AN ANALYTICALLY BASED TWO-SEX MARRIAGE MODEL AND MAXIMUM LIKELIHOOD ESTIMATES OF ITS PARAMETERS: AUSTRIA, 1979

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FOREWORD

Roughly 1.8 billion people, 42 percent of the world's population, live in urban areas today. At the beginning of the last century, the urban population of the world totaled only 25 million. According to recent United Nations estimates about 3.1 billion people, almost twice today's urban population, will be living in urban areas by the year 2000.

Scholars and policy makers often disagree when it comes to evaluating the desirability of current rapid rates of urban growth and urbanization in many parts of the globe. Some see this trend as fostering national processes of socioeconomic development, particularly in the poorer and rapidly urbanizing countries of the Third World; whereas others believe the consequences to be largely undesirable and argue that such urban growth should be slowed down.

As part of its studies of urbanization and development, the Population, Resources, and Growth Task in the Human Settlements and Services Area is examining changes in family structure and behavior during processes of structural transformation and economic development. Family formation through marriage is an integral component of such analyses, and Warren Sanderson's important contribution to the formal modeling of two-sex marriage models will become a central feature of our future research in this area.

Sanderson's paper tackles one of the major unresolved modeling problems in mathematical demography: how to describe in a parsimonious manner the matching process that underlies marriage and family formation. His elegant solution advances the current state of the art in resolving what is known among demographers as the two-sex problem.

A list of the papers in the Population, Resources, and Growth Series appears at the end of this report.

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ABSTRACT

A parameterized model of family formation, with applicability in both economics and demography is presented here. The model itself is a marriage of economic and demographic analysis. On the economic side, the model is based on a formal characterization of the "marriage market" and makes use of what economists call the "extended linear expenditure system." On the demographic side, the model makes use of the analytic nuptiality function developed by Coale and McNeil. The resulting specification with 13 easily interpretable parameters, is applied to the patterns of nuptiality observed in Austria in 1979. The plausibility of the maximum likelihood parameter estimates as well as the measures of goodness-of-fit indicate that the model can tentatively be accepted as a new tool for the study of the family.

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1. INTRODUCTION

A parameterized model of family formation, with applicability in both demographic and economic analysis is presented here and is tested against the patterns of nuptiality observed in Austria in 1979. The plausibility of the maximum likelihood parameter estimates as well as measures of goodness-of-fit indicate that the model can tentatively be accepted as a new tool for the study of the family.

The model itself is a marriage of two methodologies, one economic and one demographic. On the economic side, the model is based on a formal characterization of the "marriage market" and makes use of what economists call "systems of demand equations." On the demographic side, the model makes use of the analytic nuptiality function developed in Coale and McNeil (1972). The result is a model with 13 easily interpretable parameters.

2. MOTIVATION

There are, of course, many reasons for studying family formation. Most of these, however, proceed from a common premise: that the family is an important economic and social institution and that a better understanding of its functioning could be useful to society. This paper also begins with this premise. There are two sets of reasons though, which suggest that a study of family formation may be particularly fruitful at this time. The first set concerns the objective conditions relating to family formation whereas the second set concerns developments in economics and demography.

On the objective side, there is growing concern in Europe about below-replacement fertility levels. In many of these countries, a "family policy" has already evolved which encourages marriage and childbearing. In Austria, for example, the government currently rewards first marriages with a payment to the couple of AS 15,000. Yet the analysis of these policies has been very weak thus far, in part, because of the lack of an appropriate framework in which to study family formation. This paper provides such a framework.

There are several developments in economics and demography which make analytic modeling of family formation particularly useful now. In economics, the work of Becker and his colleagues (e.g., Becker 1974, Becker, Landes, and Michael 1977, Frieden 1974, and Santos 1975) has successfully established a new field of study, the economics of marriage and divorce. In order to supply the insights gained there to actual patterns of marriage by age that are consistent for males and females requires the sort of framework developed in this paper. Indeed, one of the important aspects of the model presented here is that it allows for the consistent application of the insights of economic theory to projections of future rates of family formation.

Another incentive for the formulation of the model was the development of large-scale economic-demographic simulation models (see Sanderson 1980 for a review of some of the better known models). These models typically were quite weak demographically. The model of family formation presented here is part of a larger research project at IIASA whose goal is to construct a model "demography" which is a more appropriate partner for the model economies which have already been developed.

Recent progress in analytic demography has also provided some motivation for this work. Analytical representations have been developed for nuptiality (one-sex)(Coale and McNeil 1972), fertility (Coale 1971), mortality (Brass 1971), and migration (Rogers, Raquillet, and Castro 1978). To create a richer analytic demographic world requires the explicit introduction of families. This is what is done in this paper.

3. PRIOR WORK ON FAMILY FORMATION

In this section, three major demographic contributions to modeling family formation are considered. In the early seventies two of them (Coale 1971, and Coale and McNeil 1972) established a very elegant one-sex marriage model. The third (Schoen 1977) presents one of the most successful previous attempts to produce an estimable two-sex marriage model.

The first step toward an analytic formulation of a nupitality function came in Coale (1971), where it was shown that first marriage frequencies for females could quite consistently be related in a relatively simple way to a standard set of first marriage rates.* To be more precise, Coale demonstrated that, in general,

$$\mu_j = \frac{c}{k} \cdot \mu_s \left(\frac{j-a}{k}\right) \tag{1}$$

where

μ_j is the first marriage rate for all females of age j (regardless of whether they are married or not)

c is the proportion of the group of women in question who ultimately ever marry

k is a parameter which (once a is fixed) controls the median age at marriage

*Coale obtained his "standard" first marriage rates from data for Swedish women during 1865-69.

