A LINEAR EXPENDITURE SYSTEM ALLOWING FOR DIRECT SUBSTITUTABILITY OF VARIOUS FOODSTUFFS

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FOREWORD

Understanding the nature and dimensions of the world food problem and the policies available to alleviate it has been the focal point of the IIASA Food and Agriculture Program since it began in 1977.

National food systems are highly interdependent, and yet the major policy options exist at the national level. Therefore, to explore these options, it is necessary both to develop policy models for national economies and to link them together by trade and capital transfers. For greater realism the models in this scheme are being kept descriptive rather than normative. In the end it is proposed to link models to twenty countries, which together account for nearly 80 percent of important agricultural attributes such as area, production, population, exports, imports and so on.

In the course of the work on the development of models of centrally planned economies, the difficulties of estimating parameters of consumer behaviour in such economies where consumer markets cannot be assumed to be in equilibrium become apparent. Since the understanding of consumer behaviour is critically important in formulating plans and designing policies that facilitate the realization of plans, we have explored alternative approaches to this problem. In this paper Leon Podkaminer proposes a consumer demand system that may be universal in its parameters in which case it may be possible to use parameter estimates based on one country for describing the demand system of another.

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PREFACE

It is widely acknowledged that the Linear Expenditure System--or any system derived on the assumption of additive utility function--rules out direct substitution effects among the commodities. When applied to the study of the structure of demand for particular foodstuffs, any such demand study violates some of the legitimate intuition about actual substitutability of various food-stuffs: the similarity of nutritional content of any two (or more) otherwise "different" commodities does--to a great extent--seem to prejudge their actual substitutability.

The paper presents a modified linear expenditure system, which, having all the advantages of the simple LES at the same time allows for direct substitutability of particular food-stuffs. The theory, related to K. Lancaster's idea of the "new approach to the demand study" appears rather fruitful. The estimates of the parameters of the system for Italy, the Netherlands, West Germany and Great Britain obtained on its basis exhibit a rather unusually high statistical significance. At the same time these estimates seem very much the same for all the four countries studied.

ACKNOWLEDGEMENTS

Thanks are due to G. Kroemer who helped in many computations. Some of the data utilized in the estimations were worked out on the basis of the studies by K. Murty of the University of Goettingen, G. Jones of the University of Oxford and U. Sichra of IIASA. G. Fischer's (IIASA) program for the FIML non-linear estimation of the parameters of LES was used.

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I. Introduction

Food policy planning is of vital importance in many countries of the world. Besides the very poor countries where hunger persists, the theme is topical wherever the governmental policies try to regulate the agricultural production and yet, at the same time, keep the consumer satisfied. Also, it is the centrally planned economies, where the need for adequate food policy planning manifests itself with the greatest urgency.

For quite a long time the subject of food policy planning has been set solely in the context of dietary considerations that - at best - lead to the recommendations concerning the use of linear programming models for the determination of "best" diets.

While recognizing the role the knowledge of the "optimum" diets may play in determining the rationing schemes for really extreme emergencies, it seems quite obvious, that granted even limited freedom of choice the consumers tend to behave in a way that is not necessarily consistent with the dietary recommendations. Supplies of foodstuffs following the linear programming models for optimum diets - if tried to be distributed through the market - and not through a pretty stiff system of rationing - may therefore result in shortages. With some foodstuffs in short supply, the additional redistribution of some goods may then take place. Also, malnutrition may occur - this time not so much as an effect of income in equality as due to unequal access to the demanded foodstuffs.

The moral to this story seems fairly obvious. Food policy planning ought to allow for consumers preferences even in the situations characterized by seemingly low levels of the satisfaction of the basic nutritional needs. (McCarthy, 1980 has even detected the presence of what seems to be "snob" attitudes in the demand for food in Pakistan, where subsistence - level consumption patterns still persist).

II. Direct Use of the Linear Expenditure System

It is widely acknowledged that the Linear Expenditure System - or some of its already known (hopefully there are more to come) modifications such as Extended, Generalized, Twice Extended(all have "static" and "dynamic" versions) may be a reasonable approach for the description of the demand for "Food" regarded as a single commodity, with indirect substitutes defined in equally broad terms as say "Durables", or "Housing"). However, the theory that the mechanism for allocation of total expenditure on food among particular foodstuffs is described by another version of LES, with specific commercially available food items or groups of items treated as separate commodities does not seem convincing. The reason for this is the additivity of the Stone - Klein utility function underlying the LES which rules out the direct substitution effects among particular goods. This however contradicts some of the legitimate intuition about actual substitutability of various foodstuffs: the similarity of the nutritional content of two "otherwise" different goods may, to a great extent, prejudge their actual substitutability. (In particular, the substitutability of various cereals and starchy products cannot be reflected properly in any traditional LES).

This fact has two implications. First, the parameters of the LES to be determined via estimation procedures cannot be assumed to be statistically independent of the errors affecting particular equations. Hence, the estimates need not be consistent. Second, the application of the LES for the determination of actual demands (or equilibrium prices) must ignore the possibility of substitution. In particular any such analysis would have to lead to the conclusion that the demand for any specific good may not fall below the respective subsistence level - whatever the price of the good's natural substitutes. Hence, with the supply of a good approaching respective subsistence demand for it, its price should--in a LES world--go to infinity, whatever the prices of the nutritional substitutes are.

