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STATISTICAL ANALYSIS OF REGIONAL GROWTH:  
CONSISTENT MODELING OF EMPLOYMENT,  
POPULATION, LABOR FORCE PARTICIPATION,  
AND UNEMPLOYMENT

Jacques Ledent

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INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS  
A-2361 Laxenburg, Austria

## FOREWORD

Declining rates of national population growth, continuing differential levels of regional economic activity, and shifts in the migration patterns of people and jobs are characteristic empirical aspects of many developed countries. In some regions they have combined to bring about relative (and in some cases absolute) population decline of highly urbanized areas; in others they have brought about rapid metropolitan growth.

The objective of the Urban Change Task in IIASA's Human Settlements and Services Area is to bring together and synthesize available empirical and theoretical information on the principal determinants and consequences of such urban growth and decline.

Within this Task a concerted effort has been made to develop a methodology, based on statistical models, that allows decision makers to formulate coherent scenarios of a region's future levels of population and employment. This paper reports on a central aspect of such a methodology: the consistent modeling of the interaction between the demand and supply sides of a regional labor market.

A list of publications in the Urban Change series appears at the end of this paper.

Andrei Rogers  
Chairman  
Human Settlements  
and Services Area

## ABSTRACT

This paper focuses on the consistency problem that arises in statistical models of regional growth from the joint and simultaneous consideration of the following four labor market variables: employment, population, the labor force participation rate, and the unemployment rate. As these variables are linked by a definitional equation, one of them must, of necessity, be derived from the others. But which of the four variables should one choose as the nonprimary variable?

A test of the four possible alternatives in connection with a simple statistical model fitted to data for the rapidly growing metropolitan area of Tucson, Arizona, reveals that the preferable choice for the nonprimary variable is the labor force participation rate.

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INTRODUCTION

In general, statistical models of regional growth include an endogenous measure of unemployment that reflects the health of the economy at hand. Typically, this measure is derived from a simple submodel that confronts the demand and supply sides of the labor market. In this paper, such a submodel is referred to as a labor market submodel.

According to several researchers who have directed their attention to the connection between migration and urban labor force dynamics (Miron 1978; Rogers 1978), the specification of a labor market submodel should stress the process whereby firms and households mutually adjust their expectations. Relying on a theory of regional growth with a mixed demand/supply orientation, the submodel should emphasize the endogenous and simultaneous determination of the following five variables:

1. employment
2. labor force
3. population
4. the unemployment rate
5. the labor force participation rate

Such a specification has been used by Chalmers and Greenwood (1978) in the context of an explanatory model and by Ledent and Gordon (1980) in the context of a simulation model.

Unfortunately, existing statistical models of regional growth do not offer a labor market submodel with a specification that follows the principles just mentioned. Thus, this author found (Ledent 1981) that

- 1) all of 23 existing models having a labor market submodel are based on an underlying theory that is exclusively demand oriented [the impact of households on economic activity through their role of labor suppliers, suggested by Borts and Stein (1964), is ignored altogether] and
- 2) only 7 of the 23 models offer an endogenous and simultaneous determination of the five aforementioned variables

This observation naturally led us to advocate the development of a more realistic labor market submodel, for which a minimal formulation--shown here as equations (1) through (12) in Table 1--was then proposed.

A problem of particular interest that arises from such a formulation concerns the coherent treatment of the five main labor market variables; a problem that was originally brought out by the realization that the derivation of the unemployment rate variable, following the course suggested by its very definition [that is by use of equation (11)], may be troublesome (Ledent 1978). One way to deal with this problem is to include equation (13) in Table 1 into the minimal formulation which then has one more equation than the number of endogenous variables. Thus, one equation must be discarded, but which one?

This paper is devoted to finding the best choice of the equation to discard. Section 1 demonstrates how the consistency problem raised by the simultaneous consideration of the five labor market variables was initially uncovered. Section 2 proposes a fundamental exposition of this problem that points

Table 1. The minimal formulation of a regional labor market model.\*

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I. EQUATIONS\*\*

1. Population Sector

$$P = \frac{\left(1 - \frac{b-d}{2}\right)P_{-1} + M}{1 + \frac{b-d}{2}} \quad (1)$$

$$b = b(w_{-1}^+, u_{-1}^-, t^-) \quad (2)$$

$$d = d \text{ (factors to be specified)} \quad (3)$$

$$M = M[\Delta E^+, (u - \bar{u})_{-1}^-, (w - \bar{w})_{-1}^+] \quad (4)$$

2. Employment Sector

$$E = WSE + A + S \quad (5)$$

$$WSE = \sum_{i=1}^x E_i \quad (6)$$

$$A = A \text{ (exogenous factors)} \quad (7)$$

$$E_i = \begin{cases} E_i(NEMP^+, P^+, \rho^+, w \frac{P}{E}^-) \\ E_i(P^+, w^+, \rho^+) \end{cases} \quad (8.1) \text{ through } (8.x)^\dagger$$

3. Real Per Capita Income

$$w = w(w^+, w_{-1}^+, \Delta E^+, \Delta P^-) \quad (9)$$

4. Demoeconomic Interface

$$\rho = \frac{LF}{P} \quad (10)$$

$$u = 1 - \frac{E}{LF} \quad (11)$$

$$\rho = \rho(u^-, w^+, m^+, t^+) \quad (12)$$

$$u = u(\bar{u}^+, u_{-1}^+, \Delta E^+, \Delta P^-) \quad (13)$$


---

\*This minimal formulation includes a redundant equation. For consistency, one equation among (4), (7), (12), and (13) must be discarded (see section 4).

\*\*An expected positive impact is denoted by a + and an expected negative impact is denoted by a -.

†The two alternative specifications apply to the goods-producing and service-producing sectors, respectively.

Table 1. Continued.

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II. VARIABLES

Endogenous Variables

P = total population

b = crude birth rate

d = crude death rate

M = net migration flow

E = total employment

A = agricultural employment

WSE = wage and salary employment

$E_i$  = employment in sector i

u = unemployment rate

w = real per capita income

$\rho$  = labor force participation rate

LF = labor force

$\Delta E = E - E_{-1}$

$\Delta P = P - P_{-1}$

$m = \frac{M}{\hat{P}} = \text{net migration rate}$

$\hat{P} = \frac{P_{-1} + P}{2}$

Exogenous Variables

NEMP = national civilian employment

S = other employment

$\bar{u}$  = national unemployment rate

$\bar{w}$  = national real per capita income

t = time trend

---

SOURCE: Ledent (1981).



to the existence of four alternative ways of closing the labor market submodel. Finally, the last two sections offer a comparison of these four alternatives that is based on qualitative considerations (in section 3) as well as an empirical analysis using data for the metropolitan area of Tucson, Arizona (in section 4).

#### 1. DERIVATION OF THE UNEMPLOYMENT RATE: THE TWO ALTERNATIVES

By definition, unemployment is an accounting concept that results from a direct comparison of the total number of persons in the labor force (LF) and the total number of persons actually employed (E). It is generally measured by the ratio (u) of the number of people unemployed to the size of the labor force, or unemployment rate

$$u = \frac{LF - E}{LF}$$

Therefore, the minimal framework proposed in the preceding section includes an unemployment variable that is derived from the above [see equation (11), Table 1].

However, a review of past statistical models of regional growth (Ledent 1981) reveals that 6 (Glickman 1972b, 1977; Klein and Glickman 1973; Adams et al. 1975; H.S. Chang 1976; Jefferson 1978; Rubin and Erickson 1980) out of the 23 models that contain a labor market submodel have adopted a less conventional treatment, justified by the nature of the unemployment rate.

As is clear from the identity above, the value of the unemployment rate follows from the comparison of the employment to labor force ratio with the value 1. In relation to this, we may here recall the well-known fact that the value of a variable A linked to two variables B and C of known value is likely to be much less accurate when the variable A depends on the difference B - C or the quotient B/C than when it depends on the addition B + C or the multiplication BC (Alonso 1964).

