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IMPLEMENTABLE MOTIVATION MECHANISMS FOR MONOPOLY AND DEEP DISEQUILIBRIA

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SUMMARY

Problems of economic reforms of centrally plannned economies are often related to the questions of price formation mechanisms. A decentralized competitive price formation would be desirable, but is usually considered infeasible because of monopolistic positions of many producers and because of deep disequilibria situations arising during a reform. The paper surveys known anti-monopolistic taxation, reward or motivation mechanisms that make decentralized price formation possible. A new, robust mechanism of a price-dependent modification of the revenue function is proposed and analysed. While such mechanisms might work well in the short-term equilibrium case, they do not necessarily motivate a long-term equilibrium; strategic problems related to attaining long-term equilibrium are discussed. For the deep disequilibria case, another type of mechanisms is needed. A price-dependent modification of the profit function is shown to result in a robust motivation mechanism implementable for deep disequilibria (without market influence). Various aspects of implementability of motivation mechanisms are discussed in the paper; it is also shown that an introduction of another agent - a distributor with properly chosen motivation - might resolve many problems of anti-monopolistic motivation.

IMPLEMENTABLE MOTIVATION MECHANISMS FOR MONOPOLY AND DEEP DISEQUILIBRIA

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1. THE PROBLEM

Recent discussions on an economic reform in Poland stressed the importance of motivation mechanisms in decentralized planned economies; similar problems occur in Hungary and Yugoslavia, in other planned economies, as well as in large corporations in market economies.

The advantages of profit maximization motives have long been recognized in planned economies. The theoretical analysis of related issues has been started by the work of Lange in 1936 and continued by many researchers. (See Lange 1964, Yunker 1975). Obviously, profit maximization and competitive price setting are socially desirable, if they are achieved by an efficiency increase; lowering costs, better resource utilization, better management, and if they result in a just distribution of profits or their use for socially needed investments. Not desirable is profit maximization achieved by a monopolistic price increase and resulting in an unjust distribution.

On the other hand, planned economies usually favour rather concentrated production. Many modern industries and services are characterized by increasing returns to scale, and the optimal production scale of a single producer is often comparable or higher than the possible demand range in a small economy, a typical case of natural monopoly. In this situation, the fear of a monopolistic price increase in one of the most important factors that prevent many planned economies from an introduction of decentralized price setting mechanisms.

Traditionally, centralized planned economy is managed by central setting of production volumes. Prices are then determined according to a mark-up rule, on the basis of average cost estimation with some reasonable profit rate. Clearly, such mechanism is often counter-efficient, it motivates cost increase, rather than decrease, and a guaranteed rate of profit does not stimulate better management or resources utilization. An alternative mechanism - a central setting of prices with production volumes determined by each producer through profit maximization* - is also not satisfactory. It requires large and precise information for price setting, leaves excuses for inefficient producers, results in unreasonable excess profits for more efficient ones, etc. More advanced price-setting systems, as in Hungary, combine centrally set accounting rules for determining prices with guideline prices from international markets; additionally antimonopolistic imports can effectively limit price increases. However, such mechanisms might not necessarily work in deep disequilibrium situations, particularly if an economy is confronted with severe problems of the balance of trade.

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* Such a central price setting has often been erroneously labelled as Lange solution; in fact, Lange solution is to set prices equal to marginal production costs and to minimize average production costs, but he did not specify central control nor motivation mechanisms that would result in such a solution. In this sense, the paper is a continuation of Lange's original ideas.

The basic monopolistic price limitation in a market economy - the threat of entry of a competitor, if the monopolist does not control basic resources - requires a fast response from the capital market in order to be effective, and has never worked perfectly in a short-term; in particular it would not work in a deep disequilibrium situation.

Recent theoretical developments for establishing competitive prices include many mechanisms such as incentive-compatible mechanisms - see, e.g., Kalai, Postlewaite and Roberts (1979), or bidding procedures for the rights to use common property resources, see, e.g. Young (1980). More related to problems of monopolistic production are the results of Weitzman (1974, 1978) and, in particular, antimonopolistic taxation or reward mechanisms as proposed by Domar (1974), Freixas (1980), Tam (1981). Such mechanisms consist of modifications of the profit function of the producer, introduced by central authority; the producer is free to set the price himself, but the mechanism induces him to set competitive price. Such mechanisms might be applicable for planned economies; in fact, many pragmatically, ad hoc devised motivation schemes has been applied in various planned economies. However, several problems arise in connection with the applicability of such mechanisms.

First is the question of robustness of a mechanism, that is, the insensitivity of its results with respect to inevitable errors in setting the parameters of the mechanism by the central authority. This question has been,to some extent, analyzed by Weitzman (1974, 1978) and Freixas (1980). In this paper, simple measures of robustness of such mechanisms are introduced and the robustness of the mechanisms of Domar, Freixas and Tam is compared; moreover, a new, more robust mechanism is introduced and analyzed.

Second is the question of short-term versus long-term properties of such mechanisms. Clearly, the parameters of any such mechanisms should be kept constant by the central authority for a reasonable time, say, a year; they can be kept constant that long if the mechanism is reasonably robust.* However, both the control authority and the producers know that these parameters will be finally changed. Will a producer be sufficiently motivated to invest and attain a long-term competitive price? Or will he rather engage in a dynamic game with the planning authority? What type of the game can he play? Will he try to hide profits and take advantage of them? These and similar questions cannot be fully analysed in this paper, however, they cannot be bypassed, and a short discussion of them is thus presented.

Third, most important for applications in Polish economy, is the question of application of such mechanisms in deep disequilibrium situation. By deep disequilibrium we understand here large production shortages (caused by inavailability of imported resources, inefficiencies and other reasons) resulting in a large gap between the demand and supply, rationing, etc. In such a situation, even competitive prices equilibrating the markets in a short term might be too high to be socially acceptable; reasons for market shortages have to be first removed before introducing competitive pricing. Thus, special motivation mechanisms are needed *that would work even in the absence of a market and nevertheless stimulate efficiency and promote finally market equilibration.* Neither of the previously analysed mechanisms has the required properties to be applicable in a deep disequilibrium situation. A new mechanism is proposed, specially constructed for deep disequilibria. Its properties are analysed, both for the cases of single and multiple producers, and single and multiple products first in deep disequilibrium, then in the presence of a market.

The above questions do not exhaust problems of applicability of motivation mechanisms: questions of central control of a disequilibrated economy and many others have also to be resolved.

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*Domar (1974) and Tam (1981) assume quite a number of parameter iterations before their mechanisms come close to competitive prices. This is not acceptable in practice - what has been, in fact, realised by both authors. Domar, being first to introduce this type of mechanism, concentrated more on basic ideas, while Tam devoted much attention to the speed of convergence of his mechanism.

However, these questions are only shortly commented upon in the final part of the paper; it is shown that another agent, an appropriately motivated distributor, might resolve best the questions of long-term properties and robustness of motivation mechanisms for the producer.

Although this paper has a theoretical character, the mathematical apparatus is kept to a necessary minimum, and the discussions of various aspects of implementability of motivation mechanisms are stressed instead.

2. NOTATION AND ASSUMPTIONS

Following Domar (1974), we consider a simple description of a market for one commodity with many consumers, where the demand is described by a function

$$q = D(p) \tag{1}$$

where q - quantity, the demand volume, and p - the price. The function D is not necessarily derived from a utility maximization of a single consumer or consumer's group; however, we assume it to be twice differentiable, convex and strictly decreasing. The demand-price elasticity is denoted here as absolutely-valued:

$$\varepsilon_D(p) = \left|\frac{dD(p)}{dp}\right| \cdot \frac{p}{D(p)} \tag{2}$$

and is assumed to be strictly increasing with p, with $\lim_{p\to 0} \varepsilon_D(p) \leq 1$, $\lim_{p\to\infty} \varepsilon_D(p) > 1$.*

* This assumption is known to be necessary for the existence of a normal monopolistic equilibrium. Not all typical demand models satisfy this assumption - neither those resulting from the Cobb-Douglas utility function, nor from linear expenditure systems, only the CES utility function with elasticity of substitution higher than 1 results in elasticity ε_D increasing to a number greater than 1. However, this assumption is not crucial - we are not that much interested in a normal monopolistic solution, we do not necessarily derive the demand from a single utility maximization and, finally, it might be argued that on a consumer market with subsistence levels (for basic needs commodities, typically described by linear expenditure systems) monopoly cannot hold at high prices, since consumers will eventually resort to self-provision.

The inverse demand $p = D^{-1}(q)$ is characterized, under above assumptions, by a convex, twice differentiable, strictly decreasing function, with elasticity $1/\varepsilon_D(D^{-1}(q))$ strictly increasing with q. The monopolistic revenue is characterized by the function

$$R(q) = q \cdot p(q) \quad ; \quad p(q) = D^{-1}(q) \tag{3}$$

and the marginal revenue has the known form

$$MR(q) = \frac{dR(q)}{dq} = p(q) \cdot \left(1 - \frac{1}{\varepsilon_D(p(q))}\right) \tag{4}$$

Similarly, we can write revenues and marginal revenues for several producers with assumed market shares, etc.

However, we consider in the initial part of the paper only a single producer, whose total production costs are characterized by a function $C(q)$, assumed to be twice differentiable, neither convex nor concave. In long-term, the long-term costs $LC(q)$ correspond to a typical case of natural monopoly, that is, $LC(q_1 + q_2) \leq LC(q_1) + LC(q_2)$; however, as substantiated by the discussion in the introductory part, we are mostly interested in a short-term analysis. Thus, a typical characteristics of a locally convex average costs $AC(q) = \frac{C(q)}{q}$ and the corresponding, also locally convex marginal costs $MC(q) = \frac{dC(q)}{dq}$ are assumed. The normal monopolistic solution would be then derived by equating $MR(q) = MC(q)$. The competitive pricing solution is derived from $p(q) = MC(q)$ and will be called 'ideal' solution, denoted by $\hat{p}, \hat{q}$, although it corresponds only to a short-term efficient allocation of resources in this simple economy.

3. MOTIVATION MECHANISMS FOR COMPETITIVE PRICE SETTING

Another agent in the economy - a central authority* - is supposed to be able either to set centrally a production quota

*We do not attach any ideological meaning to the notions of central authority or monopoly. The central authority might be a benevolent dictator, the monopoly might be natural - or all the consumers might form a cooperative in order to take advantage of increasing returns to production scale, elect a most able manager and give him a reward corresponding to a motivation mechanism selected by voting and on grounds of efficiency considerations.

$\bar{q}$, or a price $\bar{p}$, or, by using taxation, subsidy, or rewards, to influence the motivation of the producer in such a way that he himself sets the price close to the ideal level.

Observe that, if the elasticity ε_D is very high - say, $\varepsilon_D > 20$ - the central authority might choose to leave the producer maximizing his profit at the normal monopolistic solution. In fact, the price index:

$$\frac{p}{MC} = \frac{\varepsilon_D}{\varepsilon_D - 1} = \pi_M$$

would not deviate then from the ideal value 1 more than 5.5% (we omit in further text the denotation of the dependence of functions on their arguments). The price index $\frac{p}{MC}$ can be used to indirectly characterize the efficiency of a motivation mechanism. Clearly, for a given characteristics of production costs, if there were a uniform group of consumers characterized by a sufficiently smooth utility or benefit function that would have its maximum at $\frac{p}{MC} = 1$, then the loss of this benefit would be the smaller, the closer this index is to 1. Weitzman (1974) and Freixas (1980) use direct, statistic estimations of consumers benefit losses to characterize the efficiency of motivation mechanisms. However, such direct estimations are difficult to obtain for more complicated motivation mechanisms, and are substantiated only if we assume that the demand is derived from the maximization of a single benefit function. For these reasons, we use here the more simple, indirect index $\pi = \frac{p}{MC}$.

If $\varepsilon_D > 20$ and the central authority had chosen to set price $\bar{p}$, it would have to estimate MC at the ideal solution more accurately than up to 5.5% in order to obtain better results than at the normal monopolistic solution. If $\varepsilon_D > 20$ and the central authority had chosen to set production quota $\bar{q}$, it would also have to estimate MC in order to establish $\bar{q}$. Assume* that the estimate of $\bar{q}$ was initially correct and that MC has changed;

*The same result can be obtained under the reverse assumption that MC does not change but an error in its estimation has been made, leading to an error in $\bar{q}$ and in the resulting index π. There is no need, however, to perform this sensitivity analysis in order to get a result that can be obtained immediately from a much more simple and clearly equivalent reasoning. See Wierzbicki (1977) for a detailed discussion of relativity principles in sensitivity analysis.

clearly, the central authority would again have to trace the changes of MC more accurately than up to 5.5% in order to obtain better results than at the normal monopolistic solution, at which no information for the central authority is needed. Thus, in the special case when the elasticity ε_D is very high, the normal monopolistic solution might be the best mechanism if the central authority does not have precise information. However, we are more interested in cases of much lower elasticity ε_D and there are also other motivation mechanisms than the three compared above.

We do not consider here the special mechanism of Weitzman (1978) in which the central authority sets prices and additionally penalizes for deviations from centrally set quantities. Although more robust than either centrally set prices or centrally set quantities, this mechanism has the same drawbacks as both previous mechanisms: too much information required, too few natural signals that transmit the information to the central authority. We are interested in more decentralized mechanisms, where the price-setting behaviour of the producer gives natural signals to the central authority.

Domar (1974) proposed a motivation mechanism that can be interpreted as follows. The reward for the producer should be proportional to the following modified profit*

$$PD = zqp - C \; ; \quad z = \frac{\bar{\varepsilon}_D}{\bar{\varepsilon}_D - 1} \tag{6}$$

where z is set by the central authority and $\bar{\varepsilon}_D$ is a guess of the elasticity ε_D at the ideal price. Although Domar proposes a

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* More precisely, PD should not be interpreted as modified profit, but only as a reward function, while the actual reward might be covered from a small part of traditionally defined profit. Practical applications of reward functions - for example, in Hungary - show that even if additional salaries dependent on such functions are only a very small part of traditionally defined profits, they are a very strong motivation - provided that they are shared between the managers and all staff of the producing enterprise, see further discussions. The distribution of the rest of traditionally defined profit and its use for investments requires different, long-term motivations. With all these reservations, however, we shall use here the term 'modified profit' for the sake of simplicity of interpretations.

rather slowly converging iterative scheme to arrive at the right coefficient z, we shall assume that the central authority uses its best estimate of ε_D and sets the coefficient z for a possibly long time, modifying it only when really necessary. We can easily derive the price index π for this mechanism:

$$\frac{p}{MC} = \frac{(\bar{\varepsilon}_D - 1)\ \varepsilon_D}{\bar{\varepsilon}_D(\varepsilon_D - 1)} = \pi_D$$

Observe that this index deviates in a symmetrical way from 1 no matter whether $\bar{\varepsilon}_D$ was initially set incorrectly, or whether it was initially correct and then the actual ε_D has changed; the same is true also for other indices that characterize deviations from optimality, but usually in a linear approximation sense. The following robustness coefficient** of this index can be considered:

$$E_{\pi D \varepsilon D} = \frac{\partial \pi_D}{\partial \varepsilon_D} \cdot \frac{\bar{\varepsilon}_D}{\pi_D} \Big|_{\bar{\varepsilon}_D = \hat{\varepsilon}_D} = \frac{1}{\hat{\varepsilon}_D - 1} \tag{8}$$

where $\hat{\varepsilon}_D$ is the elasticity at the ideal solution. Thus, the mechanism is robust only if $\hat{\varepsilon}_D$ is reasonably large. For example, if we would like to have $\frac{p}{MC}$ deviating from 1 by no more than 5%, and if $\hat{\varepsilon}_D > 5$, we would have to estimate $\hat{\varepsilon}_D$ up to 20% accuracy, a reasonable requirement. However, if $\hat{\varepsilon}_D < 2$, we have to estimate it with a higher accuracy than the resulting accuracy of π. Moreover, the mechanism does not work if $\hat{\varepsilon}_D \leq 1$.