III. Indirect Use of the LES : Preliminaries

The proposed modification follows the idea presented by Lancaster, 1971. It is assumed that goods (foodstuffs) are "intermediaries which can produce the properties or characteristics in which consumer is actually interested". It is proposed to identify these characteristics with the amounts of particular nutrients provided by the foodstuffs consumed. In other words, the consumer's utility function $u(q_1,q_2,\ldots q_n)$ defined over the set of foodstuff's bundles $(q_1,q_2,\ldots,q_n)$ is assumed to be of the following form:

$$u(q_1,q_2,\ldots,q_n) = U(\sum_{j=i}^{n} a_{1j}q_j, \sum_{j=i}^{n} a_{2j}q_j, \ldots, \sum_{j=i}^{n} a_{mj}q_j) \quad (1)$$

where m is the number of nutrients taken into account and a_{ij} is the amount of nutrient i contained in the unit of j - th foodstuff and $U(v_1,v_2,\ldots,v_m)$ is a utility function defined over the set of vectors of nutritional characteristics $(v_1,v_2,\ldots,v_m)$ of any diet.

In other words

$$u(q_1,q_2,\ldots,q_n) = U(v_1,v_2,\ldots,v_m) \quad (2)$$

where

$$v_i = \sum_{j=i}^{n} a_{ij}q_j \qquad i = 1,2,\ldots m$$

is the total intake of i-th nutrient provided by the diet $(q_1,q_2,\ldots,q_n)$. It is worth noting that $u(q_1,q_2,\ldots,q_n)$ has to be a utility function if $U(v_1,v_2,\ldots,v_m)$ is a utility function and none of the parameters a_{ij} is negative[1]. (Still, with some negative a_{ij}, u may happen to be a utility function too).

IV. Demand System Following Stone - Geary Utility Function U

The demand system following the utility function (1) is derived from the maximization of the utility function subject to the budget constraint:

$$\text{maximize } u(q_1,q_2,\ldots,q_n) \quad (3)$$

$$\text{s.t. } \sum_{j=1}^{n} p_j q_j = y \quad (4)$$

where y is the total expenditure for food, p_j is the price of j-th food item.

Under the assumption that utility function U is of the Stone - Geary type, the constrained maximization (3) - (4) is equivalent to the following non-linear optimization problem;

[1]If $a_{ij} > o$, $\frac{\partial U}{\partial v} > o$, $\frac{\partial}{\partial v}(\frac{\partial U}{\partial v})$ is negative definite and $A = \{a_{ij}\}$

then

$$\frac{\partial u}{\partial q} = A \frac{\partial U}{\partial v}\Big|_{v = Aq} > o \qquad \text{and} \qquad \frac{\partial}{\partial q}(\frac{\partial u}{\partial q}) = A\frac{\partial}{\partial v}(\frac{\partial U}{\partial v})\Big|_{v = Aq} A^T$$

is negative definite too.

$$\text{maximize} \sum_{i=1}^{m} \beta_i \ln(v_i - \bar{v}_i) \tag{5}$$

$$\text{s.t.} \sum_{j=1}^{n} p_j q_j = y \tag{6}$$

$$\text{and } v_i = \sum_{j=1}^{n} a_{ij} q_j \qquad i = 1,2,\ldots,m \tag{7}$$

where $\bar{v}_j$ may be interpreted as the subsistence intake of j-th nutrient. The interpretation of β_i's is not, for the time being, very easy to present. Nevertheless, for obvious reasons it should be postulated that

$$\beta_i > o \quad \text{and} \quad \sum_{i=1}^{m} \beta_i = 1$$

The demand system following (5), (6), (7) is given by the following equations:

$$\beta_i - (\sum_{j=1} a_{ij} q_j - \bar{v}_i) \sum_{k=1}^{m} \mu_k = o \qquad i = 1,2,\ldots,m \tag{8}$$

and

$$p_j \sum_{l=1}^{n} \sum_{k=1}^{m} \mu_k a_{kl} = y \sum_{k=1}^{m} \mu_k a_{kj} \qquad j = 1,2,\ldots,n \tag{9}$$

where μ_k, $k=1,2,\ldots,m$ are Lagrange multipliers.

The stochastic counterpart to (8) - (9) would therefore be

$$\beta_i - (\sum_{j=i}^{m} a_{ij} q_{jt} - \bar{v}_i) \sum_{k=1}^{m} \mu_{kt} = \varepsilon_{it} \qquad i = 1,2,\ldots,m \tag{8'}$$

and

$$p_{jt} \sum_{l=1}^{n} \sum_{k=1}^{m} \mu_{kt} a_{kl} = y_t \sum_{k=1}^{m} \mu_{kt} a_{kj} = \eta_{jt} \quad j = 1,2,\ldots,m \tag{9'}$$

where t indexes the observations and ε_{it}, η_{jt} are unobservable random terms.

Since μ_{kt} are also unobservable the system (8'), (9') is not - in general - capable of being statistically estimated.

V. Identification of (8) - (9) and Estimation of Parameters under Nutritional Completeness of the Collection of Foodstuffs

To operationalize the concept it is necessary to accept the following assumption:

$$\det \{a_{ij}\} \neq 0 \tag{10}$$

(10) is equivalent to the assumption that the number of separate foodstuffs considered is equal to the number of nutrients while no foodstuff is nutritionally a perfect mix of other foodstuffs.