As a result, the prediction of the unemployment rate from previously available labor force and employment forecasts can lead to inaccurate values *all the more so because the two forecasted variables take on values that are similar.*

This statement can be illustrated as follows. On differentiating the definition of  $u$ , we have

$$\frac{\Delta u}{u} = \left(1 - \frac{1}{u}\right) \left(\frac{\Delta E}{E} - \frac{\Delta LF}{LF}\right)$$

To fix the ideas, suppose that, in a given observation or estimation period,  $E$  has been overestimated by 1.5 percent and  $LF$  has been underestimated by 1.5 percent. The application of the above formula shows that, if the true unemployment rate is equal to 4 percent, the calculated unemployment rate underestimates the true value by as much as 72 percent.

In the first approximation, the precision of the unemployment rate can be evaluated from

$$\frac{\Delta u}{u} \approx \frac{1}{u} \left(\frac{\Delta LF}{LF} - \frac{\Delta E}{E}\right)$$

a relationship that shows that the precision obtained is proportional to the reciprocal of the unemployment rate and to the difference between the precisions of the total labor force and employment estimates. It follows that the inaccuracy of the unemployment rate estimates is much less if the deviations of the labor force and employment variables from their respective true values have the same sign. Nevertheless, even if these deviations are relatively similar, the imprecision of the unemployment measure may remain important. For example, suppose that the precisions of the labor force and employment forecasts are +2 and +1.2 percent, respectively (a rather good prediction of these two variables) and that the true unemployment rate is 4 percent. The forecasted value of the unemployment rate is

then approximately 5 percent, i.e., 20 percent higher than its true value.

At this stage, we may restate the above problem in a statistical perspective: confidence intervals regarding forecasts of an unemployment rate, defined as a residual, are likely to be large, covering more than the usual range of variations of such a rate so that the forecasted point estimates may well fall outside this range.

The implication of the above for the construction of a statistical model of regional growth is clear. The endogenous derivation of the unemployment rate from a simple comparison of total labor force and employment is likely to affect the credibility of the whole model, especially if the employment measure appears as an explanatory variable in several stochastic equations. An economic-demographic model for Arizona (Battelle Columbus Laboratories 1973) provides a good illustration of this point. In this model, the unemployment rate, determined as a residual, is given the central role since most of the important linkages between endogenous variables are carried out through this variable: the unemployment rate affects age-specific fertility and net migration rates as well as sectoral wages. Under such circumstances, the low accuracy of the prediction of the endogenous variables of the model and the "noise" thus introduced tends to amplify as the forecasting period is extended. After a while, unemployment rates take on unreasonable values, thus causing the other variables of the model to behave erratically.

Possibly, the best way to attenuate the difficulty associated with the derivation of the unemployment rate directly from its definitional equation is to make this variable the dependent variable of a stochastic equation. This was done in the 6 models alluded to earlier. Therefore, the definitional equation (11) of the original minimal formulation of the labor market submodel must be replaced by a stochastic equation (13) in which the independent variables included are suggested by obvious intuitive considerations. They consist of the national unemployment

rate, the one-year lagged values of the dependent variable, and the relative changes in both employment and population.

## 2. THE CONSISTENCY PROBLEM: A FUNDAMENTAL EXPOSITION

The substitution of the stochastic equation (13) for the definitional equation (11) in determining the unemployment rate does not affect the validity of the latter, which still holds. Under such circumstances, our minimal formulation now includes one more equation than there are endogenous variables.

Clearly, this observation raises a problem of coherence between the main labor market variables, a problem that can be simply stated with the help of Figure 1. In practice, the following five aggregate variables--population, labor force, employment, the labor force participation rate, and the unemployment rate--must be predicted. No model can independently forecast all five variables since they are related by two definitional equations: those defining the labor force participation rate and the unemployment rate. Inevitably, this means that two of the five variables have to be calculated as residuals, i.e., they are to be obtained from the other three variables--labeled as primary variables--on the basis of the two aforementioned definitional equations. Perhaps the obvious candidates for residuals are the labor force participation and unemployment rates, since they are not basic numbers. When observed as a residual, however, the unemployment rate may often take on absurd values as was pointed out earlier. Thus, another choice of the residual variables appears advisable.

Fundamentally, the *consistency* problem just raised requires one to choose two variables as residuals or, equivalently, three primary variables among the five aforementioned demographic variables. Thus, ten different cases, corresponding to the alternative ways of choosing two (or equivalently three) variables among five, are possible. Among these, we can immediately rule out

- 1) the two cases in which the three primary variables are those involved in the definitions of the labor force participation rate and the unemployment rate
- 2) the other two cases in which both labor force and the labor force participation rate are included as primary variables

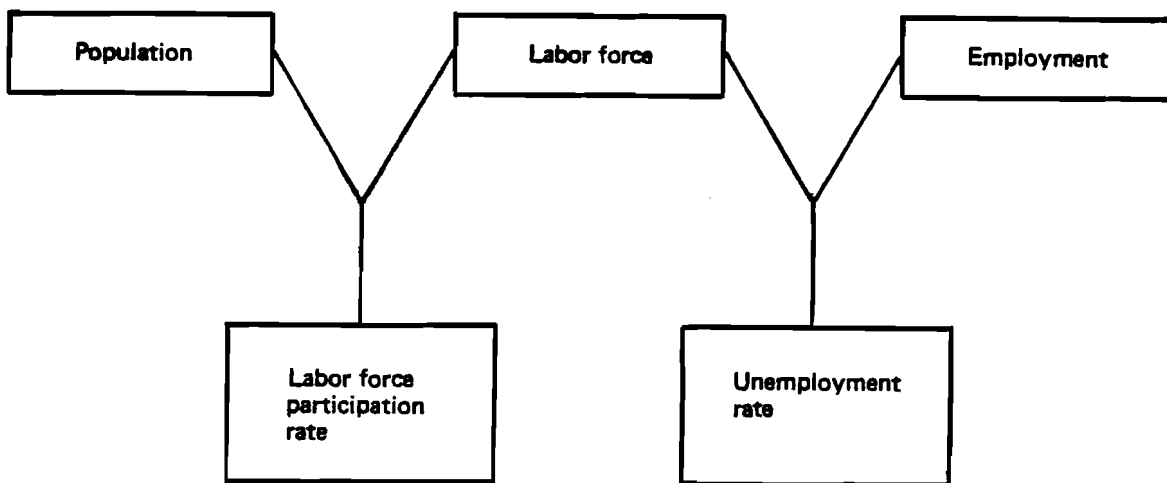


Figure 1. The basic relationships between the main labor market variables. (Source: Ledent, 1978:547.)

This leaves us with six cases which we can classify into four groups identifiable by the main residual variable:

- group A: employment
- group B: population
- group C: labor force participation rate
- group D: unemployment rate

Whereas groups B and C each contain a unique case, groups A and D contain two that have either the total labor force or the labor force participation rate as a primary variable.

Note that past statistical models of regional growth with a labor submarket have always adopted a specification corresponding to one of the three cases pertaining to groups C and D, all of which have employment and population as primary variables. None of the cases having one of these two variables taken as a nonprimary variable seems to have been used in the past (see Table 2).

Of course, in both groups A and D, the specification of the labor force participation rate as a primary variable is preferable to the specification of the labor force as a primary variable. Unlike the latter, the former allows for an explicit separation of the population size effect on the level of the labor force. Thus, it seems that the labor force variable is less important than the other four variables. Therefore, it should always be chosen as a nonprimary variable and be determined from either the identity that determines the labor force participation rate or the one that defines the unemployment rate.

Under such circumstances, the consistency problem can be reformulated. The specification of a regional demoeconometric model involves the joint and simultaneous consideration of four main variables--employment (E), population (P), the labor force participation rate ( $\rho$ ), and the unemployment rate (u)--that are linked by an identity

$$E = (1 - u)\rho P$$

obtained by combining the identities that define the labor force participation rate and the unemployment rate. Of necessity, one of these variables must be derived from the others. Since there are four alternative ways of choosing this variable, we are thus left with four alternative cases (A.b, B, C, and D.b).

This naturally leads to four variants of our minimal labor market submodel in Table 1, which are obtained by discarding one appropriate stochastic equation. This equation must have a

Table 2. The six alternative cases of the labor market submodel and corresponding existing models (E = employment; P = population; LF = labor force;  $\rho$  = labor force participation rate; u = unemployment rate).

