BEHAVIORAL STRATEGIES IN ZERO SUM GAMES
IN EXTENSIVE FORM

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Behavioral Strategies in

Zero Sum Games in Extensive Form

1. Introduction

The purpose of these notes is to present some formal properties of
behavioral strategies in relation to some comments made by Aumann and
Maschler [A-M]. In their main example, they show that the behavioral
strategy generated by the optimal mixed strategy may be dominated in terms
of security level by some other strategy.

Wilson has proposed a constructive scheme to generate a decision tree
for each player from a game tree as long as it is with perfect recall [W].
Using this scheme, one may precisely define the expected payoff conditional
on being at a given informal set and on the other players' strategies. The
security level conditional on being at a given informal set may similarly
be defined using the player's decision tree. Note that since all moves
are "sequentially played" in a decision tree, randomization is irrelevant
for the security level.

Then the mathematical counterpart of Aumann and Maschler's argument
for zero sum games in extensive form may be stated as follows: at the
information sets of a game tree, the optimal behavioral strategies may only
satisfy one part of the double minimax inequality, namely the equilibrium part.
Furthermore, the only class of games for which the double minimax inequality
seems to remain valid is the class with perfect information since, in this
case, the player's decision trees are identical.

These notes are organized as follows: the second section merely consists
of an example to illustrate our restatement of Aumann and Maschler's argument.
Some more perplexing remarks about behavioral strategies after a non optimal
move will also be presented.

2. An Example

Consider the game tree depicted in Figure 1. It may be interpreted as a one stage poker game with a high or low card and two possible raise moves or a drop move for player 1 and a drop or call move for player 2. Assume that player 1 is the maximiser.
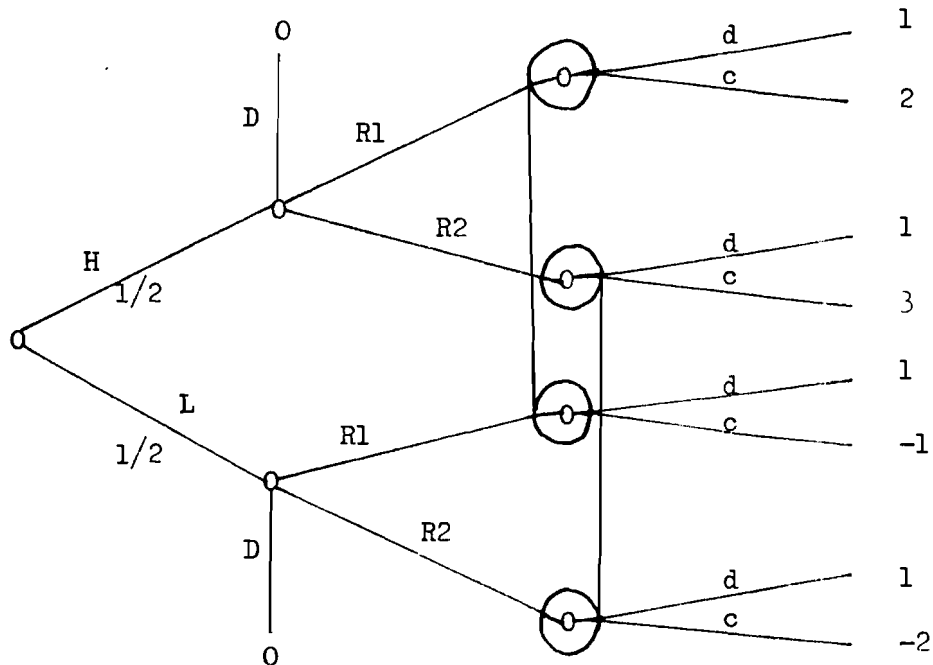
Move: Chance   Player I   Player II

Figure 1

The Game Tree

There are two information sets for player II each one including two nodes. We want to define an expected payoff conditional on each information set and a given strategy of player I. The optimal behavioral strategy for player I (which, for instance, may be obtained from the normal form) is to play R2 if the chance move is H and to play R2 with probability 2/3 and D with probability 1/3 if the chance move is L. Once player I's move is played, player II's conditional expectations may be defined by constructing his decision tree. According to Wilson's procedure, this decision tree

is described in Figure 2. Note that the probabilities on H and L are now conditional on R1 or R2. Conditional on R1, these probabilities are not defined and we shall come back to this point later on. Conditional on R2, these probabilities are easily seen to be 3/5 and 2/5 respectively.
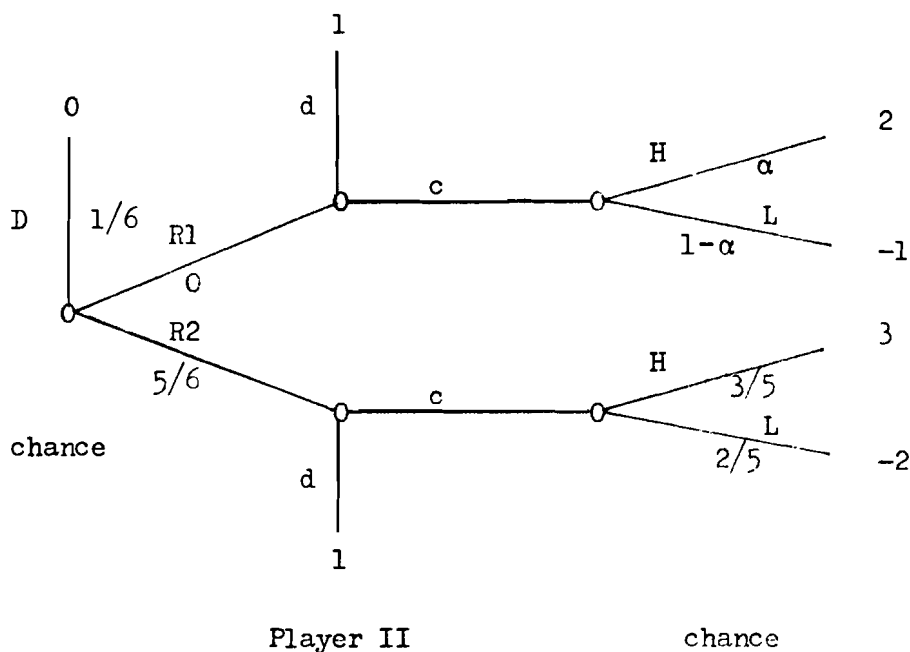