$\mu_s\left(\frac{j-a}{k}\right)$ is a function whose argument, $\frac{j-a}{k}$, is a "standard age" and whose value is the "standard" first marriage rate for females of that "standard age"

a is the age at which a consequential number of females first marry

Equation (1) is quite powerful in the sense that plausible first marriage rates for females could be generated from the "standard" rates once the three parameters a, k, and c were specified. Still, the necessity of always having to refer to a table of "standard" rates was somewhat cumbersome and inelegant. Coale and McNeil (1972) suggested a formulation which eliminated the necessity of using the standard rates. They found that first marriage rates for females could be represented by the following equation

$$\mu_j = \left(0.1946 \cdot \frac{c}{k}\right) \cdot \exp \left\{ -0.174 \cdot \left(\frac{j-a}{k} - 6.06\right) - \exp\left[-0.28881 \cdot \left(\frac{j-a}{k} - 6.06\right)\right]\right\} \qquad (2)$$

where the symbols are defined as above. Equation (2) is, in a sense, a conceptual breakthrough in analytic demography somewhat akin to the development of the Cobb-Douglas production function in economics. With equation (2), plausible patterns of first marriage rates for females are produced once the three parameters a, k, and c are given, without reference to any "standard" schedule. The parameter k may be expressed as

$$k = \frac{\bar{a} - a}{10.01} \qquad (3)$$

where $\bar{a}$ is the median age at first marriage.

Substituting the expression for k in equation (3) into equation (2) yields

$$\mu_j = \frac{1.958 \cdot c}{\bar{a} - a} \cdot \exp\left\{ -1.750 \cdot \left(\frac{j - a}{\bar{a} - a} - 0.602\right) - \exp\left[-2.905 \cdot \left(\frac{j - a}{\bar{a} - a} - 0.602\right)\right]\right\} \quad (4)$$

One important feature of this version of the Coale-McNeil specification is that the three parameters, a (the earliest age at which a consequential number of women marry), $\bar{a}$ (the median age at first marriage), and c (the proportion of women ever marrying) all have clear and useful interpretations and can all be estimated easily from age-specific marriage rate data.

Equation (4) can be used to estimate the three parameters from fragmentary data or, alternatively, given those three parameters, it can be used to produce single years of age marriage rates. What equation (4) is not designed to do is to answer questions about how female marriage rates could be expected to change when the population of potential mates changes. Questions about the impact of changes in the male age distribution on the marriage rates of females are behavioral in nature and require some additional structural formalization.

An example from economics might be useful here. Once the parameters of a production function are given, the level of output can be computed from a knowledge of the quantities of the inputs, just as in equation (4) the level of the first marriage rate can be computed from the knowledge of the women's ages. Without additional assumptions, however, the production function in general cannot reveal how the input mix would change in response to a change in input prices. In order to accomplish this, economists typically make assumptions about the nature of firm behavior (e.g., optimizing or satisficing behavior) and about the market environments in which the firm operates. Can an analogous framework be devised for use with equation (4) for the purpose of giving it more behavioral content? This question is answered in Section 4 below. Before we get there, however, it is useful to ascertain how demographers have treated the two-sex marriage problem in the past.

A good example of recent demographic work on two-sex marriage models can be found in Schoen (1977). To understand Schoen's contribution, however, requires a little preparation. Let, π_{ij} be the number of marriages of females of age j to males of age i in a given year,* F_j be the number of unmarried females of age j in the year in question, and M_i be the number of unmarried males of age i in the year in question. One approach to the problem of specifying a function which yields the age-specific numbers of marriages is to start with the hypothetical age-specific numbers of marriages which would occur if the population in question had some "standard" age-sex-marital status composition and to adjust those figures according to the observed age-sex-marital status composition of the population. The number of marriages of females of age j to males of age i could then be expressed as

$$\pi_{ij} = \pi^{s}_{ij} \cdot \eta_{ij} \tag{5}$$

where

π_{ij} is the number of marriages of females of age j to males of age i

π^{s}_{ij} is the hypothetical number of marriages of females of age j to males of age i which would be obtained if the population had some "standard" age-sex-marital status composition

η_{ij} is an adjustment factor for marriages of females of age j to males of age i which depends on the observed age-sex-marital status composition of the population

If an appropriate expression for η_{ij} could be found, then perhaps equation (5) could be used to determine age-specific numbers of marriages. Pollard (1975:70-71) suggested the following specification of η_{ij}

*The age designations can be interpreted generally to mean between exact ages i and i + Δ and between j and j + Δ. The duration of the period with which we are concerned can just as easily be other than one year. In practical work such distinctions are often quite important. At this level of discussion, however, it is useful to pick some set of conventions and retain them throughout.

$$\eta_{ij} = \frac{M_i \cdot F_j}{\sum_\ell M_\ell \cdot \alpha^m_{\ell j} + \sum_\ell F_\ell \, \alpha^f_{\ell i}} \tag{6}$$

where

$\alpha^f_{\ell i}$ is an index of the relative attractiveness of females of age ℓ to males of age i

$\alpha^m_{\ell j}$ is an index of the relative attractiveness of males of age ℓ to females of age j

Since α^f_{ji} is not necessarily equal to α^m_{ij} in Pollard's formulation, the whole set of adjustment factors, the η_{ij}, is determined once the 2 x I x J parameters,* the α^m_{ij} and the α^f_{ij} are known. This latter observation, however, can be somewhat disturbing. To see why recall that in each year there are only I x J observations on the joint male-female age structure of marriages. Therefore, at least two years worth of data are required to estimate the parameters of the Pollard adjustment factor. If one were interested in estimating the parameters for a given year, then, the Pollard specification would not be suitable.

Now let us return to Schoen (1977). In order to construct a two-sex nuptiality-mortality life table for a particular year, Schoen needed a two-sex marriage model whose parameters could be estimated from the data from the year in question. To do this, Schoen suggested a modification of Pollard's formulation:

$$\pi_{ij} = \rho_{ij} \frac{M_i \cdot F_j}{\sum_\ell \omega_{\ell j} \cdot M_\ell + \sum_\ell \omega_{i\ell} \cdot F_\ell} \tag{7}$$

where

*Here and below, I refers to the number of age groups of males, and J is the number of age groups of females considered in the problem.

$$\omega_{ij} = \frac{\rho_{ij}}{\sum_{\ell} \rho_{i\ell} + \sum_{\ell} \rho_{\ell j}} \tag{8}$$

and where the ρ_{ij} are the hypothetical marriage rates of females of age j to males of age i which would occur if the number of unmarried males and females at each age were constant.