Under (10) the following equation holds:

$$q_j = \sum_{i=1}^{m} A_{ij} V_i \qquad j = 1,2,\dots,m$$

or $$q = A^{-1}v \tag{11}$$

where $A = \{a_{ij}\}$ and $\{A_{ij}\} = A^{-1}$

The problem (5), (6), (7) may now be restated in the following manner:

$$\text{maximize} \sum_{i=1}^{m} \beta_i \ln (v_i - \bar{v}_i) \tag{12}$$

$$\text{s.t.} \sum_{i=1}^{m} p_i \left(\sum_{j=1}^{m} A_{ij} v_j\right) = pA^{-1}v = y \tag{13}$$

The demand system following (12) - (13) has the following form:

$$\tilde{p}_i v_i = \tilde{p}_i \bar{v}_i + \beta_i (y - \sum_{i=1}^{m} \tilde{p}_i \bar{v}_i) \qquad i = 1,2,\dots,m \tag{14}$$

where

$$\tilde{p} = pA^{-1} \tag{15}$$

and

$$v = Aq$$

The $\tilde{p}_i$ may be interpreted as the imputed price of the unit of i-th nutrient.

The stochastic counterpart of (14) has the following form:

$$\tilde{p}_{it} v_{it} = \tilde{p}_{it} \bar{v}_i + \beta_i (y_t - \sum_{i=1}^{m} \tilde{p}_{it} \bar{v}_i) + \varepsilon_{it} \quad i=1,2,\dots,m, \tag{16}$$

where

$$\tilde{p}_{it} = \sum_{j=1}^{m} p_{jt} A_{ji} \text{ and } v_{it} = \sum_{j=1}^{m} q_{jt} a_{ij}$$

The estimation of the parameters $\beta_1, \beta_2, \ldots, \beta_m$ and $\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_m$ in (16) does not entail any difficulty - above those present in the standard LES context.

VI. Wisdom of the Assumption (10)

The number of essential nutrients, including minerals, vitamins, fats and aminoacids necessary for a survival of a human being seems to exceed the number of nutritionally distinct food commodities (for which price and consumption statistics may be available) by several times. Moreover, the digestible nutritional contents of some foodstuffs may vary - at least with respect to some nutrients (Scrimshaw and Vernon, 1978). At a very fine level of nutritional precision the approach may therefore be not operational. However, with a more aggregate approach one could expect the approach to be operational. In actual fact the provisional checks suggest that some reasonable commodity lists comprising 15 - 20 basic foodstuffs combined with apparently reasonable lists of basic nutrients produce non-singular matrices A.

VII. Expected Result of the Estimation: A Reconciliation of Nutritional and Economic Factors in the Demand for Foodstuffs

The estimation of the parameters $\beta_i, \bar{v}_i$ should result in the specific "nutritional utility functions". While β_i's express marginal budget shares of expenditure spent on i-th nutrient, v_i expresses nothing but a "nutritional" minimum for the intake of i-th nutrient.

There seems to exist an additional possibility of checking whether the hypothesis about nutritional sources of the observable consumers behaviour is not false. The parameters of the nutritional utility functions estimated for various consumer groups which do not differ with respect to anthropological factors (height, weight, ability to synthesize some vitamins) and are subject to approximately the same environmental conditions (climate, intensity of physical work) may not exhibit too much of a difference. (Or, the differences should not be statistically significant). In particular, the estimates coming from international comparisons within pretty homogenous regions of the world (Northern and Central Europe, South-East Asia) should be reasonably close to one another.

Should the theory stand the empirical tests, one could draw workable conclusions not only for the countries that are included in the comparative study. For, it would be also possible to predict the demand patterns in the countries that either do not have reliable statistics of prices and consumption of foodstuffs, or do not operate under the conditions of market equilibria. In the latter case the recorded data on prices and consumption of foodstuffs cannot serve as a basis for any meaningful econometric estimation of any demand system (Podkaminer 1981).

Appendix 1: A Possibility of Simplifications in the Estimation Procedure.

It is not unreasonable to suppose that the "nutritional minima" to be estimated ($\bar{v}_i$) correspond to the ones the nutritional science suggests. The usual estimation procedure for LES which requires the application of the Full Information Non-Linear Maximum Likelihood starts with some provisional values for subsistence levels. By identifying these starting points with the nutritionalists' estimates for the minima one can speed up the algorithms - or even secure a chance for their actual convergence.

There is also another possibility worth attempting. Under the assumption that the nutritional minima $\bar{v}_i$ are equal to some nutritional standards, the estimation of β_i's boils down to a fairly simple model of multivariate analysis.

Appendix 2: Preliminary Estimates of the Aggregate Linear Expenditure Systems Allowing for Direct Substitutability of Main Groups of Foodstuffs for West Germany, the Netherlands, Italy and the United Kingdom.

The theory presented above has been put into a preliminary empirical testing. On the basis of the data on per capita consumption of major groups (aggregates) of foodstuffs in the EEC countries over the period 1961-1976, which were worked out by K. Murty of the University of Göthingen it has been possible to conclude time series of p.c. quantities consumed and corresponding price indexes of 6 "foodstuffs" for West Germany, the Netherlands, Italy and the United Kingdom*.

The "foodstuffs" in question are: cereals and bakery products, eggs, meat, dairy products, fish, all other food-stuffs. The nutritional unit contents of these "foodstuffs", computed by U. Sichra of IIASA comprise the following 11 characteristics: calories, low-value protein, high-value protein, fat, calcium, iron, vitamin A, thiamine, riboflavine, niacine, vitamin C. Since it is necessary to have the number of nutrients equal to the number of commodities the problem of selecting a collection

* The statistical data for the U.K. was collected with the help of G. Jones of the University of Oxford.

of the 6 out of 11 nutrients arises. Although it is imaginable to examine all 11!/6!·5! combinations of nutrients, the research reported allows for only 12 combinations that are (subjectively) thought most plausible.