Guesnerie and Laffont (1978) and Freixas (1980) considered 'linear taxation mechanisms' that can be interpreted as the maximization of the following modified profit:

$$PF = (p + \bar{t})q - C \ ; \quad \bar{t} = \frac{\overline{MC}}{\bar{\varepsilon}_D} \tag{9}$$

**Actually, it is an 'unrobustness' coefficient, the mechanism is robust if this coefficient is small.

where $\bar{\varepsilon}_D$ and $\overline{MC}$ are estimates of the elasticity ε_D and marginal cost MC at the ideal solution.* The corresponding price index is

$$\frac{P}{MC} = \frac{\varepsilon_D}{\varepsilon_D - 1} \cdot (1 - \frac{\bar{t}}{MC}) = \pi_F \tag{10}$$

and, after estimating the dependence of MC on $\bar{t}$, we obtain the robustness coefficient

$$E_{\pi FT} = \frac{d\pi_F}{d\bar{t}} \cdot \frac{\bar{t}}{\pi_F} \Bigg|_{\substack{\bar{\varepsilon}_D = \hat{\varepsilon}_D \\ \overline{MC} = \overline{MC}}} = - \frac{1}{\hat{\varepsilon}_D - 1} \left(1 - \frac{C''}{\hat{\varepsilon}_D C'' + 2\hat{p}/\hat{q}}\right) \approx$$

$$\approx - \frac{1}{\hat{\varepsilon}_D - 1} \ , \qquad \text{if } \hat{\varepsilon}_D > 1 \tag{11}$$

where $C'' = \frac{d^2C}{dq^2}$, see Appendix 1. Since $E_{T\varepsilon D} = -1$, $E_{TMC} = 1$, we have $E_{\pi F\varepsilon D} \approx \frac{1}{\hat{\varepsilon}_D - 1}$ and $E_{\pi FMC} \approx - \frac{1}{\hat{\varepsilon}_D - 1}$. Thus, the linear taxation mechanism has similar robustness as the Domar mechanism- actually, it is slightly less robust, since errors in $\bar{\varepsilon}_D$ and $\overline{MC}$ might accumulate . The linear taxation mechanism is applicable even if $\hat{\varepsilon}_D < 1$, although it is very bad if $\hat{\varepsilon}_D \approx 1$, and not really robust for $\hat{\varepsilon}_D < 2$, since we have then to estimate $\bar{t}$ with a higher accuracy than the resulting accuracy of π.

Tam (1981) considers a mechanism that is equivalent to the maximization of the following modified profit:

$$PT = (q - \bar{q})p - C \tag{12}$$

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* Observe that if PF would indeed be interpreted as a modified profit after 'taxation' (in this case, after subsidy to the producer), we would obtain a contradiction: in order to prevent a monopolistic producer from charging monopolistic prices and earning monopolistic profits, we give him a subsidy that brings his profit again to monopolistic magnitude. However, PF can be legitimately interpreted as a reward function. Another reservation to such a mechanism is that once we start to modify prices by taxation or use prices an an element of taxation, we have to go to very detailed commodity classications and the problems of aggregating prices might be severe, see also Domar (1974). These problems will be investigated in further sections.

where $\bar{q}$ is an estimate of the ideal production quantity.* Thus, Tam mechanism is a relaxed way of setting production quotas. The price index π can be obtained as:

$$\frac{p}{MC} = \frac{\varepsilon_D}{\varepsilon_D - \dfrac{1}{1 + \bar{q}/q}} = \pi_T \tag{13}$$

and, after estimating the dependence of q on $\bar{q}$, we obtain the robustness coefficient (see Appendix 1)

$$E_{\pi TQ} = \frac{d\pi_T}{d\bar{q}} \cdot \frac{\bar{q}}{\pi_T}\Big|_{\bar{q} = \hat{q}} = \frac{1}{2\hat{\varepsilon}_D} \cdot \frac{1 + \hat{\varepsilon}_D \frac{\hat{q}}{\hat{p}} C''}{1 + 0.5\, \hat{\varepsilon}_D \frac{\hat{q}}{\hat{p}} C''} \approx \frac{1}{2\hat{\varepsilon}_D} \tag{14}$$

If the quantity $\bar{q}$ is estimated from $\overline{MC}$ and $\bar{\varepsilon}_D$ by a simplified demand formula with constant elasticity, then we can show that $E_{QMC} = -\hat{\varepsilon}_D$, $E_{Q\varepsilon D} = 0$. Thus, $E_{\pi TMC} \approx \frac{1}{2}$, $E_{\pi T\varepsilon D} = 0$. This can be interpreted that the Tam mechanism is approximately twice more robust than the central quantity setting: to guarantee deviations of no more than 5% from the ideal $\pi_T = 1$, we have to estimate the production cost MC up to 10% accuracy. Moreover, Tam mechanism works even for $\hat{\varepsilon}_D < 1$.

However, all these robustness estimates are not really satisfactory. To obtain a really implementable mechanism, we should like to have deviations of π of no more than 5% for errors in parameters - such as $\bar{\varepsilon}_D$ or $\overline{MC}$ - up to at least 20%, and this for a broad range of elasticities $\hat{\varepsilon}_D$, including $\hat{\varepsilon}_D < 1$.

** The reward function PT has a serious drawback, not shared with the functions PD or PF: the maximization of PT does not necessarily promote market equilibration. We should remember that the market constraint, for a monopolistic producer, has an inequality form, $p \le p(q) = D^{-1}(q)$. Thus, if $\bar{q}$ is too large and $p(\bar{q}) < MC(\bar{q})$, we have $\partial PT/\partial q < 0$, q should be smaller than $\bar{q}$; but then $\partial PT/\partial p < 0$, p should be decreased. In this situation, the monopolistic producer is simply motivated to stop producing. Thus, the robustness coefficients derived here apply fully only for $\bar{q} \le \hat{q}$; for $\bar{q} > \hat{q}$ they might be applicable only under additional reasons to produce (survival motivation).

This can be achieved by observing, first, that the knowledge either of the ideal $\hat{\varepsilon}_D$ or of the ideal $\widehat{MC}$ would suffice for obtaining $\pi = 1$. Thus, a good mechanism should have the first derivatives of π with respect to $\bar{\varepsilon}_D$, $\overline{MC}$ equal zero at $\bar{\varepsilon}_D = \hat{\varepsilon}_D$, $\overline{MC} = \widehat{MC}$. To derive such a mechanism, we can postulate that the coefficient z in Domar mechanism (6) be dependent not only on $\bar{\varepsilon}_D$, but also on the ratio $P/\bar{p}$, where p is the actual price charged by the producer, and $\bar{p} = \overline{CM}$ is an estimate of the marginal cost at the ideal solution. Thus, the modified profit is*:

$$PW = z \cdot p \cdot q - C \quad ; \qquad z = z\,(\bar{\varepsilon}_D,\ p/\bar{p}) \tag{15}$$

and the price index π takes the form (see Appendix 2)

$$\frac{P}{CM} = \frac{\varepsilon_D}{z \cdot (\varepsilon_D - 1 + \varepsilon_Z)} = \pi_W \quad ; \qquad \varepsilon_Z = - \frac{dz}{dp} \cdot \frac{p}{z} \tag{16}$$

We assume here $\varepsilon_Z > 0$, the coefficient z diminishes with increasing p. To obtain the required properties of the mechanism, we postulate:

$$z(\bar{\varepsilon}_D,\ p/\bar{p}) \Big|_{p/\bar{p} = 1,\ \text{all } \bar{\varepsilon}_D} = 1 \quad ; \qquad \frac{\varepsilon_D}{z(\bar{\varepsilon}_D - 1 + \varepsilon_Z)} \Big|_{\varepsilon_D = \bar{\varepsilon}_D,\ \text{all } p/\bar{p}} = 1 \tag{17}$$

These requirements result also in $\varepsilon_Z \Big|_{p/\bar{p} = 1,\ \text{all } \bar{\varepsilon}_D} = 1$, and

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* Again, the maximization of PW does not promote market equilibrium, although this drawback is less serious than in the case of PT. In fact, it is easy to show (Appendix 2) that if $\bar{p} < \hat{p}$, then the producer could maximize PW by choosing $p = \bar{p} = MC(\bar{q})$ with $\bar{q} < \hat{q} = D(\hat{p})$, while not satisfying market demand. However, we assume here for simplicity sake that there are other motivations that push the producer towards satisfying market demand. Nevertheless, this drawback is a serious flaw on the implementability of the PW motivation mechanism.

can be solved as a differential equation (see Appendix 2) to obtain:

$$z(\bar{\varepsilon}_D, p/\bar{p}) = \begin{cases} \dfrac{(p/\bar{p})^{\bar{\varepsilon}_D - 1} - \bar{\varepsilon}_D}{1 - \bar{\varepsilon}_D}, & \text{if } \bar{\varepsilon}_D \neq 1 \\ 1 - \ln(p/\bar{p}), & \text{if } \bar{\varepsilon}_D = 1 \end{cases} \tag{18}$$

$$\varepsilon_Z = \begin{cases} \dfrac{(1 - \bar{\varepsilon}_D)(p/\bar{p})^{\bar{\varepsilon}_D - 1}}{(p/\bar{p})^{\bar{\varepsilon}_D - 1} - \bar{\varepsilon}_D}, & \text{if } \bar{\varepsilon}_D \neq 1 \\ \dfrac{1}{1 - \ln(p/\bar{p})}, & \text{if } \bar{\varepsilon}_D = 1 \end{cases}$$

When computing derivatives of π_W we obtain, as expected

$\left.\dfrac{\partial \pi_W}{\partial \bar{\varepsilon}_D}\right|_{\bar{\varepsilon}_D = \varepsilon_D,\ p/\bar{p} = 1} = 0$, $\left.\dfrac{\partial \pi_W}{\partial \bar{p}}\right|_{\bar{\varepsilon}_D = \varepsilon_D,\ p/\bar{p} = 1} = 0$. Moreover,

also the second derivatives are obviously zero, and only the mixed derivative $\dfrac{\partial^2 \pi_W}{\partial \bar{\varepsilon}_D \partial \bar{p}} \neq 0$. Thus, the robustness coefficient can be defined as:

$$E^2_{\pi W \varepsilon DMC} = \frac{\partial^2 \pi_W}{\partial \bar{\varepsilon}_D \partial \bar{p}} \cdot \frac{\hat{p} \cdot \hat{\varepsilon}_D}{\pi_W} = -1 \tag{20}$$

It follows that, indeed, if we estimate $\hat{\varepsilon}_D$ and $\hat{CM} = \hat{p}$ up to 20% accuracy, we obtain deviations of π_W of approximately 4% ; if the estimates of $\hat{\varepsilon}_D$ and $\hat{CM} = \hat{p}$ would be only 30% accurate, we still obtain deviations of π_W of only about 9%. For larger deviations the approximation of π_W by the robustness coefficient (20) is less accurate (see examples of computations of the results of this mechanism in Appendix 2).

It remains to comment on the possibility of iterative estimation $\bar{p}_{t+1}$ of $\hat{CM} = \hat{p}$, provided the mechanism has been used with some $\bar{p}_t$, $\bar{\varepsilon}_{Dt}$ and, as a result, some actual p_t as well as z_t and ε_{Zt} have been observed. We assume that we know $\varepsilon_D \in [\varepsilon_{Dmin}; \varepsilon_{Dmax}]$, and also $\bar{\varepsilon}_D \in [\varepsilon_{Dmin}; \varepsilon_{Dmax}]$, thus we do not change $\bar{\varepsilon}_D$, $\bar{\varepsilon}_{D,t+1} = \bar{\varepsilon}_{Dt}$. If the expected changes of p were so small that the marginal costs MC could be considered constant, we would immediately estimate

$$\left(\frac{MC}{\bar{p}_t}\right) = \begin{cases} \frac{p_t}{\bar{p}_t} z_t \left(1 - \frac{1}{\varepsilon_{Dmin}}(1 - \varepsilon_{Zt})\right), & \text{if } \frac{p_t}{\bar{p}_t} < 1 \Rightarrow \varepsilon_{Zt} < 1 \\ \frac{p_t}{\bar{p}_t} z_t \left(1 - \frac{1}{\varepsilon_{Dmax}}(1 - \varepsilon_{Zt})\right), & \text{if } \frac{p_t}{\bar{p}_t} > 1 \Rightarrow \varepsilon_{Zt} > 1 \end{cases} \tag{21}$$

$$\left(\frac{MC}{\bar{p}t}\right) = \begin{cases} \frac{p_t}{\bar{p}t} z_t \left(1 - \frac{1}{\varepsilon_{Dmax}}(1 - \varepsilon_{Zt})\right), & \text{if } \frac{p_t}{\bar{p}_t} < 1 \Rightarrow \varepsilon_{Zt} < 1 \\ \frac{p_t}{\bar{p}t} z_t \left(1 - \frac{1}{\varepsilon_{Dmin}}(1 - \varepsilon_{Zt})\right), & \text{if } \frac{p_t}{\bar{p}_t} > 1 \Rightarrow \varepsilon_{Zt} > 1 \end{cases} \tag{22}$$

If $\left(\frac{MC}{\bar{p}_t}\right)_{max} < 1$, then $\bar{p}_t$ has to be decreased conservatively, to obtain $\bar{p}_{t+1} \geqq MC_{max}$; if $\left(\frac{MC}{\bar{p}t}\right)_{min} > 1$, then $\bar{p}_t$ has to be increased, to $\bar{p}_{t+1} = MC_{min}$. If $1 \in [\left(\frac{MC}{\bar{p}_t}\right)_{min}; \left(\frac{MC}{\bar{p}_t}\right)_{max}]$, then there is not enough information for taking action, $\bar{p}_{t+1} = \bar{p}_t$. For example (taken from the results in Appendix 2), suppose we know that $\varepsilon_D \in [0.4;0.6]$ and have chosen $\bar{\varepsilon}_D = 0.5$, $\bar{p}_t = 1$. The mechanism resulted in $p_t = 1.316$ (because actual MC = 1.4, $\varepsilon_D = 0.4$, but we do not know that) and in $z_t = 0.743$, $\varepsilon_{Zt} = 1.172$. We obtain then $MC_{min} = 1.257$, $MC_{max} = 1.400$, and thus increase $\bar{p}_{t+1} = 1.257$, which results in $p_{t+1} = 1.368$, within 3% accuracy of actual marginal costs.

If we cannot assume constant marginal costs MC, we can still estimate new $\bar{p}_{t+1}$, provided we know either the second derivative $C'' = \frac{d^2C}{dq^2}$, or marginal cost elasticity $E_{C'} = \frac{C''}{MC}\hat{q}$, or some bounds for them (see Appendix 2).

Summarizing the properties of the new proposed motivation mechanism - the modified notion of profit (15), (18) - we see that it is robust, requires only a very small number of iterative changes of $\bar{p} = \overline{CM}$ and some approximate knowledge of ε_D. It works also for $\varepsilon_D < 1$, in fact, even if $\varepsilon_D \approx 0$ or even in deep disequilibrium conditions, when the constraint (1) does not apply (in both these cases it is easy to show that this mechanism becomes equivalent to the central price setting). However, it has a fairly complicated form, which might be considered as a disadvantage.*

4. SHORT-TERM VERSUS LONG-TERM PROPERTIES OF MOTIVATION MECHANISMS

In the long-term relation between the producer and the central authority, the former will undoubtedly play a game, and the latter would be forced to take this into account. Such a game can be described formally as a metagame, or hierarchical game (see Germeer 1976); the central authority could choose its strategy from a class of profit modification functions, this

*On the other hand, many ad hoc constructed motivation mechanisms in planned economies have even more complicated forms - although they are usually applied as an augmentation of the central quantity setting mechanism. This does not mean that the proposed mechanism is suggested here as the best implementable one - it does not promote market equilibrium, nor does it perform satisfactorily in long term and deep disequilibria cases.

class can be formalized and axiomatically described, the best function from this class could be chosen - probably in a much more complicated form, than the relatively simple function (15), (18). However, this is not the purpose of this paper. For the sake of implementability of motivation mechanisms, much more important is to analyze the feasibility of various strategies of players - particularly those of the producer.

First strategy that might occur to the producer is that to be rewarded independently of the centrally set reward mechanism - by hiding profits in costs and using them for his own benefit. A private producer might do so, but also he has some limits, since it is necessary to show profit to the stockholders. In decentralized planned economies, the institutional solution that prevents the manager from enjoying 'representation costs' is relatively simple: the reward is shared by the manager and all workers in the factory in the form of a thirteenth and fourteenth salary. If the manager hides profits in costs and diminishes the additional reward to the workers, he will soon be in trouble.*

The second strategy that might occur to the producer is not to hide profits, but simply not to maximize the modified profits or reward functions, for example by charging prices higher than the marginal, simulating higher costs, even if the central authority has correctly set the parameters of the motivation mechanism. This is similar to the known strategy of hiding production capacities in the quantity-setting mechanism when the manager, rewarded for a production increase from year to year, always tries to have some hidden reserves. However, in the latter case, there is usually no penalty for hiding capacities, while under a modified profit motivation mechanism, the producer must consciously forego rewards. He

* Naturally, this institutional solution has some limits: if the profits are very large, the workers might agree to building a new holiday home for the factory (or sometimes even for the manager) charging it to production costs. Thus, excessive profits are socially undesirable - even if the marginal pricing would suggest them as Pareto-optimal.

risks that the central authority will not change the parameters of the motivation mechanism next year, and the reward will be foregone without long-term effects. Again, the institutional solution of sharing the rewards with workers makes it very difficult for a manager to apply this strategy - provided that the profit is not excessive.