By introducing the concept of a rectangular population of eligibles, Schoen essentially compresses Pollard's α^m_{ij} and α^f_{ji} schedules into a single function ρ_{ij}. The ρ values are then used in two places: (i) in the creation of the weights which appear in the denominator of equation (7), and (ii) as the "standard" rates to be modified according to the age structures of the unmarried male and female populations. This insightful simplification allows the I x J number of marriages in any given year to be expressed as a function of I x J parameters, the ρ_{ij}.

The Schoen formulation is very useful because it allows the estimation of the parameters of a two-sex nuptiality function from the observed numbers of marriages in a particular year. In comparison with the Coale-McNeil specification, however, it is still very cumbersome. For example, the Coale-McNeil function requires only three parameters to produce all the single years of age marriage rates for females, say from age 15 through age 44. The Schoen formulation would require 900 parameters to perform the same task for both males and females. If we would be satisfied with an aggregation into five-year age groups the Schoen formulation would still require 36 parameters to produce the necessary 36 numbers of marriages.

The number of parameters, however, is no problem if one's purpose is to create a two-sex nuptiality-mortality life table for a given year. The nature and number of parameters becomes a consideration, however, when one has an alternative objective in mind—the production of an easily interpretable specification. For this purpose the Schoen formulation does not suffice. The parameters are the hypothetical "standard" marriage rates themselves, not characteristics of patterns of marriage rates.

The state of the demographic literature on nuptiality specifications then, is mixed. There exists an elegant one-sex nuptiality function due to Coale and McNeil but no analogous two-sex formulation. Schoen has provided a theoretically acceptable two-sex model whose parameters could be estimated from the data for a single year, but his framework is not suitable for the purpose of analytic modeling.

4. THE FORMAL MODEL

In the preceding section, the question was raised concerning whether an appropriate two-sex marriage model could be constructed by embedding the Coale-McNeil nuptiality specification into a more behaviorally oriented framework. In this section, that question is answered in the affirmative. The central notion here is that, in the aggregate, the interactions of males and females in searching for potential mates may be formally represented as a special kind of a market. This is hardly a new idea—the concept of a "marriage market" is firmly entrenched in economics, demography, and sociology. What is new here is the application of formal economic tools designed for the study of market phenomena to the demographic question at hand.

A "marriage market" is most closely related to what economists call a pure exchange economy, but even in this case there are some important differences. A "marriage market" can be thought of as a hypothetical place in which participants gather in order to search for potential mates. Each person comes endowed with a single "vow" and the exchange of "vows"—one for one—constitutes a marriage.* What makes this phenomenon of the exchange of vows an interesting one is that the participants in the market have discernable characteristics which make them more or less attractive as possible spouses. Therefore, some types of individuals will be in relatively great demand, while others will be in relatively great supply. At any moment, we would expect that the individuals in relatively great demand to have comparatively high marriage rates and to marry more selectively according to the characteristics they desire in their mates.

*I am grateful to Sherman Robinson for suggesting the relationship between a marriage market and an exchange economy and for suggesting the word "vow" for that which is exchanged.

Individuals who find themselves in relative oversupply would be expected to have comparatively low marriage rates and not to be as selective in terms of their mates' characteristics.

4.1. The Basic Framework

The foregoing suggests that the best way to consider the "marriage market" is not as a single exchange economy, but rather as a set of interrelated markets—in this simple case—for mates of different sexes and ages. A natural formulation of the problem of allocation in a set of interrelated markets is what economists call a "system of demand equations." A well-known system, which has a number of desirable features for the purpose at hand is the extended linear expenditure system (hereafter ELES) in Lluch et al. (1977). The demands of females of age j for males of various ages can be expressed in the ELES framework as:

$$D^f_{ji} = F_j \cdot \left[g_{ji}\, s_{ij} + b_{ji} \cdot (1 - \sum_i g_{ji}\, s_{ij}) \right] \quad (9)$$

$$\text{for } i' \leq i \leq i^*, \quad j' \leq j \leq j^*$$

where

D^f_{ji} is the demand of females of age j for males of age i

F_j is the number of eligible females of age j

g_{ji} and b_{ji} are parameters of the demand system

s_{ij} is the relative scarcity indicator (males as compared to females) for marriages between males of age i and females of age j

i' and j' are the initial ages, respectively, of males and females considered in the model

i^* and j^* are the terminal ages, respectively, of males and females considered in the model

The demands of males of age i for females of age j can be similarly expressed as

$$D^m_{ij} = M_i \cdot \left[\gamma_{ij}\, s_{ij} + \beta_{ij} \cdot (1 - \sum_j \gamma_{ij}\, s_{ij})\right] \tag{10}$$

for $i' \leq i \leq i^*$
$j' \leq j \leq j^*$

where

D^m_{ij} is the demand of males of age i for females of age j

M_i is the number of eligible males of age i

γ_{ij} and β_{ij} are parameters

The model is closed by imposing the restrictions that the number of females of age j who marry males of age i must always equal the number of males of age i who marry females of age j. In symbols, we must always observe that

$$D^f_{ji} = D^m_{ij} \qquad i' \leq i \leq i^*,\ j' \leq j \leq j^* \tag{11}$$

The equations above form an I x J (where $I = i^* - i' + 1$ and $J = j^* - j' + 1$) set of linear equations in I x J unknowns, the s_{ij}. Once the s_{ij} are known, the age structure of marriages can be easily computed from either equation (9) or equation (10) above. There are two disadvantages of this approach: i) there is no distinction made above between first marriages and higher order marriages, and ii) the I x J equation system requires 4 x I x J parameters—double the number used in the unwieldly Pollard formulation discussed in the previous section. The solutions to these problems are such that we may safely proceed at this point without them and return to them below when the implications of the current structure are clear.