Because the composition of particular "foodstuffs" is year--and country--specific (e.g. "meat" consists of varying quantities of pork, beef, etc.) the matrices A are also year-and country-specific. So, for each country, year and combination a separate matrix A is defined. All of them appear non-singular so that the determination of corresponding per capita intakes of the nutrients and their imputed prices does not entail any difficulty--though it does require an impressive amount of a specialized computer software (for which thanks are due to G. Kroemer of IIASA).

The proper estimation of the LES parameters was done by the application of the computer program worked out by G. Fischer of IIASA, which was run with the help of N. Narayana of IIASA.

1. There is a striking similarity of the estimates obtained for all countries considered. The parameters of the utility functions for any given combination of nutrients are practically almost the same for all countries. Additionally, even the coefficients characterizing the statistical quality of the estimates (t-statistics, $\bar{R}^2$, D-W) are--for a given combination--similar for all countries.

2. The statistical quality of the parameters of the utility functions for some combinations of nutrients is amazingly high. The values of the testing statistics obtained are typical of biological rather than economic research. Even the poorest estimates compare favorably with many pieces of econometric research that can be found in numerous demand studies.

3. The estimated parameters for some nutrients tend to take certain numerical values irrespective of the combination of the nutrients in which they appear. This, of course, constitutes a clear-cut guidance as to the final choice to be made in practical applications.

4. Although the possibility of a bias due to the imperfect operation of the estimation program may--probably--not be entirely ruled out, the results obtained seem to corroborate the hypothesis concerning the (independent of the country) "nutritional" sources of the observed behaviour of the consumer. There is little doubt that <u>any</u> of our estimated utility functions is rather superior to the estimated straight additive Stone-Geary utility functions (whether static or dynamic) for the same contexts. (Not only statistical quality seems to matter here, but also the fact that the parameters of the estimated

straight Stone-Geary utility functions for apparently very similar countries do not exhibit much of a resemblance).

5. The experiments reported seem to justify additional research. One can think of disaggregating some of the foodstuffs so as to be in a position to allow for more than 6 nutrients at one time. At the same time it may be deemed expedient to extend the list of countries to be examined. Additionally it may be worthwhile trying to select the combination of nutrients on the basis of the ability of the estimates to explain the observed demand patterns for other countries and (or) more specific foodstuffs not allowed for in the original research.

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DOCUMENTATION

The following tables are a documentation of the estimation of the linear expenditure systems allowing for nutritional substitutability of various foodstuffs for West Germany, the Netherlands, Italy and the United Kingdom.

Estimates of the parameters of the utility functions, 1st combination

Country	Nutrients											
	Calories		Crude Protein/grams		High Protein/grams		Fat grams		Calcium m.grams		Iron m.grams	
	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$
Federal Rep. of Germany	0.685 (385)	0.107 (3.13)	0.02 (264)	0 n.c.	0.011 (399)	0 n.c.	0.029 (86.5)	0 n.c.	0.249 (129)	0.130 (4.5)	res n.c.	.00007 (1.3)
Netherlands	0.674 (110)	0.003 (1.46)	0.02 (132)	0 n.c.	0.011 (941)	0 n.c.	0.026 (46)	0 n.c.	0.263 (43)	0.02 (1.7)	res n.c.	0 n.c.
Italy	0.742 (1668)	0 n.c.	0.02 (340)	0 n.c.	0.011 (890)	0.001 (1.01)	0.023 (43)	0 n.c.	0.195 (299)	0 n.c.	res n.c.	.00005 (2.07)
United Kingdom	0.671 (296)	0 n.c.	0.02 (145)	0.11 (0.6)	0.011 (122)	0 n.c.	0.026 (164)	0 n.c.	0.264 (138)	0 n.c.	res n.c.	.00005 (0.05)

t-statistics in parenthesis. n.c. = not computed (estimate on the bound or program's nicety) res. = residual $(1 - \sum_{i=1}^{5} \beta_i)$

Statistical quantity of the fit, 1st combination

Country	Equation for											
	Calories		Crude Protein		High Protein		Fat		Calcium		Iron	
	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W
Federal Rep. of Germany	0.999	2.08	0.999	1.07	0.999	1.95	0.992	0.23	0.999	1.66	0.999	2.16
Netherlands	0.998	0.72	0.999	0.71	0.999	2.26	0.993	0.63	0.992	0.65	0.993	0.82
Italy	1.000	1.16	0.999	1.72	0.999	2.36	0.991	1.04	0.997	0.88	0.995	0.39
United Kingdom	0.999	0.81	0.999	2.39	0.999	1.66	0.999	2.26	0.998	0.84	0.995	1.14

Estimates of the parameters of the utility functions, 2nd combination

Country	Nutrients											
	Calories		High Protein grams		fat grams		Calcium m grams		Iron m grams		Vitamin A Int. Units	
	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$
Federal Rep. of Germany	0.36 (322)	0.02 (0.66)	0.006 (197)	0 n.c.	0.016 (99)	0 n.c.	0.13 (63)	0.46 (3.85)	0.002 (201)	0 n.c.	res n.c.	0 (1.82)
Netherlands	0.31 (66)	0.04 (1.7)	0.005 (53)	0 (1.37)	0.012 (32)	0 n.c.	0.12 (45)	0.006 (0.3)	0.002 (51)	0 n.c.	res. n.c.	0 n.c.
Italy	0.37 (104)	0 n.c.	0.005 (54)	0 (0.65)	0.011 (101)	0 n.c.	0.09 (36)	0.06 (0.43)	0.003 (26)	0 (2.2)	res. n.c.	1.16 (0.9)
United Kingdom	0.36 (35)	4.43 (2.9)	0.006 (60)	0 n.c.	0.015 (69)	0 n.c.	0.15 (25)	28.4 (0.03)	0.002 (29)	0.01 (0.01)	res. n.c.	18.4 (0.002)

t-statistics in parenthesis. n.c. = not computed (estimate on the bound or program's nicety)