A strategy that might successfully be applied by a monopolistic producer under modified profit motivation is not to invest, or fake investments (that is, invest in production of essentially different commodities), particularly if the demand elasticity ε_D is low. In such a case, he cannot increase the quantity sold - he would have to sell at much lower prices. If he presently has a slightly smaller capacity and sells at much higher short-term marginal costs, why should he forego future profits and invest, when there is no competition forcing him to do so? However, he might have some other motivation for investments: a belief that some investments are needed for the survival of the firm, or he might be conditioned to invest by a long experience of management in the traditional system of centrally set production quota. The central authority can try to persuade him to invest by announcing new parameters of the motivation mechanism (reward function) for a few years ahead. However, short-term motivation mechanisms do not naturally stimulate the desirable investment behavior of a monopolistic producer that would result in the long-term competitive equilibrium.*

Thus, if a traditional planned economy managed by production quota is reformed and a decentralized price setting together with appropriate motivation mechanisms is introduced instead, we can expect an abrupt decrease of the demand for investments. This phenomenon has been observed in Hungary and should be judged positively, since the unrestricted demand for investments observed

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* Theoretically, we could set the parameters of the motivation mechanisms in such a way that they will correspond to the estimated long-term competitive equilibrium, and stop to bother about the time-path and investments that are necessary to achieve this equilibrium. This, however, is a purely static approach that cannot be applied in practice: the producer would probably stop the production completely and argue that the estimated long-term equilibrium is infeasible, etc.

under a central setting of production quota is a major factor that contributes to development cycles of planned economies. However, a proper motiviation mechanism for investments in monopolistic production is a difficult problem. Since this problem is related also to motivation mechanisms for market equilibration, it will be discussed in the next section.

5. MOTIVATION MECHANISMS FOR DEEP DISEQUILIBRIA

By a deep disequilibrium we understand here a case where there is a large shortage of supply, resulting in queuing or even rationing, and the market does not work properly. Such a situation might be caused by a misfunctioning of the central quantity setting mechanism accompanied by a central price setting or mark-up rules for price setting. Typically, producers in such a situation have hidden production capacities - they have invested heavily, but did not utilize the investments - and current production costs are distorted by mismanagement, material input constraints and inefficient use of basic resources. When trying to reform such an economy through introducing decentralization, a fully decentralized competitive price setting is not a feasible solution. The reason is not only that many producers might have monopolistic positions, but also that the short-term equilibrium prices, due to the distortions of costs, might be several times higher than the previous centrally set prices. When trying to equilibrate the economy by the way of short-term equilibria, wage guarantees and monetary effects might result in a very high inflation. Thus, a more desirable path towards long-term equilibrium might lead originally through deep disequilibria, and the first step on this path is to introduce motivation mechanisms that would result in a better utilization of hidden production capacities, basic resources and in removing material constraints.

Sharp central price setting without quantity setting is not an implementable mechanism in such a situation, not only because the central authority does not have all the necessary information, but also since sharp central price setting has some very undesirable effects in deep disequilibria. For example, if there are several producers of the same commodity with widely different production costs, as in the case of chemical fertilizer production in Poland, a centrally set price leads either to excessive profits of the most efficient producers, or to an instant bankruptcy of the least efficient one. The bankruptcy is not admissible, since there are market shortages, and the least efficient producer should be subsidized until other producers expand their production; thus, the prices should be differentiated in deep disequilibria.

Although it might seem to be incompatible with classical economic intuition, it is possible to construct mechanisms that leave price decisions to the producers, motivate them to set marginal prices, to use fully their capacities and to decrease costs, and at the same time allow for central control of the price range - even without the influence of the market, in deep disequilibrium, where the producer could theoretically choose any price and quantity. In order to construct such a mechanism, we utilize the idea of price-dependent modification - however, not of the revenue function, as in the modified Domar mechanism (which is equivalent in deep disequilibrium to central price setting and thus not satisfactory) but of the entire profit.

Consider the following modified profit

$$PA = \alpha(p) \cdot (pq - C) \tag{23}$$

and suppose (1) is not binding, $q \neq D(p)$.* The function α can be any sufficiently strongly decreasing function of p, however, for the purpose of an easy implementation, it is assumed that this function has the form given in Fig.1, and is defined by an upper and a lower guideline price.

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* Observe that this is a stronger assumption than simply letting $\varepsilon_D \to 0$, which would correspond only to a constant demand q independent of price p, but still binding the producer.

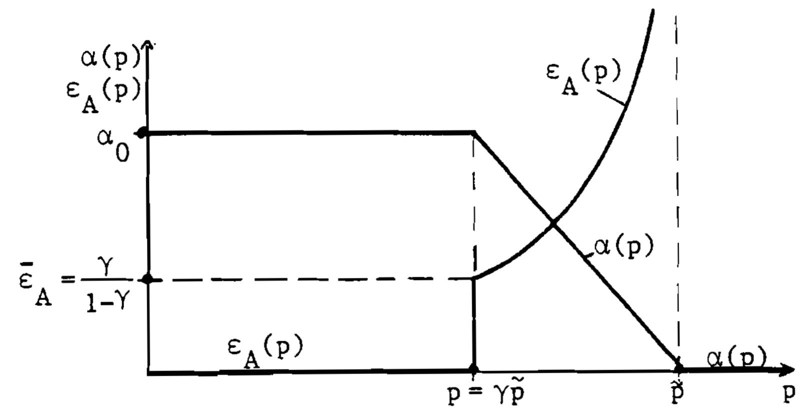

Figure 1. Function α in a price-dependent profit taxation mechanism and its elasticity ε_A, where $\tilde{p}$ - upper guideline price, $\underline{p} = \gamma\tilde{p}$ - lower guideline price $(\gamma < 1)$, α_0 - part of untaxed profit staying with the producer for low prices, $\bar{\varepsilon}_A = \frac{\gamma}{1-\gamma}$ - the increment of ε_A at its discontinuity at $p = \underline{p}$.

If the producer maximizes PA and chooses independently p and q, his solution will result from two equations (see Appendix 3):

$$\frac{\partial PA}{\partial q} = 0 \Rightarrow p = MC \tag{24}$$

$$\frac{\partial PA}{\partial q} = 0 \Rightarrow p\left(1 - \frac{1}{\varepsilon_A}\right) = AC = \frac{C}{q} \; ; \; \varepsilon_A = -\frac{d\alpha}{dp}\frac{p}{\alpha} \geq 0 \tag{25}$$

These two simple equations indicate that the producer, when maximizing PA, even in deep disequilibria behaves reasonably: he chooses the marginal price $p = MC$ and his produced quantity is such that $AC \leq MC$, that is, he is motivated to produce not less than his optimal cost capacity (defined classically as the produced quantity q such that $AC_{min} = AC = MC$). Moreover, if his capacity is orginally hidden or distorted, he is motived to extend this capacity by better management and better resource utilization - since, clearly, any extension of q at constant C, p will increase PA. How much precisely he will exceed his optimal cost capacity depends on the elasticity ε_A - that is, on the parameters of the taxation mechanism.

The particular taxation mechanism proposed in Fig.1 has a rather simple interpretation. The central authority lets the producer choose any price he wishes; but, if he exceeds the upper guidelines price $\tilde{p}$, all his profit is taxed away. Thus, if his average costs AC are higher than $\tilde{p}$ for all q, he might choose $p = AC_{min} > \tilde{p}$ and thus cover the production costs, but he is not motivated to go higher with the price - he will not obtain any profit.* If he can decrease AC_{min} below $\tilde{p}$, he will certainly do so in order to obtain some positive PA, and charge some price $p = MC > AC_{min}$, $p < \tilde{p}$. However, as long as p is close to $\tilde{p}$, a large part of his profit is taxed away, and he is motivated to further decrease AC_{min}, if only possible. First when he can decrease his costs very far below the lower guideline level (approximately, if $AC_{min} \leq (2\gamma-1)\tilde{p}$ - see Appendix 3), he chooses $p = \bar{p} = \gamma\tilde{p}$ which gives him the maximal part α_0 of his profit remaining after taxation.

The parameter α_0 does not really affect the results of the taxation mechanism and can be chosen from other considerations (at some reasonable level, say $\alpha_0 = 0.5 \div 0.8$). The parameter γ is important and expresses the uncertainty of the central authority about production costs; in the initial phase of a reform, the use of $\gamma = 0.6$ or even $\gamma = 0.5$ is suggested.** The

* Mathematically, the price is not well defined in such a case, since the elasticity ε_A, while approaching infinity if p increases to $\tilde{p}$, ceases to exist for $p > \tilde{p}$. However, if a producer has no use for further price increases, he will not charge larger prices - if only in order to show that he is not antisocial. Also, it could be just legally prohibited to charge $p > AC$ if $p > \tilde{p}$. Thus, we interpret here PA not only as a reward function - although it is a reward function, the additional salaries of the entire staff of the producer should be dependent on PA - but also as an actually modified profit. The remaining part of the actual profit is collected by central authority and might be used to stimulate investment behaviour of the producer.

** As indicated above, the producer will choose a price p in the range $\bar{p} = \gamma\tilde{p},\tilde{p}$, if his minimal average costs AC_{min} are in the range $(2\gamma-1)\tilde{p} = 2-\frac{1}{\gamma})\bar{p},\tilde{p}$. Thus, even if the central authority has rather poor information about costs and assumes that they can change 5 times, it can choose $2\gamma-1 \leq 1/5$, for example $\gamma = 0.6$, and limit the price range to $0.6\tilde{p},\tilde{p}$, which corresponds to only $\pm$ 25% price changes from the middlepoint $0.8\tilde{p}$.

parameter $\tilde{p}$ - the upper guideline price - is then a basic parameter by which the central authority influences the producer behavior, the choice of produced quantity q and the corresponding price $p = MC(q)$. However, large changes of $\tilde{p}$ are not the best strategy of the central authority in the long-term (and they are not really needed, since the admissible initial uncertainty range of the central authority is conveniently large).

This is illustrated by the examples in Fig.2 a,b and c indicating possible equilibrating strategies of the central authority. First stage is the disclosure of the actual capacity and production costs by the producer, motivated by the mechanism of maximizing PA. Initially, the cost curves of the producer were distorted by mismanagement and hidden capacities, see curves AC_D and MC_D in Fig.2a. Suspecting large hidden reserves, the central authority might choose $\tilde{p}$ on the level of recently reported production costs.* The producer chooses a produced quantity q_D and price p_D resulting from the interesection of the curves AC_D and $MC_D \cdot (1 - \frac{1}{\varepsilon_A})$. He finds that his best price P_D is very close to $\tilde{p}$ and his taxed profit PA is very low, and he starts to complain to the central authority that $\tilde{p}$ has been wrongly chosen. Here the central authority must be obstinate and announce that next year $\tilde{p}$ will not be changed and that the producer should cope with the problem on his own. At this point, the producer mobilizes his hidden reserves, improves management, and finds that his actual cost curves are AC and MC; he can increase produced quantity to q_S, lower the price to p_S, and have a respectable profit even after taxation.

The increased quantity q_S might be sufficient or not for market equilibration. If the point p_S, q_S, is already above an estimated demand curve $p = D^{-1}(q)$, Fig.2b, then the central authority can terminate rationing of the produced commodity *provided that there is some other mechanism of market equilibration that will push the producer to satisfy actual demand even if the demand increases;* such mechanisms will be discussed later.

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* Say, 5% higher in order to give some motivation for the producer; if the producer is known for high efficiency and his hidden reserves are not too large, then even up to 20% higher.

a)

b)

c)

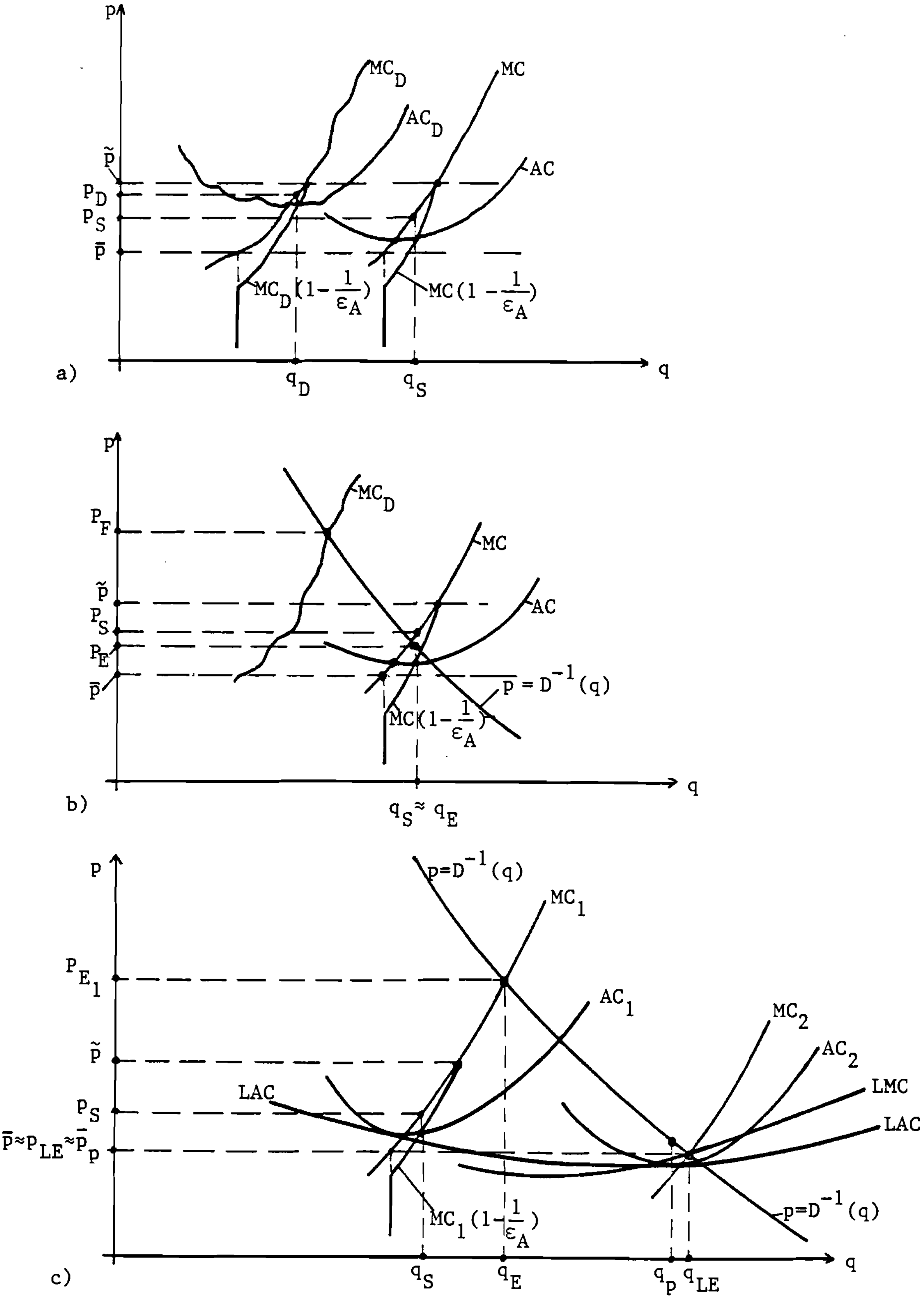

Figure 2. Equilibration under price-dependent profit taxation: a) initial phase, disclosure of true costs and capacities; b) equilibration without investment; c) equilibration through investments.

Since the initial marginal cost curve MC_D was distorted, the proposed strategy of market equilibration is clearly dominant over 'pure market' strategy that would allow the producer to go first to the price p_F on the intersection of MC_D and $p = D^{-1}(q)$.

If it is perceived that an investment is needed for market equilibration, Fig.2c, and that an increase of production scale might decrease the costs, the central authority might choose one of several strategies.

It can increase the upper guideline price $\tilde{p}$, let the producer increase his production and price, terminate rationing and apply some market-equilibration mechanism. This would result in a solution close to p_E, q_E, a short-term equilibrium. At this point, the producer could increase his modified profit PA through an investment, if $\tilde{p}$ were kept constant, but he fears that $\tilde{p}$ will again be lowered - and might choose to forego uncertain profits and keep to certain, already large profits, by not investing. Even if the central authority succeeds in convincing him to invest, the price $\tilde{p}$ would indeed have to be lowered again, and it is questionable whether this way to a long-term equilibrium through short-term equilibria with price varying up and down is really desirable from a social point of view.