Let us define

$$T_i^m \equiv \sum_j \gamma_{ij} s_{ij} \tag{12}$$

and

$$T_j^f \equiv \sum_i g_{ji} s_{ij} \tag{13}$$

These definitions are important here because the predicted numbers of marriages of males of age i to females of age j can be expressed as a simple linear function of T_i^m and T_j^f. This can be easily seen by inserting the equilibrium values of s_{ij} into either equation set (9) or (10). The result is the expression

$$\pi_{ij} = M_i \cdot F_j \cdot \left[\frac{\beta_{ij} \, g_{ji} \, (1 - T_i^m) - b_{ji} \, \gamma_{ij} \, (1 - T_j^f)}{F_j \, g_{ji} - M_i \, \gamma_{ij}} \right] \tag{14}$$

where π_{ij} is the predicted number of marriages of males of age i to females of age j.

Equation (14) is a significant simplification as compared with the equation sets (9), (10), and (11) in that the I x J predicted numbers of marriages can be produced given the parameters and only I + J intermediate variables, the T_i^m and T_j^f.

Multiplying s_{ij} by γ_{ij}, summing over j and employing equation (12) yields

$$\sum_j \gamma_{ij} \, s_{ij} = T_i^m = \sum_j \frac{M_i \, \beta_{ij} \, \gamma_{ij} \, (1 - T_i^m) - F_j \, b_{ji} \, \gamma_{ij} \, (1 - T_j^f)}{F_j \, g_{ji} - M_i \, \gamma_{ij}} \tag{15}$$

Rearranging terms and solving for T_i^m we obtain

$$T_i^m = d_i^m \sum_j T_j^f \cdot \frac{F_j\, b_{ji}\, \gamma_{ij}}{F_j\, g_{ji} - M_i\, \gamma_{ij}}$$

$$+ d_i^m \sum_j \frac{M_i\, \beta_{ij}\, \gamma_{ij} - F_j\, b_{ji}\, \gamma_{ij}}{F_j\, g_{ji} - M_i\, \gamma_{ij}} \quad (16)$$

where

$$d_i^m = \left[1 + M_i \cdot \sum_j \frac{\beta_{ij} - \gamma_{ij}}{F_j\, g_{ji} - M_i\, \gamma_{ij}} \right]^{-1} \quad (17)$$

Rewriting equation (16) in matrix notation, we have

$$[T^m] = [A_1][T^f] + [A_2] \quad (18)$$

where

$[T^m]$ is an I x 1 vector whose ith element is T_i^m

$[A_1]$ is an I x J matrix whose (i,j)th element is

$$\frac{d_i^m\, F_j\, b_{ji}\, \gamma_{ij}}{F_j\, g_{ji} - M_i\, \gamma_{ij}}$$

$[T^f]$ is a J x 1 vector whose jth element is T_j^f

$[A_2]$ is an I x 1 vector whose ith element is

$$d_i^m \sum_j \frac{M_i\, \beta_{ij}\, \gamma_{ij} - F_j\, b_{ji}\, \gamma_{ij}}{F_j\, g_{ji} - M_i\, \gamma_{ij}}$$

Analogously, we may multiply the s_{ij} by g_{ji}, sum over the i, and invoke equation (13). This results in the matrix equation

$$[T^f] = [A_3][T^m] + [A_4] \quad (19)$$

where

$[A_3]$ is a J x I matrix whose (j,i)th element is

$$\frac{d_j^f M_i \beta_{ij} g_{ji}}{M_i \gamma_{ij} - F_j g_{ji}}$$

$$\text{where} \quad d_j^f = \left[1 + F_j \cdot \sum_i \frac{b_{ji} g_{ji}}{M_i \gamma_{ij} - F_j g_{ji}} \right]^{-1} \tag{20}$$

$[A_4]$ is a J x 1 vector whose jth element is

$$d_j^f \cdot \sum_i \frac{F_j b_{ji} g_{ji} - M_i \beta_{ij} g_{ji}}{M_i \gamma_{ij} - F_j g_{ji}}$$

and

$[T^f]$ and $[T^m]$ are defined as above.

Substituting the expression for $[T^f]$ in (19) into equation (18) and solving for $[T_m]$ yields

$$[T^m] = [I - A_1 A_3]^{-1} [A_1 A_4 + A_2] \tag{21}$$

Similarly,

$$[T^f] = [I - A_3 A_1]^{-1} [A_3 A_2 + A_4] \tag{22}$$

Substituting the values of $[T^m]$ and $[T^f]$ determined in equations (21) and (22) into equation (14) yields the predicted numbers of marriages by age.

We have now shown how the interaction of the two systems of demand equations can yield a closed form solution for the predicted numbers of marriages. Next, let us turn to the two problems raised above.

4.2. The Parameters of the General Framework

As we mentioned above there are two major problems with the general framework: the large number of parameters needed for the system of demand equations and the lack of a distinction between first marriages and higher order marriages. The solutions to these two problems are related. Let us now turn to them in the order they were mentioned.

There are a number of ways to simplify the parameter structure in the current model. None of the array of alternatives is without some disadvantages, but there is one particular tack which appears to be quite promising. It is to move upward one level of conceptualization and to specify formally the underlying structure that produces the ELES parameters. The major problem with this approach is in finding a structural specification in which the upper level parameters are both readily interpretable and behaviorally meaningful. A simple solution to this problem is to base the upper level specification on the Coale-McNeil nuptiality function discussed above. What follows, then, in this section is a discussion of this upper level specification.

It is useful to begin this discussion by considering a hypothetical situation in which the marriage market is "neutral" in the sense that all the relative scarcity values, the s_{ij}, are constant. For the sake of symmetry and simplicity, a "neutral marriage market" is defined here as one in which all relative scarcity indicators are zero. The choice of this level is purely stylistic and readers who prefer alternate levels may easily alter the equations to obtain them. A more behavioral interpretation of a "neutral marriage market" is given below after the explanation of the derivation of the β and b parameters. Let us turn to this explanation.