Statistical quantity of the fit, 2nd combination

Country	Equation for											
	Calories		High Protein		Fat		Calcium		Iron		Vitamin A	
	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W
Federal Rep. of Germany	0.999	2.69	0.999	1.70	0.999	1.01	0.999	1.11	0.999	2.60	0.999	2.50
Netherlands	0.994	0.70	0.997	1.07	0.987	0.56	0.996	1.22	0.992	1.02	0.999	0.81
Italy	0.997	0.67	0.997	1.72	0.999	1.72	0.993	0.86	0.990	0.83	0.998	0.90
United Kingdom	0.997	0.09	0.999	1.11	0.999	1.56	0.999	0.79	0.999	0.85	0.999	1.31

Estimates of the parameters of the utility functions, 3rd combination

Country	Nutrients											
	High protein grams		Fat grams		Calcium m. grams		Iron m. grams		Vitamin A int. units		Thiamine m. grams	
	β	$\bar{\bar{v}}$	β	$\bar{\bar{v}}$	β	$\bar{\bar{v}}$	β	$\bar{\bar{v}}$	β	$\bar{\bar{v}}$	β	$\bar{\bar{v}}$
Federal. Rep. of Germany	0.009 (183)	0 n.c.	0.024 (146)	0 n.c.	0.20 (234)	0.001 0.97	0.003 (1005)	0 n.c.	0.75 (930)	0.05 (0.74)	res. n.c.	0 n.c.
Netherlands	0.007 (55)	0 n.c.	0.015 (11.6)	.0005 (2.1)	0.17 (101)	0 n.c.	0.003 (55)	0 n.c.	0.80 (430)	0.02 (1.15)	res. n.c.	0 n.c.
Italy	0.008 (107)	0 n.c.	0.017 (160)	0 n.c.	0.16 (89)	0.001 1.08	0.005 (46)	0 n.c.	0.80 (450)	0.11 0.59	res n.c.	0 n.c.
United Kingdom	0.010 (20.9)	0 n.c.	0.021 (21)	0 n.c.	0.23 (26)	417 (0.9)	0.003 (19.3)	0.005 (1.5)	0.73 (79.7)	143 (0.27)	res n.c.	0 n.c.

t-statistics in parenthesis. n.c. = not computed (estimate on the bound or program's nicety)

Statistical quantity of the fit, 3rd combination

Country	Equation for											
	High Protein		Fat		Calcium		Iron		Vitamin A		Thiamine	
	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W
Federal Rep. of Germany	0.999	2.67	0.999	1.93	0.999	3.04	0.999	2.40	0.999	2.78	0.999	1.83
Netherlands	0.998	1.27	0.995	1.26	0.999	1.53	0.998	1.67	0.999	1.95	0.996	1.41
Italy	0.999	2.27	0.999	1.16	0.998	1.89	0.995	1.46	0.999	1.89	0.974	2.09
United Kingdom	0.998	0.40	0.997	0.60	0.998	0.68	0.998	1.12	0.999	0.56	0.998	0.61

Estimates of the parameters of the utility functions, 4th combination

Country	Nutrients											
	Calories		Crude protein grams		High protein grams		Calcium mgr.		Iron mgr.		Niacine mgr.	
	β	$\bar{\bar{v}}$	β	$\bar{\bar{v}}$	β	$\bar{\bar{v}}$	β	$\bar{\bar{v}}$	β	$\bar{\bar{v}}$	β	$\bar{v}$
Federal Rep. of Germany	0.69 (128)	0.04 (1.51)	0.02 (104)	0 n.c.	0.01 (90)	.00002 (1.31)	0.26 (52)	0.24 (6.8)	0.004 (30)	.00005 (3.69)	res. n.c.	.00005 (1.6)
Netherlands	0.66 (82)	0 n.c.	0.02 (86)	0 n.c.	0.01 (108)	.00004 (1.96)	0.27 (31)	0.2 (2.8)	0.005 (56)	.00003 (7.8)	res. n.c.	0 n.c.
Italy	0.72 (642)	0 n.c.	0.02 (523)	0.0001 (1.12)	0.01 (129)	0 n.c.	0.19 (177)	0 n.c.	0.006 (25)	.00001 (3.24)	res. n.c.	0 n.c.
United Kingdom	0.67 (737)	998 (33.7)	0.02 (1423)	0.48 (0.5)	0.01 (102)	0.01 0.04	0.27 (297)	0 n.c.	0.0004 (57)	0.001 (0.02)	res n.c.	0 n.c.

t-statistics in parenthesis. n.c. = not computed (estimate on the bound or program's nicety)

Statistical quantity of the fit, 4th combination

Country	Equation for											
	Calories		Crude protein		High protein		Calcium		Iron		Niacine	
	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W
Federal Rep. of Germany	0.998	2.19	0.999	1.23	0.999	1.69	0.997	2.32	0.999	1.64	0.999	1.50
Netherlands	0.999	1.23	0.999	0.85	0.999	0.44	0.991	0.43	0.997	1.09	0.995	1.77
Italy	0.998	0.57	0.999	1.40	0.999	0.36	0.993	0.86	0.989	0.80	0.998	1.09
United Kingdom	0.999	0.95	1.000	1.59	0.999	1.52	0.998	0.92	0.997	1.11	0.999	0.96