Another alternative for the central authority is to continue rationing for a slightly longer time, and announce long-term guideline prices (for example, not changing them). Then the producer has an incentive to invest. However, if he is a monopolist, his best investment might be on a lower scale than required in order to bring the market to a long-term equilibrium, to the point p_{LE}, q_{LE} in Fig.2c. If the investments are centrally coordinated and short-term cost curves after investments are AC_2, MC_2, then the monopolistic producer will be forced, when maximizing modified profit PA, to choose a price p_P and quantity q_P not far from p_{LE}, q_{LE} - at which point his profits are low.*

* If the long-term marginal costs are lower at the long-term equilibrium than the long-term average costs (which is probable in the case of natural monopoly), the mechanism of maximizing PA does not work sufficiently well close to the long-term equilibrium, since it cannot result in a price that is lower than average cost. Further modifications of this mechanism, for example similar to the mechanism of maximizing pF, are then needed.

This analysis shows on one hand the difficulties related to investment motivation and market equilibration by a monopolistic producer, and on the other hand, the analysis also shows the effectiveness of price-dependent profit taxation when limiting price variations and stimulating short-term efficiency.

So far we have considered a single producer in deep disequilibria conditions. If there are several producers of the same commodity, in deep disequilibrium, they might have radically different technologies and cost curves. The central authority should then announce uniform upper and lower guideline prices $\tilde{p}$ and $\bar{p} = \gamma\tilde{p}$. Suppose the producers have already disclosed their actual cost curves and capacities when maximizing PA. Consider the least efficient producer (Fig.3a) and suppose his minimal average costs AC_{1min} are higher than $\tilde{p}$. He will then charge $p_1 = AC_{1min}$ in deep disequilibrium, have no profit, but will not go bankrupt. He knows that his production is needed only as long as other producers do not extend supply sufficiently to cover the demand - and he has some time in which to take necessary action, change technology, or to think about other production. Another producer (Fig.3b) who has minimal average costs AC_{2min} such that $(2\gamma-1)\cdot\tilde{p} < AC_{2min} < \tilde{p}$ will choose p_2 between $\gamma\tilde{p} = \bar{p}$ and $\tilde{p}$ and have moderate profits. The most efficient producer

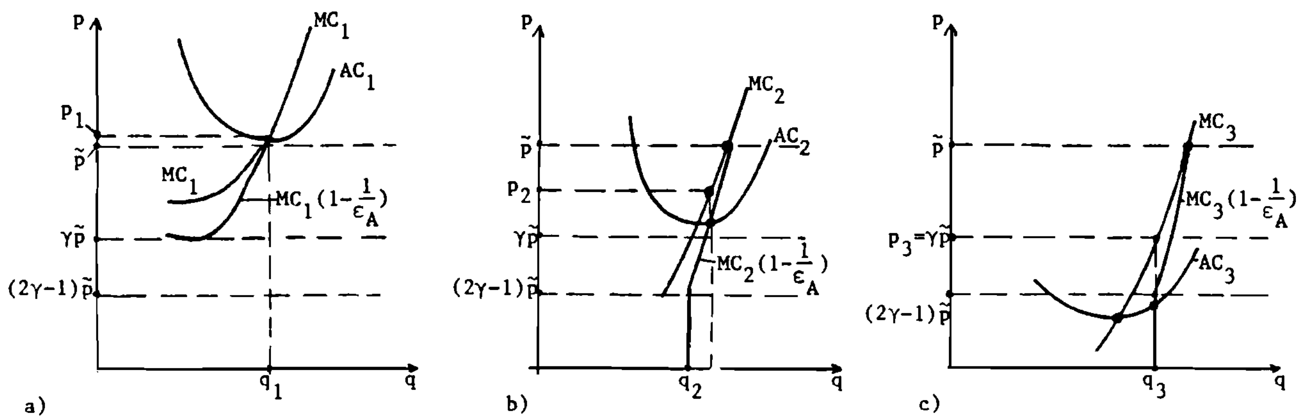

Figure 3. Three producers of the same commodity with widely differing costs.

who has minimal average costs $AC_{3min} < (2\gamma-1)\tilde{p}$ will choose $p_3 = \gamma\tilde{p} = \bar{p}$ and make very large profits. Observe that an implicit competition between these producers motivates them to invest and expand production. Thus, in the case of more than one producer, we have under the proposed mechanism a competitive situation even in the deep disequilibrium case, which makes it much easier for the central authority to coordinate investments and equilibrate the market. However, the central authority has another difficulty in this case: the producers charge different prices $p_1 \neq p_2 \neq p_3$ in deep disequilibrium.

Therefore, the central authority might create another agent - a distributor. The distributor can have many functions - beside averaging prices for the consumer, organizing distribution (including rationing), he might be reponsible for setting detailed upper and lower guideline prices for many products of the producer, having obtained from the central authority only aggregated guideline prices.*

Consider a situation in which the producer not only has multiple products, but also might sell them at differentiated prices, and guideline prices might change in time. Thus, consider separate transactions - sales of quantity q_k of some product at a price p_k, with a given upper guideline price $\tilde{p}_k$; we assume only that the coefficient γ is constant over all transactions, products and time (in an accounting period - at least for a year). We define then the revenue share y_k of a transaction, the corresponding average cost AC_k of producing q_k, the aggregated price p and guideline price $\tilde{p}$:

$$y_k = \frac{p_k q_k}{\sum_m p_m q_m} \; ; \; AC_k = \frac{y_k \cdot C}{q_k} \; ; \; p = \frac{\sum_m p_m q_m}{\sum_m q_m} \; ; \; \tilde{p} = \frac{\sum_m \tilde{p}_m q_m}{\sum_m q_m} \qquad (26)$$

* The distributor might not be needed when the products of the producer are sold not to many consumers but to other producers - for example, to other divisions of a large corporation in a market economy. Observe that this is also a disequilibrium situation - other divisions of a corporation would buy at any transfer price that is lower than the external market price. However, there is no difficulty in this situation when setting the upper guideline price $\tilde{p}$ - it might be simply the external market price, established by the buying division. The center of the corporation announces the coefficient γ and collects the price-dependent taxes. The selling division then charges (in an average transaction, see the analysis of many products and transactions) the marginal price.

and use p and $\tilde{p}$ in order to determine* the coefficient $\alpha(p)$ and its elasticity ε_A. Then, if the producer maximizes his modified profit:

$$PA = \alpha(p) \cdot (\sum_m p_m q_m - C(q_1, \ldots q_m, \ldots q_M)) \tag{27}$$

he will choose (see Appendix 3) his p_k and q_k in such a way that:

$$p_k = \overline{MC}_k \ ; \ \overline{MC}_k = (1+\delta_k) \frac{\partial C}{\partial q_k} \ ; \ \delta_k = \frac{AC_k/\tilde{p}_k - AC/\tilde{p}}{AC/\tilde{p}} \tag{28}$$

and

$$p_k(1 - \frac{1}{\varepsilon_A}) = AC_k \tag{29}$$

Observe that the price aggregation distorts slightly marginal pricing; however, this distortion is quite natural.** If, in a given transaction, he faces a favourably high upper guideline price $\tilde{p}_k$ such that, if he would set $p_k = \frac{\partial C}{\partial q_k}$, the ratio $p_k/\tilde{p}_k$ would be lower than the average ratio $p/\tilde{p}$, then he can increase the coefficient α by increasing the quantity q_k. Equations (28), (29) indicate that he should then charge a price p_k slightly lower than $\frac{\partial C_k}{\partial q_k}$ and sell a correspondingly higher quantity q_k. Conversely, if he faces an unfavourable low upper guideline price $\tilde{p}_k$, say, lower than the minimum of average costs AC_k for this transaction, he might forego the transaction at all if he has other choices. In short, in the case of many products and transactions the producer is slightly more sensitive to setting of guideline prices than it would result from marginal pricing. This

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*This accounting should be monitored by an independent agent - for example, by the bank of the producer, which is consistent with the assumption of the role of banks in the proposals of economic reform in Poland.

** In a sense similar to the situation on a market when a producer makes a transaction below his marginal costs if the sales promotes long-term market expansion or is otherwise favourable to the producer.

makes it easier for the central authority or the distributor to use the guideline prices as an instrument of control and market equilibration.

Consider now the situation, where a single producer faces the market constraint without the distributor, only under the influence of price-dependent profit taxation, and take, for simplicity, the case of one product only. If there is no threat of entry of a competitor, he has actually no incentive to satisfy market demand in short term, except in case when he has already such production capacity that his best solution without market exceeds market demand.* Since his best solution without market - see (24), (25) - is characterized by the cost ratio:

$$\varepsilon_C = \frac{MC}{AC} = \frac{\varepsilon_A}{\varepsilon_A - 1} \tag{30}$$

and $\varepsilon_C - 1 = \frac{MC - AC}{AC} = \frac{p \cdot q - C}{C}$ at this solution characterizes the rate of (unmodified) profit, he will face market constraints and satisfy the demand if the inequality $q \leq D(p)$ becomes binding and results in diminishing his rate of profit, $\varepsilon_C - 1$. Thus, the market is equilibrated if (see Appendix 3)

$$\varepsilon_C \leq \frac{\varepsilon_A}{\varepsilon_A - 1} \tag{31}$$

In this case, the producer will charge a price p and produce a quantity q resulting from two equations, the market constraint $q = D(p)$ and one of two equivalent conditions:

$$p\left(1 - \frac{\varepsilon_A(1 - \varepsilon_C) + \varepsilon_C}{\varepsilon_A + \varepsilon_D\varepsilon_C}\right) = MC \tag{32}$$

$$p\left(1 - \frac{1 - (1 - \varepsilon_C)\varepsilon_D}{\varepsilon_A + \varepsilon_D\varepsilon_C}\right) = AC \tag{33}$$

* This reservation is the same as the one made for the Tam mechanism and for the proposed modification of Domar mechanism; However, here we consider it in more detail.

where $\varepsilon_A(1 - \varepsilon_C) + \varepsilon_C \geq 0$ if (31) holds, thus $p \geq MC$. Marginal pricing $p = MC$ is obtained if $\varepsilon_C = \frac{\varepsilon_A}{\varepsilon_A - 1}$, which is implied (since $\varepsilon_A = \frac{p/\tilde{p}}{1 - p/\tilde{p}}$, see Appendix 3) by:

$$\tilde{p} = 2MC - AC \tag{34}$$

If the central authority estimates $\widehat{MC}$ wrongly ($\widehat{AC}$ is much easier to estimate) and sets $\tilde{p} > 2\widehat{MC} - \widehat{AC}$, then the producer meets demand constraint and charges a price $p > MC$, while the elasticity of the index $\pi = \frac{p}{MC}$ with respect to errors in the estimation of $\widehat{MC}$, $E_{\pi WD\widehat{MC}}$ is close to 1. If the central authority sets $\tilde{p} > 2\widehat{MC} - \widehat{AC}$, then $p = MC$, but the produced quantity does not satisfy demand. Thus, the price dependent profit taxation, while being a good mechanism for deep disequilibria, is not very robust at short-term market equilibria. Therefore, after achieving equilibrium, it is necessary either to change the motivation mechanism - or to use appropriately the services of the distributor, who was needed in deep disequilibrium but might be also used for other purposes in equilibrium.

6. THE ROLE OF A DISTRIBUTOR IN MARKET EQUILIBRATION

We assume here that the distributor is a partner of the producer, with different but not strictly antagonist motivation, and cannot fully collude with the producer.* Thus, we assume that the distributor buys from the producer at a price $p \geq MC(q)$ - resulting, for example, from the price-dependent profit taxation motivation mechanism of the producer. If we would give to the distributor full monopsomistic and monopolistic powers - the right

*Thus the distributor cannot have total power over the producer. This is guaranteed if the distributor cannot set production quota for the producer - although he can set guideline prices or even actual prices, and the producer responds then either with prices and quantity, or just with quantity. Observe that, no matter whether the distributor sets guideline prices or actual prices, he cannot buy from the producer at a price lower than the marginal production costs (on average - we do not consider here aggregation effects discussed earlier). However, he could buy from the producer at a higher price, while sharing with him the effects of the joint monopolistic position.

to buy all products from producers and sell them on the market while maximizing profit - it would be clearly socially disadvantageous. The distributor would maximize then the expression:

$$PM = p_C \cdot q - p \cdot q \tag{35}$$

where p is the price paid to the producer and p_C - the consumer price. He would choose then a solution where his marginal revenue, $MR = p_C(1 - \frac{1}{\varepsilon_D})$, equals his marginal outlay, $MO = \frac{d}{dq}(pq) = p \cdot (1 + \varepsilon_{C'})$, where $p = MC$, $\varepsilon_{C'} = \frac{C'' \cdot q}{MC}$ is the elasticity of marginal costs. Thus, he would enjoy full effect of both monopoly and monopsomy, charging a price p_C that is even higher than the normal monopolistic price of the producer. However, the central authority can choose a different motivation for the distributor. For example, he might be allowed to charge only a mark-up price $p_C = (1 + \delta)p$, where δ is a markup coefficient assumed here to estimate correctly the distribution costs;* thus, the ideal price on the market would be achieved if $p = MC$, $p_C = (1 + \delta)MC$. Since the profit of the distributor now takes the form

$$P_\delta = \delta \cdot p \cdot q$$

he is actually interested in maximizing revenue. If the demand elasticity at the ideal price is $\hat{\varepsilon}_D > 1$, then the marginal revenue is positive and the distributor can gain more by increasing quantity than by increasing price. He would buy then

*This is clearly a simplifying assumption - however, full analysis of various aspects of the functioning of a distributor cannot be achieved in this paper, since there are many other aspects that should be discussed. For example, the distributor should not have unlimited monopsomy powers-producers should be able to sell directly on the market, and it is possible to devise motivation mechanism that promote such behavior. The problems of aggregation effects of many transactions and products, of the influence of the distributor on the investments of the producer, and many others should be also investigated in detail. These problems are, however, the subject of another paper.

at p = MC and *arrive at the ideal solution without any further motivation; thus, there is actually no need to construct special motivation mechanisms for equilibria situations with* $\hat{\varepsilon}_D > 1$, *if we introduce a distributor with mark-up pricing rule.* However, if $\hat{\varepsilon}_D < 1$, the distributor would be interested in increasing price until $\varepsilon_D = 1$, decreasing the quantity sold on the market. He can do it by forming an informal coalition with the producer - simply by agreeing to pay to the producer a price $p > MC(q)$. Thus, we need special motivation mechanisms for the distributor in equilibria situations with $\hat{\varepsilon}_D \leq 1$, as well as for deep disequilibria.

As the first from such motivation mechanism, the central authority might choose a price-dependent profit taxation. For example, if the profit $P_\delta = \delta \cdot p \cdot q$ is supposed to cover, with some margin, the distribution costs, then the part of this profit that is used for additional salaries for the distributor employees and managers might be proportional to the reward function:

$$PA_\delta = \alpha(p_C) \cdot \delta \cdot p \cdot q$$

where $\alpha(p_C)$ is as in Fig 1, with $\tilde{p}_C$ and $\bar{p}_C = \gamma\tilde{p}_C$ set by the central authority.* Consider the behavior of a distributor who is starting in a deep disequilibrium while maximizing PA_δ. His constraints are:

$$p \geq MC(q) \;\; ; \;\; q \leq D(p_C) \;\; ; \;\; p_C = (1 + \delta)p \quad .$$

*In case of many products, central authority could set only aggregated upper and lower quideline prices - similarly as in the case of setting them for a producer.

Since

$$\frac{\partial PA_\delta}{\partial p} = \alpha \cdot \delta \cdot q(1 - \varepsilon_A) \;;\; \varepsilon_A = - \frac{d\alpha}{dp_C} \cdot \frac{PC}{\alpha} \tag{39}$$

hence, if $\varepsilon_A > 1$ (which is guaranteed if $\gamma > 0.5$ and $\gamma\tilde{p}_C < p_C < \tilde{p}_C$, see Appendix 4), he will always choose the minimal price at which he can buy, $p = MC(q)$. If the market constraint is not yet binding, there are shortages on the market, the constraint $p = MC(q)$ results in:

$$\frac{dPA_\delta}{dq} = \alpha \cdot \delta \cdot (p + q \cdot C'' \cdot (1 - \varepsilon_A)) \tag{40}$$

which implies a disequilibrium solution p,q satisfying two equations

$$p = MC \;;\; p = C''q(\varepsilon_A - 1) \tag{41}$$

see Fig. 4.