When all the relative scarcity indices, the s_{ij}, have value zero, the males' marriage hazard rate at age i, derived from equation (10), is given by

$$\frac{\sum_j \pi_{ij}}{M_i} = \sum_j \beta_{ij} \qquad i' \leq i \leq i^* \qquad (23)$$

where π_{ij} is, as above, the number of marriages of males of age i to females of age j. When all the relative scarcity indices have value zero, the females' marriage hazard rate at age j, derived from equation (9), is given by

$$\frac{\sum_i \pi_{ij}}{F_i} = \sum_i b_{ji} \qquad j' \leq j \leq j^* \qquad (24)$$

In order to relate the current two-sex nuptiality model to the Coale-McNeil one-sex model, it is assumed that the marriage hazard rates above (i.e., when all $s_{ij} = 0$) are consistent with marriage hazard rates generated by Coale-McNeil nuptiality functions. To be more precise, let a^m, $\bar{a}^m$, and c^m be the three parameters of the Coale-McNeil nuptiality function in equation (4) above, and let μ_i^m be the marriage rate implied by those three parameters for males of age i. Equation (23) may now be expressed as

$$\sum_j \beta_{ij} = \frac{\mu_i^m}{1 - \sum_{\ell=a^m}^{i-1} \mu_\ell^m} \qquad i' \leq i \leq i^* \qquad (25)$$

Analogously, let a^f, $\bar{a}^f$, and c^f be the three parameters of the Coale-McNeil nuptiality function for females, and let μ_j^f be the marriage rate implied by those three parameters for females of age j. Equation (24) may now be expressed

$$\sum_i b_{ji} = \frac{\mu_j^f}{1 - \sum_{\ell=a^f}^{j-1} \mu_\ell^f} \qquad j' \leq j \leq j^* \qquad (26)$$

The next question, then, is how to distribute the parameter sums across the individual components of the sums. One alternative which works well is to assume that, for each i, the β_{ij}, and for each j, the b_{ji}, are distributed according to a beta distribution whose mean value depends on the age of the individual and whose variance depends both on that mean value and on a parameter of the model. To be more precise, let

$$p_i^m \equiv \frac{\sum_j \beta_{ij} \cdot j}{\sum_j \beta_{ij}} \qquad (27)$$

and

$$p_j^f \equiv \frac{\sum_i b_{ji} \cdot i}{\sum_i b_{ji}} \tag{28}$$

It is assumed in the model that for each age, the β_{ij}'s are distributed in the age range from 18 to 65+ according to a beta distribution whose mean may be expressed as

$$p_i^m = \phi_1^m + \phi_2^m \cdot (i - 18) \tag{29}$$

where ϕ_1^m and ϕ_2^m are parameters that are to be estimated. Similarly it is assumed that the b_{ji}'s are distributed in the age range from 16 to 63+ according to a beta distribution whose mean may be written as

$$p_j^f = \phi_1^f + \phi_2^f \cdot (j - 16) \tag{30}$$

where ϕ_1^f and ϕ_2^f are parameters that are to be estimated.

The four parameters in equations (29) and (30), ϕ_1^m, ϕ_2^m, ϕ_1^f, and ϕ_2^f have natural interpretations. For example, if the marriage market were neutral, in the sense that all the s_{ij} were zero, then ϕ_1^m would be the mean age of the brides of 18-year-old males and ϕ_1^f would be the mean age of the husbands of 16-year-old females. Given ϕ_1^m and ϕ_1^f, the parameters ϕ_2^m and ϕ_2^f control the mean ages of the spouses of older males and females in a neutral marriage market. For example, in a neutral marriage market males of exact age 20 would marry women whose average age was $\phi_1^m + 2\phi_2^m$ and females of exact age 20 would marry males whose average age was $\phi_1^f + 4\phi_2^f$.

Given the age range, the beta distribution depends on two parameters. In order to determine these two parameters some additional information is required. The most natural specification would be to include in the model a parameter related to variances in mean spousal ages. This is what we have done. The model incorporates a parameter, σ, which accomplishes this. When the marriage market is neutral, σ is the standard deviation of the distribution of the ages of spouses who marry individuals who are at the midpoint of the specified age range. This notion is described in detail in Section 1 of the Appendix.

It is a straightforward task to determine the parameters of each of the beta distributions from the information at hand now. With these distributions, the known sums of the β's and b's can be allocated over the ages of potential spouses.

We can now return briefly to the concept of a neutral marriage market. When the marriage market is neutral, the proportion of eligible males of age i marrying females of age j in a given year is simply β_{ij} and analogously the proportion of eligible females of age j who marry males of age i is simply b_{ji}. It is also true that whenever the age-specific proportions marrying are these β_{ij}s and b_{ji}s, the marriage market is neutral. The parameters β_{ij} and b_{ji}, then, are similar to "standard" sets of marriage rates. The predicted marriage rates in the model are modifications of these underlying rates depending on demographic conditions. The parameter sets β_{ij} and b_{ji}, however, are themselves parameterized. Changes in socioeconomic conditions may be viewed as operating through the parameters of Coale-McNeil functions and thus affecting the underlying "standard" marriage rates or alternatively as affecting the marital pattern observed in a neutral marriage market. The effects of such changes on observed marriage rates depend not only on changes in underlying behavior (i.e., changes in the β_{ij} and b_{ji}), but also on the demographic environment as well. Changes in the demographic environment can affect observed marriage rates even when the characteristics of the neutral marriage remain unchanged, but even the same changes in demographic structure could have different impacts on observed marriage rates, if the patterns of marital behavior in case of neutrality differ in the two circumstances.