Estimates of the parameters of the utility functions, 5th combination

Country	Nutrients											
	High protein grams		Fat grams		Calcium mgr.		Iron mgr.		Riboflavine mgr.		Niacine mgr.	
	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$
Federal Rep. of Germany	0.033 (45)	.00002 (6.9)	0.10 (70)	0 n.c.	0.832 (520)	0 n.c.	0.014 (199)	0 n.c.	0.001 (118)	0 n.c.	res n.c.	.00001 (2.71)
Netherlands	0.035 (92)	.00002 (1.56)	0.09 (36)	0 n.c.	0.833 (305)	0 n.c.	0.013 (107)	0 n.c.	0.001 (77)	0 n.c.	res n.c.	0 n.c.
Italy	0.041 (62)	.00002 (2.5)	0.08 (18)	0.0003 (2.1)	0.826 (199)	0.0005 (0.4)	0.023 (122)	.00006 (1.03)	0.002 (54)	0 n.c.	res n.c.	0 n.c.
United Kingdom	0.038 (186)	0.12 (0.11)	0.085 (98)	0.50 (0.3)	0.847 (847)	0 n.c.	0.012 (113)	0 n.c.	0.002 (66)	0 n.c.	res. n.c.	0.03 (0.5)

t-statistics in parenthesis. n.c. = not computed (estimate on the bound or program's nicety)

Statistical quantity of the fit, 5th combination

Country	Equation for											
	High protein		Fat		Calcium		Iron		Riboflavine		Niacine	
	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W
Federal Rep. of Germany	0.996	2.31	0.956	0.18	0.999	0.23	0.999	1.38	0.987	0.38	0.999	1.32
Netherlands	0.999	1.23	0.980	1.96	0.999	1.60	0.999	1.37	0.998	2.18	0.993	1.63
Italy	0.998	0.73	0.922	0.35	0.999	0.55	0.999	1.47	0.995	0.35	0.999	1.55
United Kingdom	0.999	1.49	0.999	1.22	0.999	1.13	0.987	0.97	0.969	1.11	0.986	1.41

Estimates of the parameters of the utility functions, 6th combination

Country	Nutrients											
	High protein grams		Fat grams		Calcium mgr		Vitamin A int. units		Thiamine mgr		Riboflavine mgr	
	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$
Federal. Rep. of Germany	0.009 (69)	.00001 (3.75)	0.016 (12.6)	0.0003 (7.66)	0.207 (325)	0 n.c.	0.766 (1073)	1.91 (5.54)	0.003 (138)	0 n.c.	res n.c.	0 n.c.
Netherlands	0.007 (35)	.00001 (9.8)	0.014 (5.4)	0.0003 (1.9)	0.174 (117)	0 n.c.	0.804 (317)	0 n.c.	0.003 (87)	0 n.c.	res n.c.	0 n.c
Italy	0.008 (28)	.00003 (0.17)	0.017 (11)	0.0002 (0.9)	0.15 (233)	0 n.c.	0.82 (651)	0.51 0.51	0.004 (44)	0 n.c.	res. n.c.	0 n.c.
United Kingdom	0.010 (62)	0 n.c.	0.022 (70)	0 n.c.	0.221 (78)	0 n.c.	0.745 (261)	0 n.c.	0.003 (78)	0 n.c.	res. n.c.	0.0001 (1.48)

t-statistics in parenthesis. n.c. = not computed (estimate on the bound or program's nicety)

Statistical quantity of the fit, 6th combination

Country	Equation for											
	High protein		Fat		Calcium		Vitamin A		Thiamine		Riboflavine	
	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W
Federal Rep. of Germany	0.999	2.88	0.781	1.01	0.999	1.88	0.999	1.47	0.999	2.02	0.999	2.01
Netherlands	0.994	2.59	0.796	0.63	0.995	1.37	0.999	1.75	0.995	1.94	0.989	0.91
Italy	0.992	1.55	0.951	0.54	0.987	0.32	0.999	2.1	0.996	1.00	0.950	0.48
United Kingdom	0.973	0.38	0.985	0.37	0.974	0.35	0.993	0.30	0.985	0.60	0.971	1.54

Estimates of the parameters of the utility functions, 7th combination

Country	Nutrients											
	Calories		High protein grams		Iron mgr.		Thiamine mgr		Riboflavine mgr.		Vitamin C mgr.	
	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$
Federal Rep. of Germany	0.94	0	0.015	0	0.005	.00001	0.0005	0	0.0005	0	res	0.0001
	(4888)	n.c.	(333)	n.c.	(263)	(2.32)	(201)	n.c.	(182)	n.c.	n.c.	(2.5)
Netherlands	0.94	0	0.015	0	0.006	0	0.0006	0	0.0006	0	res	0
	(8872)	n.c.	(152)	n.c.	(644)	n.c.	(3.70)	n.c.	(231)	n.c.	n.c.	n.c.
Italy	0.92	0	0.012	.0001	0.007	.00003	0.0007	0	0.0006	0	res	0
	(1932)	n.c.	(84)	(2.5)	(105)	(2.5)	(30)	n.c.	(64)	n.c.	n.c.	n.c.
United Kingdom	0.94	0	0.016	0	0.005	0.04	0.0005	0.0005	0.0008	0.002	res.	0
	(11676)	n.c.	(301)	n.c.	(133)	(0.46)	(113)	(0.6)	(38)	(0.8)	n.c.	n.c.

t-statistics in parenthesis. n.c. = not computed (estimate on the bound or program's nicety)