Case	Primary Variables	Nonprimary Variables	Corresponding Existing Models	
			Name of author(s)	Variables taken exogenously
A.a.	P/LF/u	E/ $\rho$		
A.b.	P/ $\rho$ /u	E/LF		
B.	E/ $\rho$ /u	P/LF		
C.	E/P/u	LF/ $\rho$	Adams et al. (1975), H.S. Chang (1976), Jefferson (1978) Glickman (1972b, 1977) Klein/Glickman (1973), Rubin/Erickson (1980)	P -
D.a.	E/P/LF	$\rho$ /u	Puffer/Williams (1967) and Moody/Puffer (1969)*, Dagenais (1973), Salvas-Bronsard et al. (1973), Licari et al. (1973), Hall/Licari (1974) Crow et al. (1973) Glickman (1971), Peterson/Wall (1972), S. Chang (1979)	P $\rho$ -
D.b.	E/P/ $\rho$	LF/u	Klein (1969), Chau (1970)** Ghali/Renaud (1975) Ichimura (1966), Bell (1967), Crow (1969, 1973), Czamanski (1969) Glickman (1972a)	P, $\rho$ P $\rho$ -

\*In this model, the net migration component is endogenously determined but is a direct function of an exogenous economic variable.

\*\*In this model, total labor force is determined as a simple function of the (exogenous) total population, which is equivalent to assuming an exogenous labor force participation rate.

dependent variable that is, or is directly related to, the variable among the four main labor market variables chosen as nonprimary:

- variant A, corresponding to the case of employment as the residual variable, is obtained by removing either the agricultural employment equation (7) or one among the sectoral employment equations (8.1) through (8.x)
- variant B, corresponding to the case of population as the residual variable, follows from taking out the net migration equation (4)
- variant C, corresponding to the case of the labor force participation rate as the residual variable, is obtained by discarding the labor force participation rate (12) and
- variant D, corresponding to the case of the unemployment rate as the residual variable, results from the removal of the unemployment rate equation (13)

Interestingly enough, the choice of the equation to remove has major consequences for our minimal labor market submodel that are clearly revealed by a comparison of the internal structure pertaining to the four variants (see Figures 2 through 5).

- 1) The direction of the various linkages between the main labor market variables differs from one variant to another.
- 2) The exogenous information carried by the driving forces is not identically entered in all variants.

These two types of consequences are examined below.



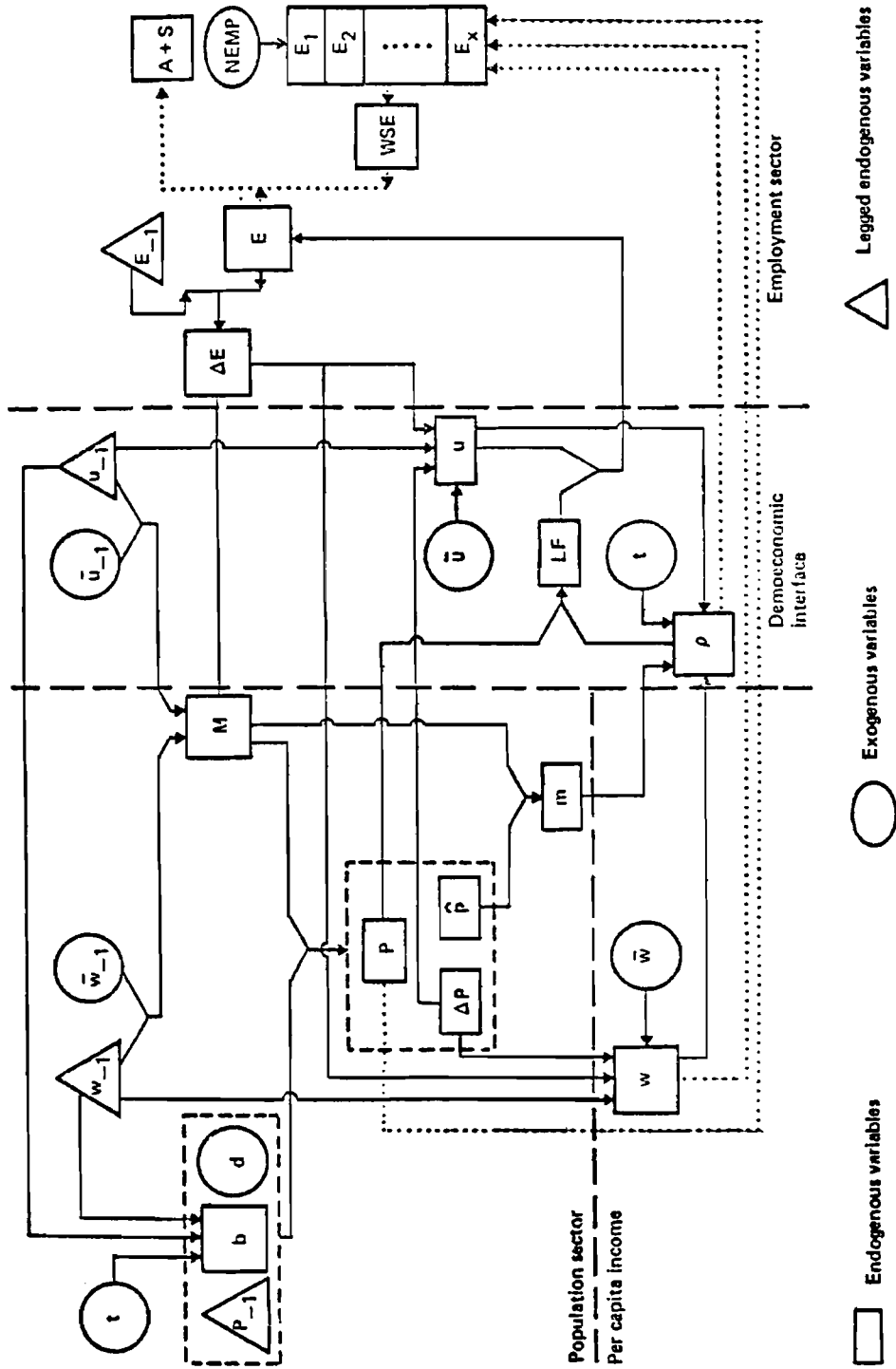


Figure 2. Structure of the minimal formulation: variant A (employment as a nonprimary variable).