That ends the discussion of the meaning of a neutral marriage market. Let us continue now with our explanation of the two level parameter structure. We have assumed that when the relative scarcity indicators are all zero, the model should produce marriage hazard rates consistent with the two Coale-McNeil nuptiality functions. It is useful to follow this line of attack and ask about what kind of behavior we would like the model to produce when the relative scarcity indicators are at other levels. As the relative scarcity of members of one sex increases across all ages, we would expect that the marriage hazard rates of the members of the opposite sex would fall. The foregoing suggests that, when all relative scarcity indicators reach some "high enough" level, the demands of females for potential spouses of all ages go to zero and similarly that, when the relative scarcity indicators reach some "low enough" level, the demands of males for potential spouses of all ages go to zero. For simplicity and symmetry, it is assumed that, when all the relative scarcity indicators take on a value of 1.0, the female demand for males of all ages is zero and that, when all relative scarcity indicators take on a value of -1.0, the male demand for females of all ages is zero.

Formally, we write

$$0 = g_{ji} + b_{ji}\left(1 - \sum_i g_{ji}\right) \qquad \begin{array}{l} i' \le i \le i^* \\ j' \le j \le j^* \end{array} \qquad (31)$$

Equations of this form can be solved to determine the values of the g_{ji}s in terms of the b_{ji}s. The solutions are

$$g_{ji} = -\frac{b_{ji}}{1 - \sum_i b_{ji}} \qquad \begin{array}{l} i' \le i \le i^* \\ j' \le j \le j^* \end{array} \qquad (32)$$

Similarly,

$$\gamma_{ij} = -\frac{\beta_{ij}}{1 - \sum_j \beta_{ij}} \qquad \begin{array}{l} i' \le i \le i^* \\ j' \le j \le j^* \end{array} \qquad (33)$$

In this manner, the γ_{ij} and g_{ji} can be computed from the β_{ij} and the b_{ji}, without the need of additional parameters.

In summary, then, the approach that has been taken to the problem of the large number of parameters in the marriage market formulation is to parameterize these parameters so that they may be generated by a smaller number of behaviorally relevant variables. This allows us to maintain simultaneously two levels of interpretation: first, the level of the overall marriage market where the underlying demographic forces work themselves out and second, the level of behavior where the response to economic and social factors is manifested.

The behavioral parameters used are all *ex ante* in nature in the sense that they can only be directly observed when the marriage market is neutral. When the marriage market is not neutral, the *ex post* or observed values of the parameters will differ from their *ex ante* values according to the extent and nature of the nonneutrality.

The use of the Coale-McNeil nuptiality function to generate the *ex ante* age-specific marriage rates provides simultaneously a problem and a useful opportunity. The Coale-McNeil nuptiality function refers only to first marriages. Since the marital behavior patterns of previously married (but currently single) people differ considerably from those of never married people, a specification based on the Coale-McNeil function without consideration of higher order marriages is theoretically unsound as well as probably being empirically unattractive.

There is, however, a relatively straightforward approach to including higher order marriages in the model. Let the remarriage rates for previously married males compared to never married males of age i be denoted by r_i^m and let the analogous ratio for females of age j be denoted by r_j^f. The relations between these ratios and age may, then, be expressed

$$r_i^m = \begin{cases} 1 + (i - 18) \cdot (\rho^m - 1)/40 & \text{if } i \leq 40 \\ \rho^m & \text{if } i > 40 \end{cases} \tag{34}$$

and

$$r_j^f = \begin{cases} 1 + (j - 16) \cdot (\rho^f - 1)/40 & \text{if } j \leq 40 \\ \rho^f & \text{if } j > 40 \end{cases} \tag{35}$$

where ρ^m and ρ^f are parameters to be estimated.

For males, the ratio of remarriage to first marriage rates is assumed to rise linearly from a value of unity at exact age 18 to a value of ρ^m at exact age 40 and to remain at that level thereafter. The parameter ρ^f has the analogous interpretation for females, except that the value of unity is attained at exact age 16. This is indeed a simple interpretation and more sophisticated ones are clearly possible. The rationale for a simple specification is given in the section on parameter estimation below.

To complete the formal model now requires only that a more precise definition be given to the variables M_i and F_j introduced in equations (9) and (10). These variables are calculated as follows

$$M_i \equiv S_i^m + r_i^m W_i^m \tag{36}$$

and

$$F_j \equiv S_j^f + r_j^f W_j^f \tag{37}$$

where

S_i^m is the number of never married males of age i

S_j^f is the number of never married females of age j

W_j^m is the number of currently single males of age i who had previously been married

W_j^f is the number of currently single females of age j who had previously been married

This ends the specification of the formal model. Let us now turn to examine whether the model provides any insight into Austrian marriage patterns observed in 1979.

5. PARAMETER ESTIMATION

The model discussed above has been fit to Austrian data for the year 1979. Austria provides an interesting case study for a number of reasons. Single year of age data on marriages by the ages of both spouses are now being published. This means that eventually a time series of parameter estimates can be generated which can be used to gain some insight into likely future trends. Such an exercise will provide a good test of the temporal stability of the estimated parameters especially in view of the irregular age structure of the Austrian population. Further, Austria has an explicit family policy so that it would be also useful to ascertain whether the framework can be used to study the effects of changes in family policy.

Since this is the first time that the parameters of a two-sex model of family formation have been estimated, it is useful to consider both the results of the estimation and the problems which arose in the process.

The major problem which arose in the estimation concerned the low first marriage rates produced by the Coale and McNeil specification for people in the upper tail of the age range. To see this clearly, let us return to equation (3) and compute the first marriage rate for previously unmarried women of age 50 under assumptions which are roughly appropriate for Austria in 1979; let the age at which an appreciable number of marriages first occurs be 16, the median age at first marriage be 21, and the proportion ever married be 90 percent. In this case, the predicted first marriage hazard rate is about 0.0007. In reality, in Austria in 1979, the first marriage hazard rate for never married 50-year-old women was 0.006,* or roughly 90 times higher than the predicted rate. For many purposes an error of ninetyfold in a tiny number may not be very important, but for the purpose of maximum likelihood estimation procedure used here which takes heteroskedasticity into account, such differences are very important indeed. These large relative differences are not limited to individuals 50 and above. Even for people in their late

*The data used in this computation are from Österreichischen Statistischen Zentralamt (1980) *Demographisches Jahrbuch Österreichs 1979*, Table 2.10, p. 37 and Table 9.07, p. 223.