Statistical quantity of the fit, combination

Country	Equation for											
	Calories		High protein		Iron		Thiamine		Riboflavine		Vitamin C	
	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W
Federal Rep. of Germany	1000	0.74	0.999	2.47	0.999	2.88	0.999	1.14	0.997	1.75	0.999	1.26
Netherlands	1000	0.93	0.999	1.49	0.997	1.53	0.999	1.63	0.999	1.84	0.999	0.90
Italy	0.999	1.25	0.998	0.88	0.999	1.01	0.992	1.71	0.986	0.83	0.998	1.02
United Kingdom	0.999	1.32	0.998	0.98	0.999	1.21	0.999	1.14	0.995	1.44	0.999	1.91

Estimates of the parameters of the utility functions, 8th combination

Country	Nutrients											
	High protein grams		Calcium mgr.		Iron mgr.		Vitamin A int. units		Thiamine mgr.		Riboflavine mgr.	
	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$
Federal Rep. of Germany	0.009 (91)	.00006 n.c.	0.21 (10.3)	0 n.c.	0.003 (76)	0 n.c.	0.77 (355)	0.97 (7.4)	0.0003 (78)	0 n.c.	res n.c.	0 n.c.
Netherlands	0.006 (23)	.00007 (3.2)	0.17 (129)	0 n.c.	0.003 (13)	0 n.c.	0.81 (581)	0 n.c.	0.0003 (61)	0 n.c.	res. n.c.	0 n.c.
Italy	0.008 (28)	.00008 (0.1)	0.15 (161)	0 n.c.	0.004 (35)	0 n.c.	0.83 (839)	0 n.c.	0.0004 (25)	0 n.c.	res n.c.	0 n.c.
United Kingdom	0.010 (23)	0 n.c.	0.23 (22)	0 n.c.	0.004 (19)	0.06 (1.6)	0.75 (68)	409 (0.7)	0.0003 (24)	0 n.c.	res n.c.	0 n.c.

t-statistics in parenthesis. n.c. = not computed (estimate on the bound or program's nicety)

Statistical quantity of the fit, 8th combination

Country	Equation for											
	High protein		Calcium		Iron		Vitamin A		Thiamine		Riboflavine	
	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W
Federal Rep. of Germany	0.999	2.78	0.999	1.67	0.947	2.70	0.999	2.28	0.970	1.87	0.999	1.79
Netherlands	0.987	2.05	0.996	1.41	0.959	0.86	0.999	1.41	0.978	1.55	0.987	0.79
Italy	0.990	1.60	0.999	1.90	0.849	0.31	0.999	0.61	0.787	1.49	0.998	1.53
United Kingdom	0.978	0.56	0.997	1.34	0.985	1.12	0.994	0.34	0.984	0.86	0.997	0.71

Estimates of the parameters of the utility functions, 9th combination

Country	Nutrients											
	High protein grams		Calcium mgr.		Iron mgr		Vitamin A int. units		Riboflavine mgr.		Niacine mgr.	
	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$
Federal Rep. of Germany	0.009 (67)	.00008 (1.3)	0.19 (57)	0 n.c.	0.003 (49)	0 n.c.	0.79 (221)	0.62 (1.84)	0.0003 (63)	0 n.c.	res n.c.	0 n.c.
Netherlands	0.007 (28)	.00005 (2.5)	0.17 (129)	0 n.c.	0.003 (16.4)	.00005 (2.46)	0.81 (579)	0 n.c.	0.0003 (92)	0 n.c.	res n.c.	.00006 (2.6)
Italy	0.008 (119)	0 n.c.	0.16 (85)	0 n.c.	0.005 (9.0)	.00005 (0.67)	0.82 (444)	0.28 (0.43)	0.003 (65)	0 n.c.	res n.c.	0 n.c.
United Kingdom	0.010 (51)	0 n.c.	0.24 (75)	897 (6.4)	0.004 (21.2)	0.24 (2.6)	0.74 (238)	0 n.c.	0.0005 (79)	0 n.c.	res n.c.	0 n.c.

t-statistics in parenthesis. n.c. = not computed (estimate on the bound or program's nicety)

Statistical quantity of the fit, 9th combination

Country	Equation for											
	High protein		Calcium		Iron		Vitamin A		Riboflavine		Niacine	
	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W
Federal Rep. of Germany	0.999	1.48	0.991	0.23	0.935	1.14	0.998	0.24	0.997	2.21	0.972	2.27
Netherlands	0.991	2.15	0.998	1.46	0.974	1.27	0.999	1.67	0.999	0.74	0.904	0.77
Italy	0,999	1.90	0.999	1.74	0.922	0.46	0.999	0.61	0.998	1.76	0.968	0.85
United Kingdom	0.967	0.37	0.996	1.64	0.984	1.13	0.999	1.64	0.996	0.59	0.981	0.68

Estimates of the parameters of the utility functions, 10th combination

Country	High protein grams		Calcium mgr.		Iron mgr.		Vitamin A int. units		Riboflavine mgr.		Vitamin C mgr.	
	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$
Federal Rep. of Germany	0.009 (668)	.00004 (1.09)	0.196 (93)	0 n.c.	0.003 (83)	0 n.c.	0.76 (365)	0 n.c.	.0003 (121)	0 n.c.	res n.c.	0 n.c.
Netherlands	0.007 (34)	.00008 (2.1)	0.178 (61)	0.01 (2.9)	0.003 (47)	.00006 (2.5)	0.79 (263)	0 n.c.	.0003 (46)	0 n.c.	res n.c.	.0002 (1.01)
Italy	0.008 (31)	0 n.c.	0.147 (31)	0 n.c.	0.005 (11)	.0003 (1.88)	0.80 (136)	0.66 (0.9)	.0003 (21)	0 n.c.	res n.c.	0 n.c.
United Kingdom	0.010 (61)	0 n.c.	0.227 (98)	0 n.c.	0.003 (32)	1.28 (1.86)	0.74 (320)	0 n.c.	.0005 (65)	0 n.c.	res n.c.	0 n.c.