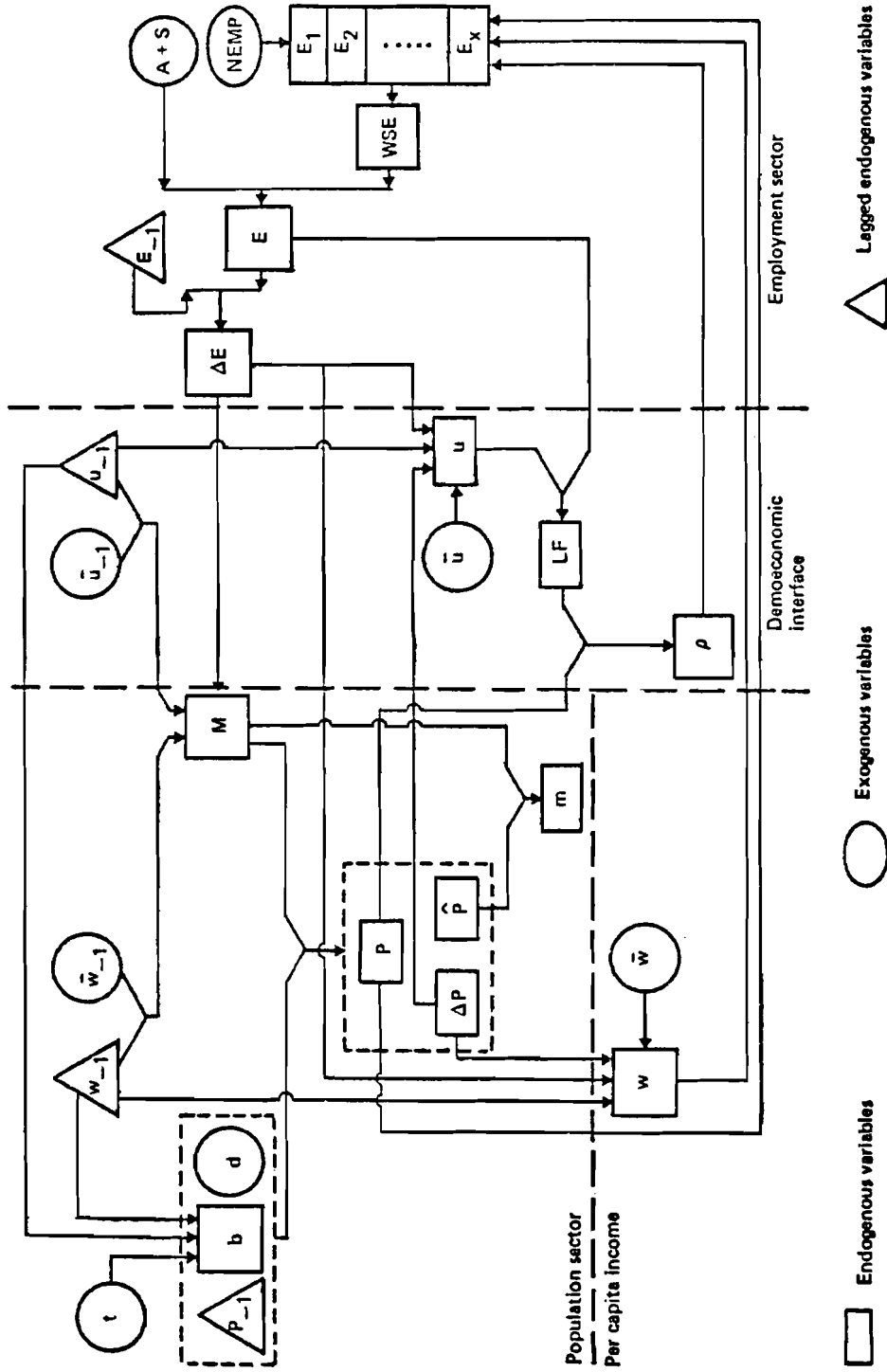


Figure 3. Structure of the minimal formulation: variant C (labor force participation rate as a nonprimary variable).



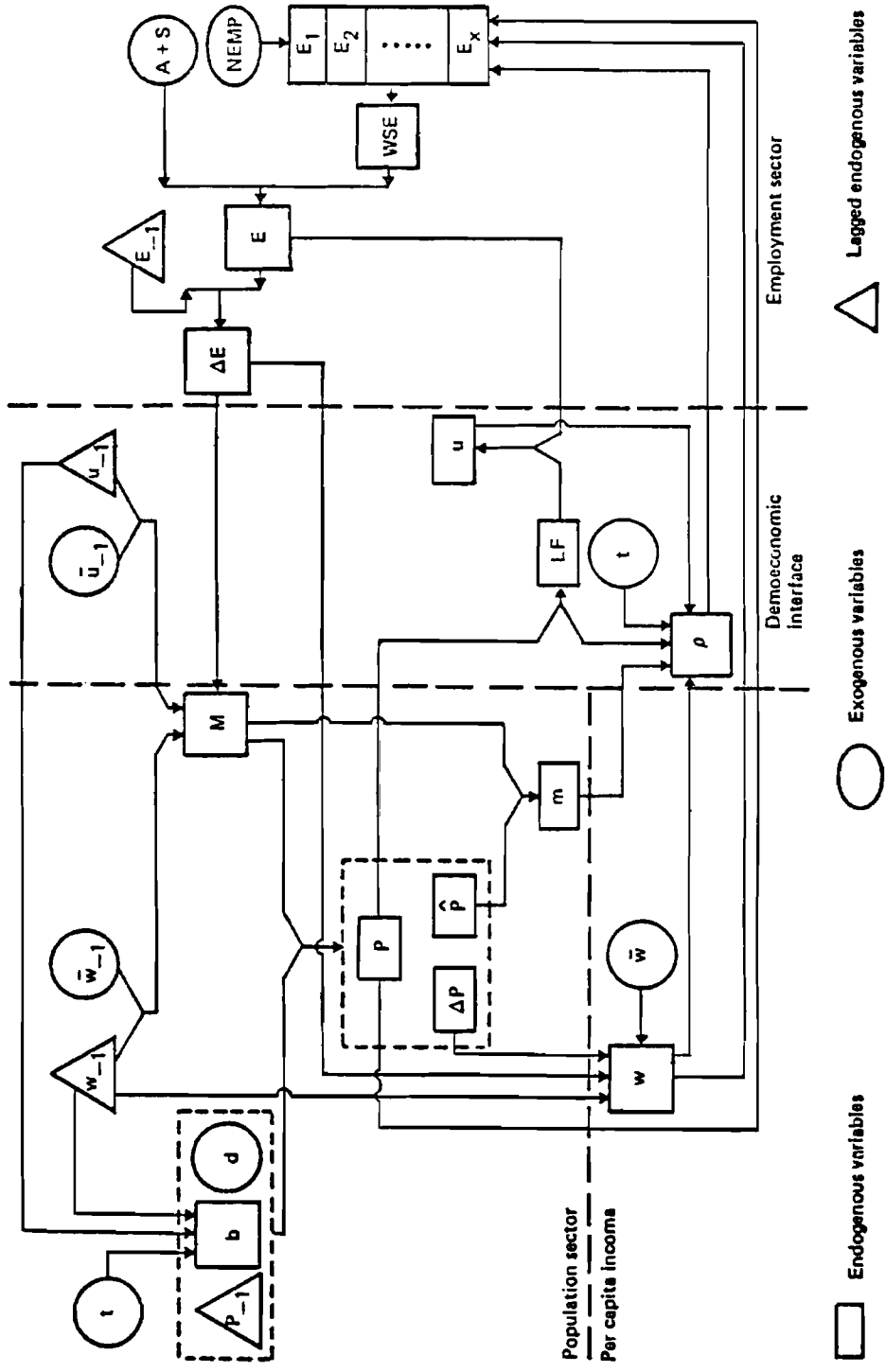


Figure 5. Structure of the minimal formulation: variant D (unemployment rate as a nonprimary variable).

### 3. A QUALITATIVE COMPARISON OF THE FOUR VARIANTS

In each variant, the direction of the linkages among the main labor market variables reflects, in fact, the way in which the two identities that define the labor force participation rate and the unemployment rate are used (see Table 3). On the basis of Alonso's (1964) point regarding the differing precision of a variable according to whether it has been calculated from a multiplication or a quotient, we might expect some sizable differences across variants.

First, the identity (10) that defines the labor force participation rate, is used as such only in variant C. After an adequate transformation, it is used in variant B to derive population and in variants A and D to derive labor force. Noting that the actual use of this identity involves a product of two variables in variants A and D rather than a quotient, as in the alternative variants, we then conclude that identity (10) introduces less inaccuracy in variants A and D than in variants B and C.

Second, the identity (11) that defines the unemployment rate is used as such only in variant D in which, as was seen earlier, it introduces a high inaccuracy resulting from the comparison of two variables (employment and labor force) that take on close values. In the other variants, this identity is used to derive employment (variant A) and labor force (variants B and C). Naturally, since the unemployment rate  $u$  intervenes through  $1 - u$ , the inaccuracy thus introduced is necessarily much smaller than in variant D, with most likely an overall low mark in variant A (where  $1 - u$  is used in a product rather than in a quotient as in variants B and C).

Therefore, combining the observations just made we here conclude that the definitional relationships (10) and (11) introduce into the model an accuracy that is, *a priori*, lowest in variant A, intermediate in variants B and C, and highest in variant D.

Table 3. The four alternative variants: specification of the equations that determine the main labor market variables.