30s, the Coale-McNeil specification underpredicts the Austrian data significantly. Further, we could hardly expect to obtain plausible estimates of the ratio of the remarriage rates to the first marriage rates, when the latter figures themselves are proportionally so inaccurate.

For the present purpose we have used a rather rough and ready solution to this problem. Let $\hat{i}$ be the first age at which the predicted marriage rate for males, μ_i [as in equations (2) and (4)], falls below 0.01 and let $\hat{j}$ be the analogous age for females. For males of age $\hat{i}$ and above we have assumed that

$$\mu_i = 0.95^{\left[\frac{i-\hat{i}+1}{k^m}\right]} \cdot \mu_{\hat{i}-1} \qquad i \geq \hat{i} \qquad (38)$$

where k^m is the k parameter in equation (2) when applied to males. Similarly for females of age $\hat{j}$ and above we have assumed that

$$\mu_j = 0.95^{\left[\frac{j-\hat{j}+1}{k^f}\right]} \cdot \mu_{\hat{j}-1} \qquad j \geq \hat{j} \qquad (39)$$

No attempt has been made to optimize over the two parameters of this correction—the 0.01 and the 0.95. They are plausible numbers and work reasonably well. Two consequences follow from this correction. First, without further modification, c^m and c^f would not longer be *ex ante* proportions ever married. To avoid this the resulting marriage rates are proportionally reduced so that c^m and c^f remain the *ex ante* proportions ever married. Second, the relationships between the k parameters and the median ages at first marriages becomes more complex than in equation (3). Since it is much simpler to estimate the ks, this is what has been done. The *ex ante* median ages at first marriage are, then, computed from the estimated parameters.

The Austrian data for 1979 are from the *Demographisches Jahrbuch Österreichs 1979* published by the Österreichischen Statistischen Zentralamt (Volume 584). Since the data by age begin where the officials of the central statistical office perceive a consequential number of marriages first to be observed, it is a great simplification, at virtually no cost in terms of the accuracy of the parameter estimates, to follow their lead and to set a^m and a^f accordingly ($a^m = 18$, $a^f = 16$). This leaves a nonlinear structure in 11 parameters to be estimated.

The next question which arises here concerns the nature of the stochastic structure appropriate for the problem at hand. The number of marriages in a single year of age cell can vary between 0 and 922 (20-year-old women marrying 22-year-old men). Clearly, it would be implausible to assume that the variance of the error term associated with a predicted value of, for example, ten marriages would be the same as the error term associated with a predicted value of five hundred marriages. A common approach to this problem of heteroskedasticity in frequency data is to assume that observations are the realizations of independent Poisson processes whose expected values are values predicted by the model. This is also a reasonable specification to assume here for three reasons: (i) all observations must be nonnegative as is indeed the case, (ii) the variances of the observations are proportional to their expected values, which is plausible, and (iii) the resulting computation procedure is sufficiently simple that estimation is not prohibitatively expensive.*

The maximum likelihood estimates of the parameters and their asymptotic standard errors are found in Table 1. The results are very encouraging. All the parameters are quite reasonable. The computed *ex ante* median ages at marriage are very sensible for Austria in 1979. The median age of males who married for the first time in 1979 was 24.6 years old and of females who married for the first time, 21.8.** From Table 1 it can be seen that, if the marriage market were neutral, the median age of marriage for males would have been 25.7 and the median age of marriage for females would have been 20.8. It appears, then, that the

*The estimation procedure is discussed in the second section of the Appendix.

**Österreichischen Statistischen Zentralamt (1979) Table 2.02, p. 34. Since the age distribution in the prime marrying ages in Austria in 1979 was relatively flat, these figures are probably good approximations to median ages at marriage.

Table 1. Maximum likelihood estimates of the parameters of the analytically based two-sex marriage model: Austria, 1979

	Males	Females
Parameters relating to first marriages in a neutral marriage market		
Age at which a consequential number of first marriages occur, a	18.0[a]	16.0[a]
Parameter controlling median age at marriage, k	0.709 (0.038)	0.460 (0.042)
Median age at marriage, $\bar{a}$	25.7[b]	20.8[b]
Proportion every marrying, c	0.815 (0.053)	0.923 (0.020)
Mean age of spouse for those marrying at earliest possible age (18 for males, 16 for females), ϕ_1	18.6 (0.095)	20.3 (0.352)
Increase in mean age of spouse for each year of age, ϕ_2	0.769 (0.051)	1.10 (0.067)
Beta distribution parameter at the midpoint of the age range σ (transform of the standard deviation of the spousal age distribution)	3.45[c] (0.097)	3.45[c] (0.097)
Parameters relating to remarriage		
Ratio of remarriage rate to first marriage rate at age 40 and beyond, ρ	5.19 (2.12)	2.71 (1.66)
Number of observations	444[d]	
Number of estimated parameters	11	
Tests of goodness-of-fit		
Pearson chi-squared statistic	0.038[e]	
Likelihood ratio statistic	0.002[e]	

Figures in parentheses are asymptotic standard errors (Cramer-Rao lower bounds).

[a]Constrained by data availability. See text for more information.

[b]Computed from parameters, not parameters themselves.

[c]Assumed to be identical for males and females for computational stability. See the first section of the Appendix for the detailed definition of this parameter and a justification of the above assumption.

[d]Observations were restricted to cells where there were ten or more marriages. Open intervals 18 and below and 65 and above for males, and 16 and below and 60 and above for females were also excluded.

[e]Test statistic has asymptotic chi-square distribution with 433 degrees of freedom. See the second section of the Appendix for definition.

nonneutrality in the marriage market leads females to marry somewhat later than they would in a neutral marriage market and males to marry somewhat earlier.