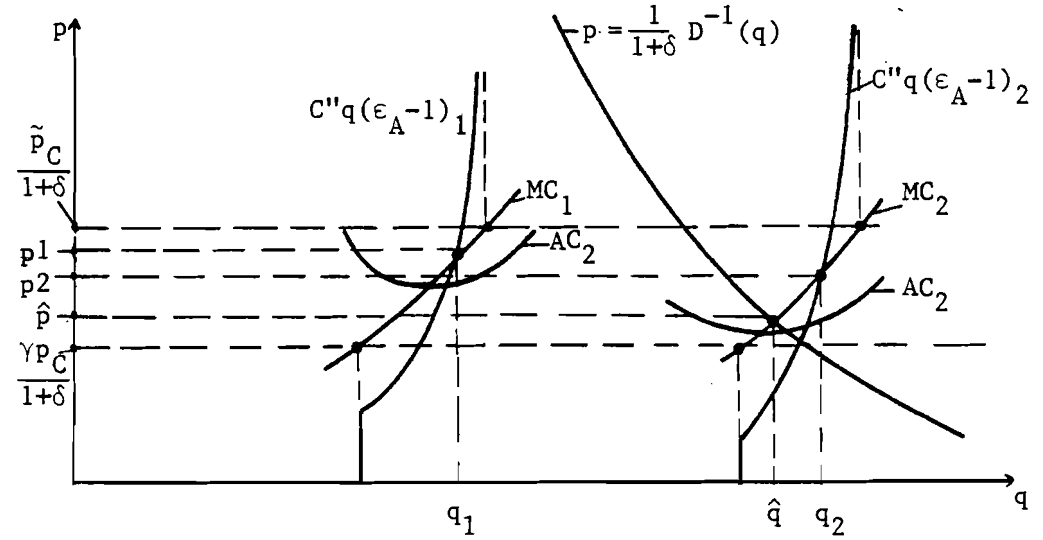

Figure 4. Market equilibrium under price dependent motivation for the distributor: p_1, q_1 - solution in deep disequilibrium; p_2, q_2 - results of an investment; $\hat{p}, \hat{q}$ - resulting ideal (short-term) solution.

However, the distributor can be also empowered to stimulate the investments of the producer. He can, for example, collect for this purpose the price-dependent taxes of the producer's profit. It can be also reasonably assumed that the distributor knows probable demand characteristics and the long-term cost characteristics of the producer much better than the central authority (he works daily with the market and with the producer). Thus, he has information needed for a choice of investments, and he can propose investment credit to the producer - if necessarv using the threat of creating a competitor in order to motivate the producer. Suppose such an investment has been made and was sufficient for covering market demand. The resulting (short-term) situation is that the quantity determined by (41), $q = \frac{MC}{(\varepsilon_A - 1)\cdot C''}$, is higher than $D(MC(1+\delta))$, see Fig. 4. Now the best solution for the distributor is precisely on the intersection of $(1+\delta)MC(q)$ and $D^{-1}(q)$, the ideal short term solution. This is because, if $q < \frac{MC}{(\varepsilon_A - 1)C''}$, the derivative $\frac{dPA_\delta}{dq}$ as defined by (40) is positive and the distributor tries to increase the supply as far as possible. *Such a solution - resulting from an intersection of two constraints that are external to the distributor - is fully robust,* does not depend on parameters of his motivation mechanism nor on other information. To see that it does not depend on the demand elasticity ε_D, we can also check the derivative $\frac{dPA_\delta}{dq}$ under the reverse assumption that $q = D(p_C)$ but $q \leq MC^{-1}\left(\frac{p_C}{1+\delta}\right)$

$$\frac{dPA_\delta}{dq} = \alpha \cdot \delta \cdot p \cdot \frac{\varepsilon_D + \varepsilon_A - 1}{\varepsilon_D} \tag{42}$$

As long as $\varepsilon_A > 1$, this derivative is positive even for $\varepsilon_D \approx 0$ and the distributor will try to buy more until he reaches $q = MC^{-1}\left(\frac{p_C}{1+\delta}\right)$, $p = MC(q)$.

In long term, the distributor will try to influence the producer to lower his marginal costs as far as possible, thus trying to achieve the long-term ideal solution.* Observe that the central authority has only to know the equilibrium price $\hat{p}_C$ up to, say $\pm$ 25%, so that $\gamma\tilde{p}_C < \hat{p}_C < \tilde{p}_C$ with $\gamma = 0.6$, and the motivation of the distributor results in fully robust ideal solution.

Although the mechanism described above is attractive in many aspects, the central authority might choose to control sharply consumer prices. In such a situation, another motivation mechanism might be used. Instead of price-dependent profit modification, we assume quantity-dependent (actually, volume-dependent, since quantities have to be then aggregated at the centrally set consumer prices) profit modification, in a sense dual to the modification represented in Fig. 1. The central authority sets then an upper guideline quantity $\tilde{q}$ and a lower guideline quantity $\bar{q} = \omega\tilde{q}$ for the distributor, and his modified profit becomes:

$$PB_\delta = \beta\ (q)\cdot\delta\cdot p\cdot q \tag{43}$$

where $\beta(q)$, represented in Fig. 5a, is equal zero if $q \leq \omega\tilde{q} = \bar{q}$, increases linearly for $\bar{q} < q < \tilde{q}$, and is equal to maximal β_0 if $q > \tilde{q}$. Its elasticity:

$$\varepsilon_B = \frac{d\beta}{dq} \cdot \frac{q}{\beta} \tag{44}$$

is decreasing with q, from very high values at q slightly larger than $\bar{q}$, to $\frac{1}{1-\omega}$ at $q \approx \tilde{q}$, then drops down to zero.

*If, at the ideal long term solution, the long-term marginal costs are lower than the long term average costs and the producer cannot make profit(which is probable in a natural monopoly case), the producer has to obtain special motivation - a subsidy similar as in the linear taxation mechanism. Such a subsidy would have to be negotiated with the central authority.

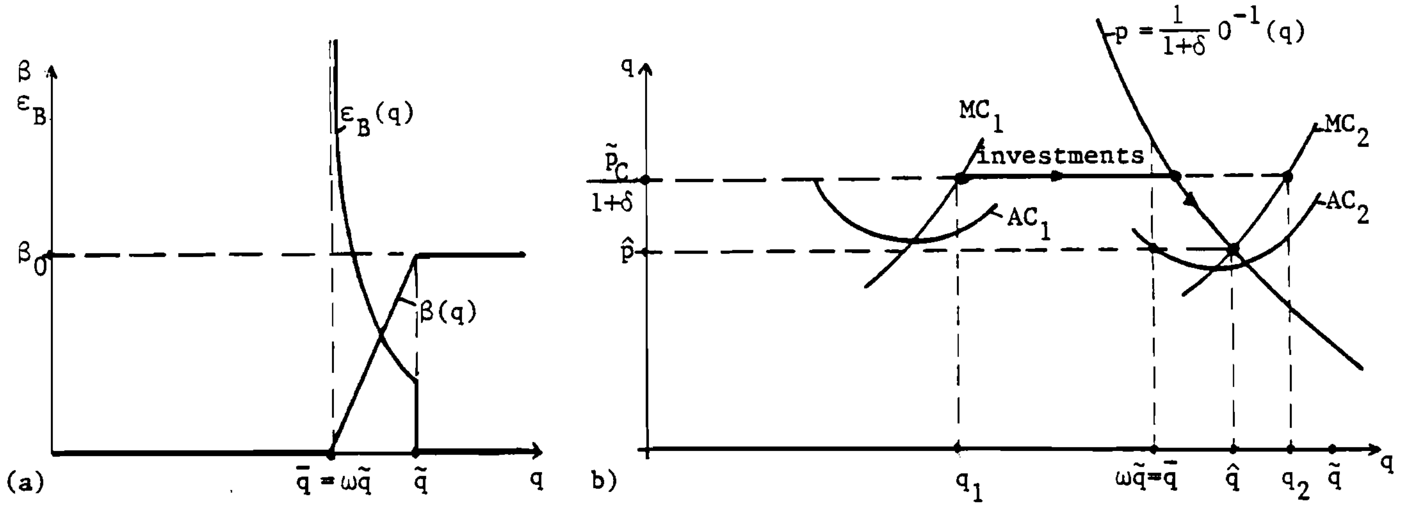

Figure 5. a) Quantity-dependent profit modification coefficient $\beta(q)$ and its elasticity $\varepsilon_B(q)$; b) Market equilibration under quantity-dependent motivation for the distributor.

If we analyze the behavior of the distributor under such motivation mechanisms, with the constraints

$$p \geq MC(q) \;\; ; \; q \leq D(p_C), \; p_C = (1+\delta)p \leq \tilde{p}_C \tag{45}$$

where $\tilde{p}_C$ is a sharply set upper-bound for consumer prices, we can see that he will rise the price to $p_C = \tilde{p}_C$, as long as he does not encounter the demand constraint. However, being quantity - motivated, he will pay to the producer $p = MC(q)$, and try to induce the producer to invest. After investments, when the demand constraint is met, he will lower the price to $\hat{p}_C = (1+\delta) \cdot MC(\hat{q}) = D^{-1}(\hat{q})$, the ideal price. This is because the derivative $\frac{dPE_\delta}{dq}$, if $p = \frac{1}{1+\delta} D^{-1}(q)$, has the form

$$\frac{dPE_\delta}{dq} = \beta \cdot \delta \cdot p \cdot \quad \frac{\varepsilon_B \varepsilon_D + \varepsilon_D - 1}{\varepsilon_D} \tag{46}$$

and is positive as long as $\varepsilon_B + 1 > \frac{1}{\varepsilon_D}$, which can be achieved for a reasonably low ε_D (say, $\varepsilon_D \geq 0.2$) by a proper choice of ω (say, $\omega \geq 0.75$). However, the ideal quantity $\hat{q}$ has then to be estimated more precisely than the ideal price $\hat{p}$ under the price-dependent mechanism.

7. CONCLUSIONS

While the motivation mechanisms for a monopolistic producer, designed for market equilibrium cases and known from various publications, can be further improved and made more robust, they are not really implementable because of their deficiencies in deep disequilibria cases, or not quite clear motivation to reach long-term equilibrium. For deep disequilibria, special mechanisms promoting the disclosure of hidden reserves can be devised; however, they are not very robust in equilibrium situations. To improve this, another agent - a distributor, empowered to promote investments by the producer - can be introduced and appropriately motivated. A system composed of a monopolistic producer motivated for deep disequilibria cases and separated from the market by a distributor, can work both in disequilibria and equilibria cases, result in robust marginal prices, and evolve towards long-term competitive equilibrium.

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APPENDIXES

APPENDIX 1. ROBUSTNESS OF MOTIVATION MECHANISMS

The Domar mechanism that maximizes PD = $\frac{\varepsilon_D}{\bar{\varepsilon}_D - 1}$ qp - C under market constraint $p = D^{-1}(q)$ results in a solution:

$$\text{(A1.1)} \qquad \frac{dPD}{dp} = 0 \Rightarrow p \cdot \frac{\bar{\varepsilon}_D}{\varepsilon_D - 1} \cdot \frac{\varepsilon_D - 1}{\varepsilon_D} = MC$$

where $\bar{\varepsilon}_D$ is an estimate of demand elasticity, ε_D - actual demand elasticity. Thus, if $\pi_D = \frac{P}{MC} = \frac{\varepsilon_D(\bar{\varepsilon}_D - 1)}{(\varepsilon_D - 1)\bar{\varepsilon}_D}$, we have

$$\frac{\partial \pi_D}{\partial \bar{\varepsilon}_D} = \frac{\varepsilon_D}{(\varepsilon_D - 1)\bar{\varepsilon}_D^2} \quad ; \quad \frac{\partial \pi_D}{\partial \varepsilon_D} = - \frac{\varepsilon_D}{(\varepsilon_D - 1)^2 \bar{\varepsilon}_D} \quad ;$$

$$\left. \frac{\partial \pi_D}{\partial \bar{\varepsilon}_D} \cdot \frac{\bar{\varepsilon}_D}{\pi_D} \right|_{\varepsilon_D = \bar{\varepsilon}_D = \hat{\varepsilon}_D} = \frac{1}{\hat{\varepsilon}_D - 1} = \left. - \frac{\partial \pi_D}{\partial \varepsilon_D} \frac{\varepsilon_D}{\pi_D} \right|_{\varepsilon_D = \bar{\varepsilon}_D = \hat{\varepsilon}_D}$$

where $\hat{\varepsilon}_D$ is the elasticity at the ideal solution. Observe the symmetry of the dependence of π_d on ε_D and $\bar{\varepsilon}_D$, which is a manifestation of more general relativity principles in sensitivity analysis. The Freixas mechanism that maximizes PF = $(p+\bar{t})q - C$, $\bar{t} - \frac{\overline{MC}}{\bar{\varepsilon}_D}$ under market constraint $p = D^{-1}(q)$ results in a solution

(A1.3) $$\frac{dPF}{dq} = 0 \Rightarrow p(1 - \frac{1}{\varepsilon_D}) + \bar{t} = MC$$

where $\overline{MC}$ is an estimate of the ideal price $\hat{p} = \hat{MC}$. Thus, $\pi_F = \frac{p}{MC} = \frac{\varepsilon_D}{\varepsilon_D - 1}(1 - \frac{\bar{t}}{MC})$. However, MC depends on $\bar{t}$; we can linearize this dependence by observing that (A1.3) can be written $p + q\frac{dp}{dq} + \bar{t} = MC$ and by using the implicit function theorem. This is equivalent to linearizing this equation for small deviations of $p, q, \bar{t}$ from $\hat{p}, \hat{q}$ and $\hat{t} = \frac{\hat{p}}{\hat{\varepsilon}_D} = -\hat{q}\frac{dp}{dq}$, which results in

(A1.4) $$2(q - \hat{q})\frac{dp}{dq} + \bar{t} - \hat{t} = (q - \hat{q})C'' \Rightarrow \frac{dq}{d\bar{t}} = \frac{1}{C'' - 2\frac{dp}{dq}}$$

where $C'' = \frac{dMC}{dq}$. Thus

(A1.4) $$\frac{dMC}{d\bar{t}} = \frac{dMC}{dq} = \frac{C''}{C'' - 2\frac{dp}{dq}}, \quad \text{where} \quad \frac{dp}{dq} = -\frac{1}{\hat{\varepsilon}}\frac{\hat{p}}{\hat{q}}$$

Since $\frac{\partial \pi_F}{\partial \bar{t}} = \frac{-\varepsilon_D}{(\varepsilon_D - 1)MC}$, $\frac{\partial \pi_F}{\partial MC} = \frac{\varepsilon_D \bar{t}}{(\varepsilon_D - 1)MC^2}$ thus

$$\left.\frac{d\pi_F}{d\bar{t}}\right|_{MC = \overline{MC} = \hat{p},\, \varepsilon_D = \bar{\varepsilon}_D = \hat{\varepsilon}_D} =$$

$$= \frac{\hat{\varepsilon}_D}{(\hat{\varepsilon}_D - 1)\hat{p}}\left(1 - \frac{C''}{\hat{\varepsilon}_D C'' + 2\hat{p}/\hat{p}}\right) \approx -\frac{\hat{\varepsilon}_D}{(\hat{\varepsilon}_D - 1)\hat{p}}, \text{ if } \hat{\varepsilon}_D > 1.$$

$$\text{(A1.5)}\qquad \frac{d\pi_F}{dt}\cdot\frac{\bar t}{\pi_F}\Bigg|_{\varepsilon_D=\bar\varepsilon_D=\hat\varepsilon_D,\ MC=\overline{MC}=\hat p} = -\frac{1}{\hat\varepsilon_D-1}\left(1-\frac{C''}{\hat\varepsilon_D C''+2\hat p/\hat p}\right)\approx$$

$$\approx -\frac{1}{\hat\varepsilon_D-1}$$

Clearly, $E_{T\varepsilon D} = \frac{d\bar t}{d\,\bar\varepsilon_D}\cdot\frac{\bar\varepsilon_D}{\bar t} = -1$, $E_{TMC} = \frac{d\bar t}{d\overline{MC}}\cdot\frac{\overline{MC}}{\bar t} = 1$,

hence the elasticities of π_F with respect to $\bar\varepsilon_D$ and $\overline{MC}$ have the same form as (A1.5).