Variable	Variant*			
	A	B	C	D
E	$E = LF(1 - u)$	(5)	(5)	(5)
u	(13)	(13)	(13)	$u = 1 - \frac{E}{LF}$
$\rho$	(12)	(12)	$\rho = \frac{LF}{P}$	(12)
P	(1)	$P = \frac{LF}{\rho}$	(1)	(1)
M	(4)	(1)**	(4)	(4)
LF	$LF = \rho P$	$LF = \frac{E}{1 - u}$	$LF = \frac{E}{1 - u}$	$LF = \rho P$

\*Variant A: employment as a nonprimary variable

B: population as a nonprimary variable

C: labor force participation rate as a nonprimary variable

D: unemployment rate as a nonprimary variable

\*\* (1) rewritten as  $M = P \left( 1 + \frac{b - d}{2} \right) - P_{-1} \left( 1 - \frac{b - d}{2} \right)$

We now turn to the second type of differences observed among the four alternative variants, one that is related to the way in which the exogenous information is incorporated. Clearly, the driving force that normally contributes to the determination of the sectoral employments (demand oriented) or the net migration flow (supply oriented) cannot be incorporated when the employment variable or, alternatively, the population variable is taken as the nonprimary variable. Therefore, in contrast to variants C and D, both of which are demand and supply oriented, variants A and B have a more restrictive orientation: a supply orientation in the case of variant A and a demand orientation in the case of variant B. Thus, from a theoretical viewpoint, it seems that variants C and D are preferable to variants A and B.

Whereas the driving force that normally contributes to the determination of population change (net migration) is taken out altogether in the case of variant B, the driving force that contributes to the determination of employment change is still at work in variant A; but it only affects the sectoral employment variables determined in an appendage to the sector that determines the main labor market variables (see Figure 2). This observation naturally suggests that one perform a slight alteration of variant A so that it takes on a mixed demand/supply orientation, thus making it as acceptable as variant C and D from a theoretical viewpoint. The leading idea here is a reintegration of the determination of the sectoral employments within the principal loop of the model. This can be achieved, for example, by substituting wage and salary employment for total employment in the equations where the latter is used as an explanatory variable (the real per capita income and unemployment rate equations). Thus, if variant A is amended in this way, only one among the four variants does not allow for a mixed demand/supply approach: variant B which does not incorporate a supply-oriented driving force.

Finally, on combining the conclusions made above, the following expectations can be put forth. First, variant C (the labor force participation rate as a nonprimary variable) should be the best performing variant. However, if amended as indicated above, variant A (employment as a nonprimary variable) should be a valid competitor. Second, variants B (population as a nonprimary variable) and D (the unemployment rate as a nonprimary variable) should be much less accurate, mainly because they incorporate less external information (variant B) or are affected by a computing problem (variant D).

#### 4. A COMPARATIVE EMPIRICAL ASSESSMENT OF THE FOUR VARIANTS

The comparative study of the four alternative variants of our labor market submodel, begun in the previous section with qualitative considerations, continues now with a quantitative analysis. For this purpose, the minimal formulation of Table 1 was fitted, using an ordinary-least-squares (OLS) procedure, to annual data for the rapidly growing metropolitan area of Tucson, Arizona, covering the period 1957-1977. The final regression equations obtained are listed in Table A1 of the Appendix. Relevant details and comments can be found in Ledent (1981).

On the basis of the estimated equations, three kinds of simulations\* were conducted:

- 1) a simulation over the whole observation period
- 2) an exhaustive series of two-year simulations over the observation period
- 3) an *ex ante* forecasting exercise for 1978 and 1979

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\*The entire computing work (equation estimation and model simulations) was carried out with the help of a single program intended for testing and simulating simultaneous-equation models: the Stochastic Simulation System (STS) developed by Schleicher (1980).



In all three experiments, the mean average percentage error (MAPE)--a statistic that reflects the discrepancy between the forecasted and actual values of a given variable--was chosen to assess the performance of all four variants.

*Ex post* Simulation 1957-1977

Table 4 sets out the MAPEs obtained from the simulation of each variant over the whole observation period. It indicates that, for 12 out of 16 selected variables (especially for the 4 main demoeconomic variables) the lowest MAPE relates to variant C. Clearly, this variant is the best performing; it is well ahead of variants D and B (D has better MAPEs than B for the four demoeconomic variables except the unemployment rate).

Naturally, variant A in its original version, is much worse: its MAPEs are generally two times higher than for any other variant. However, its amendment, presented in section 3, substantially increases its accuracy:\* the new MAPEs are generally similar to those of variant B except those relating to net migration and population variables, which are significantly better.

Regardless of the variant considered, three of the selected variables appear to have distinctively higher MAPEs amounting to 10 percent or more. They are the unemployment rate, the net migration flow, and the employment level in the construction sector.

For example, in the case of the unemployment rate, the MAPE obtained ranges from 12.6 percent (variant C) to 23.9 percent (variant D), excluding the original variant A. However, such values provide a misleading idea of how well the model replicates the past evolution of this variable, the nature of which (as was seen in section 2) substantially differs from that of most variables. Fortunately, a more illuminating assessment can

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\*The revised estimates of the real per capita income and unemployment rate equations are shown as equations (9') and (13') at the end of Table A1 in the Appendix.

Table 4. *Ex post* forecasts 1957-1977: mean average percentage errors (MAPEs) according to the alternative variants.

Variable	Variant*					Single Equation Estimation
	A		B	C	D	
	Original	Amended				
<u>Population</u>	6.86	2.55	2.90	2.20	2.55	--
Net Migration	164.19	71.35	136.38	59.90	151.88	20.62
<u>Total Employment</u>	10.99	5.03	5.08	3.42	4.40	--
Wage and Salary						
Employment	9.55	4.58	5.42	3.41	4.73	--
Manufacturing	9.54	5.82	6.74	5.34	4.80	3.48
Mining	4.70	3.21	5.50	4.24	5.02	3.24
Construction	23.11	13.79	22.42	12.58	22.61	7.23
Transportation	9.75	4.97	5.30	4.18	5.15	2.08
Trade	10.25	4.98	5.00	3.54	4.65	1.15
FIRE	15.23	8.69	9.27	8.27	9.70	5.15
Services	11.40	5.26	4.50	3.86	4.73	1.51
Government	7.82	3.90	3.16	3.41	3.92	2.09
<u>Per Capita Income</u>	5.69	3.46	4.42	3.68	4.25	1.75
<u>Labor Force</u>	10.55	4.92	4.60	3.37	4.65	--
Labor Force						
Participation Rate	5.30	2.81	2.83	2.20	2.76	0.75
Unemployment Rate	28.80	13.42	15.64	12.58	23.88	6.47

- \*Variant A: employment as a nonprimary variable  
 B: population as a nonprimary variable  
 C: the labor force participation rate as a nonprimary variable  
 D: the unemployment rate as a nonprimary variable

be obtained from Figure 6, which contrasts the trajectory of the unemployment rate implied by the *ex post* simulation of variants C and D (those with the lowest and highest MAPE for the variable concerned) with the corresponding actual evolution. In brief, Figure 6 suggests a relatively good performance of variant C although the goodness-of-fit declines significantly after 1970. Moreover, it shows the better performance of variant C *vis-à-vis* variant D.

Note that the comparison of the MAPE values across the four variants provides a striking confirmation of our earlier speculations regarding the treatment of the unemployment rate variable: the MAPE value ranges from 12.6 to 15.6 percent for the three variants (amended A, B, and C) in which it is specified as a primary variable as opposed to a 23.9 percent value for variant D in which it is specified as a residual variable.

Since the unemployment rate is used as an explanatory variable in the net migration equation of the variants with population as a primary variable (all but B), we would expect the net migration MAPE values relating to these variants to present differentials that more or less reflect those given by the unemployment rate MAPE values. As a matter of fact, the net migration MAPE value is equal to 59.9 percent for variant C, 71.4 percent for the amended variant A, and 151.9 percent for variant D. And what about the MAPE obtained with variant B in which the net migration flow is determined as a residual between total population change and natural increase? Noting

- 1) Alonso's (1964) observation that the prediction accuracy of variables derived from the difference of two others is pertinent
- 2) the external information that normally allows for a discrepancy between regional and national economic conditions is not incorporated in variant B

we would expect a relatively high MAPE value. The value actually obtained--136.4 percent--is in the neighborhood of the MAPE value obtained with variant D rather than with the other variants.