The same sort of phenomenon can be seen with respect to *ex ante* proportions ever married. Although there is no simple analog in the data, it is instructive to look at proportions ever marrying among 50-54 year olds in 1979. For males the proportion was 93.4 percent while for females it was 90.1 percent.* The estimated proportions, reported in Table 1, which would have been observed in a neutral marriage market are 81.5 percent and 92.3 percent, respectively. Apparently the nonneutrality in the marriage market not only induced men to marry at a younger age than they would have in a neutral marriage market, but induced them to marry in greater proportions as well.** The nonneutrality in the marriage market seems to be reducing female marriage frequencies when compared with the neutral marriage market situation, but given the standard error of coefficient, we cannot be confident of this.

The parameters ϕ_1 and ϕ_2 are also sensible in the context of the Austrian data. Males who married at age 18 in 1979 married women who averaged 19.0 years of age, while women who married at age 16 in 1979 married males who averaged 22.0 years.† The figures in Table 1 indicate that if the marriage market were neutral the mean spousal ages would have been 18.6 and 20.3 respectively. The observed situation is one in which very young brides marry males whose average is somewhat older than it would be if the marriage market were neutral. This is clearly consistent with the discussion above concerning the median age at marriage.

The ϕ_2 parameter tells an interesting story. For females it is not significantly different from unity, but for males it is clearly below unity. Apparently, were the marriage market neutral, 50-year-old males would marry women who averaged 43.2 years of age, whereas 50-year-old females would marry males who averaged 57.7 years. This difference in ϕ_2 parameters, however, is one of the factors that maintains the nonneutrality.

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*Österreichischen Statistischen Zentralamt (1979) Table 9.06, p. 222.

**This is precisely the sort of conjuncture the economic theory discussed in Section 2 leads one to expect.

†Österreichischen Statistischen Zentralamt (1979) Table 2.11, pp. 38-39.

The beta distribution parameter at the midpoint of the age range is the one parameter about which we have little intuition. It is discussed in more detail in the Appendix.

The ratios of remarriage rate to first marriage rate at age 40 and beyond are the only parameters in the model which do not refer to the hypothetical situation in which the marriage market is neutral. They can be used, then, to test the plausibility of the model. If we take the mean of the ratios of remarriage to first marriage rates for the age groups 40-44 and 45-49,* we obtain 4.97 for males and 1.95 for females. The ρ parameter for males, 5.19, is quite close to the observed value, whereas the ρ parameter for females, 2.71, is further away but still with one asymptotic standard deviation of the observed value. Given the uncertainty concerning the structure of the model for the upper range, the fact that the parameter estimates of ρ^m and ρ^f are quite close to the observed values is heartening and suggests the plausibility and usefulness of the analytic two-sex model.

There are two commonly used goodness-of-fit tests appropriate here: the Pearson chi-square test and the likelihood ratio test. Both test statistics are asymptotically distributed as a chi-square with 433 degrees of freedom. Since the mean of the chi-square statistic with 433 degrees of freedom is 433, the very low relative levels of the two test statistics indicate that the fit is quite good.

6. CONCLUSION

This paper presents a new, analytically based two-sex marriage model which merges theoretical specification from both economics and demography. Maximum likelihood estimates of the parameters of the model, based on Austrian data for 1979, have been computed using a procedure which takes heteroskedasticity of the residuals into account. The parameters are plausible and entirely consistent with the Austrian data. The model, then, has passed its first test and is potentially a candidate for the growing list of tools in analytic demography.

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*The sample of 444 age cells contains only a few observations at ages 50 and above.

APPENDIX

SECTION 1: THE USE OF THE BETA DISTRIBUTION

Given the fixed age range, the beta distribution can be written as a function of two parameters

$$\pi(x) = \frac{1}{B(p,q)} x^{p} (1 - x)^{q} \qquad 0 \leq x \leq 1 \qquad \text{(A1)}$$

where $\pi(x)$ is the value of the density function at x, p and q are the two parameters, and B(p,q) is a constant depending on p and q.*

We know that

$$E(x) = \frac{p}{p + q} \qquad \text{(A2)}$$

and that

$$VAR\ (x) = \frac{pq}{(p + q)^2 (p + q + 1)} \qquad \text{(A3)}$$

Now, when E(x) = 0.5, p = q and

*This discussion draws heavily on Johnson and Kotz (1970) Ch. 24.

$$\mathrm{VAR}(x) = \frac{1}{8p + 4} \tag{A4}$$

The parameter σ is the level of both p and q when E(x) = 0.5. When E(x) < 0.5, q is set equal to that joint level and equation (A2) is solved for p. When E(x) > 0.5, p is set to that joint level and quation (A2) is solved for q.

Early experiments with two σ parameters resulted in a considerable amount of instability in the estimation process and an optimum was never reached. Constraining the model to a single σ parameter caused a dramatic improvement in estimability.

SECTION 2: MAXIMUM LIKELIHOOD ESTIMATION AND MEASURES OF GOODNESS-OF-FIT

It is shown in Jennrich and Moore (1975) that maximum likelihood estimation is identical to minimizing an iteratively reweighted residual sum of squares when one is dealing with a density function in the exponential family. Since the Poisson distribution belongs to that family, maximum likelihood estimates and their standard errors are comparatively easy to obtain. Jennrich and Moore's suggestions have been followed and the reader is referred to their paper for further details.

The Pearson chi-square statistic may be written as

$$x^2 = \sum_{i=1}^{444} \left(\frac{x_i - \hat{x}_i}{\hat{x}_i} \right)^2$$

where x_i is the observed fraction of all the marriages in the 444 cells which are found in the ith cell, and $\hat{x}_i$ is the predicted fraction of marriages in the ith cell.

The likelihood ratio statistic is simply

$$G^2 = 2 \left[\sum_{i=1}^{444} x_i \ln \left(\frac{x_i}{\hat{x}_i} \right) \right]$$

where x_i and $\hat{x}_i$ are defined as above.*

*For a discussion of these two goodness-of-fit statistics, see Bishop, Fienberg, and Holland (1975), Ch. 4.

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