t-statistics in parenthesis. n.c. = not computed (estimate on the bound or program's nicety)

Statistical quantity of the fit, 10th combination

Country	High protein		Calcium		Iron		Vitamin A		Riboflavine		Vitamin C	
	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W
Federal Rep. of Germany	0.999	1.65	0.940	0.19	0.986	0.74	0.990	0.27	0.984	1.48	0.955	0.90
Netherlands	0.994	2.13	0.999	2.02	0.997	1.09	0.999	1.15	0.998	1.03	0.997	1.24
Italy	0.998	1.31	0.959	0.87	0.948	0.78	0.998	0.97	0.819	0.21	0.981	1.67
United Kingdom	0.969	0.35	0.991	0.64	0.993	1.1	0.999	0.71	0.990	0.71	0.995	0.93

Estimates of the parameters of the utility functions, 11th combination

Country	Nutrients											
	Calcium mgr.		Iron mgr.		Vitamin A int. units		Thiamine mgr		Riboflavine mgr.		Niacine mgr.	
	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$	β	$\bar{v}$
Federal Rep. of Germany	0.24 (14)	0.0005 (1.92)	0.003 (43)	0 n.c.	0.74 (42)	0 n.c.	.0003 (51)	0 n.c.	.0003 (42)	0 n.c.	res n.c.	0 n.c.
Netherlands	0.20 (35)	0.073 (3.05)	0.003 (51)	0 n.c.	0.80 (145)	0.62 (2.2)	.0003 (51)	0 n.c.	.0003 (52)	0 n.c.	res n.c.	.00005 (2.8)
Italy	0.16 (99)	0 n.c.	0.005 (31)	0 n.c.	0.83 (524)	0 n.c.	.0005 (23)	0 n.c.	.0003 (35)	0 n.c.	res. n.c.	0 n.c.
United Kingdom	0.22 (149)	0 n.c.	0.003 (104)	0.2 (1.22)	0.77 (530)	0 n.c.	.0003 (77)	0 n.c.	.0005 (53)	0 n.c.	res. n.c.	0 n.c.

t-statistics in parenthesis. n.c. = not computed (estimate on the bound or program's nicety)

Statistical quantity of the fit, 11th combination

Country	Equation for											
	Calcium		Iron		Vitamin A		Thiamine		Riboflavine		Niacine	
	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W
Federal Rep. of Germany	0.781	0.07	0.996	0.74	0.999	2.21	0.999	1.59	0.950	0.98	0.997	2.68
Netherlands	0.984	1.59	0.999	0.99	0.999	2.14	0.988	1.03	0.982	1.38	0.971	1.21
Italy	0.998	1.57	0.996	0.99	0.999	1.20	0.963	0.84	0.982	0.71	0.974	0.78
United Kingdom	0.976	0.23	0.999	1.61	0.999	1.11	0.984	0.59	0.979	0.59	0.981	0.81

Estimates of the parameters of the utility functions,12th combination

Country	Nutrients											
	Calcium mg		Iron mgr		Vitamin A int. units		Thiamine mgr.		Riboflavine mgr.		Vitamin C mgr.	
	β	$\bar{\bar{v}}$	β	$\bar{\bar{v}}$	β	$\bar{\bar{v}}$	β	$\bar{\bar{v}}$	β	$\bar{\bar{v}}$	β	$\bar{\bar{v}}$
Federal. Rep. of Germany	0.23 (57)	0.05 (7.6)	0.003 (124)	0 n.c.	0.74 (186)	0 n.c.	0.0003 (151)	0 n.c.	0.0003 (84)	0 n.c.	res. n.c.	.0001 0.97
Netherlands	0.18 (45)	0.016 (2.7)	0.003 (47)	0 n.c.	0.79 (203)	0.18 (2.87)	0.0003 (51)	0 n.c.	0.0003 (46)	0 n.c.	res. n.c.	0 n.c.
Italy	0.15 (95)	0 n.c.	0.005 (174)	0 n.c.	0.81 (497)	0 n.c.	0.0005 (21)	0 n.c.	0.0003 (29)	0 n.c.	res. n.c.	0 n.c.
United Kingdom	0.24 (38)	181 (2.32)	0.003 (15.1)	0.3 (0.6)	0.73 (120)	0 n.c.	0.0003 (59)	0 n.c.	0.0005 (43)	0 n.c.	res. n.c.	0 n.c.

t-statistics in parenthesis. n.c. = not computed (estimate on the bound or program's nicety)

Statistical quantity of the fit,12th combination

Country	Equation for											
	Calcium		Iron		Vitamin A		Thiamine		Riboflavine		Vitamin C	
	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W	$\bar{R}^2$	D-W
Federal Rep. of Germany	0.996	2.43	0.999	0.86	0.999	2.86	0.999	1.59	0.989	0.66	0.995	1.70
Netherlands	0.992	1.34	0.994	1.30	0.999	1.43	0.993	1.19	0.985	1.04	0.996	2.47
Italy	0.976	0.92	0.998	1.66	0.999	0.33	0.973	1.42	0.907	0.22	0.988	1.86
United Kingdom	0.991	1.04	0.970	0.37	0.997	0.83	0.976	0.46	0.964	0.57	0.989	0.59