Figure 2

Player II's Decision

We are now in a position to define player II's expected payoff conditional on R2 being played and on player I's strategy. The expected payoff is 1 if he plays d and $\frac{3}{5} \cdot 3 - \frac{2}{5} \cdot 2 = 1$ if he plays c. His optimal behavioral stragegy at this information set is to play d with probability 2/3 and c with probability 1/3. As such, it maximizes his expected payoff conditional on R2 being played and on player I's optimal strategy.

What is the security level associated with his optimal strategy given that R2 was played? If player I's strategy was to play R2 only if the outcome of the chance move is H, then player II's expectation, given his own optimal strategy and conditional on R2 being played and on player I's new strategy, would now be $\frac{1}{3}[1.3 - 0.2] + \frac{2}{3} \cdot 1 = \frac{5}{3}$ which is worse than 1 (remember that player II is the minimiser).

Let (s, t) denote behavioral strategies for player I and II respectively and (s*, t*) denote the optimal strategies. Let E be a player's information set and $V(s^*, t^*| E)$ be his conditional payoff at this information set. Assume that this information set belongs to player I. Then it is easy to show that:

$$(i) \quad \text{for all } s, \quad V(s, t^*| E) \le V(s^*, t^*| E),$$

whereas the following inequality may not be true as shown in the preceding example:

$$(ii) \quad \text{for all } t, \quad V(s^*, t^*| E) \le V(s^*, t| E) \quad .$$

Thus, in terms of conditional payoffs, only the equilibrium part of the double minimax inequality remains satisfied. Note that as soon as the players are called simultaneously to make a move in a game tree, it is likely to result in different security levels for each of them since, according to our definition of conditional payoffs at an information set, it cannot be increased by means of randomization. Thus one would expect that the only class of games for which (i) and (ii) could be satisfied at each information set are games with perfect information. And indeed, one can easily show that for such games, (i) and (ii) remain true since then both players have the same decision tree.

This would suggest that the minimax principle is essentially appropriate for the "static" normal form (and Von Neumann and Morgenstern do claim repeatedly that they are building a static theory [N-M]). Subsequently, "optimal" behavioral strategies in zero sum extensive games may ordinarily only qualify as "equilibrium" strategies in a "non-zero sum" sense. But would the equilibrium principle, with its well known pitfalls, be actually suitable for "dynamic" theory of extensive games? The following remarks are presented so as to point out some difficulties in the dynamic aspect of the equilibrium principle.

Consider again the poker example but now assume that player II is at the other information set; that is, assume that Rl was played. Player II's equilibrium behavioral strategy is degenerated and may be any convex combination of the following two strategies: the first one consists of playing d with probability $\frac{1}{2}$ and c with probability $\frac{1}{2}$, and the second one of playing d with probability $\frac{1}{3}$ and c

with probability $\frac{2}{3}$. As we said earlier, player II's expectation conditional
on Rl being played and on player I's equilibrium strategy is not mathematically
defined since it is conditioned on any event with zero probability. Thus,
inequality (i) is not defined. Nevertheless, let us try to interpret player
II's equilibrium strategy by working backward. Clearly it does not guarantee
him his conditional security level which is 1. If it were to maximize his
expected payoff conditional on Rl being played, then this would imply that
the probabilities on H and L would be 2/3 and 1/3 respectively. This, in turn,
would imply that player I would have chosen Rl with a probability, say, k
if the chance move is H and with a probability k/2 is the chance move is L.
This seems to be a very definite statement to make since the only thing that
player II knows about player I's strategy is that he made a mistake. How he
made it is certainly a matter of opinion and not a mathematical fact (or would
there be anything like an "optimal" mistake?). Thus, it seems unjustified to
say that player II's equilibrium strategy maximizes his expected payoff
conditional on Rl being played. The only rationale for player II's equili-
brium strategy appears to be that it makes move Rl unattractive to player I
and thus should enforce him to play his equilibrium strategy. As such, it
may be interpreted as a good threat to deter player I from a deviating behavior.
However, once player I did deviate, this threat has no theoretical justification
any longer. (To carry out this threat may be worthwhile in a repeated situation
but this is a one shot game.)

Thus, the equilibrium principle, just as the minimax principle, appears
to be time dependent, though in a much weaker sense; that is, outside the
"equilibrium paths" of the game tree.

References

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