The Tam mechanism that maximizes $PT = (q-\bar q)p - C$ should be considered, actually, under the inequality market constraint $p \le D^{-1}(q)$ or $q \le D(p)$ since this constraint might not be active at the maximum of PT. If $\bar q > q$ and thus $D^{-1}(\bar q) < MC(\bar q)$, we would have $\frac{\partial PT}{\partial q}\Big|_{q=\bar q,\, p\le D^{-1}(\bar q)} < 0$, hence q should be decreased starting from $\bar q$; but if $q < \bar q$, then $\frac{\partial PT}{\partial p} < 0$ and p should also be decreased - thus the constraint $p \le D^{-1}(q)$ cannot be active. However, we can analyze the robustness of Tam mechanism under the assumption that $\bar q \le \hat q$ which results in the activity of market constraint, $p = D^{-1}(q)$, or just assume that other motivations (survival of the firm, etc.) would result in $p = D^{-1}(q)$ even if $\bar q > \hat q$. Under this simplifying assumption we obtain:

$$\text{(A1.6)}\qquad p\left(1-\frac{1}{\varepsilon_D}\left(1-\frac{\bar q}{q}\right)\right) = MC$$

and $\pi_T = \frac{p}{MC} = \frac{\varepsilon_D}{\varepsilon_D - 1 + \bar q/q}$. Similar to (A1.3), here we have to approximate the dependence of q on $\bar q$. Since (A1.6) can be written as $MC = p + q\frac{dp}{dq} - \bar q\frac{dp}{dq}$, we use the implicit function theorem or, equivalently, linearize this equation for small deviations from $q = \bar{q} = \hat{q}$, $MC = p = \hat{p}$ which yields

$$(A1.7) \qquad (q-\hat{q})C'' = 2(q-\hat{q})\frac{dp}{dp} - (\bar{q}-\hat{q})\frac{dp}{dP} \Rightarrow \frac{dq}{d\bar{q}} = \frac{\frac{dp}{dq}}{2\frac{dp}{dq}-C''} = \frac{1}{2+C''\varepsilon_D\hat{p}/\hat{p}}$$

Now

$$(A1.8) \qquad \left.\frac{d\pi_T}{d\bar{q}}\right|_{q=\bar{q}=\hat{q},\varepsilon_D=\hat{\varepsilon}_D} = -\frac{1}{\bar{q}\bar{\varepsilon}_D}(1-\frac{dq}{d\bar{q}}) = \frac{1}{\hat{q}\hat{\varepsilon}_D}\;\frac{1+C''\hat{\varepsilon}_D\hat{q}/\hat{p}}{2+C''\hat{\varepsilon}_D\hat{q}/\hat{p}}$$

which results in $\left.\frac{d\pi_T}{d\bar{q}}\cdot\frac{\bar{q}}{\pi_T}\right|_{q=\bar{q}=\hat{q},\bar{\varepsilon}_D=\hat{\varepsilon}_D} = -\frac{1}{2\hat{\varepsilon}_D}\cdot\frac{1+C''\hat{\varepsilon}_D\hat{q}/\hat{p}}{1+0.5C''\hat{\varepsilon}_D\hat{q}/\hat{p}} \approx$

$$\approx -\frac{1}{2\hat{\varepsilon}_D} \quad ,$$

if C" or ε_D are sufficiently small. However, the quideline quantity $\bar{q}$ has somehow to be estimated . Suppose the demand has a simple constant elasticity form, $p = D^{-1}(q) = \hat{p}(q/\hat{q})^{-\hat{\varepsilon}_D}$ and consider what deviation $q-\hat{q} = \bar{q}-\hat{q}$ would be caused by setting $p = \overline{MC} \neq \hat{p}$ or $\bar{\varepsilon}_D \neq \hat{\varepsilon}_D$. By implicit function theorem, we then obtain

$$\left.\frac{\partial\bar{q}}{\partial\overline{MC}}\right|_{q=\hat{q},\varepsilon_D=\hat{\varepsilon}_D} = -\frac{\hat{\varepsilon}_D\hat{q}}{\hat{p}} \quad \text{and} \quad \left.\frac{\bar{q}}{\bar{\varepsilon}_D}\right|_{\bar{q}=\hat{q},\varepsilon_D=\hat{\varepsilon}_D} = 0,$$

which implies

$$\frac{\partial\bar{q}}{\partial\overline{MC}}\cdot\frac{\overline{MC}}{\bar{q}} = -\hat{\varepsilon}_D,\; \frac{\partial\bar{q}}{\partial\bar{\varepsilon}_D}\cdot\frac{\bar{\varepsilon}_D}{\bar{q}} = 0 \;, \quad \text{and} \quad \frac{\partial\pi_T}{\partial\overline{MC}}\cdot\frac{\overline{MC}}{\pi_T} \approx \frac{1}{2} \;.$$

APPENDIX 2. A MODIFIED ROBUST MOTIVATION MECHANISM

Consider the modified Domar mechanism with PW = zqp-C, where z is a function of $\bar{\varepsilon}_D, \bar{p} = \overline{MC}$ and actual price p. Although, generally, $p \leq D^{-1}(q)$ should be considered as the market constraint, assume $p = D^{-1}(q)$ for initial analysis. Then

$$\text{(A2.1)} \qquad \frac{DPW}{dq} = 0 \Rightarrow zp(1 - \frac{1}{\varepsilon_D}(1-\varepsilon_z)) = MC$$

where $\varepsilon_z = -\frac{dz}{dp}\frac{p}{z}$. Postulate $z(1-\frac{1}{\varepsilon_D}(1-\varepsilon_z)) = 1$ for $\varepsilon_D = \bar{\varepsilon}_D$ and for all p. Since this can be rewritten as $z(\bar{\varepsilon}_D - 1) - \bar{\varepsilon}_D = \frac{dz}{dp}p$ we obtain a differential equation:

$$\text{(A2.2)} \qquad -\frac{dp}{p} = \frac{dz}{z(1-\bar{\varepsilon}_D)+\bar{\varepsilon}_D}$$

If we also postulate $z(1-\frac{1}{\varepsilon_D}(1-\varepsilon_z) = 1$ for all ε_D and for $p = \bar{p} = \overline{MC}$, we obtain $\varepsilon_z = 1$ and $z = 1$ for all ε_D and $p = \bar{p}$. Thus, the differential equation (A2.2) has a boundary condition:

$$\text{(A2.3)} \qquad z = 1 \quad \text{if} \quad p = \bar{p}$$

When solving (A2.2) with (A2.3) for two cases $\varepsilon_D \neq 1$ and $\varepsilon_D = 1$, the following solutions are obtained:

$$\text{(A2.4)} \qquad z = \begin{cases} \dfrac{(p/\bar{p})^{\bar{\varepsilon}_D - 1} - \bar{\varepsilon}_D}{1 - \bar{\varepsilon}_D} & , \quad \bar{\varepsilon}_D \neq 1 \\ 1 = \ln(p/\bar{p}) & , \quad \bar{\varepsilon}_D = 1 \end{cases}$$

$$\text{(A2.5)} \qquad \varepsilon_Z = \begin{cases} \dfrac{(1-\bar{\varepsilon}_D)(p/\bar{p})^{\bar{\varepsilon}_D - 1}}{(p/\bar{p})^{\bar{\varepsilon}_D - 1} - \bar{\varepsilon}_D} , & \bar{\varepsilon}_D \neq 1 \\[2ex] \dfrac{1}{1 - \ln(p/\bar{p})} , & \bar{\varepsilon}_D \neq 1 \end{cases}$$

Observe that if $p/\bar{p} < 1$, then $\varepsilon_Z < 1$, and if $p/\bar{p} > 1$, then $\varepsilon_Z > 1$. Now, if we define $\pi_W = \frac{P}{MC}$, we have

$$\text{(A2.6)} \qquad \pi_W = \frac{\varepsilon_D}{z(\varepsilon_D - 1 + \varepsilon_Z)}$$

However, both z and ε_Z depend on $p/\bar{p} = \pi_W \cdot y$, where $y = MC/\bar{p}$. Taking this into account and combining (A2.4), (A2.5), (A2.6) we obtain (at $\bar{\varepsilon}_D \neq 1$, $\varepsilon_D \neq 1$; for $\bar{\varepsilon}_D = 1$, a similar analysis applies)

$$\text{(A2.7)} \qquad \pi_W + \frac{\bar{\varepsilon}_D - \varepsilon_D}{\bar{\varepsilon}_D(\varepsilon_D - 1)} \, y^{\bar{\varepsilon}_D - 1} \, \pi_W^{\bar{\varepsilon}_D} - \frac{\varepsilon_D(\bar{\varepsilon}_D - 1)}{\bar{\varepsilon}_D(\varepsilon_D - 1)} = 0$$

By the implicit function theorem, $\frac{\partial \pi_W}{\partial y}$ is the solution of an equation obtained by the differentiation of (A2.7) in y:

$$\text{(A2.8)} \qquad \frac{\partial \pi_W}{\partial y}\left(1 + \frac{\bar{\varepsilon}_D - \varepsilon_D}{\varepsilon_D - 1} \, \pi_W^{\bar{\varepsilon}_D - 1} \, y^{\bar{\varepsilon}_D - 1}\right) - \frac{(\bar{\varepsilon}_D - \varepsilon_D)(\bar{\varepsilon}_D - 1)}{\bar{\varepsilon}_D(\varepsilon_D - 1)} \, \pi_W^{\bar{\varepsilon}_D} \, y^{\bar{\varepsilon}_D - 2} = 0$$

which implies $\frac{\partial \pi_W}{\partial y} = 0$ for $\bar{\varepsilon}_D = \varepsilon_D$, all y. Taking this into account and differentiating (A2.8) once more, we also obtain $\frac{\partial^2 \pi_W}{\partial y^2} = 0$ for $\bar{\varepsilon}_D = \varepsilon_D$, all y - and we really do not need to do it, since we have actually postulated $\pi_W = 1$ at $\bar{\varepsilon}_D = \varepsilon_D$, all y and $\pi_W = 1$ at $y = 1$, all $\bar{\varepsilon}_D$ - thus all derivatives $\frac{\partial^n \pi W}{\partial y^n} = 0$ at $\bar{\varepsilon}_D = \varepsilon_D$ and all derivatives $\frac{\partial^n \pi W}{\partial \bar{\varepsilon}_D^n} = 0$ at $y = 1$. However, (A2.8) can be used to derive the mixed derivative $\frac{\partial^2 \pi_W}{\partial \bar{\varepsilon}_D \partial y}$;

by differentiating (A2.8) with respect to $\bar{\varepsilon}_D$, taking into account that other derivatives are zero, we obtain

$$\left.\frac{\partial^2 \pi_W}{\partial \bar{\varepsilon}_D \partial y}\right|_{\bar{\varepsilon}_D = \varepsilon_D\ y = 1} = -\frac{1}{\bar{\varepsilon}_D},$$

which results in

$$\text{(A2.9)} \qquad \left.\frac{\partial^2 \pi_W}{\partial \bar{\varepsilon}_D \partial y} \cdot \frac{\bar{\varepsilon}_D \cdot y}{\pi_W}\right|_{\bar{\varepsilon}_D = \varepsilon_D, y = 1} = -1$$

Thus, for example, 20% changes of both $\bar{\varepsilon}_D$ as compared to ε_D and $\overline{MC} = \bar{p}$ as compared to MC would result, at first approximation, in only 4% change of π_W. However, this is only a local estimate and it is necessary to check its validity for larger changes of $\bar{\varepsilon}_D, \bar{p}$. This can be done by solving (A2.7). While (A2.7) can be solved only numerically for a general case of any positive $\bar{\varepsilon}_D$, there are three cases when it take a more simple form. If $\bar{\varepsilon}_D = 0.5$, we obtain a quadratic equation with respect to $x = \pi_W^{0.5}$ with the positive solution:

$$\text{(A2.10)} \qquad \pi_W = \frac{(1-2\varepsilon_D)^2 + 2(1-\varepsilon_D)\varepsilon_D y + (1-2\varepsilon_D)\left((1-2\varepsilon_D)^2 + 4(1-\varepsilon_D)\varepsilon_D y\right)^{0.5}}{2(1-\varepsilon_D)^2 y}\ ; \ \bar{\varepsilon} = 0.5$$

and can check the robustness of the mechanism by constructing a table:

$\bar{\varepsilon}_D = 0.5$							
$y=\frac{MC}{\bar{p}}$	0.4	0.6	0.8	1.0	1.2	1.4	1.6
ε_D	$\pi_W = \frac{P}{MC}$						
0.1	2.192	1.531	1.200	1.000	0.866	0.771	0.698
0.2	1.873	1.392	1.148	1.000	0.899	0.826	0.770
0.3	1.555	1.255	1.098	1.000	0.932	0.882	0.843
0.4	1.258	1.123	1.048	1.000	0.966	0.94=	0.919
0.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.6	0.795	0.891	0.954	1.000	1.036	1.064	1.088
0.7	0.643	0.797	0.911	1.000	1.073	1.134	1.186
0.8	0.534	0.719	0.871	1.000	1.112	1.210	1.298
1.0	0.400	0.600	0.800	1.000	1.200	1.400	1.600

We observe that if, indeed, changes of $\bar{\varepsilon}_D$ and $\bar{p}$ are not more than 20% or even 30%, the elasticity (A2.9) gives reasonable estimates of the changes of π_W. If $\bar{\varepsilon}_D = 1$, then (A2.7) becomes:

$$\text{(A2.11)} \qquad \pi_W\left(1+\frac{1-\varepsilon_D}{\varepsilon_D}\ln\,(\pi_W\cdot y)\right) = 1 \quad ; \quad \bar{\varepsilon}_D = 1$$

which, though a simpler equation, still have to be solved numerically. A similar table as presented above has been constructed and confirms the conclusions on the validity of the approximation of π_W by the elasticity (A2.9). The same qualitative result was obtained by constructing a corresponding table for $\bar{\varepsilon}_D = 2$, in which case (A2.7) is a quadratic equation in π_W with the positive solution:

$$\text{(A2.12)} \qquad \pi_W = \frac{(\varepsilon_D-1) - (1-\varepsilon_D(\varepsilon_D-2)(y-1))^{0.5}}{(\varepsilon_D-2)y} \quad ; \quad \bar{\varepsilon}_D = 2$$

There is no reason to check the robustness of π_W for higher $\bar{\varepsilon}_D$, say, $\bar{\varepsilon}_D = 5$, since then the unmodified Domar mechanism is reasonably robust.

The above analysis also shows that there is no necessity to adjust $\bar{\varepsilon}_D$ too precisely in the modified Domar mechanism. One $\bar{\varepsilon}_D$ is roughly estimated, the guideline price $\bar{p} = \overline{MC}$ can be used as the main parameter of the mechanism. The way of an iterative correction of $\bar{p}$ under the assumption of constant MC has been discussed in Section 3. If MC are not constant, we must know an estimate of $C'' = \frac{dMC}{dq}$ or of $E_{C'} = \frac{C''\cdot\hat{q}}{MC}$ to construct an iterative correction of $\bar{p}$. Suppose we take the simplest approximation of demand by assuming constant elasticity ε_D and define $x = \frac{q}{\hat{q}} - 1 = -\varepsilon_D(\frac{p}{\hat{p}} - 1)$. Now we would like to compute such x that

$$\text{(A2.13)} \qquad MC \approx MC + C''(\hat{q}-q) = MC(1-\varepsilon_{C'}x) = \hat{p}$$

which results in

$$\text{(A2.14)} \qquad \frac{p}{\hat{p}} = \frac{p}{MC(1-\varepsilon_{C'}x)} \qquad ; \qquad \frac{p}{\hat{p}} = 1 - \frac{x}{\varepsilon_D}$$

Jointly, these two equations give a quadratic equation in x, with the solution

$$\text{(A2.15)} \qquad x = \frac{\varepsilon_{C'}\varepsilon_D + 1 - ((\varepsilon_{C'}\varepsilon_D + 1)^2 - 4\varepsilon_{C'}\varepsilon_D(1-\pi_W))^{0.5}}{2\varepsilon_{C'}}$$

The other solution of this quadratic equation does not satisfy the requirement that x be negative if $\pi_W > 1$. Here, $\pi_W = \frac{p}{MC}$ cannot really be observed, only estimated from (A2.6), since z and ε_Z can be computed from observed data, and we are supposed to know approximate ε_D. Knowing x, we next choose $\bar{p}_{t+1}$ to approximate $\hat{p} = MC(1+\varepsilon_{C'},x)$:

$$\text{(A2.16)} \qquad \frac{\bar{p}_{t+1}}{\bar{p}_t} = \frac{1}{\pi_{W,t}} \frac{p_t}{\bar{p}_t}(1-\frac{1}{2}(\varepsilon_{C'}\varepsilon_D+1-((\varepsilon_{C'}\varepsilon_D+1)^2 - 4\varepsilon_{C'}\varepsilon_D(1-\pi_{W,t}))^{0.5})$$

Since we might be uncertain about true values of $\varepsilon_D \in [\varepsilon_{Dmin};\varepsilon_{Dmax}]$ $\varepsilon_{C'} \in [\varepsilon_{C'min};\varepsilon_{C'max}]$, we would have to compute $\pi_{W,t}$ and $\bar{p}_{t+1}/\bar{p}_t$ under various assumptions concerning ε_D and $\varepsilon_{C'}$ and then choose the most conservative estimate of $\bar{p}_{t+1}/\bar{p}_t$. Nevertheless, the information obtained that way might still be valuable for iterative corrections of $\bar{p}$. For example, suppose true parameters were $\varepsilon_{C'} = 1.5$, $\varepsilon_D = 0.8$, $\pi_{W,t} = 1.21$, $MC_t/\bar{p}_t = 1.4$, but we do not know them. We have used $\bar{\varepsilon}_D = 1.0$ and, after choosing some $\bar{p}_t$, have observed $p_t/\bar{p}_t = 1.69$, $z = 0.537$, $\varepsilon_Z = 1.428$. If true parameters were known, we would compute $\bar{p}_{t+1}/\bar{p}_t = 1.552$ and obtain $\bar{p}_{t+1} = \hat{p}$. However, suppose we only know that $\varepsilon_D \in [0.4;1.2]$ and $\varepsilon_{C'} \in [1.0;2.0]$. We can them compute four different estimates of $\bar{p}_{t+1}/\bar{p}_t = 1.458;\ 1.530;\ 1.790;\ 1.823$. When using the most conservative estimate $\bar{p}_{t+1}/\bar{p}_t = 1.458$, we finally obtain $\pi_{W,t+1} = 1.036$, a reasonable result with such uncertain data.

