sáez, A., De la Fuente, J. Real-driving emissions of circulating  
456 Spanish car fleet in 2015 using RSD Technology. *Science of The Total Environment*, 2017, 576,  
457 pp.193-209. <https://doi.org/10.1016/j.scitotenv.2016.10.049>
- 458 (15) Chen, Y., Borken-Kleefeld, J. NOx emissions from diesel passenger cars worsen with age.  
459 *Environmental science & technology*, 2016, 50(7), pp.3327-3332.  
460 <https://doi.org/10.1021/acs.est.5b04704>
- 461 (16) Borken-Kleefeld, J. and Chen, Y. New emission deterioration rates for gasoline cars–  
462 Results from long-term measurements. *Atmospheric Environment*, 2015, 101, pp.58-64.  
463 <https://doi.org/10.1016/j.atmosenv.2014.11.013>
- 464 (17) Bishop, G.A., Stedman, D.H., Ashbaugh, L., 1996. Motor Vehicle Emissions Variability.  
465 *Journal of the Air & Waste Management Association* 46 (7): 667–75.  
466 <https://doi.org/10.1080/10473289.1996.10467501>.
- 467 (18) Rushton, C. E., Tate, J. E., Shepherd, S. P., Carslaw, D. C. Inter-instrument comparison of  
468 remote sensing devices and a new method for calculating on-road nitrogen oxides emissions and  
469 validation of vehicle-specific power, *J Air Waste Management*, 2018, 68:2, 111-122.  
470 <https://doi.org/10.1080/10962247.2017.1296504>
- 471 (19) Carslaw, D., Farren, N., Borken-Kleefeld, J., Sjödin, Å. “Study on the Durability of  
472 European Passenger Car Emission Control Systems Utilizing Remote Sensing Data.” C 387.  
473 Gothenburg, Sweden: IVL Swedish Environmental Research Institute, 2019.
- 474 (20) Kuhn, M., Johnson, K. *Applied predictive modeling* (Vol. 26). New York: Springer, 2013.
- 475 (21) Dallmann, T., Bernard, Y., Tietge, U., Muncrief, R. Remote sensing of motor vehicle  
476 emissions in London. *The International Council on Clean Transportation*, white paper, 2018.  
477 <https://www.theicct.org/publications/true-london-dec2018>
- 478 (22) Jiménez, J.L. “Vehicle Specific Power: A Useful Parameter for Remote Sensing and  
479 Emission Studies.” presented at the 9th CRC On-Road Vehicle Emissions Workshop, San Diego,  
480 April 21, 1999.
- 481 (23) US-EPA OTAQ. “Development of Emission Rates for Light-Duty Vehicles in the Motor  
482 Vehicle Emissions Simulator (MOVES2010).” Final Report EPA-420-R-11-011. Washington,  
483 DC, USA.: US Environmental Protection Agency., 2011.  
484 <http://www.epa.gov/otaq/models/moves/documents/420r11011.pdf>.



- 485 (24) Sjödin, Å, Borken-Kleefeld, J., Carslaw, D., Tate, J., G.-M. Alt, J. Dela Fuente, Y.,  
486 Bernard, P McClintock, R Gentala, N Vescio, and S Hausberger. “Real-Driving Emissions from  
487 Diesel Passenger Cars Measured by Remote Sensing and as Compared with PEMS and Chassis  
488 Dynamometer Measurements - CONOX Task 2 Report.” C 294. Gothenburg, Sweden: IVL  
489 Swedish Environmental Research Institute, 2018.
- 490 (25) Borken-Kleefeld, J., Bernard, Y., Carslaw, D., Sjödin, Å. “Contribution of Vehicle Remote  
491 Sensing to In-Service/Real Driving Emissions Monitoring - CONOX Task 3 Report.”  
492 Gothenburg, Sweden: IVL Swedish Environmental Research Institute, 2018.
- 493 (26) Grange, S.K., Farren, N.J., Vaughan, A.R., Rose, R.A. and Carslaw, D., 2019. Strong  
494 temperature dependence for light-duty diesel vehicle NOx emissions. *Environmental science &*  
495 *technology*.
- 496 (27) Bishop, G.A. Stedman, D.H., 2008. A decade of on-road emissions measurements.  
497 *Environmental Science & Technology*, 42(5), pp.1651-1656.
- 498 (28) Claudio Mazzoleni, Hampden D. Kuhns, Hans Moosmüller, Robert E. Keislar, Peter W.  
499 Barber, Norman F. Robinson, and John G. Watson, Djordje Nikolic. On-Road Vehicle  
500 Particulate Matter and Gaseous Emission Distributions in Las Vegas, Nevada, Compared with  
501 Other Areas. *J. Air & Waste Manage. Assoc.* 54:711–726, 2004.