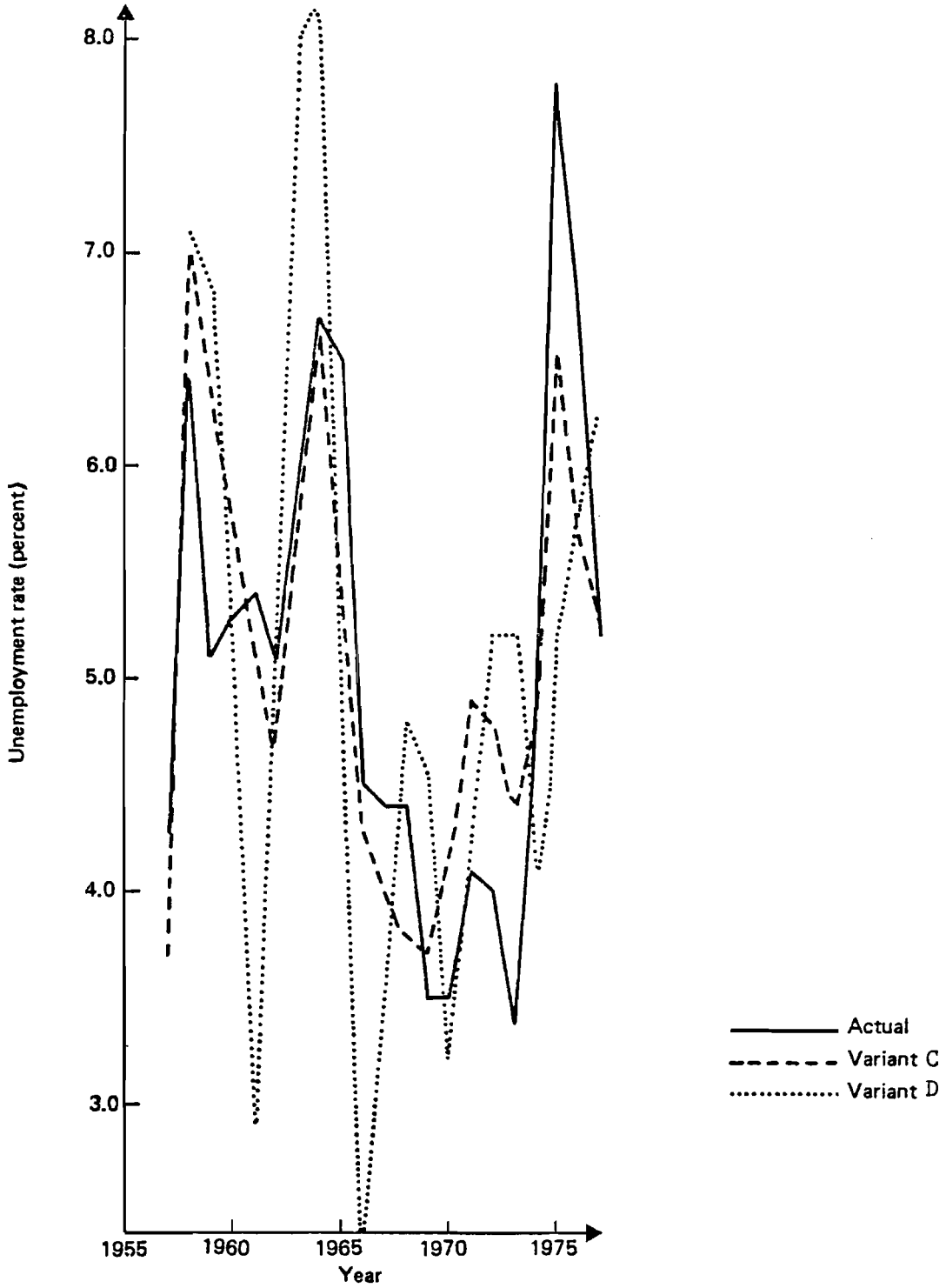


Figure 6. Unemployment rate, 1957-1977: *ex post* simulation (variants C and D) versus actual evolution.

The MAPE values concerning the net migration variable are even larger than those for the unemployment variable; they range from 59.9 to 151.9 percent as opposed to 12.6 to 23.9 percent in the latter case. Undoubtedly, these rather high MAPEs can be attributed to net migration being a *flow* variable rather than a *stock* variable (such as population, employment, labor force, etc.). Again the MAPE statistic allows a comparison among variants but does not provide a clear understanding of how well each variant replicates the past evolution of the variable concerned. Figure 7, which contrasts the trajectory of the net migration flow implied by the simulation of variants C and D (those with the lowest and highest MAPE for this variable), suggests conclusions similar to those in Figure 6: in particular, variant C reproduces the actual variations of the net migration flow rather closely until 1970, after which it simply follows an averaging path.

Finally, turning to the third variable with a large MAPE, construction employment\*--of which population change is an explanatory variable--we have MAPE differentials across variants that reflect the MAPEs observed for net migration flow. The MAPEs obtained with variants A (amended version) and C are half the values obtained with variants B and D.

It is worth noting the result that is correlated with the relatively poorer performance of variant B. Quite consistently over the simulation period, the solution of variant B was harder to obtain than the solution of the alternative variants. On average, the convergence of the Gauss-Seidel iterative method that underlies the simulation procedure used here required 52 iterations in the case of variant B as opposed to 7 to 15 in the case of the other variants (including the original version of variant A). Therefore, whereas the specification of the unemployment rate as a nonprimary variable was thought to be quite

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\*The rather high MAPE obtained for this variable is essentially the consequence of the relatively poor performance of the estimated regression equation.

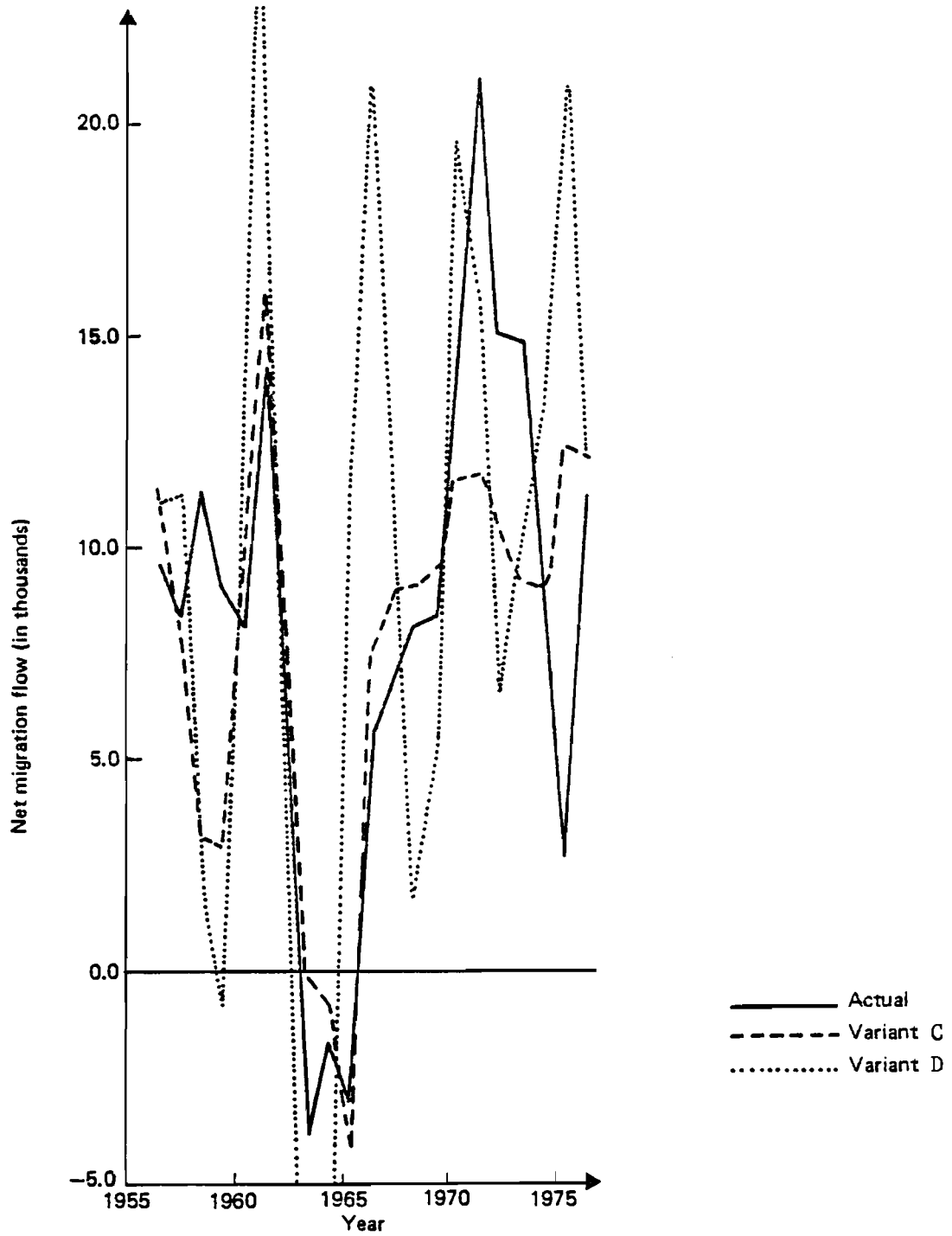


Figure 7. Net migration flow, 1957-1977: *ex post* simulation (variants C and D) versus actual evolution.