Thus, the modified Domar mechanism has many desirable theoretical properties: it is robust and can be interatively corrected even with uncertain data. However, this does not mean that this is a really implementable mechanism. Besides questions of long-term motivation, of price aggregation, etc., that are not discussed here, the mechanism has also a theoretical drawback, the same as the Tam mechanism, that it does not promote market equilibration if $\bar{p} < \hat{p}$. In fact, consider the market constraint in the form $p \leq D^{-1}(q)$ and let $\bar{p} < \hat{p} = D^{-1}(\hat{q})$. The first-order derivatives of PW at $p = \bar{p}$ are

$$\text{(A2.17)} \qquad \left.\frac{\partial \pi_W}{\partial p}\right|_{p=\bar{p}} = \left(\frac{dz}{dp} \cdot p \cdot q + z \cdot q\right)_{p=\bar{p}} = zq(1-\varepsilon_z)_{p=\bar{p}} = 0$$

$$\text{(A2.18)} \qquad \left.\frac{\partial \pi_W}{\partial q}\right|_{p=\bar{p}} = (z \cdot p - MC)_{p=\bar{p}} = \bar{p} - MC$$

Thus, the producer can behave in this situation as if the price $\bar{p}$ were centrally set, that is, choose $p = \bar{p}$ and $q = \bar{q}$ such that $MC(\bar{q}) = \bar{p}$, as long as this $\bar{q} < D(\bar{p})$ - which happens precisely if $\bar{p} < \hat{p} = D^{-1}(\hat{q})$, implying $\bar{q} = MC^{-1}(\bar{p}) < MC^{-1}(\hat{p}) = \hat{q} = D(\hat{p}) < D(\bar{p})$ under the assumption of locally invertible, rising marginal cost.

This also shows that in deep disequilibrium the modified Domar mechanism is equivalent to central price setting. In the case of a very low market elasticity, when $\varepsilon_D \to 0$ and $q = D(p)$ is approximately constant, it is easy to see from (A2.4) that $z \to \bar{p}/p$, thus $PW \to \bar{p}q - C$ - again equivalent to the central price setting of $\bar{p}$. The only difference between deep disequilibrium and inelastic demand is that the producer is assumed to meet the demand in the latter case.

APPENDIX 3. PRICE-DEPENDENT TAXATION MECHANISM FOR DEEP DISEQUILIBRIA

If the reward function (or the actual profit remaining with the producer after taxation) is $PA = \alpha(pq-C)$, where α is a function of p and $\varepsilon_A = -\frac{d\alpha}{dp}\frac{p}{\alpha} \geq 0$, and if $p < D^{-1}(q)$, the market constraint is not active, then

(A3.1) $\quad \frac{\partial PA}{\partial p} = \frac{d\alpha}{dp}(pq-C) + \alpha q = \alpha q(1-\varepsilon_A(1-\frac{AC}{p}))$; $AC = \frac{C}{q}$

and

(A3.2) $\quad \frac{\partial PA}{\partial p} = 0 \Rightarrow p(1-\frac{1}{\varepsilon_A}) = AC$

where

(A3.3) $\quad \frac{\partial PA}{\partial q} = \alpha(p-MC)$; $MC = \frac{dC}{dq}$

and

(A3.4) $\quad \frac{\partial PA}{\partial q} = 0 \Rightarrow p = MC$

Now, suppose $\alpha(p)$ has the form given in Fig.1. Thus

(A3.5) $$\alpha = \begin{cases} \alpha_0 , & \text{if} \quad p \leq \bar{p} = \gamma\tilde{p} \\ \frac{\alpha_0}{1-\gamma}(1-\frac{p}{\tilde{p}}) , & \text{if} \quad \gamma\tilde{p} = \bar{p} < p < \tilde{p} \\ 0 , & \text{if} \quad \tilde{p} \leq p \end{cases}$$

and

(A3.6) $$\varepsilon_A = \begin{cases} 0 , & \text{if} \quad p < \bar{p} = \gamma\tilde{p} \\ \frac{p/\tilde{p}}{1-p/\tilde{p}} , & \text{if} \quad \gamma\tilde{p} = \bar{p} < p < \tilde{p} \\ \text{undefined}, & \text{if} \quad \tilde{p} \leq p \end{cases}$$

Observe that at $p = \bar{p} = \gamma\tilde{p}$ ε_A increases from zero to $\frac{\gamma}{1-\gamma}$, and that $\varepsilon_A \to \infty$ if $p \to \tilde{p}_-$. Moreover, $1 - \frac{1}{\varepsilon_A} = 2 - \frac{\tilde{p}}{p}$ if $\bar{p} < p < \tilde{p}$. Thus (A3.2) and (A3.4) yield $p(2-\frac{\tilde{p}}{p}) = 2p - \tilde{p} = AC$ with $p = MC$, which results in

(A3.7) $\quad MC = p = \frac{1}{2}(AC+\tilde{p}) \quad$ if $\quad \bar{p} < p < \tilde{p}$

This simple result gives the possibility of an easy estimation of $MC = p$ and $AC = 2p - \tilde{p}$ once $\tilde{p}$ is centrally set and p is observed. Moreover, suppose $AC_{min} > (2\gamma-1)\tilde{p}$; then $\frac{1}{2}(AC+\tilde{p}) > \frac{1}{2}(AC_{min}+\tilde{p}) > \gamma\tilde{p}$, and $p > \gamma\tilde{p} = \bar{p}$. Clearly $AC_{min} < \tilde{p}$ results in $MC = p < \tilde{p}$. Thus;

(A3.8) $$AC_{min} < \tilde{p} < \frac{1}{2\gamma-1} AC_{min} \Rightarrow \bar{p} < p < \tilde{p} \text{ , if } \gamma > 0.5.$$

Therefore, by choosing, say, $\gamma = 0.6$ we admit a wide range of uncertainty for AC_{min}: it can change between $0.2\tilde{p}$ and $\tilde{p}$, still resulting in p between $0.6\tilde{p}$ and $\tilde{p}$. If we choose $\gamma = 0.5$, we even have $0 < AC_{min} < \tilde{p} \Rightarrow \bar{p} < p < \tilde{p}$. Thus, when introducing price-dependent profit taxation without much information about cost characteristics, it is best to choose γ equal or close to 0.5 and $\tilde{p}$ obviously not smaller than AC_{min} (for example, on the level of recently reported average costs).

On the other hand, (A3.7) defines how far the actual produced quantity q exceeds the optimal cost capacity q_0 defined by

(A3.9) $$\min_{q} AC(q) = AC(q_0) = AC_{min}$$

In order to approximate the quantity q, assume simple quadratic approximation of production cost around q_0

(A3.10) $$C = C_0 + C_0'(q-q_0) + \frac{1}{2}C_0''(q-q_0)^2$$

which results in approximations of average and maximal costs

(A3.11) $$MC = C_0' + C_0''(q-q_0) \quad ; \quad AC = C_0' + \frac{1}{2}\frac{C_0'}{q_0}(q-q_0)^2$$

where $C_0' = AC_{min} = C_0/q_0$. Substituting (A3.11) by (A3.7) we obtain a quadratic equation for $(q-q_0)$ and choose a solution of this equation that satisfies $q-q_0 = 0$ if $\tilde{p} = AC_{min}$

$$\text{(A3.12)} \qquad q - q_0 = 2q_0(1-(1-(\frac{\tilde{p}}{AC_{min}}-1)/2\varepsilon_{C'})^{0.5}) \quad ; \varepsilon_{C'} = \frac{C_0''q_0}{C_0'}$$

This, if the marginal cost elasticity $\varepsilon_{C'}$ is, for example, equal to 1, then $\tilde{p} = 3AC_{min}$ yields $q = 3q_0$. For smaller $\tilde{p}/AC_{min}$, a more realistic approximation of (A3.12) is

$$\text{(A3.13)} \qquad q - q_0 \approx \frac{\tilde{p}-AC_{min}}{2C_0''} = q_0(\frac{\tilde{p}}{AC_{min}} - 1)/2\varepsilon_{C'} \quad ; \frac{\partial q}{\partial \tilde{p}} = \frac{1}{2C_0''}$$

Both (A3.12) and (A3.13) hold if $\tilde{p} \geq AC_{min}$, implying $q \geq q_0$. If $\tilde{p} < AC_{min}$, since ε_A is not defined at $p \geq \tilde{p}$, the producer could theoretically choose any price and any produced quantity. However, we assume that he will behave rationally and try to minimize his average costs in order to check whether a profitable production is at all possible; this results in $q = q_0$ and $p = AC_{min}$ if $\tilde{p} < AC_{min}$ (which could be formally obtained by additionally defining $\varepsilon_A = +\infty$ and $1 - \frac{1}{\varepsilon_A} = 1$ for $p > \tilde{p}$, see Fig.2,3).

Therefore, if there are several producers with widely different production costs and subject to uniform price-dependent profit taxation, with the same $\tilde{p}$ and $\bar{p} = \gamma\tilde{p}$, then their profits will be sharply limited by the taxation mechanisms. The least efficient with $AC_{1min} \geq \tilde{p}$ will charge the price $p_1 = AC_{1min}$ and have no profit. The more efficient with $(2\gamma-1)\tilde{p} < AC_{2min} < \tilde{p}$ will charge a price p_2 that can be determined by combining (A3.11) and (A3.13):

$$\text{(A3.14)} \qquad p_2 = MC_2 \approx \frac{\tilde{p}+AC_{2min}}{2} = AC_{2min}\,\frac{\eta_2^{-1}-1}{2} \; ; \; \eta_2 = \frac{AC_{2min}}{p}$$

and will produce a quantity

$$\text{(A3.15)} \qquad q_2 \approx q_{20}\left(1 + \frac{\eta_2^{-1}-1}{2\varepsilon_{C_2'}}\right)$$

while his profit remaining after taxation can be estimated by:

$$\text{(A3.16)} \qquad PA_2 \approx AC_{2min}\, q_{20}\, \frac{\alpha_0}{4(1-\gamma)} \cdot \frac{(1-\eta_2)^2}{\eta_2}\left(1 + \frac{\eta_2^{-1}-1}{2\varepsilon_{C_2'}}\right)$$

and the corresponding profit rate PA_2/AC_2q_2 by:

$$\text{(A3.17)} \qquad \frac{PA_2}{AC_2q_2} \approx \frac{\alpha_0}{4(1-\gamma)} \cdot \frac{(1-\eta_2)^2}{\eta_2}$$

This is illustrated by the following table (with $\gamma = 0.5$, $\alpha_0 = 1$):

$\eta = \frac{AC_{min}}{p}$	1.000	0.667	0.500	0.333	0.250	0.200
$\frac{p}{AC_{min}}$	1.000	1.250	1.500	2.000	2.500	3.00
$\alpha(p)$	0.000	0.333	0.500	0.667	0.750	0.800
$\frac{PA}{AC \cdot q}$	0.000	0.083	0.250	0.667	1.125	1.600

Even if, for example, $\tilde{p}$ is 3 times larger than AC_{min}, $\eta = 0.333$, the producer charges the price p only twice AC_{min}, and has a remaining profit rate of 66.7% - instead of charging the price $p = \tilde{p}$ and having 200% profit rate, which would be obtained in this case if $p = \tilde{p}$ were centrally set. The above results are approximate; more precise results can be obtained by using (A3.12) instead of (A3.13), which limits the profit rates of the producer even more strongly.

If $\gamma > 0.5$, then it might happen that some producer would have $AC_{3min} < (2\gamma-1)\tilde{p}$ and charge the price $p = \gamma\tilde{p} = \bar{p}$, thus keeping the largest part of profit α_0 to himself. In any case, the above examples show that the central authority can influence producer behavior by choosing an upper guideline price $\tilde{p}$ - but this influence is much softer and more robust than when simply centrally setting the price $\tilde{p}$.

Consider now a case of various transactions and products of a single producer. Suppose that, in a single transaction, he sells a quantity q_k of a product at a price p_k, while the upper guideline price for this product and transaction was established at $\tilde{p}_k$. Consider first a situation in which the quantity q_k was externally fixed and suppose the producer chooses p_k in order to maximize

$$\text{(A3.18)} \qquad PA = \alpha(p) \left(\sum_{m=1}^{M} p_m q_m - C(q_2, \ldots, q_k, \ldots, q_M) \right)$$

where $\alpha(p)$ is defined as in (A3.5) with p being some function of p_k (and $\tilde{p}$ - of $\tilde{p}_k$). Then the condition

(A3.19) $$\frac{\partial PA}{\partial p_k} = \frac{\partial \alpha}{\partial p}\frac{\partial p}{\partial p_k}\left(\sum_{m=1}^{M} p_m q_m - C\right) + \alpha q_m = 0$$

can be transformed to

(A3.2) $$p_k\left(1 - \frac{y_k}{\varepsilon_A \varepsilon_{p_k}}\right) = AC_k$$

where

(A3.21) $$AC_k = \frac{y_k\ C}{q_k} \quad ; \quad y_k = \frac{p_k q_k}{\sum_{m=1}^{M} p_m q_m} \quad ; \quad \varepsilon_{p_k} = \frac{\partial p}{\partial p_k}\frac{p_k}{p}$$

Observe that y_k is the revenue share of the k-th transaction and AC_k is the average cost related to the k-th transaction through this revenue share. To obtain the same result as in (A3.2) we would have to define the dependence of p on p_k in such a way that $\varepsilon_{p_k} = y_k$. But this is obtained by the most natural way of price aggregation:

(A3.22) $$p = \frac{\sum_{m=1}^{M} p_m q_m}{\sum_{m=1}^{M} q_m} \quad ; \quad \tilde{p} = \frac{\sum_{m=1}^{M} \tilde{p}_m q_m}{\sum_{m=1}^{M} q_m}$$

Since then $\frac{\partial p}{\partial p_k} = q_k / \sum_{m=1}^{M} q_m$ and $\varepsilon_{p_k} = y_k$, which results in:

(A3.23) $$p_k\left(1 - \frac{1}{\varepsilon_A}\right) = AC_k$$

which is easily shown to be equivalent to:

(A3.24) $$p\left(1 - \frac{1}{\varepsilon_A}\right) = AC, \quad AC = C / \sum_{m=1}^{M} q_m$$

If the producer, in deep disequilibrium conditions, can also influence the quantities q_k which he sells, he will choose them to satisfy $\frac{\partial PA}{\partial q_k} = 0$. This, if $\alpha(p)$ did not depend on q_k, would result in $p_k = \frac{\partial C}{\partial q_k}$, that is in ideal marginal pricing. However, $p/\tilde{p}$ as aggregated by (A3.22) does not depend on q_k only if $p_k/\tilde{p}_k = p/\tilde{p}$, that is, if the relative price level of the k-th transaction coincides with the average price level.
In a more general case we have

$$\text{(A3.25)} \qquad \frac{\partial(p/\tilde{p})}{\partial q_k} = \tilde{p}_k \frac{(p_k/\tilde{p}_k - p/\tilde{p})}{\sum_{m=1}^{M} \tilde{p}_m q_m}$$

Thus, if $p_k/\tilde{p}_k < p/\tilde{p}$, an increase of q_k favourably influence (increase it, since it decreases $p/\tilde{p}$). In other words, if the producer is offered at the k-th transaction a favourably high upper guideline price $\tilde{p}_k$, he will increase the quantity q_k and change the price p_k when compared to the marginal price $\frac{\partial C}{\partial q_k}$, while keeping to the average pricing rule (A3.23). To determine how far he will distort the marginal pricing rule, we compute:

$$\text{(A3.26)} \qquad \frac{\partial PA}{\partial q_k} = \frac{\partial \alpha}{\partial p} \cdot \tilde{p} \cdot \frac{\partial(p/\tilde{p})}{\partial q_k} \left(\sum_{m=1}^{M} p_m q_m - C \right) + \alpha \cdot \left(p_k - \frac{\partial C}{\partial q_k} \right) = 0$$

which can be solved (since

$$\frac{\tilde{p}}{p} \left(\sum_{m=1}^{M} p_m q_m - C \right) = \sum_{m=1}^{M} \tilde{p}_m q_m \left(1 - \frac{AC_k}{p_k} \right) = \frac{1}{\varepsilon_A} \sum_{m=1}^{M} \tilde{p}_m q_m \quad \text{and} \quad \frac{d\alpha}{dp} \cdot \frac{p}{\alpha} = - \varepsilon_A)$$

to obtain

$$\text{(A3.27)} \qquad \frac{p}{\tilde{p}} = \frac{MC_k}{\tilde{p}_k} \quad ; \quad MC_k = \frac{\partial C}{\partial q_k}$$

Thus, the producer should increase q_k until $\frac{\partial C}{\partial q_k} = MC_k$ has the same proportion to $\tilde{p}_k$ as p has to $\tilde{p}$. Since $1-\frac{1}{\varepsilon_A} = 2-\frac{\tilde{p}}{p}$ if ε_A has the form (A3.6), hence p_k can be determined from (A3.23) as

$$\text{(A3.28)} \qquad p_k = MC_k \cdot \frac{AC_k}{2MC_k-\tilde{p}_k} = MC_k(1+\delta_k)$$

where

$$\text{(A3.29)} \qquad \delta_k = \frac{AC_k/\tilde{p}_k - AC/\tilde{p}}{AC/\tilde{p}} = \frac{\eta_k-\eta}{\eta}$$

The above equations should be interpreted as follows. The producer expects average price $p,\tilde{p}$ and total costs C,AC. If he has to determine the quantity q_k and the price p_k for a transaction with a given $\tilde{p}_k$, he determines first the quantity q_k such that $MC_k(q_k) = \frac{p}{\tilde{p}}\,\tilde{p}_k$ and sets, as a first approximation, $p_k = MC_k(q_k)$. He can then check the average cost related to this transaction, AC_k, and compare it to $\tilde{p}_k$. If $\eta_k = AC_k/\tilde{p}_k$ turns out to be smaller than $\eta = AC/p$, then he knows that $\delta_k < 0$, p_k can be set smaller than MC_k and thus favourably influence the ratio $p/\tilde{p}$. Clearly, this is only the first approximation: such a process should be iteratively repeated, the expected price p should be adjusted and thus the quantity q_k changed, etc. However, we do not discuss these questions fully here, as a full analysis would require assumptions about how the expectations of the producer regarding prices - in particular guideline prices $\tilde{p}_t$ - are formed.