a serious problem affecting the performance of a demoeconometric model, the above results suggest that an equally important and perhaps more troublesome problem follows from the specification of the population variable as a nonprimary variable, that is, the derivation of the net migration flow as a residual.

#### Two-year *ex post* Simulations

In addition to the simulation over the whole observation period, a series of 20 simulations for two consecutive years T and T+1 (T = 1957, 1958, ..., 1976) was performed for each of the four variants (for both the original and amended versions of variant A). In all cases, the MAPE statistic relating to selected variables was calculated for each of the 20 two-year simulations and then an average value was derived, one that is shown in Table 5.

No figures appear in the column for variant B. The reason is that the corresponding model could not be solved in 13 out of 20 instances within a maximal number of iterations fixed at 1000\*, a finding that is hardly surprising in light of the relatively higher difficulty encountered earlier for simulating variant B over the whole simulation period. The smaller the discrepancy between the actual and simulated values (those obtained from the simulation of variant B over the observation period) of the main labor market variables in a given year, the less difficult the convergence of the model for a two-year simulation of the model, starting in the next year. This speculation was confirmed as convergence (which often required several hundred iterations) was obtained only for the first four simulations (T = 1957, 1958, 1959, and 1960) and three simulations at the turn of the seventies (T = 1969, 1970, and 1971), i.e., periods for which the simulated values of variant B (simulation over the whole observation period) were comparatively closer to the actual ones.

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\*By contrast, the two-year simulations relating to the other variants rarely required more than 20 iterations.

Table 5. Two-year *ex post* forecasts over the period 1957-1977: average values of the mean average percentage errors according to the alternative variants.

Variable	Variant*				
	A		B	C	D
	Original	Amended			
<u>Population</u>	0.81	0.80	--	0.70	1.20
Net Migration	46.31	42.03	--	33.42	69.60
<u>Total Employment</u>	2.22	2.26	--	2.17	2.28
Wage and Salary Employment	2.30	2.12	--	2.03	2.21
Manufacturing	3.13	5.84	--	5.22	4.63
Mining	4.32	4.08	--	3.55	3.65
Construction	6.90	7.44	--	8.87	11.49
Transportation	2.19	1.47	--	3.59	2.67
Trade	2.50	2.13	--	2.10	2.02
FIRE	4.90	4.24	--	5.98	7.21
Services	2.88	3.36	--	1.85	2.22
Government	2.61	3.12	--	2.17	2.32
<u>Per Capita Income</u>	2.95	2.89	--	2.42	2.58
<u>Labor Force</u>	1.83	1.95	--	1.86	1.87
Labor Force Participation Rate	1.30	1.48	--	1.50	1.15
Unemployment Rate	12.57	7.33	--	7.73	20.83

\*Variant A: employment as a nonprimary variable  
 B: population as a nonprimary variable  
 C: the labor force participation rate as a nonprimary variable  
 D: the unemployment rate as a nonprimary variable



The average values of the MAPEs obtained for variants other than B, are substantially lower than the MAPEs obtained earlier when simulating the alternative variants over the whole observation period. Actually, the increase in goodness-of-fit obtained when going from a simulation over the whole period to the two-year simulations is relatively homogeneous across variants although the gain is proportionately higher in the case of the amended version of variant A\*: the goodness-of-fit in this variant is, as before, worse than for variant C but is now better than for variant D.

#### *Ex ante* Forecasts 1978-1979

We have tested the ability of the four alternative variants to replicate the past evolution of Tucson's main labor market variables. We have also performed another test to assess the ability of the four alternative variants to predict the future rather than the past evolution of these variables.

Let us recall that the period of observation chosen for the estimation of the stochastic equations was 1957-1977. On the basis of these equations, for each variant, we generated forecasts for 1978 and 1979, naturally attributing the exogenous variables to the values actually observed in those two years. Consequently, the comparison of the forecasted values of the endogenous variables with the corresponding actual values provides a true indication of the forecasting ability of each of the variants (the error introduced by the *ex ante* prediction of the exogenous variables being removed).

Table 6, which shows the MAPE values obtained for each variant, suggests that variant A (both original and amended versions) performs slightly better than variants C and D and much better than variant B (for which convergence was obtained only after several hundred iterations). This superiority of

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\*Note that the original version of this variant performs as well as the amended version in this case.

Table 6. *Ex ante* forecasts 1978-1979: mean average percentage errors according to the alternative variants.

Variable	Variant*				
	A		B	C	D
	Original	Amended			
<u>Population</u>	1.3	1.1	4.1	0.9	0.4
Net Migration	76.5	53.8	218.2	42.5	47.1
<u>Total Employment</u>	1.4	2.2	8.7	4.0	4.4
Wage and Salary Employment	3.5	4.0	9.9	4.8	5.3
Manufacturing	9.9	10.0	13.2	11.7	11.0
Mining	5.7	5.6	6.6	5.6	5.5
Construction	26.0	27.3	44.2	28.0	31.0
Transportation	4.8	5.1	8.1	6.5	5.7
Trade	3.1	3.7	10.2	4.7	5.0
FIRE	3.8	4.1	10.1	4.4	5.8
Services	4.9	5.5	13.3	6.2	6.9
Government	6.2	6.0	3.2	5.7	5.4
<u>Per Capita Income</u>	4.0	3.2	1.6	4.0	3.6
<u>Labor Force</u>	0.1	0.7	7.1	2.3	2.0
Labor Force Participation Rate	1.5	1.7	3.2	3.2	2.2
Unemployment Rate	36.4	46.0	48.7	49.2	73.9

- \*Variant A: employment as a nonprimary variable
- B: population as a nonprimary variable
- C: the labor force participation rate as a nonprimary variable
- D: the unemployment rate as a nonprimary variable

variant A over variant C, which reverses the conclusion obtained from the preceding experiments, is not surprising, however.\* The relatively better performance of variant C in the case of the two-year simulation series was an average result, i.e., it was not observed for all of the 20 simulations performed. It is clear that, in other circumstances, we could have found variant C or perhaps variant D (but not variant B) to have the best *ex ante* forecasting record.

Summarizing the findings of the various experiments reported above, we find that variant B (with a nonprimary population variable and a residual net migration) is harder to simulate as well as substantially less reliable than the other variants.\*\* Among these, variant D is the least accurate because of the imprecision introduced by the specification of the unemployment rate as a nonprimary variable. Depending on the circumstances, the most accurate set of forecasts can be obtained from either variant C or the amended version of variant A. But, as suggested by the results of our simulations over the whole observation period, variant C (the labor force participation rate as a nonprimary variable) is likely to perform better than amended variant A (employment as a nonprimary variable).

Finally, comparing the above findings with the qualitative considerations developed earlier in section 3, we see that our empirical assessment not only broadly confirms our expectations about the comparative performance of the four alternative variants but also suggests two additional results: the comparatively lower performance of variant B and the slight superiority of variant C over variant A (amended version).