Now consider again the price-dependent taxation $PA = \alpha(p)\cdot(pq-C)$, with one producer and one product, but with possible market constraint, $q \leq D(p)$. The Lagrange function $L = \alpha(pq-C) + \lambda(D-q)$ has stationary points in p and q, if

(A3.30) $\lambda = \alpha(p-MC) \quad ; \quad p(1-\varepsilon_A) + \varepsilon_A AC = \frac{\varepsilon_D}{\alpha}\lambda$

The first condition indicates that $\lambda \geq 0$ only if

(A3.31) $p \geq MC$

and only under this condition can the constraint $q \leq D(p)$ be active. If $q < D(p)$, then $\lambda = 0$ and $p = MC$. The second condition results in:

(A3.32) $p(1-\varepsilon_A-\varepsilon_D) + \varepsilon_A AC + \varepsilon_D MC = 0$

which, with $\frac{MC}{AC} = \varepsilon_C$, can be rewritten in two equivalent forms:

(A3.33) $p(1 - \frac{\varepsilon_A(1-\varepsilon_C)+\varepsilon_C}{\varepsilon_A+\varepsilon_D\varepsilon_C} = MC \Leftrightarrow p(1 - \frac{1-(1-\varepsilon_C)\varepsilon_D}{\varepsilon_A+\varepsilon_D\varepsilon_C}) = AC$

Observe that $p \geq MC$ is equivalent to $\varepsilon_A(1-\varepsilon_C) + \varepsilon_C \geq 0$, or:

(A3.34) $\varepsilon_C \leq \frac{\varepsilon_A}{\varepsilon_A-1}$

Since we have $\varepsilon_A = \frac{p/\tilde{p}}{1-p/\tilde{p}}$, thus $\frac{\varepsilon_A}{\varepsilon_A-1} = \frac{p}{2p-\tilde{p}}$; we obtain $\varepsilon_C = \frac{\varepsilon_A}{\varepsilon_A-1}$, $\hat{p} = \hat{MC}$ if $2\hat{p} - \tilde{p} = \hat{AC}$, or:

(A3.35) $\tilde{p} = 2\hat{MC} - \hat{AC}$

If $\tilde{p}$ is set wrongly, $\tilde{p} > 2\hat{MC} - \hat{AC}$, then $\frac{\hat{MC}}{2\hat{MC}-\tilde{p}} > \frac{\hat{MC}}{\hat{AC}} = \hat{\varepsilon}_C$. Thus, $p = \hat{MC}$ will not be chosen, because then $\varepsilon_A(1-\hat{\varepsilon}_C) + \hat{\varepsilon}_C > 0$ and $p > \hat{MC}$. If $\tilde{p} < 2\hat{MC} - \hat{AC}$, then $\frac{\hat{MC}}{2\hat{MC}-\tilde{p}} < \hat{\varepsilon}_C$, which would indicate $p < \hat{MC}$; but this implies that the market constraint will not be active, $\lambda = 0$ amd some $p = \hat{MC}$ MC will be chosen without producing a quantity that equilibrates the market.

Suppose $\tilde{p} \geq 2\hat{MC} - \hat{AC}$ and consider the robustness of the price ratio:

$$\text{(A3.36)} \qquad \pi_A = \frac{P}{MC} = \frac{\varepsilon_A/\varepsilon_C + \varepsilon_D}{\varepsilon_A + \varepsilon_D - 1}$$

We then have $\frac{\partial \pi_A}{\partial \varepsilon_A} \cdot \frac{\varepsilon_A}{\pi_A} = - \frac{1}{\varepsilon_A + \varepsilon_D - 1}$ at $\varepsilon_A = \frac{\varepsilon_C}{\varepsilon_C - 1}$ resulting in $\pi_A = 1$.

Moreover, $\frac{\partial \varepsilon_A}{\partial (p/\tilde{p})} \cdot \frac{p/\tilde{p}}{\varepsilon_A} = \frac{1}{1-p/\tilde{p}}$. To simplify analysis, assume $\varepsilon_A >> \varepsilon_D$; then (A3.33) becomes $p(\varepsilon_A - 1) \approx \varepsilon_A AC$ or $AC \approx 2p - \tilde{p}$, $p \approx \frac{1}{2}(\tilde{p} + AC)$, with $\frac{\partial (p/\tilde{p})}{\partial \tilde{p}} \cdot \frac{p}{p/p} = - \frac{AC}{2p}$. Thus we obtain (with $\frac{1}{\varepsilon_A - 1 + \varepsilon_D} \approx \frac{1}{\varepsilon_A - 1}$) :

$$\text{(A.37)} \qquad \left. \frac{\partial \pi_A}{\partial \tilde{p}} \frac{\tilde{p}}{\pi_A} \right|_{\tilde{p} = 2\hat{MC} - \hat{AC}} \approx \frac{\hat{AC}(2\hat{MC} - \hat{AC})}{2\hat{MC}(3\hat{MC} - 2\hat{AC})} = \frac{2\hat{\varepsilon}_C - 1}{2\hat{\varepsilon}_C(3\hat{\varepsilon}_C - 2)} \approx \frac{1}{2},$$

if $\hat{\varepsilon}_C \approx 1$.

Since $\frac{\partial \tilde{p}}{\partial \hat{MC}} = 2$ and $\frac{\partial \tilde{p}}{\partial \hat{MC}} \cdot \frac{\hat{MC}}{\tilde{p}} = \frac{2\hat{\varepsilon}_C}{2\hat{\varepsilon}_C - 1} \approx 2$, if $\hat{\varepsilon}_C \approx 1$. we see that

$$\text{(A3.38)} \qquad E_{\pi AMC} = \left. \frac{\partial \pi_A}{\partial \overline{MC}} \frac{\overline{MC}}{\pi_A} \right|_{\overline{MC} = \hat{MC}} \approx 1$$

if $\hat{\varepsilon}_C$ is not too far from 1. Thus the price dependent profit taxation mechanism is not very robust, if the market constraint is active.

APPENDIX 4. MOTIVATION MECHANISMS FOR A DISTRIBUTOR

First consider a distributor buying from several producers. No matter whether he sets the buying price p_B, or upper and lower guideline prices for a profit taxation mechanism, he will buy quantities q_i resulting from

$$p_B = MC_i(q_i) \tag{A4.1}$$

Hence his total outlay is

$$TO = \sum_i MC_i(q_i) \cdot q_i \tag{A4.2}$$

Let $q = \sum_i q_i$; then $\frac{dq}{dp_B} = \sum_i \frac{dq_i}{dp_B} = \sum_i \frac{1}{C_i''}$, where $C_i'' = \frac{dMC_i(q_i)}{dq_i}$.

Denote $\bar{C}'' = \left(\sum_i \frac{1}{C_i''}\right)^{-1}$; then $\frac{dp_B}{dq} = \bar{C}''$. The marginal outlay of the distributor is:

$$MO = \frac{d}{dq}(p_B q) = p_B\left(1+\frac{\bar{C}''\,q}{C'}\right) = p_B(1+\varepsilon_{C'}) \tag{A4.3}$$

where $\varepsilon_{C'} = \frac{q\bar{C}''}{MC_i}$ does not depend on the number of producers, only on the distribution of their production costs; if all producers have the same cost curves, then $\varepsilon_{C'} = \frac{qC''}{MC}$ is the elasticity of marginal costs. It can easily be seen that, for typical cost curves, $\varepsilon_{C'} \approx \frac{2C(0)}{C(q_0)} < 2$.

If the distributor has full monopoly and monopsomy power, he will maximize $(p_C - p_B)q$ on a market with $p_C = D^{-1}(q)$, which results in

$$MR = MO \Rightarrow p_C\left(1-\frac{1}{\varepsilon_D}\right) = p_B(1+\varepsilon_{C'}) \tag{A4.4}$$

with $p_C >> p_B = MC$. However, we can limit the power of the distributor by allowing him to charge only mark-up prices, $p_C = (1+\delta)p_B$, where δ is a given coefficient (approximating distribution costs). His profit then takes the form (we denote $p_B = p$ for simplicity):

$$(A4.5) \qquad p_\delta = \delta \cdot p \cdot q$$

and his behavior (under market constraint $p + D^{-1}(q)$) is determined by the derivative:

$$(A4.6) \qquad \frac{dp_\delta}{dq} = \delta \cdot MR = \delta \cdot p(1-\frac{1}{\varepsilon_D})$$

If $\varepsilon_D > 1$ for all $p = D^{-1}(q)$, then the distributor is motivated to increase q as far as possible. Thus, he will buy at the lowest price $p = MC(q)$ which, together with $p = D^{-1}(q)$, determines the maximal q - at the ideal short-term market equilibrium. If $\varepsilon_D \leq 1$ for some $p = D^{-1}(q)$, $p > MC(q)$, the distributor will choose the price p and quantity q which result in $\varepsilon_D=1$, $MR=0$ - even if he then has to pay the producer price p higher than $MC(q)$.

Thus, consider a reward function for the distributor in the form

$$(A4.7) \qquad PA_\delta = \alpha(pc) \cdot \delta \cdot p \cdot q \quad ; \quad p_C = (1+\delta)p$$

where α has the same form as in Appendix 3. Here we consider the market constraint in the form $p_C \leq D^{-1}(q)$ and the buying constraint in the form $p \geq MC(q)$, in order to check whether the reward function (A4.7) promotes market equilibration.

Since

$$(A4.8) \qquad \frac{\partial PA_\delta}{\partial q} = \alpha \cdot \delta \cdot p > 0 \quad ; \quad \frac{\partial PA_\delta}{\partial p} = \alpha \cdot \delta \cdot q(1-\varepsilon_A) < 0$$

where $\varepsilon_A = -\frac{d\alpha}{dp_C} \cdot \frac{p_C}{\alpha} > 1$ if $\gamma > 0.5$ (since $\varepsilon_A = \frac{p_C/\tilde{p}_C}{1-p_C/\tilde{p}_C}$ with $p(\tilde{p}_C > \gamma)$, hence the distributor is motivated to increase q and decrease p. By doing this, he first meets the buying constraint $p = MC(q)$; we assume that the marginal costs (in short-term) are rising with q. This constraint results in

$$(A4.9) \qquad \frac{dPA_\delta}{dq} = \alpha \cdot \delta \cdot (p+q\ C''(1-\varepsilon_A))$$

If the production capacities are too low to satisfy market demand and $\frac{dPA_\delta}{dq}$ becomes zero at some $q < D(p_C)$, then the distributor chooses a disequilibrium solution:

$$(A4.10) \qquad p = MC \ ; \ q = \frac{MC}{C''(1-\varepsilon_A)} \quad \Rightarrow \quad \varepsilon_{C'} = \frac{1}{1-\varepsilon_A}$$

However, he is still motivated to increase q if he can achieve this by promoting investments and thus lower marginal costs.
If the production capacities are already sufficient for market equilibration and (A4.9) stays positive until $q = D(p_C)$, he will choose the ideal solution $p_C = (1+\delta)MC$, $q = D(p_C)$, no matter what the parameters of his profit taxation mechanism are (as long as $(2\gamma-1)\tilde{p}_C < p_C < \tilde{p}_C$, see Appendix 3). This can be shown in various ways; the simplest one is to compute the derivative $\frac{dPA_\delta}{dq}$ under the constraint $p_C = (1+\delta)p = D^{-1}(q)$, without the constraint $p = MC(q)$:

$$(A4.11) \qquad \frac{dPA_\delta}{dq} = \alpha \cdot \delta \cdot p \cdot \frac{\varepsilon_D + \varepsilon_A - 1}{\varepsilon_D} > 0$$

if $\varepsilon_A > 1$. Thus, under a given pair p_C, q on the market, the distributor is motivated to increase q and decrease p_C until he reaches $p_C = (1+\delta)MC(q)$.

If the central authority sets the consumer price sharply by $p_C \leq \tilde{p}_C$, but uses volume-dependent taxation to establish the distributor motivation:

$$(A4.12) \qquad PB_\delta = \beta(q) \cdot \delta \cdot p \cdot q$$

with

$$(A4.13) \qquad \beta(q) = \begin{cases} 0 & , \quad q \leq \bar{q} = \omega\tilde{q} \\ \beta_0 \frac{q/\tilde{q} - \omega}{1 - \omega} & , \quad \bar{q} = \omega\tilde{q} < q < \tilde{q} \\ \beta_0 & , \quad \tilde{q} \leq q \end{cases}$$

and

$$(A.14) \qquad \frac{d\beta}{dq} \cdot \frac{q}{} - \varepsilon_B = \begin{cases} \text{undefined} & , \quad q \leq \bar{q} = \omega\tilde{q} \\ \frac{q/q}{q/q - \omega} & , \quad \bar{q} = \omega\tilde{q} < q < \tilde{q} \\ 0 & , \quad \tilde{q} \quad q \end{cases}$$

then $\frac{\partial PB_\delta}{\partial p} = \beta \cdot \delta \cdot q > 0$, the distributor increases $p_C = (1+\delta)p$ to $\tilde{p}_C$ and

$$(A4.15) \qquad \frac{\partial PB_\delta}{\partial q} = \beta \cdot \delta \cdot p(1+\varepsilon_B) > 0$$

the distributor increases the volume until he meets the buying constraint, $MC(q) \leq p = \frac{\tilde{p}_C}{1+\delta}$, or the market constraint $q \leq D(p_C)$. If the buying constraint is met first, a disequilibrium solution

$q = MC^{-1}(\frac{\tilde{p}_C}{1+\delta})$ is obtained; but the distributor is motivated to promote investments and equilibriate the market. If the market constraint is met first, then:

$$(A4.16) \qquad \frac{dPB_\delta}{dq} = \beta \cdot \delta \cdot p(1+\varepsilon_B - \frac{1}{\varepsilon_D})$$

and $\frac{dPB_\delta}{dq} > 0$ if $\varepsilon_B + 1 > \frac{1}{\varepsilon_D}$. Since $\varepsilon_B \geq \frac{1}{1-\omega}$ as long as $\bar{q} < q < \tilde{q}$, hence if we choose $\omega = 0.75$, $\varepsilon_B \geq 4$, we have $\frac{dPB_\delta}{dq} > 0$ for all $\varepsilon_D > 0.2$. Under these conditions, market constraint will result in further increase of q and decrease of p_C until the buying constraint, $MC(q) \leq p = \frac{p_C}{1+\delta}$ is attained. This corresponds again to a locally fully robust ideal equilibrium solution.