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\*Comparison of Tables 5 and 6 suggests that the *ex ante* forecasting ability of our statistical model is substantially lower than its *ex post* forecasting ability, especially for some variables such as manufacturing and construction employments, the net migration flow, and the unemployment rate. This finding, undoubtedly, reflects the inability of our statistical model to predict the development of the peculiar economic conditions that took place in Tucson over the period 1978-79 (see Ledent 1981).

\*\*We assume that variant A is implemented in its amended version.

## SUMMARY AND CONCLUSION

This paper has demonstrated the existence of a severe accuracy problem that affects the consistent modeling of the aggregate variables of a regional labor market (employment, labor force, and population). In brief, these variables and the two that are normally derived from them--the unemployment rate and the labor force participation rate--cannot be derived independently. Two among them must be determined, as residuals, from the others; these are labor force and one other variable.

We have shown that the choice of the second residual variable strongly affects the accuracy of the labor market submodel considered. The choice of the unemployment rate and that of population should be avoided; a conclusion initially derived from qualitative considerations and later confirmed through empirical testing. The second residual should be employment or, preferably, the labor force participation rate.

Of course, the modeling problem treated here is not restricted to the particular situation examined in this paper (the regional labor market). It probably has a more general bearing that concerns modeling situations in which several variables are linked by one or several definitional equations. However, it is doubtful that there are many cases in which the accuracy issue can be as acute as in the case dealt with here where one definitional equation involves a *comparison* (quotient) of two variables that take on close values.

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## APPENDIX

Table A1 simply lists the final regression equations obtained from fitting the minimal formulation of Table 1 to annual data for the metropolitan area of Tucson, Arizona.

Note here the existence of a few modifications brought to the minimal framework of Table 1. First, owing to the smallness of agricultural activities in Tucson, no agricultural employment equation could be adequately estimated. Hence, since the sum of the agricultural and other employments was found to be relatively constant over time, equation (7) in Table 1 was replaced by an equation in which total employment is a simple function of total wage and salary employment. Second, a few variables that reflect some special features of the Tucson economy were introduced

X = dummy variable (= 1 since 1964, 0 otherwise)

PC = real price of copper

Y = dummy variable (= 1 in 1961 and 1962, 0 otherwise)

PS = square of the population level

The interpretation of all the other variables is the same as in Table 1.

In each regression equation, the statistic appearing between parentheses below each coefficient is the corresponding t-statistic. With 21 observations (i.e., 20 degrees of freedom), the critical values of this statistic for a two-tail test are 1.729 at the 10 percent level, 2.093 at the 5 percent level, and 2.861 at the 1 percent level. The other statistics shown are the coefficient of determination ( $R^2$ ), the corrected coefficient of determination ( $\bar{R}^2$ ), the mean average percentage error (MAPE), the Durbin-Watson statistic (DW), and the coefficient of autocorrelation ( $\rho$ ).

Table A1. Application to Tucson, 1957-1977: OLS equation estimates.

Equation Estimates	Equation Number	R <sup>2</sup>	$\bar{R}^2$	MAPE	DW	$\rho$
<u>Population Sector</u>						
• Crude birth rate						
$b = 0.9881[b]_{-1} + 0.2825 \times 10^{-3}[u]_{-1} + 0.3875 \times 10^{-3}[w]_{-1}$ (37.19) (2.36) (2.03)	(2)	.976	.973	2.89	0.87	0.56
• Crude death rate						
$d = 1.603 \times 10^{-3} + 0.8543[d]_{-1} - 1.248 \times 10^{-6} \hat{p}$ (1.36) (5.42) (1.47)	(3)	.630	.589	2.10	1.93	0.02
• Net migration flow						
$M = 15.22 + 1.544\Delta E_1 + 1.105\Delta E_3 - 5.161[u]_{-1} + 3.651[\bar{u}]_{-1} + 4.688[w - \bar{w}]_{-1}$ (7.94) (3.43) (3.70) (9.87) (7.58) (1.60)	(4)	.932	.909	20.62	-1.84	-0.37
<u>Employment Sector</u>						
• Total employment						
$E = 14.08 + 1.005WSE$ (23.09) (169.78)	(7)	.999	.999	0.58	0.43	0.80
• Manufacturing employment						
$E_1 = -9.865 + 0.2409 \times 10^{-3}NEMP + 33.61\rho - 1.990X - 1.012 \frac{WP}{E}$ (3.73) (4.83) (3.35)	(8.1)	.955	.944	3.48	1.69	0.14
• Mining employment						
$E_2 = -3.706 + 0.7735[E_2]_{-1} + 0.0790PC + 0.2192 \frac{WP}{E}$ (4.82) (21.64) (6.34) (2.01)	(8.2)	.992	.991	2.96	2.08	-0.05
• Construction employment						
$E_3 = 1.806 + 0.7985E_2 + 1.447Y + 0.1407(P - [P]_{-1}) + 0.05188([P]_{-1} - [P]_{-2})$ (3.57) (8.41) (2.30) (3.78) (1.44)	(8.3)	.920	.900	7.23	1.82	0.06
• Employment in transportation and communication						
$E_4 = -3.626 + 0.3554[E_4]_{-1} + 4.348 \times 10^{-3}P + 17.39\rho$ (4.23) (2.98) (3.39) (4.49)	(8.4)	.974	.969	2.08	2.06	-0.04
• Trade employment						
$E_5 = -11.23 + 0.1011 \times 10^{-3}PS + 38.24\rho + 2.652w$ (3.02) (28.95) (6.67) (6.16)	(8.5)	.998	.998	1.15	1.39	0.30

• Employment in finance and real estate

$$E_6 = -3.967 + 0.01765P + 8.077p - 0.09982u \quad (8.6) \quad .969 \quad .963 \quad 5.15 \quad 0.92 \quad 0.52$$

(3.56) (11.91) (1.96) (1.95)

• Service employment

$$E_7 = -6.750 + 0.1120 \times 10^{-3}PS + 19.55p + 1.541w \quad (8.7) \quad .998 \quad .998 \quad 1.51 \quad 1.13 \quad 0.42$$

(4.46) (33.85) (3.50) (3.78)

• Government employment

$$E_8 = -14.44 + 0.6395[E_8]_{-1} + 23.72 \times 10^{-3}P + 4.475w + 0.4162u \quad (8.8) \quad .996 \quad .995 \quad 2.09 \quad 2.25 \quad -0.21$$

(3.61) (4.96) (1.69) (3.77) (2.58)

Real Per Capita Income

$$w = -2.361 + 0.9585[w]_{-1} + 2.516 \frac{E}{[E]_{-1}} - 4.127m \quad (9) \quad .972 \quad .967 \quad 1.75 \quad 2.50 \quad -0.31$$

(3.66) (22.88) (3.89) (3.29)

Demoeconomic Interface

• Labor force participation rate

$$\rho = 0.03193 + 0.7278[\rho]_{-1} + 0.02049w + 0.1587m \quad (12) \quad .980 \quad .976 \quad .075 \quad 1.62 \quad 0.10$$

• Unemployment rate

$$u = 2.866 + 0.7063[u]_{-1} + 0.5205(\bar{u} - [\bar{u}]_{-1}) - 21.77 \frac{E}{[E]_{-1}} + 20.53 \frac{P}{[P]_{-1}} \quad (13) \quad .885 \quad .857 \quad 6.47 \quad 1.60 \quad 0.14$$

(0.46) (5.67) (3.87) (4.94)

ALTERNATIVE EQUATIONS FOR AMENDED VARIANT A

$$w = -1.939 + 0.9771[w]_{-1} + 2.037 \frac{WSE}{[WSE]_{-1}} - 3.739m \quad (9') \quad .970 \quad .964 \quad 1.80 \quad 2.49 \quad -0.31$$

(3.29) (3.55) (2.96)

$$u = 0.8603 + 0.7248[u]_{-1} + 0.5248(\bar{u} - [\bar{u}]_{-1}) - 18.83 \frac{WSE}{[WSE]_{-1}} + 19.54 \frac{P}{[P]_{-1}} \quad (13') \quad .881 \quad .851 \quad 5.82 \quad 1.81 \quad -0.03$$

(0.14) (5.76) (3.82) (4.77) (2.84)

SOURCE: Ledent (1981).

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